Exploring the hadron-quark phase transition in neutron stars

Pianpian Qin (秦翩翩)

Collaborators: Zhan Bai, Sibo Wang, Chencan Wang, Si-xue Qin

Phys. Rev. D ***107***, *103009 (2023)*

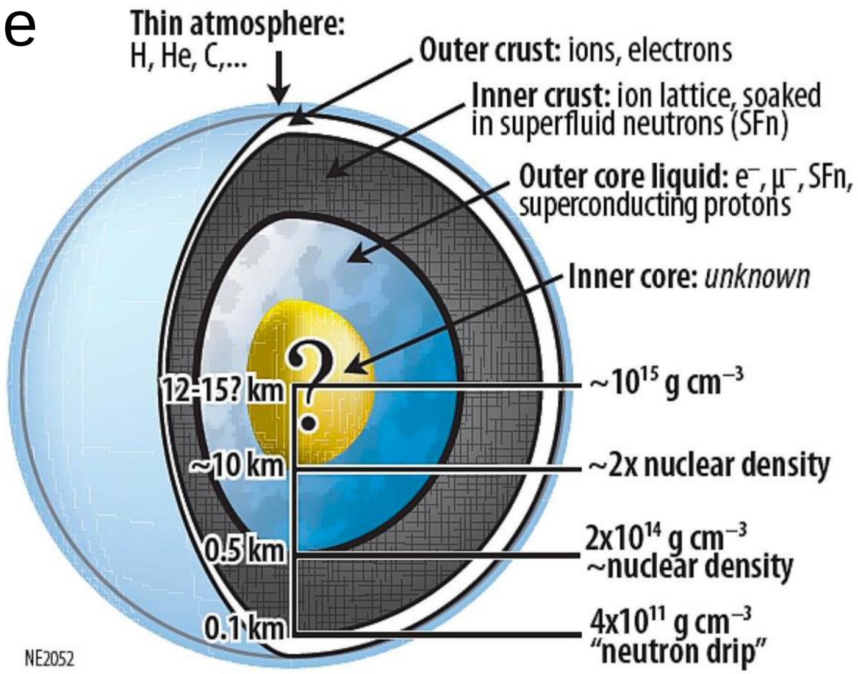
patia413@cqu.edu.cn

Contents

Neutron Star

- One of the most compact objects in universe

$$\rho_c = 5\sim10\ \rho_0$$



- Internal structure and composition
  - **Atmosphere**
  - **Crust:** nucleons and electrons
  - **Outer core:** nucleons, electrons, and muons
  - **Inner core:** hyperons, meson condensates, quark matter

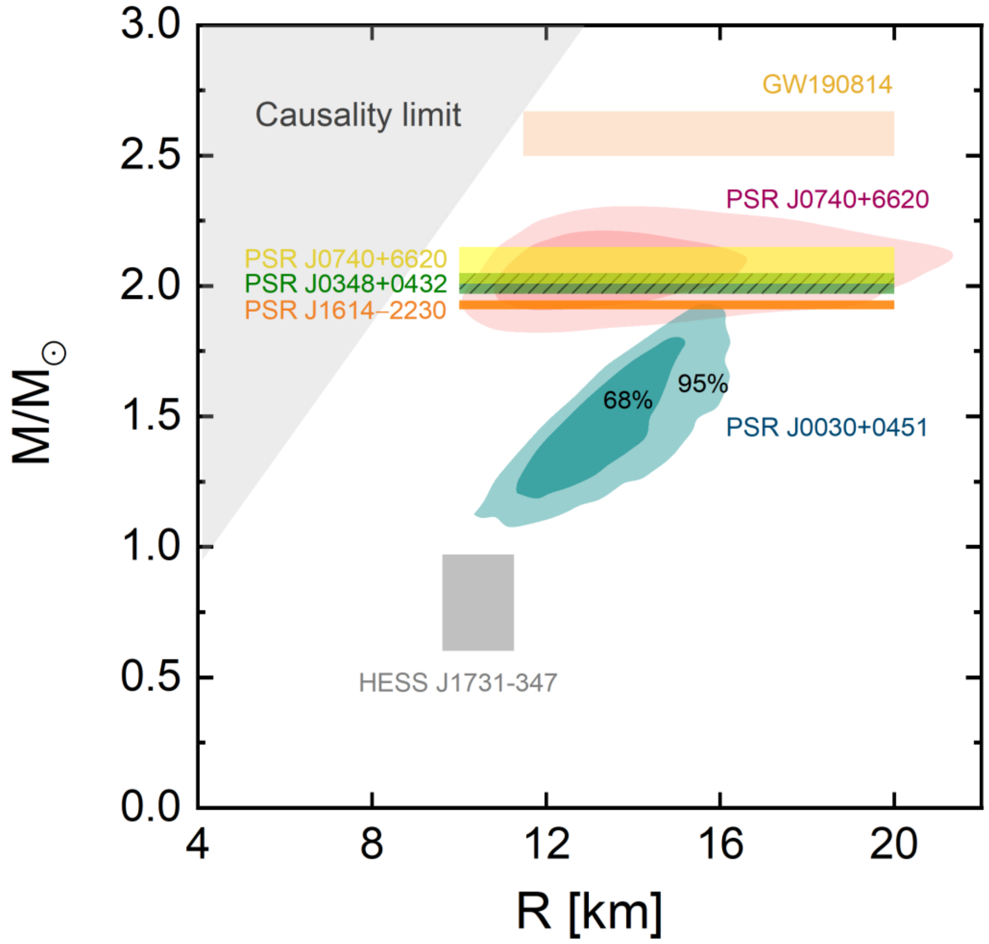
Hadron-quark phase transition

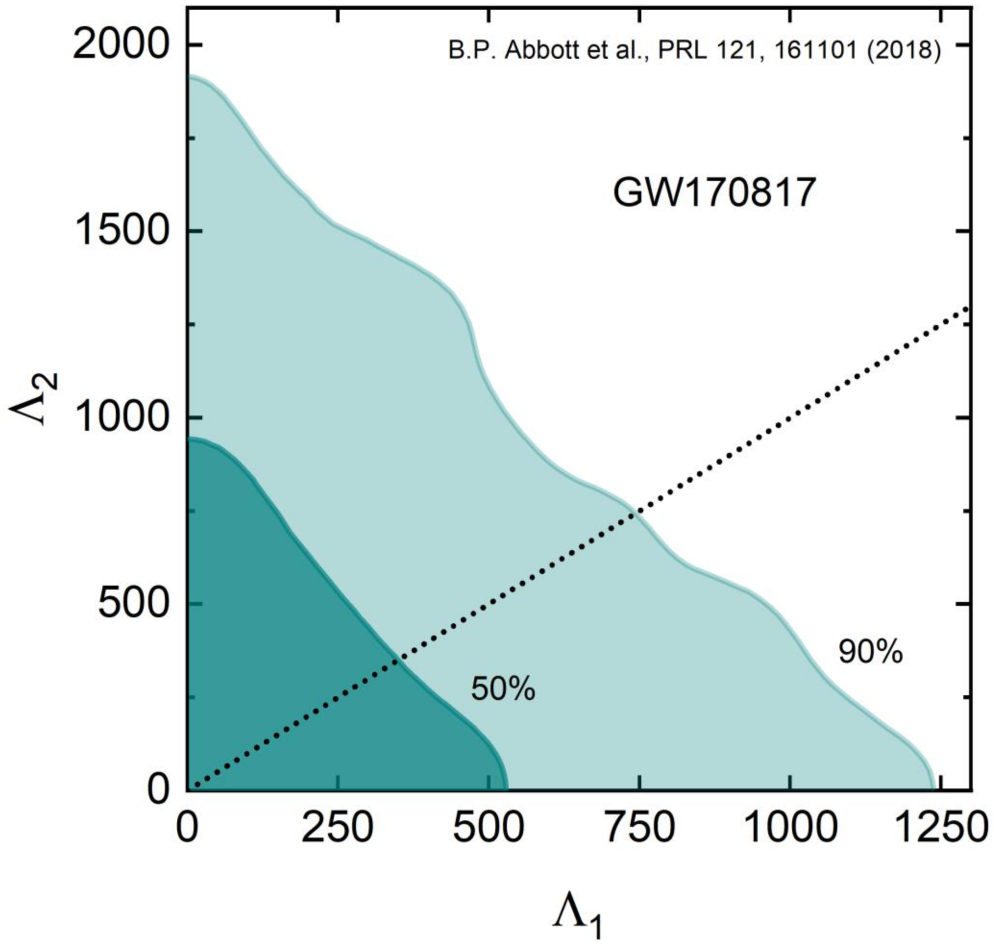
↓

Astronomical Observation → EOS ← Theoretical Method

Astronomical observation

- Massive neutron stars, joint mass-radius observation, and GW





Theoretical methods at finite density

☐ Nuclear Matter

*G.F. Burgio et al., PPNP **120**, 103879 (2021)*

Density functional theory (DFT)	Effective NN interaction	Skyrme, Gogny, RMF, RHF etc.
Ab initio method	Realistic NN interaction	(R)BHF, Variational, Mont-Carlo etc.

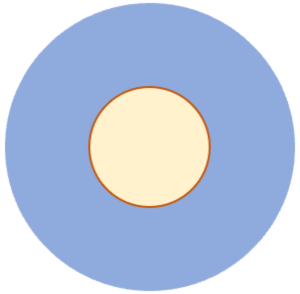
☐ Quark Matter

*C.D. Roberts et al., PPNP **45**, S1 (2000)*

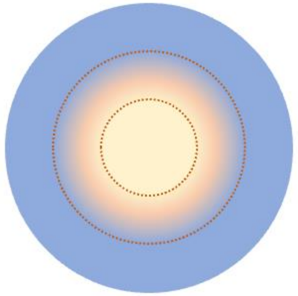
Phenomenological model	MIT bag model，NJL model，quark-meson model
Continuum field theory	Dyson-Schwinger equation (DSE) approach, FRG

Construction schemes

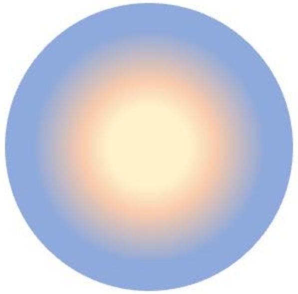
First-order



First-order



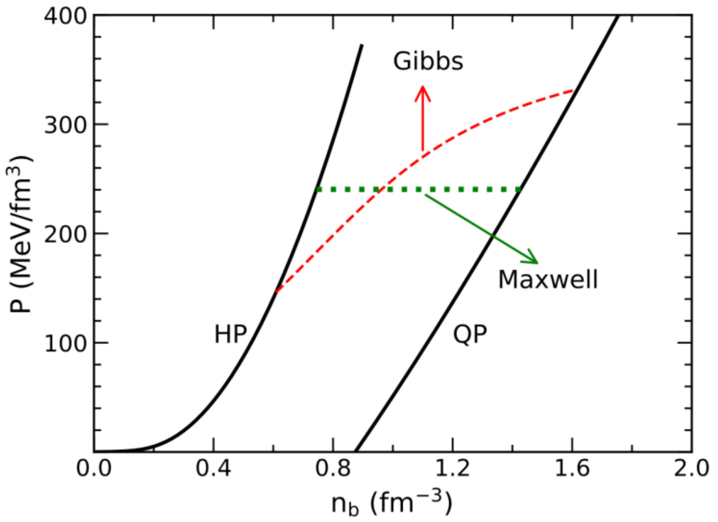
Crossover



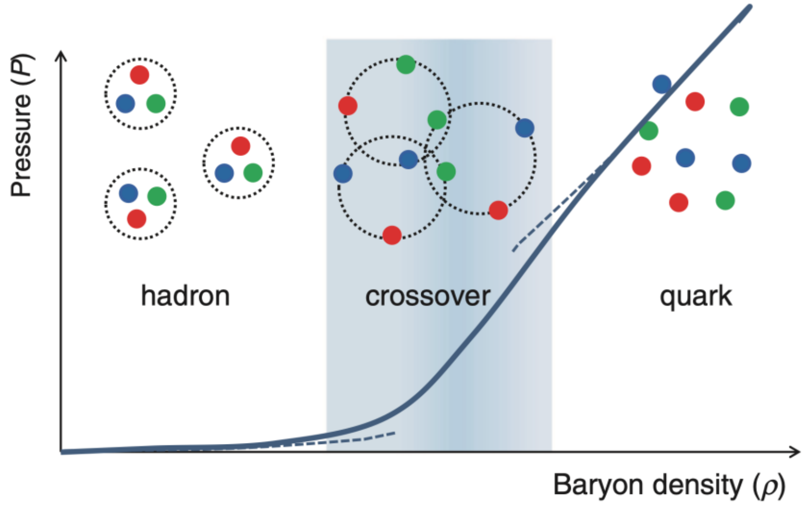
Maxwell construction

Gibbs construction

3-window construction



*N. K. Glendenning, PRD **46**, 1274 (1992)*



*K. Masuda et al., PTEP **7**, 073D01 (2013)*

Research status

Hadron ↓ / Quark →

	Phenomenological model	Continuum field theory
DFT	*Schertler (1999): RMF+NJL* *Agrawal (2010): MF+MIT/NJL* *Logoteta (2013): RMF+NJL* *Orsaria (2013): NRMF+NJL* *Wu (2017,2018): RMF+NJL* *Ju (2021): QMC+MIT* *Huang (2022): RMF+NJL*	*Bai (2018): RMF+DSE* *Bai (2021): RMF+DSE*
Ab initio	*Klahn (2007): RBHF+NJL* *Agrawal (2010): APR+MIT/NJL* *Masuda (2013): APR+NJL* *Kojo (2015): APR+NJL* *Li (2015): BHF+NJL* *Baym (2019): APR+NJL*	*Chen (2011): BHF+DSE* *Chen (2012): BHF+DSE* *Chen (2015): BHF+DSE* *Chen (2016): BHF+DSE* *Qin (2023): RBHF+DSE*

This work： exploring the possible hadron-quark phase transition in neutron stars with the RBHF+DSE methods in relativistic framework.

Contents

Nuclear matter：Relativistic-Brueckner-Hartree-Fock（RBHF）

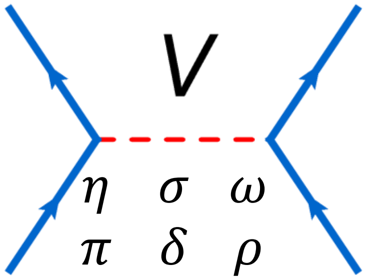
- Realistic NN interaction: Bonn potential

R. Brockmann et al., PRC ***42****, 1965 (1990)*

$$\mathscr{L}^{(pv)} = -\frac{f_{ps}}{m_{ps}}\bar{\psi}\gamma^5\gamma^\mu\psi\partial_\mu\varphi^{(ps)},$$

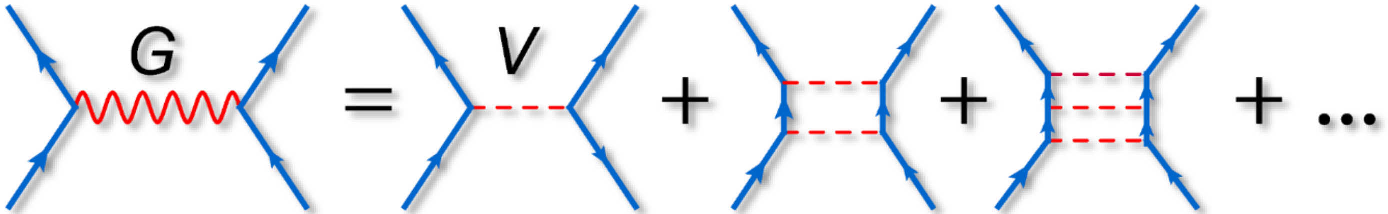
$$\mathscr{L}^{(s)} = g_s\bar{\psi}\psi\varphi^{(s)},$$

$$\mathscr{L}^{(v)} = -g_v\bar{\psi}\gamma^\mu\psi\varphi_\mu^{(v)} - \frac{f_v}{4M}\bar{\psi}\sigma^{\mu\nu}\psi\left(\partial_\mu\varphi_\nu^{(v)} - \partial_\nu\varphi_\mu^{(v)}\right),$$



- The effective NN interaction: G matrix

K.A. Brueckner et al., PR ***95****, 217 (1954)*



- The ground state energy of nuclear matter

B.D. Day, RMP ***39****, 719 (1967)*

S. Wang et al., PRC ***105****, 054309 (2022)*

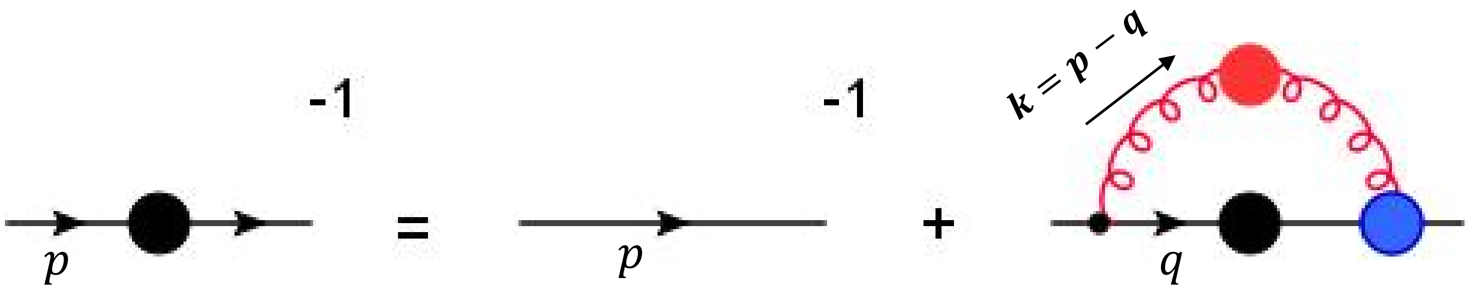
Quark matter：Dyson-Schwinger Equation (DSE) approach

◻ QCD Lagrangian

$$\mathcal{L}_{QCD} = \sum_{f=1}^{N_f} \bar{\psi}^f \left(i\gamma_\mu D^\mu - m_0^f\right)\psi^f - \frac{1}{4}F_{\mu\nu a}F^{\mu\nu a}$$

$$D^\mu = \partial^\mu - ig_0 A_a^\mu t_a = \partial^\mu - ig_0 A^\mu \qquad F_a^{\mu\nu} = \partial^\mu A_a^\nu - \partial^\nu A_a^\mu + g_0 f_{abc} A_b^\mu A_c^\nu$$

◻ DSE for quark propagator



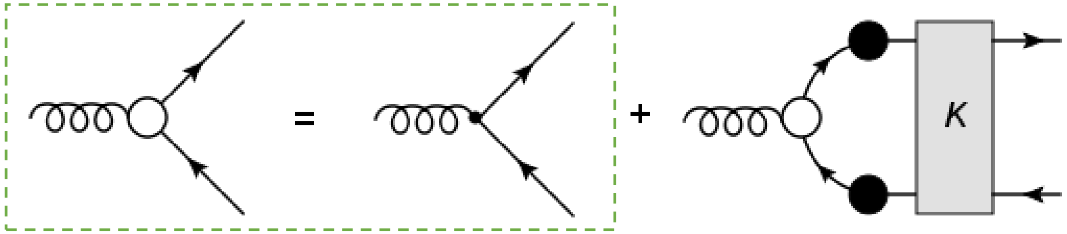
$$S(p;\mu)^{-1} = Z_2\left(i\boldsymbol{\gamma}\cdot\boldsymbol{p} + i\gamma_4\tilde{p}_4 + m_q\right) + \int \frac{d^4q}{(2\pi)^4}\mathcal{G}(k^2;\boldsymbol{\mu})D_{\rho\sigma}^{\text{free}}(k)\gamma_\rho S(q;\mu)\Gamma_\sigma(q,p;\mu)$$

Truncation scheme and Gluon model

◻ Quark-gluon vertex: bare vertex approximation

*H. J. Munczek, PRD **52**, 4736 (1995)*

$$\Gamma_\sigma(q,p;\mu) = \gamma_\sigma$$

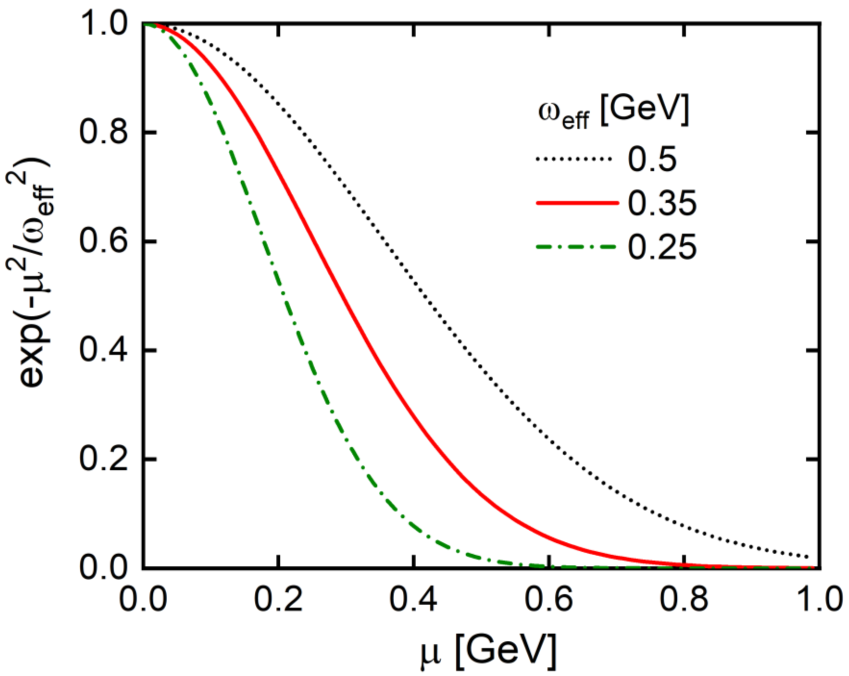


◻ Gluon: Gaussian model

$$\frac{\mathcal{G}(k^2)}{k^2} = \frac{4\pi^2 D}{\omega^6} k^2 e^{-k^2/\omega^2}$$

$$\zeta \coloneqq D\omega = \text{const.}$$

$$\frac{\mathcal{G}(k^2;\boldsymbol{\mu})}{k^2} = \frac{4\pi D}{\omega^6} k^2 e^{-k^2/\omega^2} \boldsymbol{e^{-\mu^2/\omega_{\mathrm{eff}}^2}}$$



ω_{eff}: effective interaction width

*P. Maris et al., PRC **60**, 055214 (1999)*
*H. Chen et al., PRD **84**, 105023 (2011)*

EOS $(P, \mathcal{E}, \rho, \mu_B)$

H. Chen et al., PRD **78**, 116015 (2008)
H. Chen et al., PRD **84**, 105023 (2011)

☐ The quark number density

$$f_q(|\boldsymbol{p}|;\mu) = \frac{1}{4\pi}\int_{-\infty}^{\infty} dp_4 \mathrm{tr_D}[-\gamma_4 \boldsymbol{S_q(p;\mu)}]$$

$$\boldsymbol{n_q(\mu)} = 6\int \frac{d^3p}{(2\pi)^3} f_q(|\boldsymbol{p}|;\mu)$$

☐ The EOS for the quark matter

$$P_q(\mu_q) = P_q(\mu_{q,0}) + \int_{\mu_{q,0}}^{\mu_q} d\mu\, \boldsymbol{n_q(\mu)}$$

$$\boldsymbol{P_Q(\mu_u,\mu_d,\mu_s)} = \sum_{q=u,d,s} P_q(\mu_q)$$

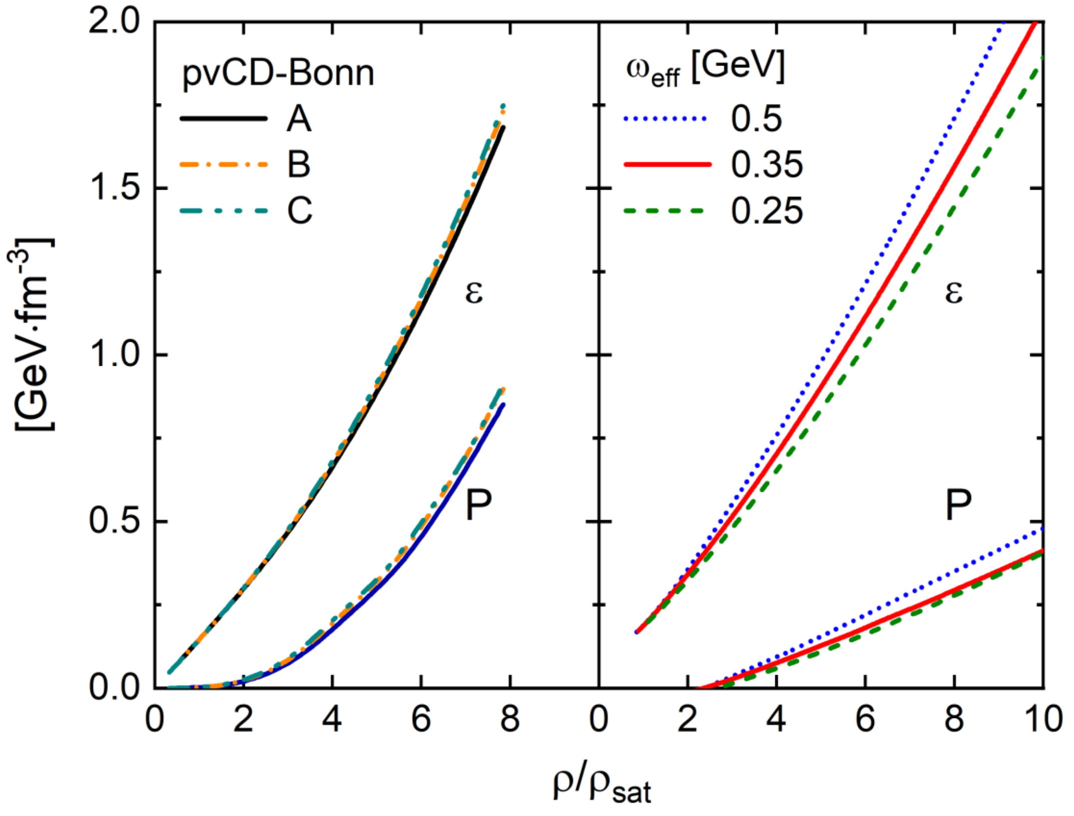
$$(P,\mu_B) \xrightarrow{\text{Thermo−relation}} (P,\boldsymbol{\varepsilon},\boldsymbol{\rho},\mu_B)$$

Contents

- Introduction
- Theoretical framework
  - Nuclear matter: RBHF theory
  - Quark matter: DSE approach
- Results and discussions
  - EOS for hybrid star
  - Mass-Radius relation, tidal deformability
- Summary and prospects

EOSs for pure hadron star and pure quark star

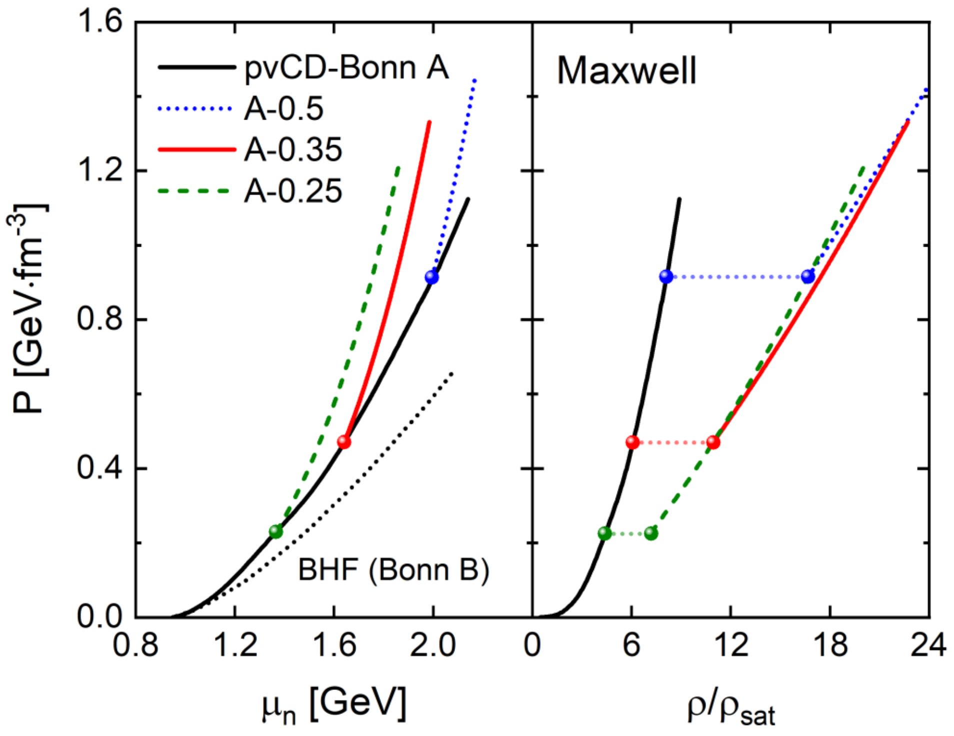
□ $P-\rho$ relation and $\varepsilon-\rho$ relation



- The differences of the results from pvCD-Bonn A, B, and C are negligible
- With a larger ω_{eff}, the energy density and pressure become larger.

EOSs for hybrid stars under **Maxwell construction**

$P - \mu_B$ relation and $P - \rho$ relation

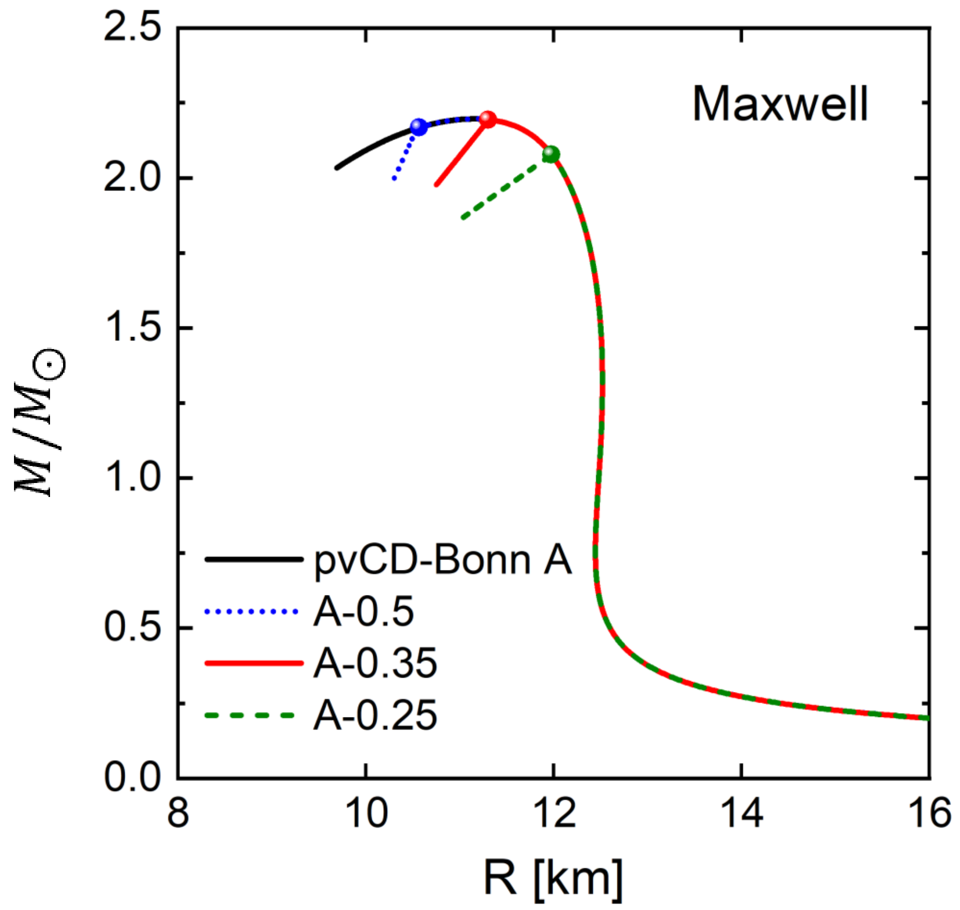


Thermodynamic parameters at pt. points

Model	$\mu_{n,c}$ [GeV]	$P(\mu_{n,c})$ $[\mathrm{GeV}\cdot\mathrm{fm}^{-3}]$	ρ_H $[\rho_{sat}]$	ρ_Q $[\rho_{sat}]$
A-0.5	1.995	0.916	8.105	16.689
A-0.35	1.640	0.471	6.078	10.940
A-0.25	1.366	0.225	4.379	7.189
BHF-0.35	1.416	0.193	3.556	5.744

- For a larger ω_{eff} , the critical baryon chemical potential and density region are higher, and the corresponding pressure is larger

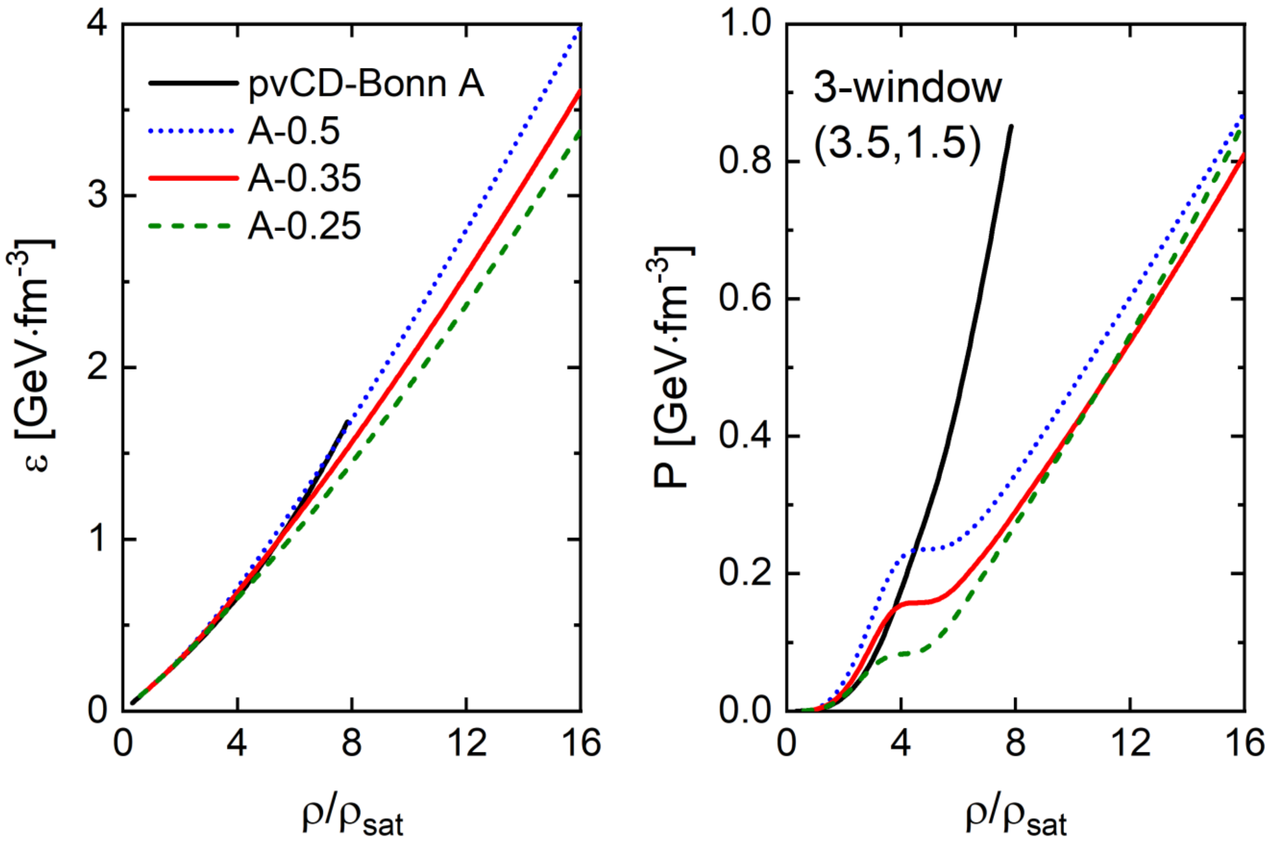
The M-R relation for hybrid star under Maxwell construction



→ with the present models, there is no stable quark core inside the neutron star under the Maxwell construction

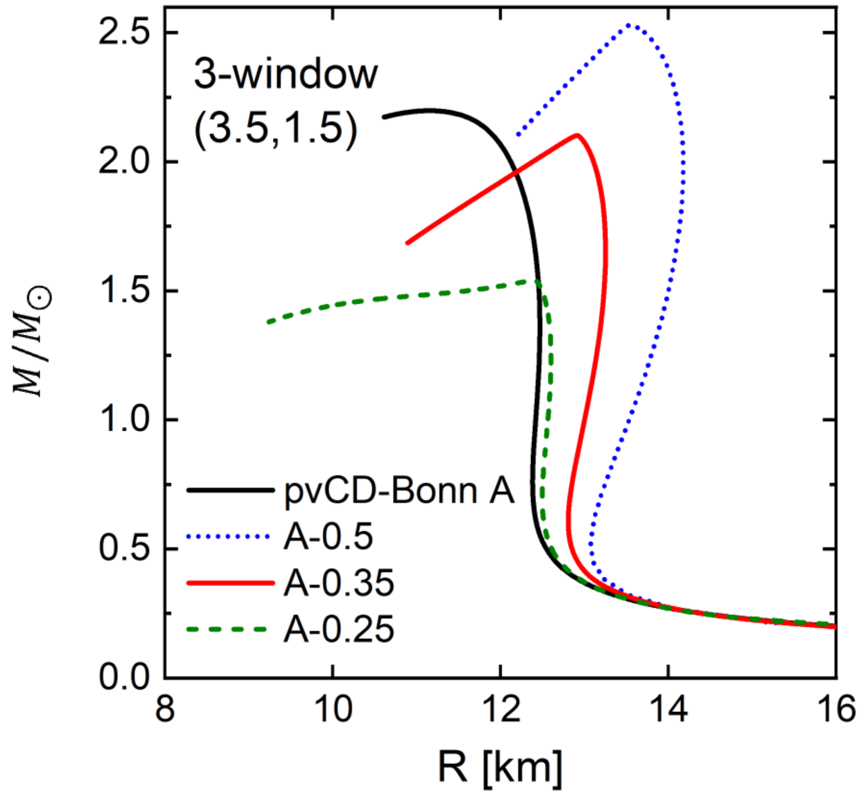
EOSs for hybrid stars under 3-window construction

- $\varepsilon - \rho$ relation and $P - \rho$ relation



→ For a lager ω_{eff}, the energy density and pressure are larger in the crossover region.

The M-R relation for hybrid star under 3-window construction



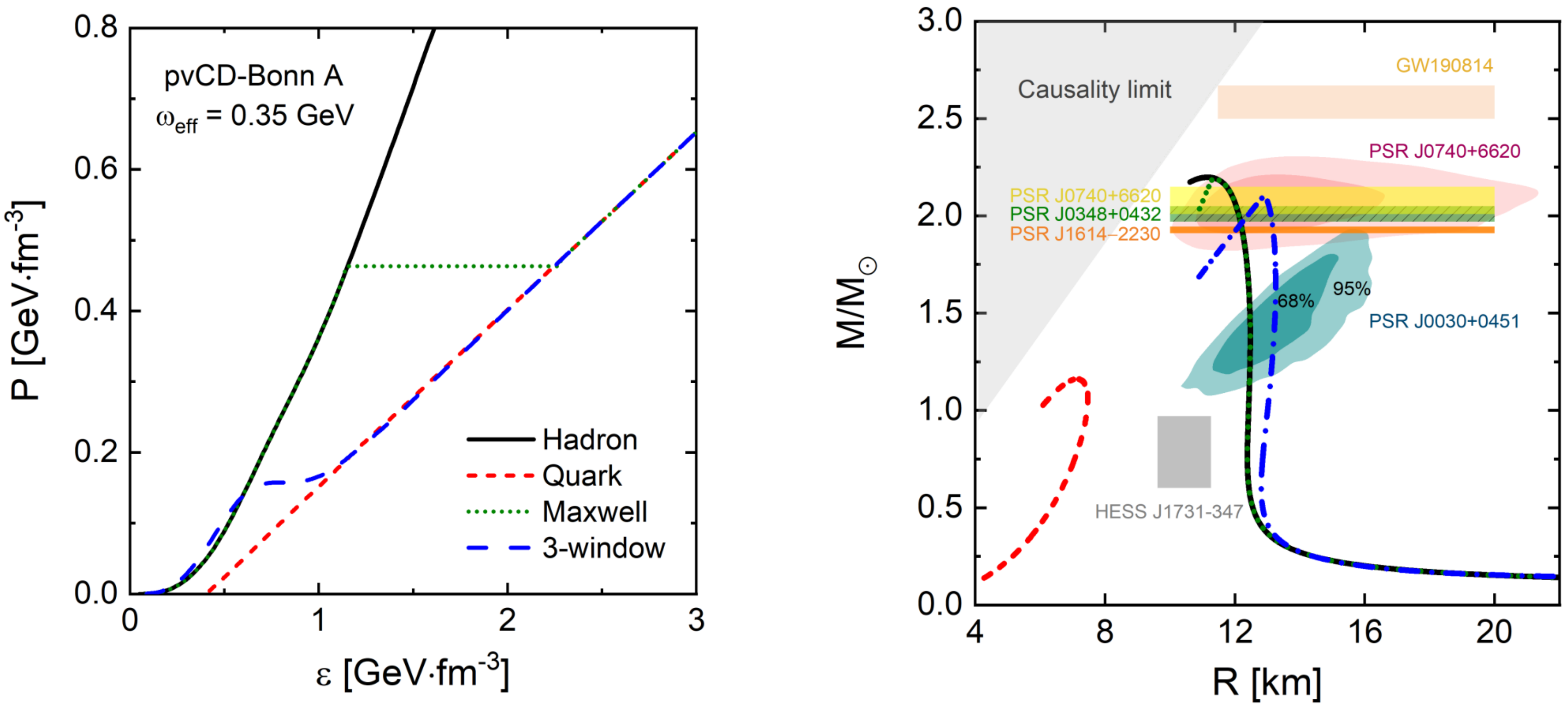
Maximum mass and corresponding radius

Model	$M_{\mathbf{max}}$ $[M_\odot]$	$R_{M_{\mathbf{max}}}$ [km]	$R_{1.4M_\odot}$ [km]
pvCD-Bonn A	2.198	11.15	12.47
A-0.5	2.530	13.54	13.93
A-0.35	**2.102**	12.91	13.20
A-0.25	1.539	12.39	12.57

→ For $\omega_{\mathrm{eff}} = 0.35\ \mathrm{GeV}$, the maximum mass of the hybrid star is $2.1M_\odot$, which is consistent with the current constraints from astrophysical observation.

Pure hadron star, pure quark star, and hybrid stars

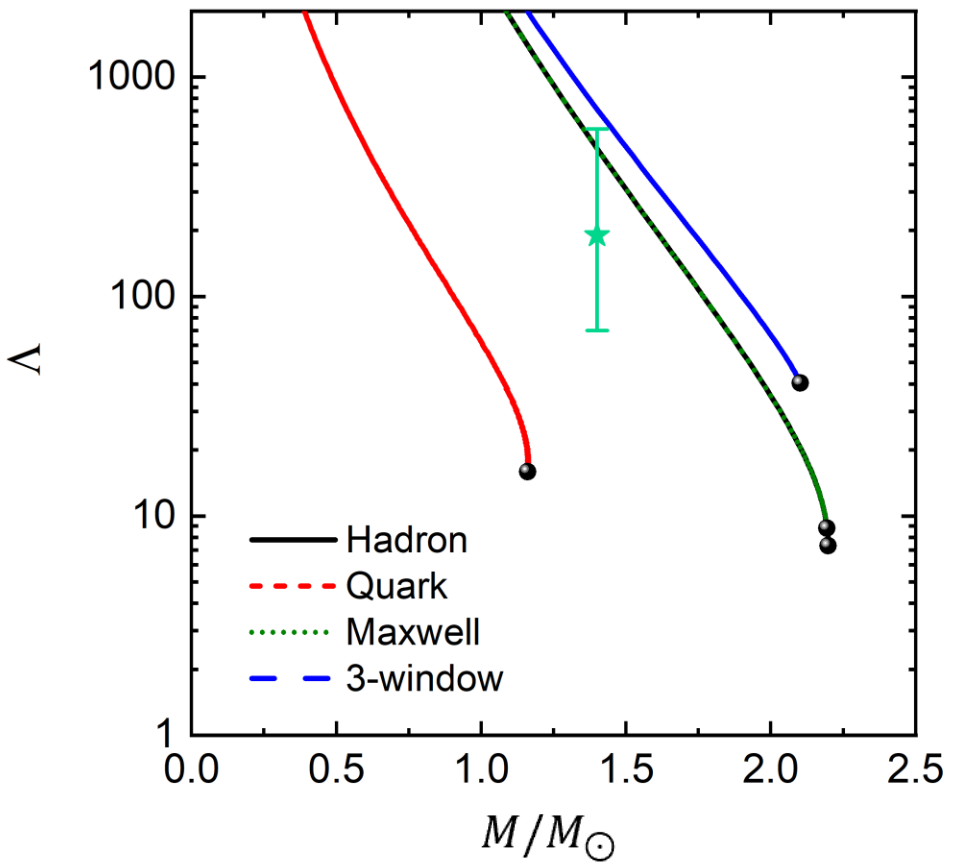
- The EOS and the M-R relation



→ the hybrid star under the 3-window construction is consistent with the joint mass and radius observations of neutron stars

Pure hadron star, pure quark star, and hybrid stars

- Tidal deformability



→The tidal deformability of the hybrid star under the 3-window construction is

$\Lambda_{1.4\mathrm{M}_{\odot}} = 700$, slightly larger than the constraint $\Lambda_{1.4M_{\odot}} = 190^{+390}_{-120}$ from GW170817

Summary and prospects

- RBHF + DSE is used to explore the hadron-quark phase transition in NS
  - **First-order phase transition** (Maxwell construction), **negative**
  - **Crossover** (3-window construction), **positive**

- Further exploration
  - Gibbs construction for the first-order phase transition
  - Interaction models in DSE approach

Hadron-quark phase transition：EOS for hybrid star

- Maxwell construction

$$P_H(\mu_{B,c}) = P_Q(\mu_{B,c})$$

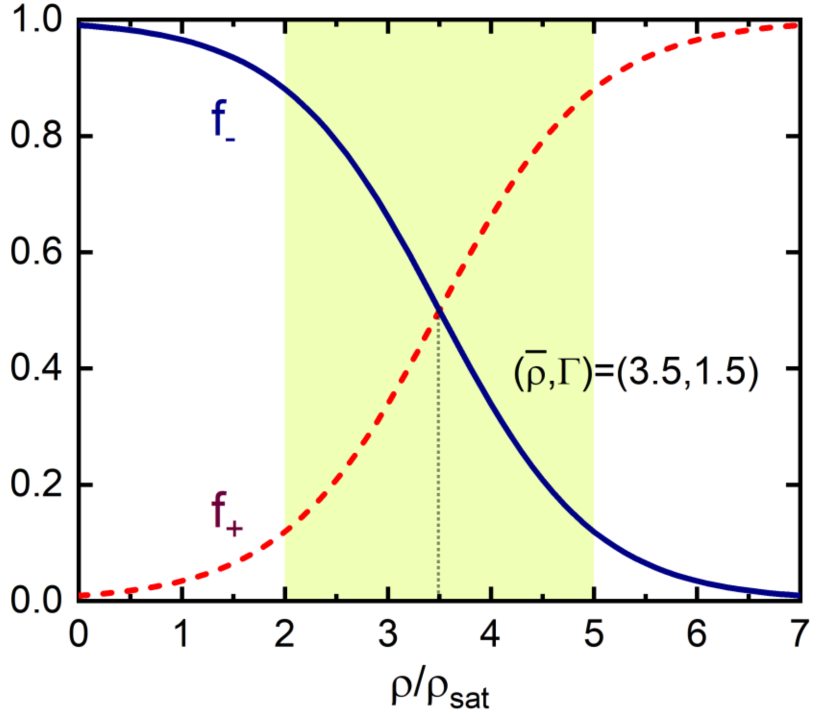
$$\varepsilon(\mu_B) = \begin{cases} \varepsilon_H, & \text{if } \mu_B < \mu_{B,c} \\ \varepsilon_Q, & \text{if } \mu_B > \mu_{B,c} \end{cases}$$

- 3-window construction

$$\varepsilon(\rho) = f_-(\rho)\varepsilon_H(\rho) + f_+(\rho)\varepsilon_Q(\rho)$$

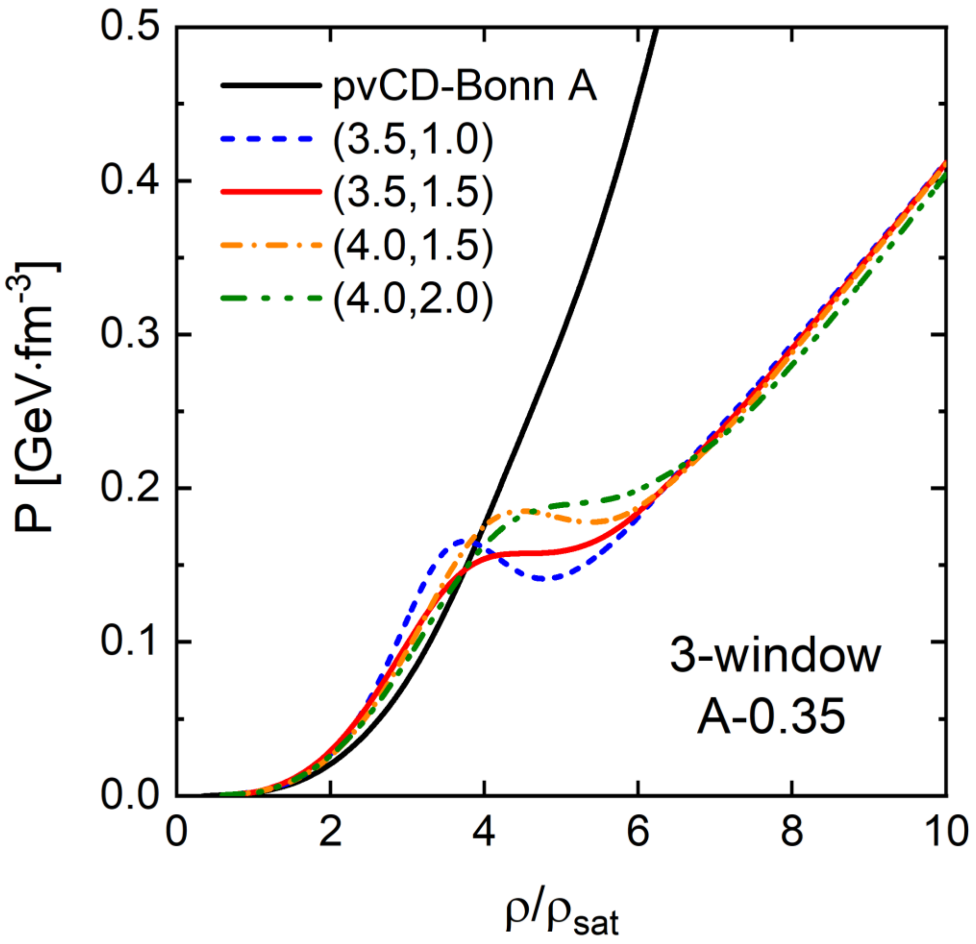
$$f_\pm = \frac{1}{2}(1 \pm \tanh(\frac{\rho - \bar{\rho}}{\Gamma}))$$

$$(\varepsilon, \rho) \xrightarrow{\text{Thermo−relation}} (\boldsymbol{P}, \varepsilon, \rho, \boldsymbol{\mu_B})$$



EOS：3-window construction and parameters

- Different $(\bar{\boldsymbol{\rho}}, \boldsymbol{\Gamma})$ and the density dependent of pressure for hybrid stars



- Selection criteria for $(\bar{\boldsymbol{\rho}}, \boldsymbol{\Gamma})$
  - ✓ Density range → conforms to the model
  - ✓ Stable star → EOS is monotonic
  - ✓ Massive star → EOS is stiff enough

→ $(\bar{\boldsymbol{\rho}}, \boldsymbol{\Gamma}) = (\mathbf{3.5}, \mathbf{1.5})$

The relations to obtain EOS $(\boldsymbol{P}, \boldsymbol{\mathcal{E}}, \boldsymbol{\rho}, \boldsymbol{\mu_B})$

- Thermo-relations

$$\rho(\mu) = \frac{\partial P(\mu)}{\partial \mu}$$

$$\mu = (\varepsilon + P)/\rho$$

$$P = \rho^2 \frac{\partial(\varepsilon/\rho)}{\partial \rho}$$

- β equilibrium, charge equilibrium

$$\mu_d = \mu_u + \mu_e = \mu_s,$$

$$\frac{2\rho_u - \rho_d - \rho_s}{3} - \rho_e - \rho_{\mu^-} = 0.$$

- From quark to hadron

$$\rho_B = \frac{1}{3}(\rho_u + \rho_d + \rho_s),$$

$$\mu_B = \mu_u + 2\mu_d.$$

Neutron star properties

- Tolman-Oppenheimer-Volkov (TOV), to obtain M-R relation

$$\frac{dP(r)}{dr} = -\frac{[\varepsilon(r) + P(r)][M(r) + 4\pi r^3 P(r)]}{r^2[1 - 2M(r)/r]}$$

$$\frac{dM(r)}{dr} = 4\pi r^2 \varepsilon(r)$$

- To solve tidal deformability

$$r\frac{dy(r)}{dr} + y^2(r) + y(r)F(r) + r^2 Q(r) = 0$$

$$\Lambda = \frac{2}{3} k_2 C^{-5}$$

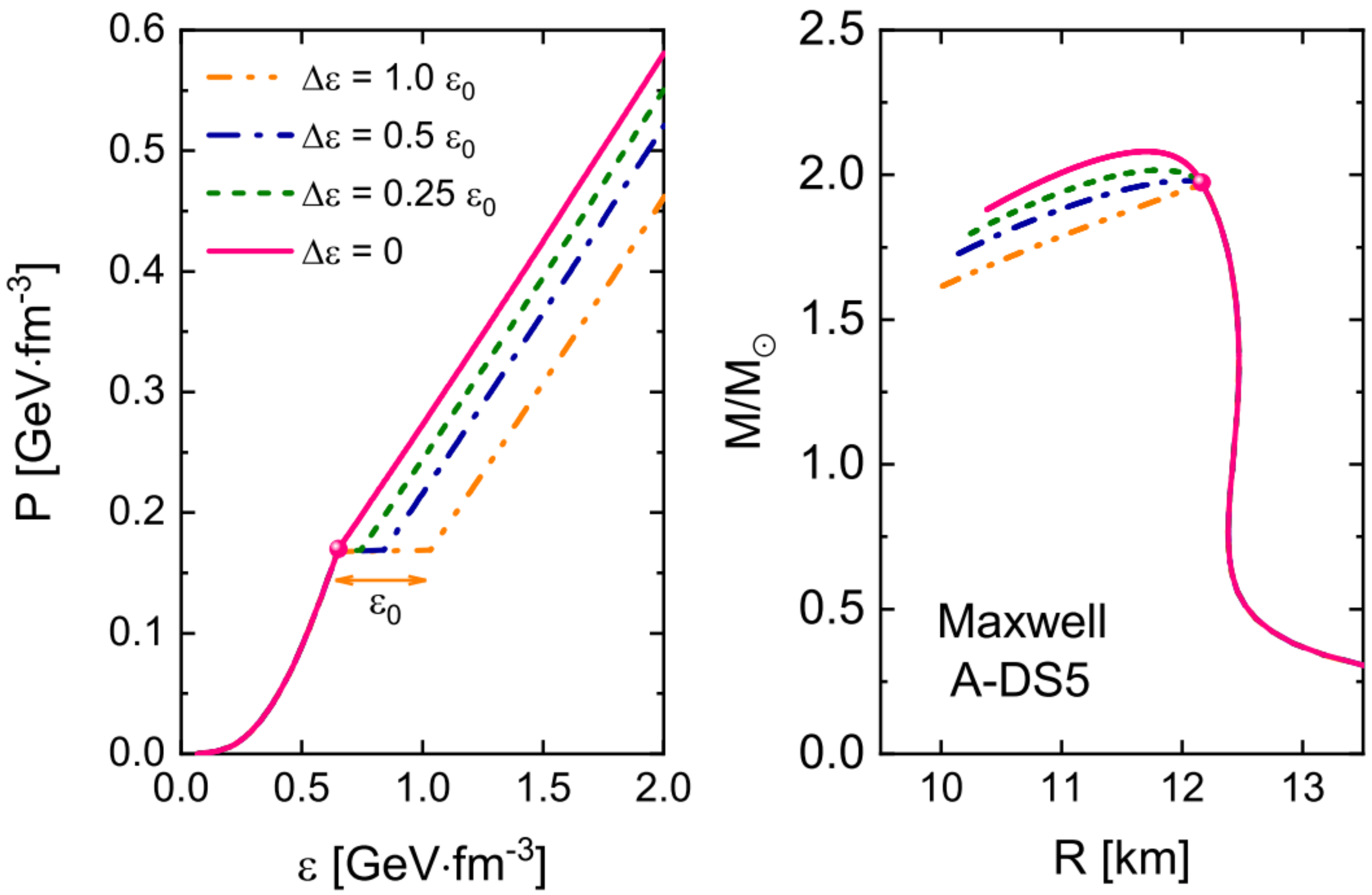
$$F(r) = \left[1 - \frac{2M(r)}{r}\right]^{-1} \{1 - 4\pi r^2[\mathcal{E}(r) - P(r)]\}$$

$$Q(r) = \left\{4\pi\left[5\mathcal{E}(r) + 9P(r) + \frac{\mathcal{E}(r) + P(r)}{\partial P/\partial \mathcal{E}}\right] - \frac{6}{r^2}\right\} \times \left[1 - \frac{2M(r)}{r}\right]^{-1} - \left[\frac{2M(r)}{r^2} + 8\pi r P(r)\right]^2 \times \left[1 - \frac{2M(r)}{r}\right]^{-2}$$

$$C = M/R$$

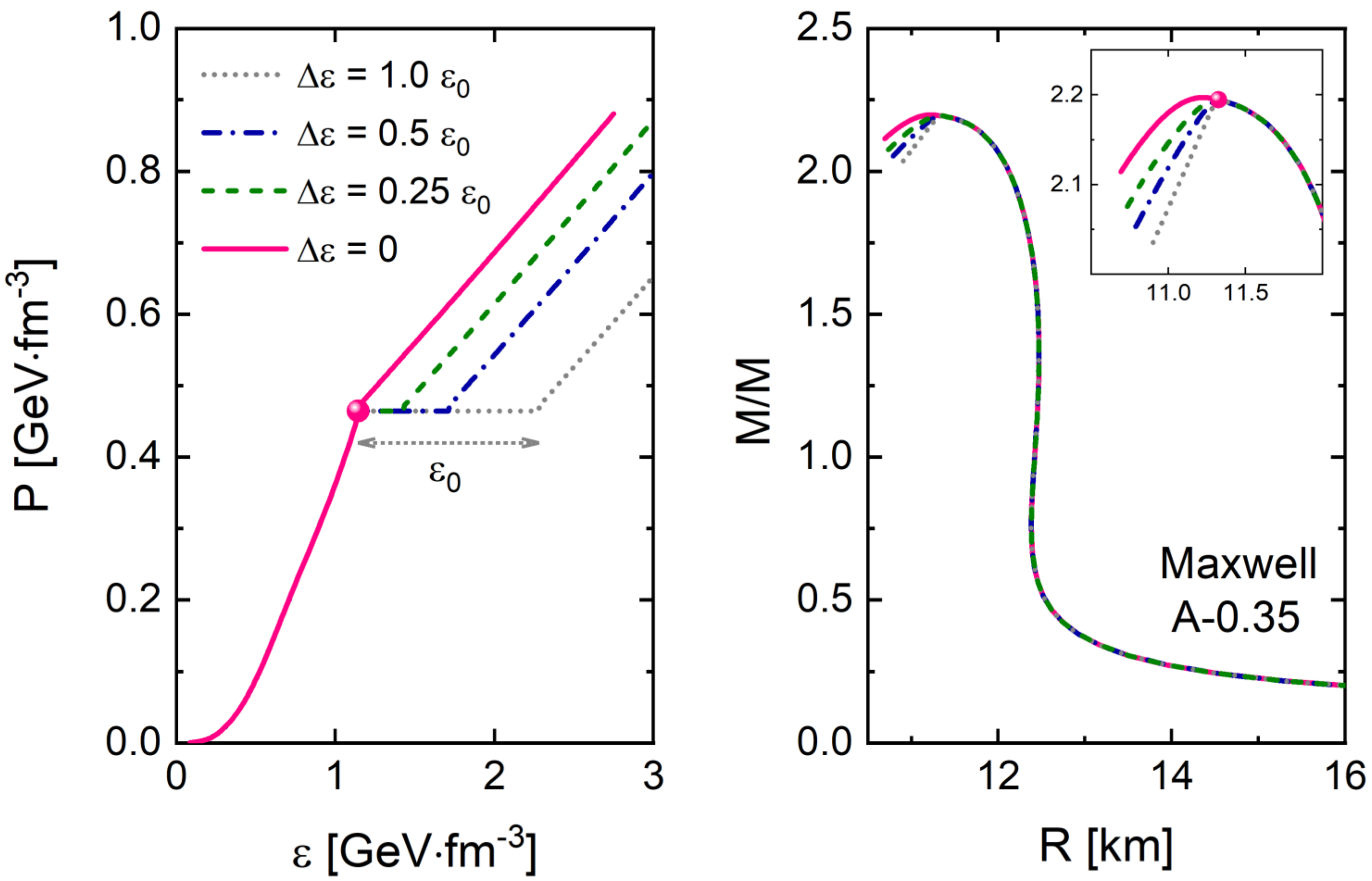
$$k_2 = \frac{8C^5}{5}(1-2C)^2[2 - y_R + 2C(y_R - 1)] \times \{6C[2 - y_R + C(5y_R - 8)] + 4C^3[13 - 11y_R + C(3y_R - 2) + 2C^2(1 + y_R)] + 3(1-2C)^2[2 - y_R + 2C(y_R - 1)]\ln(1 - 2C)\}^{-1},$$

Latent heat and M-R relation



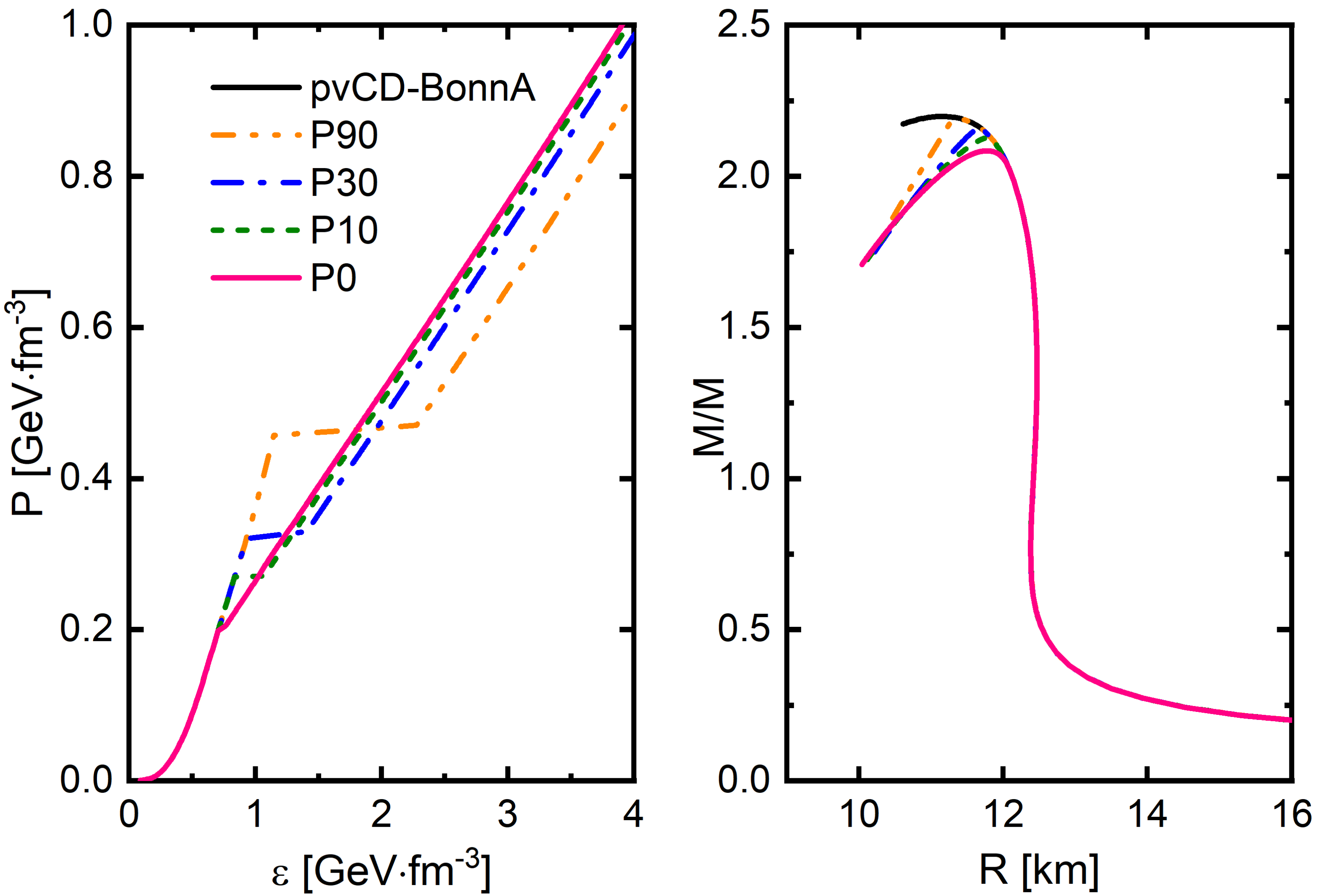
$\omega_{\text{eff}} = 0.22\text{GeV}$

Latent heat and M-R relation



$\omega_{\mathrm{eff}} = 0.35\mathrm{GeV}$

Vacuum pressure and M-R relation



$\omega_{\mathrm{eff}} = 0.35\mathrm{GeV}$