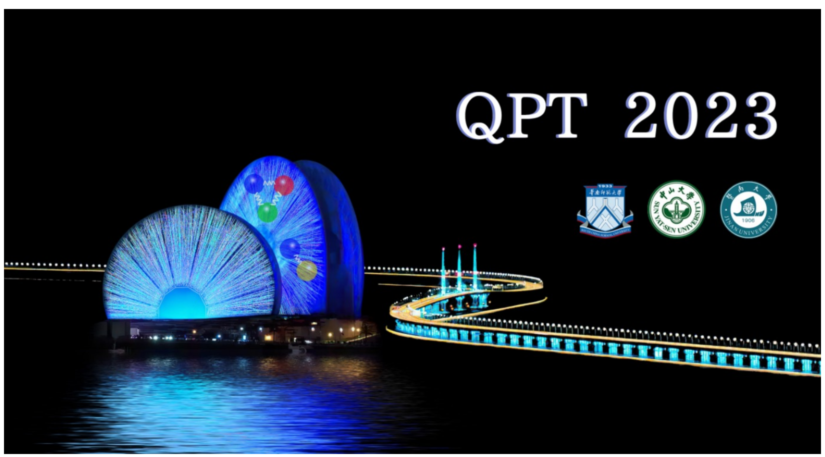
Pion dynamics in a soft-wall AdS/QCD model

Cao Xuanmin （操宣敏）

Jinan University （暨南大学）

Based on: Cao X, Baggioli Matteo, Liu H(刘绘) and Li D (李丹凝)，JHEP12 **2022** 113



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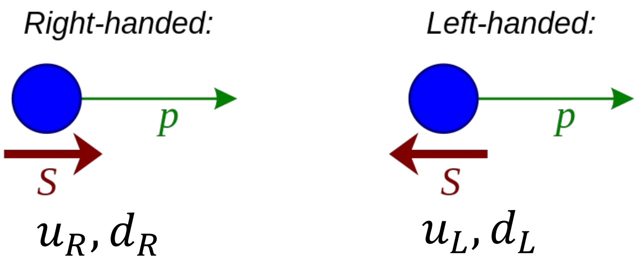
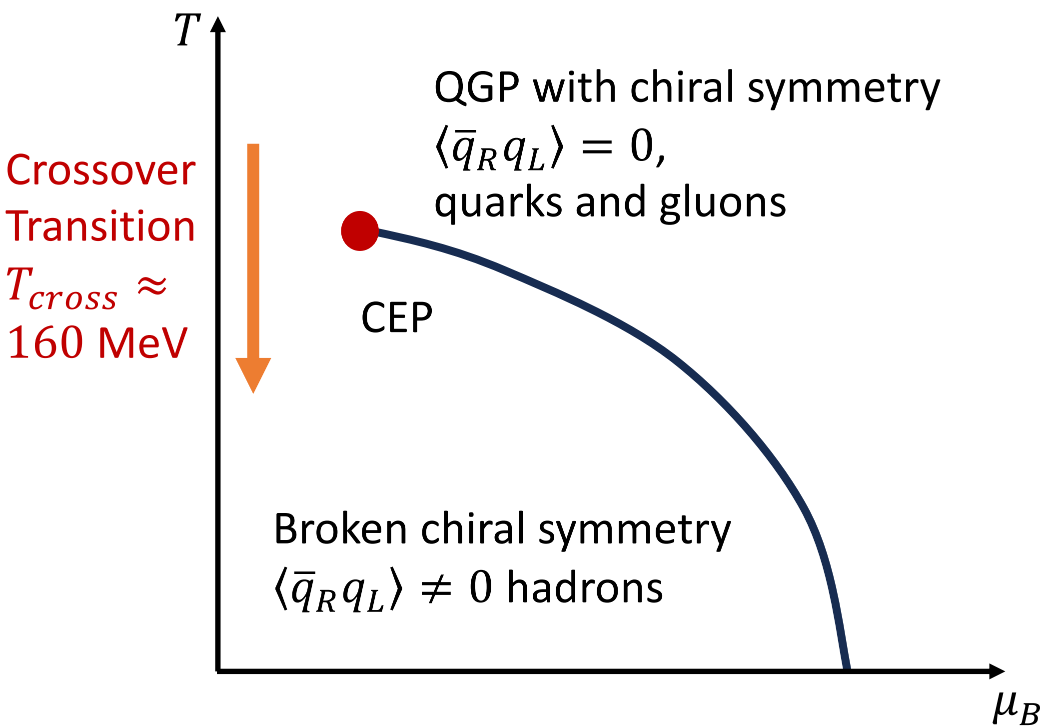
Background & Motivation



- Massless quark ($m_d = m_u = 0$) :
  $$\begin{pmatrix} u_R' \\ d_R' \end{pmatrix} = U_R \begin{pmatrix} u_R \\ d_R \end{pmatrix},$$
  $$\begin{pmatrix} u_L' \\ d_L' \end{pmatrix} = U_L \begin{pmatrix} u_L \\ d_L \end{pmatrix}$$

$$SU(2)_R \times SU(2)_L \approx O(4)$$

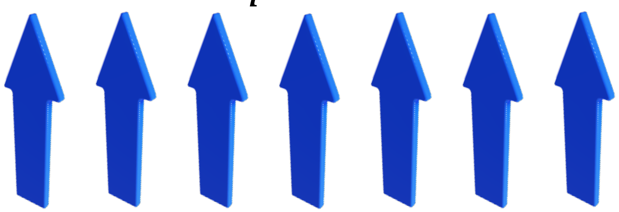


- The chiral symmetry breaking is a second order phase transition

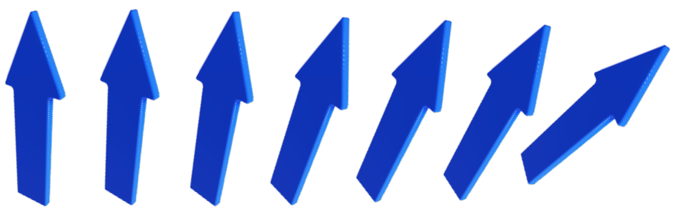
Background & Motivation

Comparing to Ising model

- $T < T_c (or\ T_{cp})$ Normal phase (chiral symmetry breaking)

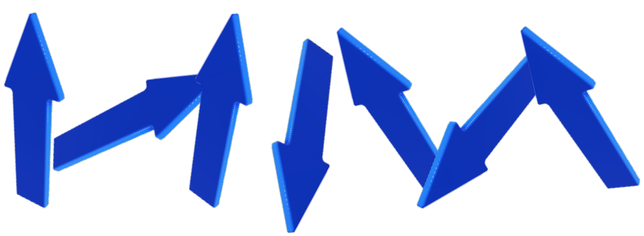


Ordered state. $\langle \bar{q}_R q_L \rangle = \bar{\sigma} \mathrm{I}_{2\times 2}$ similar to magnetization M



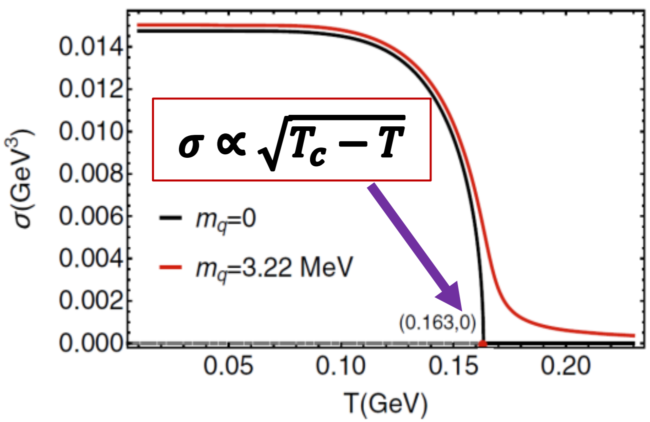
The slow modulation of the $SU_A(2)$ phase of $\bar{q}_R q$ is a pion, $\bar{q}_R q_L = \Sigma e^{i\,2\,\varphi}, \varphi = \pi^a t^a$

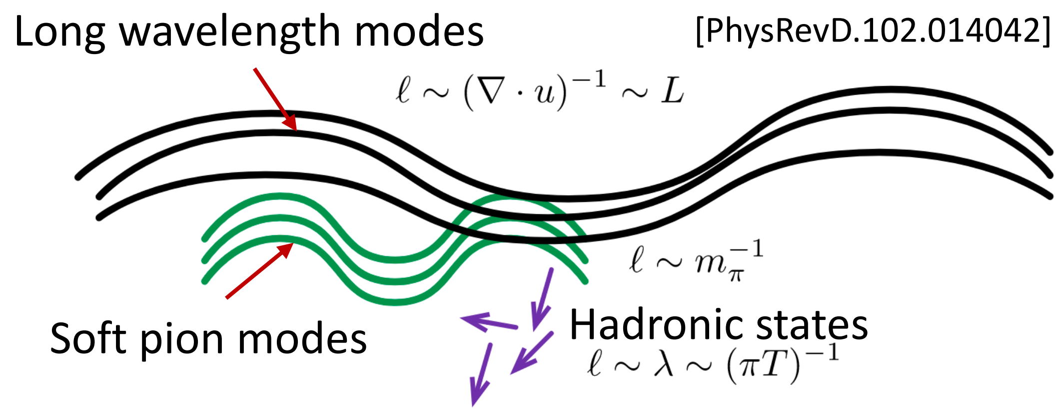
- $T > T_c (or\ T_{cp})$ Disordered phase (chiral symmetry restored)

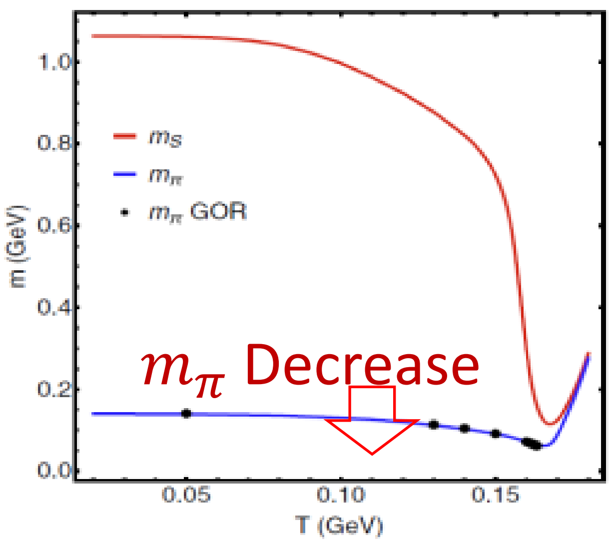


Pion propagation is frustrated

Background & Motivation







Our previous works [PhysRevD.102.126014,JHEP08(2021)005]

- GMOR relation (Black dots in Fig.) [PhysRevLett.95.261602]

$$m_\pi^2 = \frac{(m_u + m_d)\langle \bar{q} q \rangle}{f_\pi^2} + O(m_{ud}^2)$$

- The damping Ω in the limit of soft explicit breaking satisfies the following relation:

$$\Omega = D_\varphi m^2$$

- A useful phenomenological parameter:

$$\tau^2 = D_\varphi / D_A,$$

when $T \to 0$, $\tau^2 \to \frac{3}{4}$ Universal results?

2. Soft-wall AdS/QCD model

[PhysRevD.74.015005, PhysRevLett.95.261602]

Action: $S = \int d^5x \sqrt{g} e^{-\Phi(z)} \mathrm{Tr}\left\{|D_M X|^2 - V(|X|) - \frac{1}{4g_5^2}(F_L^2 + F_R^2)\right\}$

$X = (\chi + S)t^0 e^{-i2\pi^a t^a}, t^0 = \frac{I_2}{2}, t^a = \sigma^i/2$ $\quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu].$

TABLE I. Operators/fields of the model.

4D: $\mathcal{O}(x)$	5D: $\phi(x, z)$	p	Δ	$(m_5)^2$
$\bar{q}_L \gamma^\mu t^a q_L$	$A^a_{L\mu}$	1	3	0
$\bar{q}_R \gamma^\mu t^a q_R$	$A^a_{R\mu}$	1	3	0
$\bar{q}^\alpha_R q^\beta_L$	$(2/z)X^{\alpha\beta}$	0	3	-3

This model can describe both Regge Trajectorie behavior and spontaneous chiral symmetry breaking【*Physics Letters B* 762, 86–95 (2016).】

$(\Delta - p)(\Delta + p - d) = m_5^2$

$\chi(z \to 0) = m_q \zeta z + \frac{\sigma}{\zeta} z^3 + \cdots$

$\langle \bar{q}q \rangle$ chiral condensate

$m_u = m_d = m_q$ quark mass

Soft-wall AdS/QCD model

- EOM with $q = (\omega, 0,0, k)$:

$$a_0'' + (A' - \Phi')a_0' - \left(\frac{k^2 + e^{2A} g_5^2 \Sigma^2}{f}\right) a_0 - \frac{\omega k a_0 + i\omega e^{2A} g_5^2 \Sigma^2 \varphi}{f} = 0,$$

$$a_3'' + \left(A' + \frac{f'}{f} - \Phi'\right) a_3' + \left(\frac{\omega^2 - e^{2A} g_5^2 \Sigma^2 f}{f^2}\right) a_3 + \frac{\omega k a_3 + ike^{2A} g_5^2 \Sigma^2 f \varphi}{f^2} = 0,$$

and

$$\varphi'' + \left(3A' + \frac{f'}{f} - \Phi' + \frac{2\Sigma'}{\Sigma}\right)\varphi' + \left(\frac{\omega^2 - k^2 f}{f^2}\right)\varphi - i\left(\frac{\omega a_0 + k f a_3}{f^2}\right) = 0.$$

$$\Longrightarrow \quad ikfa_3' + i\omega a_0' - e^{2A}\Sigma^2 g_5^2 f\varphi' = 0.$$

- On-shell action:

$$S_{\text{on}} = -\frac{1}{2g_5^2}\int dq^4 \Big\{ e^{A(z)-\Phi(z)} \big[a_0(-q,z) a_0{}'(q,z) - a_3(-q,z) f(z) a_3{}'(q,z) \big] + e^{3A-\Phi} g_5^2 f(z) \varphi(-q,z) \times \Sigma(z)^2 \varphi'(q,z) \Big\}_{z=\epsilon},$$

Soft-wall AdS/QCD model

- Asymptotical expansion near UV boundary ($m_q \neq 0$):

$$a_0(z) = a_{t0} + a_{tl} z^2 \ln(z) + a_{t2} z^2 + \mathcal{O}(z^3),$$
$$a_3(z) = a_{x0} + a_{xl} z^2 \ln(z) + a_{x2} z^2 + \mathcal{O}(z^3),$$
$$\varphi(z) = \varphi_0 + \varphi_l z^2 \ln(z) + \varphi_2 z^2 + \mathcal{O}(z^3),$$

with

$$\varphi_2 = \frac{i(a_{t2}\omega + a_{x2}k)}{g_5^2 (m_q \zeta)^2}.$$

- Thus, $S_{\text{on}} = -\int dq^4 \left\{ \frac{a_{t0}(-q) a_{t2}(q)}{g_5^2} - \frac{a_{x0}(-q) a_{x2}}{g_5^2} + \varphi_0(-q)\varphi_2(q)(m_q\zeta)^2 \right\}$

Integration constants:		Sources/expectation for
a_{t0} and a_{x0}	$\frac{a_{t2}}{g_5^2}$ and $\frac{a_{x2}}{g_5^2}$	A_μ
$2\varphi_0 m_q \zeta$	$\frac{1}{2}\varphi_2 m_q \zeta$	π

- Therefore, the two-point Retarded correlator:[JHEP09(2002)042]

$$G_{\varphi\varphi}(q) = \frac{\delta^2 S_{on}}{\delta J_\varphi(-q)\delta J_\varphi(q)} = \frac{1}{(2m_q\bar{\sigma})^2} \frac{2i[a_{t2}(q)\omega + a_{x2}(q)k]}{g_5^2 \varphi_0(q)}$$

Soft-wall AdS/QCD model

- Asymptotical expansion near UV boundary ($m_q \neq 0$):

$$\varphi(z) = z^{-2}\left\{\bar{\varphi}_0 + \frac{1}{2}(q^2 - 2\mu_g^2 + \mu_c^2)\bar{\varphi}_0 z^2 \ln(z) + \bar{\varphi}_2 z^2 + \mathcal{O}(z^3)\right\},$$

$$\text{with } \bar{\varphi}_0 = -\frac{i(a_{t2}\omega + a_{x2}k)}{g_5^2(\bar{\sigma}/\zeta)^2}.$$

- On-shell action:

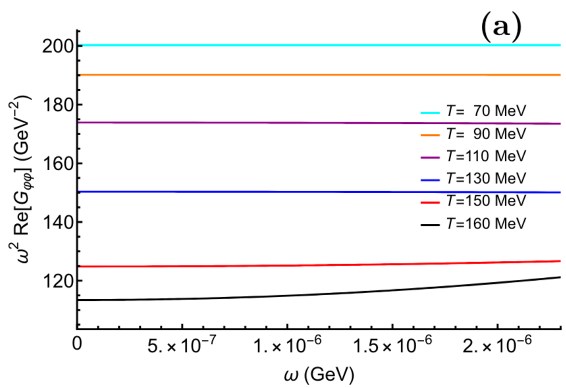
$$S'_{\rm on} = -\int dq^4 \left\{\frac{a_{t0}(-q)a_{t2}(q)}{g_5^2} - \frac{a_{x0}(-q)a_{x2}}{g_5^2} - \bar{\varphi}_0(-q)\bar{\varphi}_2(q)(\bar{\sigma}/\zeta)^2\right\}.$$

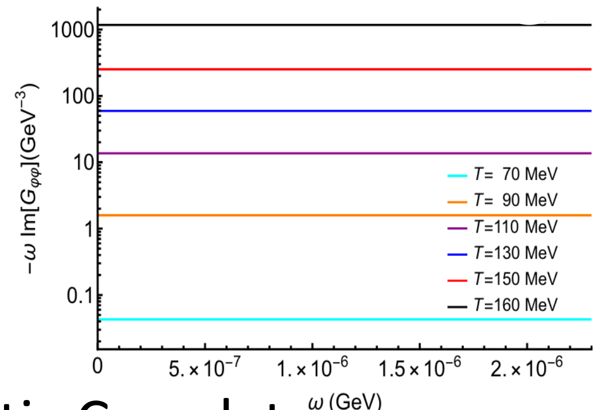
- At this consequence, $\frac{2\overline{\varphi}_0(q)\overline{\sigma}}{\zeta}$ has to be identified as the source of pion operator and $\frac{\overline{\sigma}}{2/\zeta\overline{\varphi}_2(q)}$ its expectation value.

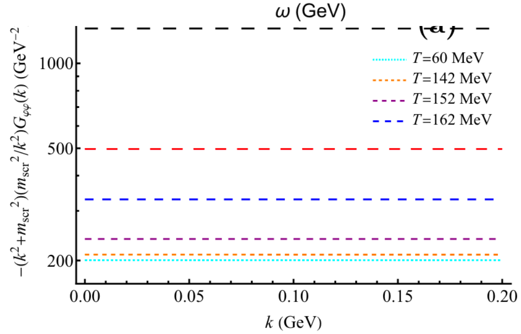
$$\Longrightarrow G_{\varphi\varphi}(q) = \frac{\delta^2 S_{on}}{\delta J_\varphi(-q)\delta J_\varphi(q)} = \frac{\bar{\varphi}_2(q) g_5^2}{2i[a_{t2}(q)\omega + a_{x2}(q)k]}$$

3. Numerical results

- Correlators: $G_{\varphi\varphi}(\omega, k=0) = \frac{1}{\chi_Q\,\omega^2} - \frac{i}{\omega}\Xi + \ldots$ Pion decay constant: $f_t^2 = \chi_Q$





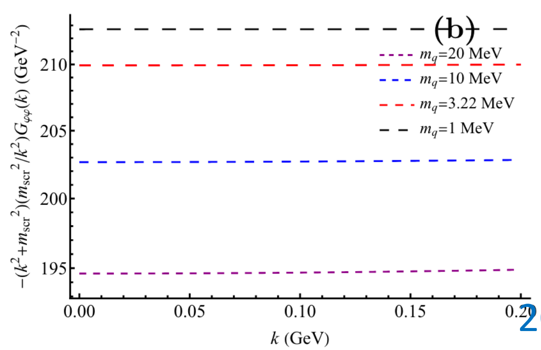


- Static Correlators:

$$G_{\varphi\varphi}(\omega=0,k) = -\frac{\chi_{\varphi\varphi}}{k^2+k_0^2} \qquad f_s^2 = 1/\chi_{\varphi\varphi}$$

$$\sigma_Q \equiv -\lim_{\omega\to 0}\frac{1}{\omega}\mathrm{Im}G_{J_Q J_Q}(\omega, k=0)$$

More details in [10.1103/PhysRevLett.128.141601]



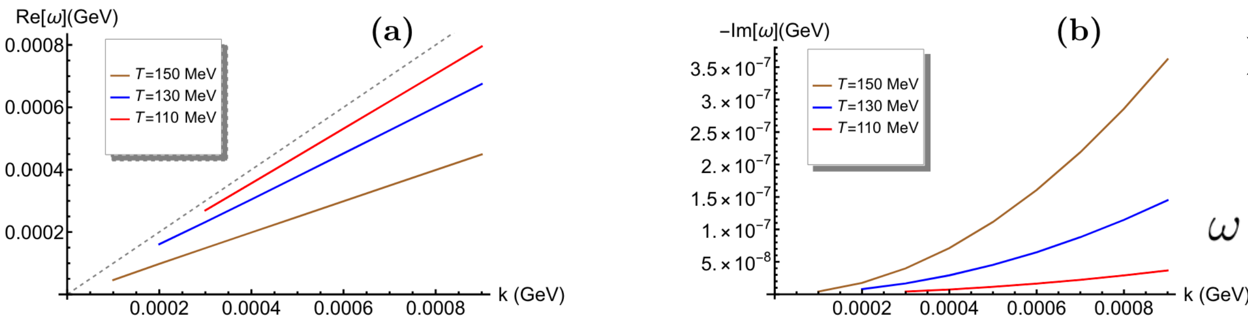
Goldstone Diffusivity,

$$D_\varphi = \frac{\Xi}{\chi_{\varphi\varphi}}$$

Diffusion constant of conserved charge

$$D_Q = \sigma_Q/\chi_Q$$

Numerical results

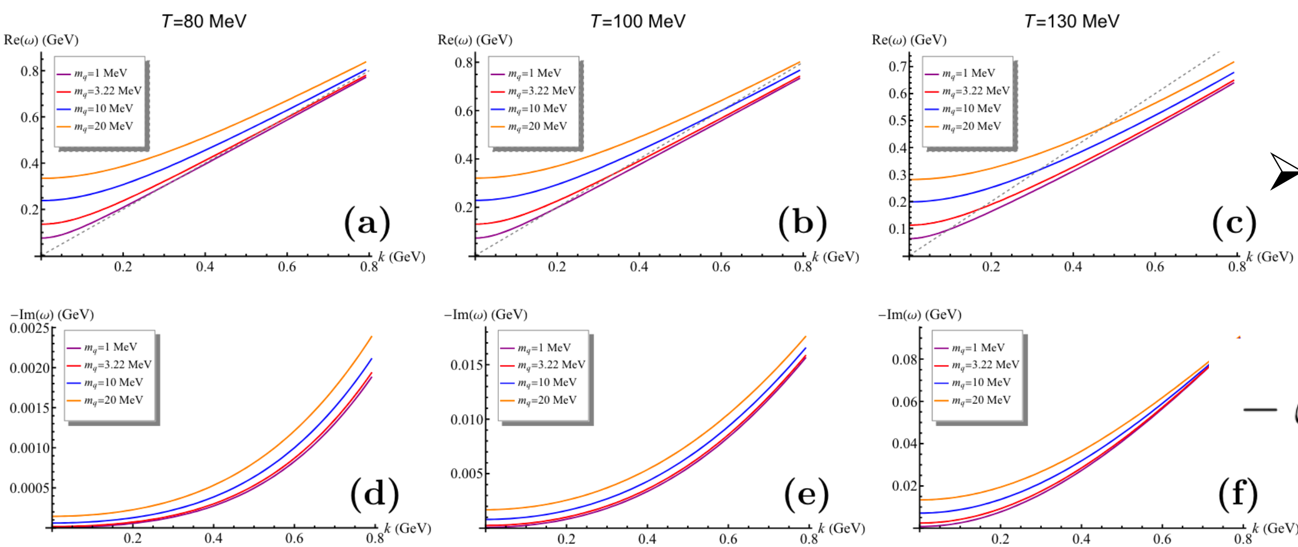


- Dispersion relation in the chiral limit:

$$\omega = \pm v\, k - \frac{i}{2} D_A\, k^2 + \ldots$$

Attenuation constant:

$$D_A = D_5 + D_\varphi$$

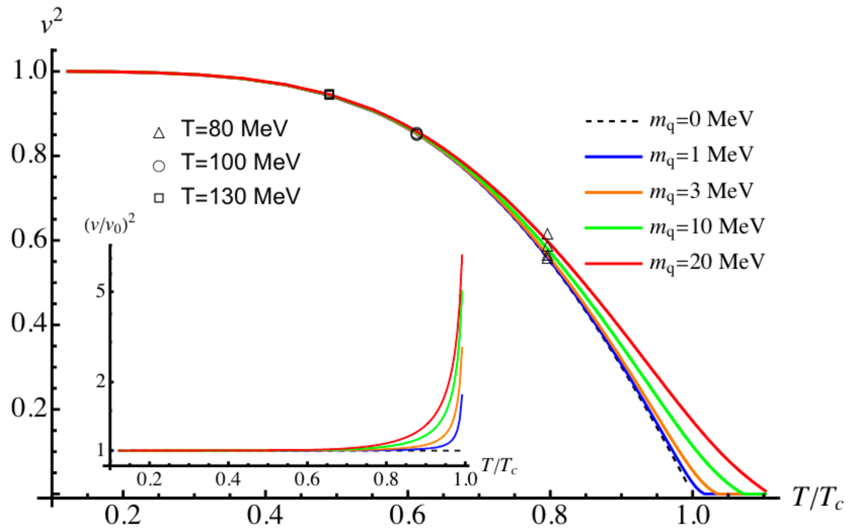


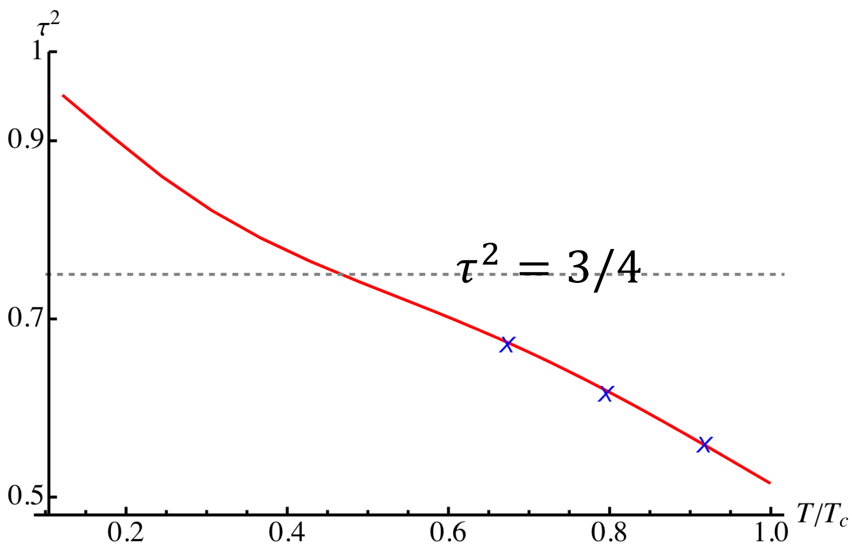
- Dispersion relation with finite quark mass:

$$-\omega^2 + \omega_k^2 - i\,\omega\,\Gamma_k = 0$$

$$\omega_k^2 = v^2 \left(k^2 + m_{\rm scr}^2\right) + \ldots \qquad \Gamma_k = D_\varphi m_{\rm scr}^2 + (D_5 + D_\varphi)\, k^2$$

Numerical results





- Speed of sound

$$\omega_k^2 = v^2 \left(k^2 + m_{\mathrm{scr}}^2\right) + \ldots$$

$$v_0^2 \propto (T_c - T)\,.$$

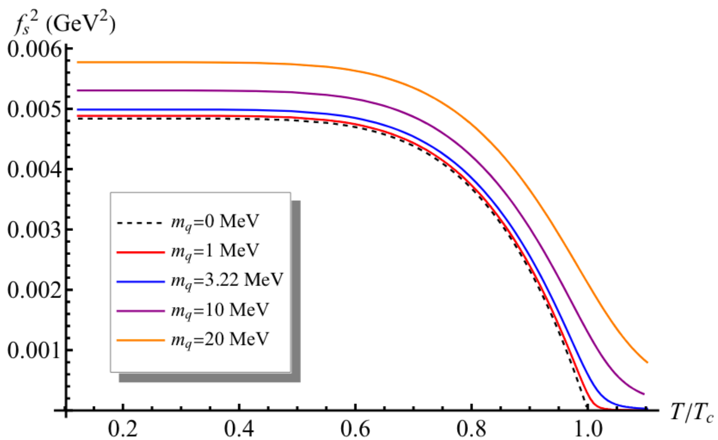
- A useful phenomenological parameter:

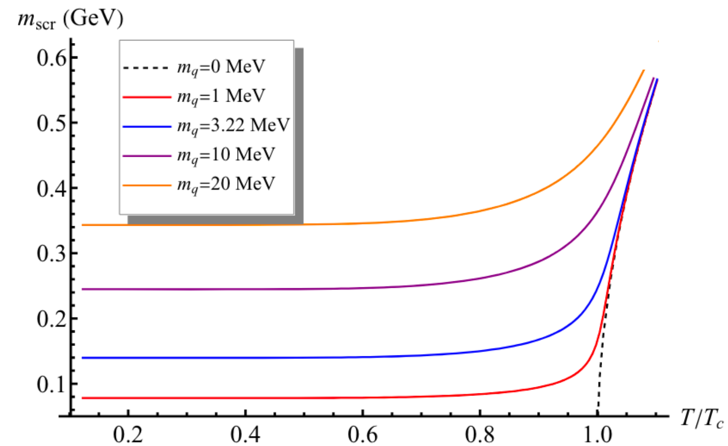
$$\tau^2 \equiv \frac{D_\varphi}{D_\varphi + D_5} = \frac{D_\varphi}{D_A}$$

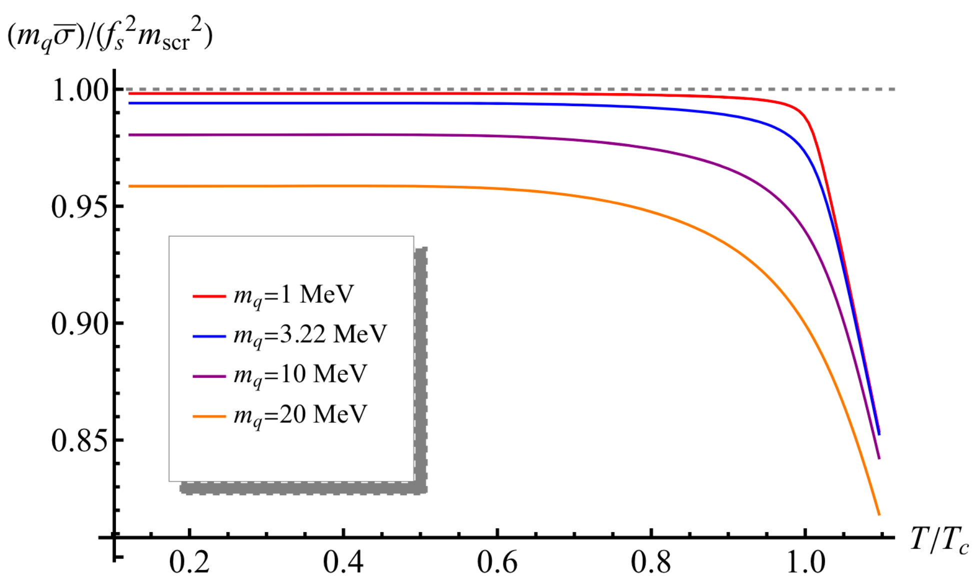
[10.1103/PhysRevD.106.056012]

Is it an effect of strong coupling? Is it an effect of large-N?

Numerical results



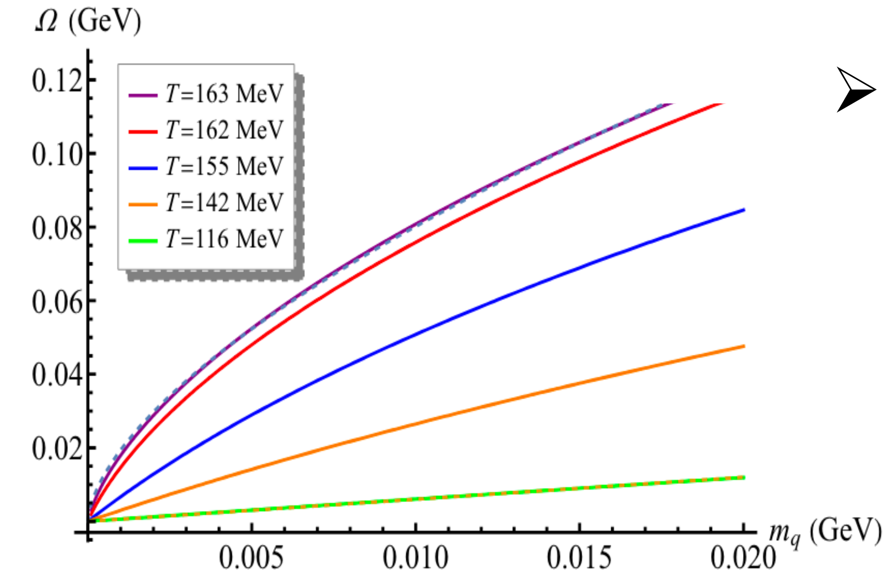


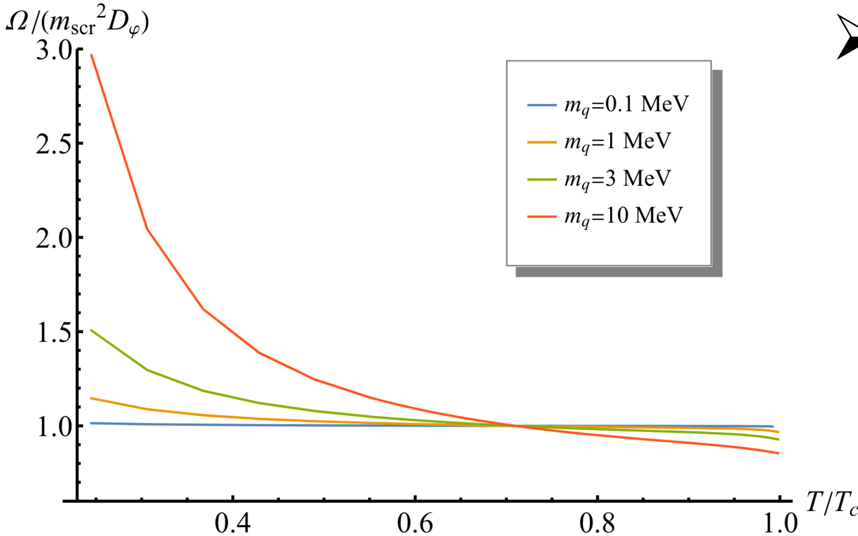


- The bound of the GMOR relation

$$m_{\rm scr}^2 f_s^2 \geq 2 m_q \bar{\sigma} \, .$$

Numerical results





- Damping (or thermal width at k=0)

  Low temperature $T << T_c$

  $$\Omega \propto m_q$$

  Near critical temperature $T \sim T_c$

  $$\Omega \propto m_q^{\nu z/(\beta\delta)}$$

- Universal damping relation

  $$\Omega = D_\varphi m_{\rm scr}^2$$

  With large quark mass, this relation appears to be violated in both directions

Outlook

- It would be interesting to compute the effects of a finite pion mass on the QCD transport properties such as the shear viscosity or the axial conductivity and investigate further the scaling behaviors of the transport coefficients close to the critical point;

- In our analysis the dynamics and role of the amplitude mode (the σ meson) has been completely neglected. Close to the critical temperature, the effects of the amplitude mode could be dramatic;

- Finally, it would be fruitful to extend our analysis to the time dependent dynamics and analyze the thermalization properties of this system near the critical point.

Thanks for your attention!