Correlations and fluctuations in nonuniform-temperature system

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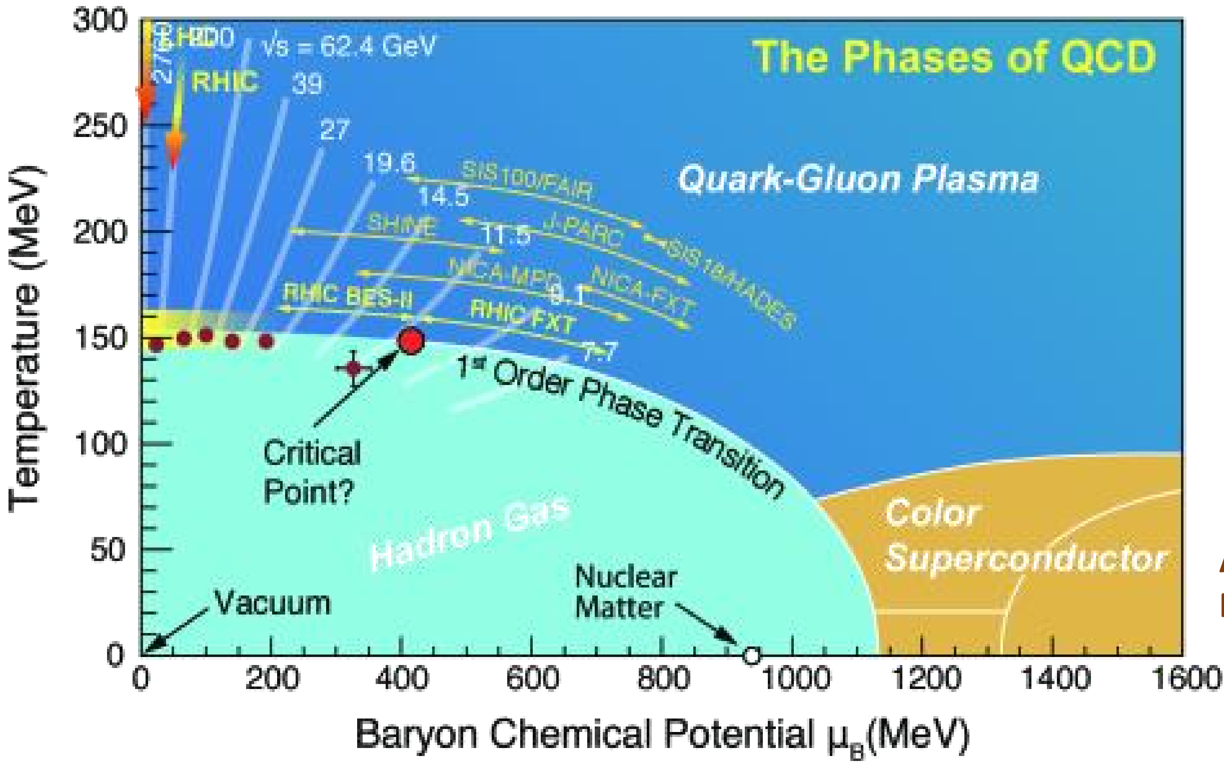
Zheng,Jiang, Phys. Rev. D 104, 016031(2021), and work in preparation

QCD phase diagram



A.Bzdak, et al, Phys. Rep. 853,1 (2020)

- **Lattice simulation**
  small μ **crossover**
- **Effective theories (P)NJL, QM, FRG, DSE...)**
  large μ **1st order**
- **Experimental facilities**
  RHIC (BES I & II), FAIR, NICA, HIAF

Questions:

Location of CP?

The 1st order phase transition?

Signals?

The signals for the CP

σNN

Due to the increase of σ's correlation length, the interaction contribute a singular part to the fluctuations of particle as one approaches the CP

M. Stephanov, PRL 102, 032301(2009)

- **Without this interaction**
  statistical fluctuations give

$$\langle \Delta n_p \Delta n_k \rangle = \left\langle (\Delta n_p)^2 \right\rangle \delta_{pk} = \langle n_p \rangle \delta_{pk}$$

- **With this interaction**
  fluctuating σ → fluctuating mass of N
  particle number correlation becomes

$$\langle \Delta n_p \Delta n_k \rangle = v_p^2 \delta_{pk} + \frac{G^2}{T} \frac{v_p^2 v_k^2}{\omega_p \omega_k} \xi^2$$

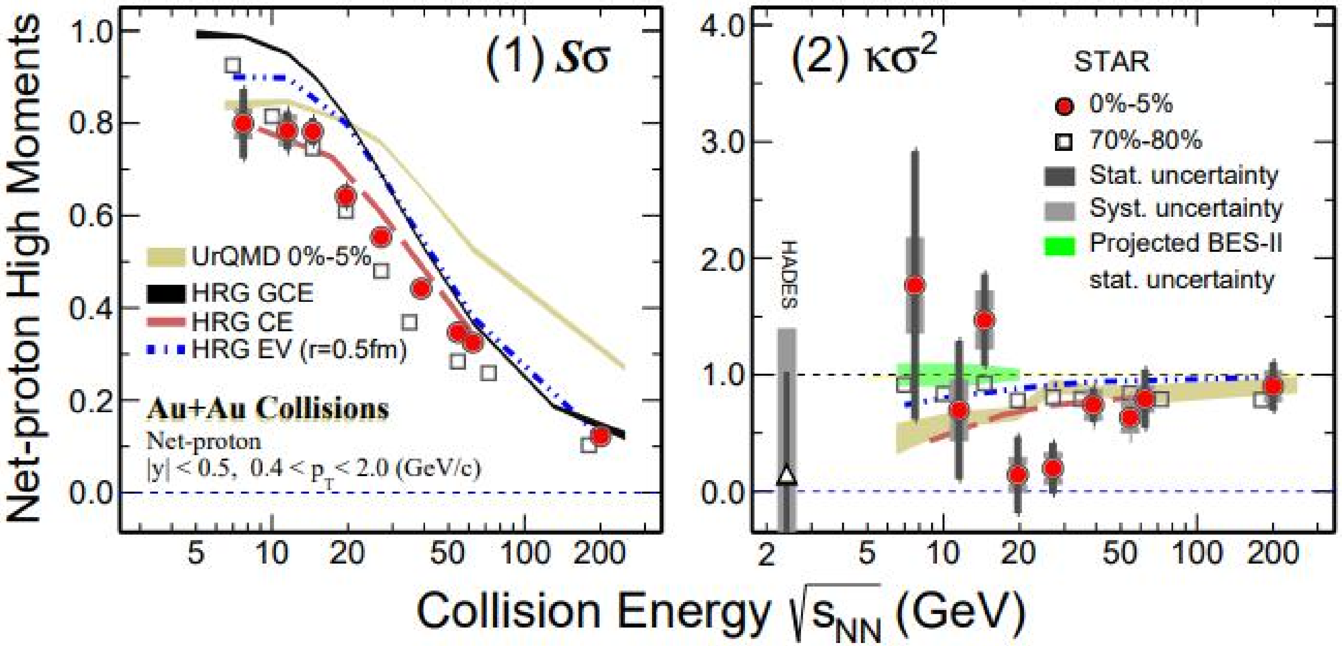
- **Stronger signal:**
  higher-order cumulants

$$\omega_3(N_p)_\sigma \approx 6\left(\frac{\tilde{\lambda}_3}{4}\right)\left(\frac{g}{10}\right)^3\left(\frac{\xi}{1\ \mathrm{fm}}\right)^{9/2},$$

$$\omega_4(N_p)_\sigma \approx 46\left(\frac{2\tilde{\lambda}_3^2 - \tilde{\lambda}_4}{50}\right)\left(\frac{g}{10}\right)^4\left(\frac{\xi}{1\ \mathrm{fm}}\right)^7.$$

STAR BES: Cumulant ratios

J. Adam et al. (STAR Collaboration), PRL 126, 092301 (2021)



$$S\sigma = \frac{\langle (N - \langle N \rangle)^3 \rangle}{\sigma^2}$$

$$\kappa\sigma^2 = \frac{\langle (N - \langle N \rangle)^4 \rangle}{\sigma^2} - 3$$

- Nonmonotonic deviations of $\kappa\sigma^2$ at around $\sqrt{s_{NN}} \sim 20\ GeV$.
- Could not be quantitatively explained by any models such as UrQMD, HRG, and equi/non-equi critical models

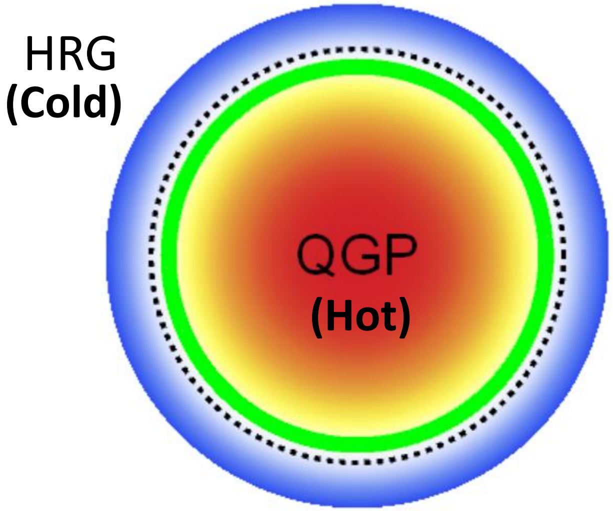
Theoretical progress

➢ **Dynamical effects, the finite-size effects.**

B. Berdnikov and K. Rajagopal, PRD 61, 105017 (2000).
M. Stephanov, PRL 102, 032301 (2009) and 107, 052301 (2011).
S. Mukherjee, R. Venugopalan, and Y. Yin, PRC 92, 034912 (2015).
L. Jiang, P. Li, and H. Song, PRC 94, 024918 (2016).
L. Jiang, S. Wu, and H. Song, NPA 967, 441(2017).
S. Wu, Z. Wu, and H. Song, PRC 99, 064902 (2019).
M. Stephanov and Y. Yin, PRD 98, 036006 (2018).
L. Du, U. Heinz, K. Rajagopal, and Y. Yin PRC 102, 054911 (2020).
L. Jiang, H. Stoecker, and J.-H. Zheng, EPJC 83, 117(2023).....

Langevin dynamics,
Fokker-Planck equations
Hydro+, Hydro+CP EOS

Memory effects,
Critical slowing down



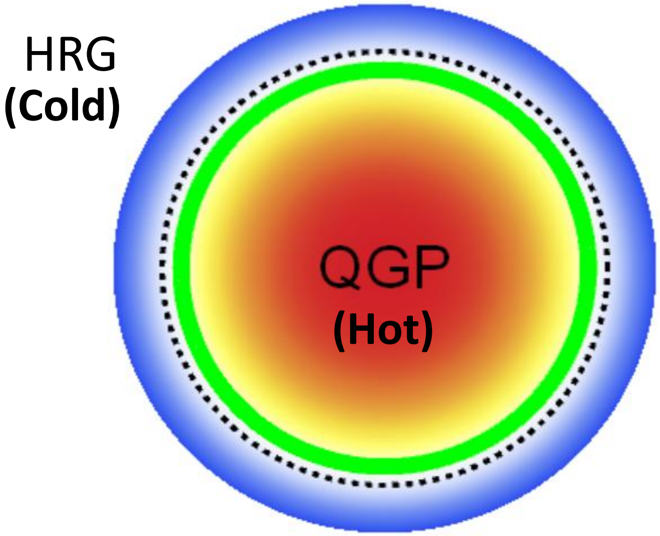
A slice of the fireball

Phase transition occurs in a narrow 2D shell.

Question: What is the effects of spatial-nonuniform temperature distribution on the phase transition and the related fluctuations?

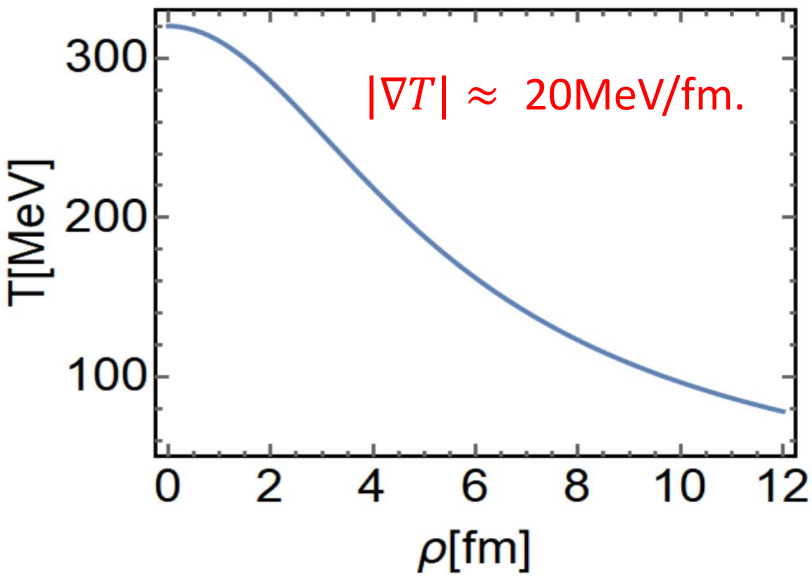
The focus of this talk!!

Assumptions



Assumptions:

- **Boost invariant, 2D Disk**
- **Markov process**
- **Fast relaxation**
- **Local equilibrium**
- **Uniform chemical potential**



Temperature profile from Gubser flow

$$T(\rho) = \frac{C}{t}\frac{(2qt)^{2/3}}{[1+2q^2(t^2+\rho^2)+q^4(t^2-\rho^2)^2]^{1/3}}.$$

with *C = 2.8, q = 1/4.3fm and t=1fm.*

S. Gubser, PRD 82, 085027 (2010)

Theoretical Framework

Jiang, Yang, Zheng in preparation

Partition function (local equilibrium)

$$Z = \mathrm{Tr}\exp\left\{-\int d^3\boldsymbol{r}\frac{\mathcal{H}(\pi,\sigma)}{T(\boldsymbol{r})}\right\}$$

$$\mathcal{H}(\pi,\sigma) = \pi^2/2 + (\nabla\sigma)^2/2 + \mathcal{V}(\sigma).$$

Path integral form: $Z = \int_{\mathrm{periodic}} [d\sigma] e^{S[\sigma]}.$

The action:

$$S[\sigma] = -\int_0^1 d\tau \int d^3\boldsymbol{r}\left[\frac{T(\partial_\tau\sigma)^2}{2} + \frac{(\nabla\sigma)^2}{2T} + \frac{\mathcal{V}(\sigma)}{T}\right]$$

minimal action principle:

$$\left.\frac{\delta S}{\delta\sigma}\right|_{\sigma=\sigma_c} = 0.$$

2D-disk case

$$-\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial\sigma}{\partial\rho}\right) + \frac{\hat{L}_z^2}{\rho^2}\sigma + \frac{1}{T}\frac{\partial\sigma}{\partial\rho}\frac{\partial T}{\partial\rho} + \underline{\eta_1 + \eta_2\sigma + \eta_4\sigma^3} = 0$$

T gradient term

Ising parameterization

The stable solution σ_c

Jiang, Yang, Zheng in preparation

Ising-like potential

$$\mathcal{V}(\sigma) = \eta_1\sigma + \eta_2\sigma^2/2 + \eta_4\sigma^4/4$$

$$\eta_1 = 0.5 fm^{-2} \times (T - T_c),$$

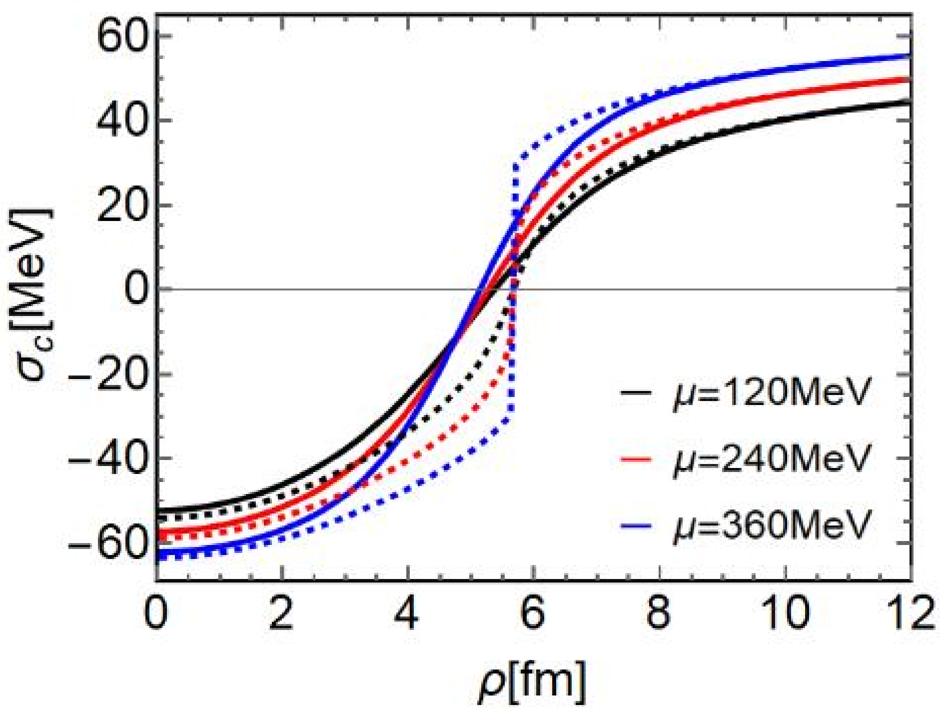
$$\eta_2 = -0.5\, fm^{-1} \times (\mu - \mu_c)$$

$$\eta_4 = 14.4$$

CP:

$(T_c, \mu_c) = (170, 240)$ **MeV.**

Dashed lines are minia of V(σ)



- $\sigma_c(x)$ changes its sign at higher T region.
- The discontinuity of first order phase transition is rounded.

The fluctuations around σ_c

Jiang, Yang, Zheng in preparation

Expanding around the base sigma field

$$\sigma(\boldsymbol{r}) = \sigma_c(\rho) + \tilde{\sigma}(\boldsymbol{r})$$

we have

$$S[\sigma_c(r) + \tilde{\sigma}(\boldsymbol{r})] = S[\sigma_c(r)] + \Delta S[\tilde{\sigma}(\boldsymbol{r})]$$

where the fluctuating part reads

$$\Delta S[\tilde{\sigma}(\boldsymbol{r})] = -\int_0^1 d\tau \int d^3\boldsymbol{r} \left(\frac{1}{2}\tilde{\sigma}\hat{\mathcal{O}}\tilde{\sigma} + \frac{\lambda_3}{3}\tilde{\sigma}^3 + \frac{\lambda_4}{4}\tilde{\sigma}^4\right),$$

Gaussian part

$$\hat{\mathcal{O}} = \overleftarrow{\partial_\tau} T \overrightarrow{\partial_\tau} + \overleftarrow{\nabla}\frac{1}{T}\cdot\overrightarrow{\nabla} + \frac{(\eta_2 + 3\eta_4\sigma_c^2)}{T},$$

Non-Gaussian part

$$\lambda_3 = \frac{3\eta_4\sigma_c}{T}, \qquad \lambda_4 = \frac{\eta_4}{T}.$$

solution to the fluctuations

Jiang, Yang, Zheng in preparation

Complete basis (of the Hermitian operator $\hat{O}$) expansion

$$\tilde{\sigma}(\boldsymbol{r},\tau)=\sum_{n,l=-\infty}^{\infty}\sum_{k}c_{nkl}R_{nkl}(\rho)\,\Theta_l(\theta)\,e^{-i\omega_n\tau},$$

$\Theta_l(\theta) \rightarrow \frac{e^{il\theta}}{\sqrt{2\pi}}$ $\qquad \omega_n \rightarrow 2\pi n$

Gaussian part:

$$\begin{aligned}\Delta S_0[\tilde{\sigma}] &= -\frac{1}{2}\int_0^1 d\tau\int d^3\boldsymbol{r}\,\tilde{\sigma}\hat{O}\tilde{\sigma}\\ &= -\frac{1}{2}d_z\sum_{nkk'l}c^*_{nk'l}c_{nkl}\int d\rho[\sqrt{\rho}R^*_{nk'l}]\hat{O}_{nl}[\sqrt{\rho}R_{nkl}]\\ &= -\frac{1}{2}d_z\sum_{nkl}c^*_{nkl}\varepsilon_{nkl}c_{nkl},\end{aligned}$$

$$\hat{O}_{nl}=T\omega_n^2+\overleftarrow{\partial}_\rho\frac{1}{T(\rho)}\overrightarrow{\partial}_\rho-\frac{1}{2\rho}\frac{1}{T^2(\rho)}\partial_\rho T(\rho)-\frac{1}{4\rho^2}\frac{1}{T(\rho)}+\frac{1}{T(\rho)}\frac{l^2}{\rho^2}+\frac{(\eta_2+3\eta_4\sigma_c^2)}{T}$$

Two-point correlation:

$$\langle\tilde{\sigma}(\boldsymbol{r},\tau)\tilde{\sigma}(\boldsymbol{r}',\tau)\rangle=\sum_{nkl}\frac{R_{nkl}(\rho)\,R_{nkl}(\rho')\,\Theta_l(\theta)\,\Theta_l^*(\theta')}{d_z\varepsilon_{nkl}}.$$

non-zero l mode

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Introducing
$$\tilde{\sigma}_l(\rho,\tau) \equiv \int dz \int_0^{2\pi} d\theta \tilde{\sigma}(\boldsymbol{r},\tau)\, e^{-il\theta}$$
(anisotropic)

The particle number fluctuations:

$$\delta N_l(\rho) \equiv \rho\Delta\rho \int dz \int_0^{2\pi} d\theta e^{-il\theta} \delta n(\boldsymbol{r}) \propto \rho\tilde{\sigma}_l$$

$$\langle \delta N_{l_1}(\rho)\,\delta N_{l_2}(\rho)\rangle \propto \rho^2 \langle \tilde{\sigma}_{l_1}(\rho,0)\,\tilde{\sigma}_{l_2}(\rho,0)\rangle\,,$$

$$\langle \delta N_{l_1}(\rho)\,\delta N_{l_2}(\rho)\,\delta N_{l_3}(\rho)\rangle \propto \rho^3 \langle \tilde{\sigma}_{l_1}(\rho,0)\,\tilde{\sigma}_{l_2}(\rho,0)\,\tilde{\sigma}_{l_3}(\rho,0)\rangle\,,$$

$$\langle \delta N_{l_1}(\rho)\,\delta N_{l_2}(\rho)\,\delta N_{l_3}(\rho)\,\delta N_{l_4}(\rho)\rangle \propto \rho^4 \langle \tilde{\sigma}_{l_1}(\rho,0)\,\tilde{\sigma}_{l_2}(\rho,0)\,\tilde{\sigma}_{l_3}(\rho,0)\,\tilde{\sigma}_{l_4}(\rho,0)\rangle$$

- The expectation of $\tilde{\sigma}_l(\rho,\tau)$ is zero, while the correlations are nonzero.

- Comparison with two limiting cases:
  1. Zero-momentum mode approximation (strong correlation):

     only $l = 0$ mode is reserved (comoving for all the spatial points)
  2. Statistical fluctuations (no correlation):

     The correlations are l independent.

2-point correlation

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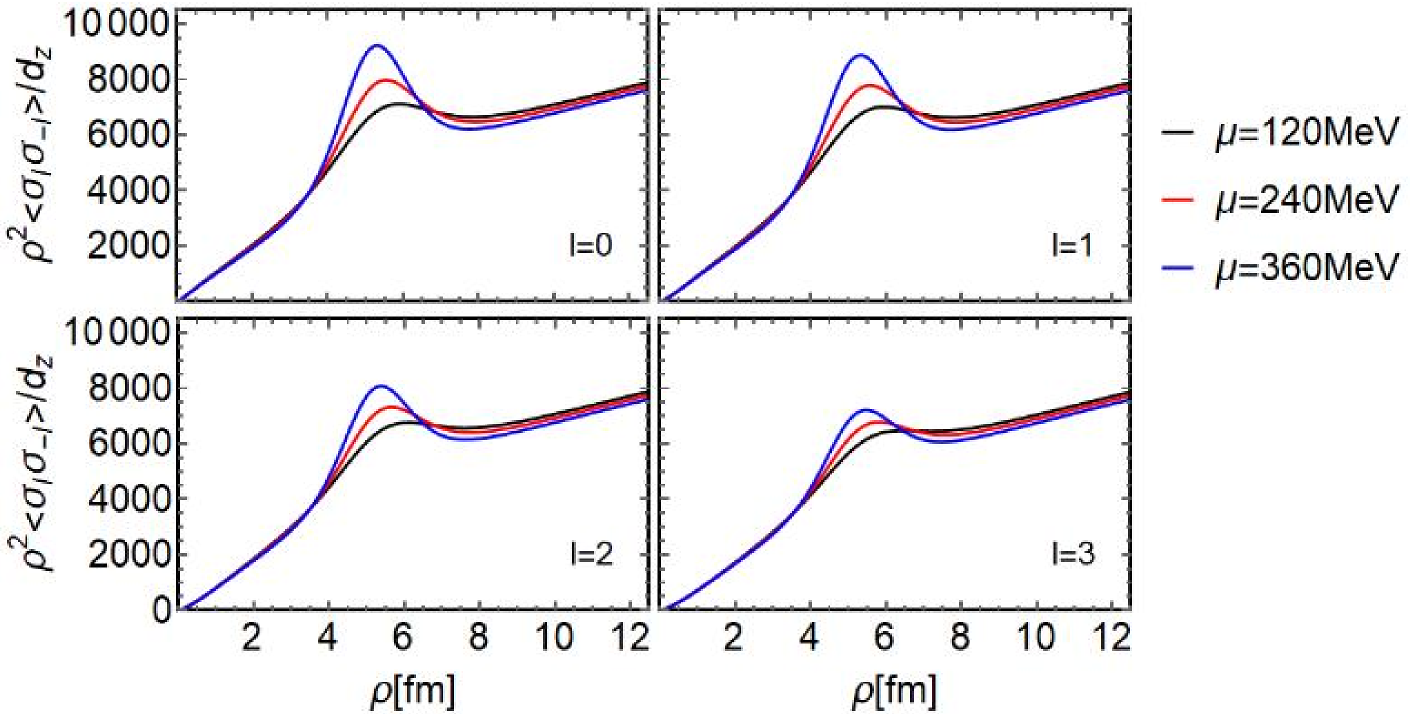
$$\left\langle \tilde{\sigma}_{-l}\left(\rho,\tau\right)\tilde{\sigma}_{l'}\left(\rho',\tau'\right)\right\rangle = \left\langle \tilde{\sigma}_{l}^{*}\left(\rho,\tau\right)\tilde{\sigma}_{l'}\left(\rho',\tau'\right)\right\rangle = 2\pi d_z \delta_{l'l} \sum_{n=-\infty}^{\infty} G_{nl}(\rho,\rho') e^{i\omega_n(\tau-\tau')}.$$

with $G_{nl}(\rho,\rho') = \sum_k R_{nkl}\left(\rho\right) R_{nkl}\left(\rho'\right)/\varepsilon_{nkl}.$



The nonzero l–modes are unique and reflect the spatial correlation of critical fluctuations, which is also detectable.

3-point correlation

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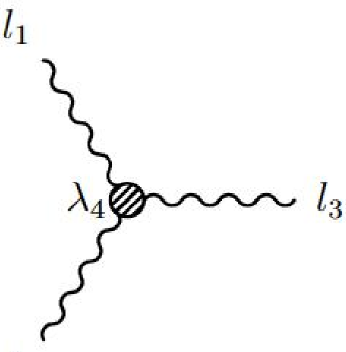
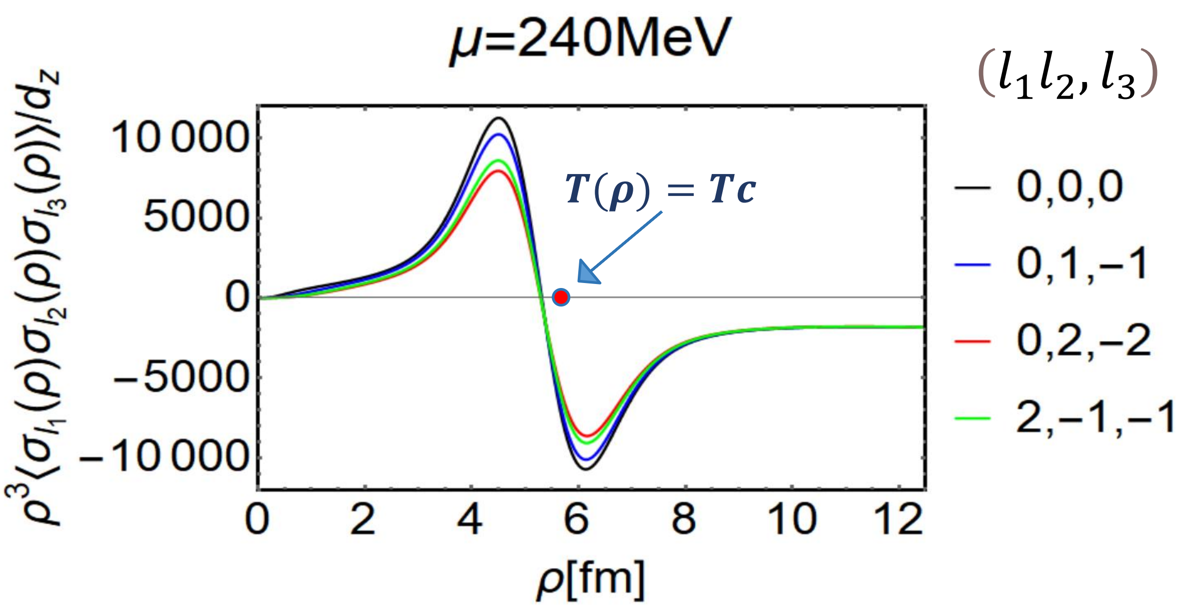
$$\langle\tilde{\sigma}_{l_1}(\rho,0)\,\tilde{\sigma}_{l_2}(\rho,0)\,\tilde{\sigma}_{l_3}(\rho,0)\rangle = -12\pi d_z \eta_4 \delta_{l_1+l_2+l_3,0} \times$$
$$\sum_{n_1,n_2=-\infty}^{\infty} \int d\rho' \frac{\rho'\sigma_c(\rho')}{T(\rho')} G_{n_1 l_1}(\rho,\rho') G_{n_2 l_2}(\rho,\rho') G_{n_1+n_2,l_3}(\rho,\rho'),$$



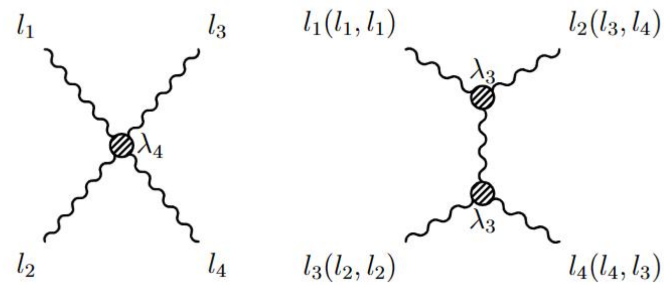


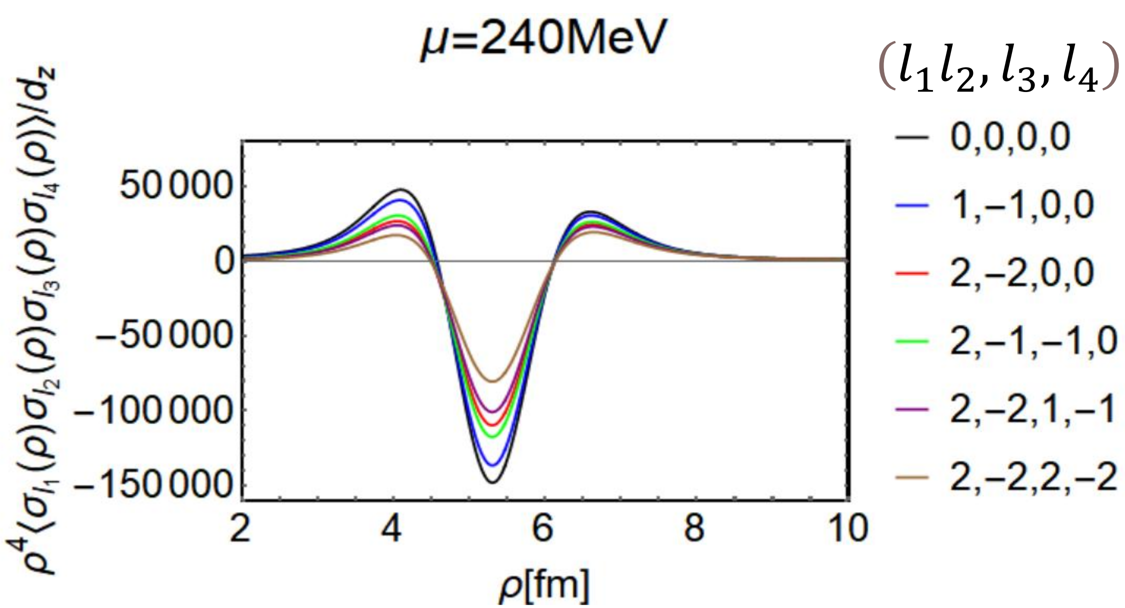
The higher-order flucs are more sensible to the PT, different l-mode combinations are comparable.

4-point correlation

Jiang, Yang, Zheng in preparation

$$
\begin{aligned}
&\langle \tilde{\sigma}_{l_1}(\rho,0)\,\tilde{\sigma}_{l_2}(\rho,0)\,\tilde{\sigma}_{l_3}(\rho,0)\,\tilde{\sigma}_{l_4}(\rho,0)\rangle \\
&= -12\pi d_z \eta_4 \delta_{l_1+l_2+l_3+l_4,0} \\
&\quad \times \sum_{n_1,n_2,n_3} \int d\rho' \frac{\rho'}{T(\rho')} G_{n_1 l_1}(\rho,\rho') G_{n_2 l_2}(\rho,\rho') G_{n_3 l_3}(\rho,\rho') G_{n_1+n_2+n_3,l_4}(\rho,\rho') \\
&\quad + 72\pi d_z \eta_4^2 \delta_{l_1+l_2+l_3+l_4,0} \\
&\quad \times \sum_{n_1,n_2,n_3} \int d\rho_1 \int d\rho_2 \frac{\rho_1 \sigma_c(\rho_1)}{T(\rho_1)} \frac{\rho_2 \sigma_c(\rho_2)}{T(\rho_2)} \\
&\quad \times \Big[G_{n_1 l_1}(\rho,\rho_1) G_{n_2 l_2}(\rho,\rho_1) G_{n_3 l_3}(\rho,\rho_2) G_{n_1+n_2+n_3,l_4}(\rho,\rho_2) G_{n_1+n_2,l_1+l_2}(\rho_1,\rho_2) \\
&\quad + G_{n_1 l_1}(\rho,\rho_1) G_{n_2 l_3}(\rho,\rho_1) G_{n_3 l_2}(\rho,\rho_2) G_{n_1+n_2+n_3,l_4}(\rho,\rho_2) G_{n_1+n_2,l_1+l_3}(\rho_1,\rho_2) \\
&\quad + G_{n_1 l_1}(\rho,\rho_1) G_{n_2 l_4}(\rho,\rho_1) G_{n_3 l_2}(\rho,\rho_2) G_{n_1+n_2+n_3,l_3}(\rho,\rho_2) G_{n_1+n_2,l_1+l_4}(\rho_1,\rho_2) \Big]
\end{aligned}
$$





Summary

- Construct the framework for the nonuniform-T effects in a 2D disk, solve σ_c, and the (non-) Gaussian fluctuations around it.

  - phase transition is forced to a higher T region.
  - the divergence at CP and the discontinuities of the 1st order phase transition disappear.
  - the nonzero l-modes are important in the spatially nonuniform system, and are detectable.

- Future work:
  - more realistic nonuniform-T system.
  - combination of nonuniform-T effects and dynamical effects.
  - …

Thank you!

Back up

The fourth-order cumulants at a fixed freezeout temperature