第十五届QCD相变与相对论重离子物理研讨会

Vortical effects on chiral and deconfinement transition

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Rotation and magnetic field

Magnetic field	Rotation
Pseduo-vector	Pseduo-vector
Polarize J	Polarize J
Chiral transportation	Chiral transportation
Anomalous effects	Anomalous effects
Chiral catalysis	Chiral inhibition
Inverse chiral catalysis	?

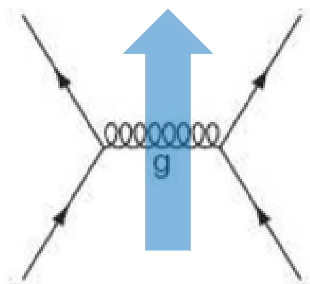
Quark fluctuation In Gluon propagator

Chiral Vortical catalysis

Chiral condensation

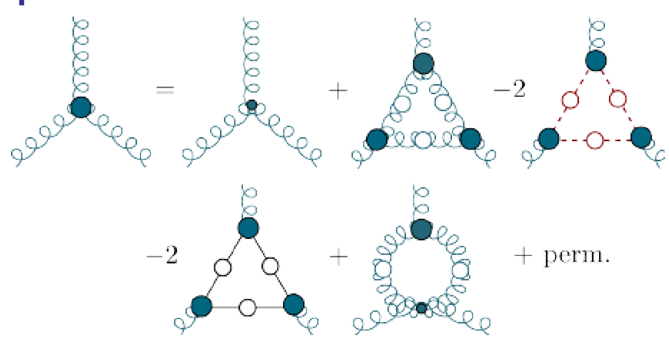
- With effective model, such as NJL, chiral condensation, which is typically non-perturbative, is studied intuitively by the 4-fermion interaction and mean-filed approx.

- All the gluon contributions is included in the coupling constant G.



$G(\omega)$

- We should deal with analytical integration of 6 to 8 Bessel function multiplications



$$D_{\mu\nu}(x,y) = \frac{i}{(2\pi)^3}\sum_{n,\lambda}\int dk_0 dk_z k_t dk_t e^{ik_z(z-\zeta)} e^{in(\phi-\theta)}$$
$$\times e^{-i(k_0-n\omega)(t-s)} \frac{A_{\mu,\lambda}(k_t,n,k_z;\rho)A^*_{\nu,\lambda}(k_t,n,k_z;r)}{k_0^2 - E_k^2 + i\eta}$$
$$= \frac{i}{(2\pi)^3}\sum_{n,\lambda}\int dk_0 dk_z k_t dk_t e^{ik_z(z-\zeta)} e^{in(\phi-\theta)}$$
$$\times e^{-ik_0(t-s)} \frac{A_{\mu,\lambda}(k_t,n,k_z;\rho)A^*_{\nu,\lambda}(k_t,n,k_z;r)}{(k_0-n\omega)^2 - E_k^2 + i\eta} \qquad (34)$$

$$D^n_{\mu\nu}(k_t,k_z;\rho,r) = \sum_{\lambda} A_{\mu,\lambda}(k_t,n,k_z;\rho)A^*_{\nu,\lambda}(k_t,n,k_z;r)$$
$$= \frac{1}{4}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & M_n^{++} & -iM_n^{+-} & 0 \\ 0 & iM_n^{-+} & M_n^{--} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \qquad (35)$$
$$+\frac{E_{kt}^2}{4m^2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & M_n^{--} & -iM_n^{-+} & -2i\frac{k_t k_z}{E_{kt}^2}N_n^{1-} \\ 0 & iM_n^{+-} & M_n^{++} & 2\frac{k_t k_z}{E_{kt}^2}N_n^{1+} \\ 0 & 2i\frac{k_t k_z}{E_{kt}^2}N_n^{2-} & 2\frac{k_t k_z}{E_{kt}^2}N_n^{2+} & 4\frac{E_z^2}{E_{kt}^2}\Pi_n \end{pmatrix}$$
$$+\frac{k_t E_k}{2m^2}\begin{pmatrix} \frac{2k^2}{k_t E_k}\Pi_n & -iN_n^{2-} & -N_n^{2+} & -2\frac{k_z}{k_t}\Pi_n \\ iN_n^{1-} & 0 & 0 & 0 \\ -N_n^{1+} & 0 & 0 & 0 \\ -2\frac{k_z}{k_t}\Pi_n & 0 & 0 & 0 \end{pmatrix}$$

$$M_n^{++} = Z_n^+(\rho,\phi)Z_n^{+*}(r,\theta)$$
$$M_n^{+-} = Z_n^+(\rho,\phi)Z_n^{-*}(r,\theta)$$
$$M_n^{-+} = Z_n^-(\rho,\phi)Z_n^{+*}(r,\theta)$$
$$M_n^{--} = Z_n^-(\rho,\phi)Z_n^{-*}(r,\theta),$$

$$N_n^{1+} = Z_n^+(\rho,\phi)J_n(r)$$
$$N_n^{1-} = Z_n^-(\rho,\phi)J_n(r)$$
$$N_n^{2+} = J_n(\rho)Z_n^{+*}(r,\theta)$$
$$N_n^{2-} = J_n(\rho)Z_n^{-*}(r,\theta)$$

$$\Pi_n = J_n(\rho)J_n(r)$$

$$Z_n^+(\rho,\phi) = J_{n-1}(\rho)e^{-i\phi} + J_{n+1}(\rho)e^{i\phi}$$

$$Z_n^-(\rho,\phi) = J_{n-1}(\rho)e^{-i\phi} - J_{n+1}(\rho)e^{i\phi}$$

Rotating system

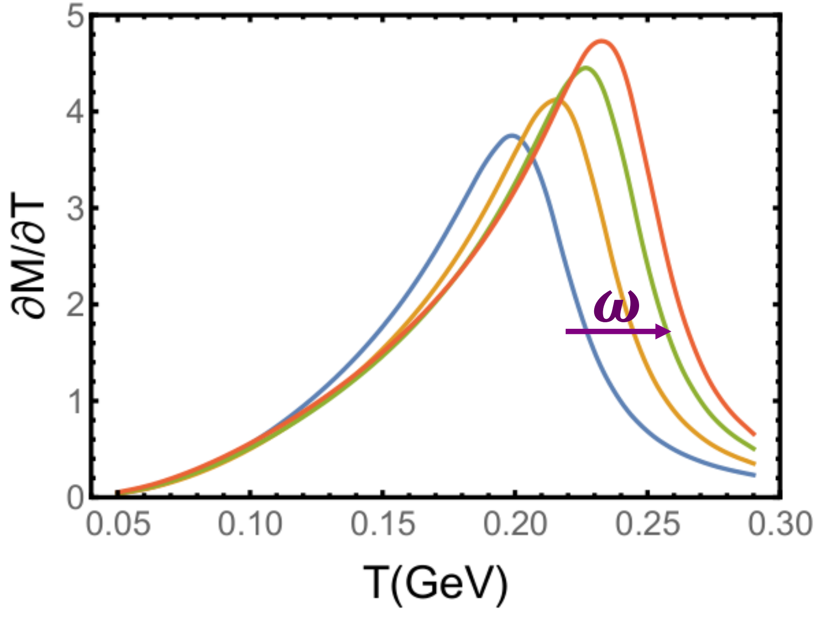
- Consider a pure gluon system. Introduce both background color magnetic field and rotation into the eigen equation. Calculate the ω-dependent QCD coupling constant.
- Summation over all the energy levels of gluon. Extract the QCD coupling constant as a function of the rotation speed.

$$G(\omega) = G_0(1 + 0.32\frac{\omega}{\Lambda_{NJL}})$$

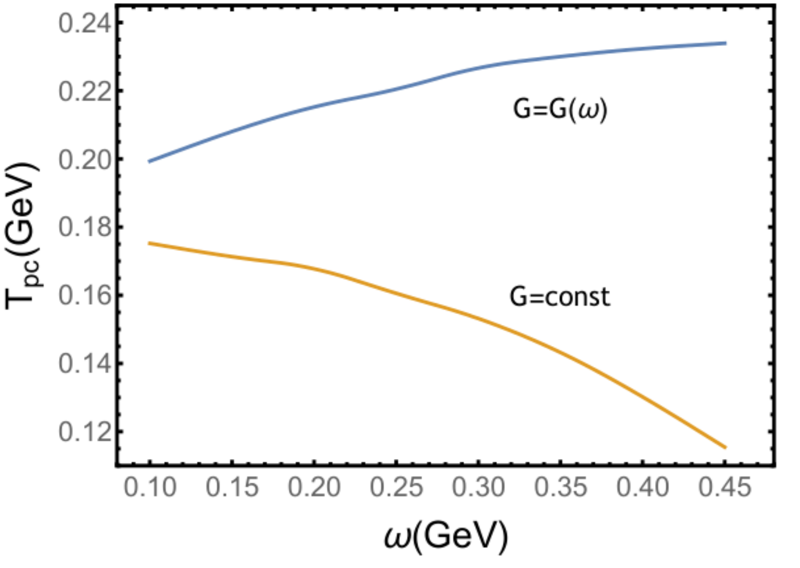
See N.K.Nielsen, Am.J. Phys.49, 1171(1981);
R.A.Schneider, Phys. Rev. D 66, 036003(2002);
R.A.Schneider, Phys.Rev.D 67,057901(2003)
Yin Jiang, EPJ.C 82 (2022) 10, 949 ; e-Print: 2108.09622 [hep-ph]

Chiral condensate

- Replace the NJL coupling G with $G(\omega)$, in mean field approximation



approaches saturation eventually because of the model cutoff

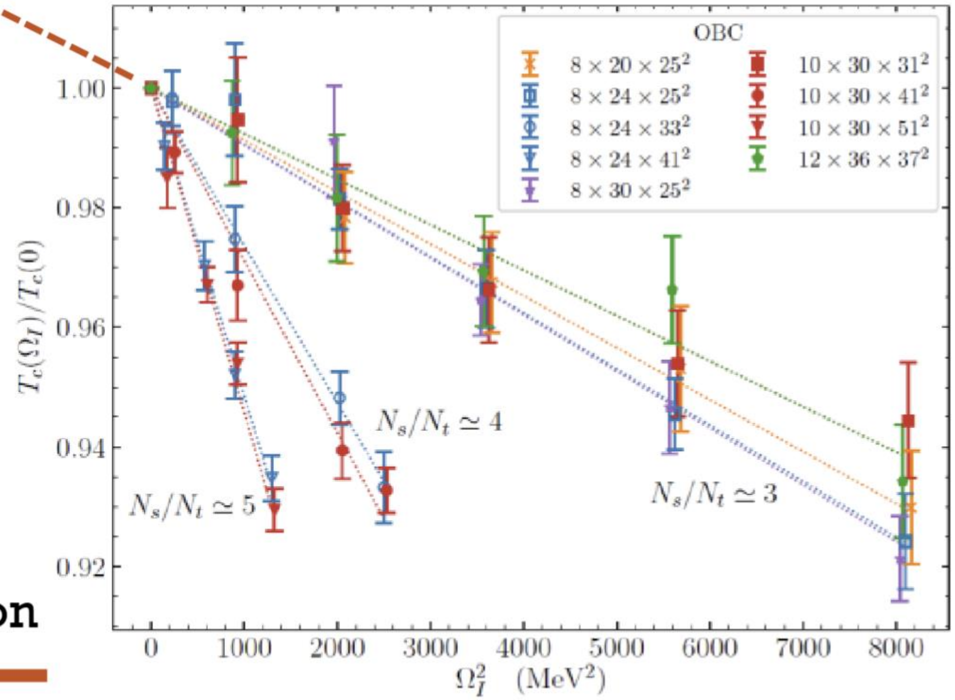


The same behavior has also been observed in recent lattice QCD simulation for deconfinement transition. (V. V. Braguta, A. Y. Kotov, D. D. Kuznedelev and A. A. Roenko, Phys. Rev. D 103, no.9, 094515 (2021))

- Lattice result with imaginary rotation

analytical continuation

$\Omega_I^2 \leftrightarrow -\Omega^2$



Real rotation

Braguta V V, Kotov A Y, Kuznedelev D D, et al. arXiv:2110.12302, 2021.

What's more

- It is better to check the running coupling as function of angular velocity with traditional perturbative computation.
- The increasing trend appears not so significant. It is not so convincible that the chiral symmetry will not restore at very large angular velocity even at very high temperature.
- Go beyond NJL model. The QCD vacuum is controlled by some non-trivial gluon configurations. The vacuum structure may be modified by the rotation.
- Gluon fluctuations around the non-trivial gluon profile will be changed by the finite-size and rotation polarization effects as well.
- These motivate us to investigate the gauge field seriously in a globally rotating system and switch to deconfinement transition.

Rotation and color confinement

http://arxiv.org/abs/2312.06166

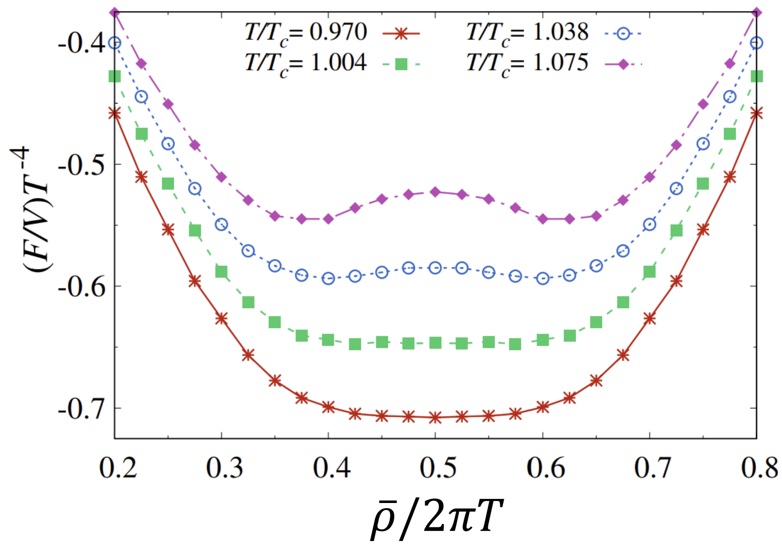
Understand confinement with KvBLL CALORON

- As a solution of the Yang-Mills equation, KvBLL caloron is good because 1) color neutrality; 2) nonzero topological charge; 3) confinement; 4) periodic along imaginary time axis.
- Focus on the SU(2) gauge group case from now on. A potential non-trivial gluon field which may be responsible for the confinement is Caloron

$$A_4^{calron}(r \to +\infty) = \bar{\rho}\frac{\tau_3}{2}$$

$$F_p(T,\omega) \quad = \frac{1}{3(2\pi)^2 T}\bar{\rho}^2(2\pi T - \bar{\rho})^2.$$

$$F_{np}(T) = -c[|\bar{\rho}|^3(\frac{\Lambda}{\pi T})^{\frac{22|\bar{\rho}|}{6\pi T}} + |2\pi T - \bar{\rho}|^3(\frac{\Lambda}{\pi T})^{\frac{22|2\pi T - \bar{\rho})|}{6\pi T}}]$$



$$L = \mathcal{P}e^{i\int_0^\beta dx_4 A_4} \quad : \frac{\bar{\rho}}{2\pi T} = 0.5, Tr(L) = 0$$ confinement

Spin the pure gluon system

- Real angular velocity.
- Caloron and anti-caloron as the background color field.
- Hard boundary. Finite size effect in the perturbative part.
- Running coupling.

Once a good semi-classical solution obtained

- Compute the thermodynamic potential.
- Minimize the potential and compute Polyakov loop.

1. Nonperturbative part from the caloron.

$$
\begin{aligned}
F_{np}(T,\omega) \quad &= -\frac{c}{2}[sgn(\bar{\rho})(\bar{\rho}+i\omega)^3(\frac{\Lambda}{\pi T})^{\frac{22sgn(\bar{\rho})(\bar{\rho}+i\omega)}{6\pi T}} \\
&+sgn(\bar{\rho}_c)(\bar{\rho}_c+i\omega)^3(\frac{\Lambda}{\pi T})^{\frac{22sgn(\bar{\rho}_c)(\bar{\rho}_c+i\omega)}{6\pi T}}] \\
&-\frac{c}{2}[sgn(\bar{\rho})(\bar{\rho}-i\omega)^3(\frac{\Lambda}{\pi T})^{\frac{22sgn(\bar{\rho})(\bar{\rho}-i\omega)}{6\pi T}} \\
&+sgn(\bar{\rho}_c)(\bar{\rho}_c-i\omega)^3(\frac{\Lambda}{\pi T})^{\frac{22sgn(\bar{\rho}_c)(\bar{\rho}_c-i\omega)}{6\pi T}}]
\end{aligned}
$$

2. Perturbative part of the thermodynamic potential.

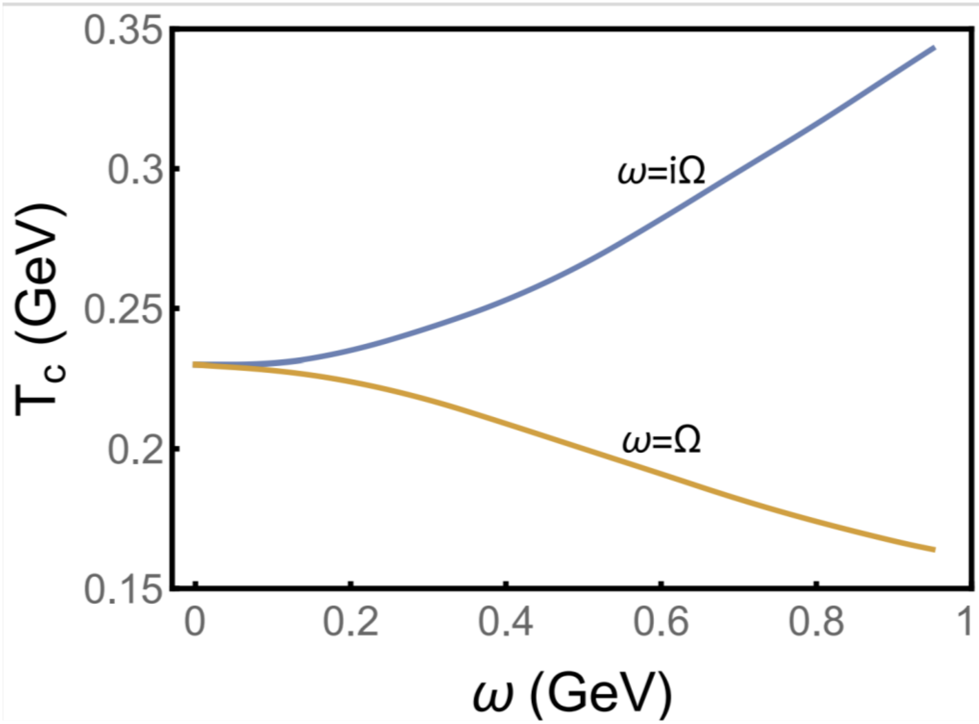
$$
F_p^{\omega}(T,\omega) = -\sum_{\substack{s,m=1\\ n=-\infty}}^{+\infty} \frac{e^{\frac{sn\omega}{T}}}{\pi^2 sR^3} \frac{4\xi_n^{(m)} cos(s\frac{\bar{\rho}}{T})}{J_{n+1}(\xi_n^{(m)})^2} K_1(s\frac{\xi_n^{(m)}}{TR})
$$

3. Mystical running coupling.

$$
g(\omega) = (1 + 0.1\omega/\Lambda)g
$$

➢ Which of them gives vortical catalysis?

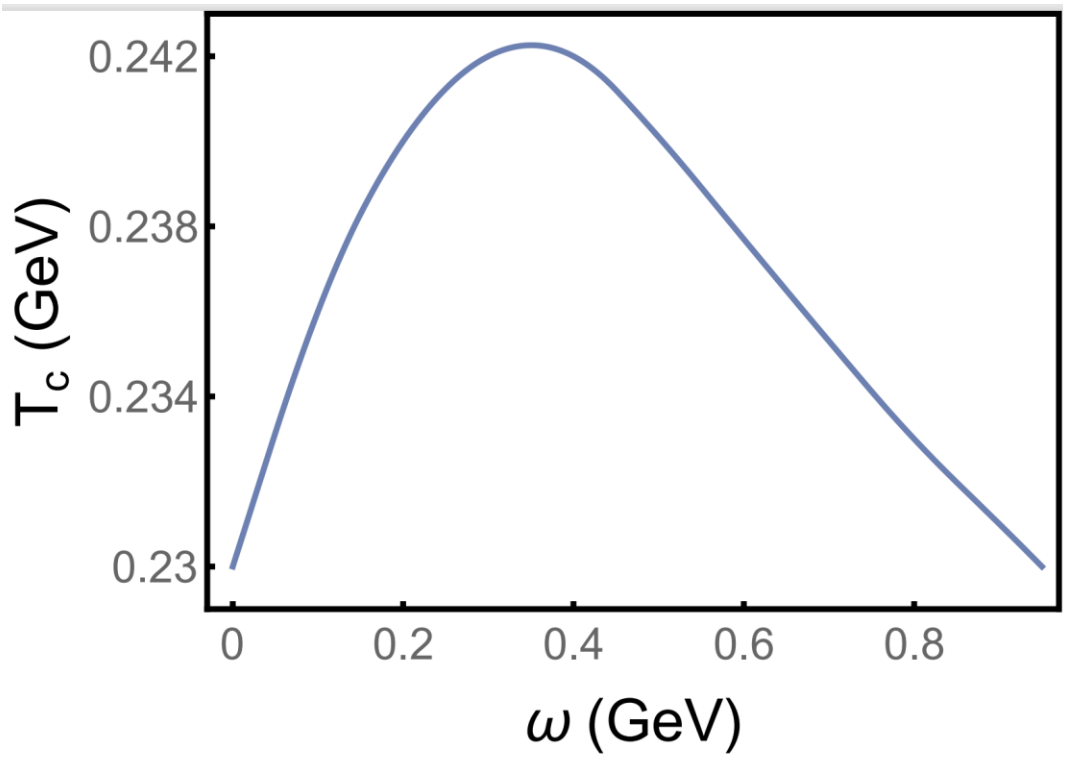
Constant running coupling



Bag is broken by rotation.

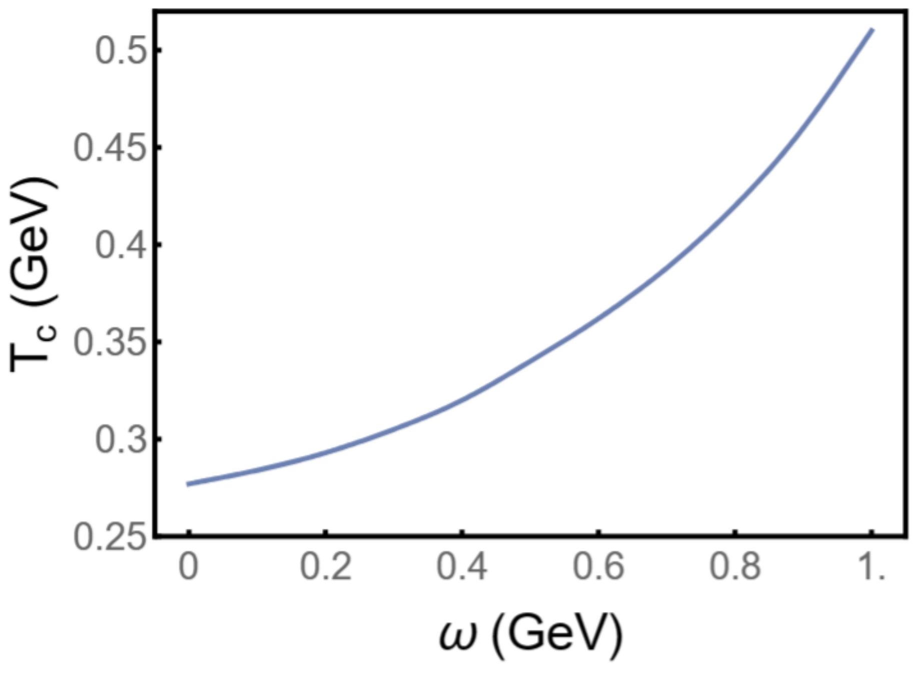
- Rotation helps to free color charge.
- The perturbation part will not be helpful to confine the color charge.
- Finite-size and polarization help to free color charge.

Running coupling constant



- Competition between running coupling and the other two contributions.
- The increasing range is short and unsignificant. It may disappear if the coupling dependence on rotation is weaker.

When the running coupling dominates



- Running coupling helps to confine color charge.
- *The only ambiguity in this computation.*

Construct KvBLL CALORON with dyons

- As a solution of the Yang-Mills equation, single dyon is NOT good because it is color charged;
- Combine several dyons, i.e. M and L in SU(2) case.

1) Comb(gauge transform) dyons to make them have the same asymptotic behavior at spatial infinity.
2) Superpose them using ADHM construction.

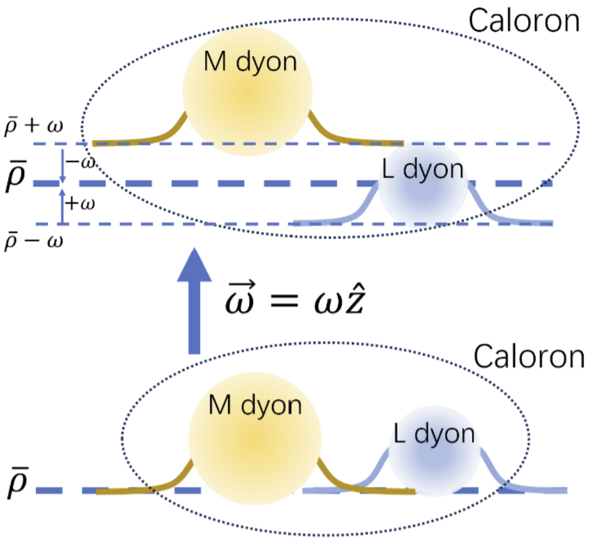
M and L dyon In hedge-hog gauge

$$A_4^a = \pm n_a\left(\frac{1}{r} - \rho\ \coth(\rho r)\right)$$

$$A_m^a = \epsilon_{amk} n_k\left(\frac{1}{r} - \rho\ \mathrm{csch}(\rho r)\right).$$

where $n_i = x_i/r$ and $r = |\vec{x}|$.

$$A_4^{calron}(r \to +\infty) = \bar{\rho}\frac{\tau_3}{2}$$



$$L = \mathcal{P}e^{i\int_0^\beta dx_4 A_4} : \frac{\bar{\rho}}{2\pi T} = 0.5, Tr(L) = 0$$ confinement

Outlook

- Achieved in these works
- With a novel method it is shown that the effective coupling will become larger when the rotation becomes faster.
- The pseudo critical temperature of the chiral restoration increases with rotation and approaches saturation eventually which may be induced by the model cutoff.
- Modified QCD vacuum and fluctuation contribution(finite size and polarization) are not powerful enough to enhance the critical temperature.
- The increase coupling constant may be the only reason to give us vortical catalysis.

- Double check the coupling running behavior.
- Consider dyon ensemble beyond dilute limit.
- Compute spatial dependent results to compare with lattice QCD.

Thank you for your attention!