Institute of Theoretical Physics
Chinese Academy of Sciences

Chiral Effective Field Theory

For QCD and Electroweak Dynamics

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第八届手征有效场论研讨会 @ 河南开封

10-28, 2023

Outline

- Overview on chiral symmetry
- QCD chiral perturbation theory

[Hao Sun, Yi-Ning Wang, **J.H.Yu**, in préparation]
[Xuan-He Li, Chuan-Qiang Song, Hao Sun, **J.H.Yu**, in préparation]

- Electroweak chiral Lagrangian

[Hao Sun, Ming-Lei Xiao, **J.H.Yu**, 2206.07722]
[Hao Sun, Ming-Lei Xiao, **J.H.Yu**, 2210.14939]
[Hao Sun, Yi-Ning Wang, **J.H.Yu**, 2211.11598]

- Nuclear chiral effective theory

[Hao Sun, Yi-Ning Wang, **J.H.Yu**, in préparation]
[Yong-Kang Li, Yi-Ning Wang, **J.H.Yu**, in préparation]

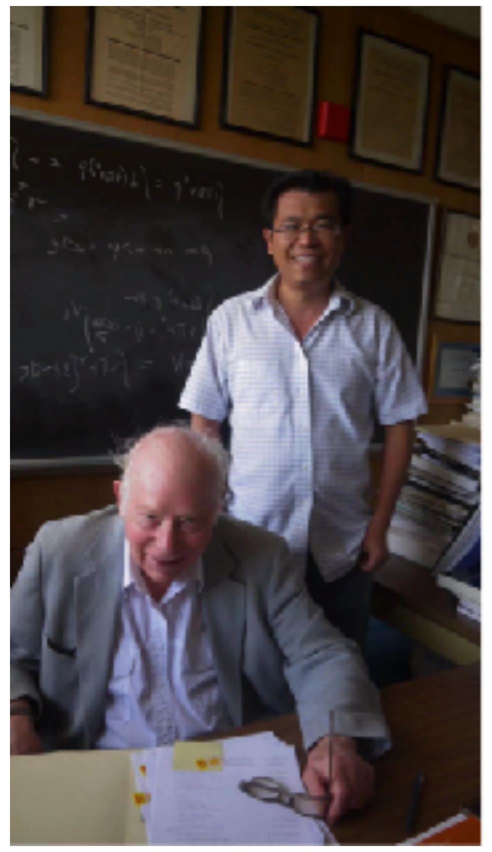
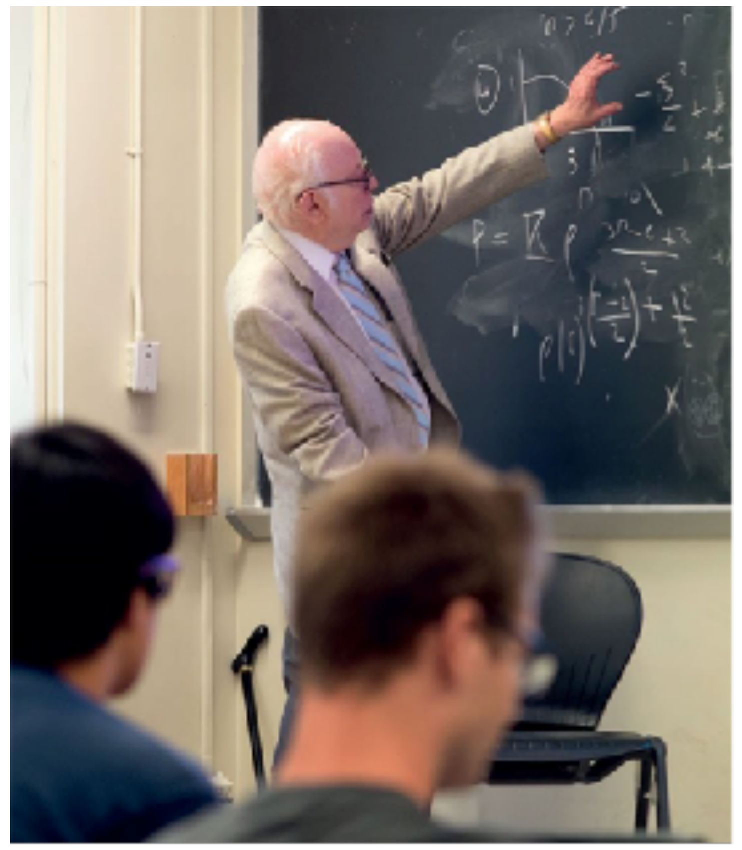
- Axion effective field theory

[Huayang Song, Hao Sun, **J.H.Yu**, 2305.16770]
[Huayang Song, Hao Sun, **J.H.Yu**, 2306.05999]

- Summary

Outline

- This talk is dedicated to the memory of Steven Weinberg (1933 - 2021)

[Weinberg 1933.5.3 - 2021.7.23]

- Review chiral Lagrangian at various scales: QCD, electroweak, TeV, nuclei scale

Physica **96A** (1979) 327–340 © *North-Holland Publishing Co.*

PHENOMENOLOGICAL LAGRANGIANS*

STEVEN WEINBERG
Lyman Laboratory of Physics, Harvard University

A MODEL OF LEPTONS*

Steven Weinberg†
Laboratory for Nuclear Science and Physics Department,
Massachusetts Institute of Technology, Cambridge, Massachusetts
(Received 17 October 1967)

Nuclear forces from chiral lagrangians

Steven Weinberg [1]
Theory Group, Department of Physics, University of Texas, Austin, TX 78712, USA

Received 14 August 1990

PHYSICAL REVIEW VOLUME 166, NUMBER 5 25 FEBRUARY 1968

Nonlinear Realizations of Chiral Symmetry*

Steven Weinberg†
Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts
(Received 25 September 1967)

VOLUME 40, NUMBER 4 PHYSICAL REVIEW LETTERS 23 JANUARY 1978

A New Light Boson?

Steven Weinberg
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138
(Received 6 December 1977)

It is pointed out that a global U(1) symmetry, that has been introduced in order to preserve the parity and time-reversal invariance of strong interactions despite the effects of instantons, would lead to a neutral pseudoscalar boson, the "axion," with mass roughly of order 100 keV to 1 MeV. Experimental implications are discussed.

EFFECTIVE CHIRAL LAGRANGIANS FOR NUCLEON–PION INTERACTIONS AND NUCLEAR FORCES

Steven WEINBERG*
Theory Group, Department of Physics, University of Texas, Austin, TX 78712, USA

Received 2 April 1991

Outline

Among 15 top-cited papers, 7 are relevant to chiral symmetry, 5 relevant to effective field theory

311 results | cite all   Citation Summary   Most Cited

A Model of Leptons #1
Steven Weinberg (MIT, LNS) (Nov, 1967)
Published in: *Phys.Rev.Lett.* 19 (1967) 1264-1266
links DOI cite claim reference search 14,151 citations

The Cosmological Constant Problem
Steven Weinberg (Texas U.) (May, 1988)
Published in: *Rev.Mod.Phys.* 61 (1989) 1-23
DOI cite claim

A New Light Boson?
Steven Weinberg (Harvard U.) (Dec, 1977)
Published in: *Phys.Rev.Lett.* 40 (1978) 223-226
pdf DOI cite claim

Phenomenological Lagrangians
Steven Weinberg (Harvard U. and Harvard-Smithsonian Ctr. Astrophys.) (O
Published in: *Physica A* 96 (1979) 1-2, 327-340 • Contribution to: Sympos
Occasion of his 60th Birthday, 327-340
DOI cite claim

Implications of Dynamical Symmetry Breaking
Steven Weinberg (Harvard U.) (Sep, 1975)
Published in: *Phys.Rev.D* 13 (1976) 974-996, *Phys.Rev.D* 19 (1979) 1277-
DOI cite claim

Broken Symmetries
Jeffrey Goldstone (Cambridge U.), Abdus Salam (Imperial Coll., London), Steven W
1962)
Published in: *Phys.Rev.* 127 (1962) 965-970
DOI cite claim

Baryon and Lepton Nonconserving Processes
Steven Weinberg (Harvard U.) (1979)
Published in: *Phys.Rev.Lett.* 43 (1979) 1566-1570
pdf DOI cite claim

Natural Conservation Laws for Neutral Currents
Sheldon L. Glashow (Harvard U.), Steven Weinberg (Harvard U.) (Aug, 1976)
Published in: *Phys.Rev.D* 15 (1977) 1958
pdf DOI cite claim

Hierarchy of Interactions in Unified Gauge Theories
H. Georgi (Harvard U.), Helen R. Quinn (Harvard U.), Steven Weinberg (Harvard U.)
Published in: *Phys.Rev.Lett.* 33 (1974) 451-454
DOI cite claim

Pion scattering lengths #10
Steven Weinberg (UC, Berkeley) (Jun, 1966)
Published in: *Phys.Rev.Lett.* 17 (1966) 616-621
DOI cite claim reference search 1,808 citations

Supergravity as the Messenger of Supersymmetry Breaking #11
Lawrence J. Hall (UC, Berkeley), Joseph D. Lykken (Texas U.), Steven Weinberg (Texas U.) (1983)
Published in: *Phys.Rev.D* 27 (1983) 2359-2378
DOI cite claim reference search 1,629 citations

Cosmological Lower Bound on Heavy Neutrino Masses #12
Benjamin W. Lee (Fermilab), Steven Weinberg (Stanford U., Phys. Dept.) (May, 1977)
Published in: *Phys.Rev.Lett.* 39 (1977) 165-168
pdf links DOI cite claim reference search 1,566 citations

Gauge and Global Symmetries at High Temperature #13
Steven Weinberg (Harvard U.) (Mar, 1974)
Published in: *Phys.Rev.D* 9 (1974) 3357-3378
DOI cite claim reference search 1,554 citations

Nuclear forces from chiral Lagrangians #14
Steven Weinberg (Texas U.) (Oct 9, 1990)
Published in: *Phys.Lett.B* 251 (1990) 288-292
DOI cite claim reference search 1,551 citations

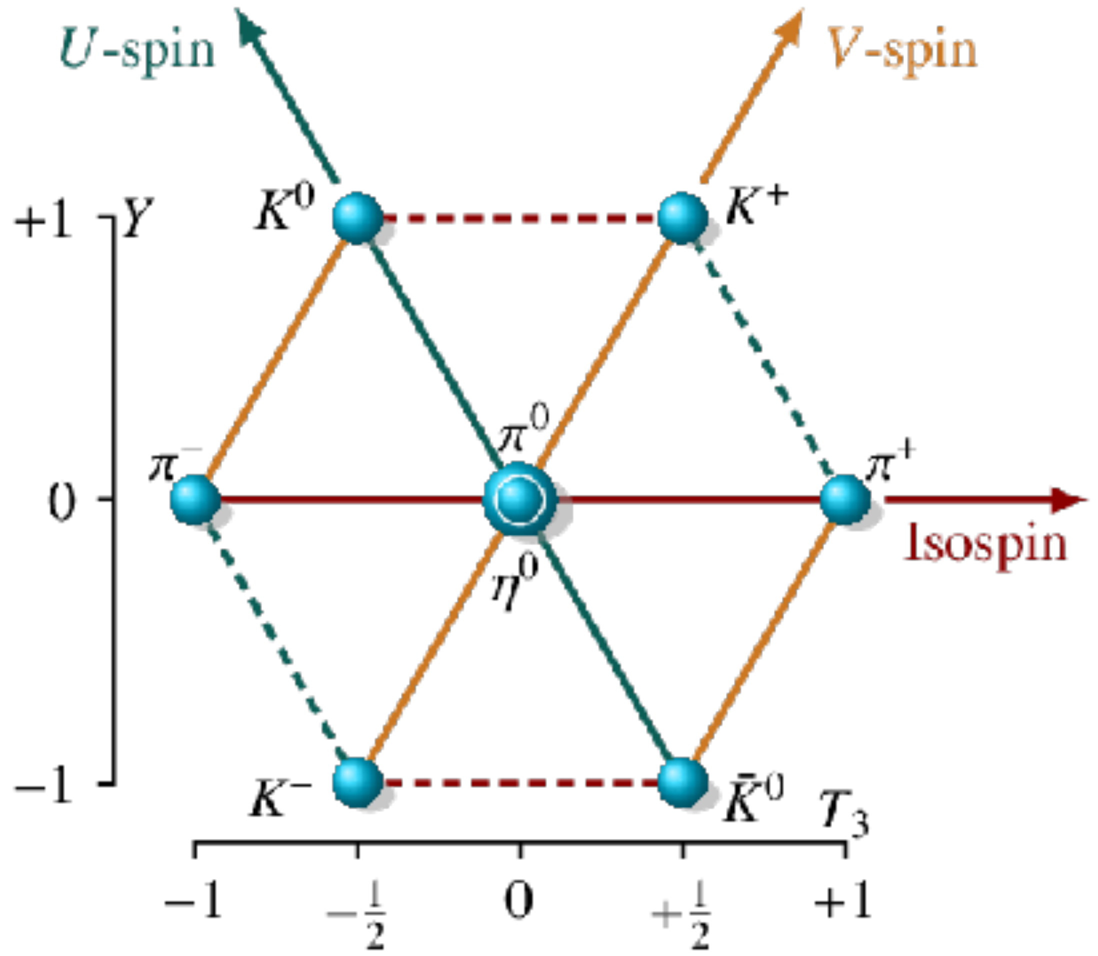
Effective chiral Lagrangians for nucleon - pion interactions and nuclear forces #15
Steven Weinberg (Texas U.) (Apr 1, 1991)
Published in: *Nucl.Phys.B* 363 (1991) 3-18
pdf DOI cite claim reference search 1,465 citations

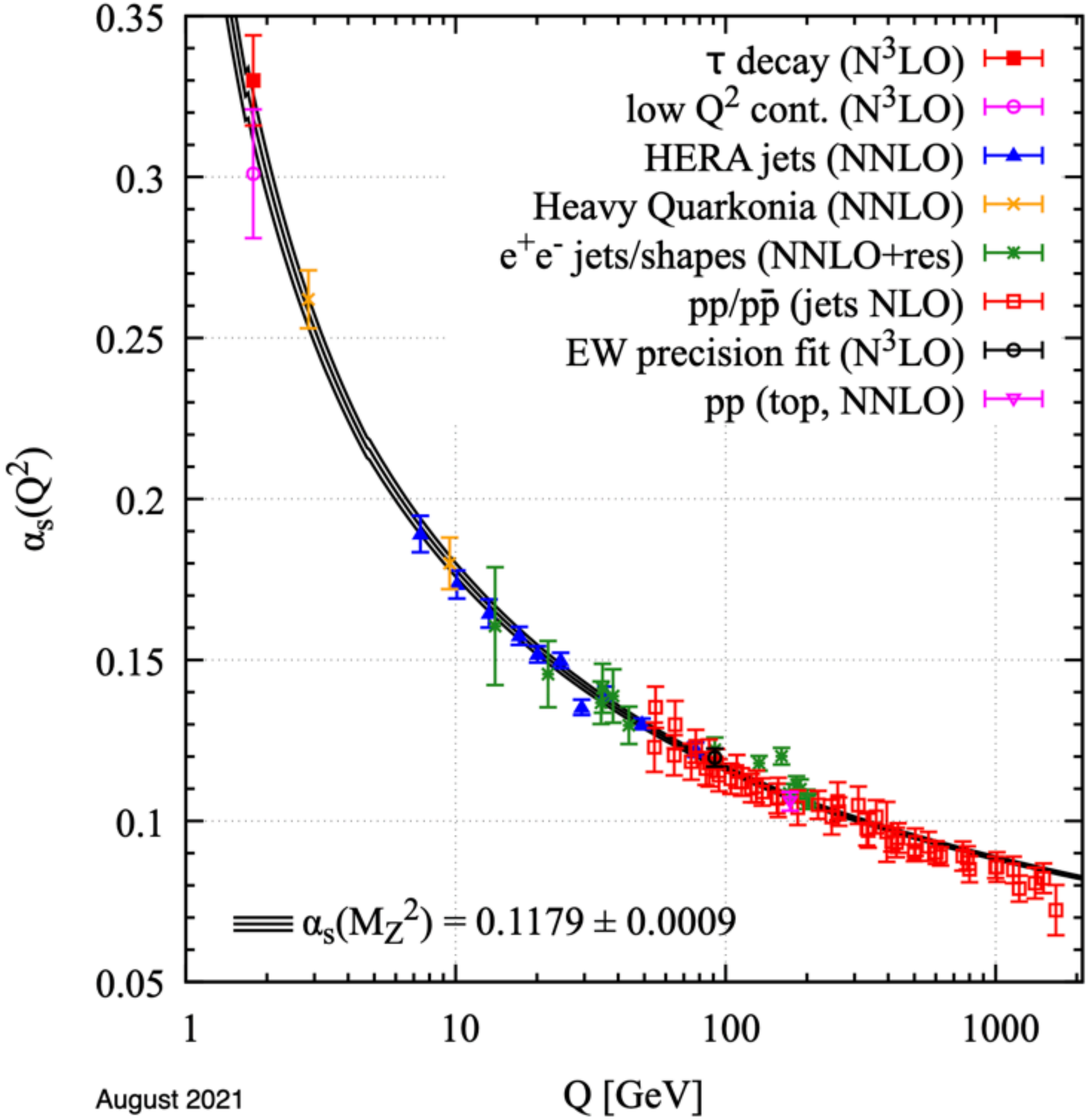
Overview on Chiral Symmetry

QCD at high and low scales

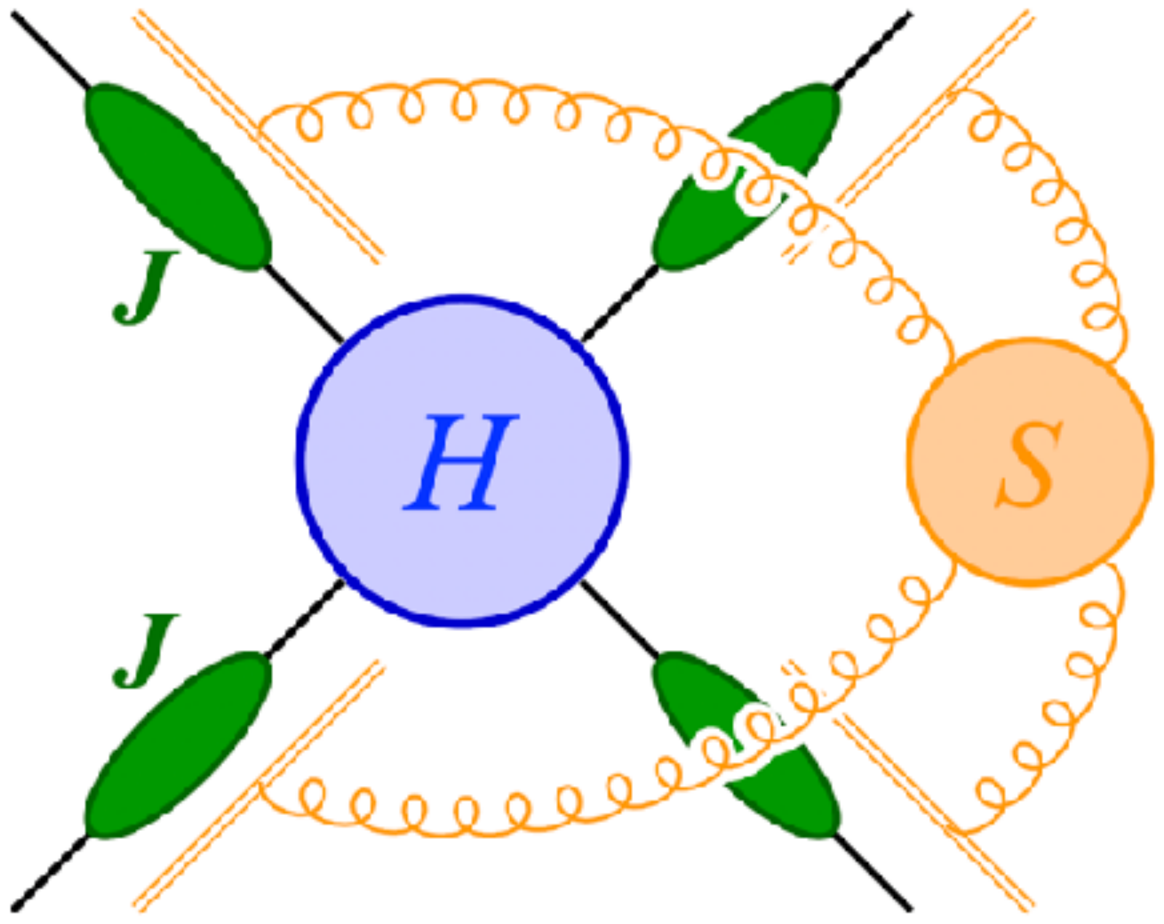
Perturbative QCD vs non-perturbative QCD

Symmetry





Factorization



Symmetries in QCD

$$\mathcal{L}_{\rm QCD} = \sum_q \bar{q}\left(i\not{D} - m_q e^{i\theta_q}\right)q - \frac{1}{4}G_a^{\mu\nu}G^a_{\mu\nu} - \theta\frac{\alpha_s}{8\pi}G_a^{\mu\nu}\tilde{G}^a_{\mu\nu}$$

Scale symmetry

$$x^\mu \to \lambda x^\mu,$$

$$\psi_q(x) \to \lambda^{3/2}\psi_q(\lambda x), \quad A^a_\mu(x) \to \lambda A^a_\mu(\lambda x)$$

Anomalous: trace anomaly

$$\partial_\mu S^\mu = \Theta^\mu_\mu = -\frac{\beta}{2g_s}F^{(a)}_{\mu\nu}F^{(a)\mu\nu} + \sum_q m_q \bar{\psi}_q \psi_q$$

$$m_N \bar{u}(\mathbf{p})u(\mathbf{p}) = \langle N(\mathbf{p})|\theta^\mu_\mu|N(\mathbf{p})\rangle$$
$$= \left\langle N(\mathbf{p})\middle|\frac{\beta_{\rm QCD}}{2g_3}F^a_{\mu\nu}F^{\mu\nu}_a + \sum_q m_q\bar{q}q\middle|N(\mathbf{p})\right\rangle$$

90% proton mass from gluon dynamics

Chiral symmetry

$$\Lambda_{\rm QCD} \gg m_q$$

SU(3) x SU(3) $\begin{pmatrix} u \\ d \\ s \end{pmatrix} \to U_{V,A}\begin{pmatrix} u \\ d \\ s \end{pmatrix}$

U(1)B $\psi_q \to e^{i\alpha}\psi_q$

U(1)A $\psi_q \to e^{i\alpha\gamma_5}\psi_q$

Anomalous: axial anomaly

$$\partial^\mu J^{(0)}_{5\mu}(x) = \frac{3\alpha_s}{4\pi}F^a_{\mu\nu}\tilde{F}^{\mu\nu}_a + \sum_q m_q\bar{q}\gamma_5 q$$

$$\mathcal{L}_{\rm det} = (-1)^{N_f}K^{-5}\langle\bar{u}_L u_R\rangle\left\langle\bar{d}_L d_R\right\rangle\langle\bar{s}_L s_R\rangle e^{i2\theta_{\eta'}} \sim \Lambda^2_{\rm QCD}\,\eta'^2$$

Eta-prime mass from instanton dynamics

Chiral symmetry **8 Goldstone bosons from SSB**

$$\langle 0|\left(\bar{\mathbf{q}}_L\mathbf{q}_R + \bar{\mathbf{q}}_R\mathbf{q}_L\right)|0\rangle \neq 0$$

$$\mathbf{\Phi} \equiv \frac{\vec{\lambda}}{\sqrt{2}}\vec{\phi} = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta \end{pmatrix}$$

Lagrangian for low energy mesons

Most general scalar Lagrangian

Current algebra, PCAC, sigma model, NJL model …

$$\mathcal{L}_\pi = \frac{1}{2}\partial_\mu \pi^a \partial^\mu \pi^a + b_0 \pi^a \pi^a$$ quadratic in pion fields

$$+ b_1 (\pi^a \pi^a)^2 + \ldots$$ quartic, no derivative

$$+ c_1 (\partial_\mu \pi^a \pi^a)^2 + c_2 (\partial_\mu \pi^a \partial^\mu \pi^a)(\pi^b \pi^b) + \ldots$$ quartic, two derivatives

$$+ \ldots$$ quartic, four derivatives, …

Chiral symmetry

Flavor symmetry relates matrix elements of multiplets

$\alpha \to \beta$ ⟷ (At low energy) $\alpha + n_1 \pi \to \beta + n_2 \pi$

Goldberger-Trieman, Callan-Trieman, Adler-Weisberger, etc

Adler Zero condition

$b_k = 0$ for $k = 1, \ldots$

$T(\alpha + \phi(p), \beta) = -\frac{p_\mu}{F} R^\mu(p) \xrightarrow{p \to 0} 0$

Chiral Ward Identity

$c_1 = -c_2$

PCAC relation

$$c_1 = -c_2 = \frac{1}{6 f^2}$$

$$\langle 0 | j^a_{5\mu}(x) | \pi^b(p) \rangle = \delta^{ab}\, i p_\mu F e^{-ip\cdot x}$$

Goldstone EFT and power counting

Construct generic EFT for Goldstone at IR broken phase

PHYSICAL REVIEW VOLUME 166, NUMBER 5 25 FEBRUARY 1968

Nonlinear Realizations of Chiral Symmetry*

STEVEN WEINBERG†

Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts

(Received 25 September 1967)

Shift symmetry:

$$\pi \to \pi + \epsilon + \cdots$$

Goldstone mode is a fluctuation around the background in the direction of broken generator

Gapless mode

Weakly coupled at IR

Non-linear transform under G/H

No interaction at long-wave limit

$$\mathscr{L}_{\text{EFT}} = \mathscr{L}_{p^2} + \mathscr{L}_{p^3} + \mathscr{L}_{p^4} + \mathscr{L}_{p^5} + \mathscr{L}_{p^6} + \ \ldots$$

$$\frac{f^2}{4}\langle D_\mu \mathbf{U}^\dagger D^\mu \mathbf{U}\rangle$$

$$\Pi_{\hat a} \to \Pi_{\hat a}^{(g_\mathcal{H})} = \left(e^{i\alpha_a t_\pi^a}\right)_{\hat a}^{\ \hat b}\Pi_{\hat b}$$

$$\Pi_{\hat a} \to \Pi^{(g_{\mathcal{G}/\mathcal{H}})}{}_{\hat a} = \Pi_{\hat a} + \frac{f}{\sqrt{2}}\alpha_{\hat a} + \mathcal{O}\left(\alpha\frac{\Pi^2}{f} + \alpha\frac{\Pi^3}{f^2}\ldots\right)$$

$$U\left[\Pi\right] = e^{i\frac{\sqrt{2}}{f}\Pi_{\hat a}(x)\widehat{T}^{\hat a}}$$

$$g \cdot U\left[\Pi\right] = U\left[\Pi^{(g)}\right]\cdot h\left[\Pi;\, g\right] \qquad h\left[\Pi;\, g\right] = e^{i\zeta_a[\Pi;g]T^a}$$

Power counting: Derivative expansion

Coset Construction

[Callan, Coleman, Wess, Zumino, 1969]

CCWZ chiral Lagrangian

Define the nonlinear Goldstone matrix

[Callan, Coleman, Wess, Zumino, 1969]

$$\Omega(\Pi) \equiv \exp\left[\frac{i}{2f}\Pi(x)\right] \to \Omega(\Pi^{(\mathfrak{g})}) = \mathfrak{g}\Omega(\Pi)\mathfrak{h}^{-1}(\Pi;\mathfrak{g})$$

$[T_{\hat{a}}, T_{\hat{b}}] \propto T_c$

CCWZ Coset

$$-i\Omega^\dagger\partial_\mu\Omega = d_\mu^{\hat{a}}T^{\hat{a}} + E_\mu^a T^a \equiv d_\mu + E_\mu$$

$$d_\mu \to \mathfrak{h}d_\mu\mathfrak{h}^{-1}, \quad E_\mu \to \mathfrak{h}E_\mu\mathfrak{h}^{-1} - ih\partial_\mu\mathfrak{h}^{-1}$$

$$\partial_\mu \to D_\mu = \partial_\mu + iA_\mu$$
$$A_\mu = A_\mu^{\hat{a}}T^{\hat{a}} + A_\mu^a T^a$$

Building block

$$d_\mu(\Pi), \quad E_{\mu\nu}(\Pi) \quad \nabla_\mu \equiv \partial_\mu + iE_\mu$$
$$f_{\mu\nu} = \Omega^\dagger F_{\mu\nu}\Omega = f_{\mu\nu}^{-\hat{a}}T^{\hat{a}} + f_{\mu\nu}^{+a}T^a$$

Symmetric Coset

$$\Omega \to \mathfrak{g}\Omega\mathfrak{h}^{-1}, \quad \Omega \to \mathfrak{h}\Omega\mathfrak{g}_{\mathcal{R}}^{-1}$$

$$U \equiv \Omega^2 \to \mathfrak{g}\,U\mathfrak{g}_{\mathcal{R}}^{-1}$$

$$D_\mu U \equiv \partial_\mu U + iA_\mu U - iUA_\mu^{(R)}$$
$$A_\mu^{(R)} \equiv A_\mu^a T^a - A_\mu^{\hat{a}}T^{\hat{a}}$$

$$E_{\mu\nu} = -i[u_\mu, u_\nu] + f_{\mu\nu}^+$$
$$d_\mu = u_\mu$$

$$u_\mu = i\Omega(D_\mu U)^\dagger\Omega \qquad D_\mu$$
$$f_{\mu\nu}^\pm = \frac{1}{2}\left(f_{\mu\nu} \pm f_{\mu\nu}^{(R)}\right) = \Omega^\dagger F_{\mu\nu}\Omega \pm F_{\mu\nu}^{(R)}$$

Building block

$$\Omega(\Pi) \equiv \begin{bmatrix} u(\Pi) & 0 \\ 0 & u^\dagger(\Pi) \end{bmatrix} \qquad u \to \sqrt{\mathfrak{g}_L U \mathfrak{g}_R^\dagger} = \mathfrak{g}_L\,u\mathfrak{h}^{-1} = \mathfrak{h}^{-1}\,u\mathfrak{g}_R$$

$$U(\Pi) \equiv u^2(\Pi) \longrightarrow \mathfrak{g}_L\mathbf{U}(\Pi)\mathfrak{g}_R^\dagger$$

QCD Chiral Lag

$$u_\mu \to \mathfrak{h}\,u_\mu\,\mathfrak{h}^{-1}$$

$$u_\mu = \mathrm{i}\left[u^\dagger\left(\partial_\mu - \mathrm{i}r_\mu\right)u - u\left(\partial_\mu - \mathrm{i}l_\mu\right)u^\dagger\right]$$

$$\chi_\pm = u^\dagger\chi u^\dagger \pm u\chi^\dagger u,$$
$$f_{\mu\nu}^\pm = uf_{\mu\nu}^{\mathrm{L}}u^\dagger \pm u^\dagger f_{\mu\nu}^{\mathrm{R}}u,$$

EW Chiral Lag

$$\mathbf{V}_\mu \to \mathfrak{g}_L\,\mathbf{V}\,\mathfrak{g}_L^{-1}$$

$$\mathbf{V}_\mu(x) = i\mathbf{U}(x)D_\mu\mathbf{U}(x)^\dagger$$

$$\mathbf{T} = \mathbf{U}\mathcal{T}_R\mathbf{U}^\dagger \quad \longrightarrow \quad \mathfrak{g}_L\mathbf{T}\mathfrak{g}_L^\dagger \qquad \hat{W}_{\mu\nu} \longrightarrow \mathfrak{g}_L\hat{W}_{\mu\nu}\mathfrak{g}_L^\dagger$$

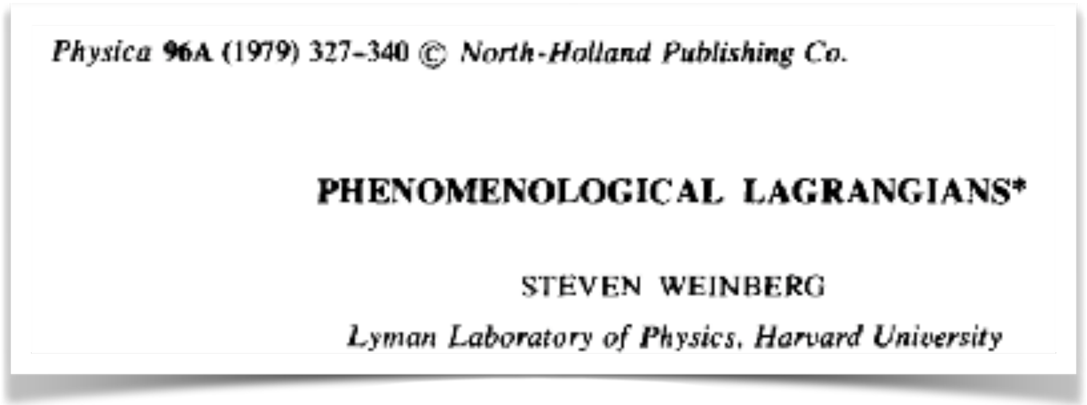
$$\mathbf{Y} = \mathbf{U}\mathcal{Y}_R\mathbf{U}^\dagger \quad \longrightarrow \quad \mathfrak{g}_L\mathbf{Y}\mathfrak{g}_L^\dagger \qquad \hat{B}_{\mu\nu} \longrightarrow \mathfrak{g}_R\hat{B}_{\mu\nu}\mathfrak{g}_R^\dagger.$$

QCD Chiral Perturbation Theory

[Hao Sun, Yi-Ning Wang, **J.H.Yu**, in préparation]

[Xuan-He Li, Chuan-Qiang Song, Hao Sun, **J.H.Yu**, in préparation]

Weinberg's Folk theorem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PHENOMENOLOGICAL LAGRANGIANS*

STEVEN WEINBERG

Lyman Laboratory of Physics, Harvard University

Weinberg developed a systematic procedure on effective Lagrangian in 1979

[Weinberg 1933 - 2021]

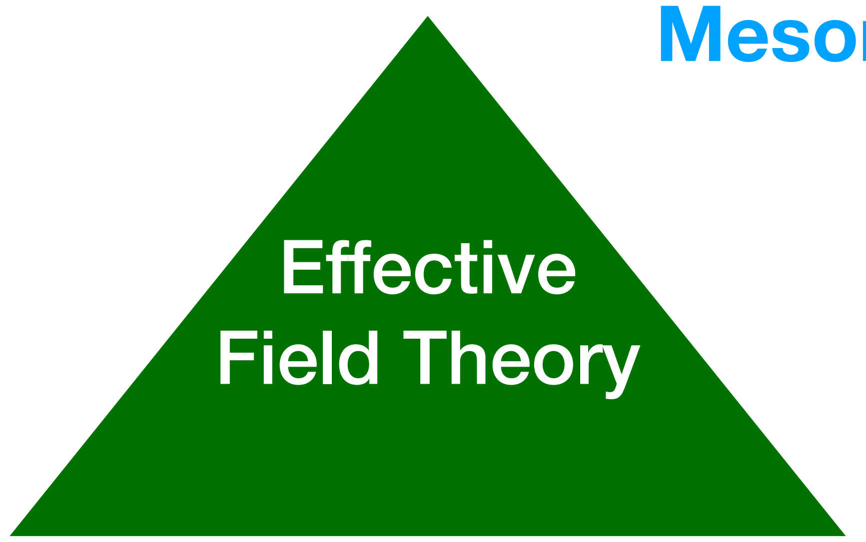
Weinberg's Folk theorem

a folk theorem: "if one writes down the most general possible Lagrangian, including *all* terms consistent with assumed symmetry principles, and then calculates matrix elements with this Lagrangian to any given order of perturbation theory, the result will simply be the most general possible S-matrix consistent with perturbative unitarity, analyticity, cluster decomposition, and the assumed symmetry properties."

$$\mathcal{L}_2 = \frac{F^2}{4}\mathrm{Tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) + \frac{F^2}{4}\mathrm{Tr}\left(\chi U^\dagger + U\chi^\dagger\right)$$

Proper Degree of freedom

Meson and baryon, external source



Global and local symmetry

SU(2) x SU(2) / SU(2)

Power Counting scheme

$$\mathcal{L}_{\mathrm{eff}} = \mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_6 + \cdots$$

Weinberg power counting

Based on naive dimensional analysis (NDA) $p^2/(4\pi f)^2$

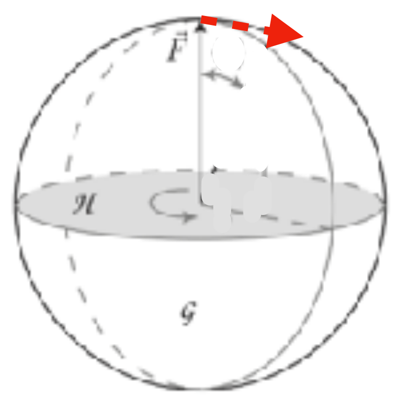
$$\mathcal{L}_0 = \frac{f^2}{4}\mathrm{Tr}\,\partial U^\dagger \partial U = \mathrm{Tr}\,\partial\pi\partial\pi + \frac{1}{3f^2}\mathrm{Tr}[\partial\boldsymbol{\pi},\boldsymbol{\pi}]^2 + \dots$$

$T =$ 

$$\mathcal{L}_{2,4\pi} \simeq \frac{p^2\pi^4}{f^2} \qquad 2\times\ \mathcal{L}_{2,4\pi} \simeq \frac{p^4\pi^4}{f^4}\int\frac{d^4k}{(2\pi)^4}\frac{1}{(k^2)^2} \qquad \mathcal{L}_{4,4\pi}$$

$$\simeq \frac{p^4\pi^4}{f^4}\frac{1}{(4\pi)^2}\log(\Lambda^2_{\chi SB}/\kappa^2)$$

Curved field space



NDA

$$\mathcal{L} = \Lambda^2 f^2\left[\frac{1}{4\Lambda^2}\mathrm{Tr}\,\partial_\mu U^\dagger\partial^\mu U + \frac{1}{\Lambda^4}\left(\mathrm{Tr}\left(\partial_\mu U^\dagger\partial^\mu U\right)\right)^2 + \cdots\right]$$

$$\Lambda \sim 4\pi f_\pi$$

Up to given order, only finite # of LEC are needed to renormalize the EFT

Naive dimensional Analysis

$$f^2\Lambda^2\left[\frac{\partial}{\Lambda}\right]^{N_p}\left[\frac{\phi}{f}\right]^{N_\phi}\left[\frac{A}{f}\right]^{N_A}\left[\frac{\psi}{f\sqrt{\Lambda}}\right]^{N_\psi}\left[\frac{g}{4\pi}\right]^{N_g}\left[\frac{y}{4\pi}\right]^{N_y}\left[\frac{\lambda}{16\pi^2}\right]^{N_\lambda}$$

Weinberg power counting

$$D = 2 + 2N = 2 + 2\left(L + \sum_d \frac{d-2}{2}V_d\right)$$

V_d = # vertices with d derivatives

I = # internal lines

L = # loops

Building blocks in ChPT

Two kinds of parametrizations $\mathcal{G} = SU(2)_L \times SU(2)_R \to \mathcal{H} = SU(2)_V$ $\mathcal{L} = \mathcal{L}^0_{\rm QCD} + \bar{q}\gamma^\mu(v_\mu + \gamma_5 a_\mu)q - \bar{q}(s - i\gamma_5 p)q$

$$\Omega(\Pi) \equiv \begin{bmatrix} u(\Pi) & 0 \\ 0 & u^\dagger(\Pi) \end{bmatrix} \to \begin{pmatrix} \mathfrak{g}_L & 0 \\ 0 & \mathfrak{g}_R \end{pmatrix} \begin{pmatrix} u(\Pi) & 0 \\ 0 & u^\dagger(\Pi) \end{pmatrix} \begin{pmatrix} \mathfrak{h}^{-1} & 0 \\ 0 & \mathfrak{h}^{-1} \end{pmatrix} \qquad u \to \sqrt{\mathfrak{g}_L U \mathfrak{g}_R^\dagger} = \mathfrak{g}_L\, u \mathfrak{h}^{-1} = \mathfrak{h}^{-1} u \mathfrak{g}_R$$

Symmetric coset (global)

$$U(\Pi) \equiv u^2(\Pi) \longrightarrow \mathfrak{g}_L \mathrm{U}(\Pi) \mathfrak{g}_R^\dagger$$

External source

$$\chi = 2B_0(s + ip)$$

$$f^R_{\mu\nu} \equiv \partial_\mu r_\nu - \partial_\nu r_\mu - i[r_\mu, r_\nu]$$

$$f^L_{\mu\nu} \equiv \partial_\mu l_\nu - \partial_\nu l_\mu - i[l_\mu, l_\nu]$$

Covariant derivative and chiral connection

$$D_\mu X \equiv \partial_\mu X - i r_\mu X + i X\, l_\mu$$

$$U, D_\mu, \chi, f^L_{\mu\nu} U, U f^R_{\mu\nu}$$

Transform under local symmetry $X \to h\,X\,h^\dagger$

$$u_\mu = \mathrm{i}\left[u^\dagger\left(\partial_\mu - \mathrm{i}r_\mu\right)u - u\left(\partial_\mu - \mathrm{i}l_\mu\right)u^\dagger\right]$$

$$u_\mu \to \mathfrak{h}\, u_\mu\, \mathfrak{h}^{-1} \qquad B \to \mathfrak{h}\, B\, \mathfrak{h}^{-1}$$

External source

$$\chi_\pm = u^\dagger \chi u^\dagger \pm u \chi^\dagger u\,,$$

$$f^\pm_{\mu\nu} = u f^{\rm L}_{\mu\nu} u^\dagger \pm u^\dagger f^{\rm R}_{\mu\nu} u\,,$$

Covariant derivative and chiral connection

$$D_\mu X = \partial_\mu X + [\Gamma_\mu, X] \qquad \Gamma_\mu = \frac{1}{2}\left[u^\dagger\left(\partial_\mu - \mathrm{i}r_\mu\right)u + u\left(\partial_\mu - \mathrm{i}l_\mu\right)u^\dagger\right]$$

$$[D_\mu, D_\nu]\, A = \frac{1}{4}\left[[u_\mu, u_\nu], A\right] - \frac{\mathrm{i}}{2}\left[f^+_{\mu\nu}, A\right]$$

$$\Gamma^{\mu\nu} = \nabla^\mu \Gamma^\nu - \nabla^\nu \Gamma^\mu - [\Gamma^\mu, \Gamma^\nu] = \frac{1}{4}[u^\mu, u^\nu] - \frac{i}{2} f_+^{\mu\nu}$$

$$u_\mu, D_\mu, \chi^\pm, f^\pm$$

Operator redundancies

Equation of motion (field redefinition)

$$D_\mu u^\mu = \chi_- - \frac{1}{3}\langle \chi_- \rangle$$

$$i\gamma^\mu D_\mu N = (M - \frac{g_A}{2}\gamma^5\gamma^\mu u_\mu)N\,,$$
$$-iD_\mu \overline{N}\gamma_\mu = \overline{N}(M - \frac{g_A}{2}\gamma^5\gamma^\mu u_\mu)\,.$$

$$(\overline{N}D^\mu u_\mu \Gamma N) \to 0\,,$$
$$(\overline{N}\gamma^\mu \overleftrightarrow{D}_\mu N) \to (\overline{N}(M - \frac{g_A}{2}\gamma^5\gamma^\mu u_\mu)N)\,,$$
$$(\overline{N}\gamma^5\gamma^\mu \overleftrightarrow{D}_\mu N) \to (\overline{N}\gamma^5(M - \frac{g_A}{2}\gamma^5\gamma^\mu u_\mu)N)\,,$$

Integration by part (momentum conservation)

$$D^{\mu_1}D^{\mu_2}\ldots D^{\mu_n}(\overline{N}\Gamma\Pi N) \to (\overline{N}\Gamma D^{\mu_1}D^{\mu_2}\ldots D^{\mu_n}\Pi N)\,,$$

Ferz identity (Schouten identity)

$$(\overline{N}^\alpha \Gamma_{1\alpha\beta}N^\lambda)(\overline{N}^\rho \Gamma_{2\rho\lambda}N^\beta) \to (\overline{N}^\alpha \Gamma_{3\alpha\beta}N^\beta)(\overline{N}^\rho \Gamma_{4\rho\lambda}N^\lambda)\,,$$

$$(\overline{N}^i \Gamma_1 \tau^I{}_{il}N^l)(\overline{N}^k \Gamma_2 \tau^I{}_{kj}N^j) \to (\overline{N}^i \Gamma_1 N_j)(\overline{N}^j \Gamma_2 N_i) - (\overline{N}^i \Gamma_1 N_i)(\overline{N}^j \Gamma_2 N_j)\,.$$

Cayley-Hamilton relation (trace basis)

$$\begin{aligned}&-\langle AD\rangle\langle BC\rangle - \langle AC\rangle\langle BD\rangle - \langle AB\rangle\langle CD\rangle\\ &+\langle ABCD\rangle + \langle ACBD\rangle + \langle ABDC\rangle + \langle ACDB\rangle + \langle ADBC\rangle + \langle ADCB\rangle\\ &=0\,,\end{aligned}$$

$$\begin{aligned}&T_1 = \langle AC\rangle\langle BD\rangle\,, \quad T_2 = \langle AB\rangle\langle CD\rangle\,,\\ &T_3 = \langle ABCD\rangle\,, \quad T_4 = \langle ABDC\rangle\,, \quad T_5 = \langle ACBD\rangle\\ &T_6 = \langle ACDB\rangle\,, \quad T_7 = \langle ADBC\rangle\,, \quad T_8 = \langle ADCB\rangle\,.\end{aligned}$$

Operator as spinor Young tensor

[Li, Ren, Xiao, **Yu**, Zheng, 2201.04639]
[Li, Ren, Xiao, **Yu**, Zheng, 2012.09188]
[Li, Ren, Xiao, **Yu**, Zheng, 2007.07899]
[Li, Ren, Shu, Xiao, **Yu**, Zheng, 2005.00008]

Operator

$$(\psi_1\psi_2)(D^\mu F_{R3}^{\nu\lambda})(D_\mu F_{R4\nu\lambda})$$

$$D^{r_i}\phi_i \Leftrightarrow \lambda_i^{r_i}\tilde{\lambda}^{i,r_i}, \quad D^{r_i-1}F_{L/R\,i} \Leftrightarrow \lambda_i^{r_i\pm1}\tilde{\lambda}^{i,r_i\mp1},$$
$$D^{r_i-1/2}\psi_i^{(\dagger)} \Leftrightarrow \lambda_i^{r_i\pm1/2}\tilde{\lambda}^{i,r_i\mp1/2},$$

↓

Spinor Tensor → **Symmetrize indices**

$$\mathcal{O}_N^{(d)} = (\epsilon^{\alpha_i\alpha_j})^{\otimes n}(\tilde{\epsilon}_{\dot{\alpha}_i\dot{\alpha}_j})^{\otimes\tilde{n}}\prod_{i=1}^N (D^{r_i-|h_i|}\Psi_{i,a_i})^{\dot{\alpha}_i^{r_i+h_i}}_{\alpha_i^{r_i-h_i}}$$

$$D_{[\alpha\dot{\alpha}}D_{\beta]\dot{\beta}} = D_\mu D_\nu \sigma^\mu_{[\alpha\dot{\alpha}}\sigma^\nu_{\beta]\dot{\beta}} = -D^2\epsilon_{\alpha\beta}\epsilon_{\dot{\alpha}\dot{\beta}} + \frac{i}{2}[D_\mu, D_\nu]\epsilon_{\alpha\beta}(\bar{\sigma}^{\mu\nu})_{\dot{\alpha}\dot{\beta}},$$
$$D_{[\alpha\dot{\alpha}}\psi_{\beta]} = D_\mu\sigma^\mu_{[\alpha\dot{\alpha}}\psi_{\beta]} = -\epsilon_{\alpha\beta}(D_\mu\sigma^\mu\psi)_{\dot{\alpha}},$$
$$D_{[\alpha\dot{\alpha}}F_{L\beta]\gamma} = \frac{i}{2}D_\mu F_{L\nu\rho}\sigma^\mu_{[\alpha\dot{\alpha}}\sigma^{\nu\rho}_{\beta]\gamma} = iD^\mu F_{L\mu\nu}\epsilon_{\alpha\beta}\sigma^\nu_{\gamma\dot{\alpha}},$$

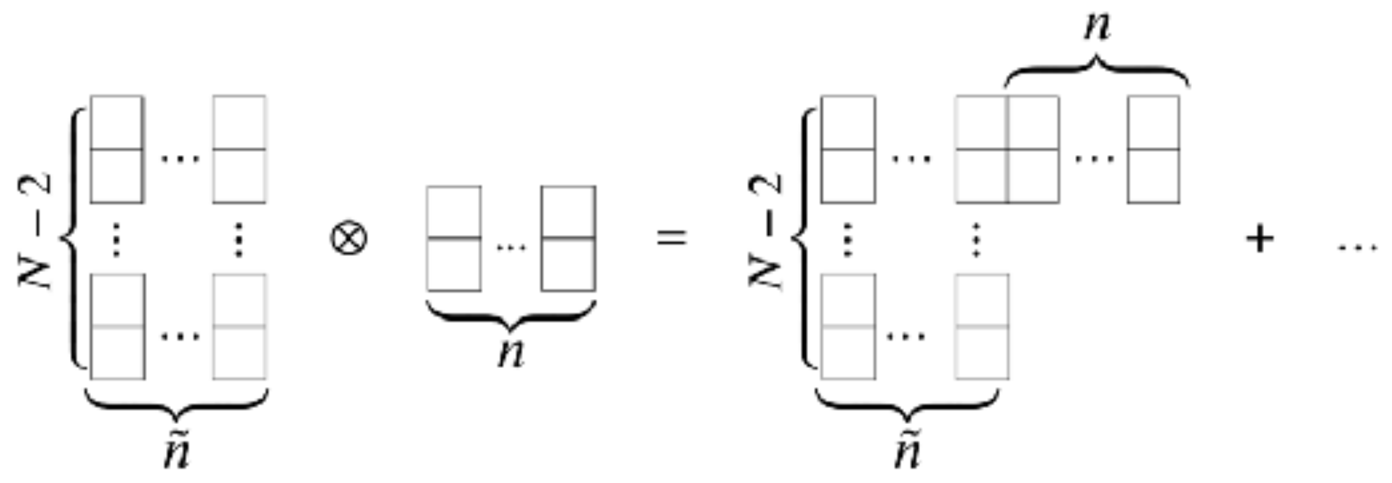
$$(D\psi)_{\alpha\beta\dot{\alpha}} = -\frac{1}{2}\epsilon_{\alpha\beta}(\not{D}\psi)_{\dot{\alpha}} + \frac{1}{2}(D\psi)_{(\alpha\beta)\dot{\alpha}}$$

$$(\epsilon^{\alpha_i\alpha_j})^{\otimes n}(\tilde{\epsilon}_{\dot{\alpha}_i\dot{\alpha}_j})^{\otimes\tilde{n}}\prod_{i=1}^N \lambda_i^{r_i-h_i}\tilde{\lambda}^{i,r_i+h_i}$$

↓

SL(2,C) x SU(N)

$$\lambda_i \to \sum_j U_i^j\lambda_j, \quad \tilde{\lambda}^i \to \sum_k U^{\dagger i}{}_k\tilde{\lambda}^k,$$



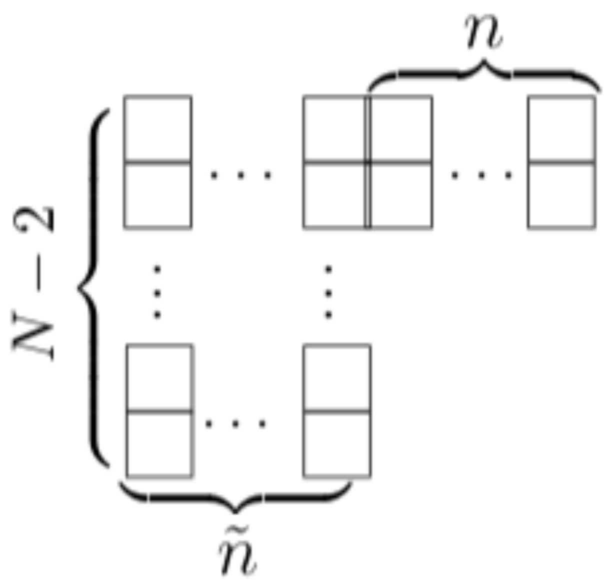
↓

Momentum conservation

$$\delta^{(4)}\left(\sum_{i=1}^N \lambda_i\tilde{\lambda}_i\right)$$

$$\sum_i \lambda_i\tilde{\lambda}^i \to \sum_i\sum_j\sum_k U_i^j U^{\dagger i}{}_k\lambda_j\tilde{\lambda}^k = \sum_j \lambda_j\tilde{\lambda}^j$$

SSYT

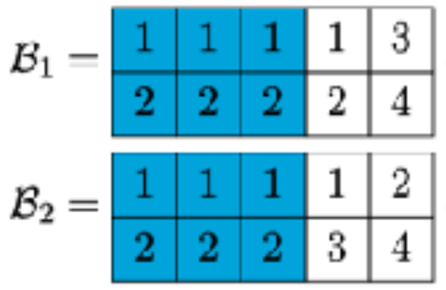


$$\begin{array}{|c|}\hline i \\ \hline j \\ \hline\end{array} \Leftrightarrow \langle ij\rangle$$

$$\{\overbrace{1,\ldots,1}^{\#1},\overbrace{2,\ldots,2}^{\#2},\ldots,\overbrace{N,\ldots,N}^{\#N}\}$$

$$\#i = \tilde{n} - 2h_i$$

→ **On-shell Amplitude**



$$\mathcal{B}_1 \sim [34]^2\langle12\rangle\langle34\rangle = (\psi_1\psi_2)(D^\mu F_{R3}^{\nu\lambda})(D_\mu F_{R4\nu\lambda}),$$
$$\mathcal{B}_2 \sim [34]^2\langle13\rangle\langle24\rangle = (\psi_1\sigma^{\rho\lambda}\psi_2)(D_\lambda F_{R3}^{\mu\nu})(D_\rho F_{R4\mu\nu}),$$

On-shell Amplitude correspondence

Adler zero condition for Goldstone Boson

Chiral symmetry (PCAC)

$$\alpha \to \beta \quad \xleftrightarrow{\text{At low energy}} \quad \alpha + n_1\pi \to \beta + n_2\pi$$

Goldberger-Trieman, Callan-Trieman, Adler-Weisberger, etc

Adler Zero condition [Adler, 1965]

$$T(\alpha + \phi(p), \beta) = -\frac{p_\mu}{F} R^\mu(p) \xrightarrow{p \to 0} 0$$

Amplitude (soft limit of external leg s)

$$\mathcal{A}(1,\ldots,N,s) \xrightarrow{p_s \to 0} \begin{cases} \left(S^{(0)}(s) + S^{(\text{sub})}(s)\right)\mathcal{A}(1,\ldots,N) \\ \mathcal{O}(p_s^\sigma) \qquad \text{for Goldstone Boson} \end{cases}$$

$$\{-1/2, -1/2, 1, 0, 0\} \quad \begin{array}{|c|c|c|c|}\hline 1&1&1&4\\\hline 2&2&2&5\\\hline 4&5\\\cline{1-2}\end{array}, \quad \begin{array}{|c|c|c|c|}\hline 1&1&1&2\\\hline 2&2&5&5\\\hline 4&4\\\cline{1-2}\end{array}, \quad \begin{array}{|c|c|c|c|}\hline 1&1&1&2\\\hline 2&2&4&4\\\hline 5&5\\\cline{1-2}\end{array}, \quad \begin{array}{|c|c|c|c|}\hline 1&1&1&2\\\hline 2&2&4&5\\\hline 4&5\\\cline{1-2}\end{array},$$

Expand the soft-limit amplitude into the SSYT basis

Put constraints on the SSYT basis

$$\mathcal{B}_i^{(N)}(p_\pi \to 0) = \sum_{l=1}^{d_N} \mathcal{K}_{il} \mathcal{B}_l^{(N)}$$

$$\begin{array}{|c|c|c|c|}\hline 1&1&1&4\\\hline 2&2&2&5\\\hline 4&5\\\cline{1-2}\end{array}, \quad \begin{array}{|c|c|c|c|}\hline 1&1&1&2\\\hline 2&2&4&5\\\hline 4&5\\\cline{1-2}\end{array},$$

[Sun, Xiao, **Yu**, 2210.14939]

[Sun, Xiao, **Yu**, 2206.07722]

[Low, Shu, Xiao, Zheng, 2022]

Chiral symmetry breaking: spurion technique

Chiral Lagrangian for QCD and beyond

$$\mathscr{L}_{\rm EFT} = \mathscr{L}_{p^2} + \mathscr{L}_{p^3} + \mathscr{L}_{p^4} + \mathscr{L}_{p^5} + \mathscr{L}_{p^6} + \ldots$$

Pure Meson sector

- p2 order: [Weinberg, 1979]
- p4 order: [Gasser, Leutwyler, 1984, 1985]; [Fearing, Scherer 1994]
- p6 orde: [Bijnens, Colangelo, Ecker, 1999]; [Jiang, Ge, Wang, 2014] CP-odd
- p8 orde: [Bijnens, Hermansson, Wang, 2018]; [Sun, Wang, **Yu**, in préparation] C+P+ C+P- C-P+ C-P-

Meson-Baryon sector

- SU(2) p3: [Krause, 1990]; [Ecker, 1994]
- SU(2) p4: [Fettes, Meisner, Mojzis, Steininger, 2000]
- p3 order: [Oller, Verbeni, Prades, 2006]; [Frink, Meisner, 2006]
- p4 order: [Jiang, Chen, Liu, 2017]; [Li, Song, Sun, **Yu**, in préparation] C+P+ C+P- C-P+ C-P-
- p5 order: [Song, Sun, **Yu**, in préparation]

Classifying operators by CP symmetry

Parity and charge conjugation are the outer automorphism of the Lorentz and internal symmetries

$$SO(4) \rtimes \{1, \mathcal{P}\} = O(4) = SO(4) \sqcup O_-(4)$$

$$I \rtimes \{1, \mathcal{C}\} = I \sqcup I_-.$$

[Hao Sun, Yi-Ning Wang, **J.H.Yu**, 2211.11598]

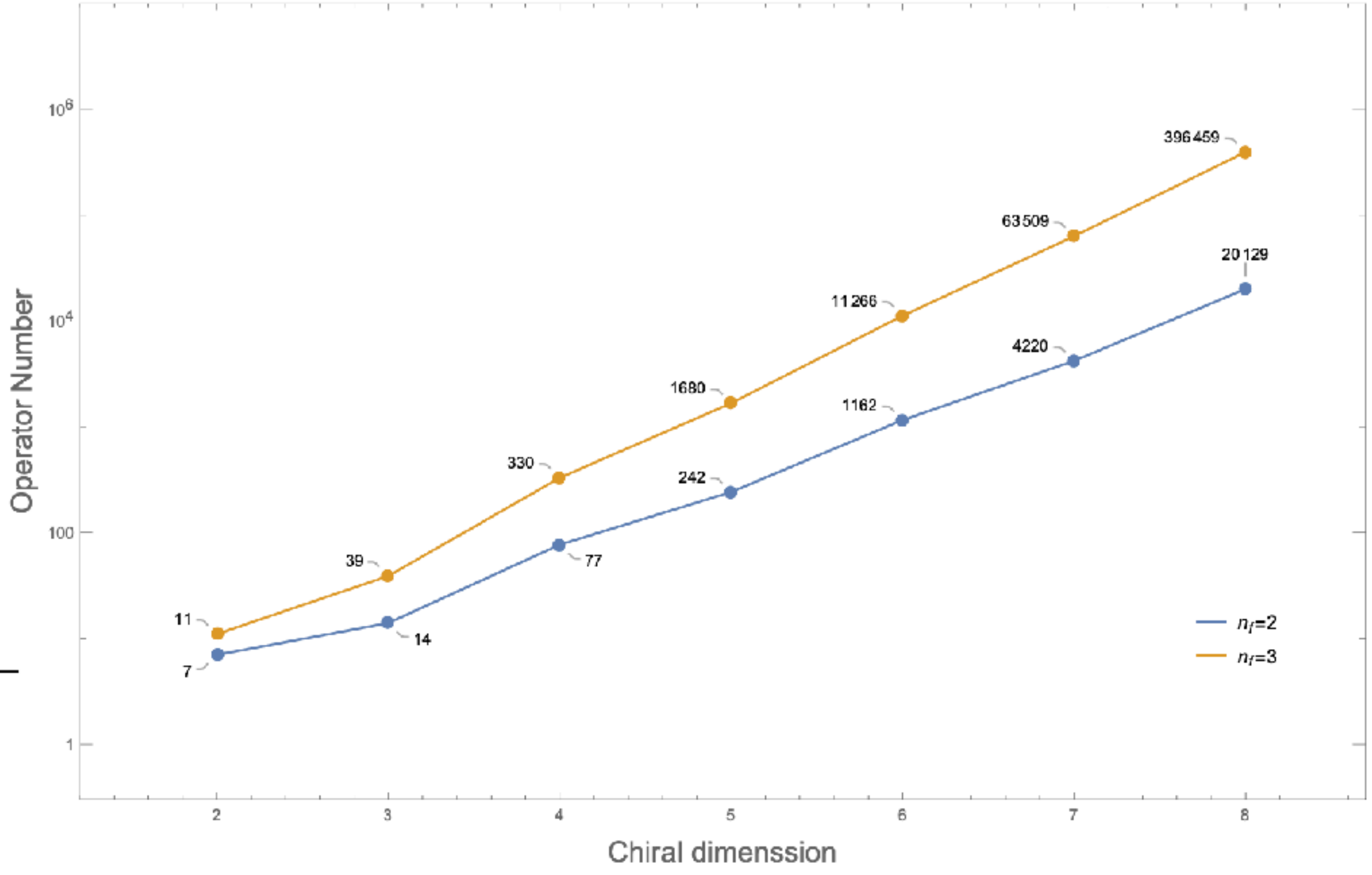
[Sun, Wang, **Yu**, in préparation]

Hilbert series

$$\mathcal{H}^{C^\pm P^\pm}(D, \phi) \equiv \int_G d\mu(g) \left(\sum_{C^\pm P^\pm} \frac{\text{PE}(\phi \chi_{R_\phi}(D, g_{\{C^\pm P^\pm\}}))}{P(D, g_{\{C^\pm P^\pm\}})} \right)$$

Group Branch	$SO(4)$	$O_-(4)$
integral variable	$a_+ = a = (a_1, a_2)$	$a_- = a_1$
reparametrization	$\bar{a}_+ = a$	$\bar{a}_- = (a_1, 1)$
Haar measure	$d\mu_{SO(4)}(a)$	$d\mu_{Sp(2)}(a_-)$
Group Branch	$U(1)$	$U_-(1)$
integral variable	$x_+ = x$	$x_- = x$
reparametrization	$\bar{x}_+ = x$	$\bar{x}_- = x$
Haar measure	$d\mu_{U(1)}(x)$	$d\mu_{U(1)}(x)$
Group Branch	$SU(2)$	$SU_-(2)$
integral variable		
reparametrization		
Haar measure		
Group Branch	$SU(N)$	
integral variable	$z = (z_1, ...,$	
reparametrization	$\bar{z}_+ = z$	
Haar measure	$d\mu_{SU(N)}($	

Fields	$SU(N_f)_V$	Intrinsic Parity	Charge Conjugation	Chiral Dim
u_μ	adjoint	$-$	u_μ^T	1
$\Sigma_\pm$	adjoint	$\pm$	$\Sigma_\pm^T$	2
$\langle\Sigma_\pm\rangle$	singlet	$\pm$	$\langle\Sigma_\pm\rangle$	2
$f_{\pm\mu\nu}$	adjoint	$\pm$	$\mp f_{\pm\mu\nu}^T$	2



Operator Bases for Generic EFT up to All Order

Amplitude Basis Construction for Effective Field Theory [Li, Ren, Xiao, **Yu**, Zheng, 2201.04639]

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Welcome to the HEPForge Project: ABC4EFT

This is the website for the Mathematica package: Amplitude Basis Construction for Effective Field Th

Package

This package has the following features:

- It provides a general procedure to construct the independent and complete operator bases for generic L invariant effective field theory, given any kind of gauge symmetry and field content, up to any mass dim
- Various operator bases have been systematically constructed to emphasize different aspects: operator independence (y-basis), flavor relation (p-basis) and conserved quantum number (j-basis).
- It provides a systematic way to convert any operator into our on-shell amplitude basis and the basis cor can be easily done.

Authors

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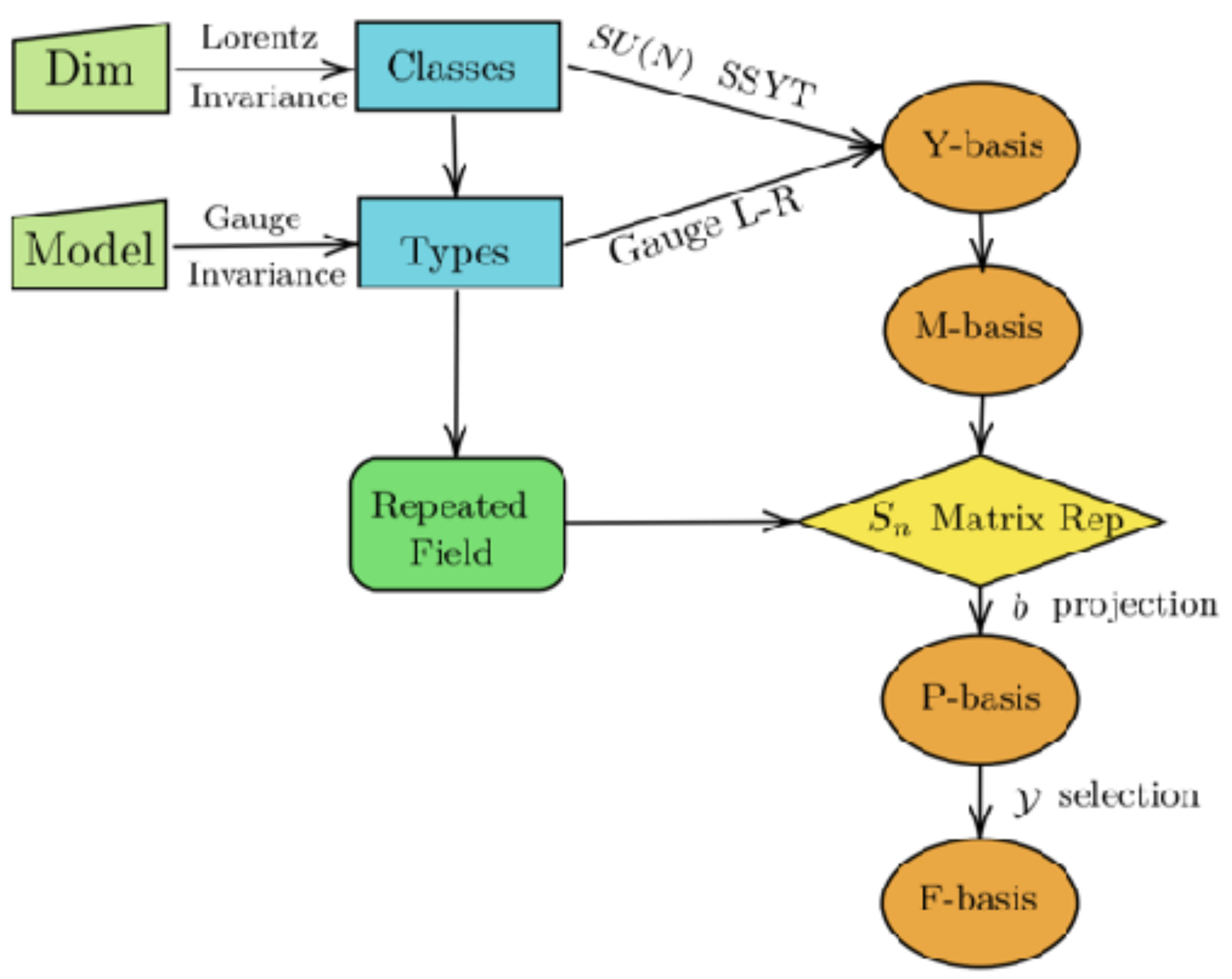
- Jiang-Hao Yu (professor at ITP-CAS)
- Yu-Hui Zheng (5th-year graduate student at ITP-CAS)

https://abc4eft.hepforge.org/



Fully Automatic

- Standard model EFT
- Low energy EFT
- Dark matter EFT
- Sterile neutrino EFT
- Gravity EFT
- Axion EFT
- …

Electroweak Chiral Lagrangian

[Hao Sun, Ming-Lei Xiao, **J.H.Yu**, 2206.07722]

[Hao Sun, Ming-Lei Xiao, **J.H.Yu**, 2210.14939]

[Hao Sun, Yi-Ning Wang, **J.H.Yu**, 2211.11598]

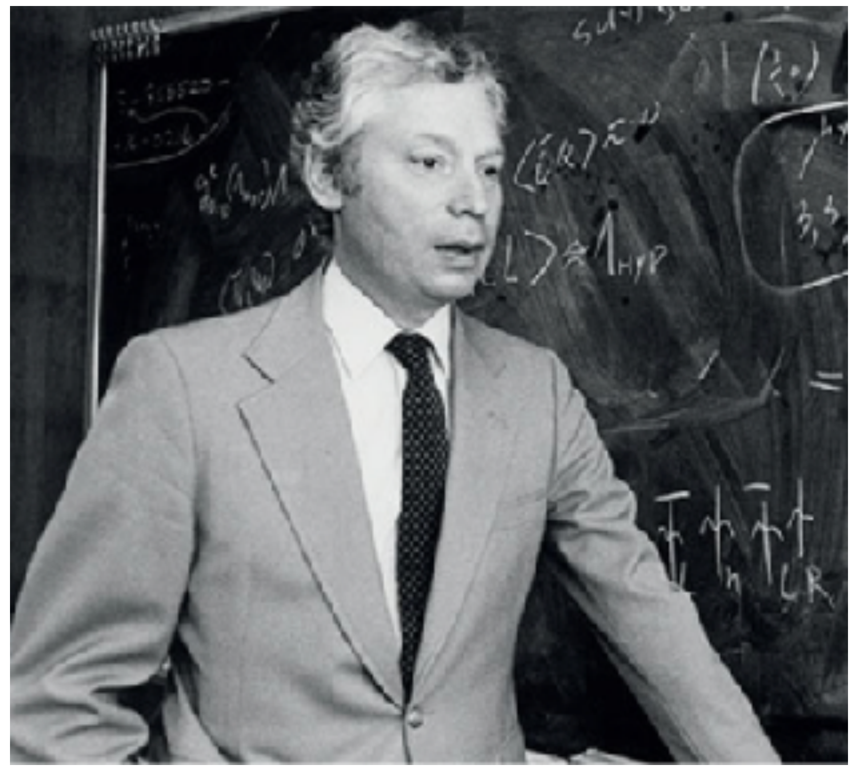
Weinberg's Standard Model

Electroweak unification inspired by QCD chiral dynamics

A MODEL OF LEPTONS*

Steven Weinberg†
Laboratory for Nuclear Science and Physics Department, Massachusetts Institute of Technology, Cambridge, Massachusetts

(Received 17 October 1967)



[Weinberg 1933 - 2021]

My starting point in 1967 was the old aim, going back to Yang and Mills, of developing a gauge theory of the strong interactions, based on the symmetry group $SU(2) \times SU(2)$

Then it suddenly occurred to me that this was a perfectly good sort of theory, but I was applying it to the wrong kind of interaction. The right place to apply these ideas was not to the strong interactions, but to the weak and electromagnetic interactions.

Weinberg 2004

$SU(2)_L \otimes SU(2)_R \to SU(2)_{L+R}$ **Symmetry**

$$\Phi = \begin{pmatrix} \Phi^+ \\ \Phi^0 \end{pmatrix} = \frac{1}{\sqrt{2}}(v+H)U(\vec{\varphi})\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\mathcal{L}_\Phi = (D_\mu \Phi)^\dagger D^\mu \Phi - \lambda\left(|\Phi|^2 - \frac{v^2}{2}\right)^2$$

$$\Sigma \equiv (\Phi^c, \Phi) = \begin{pmatrix} \Phi^{0*} & \Phi^+ \\ -\Phi^- & \Phi^0 \end{pmatrix} = \frac{1}{\sqrt{2}}(v+H)U(\vec{\varphi})$$

$$\mathcal{L}_\Phi = \frac{1}{2}\mathrm{Tr}\left[(D^\mu\Sigma)^\dagger D_\mu \Sigma\right] - \frac{\lambda}{4}\left(\mathrm{Tr}\left[\Sigma^\dagger\Sigma\right] - v^2\right)^2$$

Gell-mann-Levi model

$$\mathcal{L}_\Phi = \frac{v^2}{4}\left(1+\frac{h}{v}\right)^2 \langle D_\mu \mathbf{U}^\dagger D^\mu \mathbf{U}\rangle + \frac{1}{2}\left(\partial_\mu h \partial^\mu h - m_h^2 h^2\right) - \frac{m_h^2}{2v}h^3 - \frac{m_h^2}{8v^2}h^4$$

Custodial symmetry:

$$\mathcal{L}_2 = \frac{v^2}{4}\mathrm{Tr}\left(D_\mu U^\dagger D^\mu U\right) \xrightarrow{U=1} \mathcal{L}_2 = M_W^2 W_\mu^\dagger W^\mu + \frac{1}{2}M_Z^2 Z_\mu Z^\mu$$

$$\mathrm{M_W} = \mathrm{M_Z}\cos\theta_W = \tfrac{1}{2}\,\mathrm{g\,v}$$

Electroweak chiral Lagrangian

Standard Model Effective Field Theory → (Matching / Running) → Low Energy Effective Field Theory

approximate custodial symmetry
SU(2) x SU(2)

$$\Sigma \equiv (\Phi^c, \Phi) = \begin{pmatrix} \Phi^{0*} & \Phi^+ \\ -\Phi^- & \Phi^0 \end{pmatrix} \to g_L \, \Sigma \, g_R^\dagger$$

$$\langle \Sigma \rangle = \begin{pmatrix} v & 0 \\ 0 & v \end{pmatrix} \neq 0$$

approximate chiral symmetry
SU(3) x SU(3)

$$\mathbf{q}_L \to g_L \, \mathbf{q}_L \,, \quad \mathbf{q}_R \to g_R \, \mathbf{q}_R \,,$$

$$\langle 0 | (\bar{\mathbf{q}}_L \mathbf{q}_R + \bar{\mathbf{q}}_R \mathbf{q}_L) | 0 \rangle \neq 0$$

Electroweak Chiral Lagrangian

$$\mathbf{\Phi} \equiv \frac{1}{\sqrt{2}} \vec{\sigma} \cdot \vec{\varphi} = \begin{pmatrix} \frac{1}{\sqrt{2}} \varphi^0 & \varphi^+ \\ \varphi^- & -\frac{1}{\sqrt{2}} \varphi^0 \end{pmatrix}$$

SM fields and Goldstone

QCD Chiral Lagrangian

$$\mathbf{\Phi} \equiv \frac{\vec{\lambda}}{\sqrt{2}} \vec{\phi} = \begin{pmatrix} \frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}} \eta \end{pmatrix}$$

meson and baryon

SM Fermion masses from Higgs VEV

Baryon masses around cutoff scale from Trace anomaly

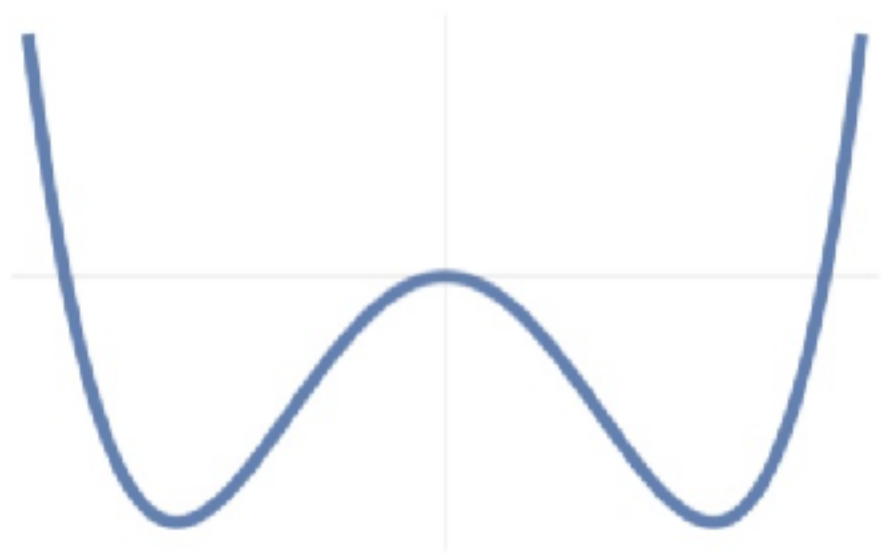
Which EFT? SMEFT or HEFT

Does the SMEFT cover all kinds of new physics scenarios?

[Agrawal, Saha, Xu, **Yu**, Yuan, 1907.02078]

Depending on nature of the Higgs boson and decoupling feature of new particle

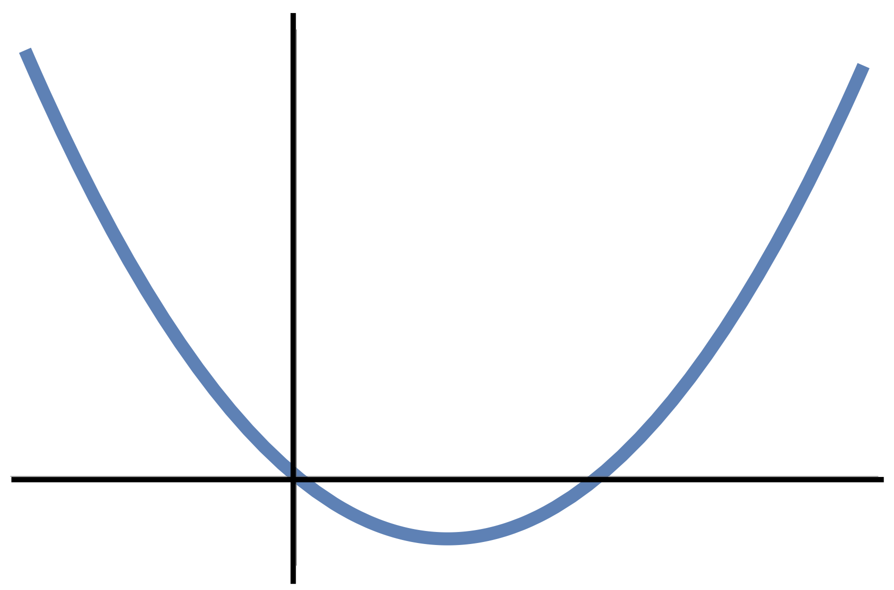
Landau-Ginzburg Higgs



$$V(\phi) = -m^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2$$

Fundamental particle

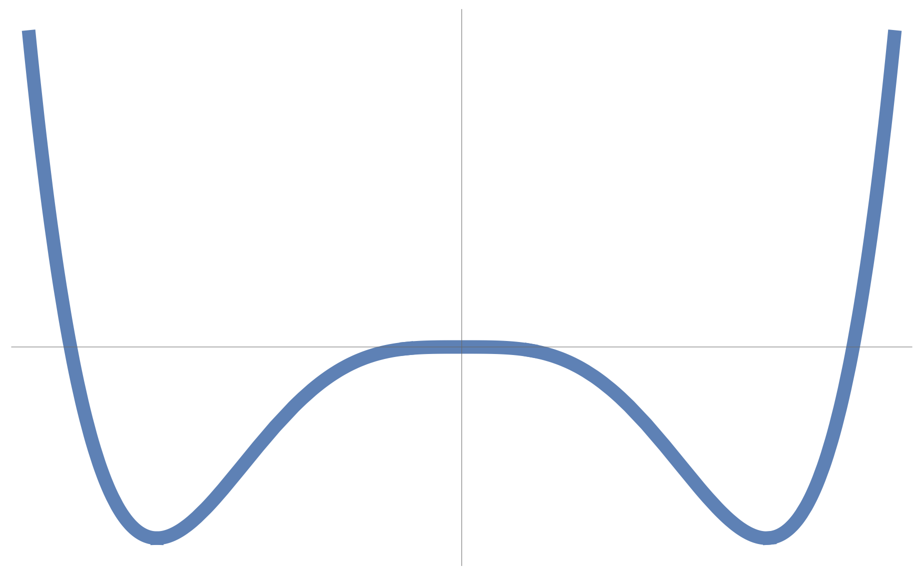
Tadpole-induced Higgs



$$V(\phi) = -\mu^3\sqrt{\phi^\dagger\phi} + m^2\phi^\dagger\phi$$

Partial Fundamental (condensate)

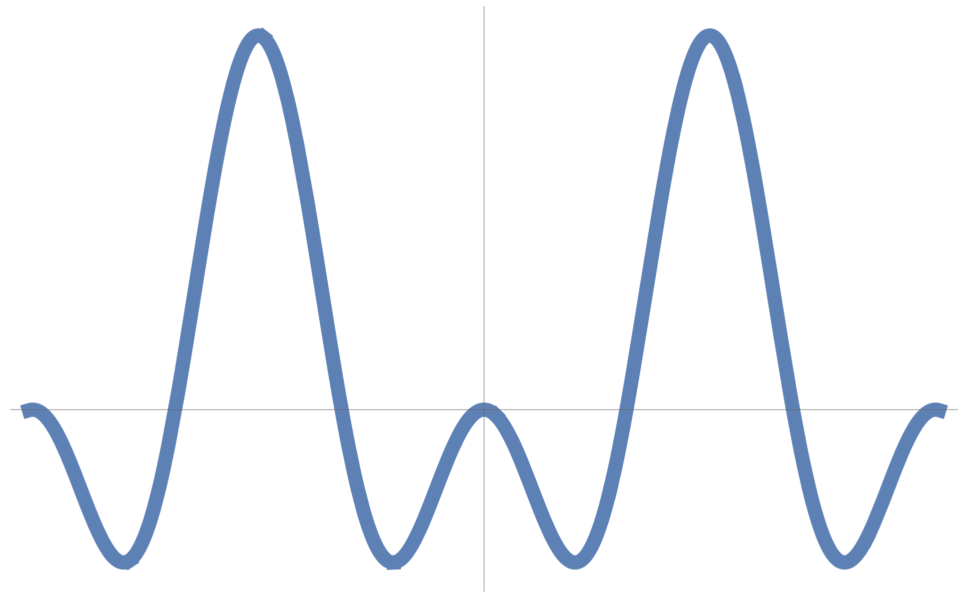
Coleman Weinberg Higgs



$$V(\phi) = \lambda(\phi^\dagger\phi)^2 + \epsilon(\phi^\dagger\phi)^2 \log\frac{\phi^\dagger\phi}{\mu^2}$$

Conformal particle

Pseudo-Goldstone Higgs



$$V(\phi) = -a\sin^2(\phi/f) + b\sin^4(\phi/f)$$

Composite particle

Also [Falkowski, Rattazzi 2019]

[Cohen, Craig, Lu, Sutherland, 2021]

Ingredients of electroweak chiral Lagrangian

Three ingredients: field, symmetry and power counting

building blocks	spinor-helicity	Lorentz group	$SU(2)_L$	$SU(3)_C$	d_χ
L_L	$L_{L\alpha}$	$(\frac{1}{2},0)$	Fundamental	Singlet	1
L_R	${L_R}^{\dot\alpha}$	$(0,\frac{1}{2})$	Fundamental	Singlet	1
Q_L	$Q_{L\alpha}$	$(\frac{1}{2},0)$	Fundamental	Fundamental	1
Q_R	${Q_R}^{\dot\alpha}$	$(0,\frac{1}{2})$	Fundamental	Fundamental	1
W_L	${W_L}^I_{\alpha\beta}\tau^I$	$(1,0)$	Adjoint	Singlet	2
W_R	${W_R}^{I\dot\alpha\dot\beta}\tau^I$	$(0,1)$	Adjoint	Singlet	2
G_L	$G_{L\alpha\beta}$	$(1,0)$	Singlet	Adjoint	2
G_R	${G_R}^{\dot\alpha\dot\beta}$	$(0,1)$	Singlet	Adjoint	2
B_L	$B_{L\alpha\beta}$	$(1,0)$	Singlet	Singlet	2
B_R	${B_R}^{\dot\alpha\dot\beta}$	$(0,1)$	Singlet	Singlet	2
$\mathbf{V}^\mu \sim D^\mu\Pi$	$(D\phi^I)_{\dot\alpha\beta}\tau^I$	$(\frac{1}{2},\frac{1}{2})$	Adjoint	Singlet	1
D^μ	$D_{\alpha\dot\beta}$	$(\frac{1}{2},\frac{1}{2})$	Singlet	Singlet	1
$\mathbf{T}$	$\mathbf{T}^T\tau^I$	$(0,0)$	Adjoint	Singlet	0

$$u_\mu = iu\,(D_\mu U)^\dagger\, u = -iu^\dagger D_\mu U u^\dagger \longrightarrow \mathfrak{g}_\mathcal{H} u_\mu \mathfrak{g}_\mathcal{H}^\dagger$$
$$f_\pm^{\mu\nu} = u^\dagger \hat{W}^{\mu\nu} u \pm u\hat{B}^{\mu\nu}u^\dagger \longrightarrow \mathfrak{g}_\mathcal{H} f_\pm^{\mu\nu} \mathfrak{g}_\mathcal{H}^\dagger$$
$$\mathcal{T} = u\mathcal{T}_R u^\dagger \longrightarrow \mathfrak{g}_\mathcal{H}\mathcal{T}\mathfrak{g}_\mathcal{H}^\dagger$$
$$u^\dagger\psi_L \longrightarrow \mathfrak{g}_\mathcal{H} u^\dagger\psi_L$$
$$u\psi_R \longrightarrow \mathfrak{g}_\mathcal{H} u\psi_R$$
$$\mathcal{Y} = u\mathcal{Y}_R u^\dagger \longrightarrow \mathfrak{g}_\mathcal{H}\mathcal{Y}\mathfrak{g}_\mathcal{H}^\dagger$$

$$\mathbf{V}_\mu(x) = i\mathbf{U}(x)D_\mu\mathbf{U}(x)^\dagger, \longrightarrow \mathfrak{g}_L\mathbf{V}_\mu\mathfrak{g}_L^\dagger$$
$$\hat{W}_{\mu\nu} \longrightarrow \mathfrak{g}_L\hat{W}_{\mu\nu}\mathfrak{g}_L^\dagger$$
$$\hat{B}_{\mu\nu} \longrightarrow \hat{B}_{\mu\nu}$$
$$\mathbf{T} = \mathbf{U}\mathcal{T}_R\mathbf{U}^\dagger \longrightarrow \mathfrak{g}_L\mathbf{T}\mathfrak{g}_L^\dagger$$
$$\psi_L \longrightarrow \mathfrak{g}_L\psi_L$$
$$\mathbf{U}\psi_R \longrightarrow \mathfrak{g}_L\mathbf{U}\psi_R$$
$$\mathbf{Y} = \mathbf{U}\mathcal{Y}_R\mathbf{U}^\dagger \longrightarrow \mathfrak{g}_L\mathbf{Y}\mathfrak{g}_L^\dagger$$

Chiral dimension (NDA)

$$d_\chi = d_i + k_i + \frac{F_i}{2} + V_i = 2L_i + 2.$$

$$\frac{p^2}{16\pi^2\mathbf{v}^2} \sim \frac{g^2}{(4\pi)^2}, \frac{y^2}{(4\pi)^2}, \frac{\lambda}{(4\pi)^2} \ll 1\,.$$

Spurion technique

The SU(2) spurion is introduced to parametrize custodial symmetry breaking

$$\tau^{K}{}^{i}_{l}\tau^{M}{}^{k}_{j}\epsilon^{IJM}\mathbf{T}^{J}\mathbf{T}^{K}W_{L}{}^{I}{}_{\mu\nu}(L_{Lp_i}\sigma_{\lambda}{}^{\nu}L^{\dagger}_{Rr}{}^{j})(Q_{Lsak}\sigma^{\mu\lambda}Q^{\dagger}_{Rt}{}^{al})$$
$$\tau^{J}{}^{k}_{l}\mathbf{T}^{I}\mathbf{T}^{J}W_{L}{}^{I}{}_{\mu\nu}(L_{Lp_i}\sigma^{\mu\nu}L^{\dagger}_{Rr}{}^{i})(Q_{Lsak}Q^{\dagger}_{Rt}{}^{al})$$

$$t_i \in \mathbf{2} \sim \square$$
$$\epsilon_{ij}t^j \in \overline{\mathbf{2}} \sim \square$$
$$t^I \tau^{Ik}_{i}\epsilon_{kj} \in \mathbf{3} \sim \square\square$$

$$\mathbf{T}^I \tau^{Ik}{}_i \epsilon_{kj} \in \boxed{i\;|\;j},$$

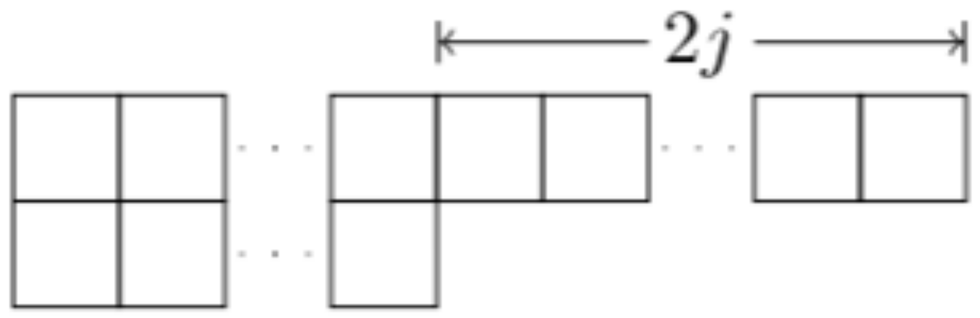
$$\mathbf{T}^{\{I_1}\ldots\mathbf{T}^{I_j\}} \in \text{spin } j$$

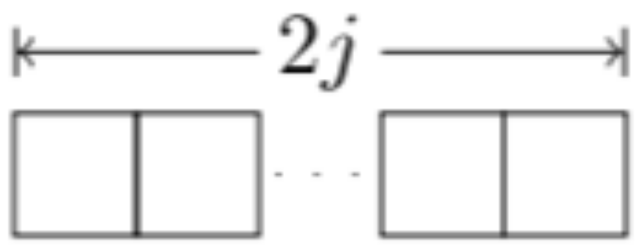
$$\begin{array}{ccccccc} \mathbf{T}^I\mathbf{T}^J & = & \mathbf{T}^2\delta^{IJ} & + & \mathbf{T}^{[I}\mathbf{T}^{J]} & + & \mathbf{T}^{(I}\mathbf{T}^{J)}, \\ \mathbf{3}\otimes\mathbf{3} & = & \mathbf{1} & + & \mathbf{3} & + & \mathbf{5}. \end{array}$$

Littlewood-Richarson rules

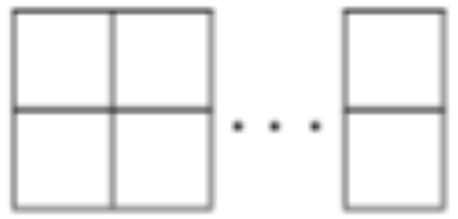
Symmetric highest weight

$$\epsilon^{IJK}\mathbf{T}^I\mathbf{T}^J A^K$$





Gauge Singlet

$SU(2) \sim$ 

[Sun, Xiao, **Yu**, 2206.07722]

Results at LO/NLO/NNLO

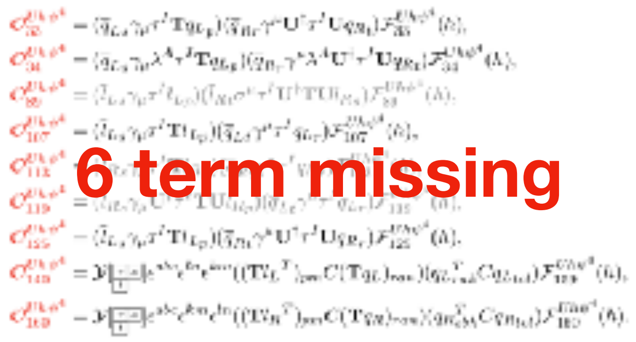
Compared with literatures, new 6 (9) operators found at NLO $\mathscr{L}_{\rm EFT} = \mathscr{L}_{p^2} + \mathscr{L}_{p^3} + \mathscr{L}_{p^4} + \mathscr{L}_{p^5} + \mathscr{L}_{p^6} + \ldots$

LO Lagrangian [Weinberg, 1979]

NLO bosonic [Appelquist, Bernard, 1980] [Longhitano, 1980, 1981] [Feruglio, 1993]

NLO 2-fermion [Buchalla, Cata, Krause, 2014]

NLO 4-fermion [Buchalla, Cata, Krause, 2014] [Pich, Rosell, Santos, Sanz-Cillero, 2015, 2018] [Sun, Xiao, **Yu**, 2206.07722]



NNLO Basis [Sun, Xiao, **Yu**, 2210.14939]

Classes	$\mathcal{N}_{\rm type}$	$\mathcal{N}_{\rm term}$	$\mathcal{N}_{\rm operator}$
UhD^4	$3+6+0+0$	15	15
X^2Uh	$6+4+0+0$	10	10
$XUhD^2$	$2+6+0+0$	8	8
X^3	$4+2+0+0$	6	6
ψ^2UhD	$4+8+0+0$	13(16)	$13{n_f}^2 \quad (16{n_f}^2)$
ψ^2UhD^2	$6+10+0+0$	60(80)	$60{n_f}^2 \quad (80{n_f}^2)$
ψ^2UhX	$7+7+0+0$	22(28)	$22{n_f}^2 \quad (28{n_f}^2)$
ψ^4	$12+24+4+8$	117(160)	$\frac{1}{4}{n_f}^2(31-6n_f+335{n_f}^2) \quad ({n_f}^2(9-2n_f+125{n_f}^2))$
Total	123	261(313)	$\frac{335{n_f}^4}{4}-\frac{3{n_f}^3}{2}+\frac{411{n_f}^2}{4}+39 \quad (39+133{n_f}^2-2{n_f}^2-2{n_f}^3+125{n_f}^4)$ $\mathcal{N}_{\rm operatrs}(n_f=1)=224(295), \quad \mathcal{N}_{\rm operatrs}(n_f=3)=7704(11307)$

$$\mathcal{O}^{Uh\psi^4}_{33} = (\bar{q}_{L_s}\gamma_\mu\tau^I\mathbf{T}q_{L_p})(\bar{q}_{R_r}\gamma^\mu\mathbf{U}^\dagger\tau^I\mathbf{U}q_{R_t})\mathcal{F}^{Uh\psi^4}_{33}(h),$$

$$\mathcal{O}^{Uh\psi^4}_{34} = (\bar{q}_{L_s}\gamma_\mu\lambda^A\tau^I\mathbf{T}q_{L_p})(\bar{q}_{R_r}\gamma^\mu\lambda^A\mathbf{U}^\dagger\tau^I\mathbf{U}q_{R_t})\mathcal{F}^{Uh\psi^4}_{34}(h),$$

$$\mathcal{O}^{Uh\psi^4}_{89} = (\bar{l}_{L_s}\gamma_\mu\tau^I l_{L_p})(\bar{l}_{R_t}\sigma^\mu\tau^I\mathbf{U}^\dagger\mathbf{T}\mathbf{U}l_{R_s})\mathcal{F}^{Uh\psi^4}_{89}(h),$$

$$\mathcal{O}^{Uh\psi^4}_{107} = (\bar{l}_{L_s}\gamma_\mu\tau^I\mathbf{T}l_{L_p})(\bar{q}_{L_t}\gamma^\mu\tau^I q_{L_r})\mathcal{F}^{Uh\psi^4}_{107}(h),$$

$$\mathcal{O}^{Uh\psi^4}_{113} = (\bar{l}_{R_s}\gamma_\mu\tau^I\mathbf{T}l_{R_p})(\bar{q}_{R_t}\gamma^\mu\tau^I q_{R_r})\mathcal{F}^{Uh\psi^4}_{113}(h),$$

$$\mathcal{O}^{Uh\psi^4}_{119} = (\bar{l}_{R_s}\gamma_\mu\mathbf{U}^\dagger\tau^I\mathbf{T}\mathbf{U}l_{R_p})(\bar{q}_{L_t}\gamma^\mu\tau^I q_{L_r})\mathcal{F}^{Uh\psi^4}_{119}(h),$$

$$\mathcal{O}^{Uh\psi^4}_{125} = (\bar{l}_{L_s}\gamma_\mu\tau^I\mathbf{T}l_{L_p})(\bar{q}_{R_t}\gamma^\mu\mathbf{U}^\dagger\tau^I\mathbf{U}q_{R_r})\mathcal{F}^{Uh\psi^4}_{125}(h),$$

$$\mathcal{O}^{Uh\psi^4}_{140} = \mathcal{Y}\left[\begin{smallmatrix} r & s \\ t & \end{smallmatrix}\right]\epsilon^{abc}\epsilon^{ln}\epsilon^{km}((\mathbf{T}{l_L}^T)_{pm}C(\mathbf{T}q_L)_{ran})({q_L}^T_{rak}Cq_{L_{tcl}})\mathcal{F}^{Uh\psi^4}_{159}(h),$$

$$\mathcal{O}^{Uh\psi^4}_{160} = \mathcal{Y}\left[\begin{smallmatrix} r & s \\ t & \end{smallmatrix}\right]\epsilon^{abc}\epsilon^{km}\epsilon^{ln}((\mathbf{T}{l_R}^T)_{pm}C(\mathbf{T}q_R)_{ran})({q_R}^T_{sbk}Cq_{R_{tcl}})\mathcal{F}^{Uh\psi^4}_{160}(h).$$

Nature of the Higgs Boson

Before the Higgs discovery, and after …

What is dynamics at EW scale?

Weak dynamics @ EW scale

SM, SUSY, etc

Strong dynamics @ EW scale

Technicolor, etc

Implications of dynamical symmetry breaking

Steven Weinberg
Phys. Rev. D **13**, 974 – Published 15 February 1976

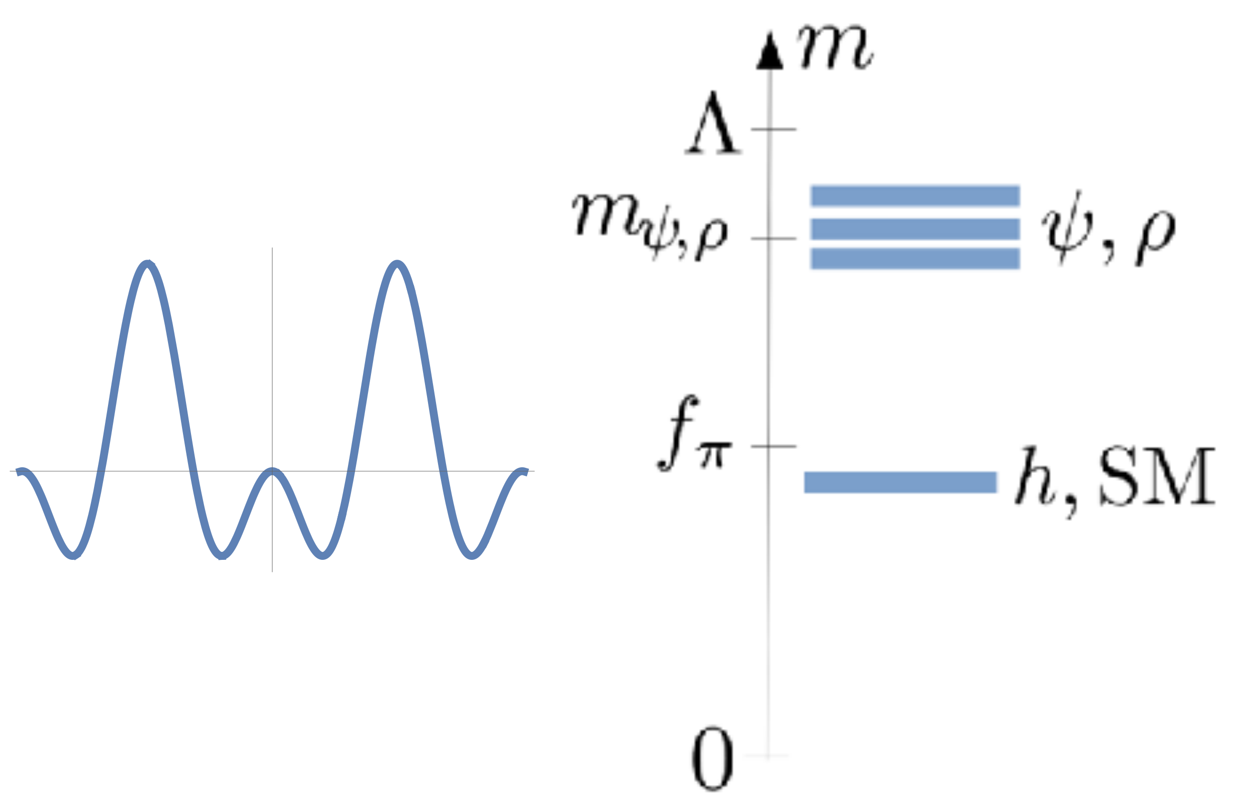
What is dynamics at TeV scale?

Fundamental

weak dynamics at TeV

Composite

strong dynamics at TeV



EFT for Psuedo-Goldstone Higgs

Composite Higgs, neutral naturalness, little Higgs, etc

Composite Higgs

[Agashe, Contino, Pomarol 2004]

Minimal Neutral Naturalness

[Xu, **Yu**, Zhu, **2018**]

$\mathcal{G}$	$\mathcal{H}$	C	N_G	$\mathbf{r}_{\mathcal{H}} = \mathbf{r}_{SU(2)\times SU(2)}$ $(\mathbf{r}_{SU(2)\times U(1)})$
SO(5)	SO(4)	✓	4	$\mathbf{4} = (\mathbf{2},\mathbf{2})$
SO(6)	SO(5)	✓	5	$\mathbf{5} = (\mathbf{1},\mathbf{1}) + (\mathbf{2},\mathbf{2})$
SU(3) × U(1)	SU(2) × U(1)		5	$\mathbf{2}_{\pm 1/2} + \mathbf{1}_0$
SU(4)	Sp(4)	✓	5	$\mathbf{5} = (\mathbf{1},\mathbf{1}) + (\mathbf{2},\mathbf{2})$
SO(7)	SO(6)	✓	6	$\mathbf{6} = 2\cdot(\mathbf{1},\mathbf{1}) + (\mathbf{2},\mathbf{2})$
SO(8)	SO(7)	✓	7	$\mathbf{7} = 3\cdot(\mathbf{1},\mathbf{1}) + (\mathbf{2},\mathbf{2})$
SU(4) × U(1)	SU(3) × U(1)		7	$\mathbf{3}_{-1/3} + \bar{\mathbf{3}}_{+1/3} + \mathbf{1}_0 = 3\cdot\mathbf{1}_0 + \mathbf{2}_{\pm 1/2}$
SO(7)	G_2	✓*	7	$\mathbf{7} = (\mathbf{1},\mathbf{3}) + (\mathbf{2},\mathbf{2})$
SO(9)	SO(8)	✓	8	$\mathbf{8} = 2\cdot(\mathbf{2},\mathbf{2})$
SU(4)	$[SU(2)]^2 \times U(1)$	✓*	8	$(\mathbf{2},\mathbf{2})_{\pm 2} = 2\cdot(\mathbf{2},\mathbf{2})$
$[SU(3)]^2$	SU(3)		8	$\mathbf{8} = \mathbf{1}_0 + \mathbf{2}_{\pm 1/2} + \mathbf{3}_0$
Sp(6)	Sp(4) × SU(2)	✓	8	$(\mathbf{4},\mathbf{2}) = 2\cdot(\mathbf{2},\mathbf{2})$
SU(5)	SU(4) × U(1)	✓*	8	$\mathbf{4}_{-5} + \bar{\mathbf{4}}_{+5} = 2\cdot(\mathbf{2},\mathbf{2})$
$[SO(5)]^2$	SO(5)	✓*	10	$\mathbf{10} = (\mathbf{1},\mathbf{3}) + (\mathbf{3},\mathbf{1}) + (\mathbf{2},\mathbf{2})$
SO(7)	SO(5) × U(1)	✓*	10	$\mathbf{10}_0 = (\mathbf{3},\mathbf{1}) + (\mathbf{1},\mathbf{3}) + (\mathbf{2},\mathbf{2})$
SO(7)	$[SU(2)]^3$	✓*	12	$(\mathbf{2},\mathbf{2},\mathbf{3}) = 3\cdot(\mathbf{2},\mathbf{2})$
SU(5)	SO(5)	✓*	14	$\mathbf{14} = (\mathbf{3},\mathbf{3}) + (\mathbf{2},\mathbf{2}) + (\mathbf{1},\mathbf{1})$
SU(6)	Sp(6)	✓*	14	$\mathbf{14} = 2\cdot(\mathbf{2},\mathbf{2}) + (\mathbf{1},\mathbf{3}) + 3\cdot(\mathbf{1},\mathbf{1})$
$[SO(6)]^2$	SO(6)	✓*	15	$\mathbf{15} = (\mathbf{1},\mathbf{1}) + 2\cdot(\mathbf{2},\mathbf{2}) + (\mathbf{3},\mathbf{1}) + (\mathbf{1},\mathbf{3})$
SO(9)	SO(5) × SO(4)	✓*	20	$(\mathbf{5},\mathbf{4}) = (\mathbf{2},\mathbf{2}) + (\mathbf{1}+\mathbf{3},\mathbf{1}+\mathbf{3})$

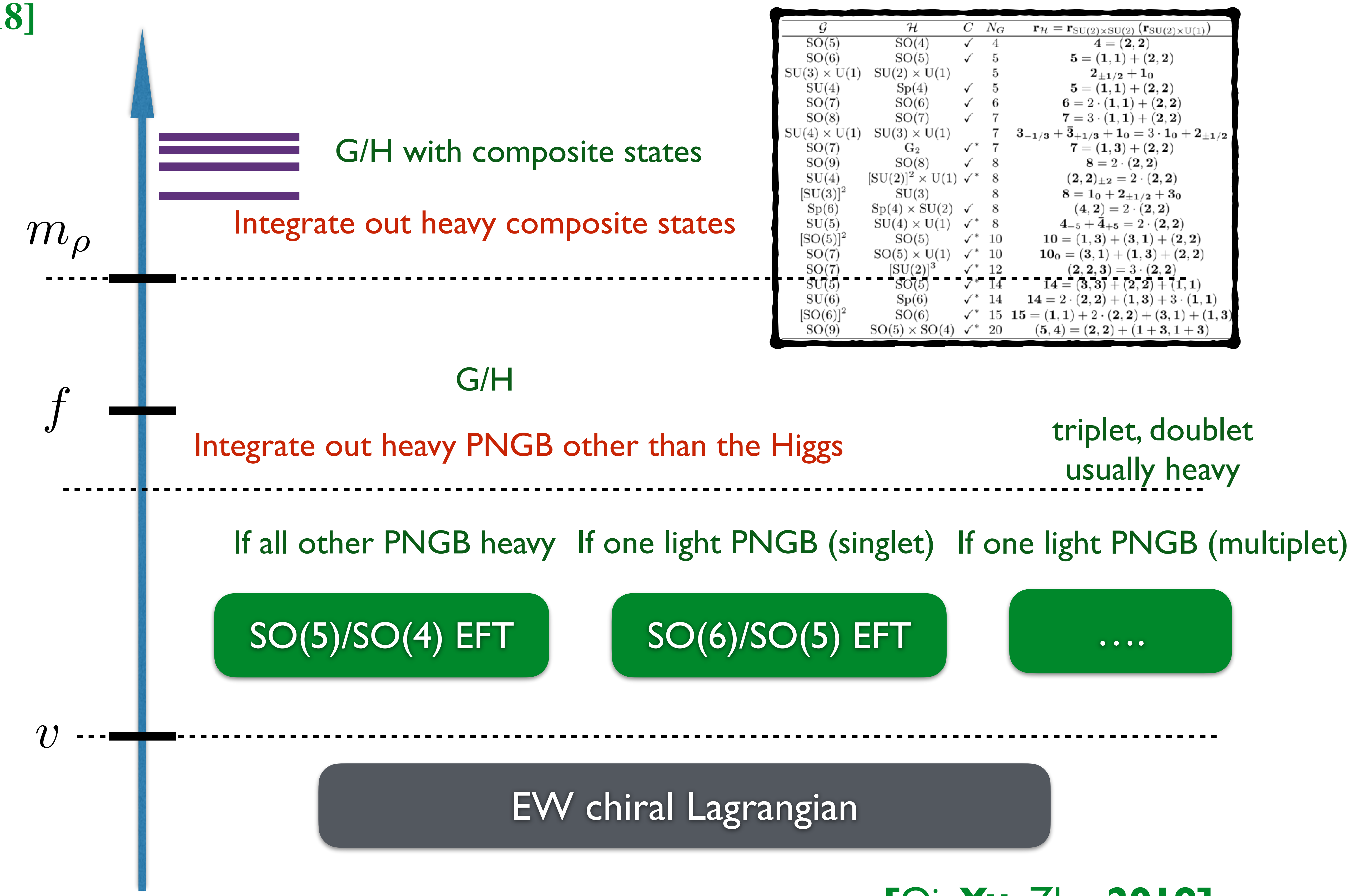
[Csaki, et. al, 2015]

Twin Higgs

[Chacko, Goh, Harnik 2006]

Little Higgs

[Arkani-hamed, et.al. 2000]

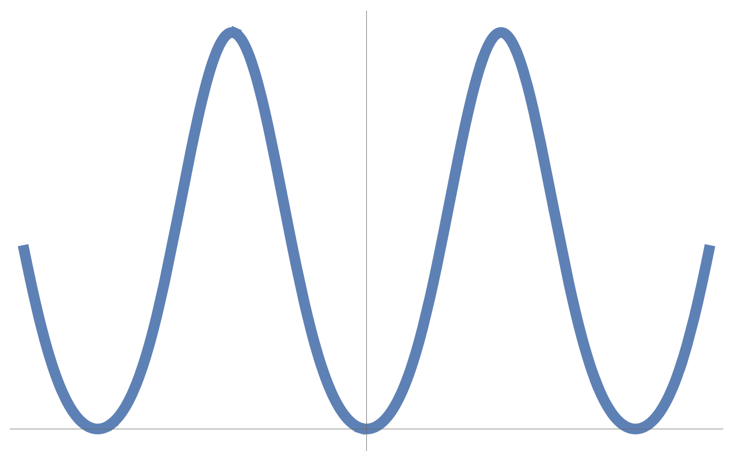


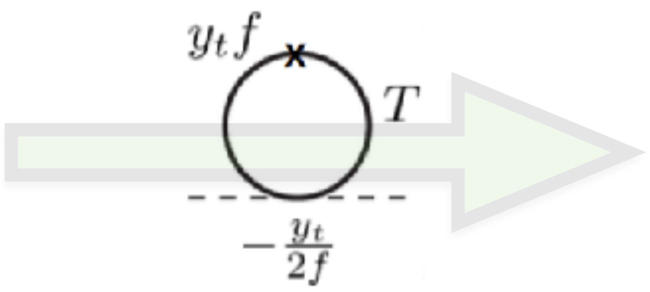
[Qi, **Yu**, Zhu, **2019**]

Vacuum misalignment and effective potential

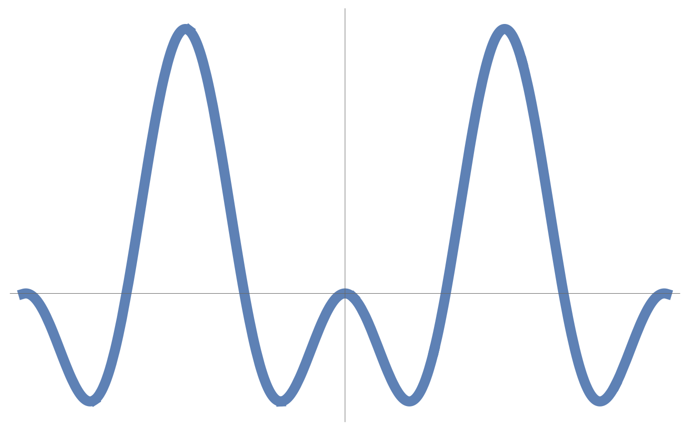
Top partner to solve little hierarchy problem

Higgs potential with only boson sector

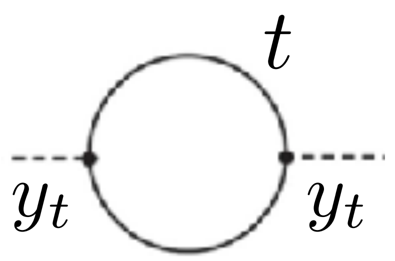
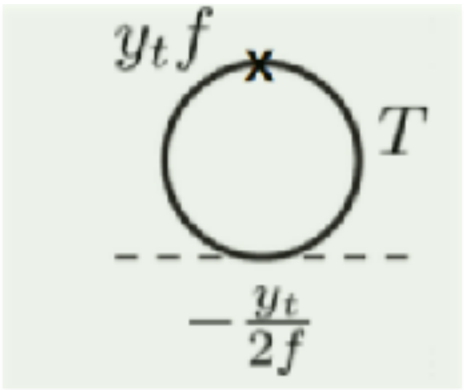




EW symmetry breaking



$$V(\phi) = -a \sin^2(\phi/f) + b \sin^4(\phi/f)$$

$-(100\ \text{GeV})^2 =$  $+$  $+$ 

$$= (m^0)^2 + \frac{\Lambda^2}{16\pi^2}(-6y_t^2 + 6y_t^2) + \frac{6y_t^2}{16\pi^2} m_{t'}^2 \log\frac{\Lambda^2}{m_{t'}^2}$$

Little Higgs

Twin Higgs

Composite Higgs

Neutral Naturalness

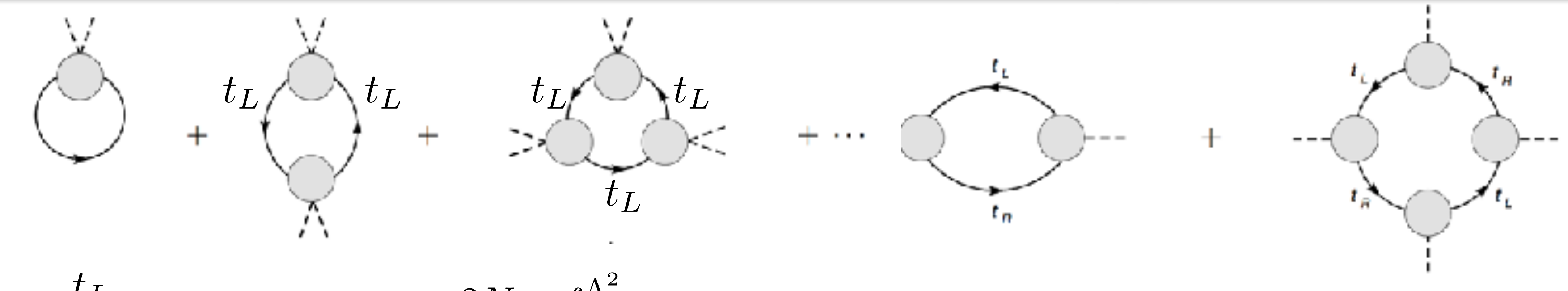
Gauge Higgs

Composite Higgs | Left-Right Z2 | Twin Higgs | Minimal NN

$$\mathcal{L}_{\text{eff}} = \bar{t}_L \not{p}\Pi_{t_L}(p^2) t_L + \bar{t}_R \not{p}\Pi_{t_R}(p^2) t_R - (\bar{t}_L \Pi_{t_L t_R}(p^2) t_R + \text{h.c.})$$

$$\Pi_{t_L}(-Q^2) = \Pi_{0t_L}(-Q^2) + \Pi_{1t_L}(-Q^2)\, s_h^2 + \Pi_{2t_L}(-Q^2)\, s_h^4 + \cdots$$

Coleman-Weinberg effective potential

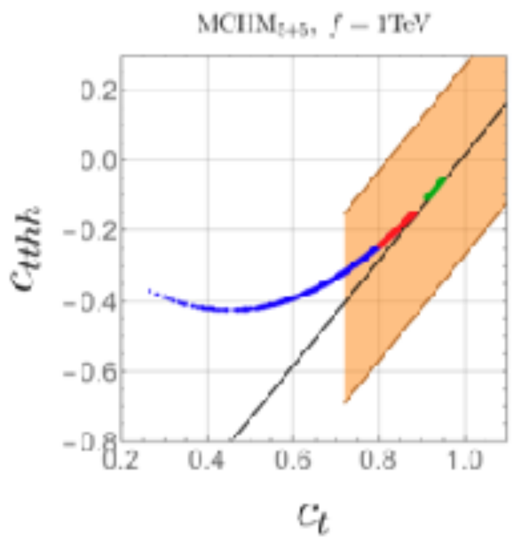


$$V(h) = -\frac{2N_c}{16\pi^2}\int_0^{\Lambda^2} dQ^2 Q^2 \log[\Pi_{t_L}\Pi_{t_R} \cdot Q^2 + \Pi_{t_L t_R}^2]$$

$$= -\gamma_f s_h^2 + \beta_f s_h^4 + \cdots$$

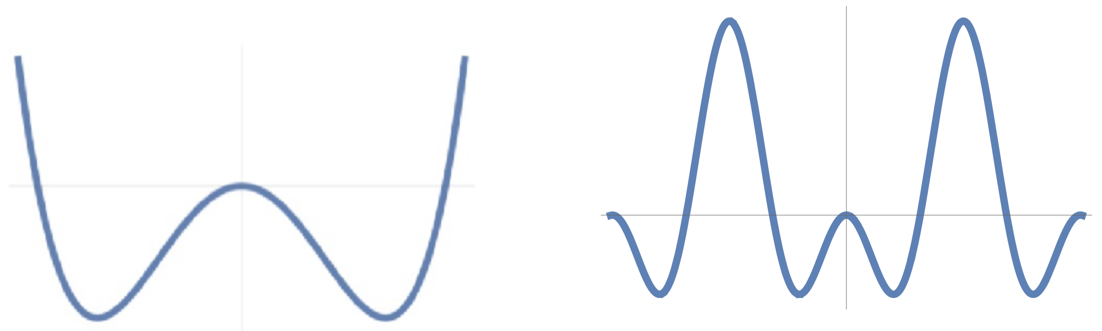
EW Chiral Lagrangian

Higgs nonlinearity



[Li, Xu, **Yu**, Zhu, **2019**]

Shape of Higgs potential



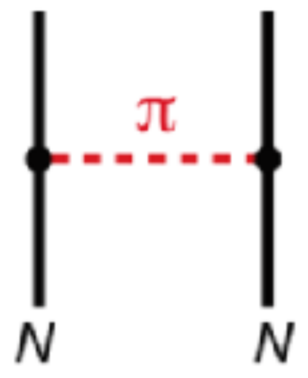
[Agrawal, Saha, Xu, **Yu**, Yuan, **2019**]

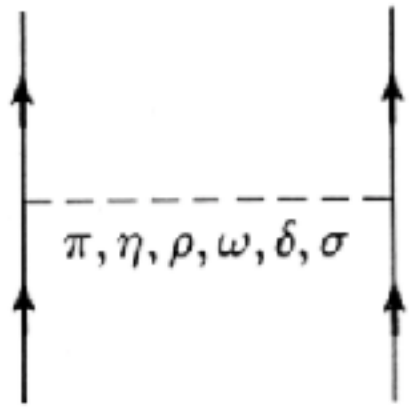
Nuclear Chiral Effective Theory

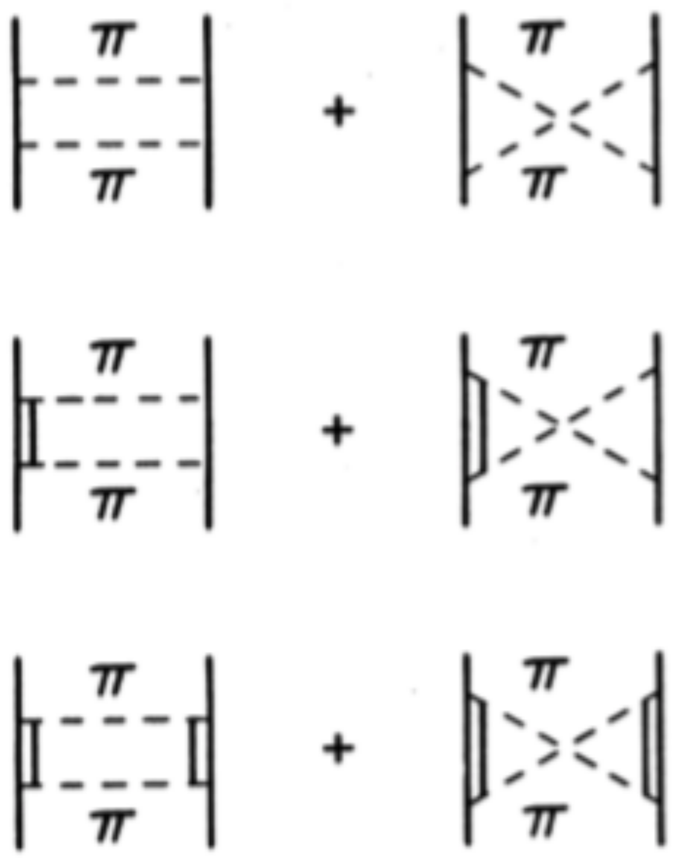
[Hao Sun, Yi-Ning Wang, **J.H.Yu**, in préparation]

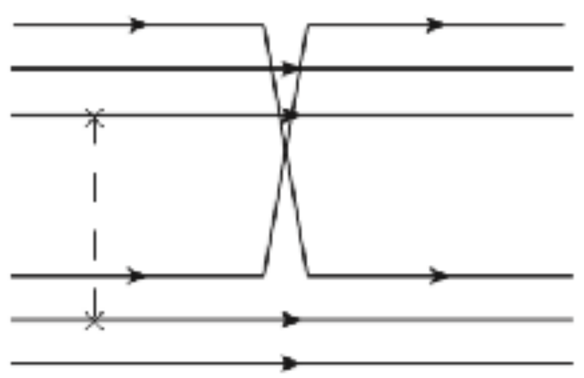
[Yong-Kang Li, Yi-Ning Wang, **J.H.Yu**, in préparation]

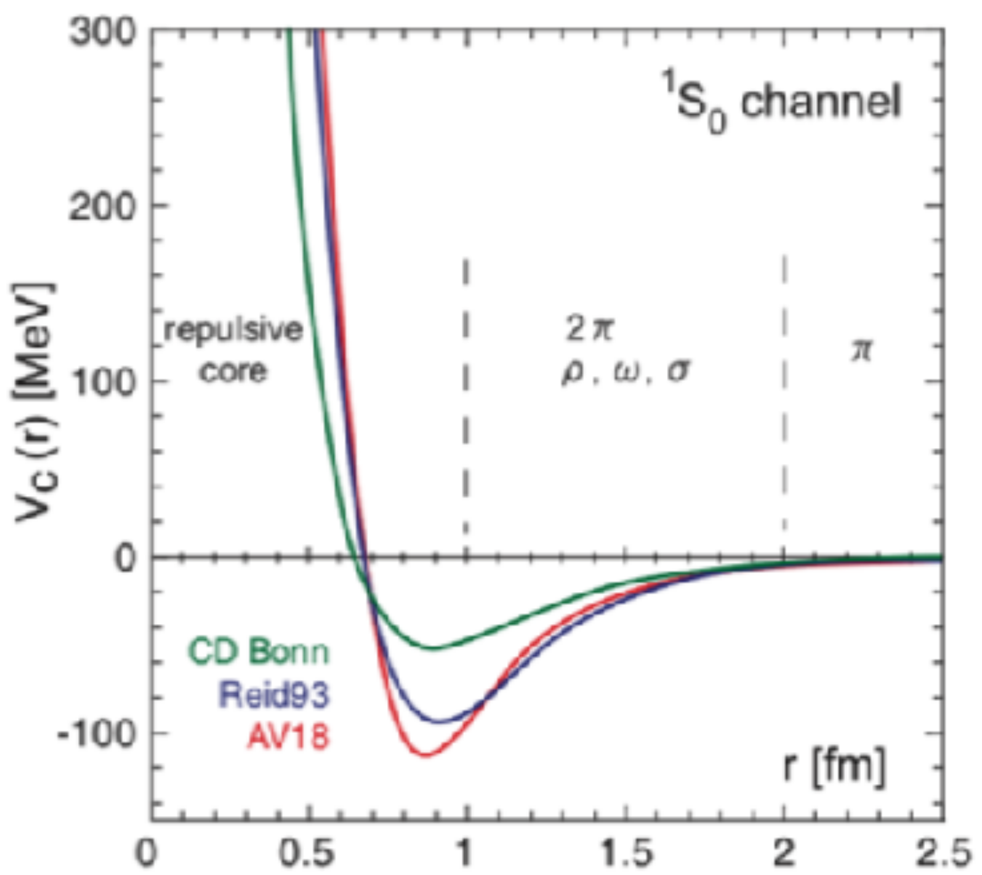
Historical overview on nuclear force

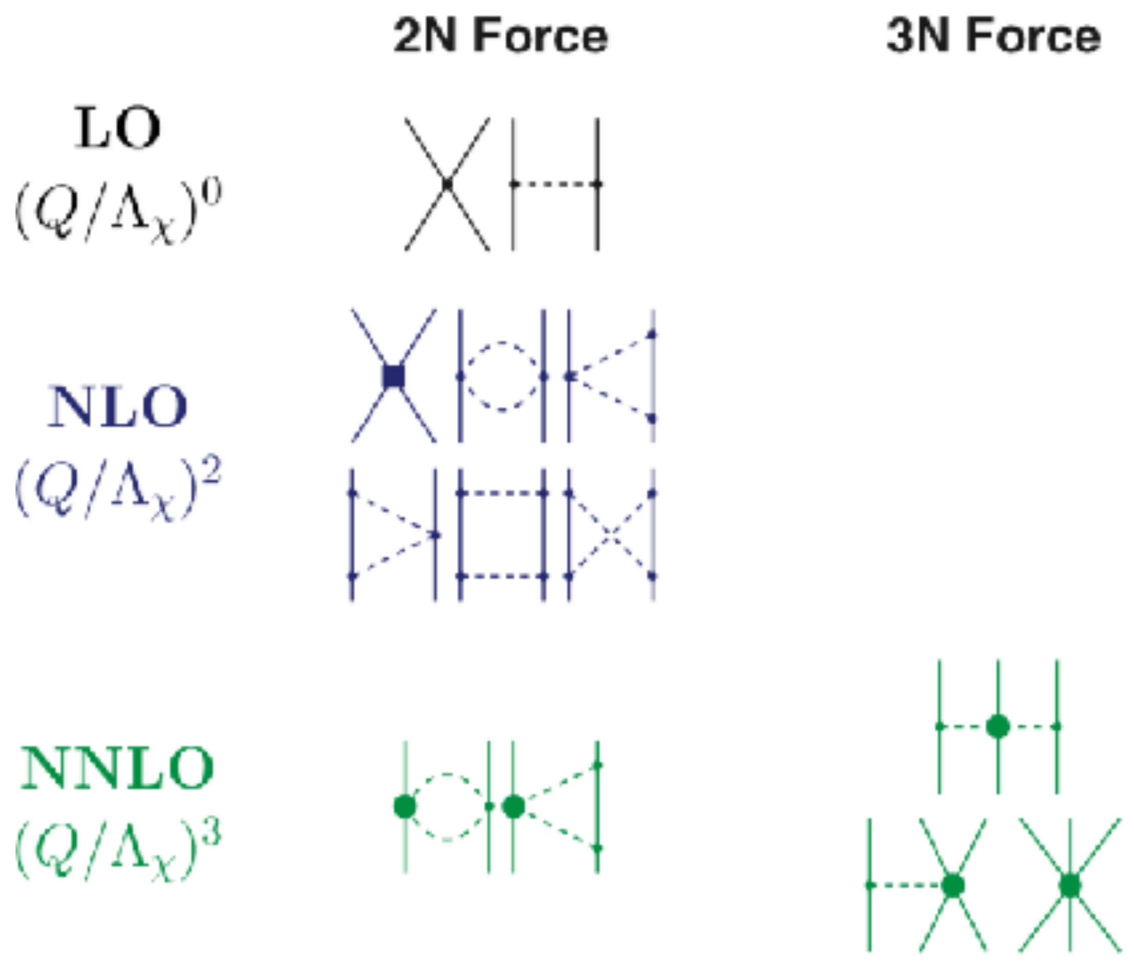












Yukawa Pion Theory
1935

One-Boson Exchange Model
1936 - 1960

Two-Pion Exchange
1950 - 1980

N-N from quark/chiral-bag model
1970 - 1980

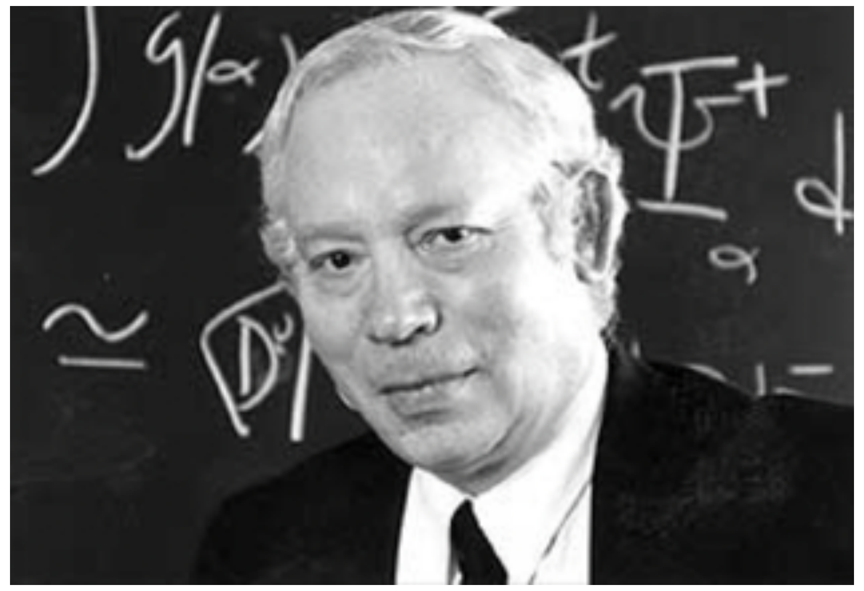
High precision potential
1990 -

Weinberg nuclear chiral EFT
1990 -

Proca, Kemmer, Moller, Rosenfeld and Schwinger, Pauli, …

Taketani, Nakamura Sasaki, Bruckner, Watson, …

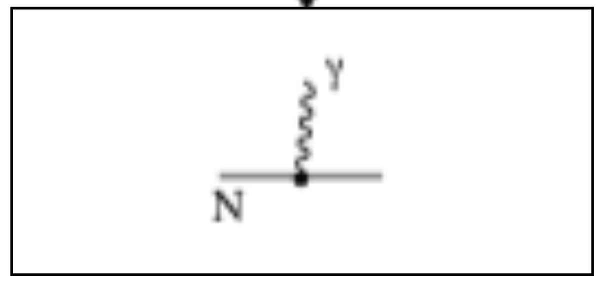
AV18, CD Bonn, Nijm, Reid93 …

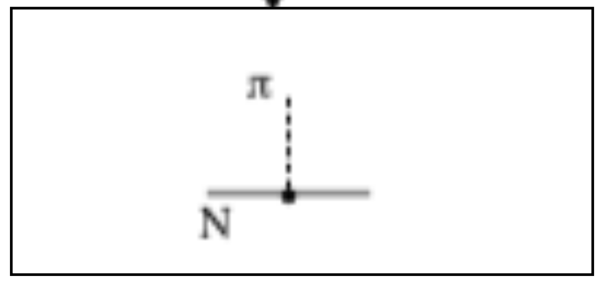


Chiral Nuclear Force

Meson Exchange Model

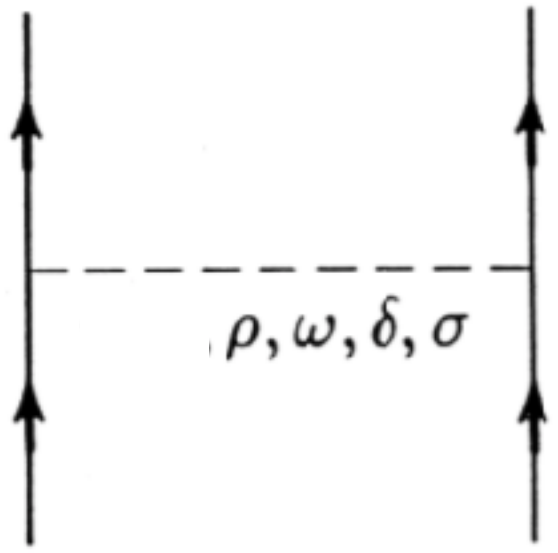
$$\mathcal{L}_\sigma = \overline{N_L}\, i\not{D}\, N_L + \overline{N_R}\, i\not{D}\, N_R - g\overline{N_R}\,\Sigma\, N_L - g\overline{N_L}\,\Sigma^\dagger\, N_R$$

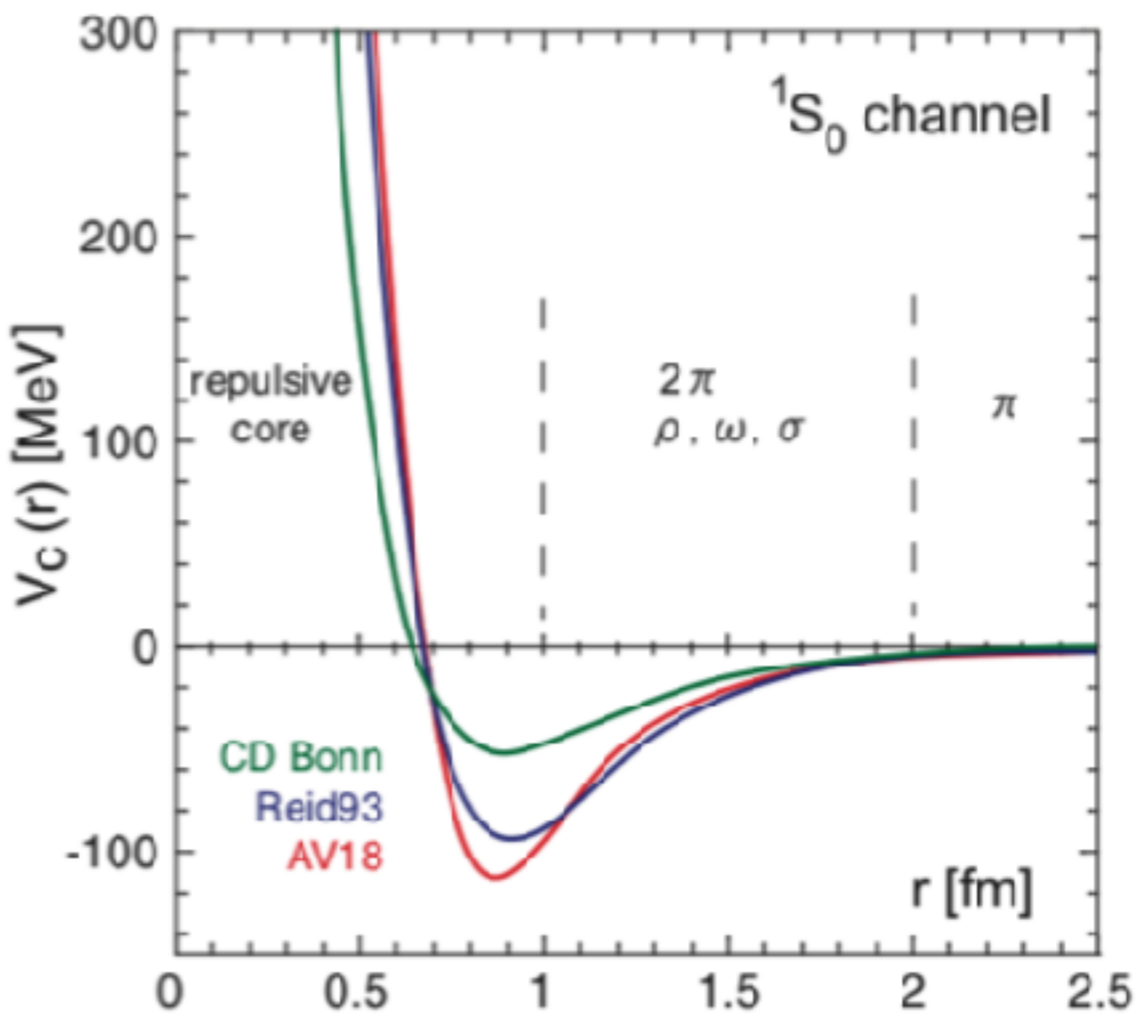




$g_A = 1$ $\quad m_N = gv \equiv g_{\pi NN} F_\pi$

Modeling only



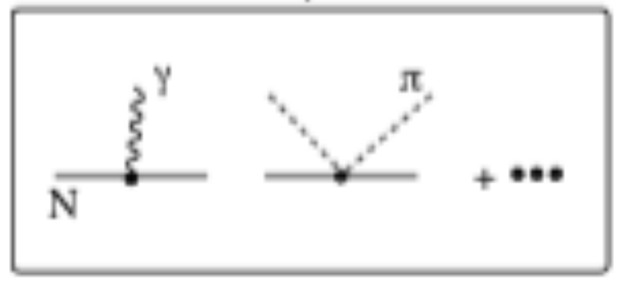


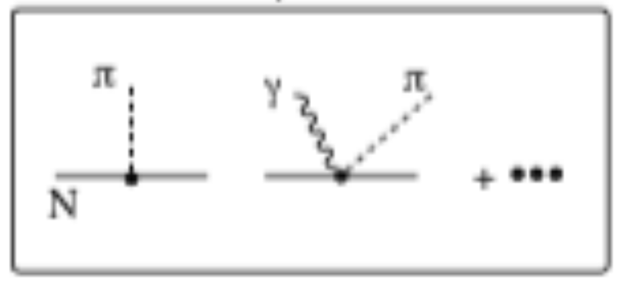
Chiral EFT

$$\mathcal{L}^{(1)}_{\pi N} = \bar{\psi}\left(i\not{D} - m_N + \tfrac{1}{2} g_A \gamma_\mu \gamma_5 u^\mu\right)\psi$$

$$M_N g_A(0) = F_\pi\, g_{\pi NN}$$

$$g_A \simeq 1.27,\ g_{\pi NN} \simeq 13.40$$

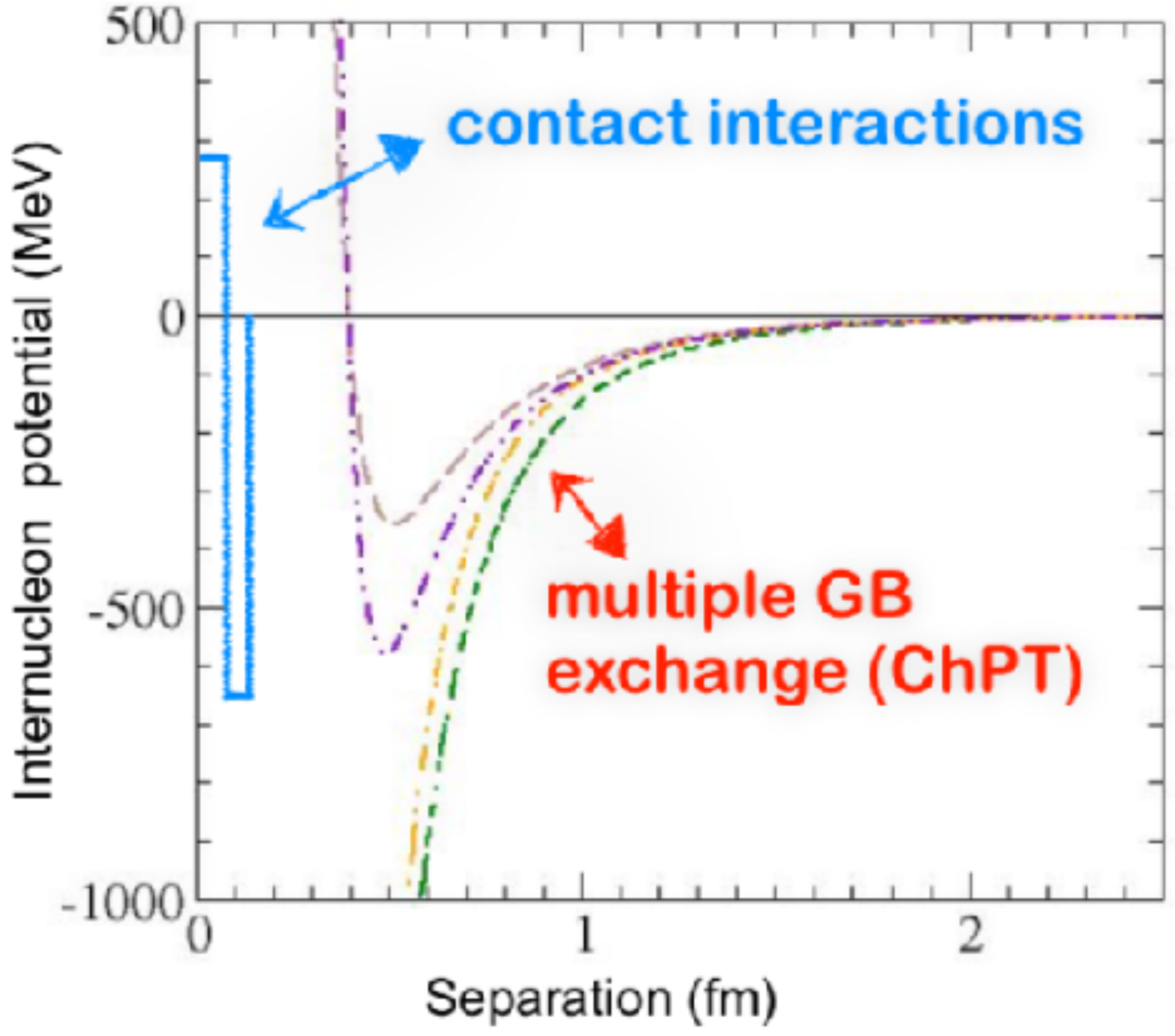




Goldberger-Treiman Relation

Perturbative derivative expansion

Repulsive central



Weinberg's nuclear force

Hard-core nucleon-nucleon interaction

Nuclear forces from chiral lagrangians

Steven Weinberg [1]
Theory Group, Department of Physics, University of Texas, Austin, TX 78712, USA

Received 14 August 1990

EFFECTIVE CHIRAL LAGRANGIANS FOR NUCLEON–PION INTERACTIONS AND NUCLEAR FORCES

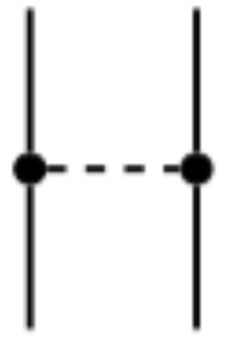
Steven WEINBERG*
Theory Group, Department of Physics, University of Texas, Austin, TX 78712, USA

Received 2 April 1991

[Weinberg 1933 - 2021]

Weinberg power counting

$$D = 2 - A + 2L + \sum_d V_d \left(d - 2 + \frac{f}{2} \right)$$



= 2(1-2+2/2) = 0

= (0-2+4/2) = 0

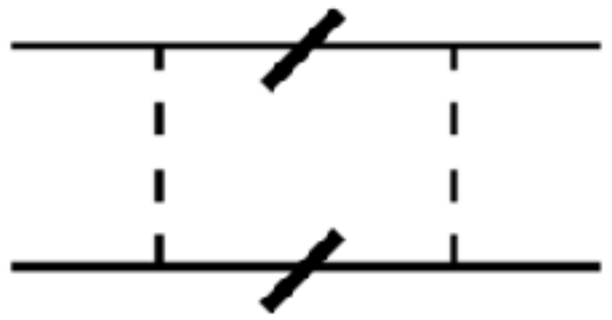
It had taken me a decade to realize that four divided by two is two. This sort of interaction is just the kind of hard-core nucleon-nucleon interaction that nuclear physicists had always known would be needed to understand nuclear forces. But now we had a rationale for it.

Weinberg 2021

2PI Diagrams

Nuclear potential from Irre. 2PI only

2PR Diagrams

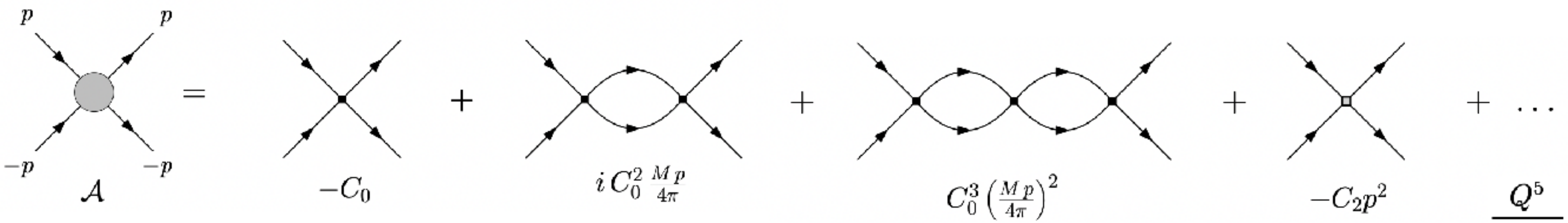


Breakdown in perturbation theory = nuclear bound states

Resumed by Schrodinger equation

Pionless effective field theory

At low energy, the NN contact interaction shows non-perturbative nature (nuclei bound states)



$$\mathcal{A} = -C_0 + iC_0^2 \frac{Mp}{4\pi} + C_0^3 \left(\frac{Mp}{4\pi}\right)^2 - C_2 p^2 + \dots$$

$$\frac{Q^5}{4\pi M} \times C_0 \left(\frac{M}{Q^2}\right)\left(\frac{M}{Q^2}\right) C_0 \sim C_0^2 \frac{MQ}{4\pi}$$

Infrared enhancement!

Weinberg, 1991

Natural scattering length $|a| \lesssim 1/\Lambda$

$$\mathcal{A} = -\frac{4\pi a}{M}\left[1 - iap + \left(\frac{1}{2}ar_0 - a^2\right)p^2 + O(p^3/\Lambda^3)\right]$$

$C_0 \sim 4\pi a/M$ **Irrelevant** $\mathcal{O}(p^0)$

$$C_0^2 \frac{MQ}{4\pi} \sim \left(\frac{4\pi a}{M}\right)^2 \frac{MQ}{4\pi} \sim \frac{4\pi a^2}{M} Q$$

$C_2 = \frac{\pi a^2}{m} r_0$ $\mathcal{O}(p^2)$

Kaplan, Savage, Wise 1998

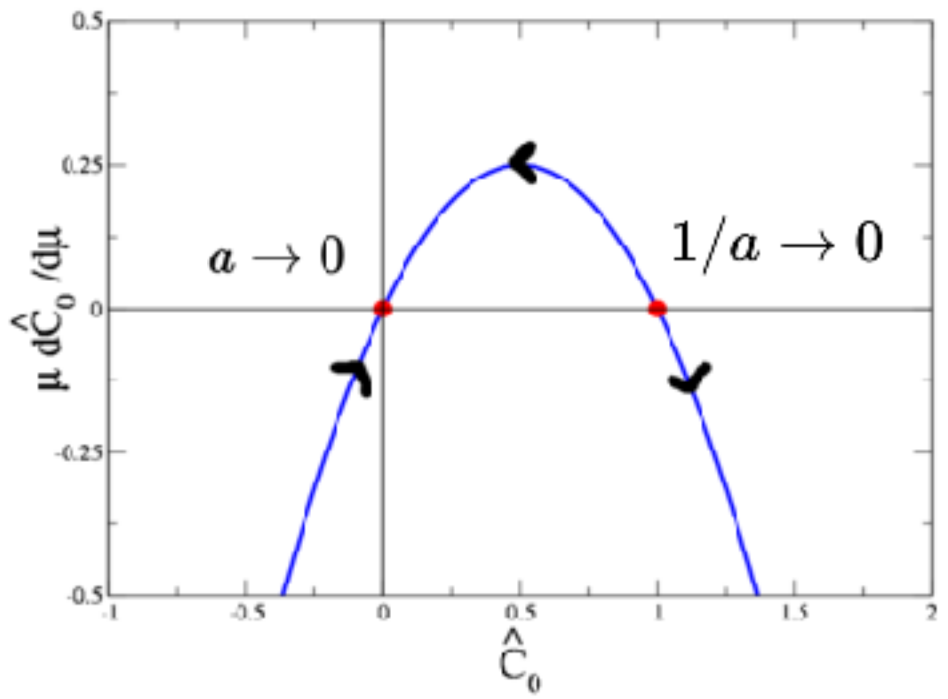
Unnatural scattering length $|a| \sim p^{-1} \gg 1/\Lambda$

$$\mathcal{A} = -\frac{4\pi}{M}\frac{1}{(1/a + ip)}\left[1 + \frac{r_0/2}{(1/a+ip)}p^2 + \frac{(r_0/2)^2}{(1/a+ip)^2}p^4 + \dots\right]$$

$C_0 \sim 4\pi/MQ$ **Relevant** $\mathcal{O}(p^{-1})$

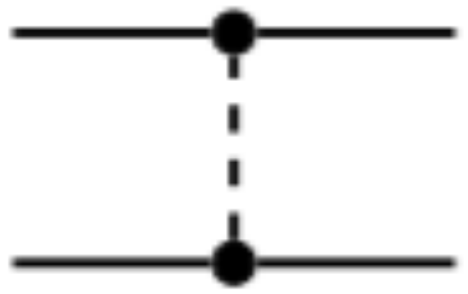
$$C_0^2 \frac{MQ}{4\pi} \sim \left(\frac{4\pi}{MQ}\right)^2 \frac{MQ}{4\pi} \sim \frac{4\pi}{MQ}$$

$C_2 = \frac{\pi}{m}\frac{r_0}{Q^2}$ $\mathcal{O}(p^0)$



Chiral effective field theory

Weinberg power counting $\mu = 2 + 2\ell - r + \sum_i V_i \left(d_i + \frac{1}{2} n_i - 2\right)$ [Weinberg, 1990/1991]

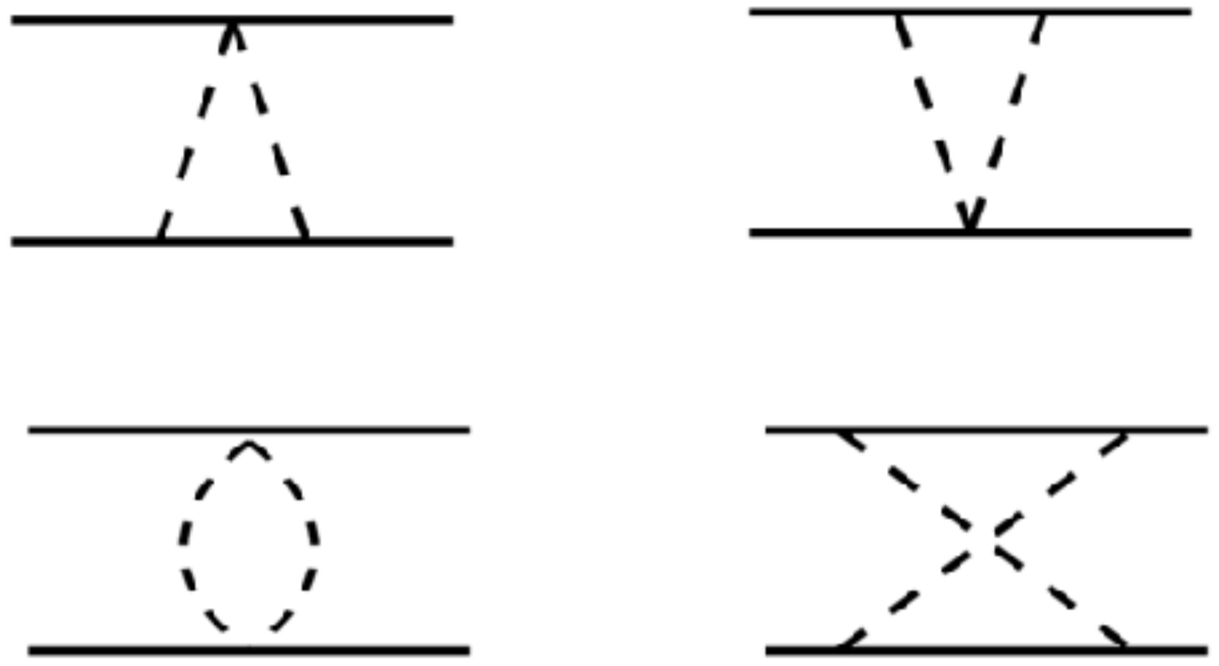


Dim = 2(1-2+2/2) = 0

$$V_{1\pi} = -\left(\frac{g_A}{2F_\pi}\right)^2 \frac{\vec{\sigma}_1 \cdot \vec{q}\, \vec{\sigma}_2 \cdot \vec{q}}{q^2 + M_\pi^2} \vec{\tau}_1 \cdot \vec{\tau}_2 \sim \mathcal{O}(1)$$

2PI diagrams

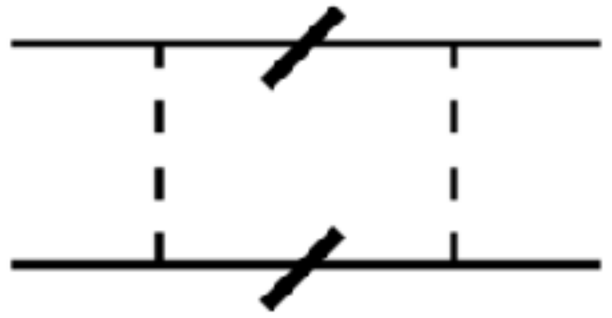
Dim = 2+2-2+2(1-2+2/2) = 2



1. calculate nuclear potential from irreducible diagrams

pinch diagrams subtracted

2PR Diagrams



$$\sim \left(\frac{g_A}{F_\pi}\right)^2 \frac{Q}{\Lambda_{NN}} \qquad \Lambda_{NN} = \frac{4\pi F_\pi^2}{g_A^2 m_N} \sim f_\pi$$

$$I \sim \int \frac{d^dq}{(2\pi)^d} \frac{1}{q^0 - \frac{\vec{p}^2 - \vec{q}^2}{2M} - i\epsilon} \frac{1}{-q^0 + \frac{\vec{p}^2 - \vec{q}^2}{2M} + i\epsilon} \frac{1}{(q+p)^2 + i\epsilon} \frac{1}{(q-p)^2 + i\epsilon}$$

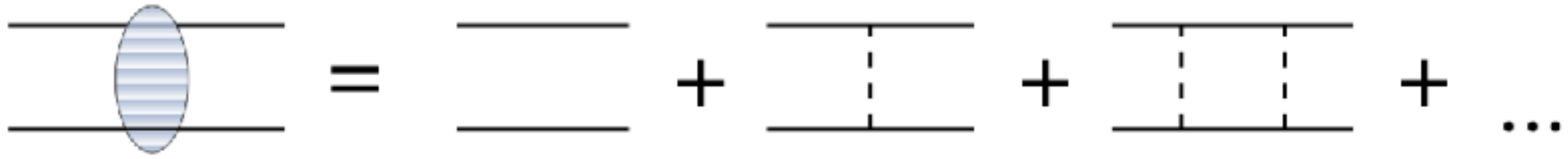
nucleon pole

Pinch singularity

$$\frac{Q^3}{16\pi^2}(2\pi) \frac{2M_N}{\vec{p}^2 - \vec{q}^2 + i\epsilon}$$

mN enhancement

2. Truncated nuclear potential is iterated to all order



Solve Schrodinger equation

Power counting schemes

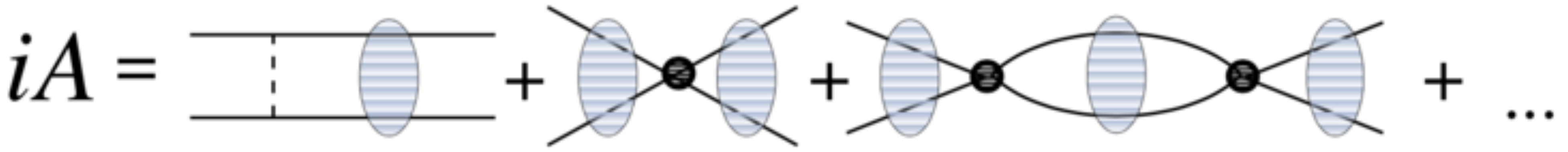
Complicated due to non-perturbative natures and renormalization problems

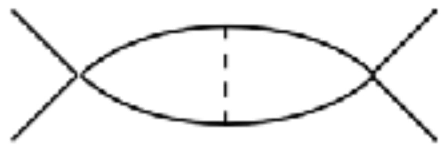
Weinberg Scheme

$V^{\rm LO}_{\rm Weinberg} \sim \mathcal{O}(1), \quad V^{\rm NLO}_{\rm Weinberg} \sim \mathcal{O}(p^2)$

[i.e. scaling of C_{2n} according to NDA (~ O(1))]

Weinberg, 1991





Renormalization problem!

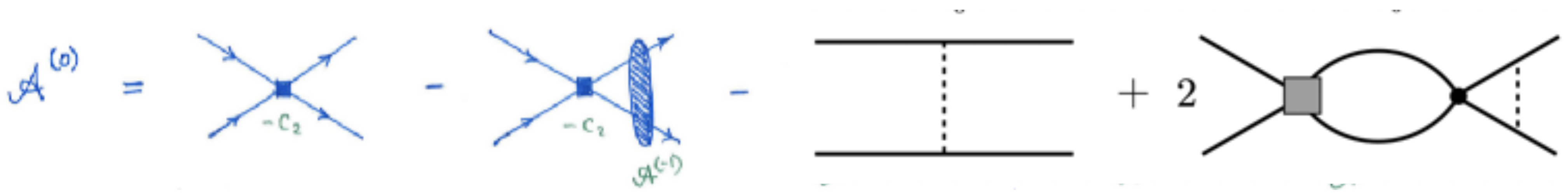
KSW Scheme

$V^{\rm LO}_{\rm KSW} \sim \mathcal{O}(p^{-1}), \quad V^{\rm NLO}_{\rm KSW} \sim \mathcal{O}(1)$

[i.e. scaling of C_{2n} as $C_{2n} \sim O(p^{-1-n})$]

Kaplan, Savage, Wise 1998

Pion are perturbative



Converge problem!

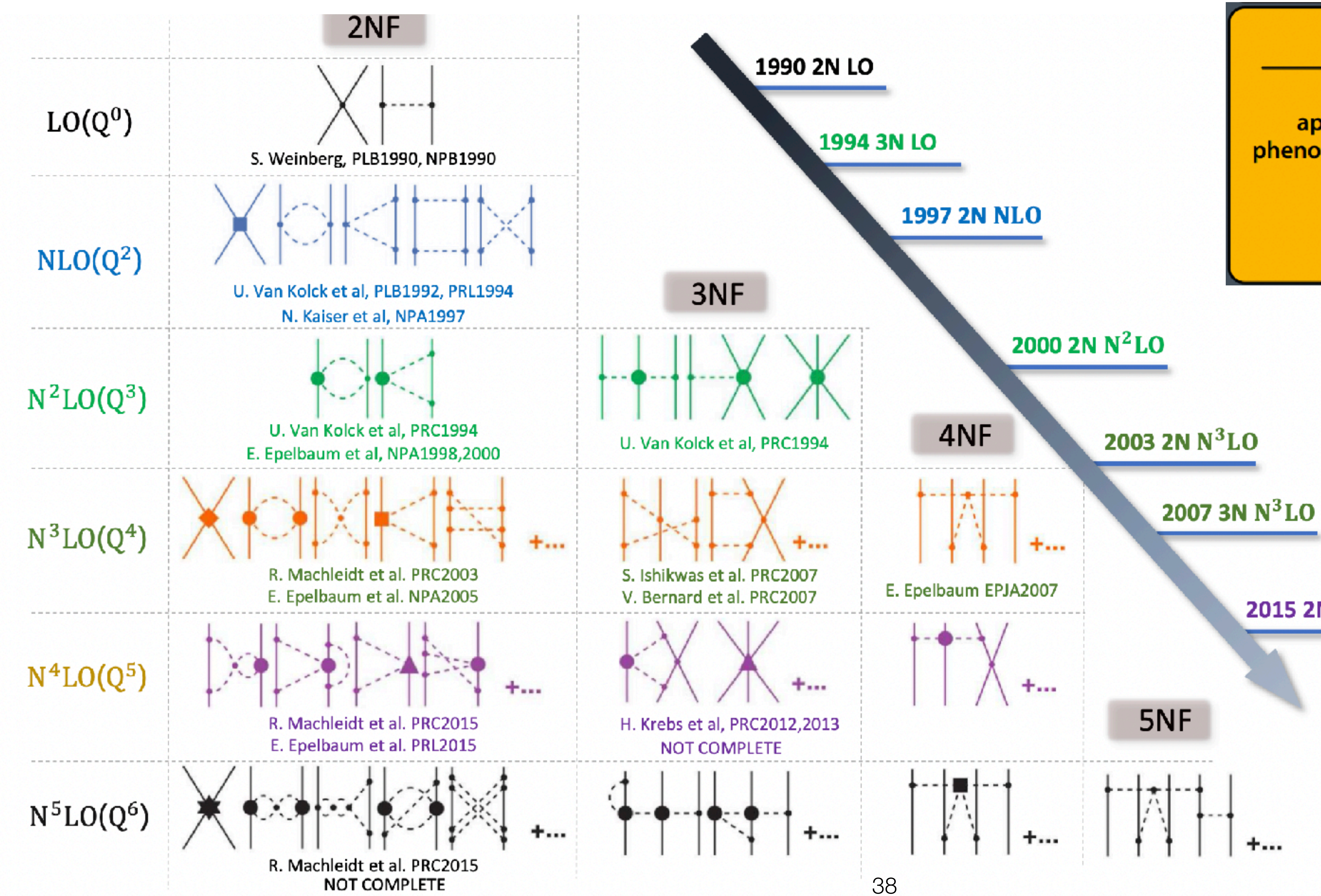
Modified Weinberg

[Nogga, Timmermans, van Kolck, 2005]
[Epelbaum, Gegelia, 2012]
[S. Wu, B. W. Long, 2019]

$$V_{\rm LO}^{\rm WPC}(\mathbf{p},\mathbf{p}') = \frac{g_A^2}{4f_\pi^2}\tau_1\cdot\tau_2\frac{(\sigma_1\cdot\mathbf{q})(\sigma_2\cdot\mathbf{q})}{m_\pi^2+\mathbf{q}^2} + \tilde{C}_{^1S_0} + \tilde{C}_{^3S_1} \;\longrightarrow\; V_{\rm LO}^{\rm MWPC}(\mathbf{p},\mathbf{p}') = V_{\rm LO}^{\rm WPC}(\mathbf{p},\mathbf{p}') + (\tilde{C}_{^3P_0} + \tilde{C}_{^3P_2})pp'$$

Solve both ! ?

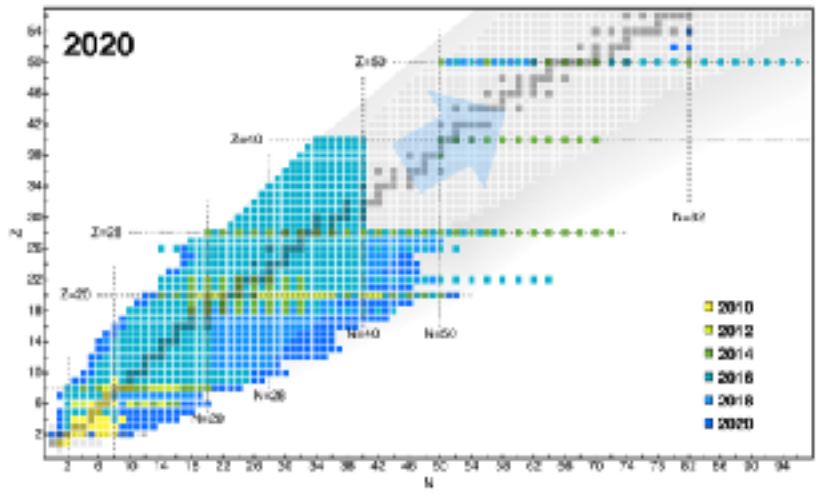
High precision nuclear force



many-body theory

exact	QMC, NCSM, ...
approximate	CC, IMSRG, MBPT, SCGF, ...
phenomenological	SM, DFT, ...

renormalization group
(SRG, Okubo-Lee-Suzuki, ...)



[from L. S. Geng's slides]

Weinberg scheme

[Entem, Machleidt, Nosyk, 2020]

NN and 3N Operators

Nucleon-nucleon sector

LO

[Weinberg 1990]   [Weinberg 1991]

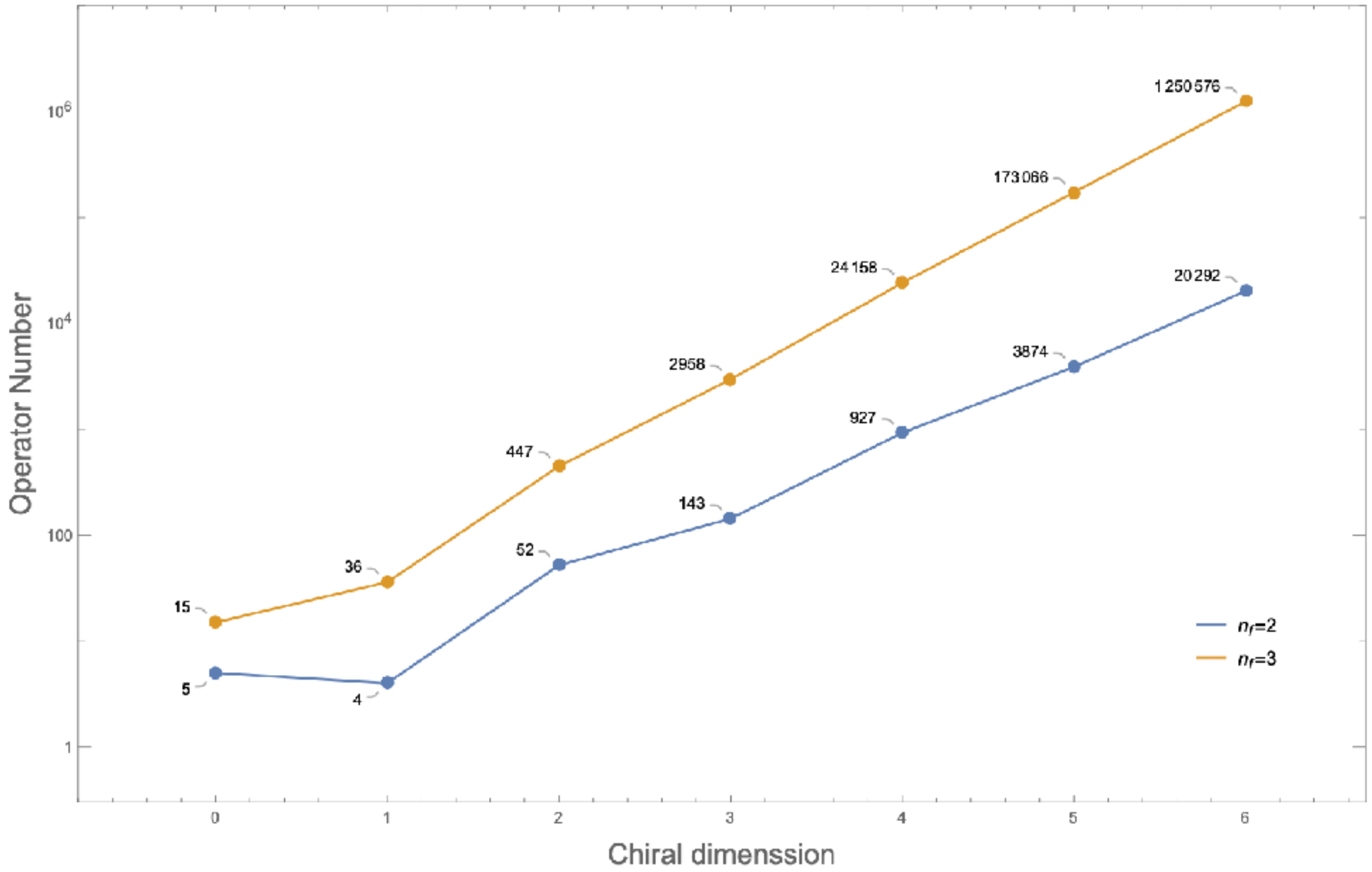
[van Kolck, Ordonez, 1992]

NLO

[Girlanda, Pastore, Schiavilla, Viviani, 2010]

[Petschauer, Kaiser, 2013]   [Xiao, Geng, Ren, 2019]

NNLO



3 nucleon sector

[Petschauer, Haidenbauer, Kaiser, Meisner, Weise, 2016]

[Nasoni, Filandri, Girlanda, 2023]

In order to obtain the most general contact Lagrangian in flavor SU(3), we follow the same procedure as used for the four-baryon contact terms in Ref. [47]. Generalizing these construction rules straightforwardly to six-baryon contact terms, we end up with a (largely) overcomplete set of terms for the leading covariant Lagrangian:

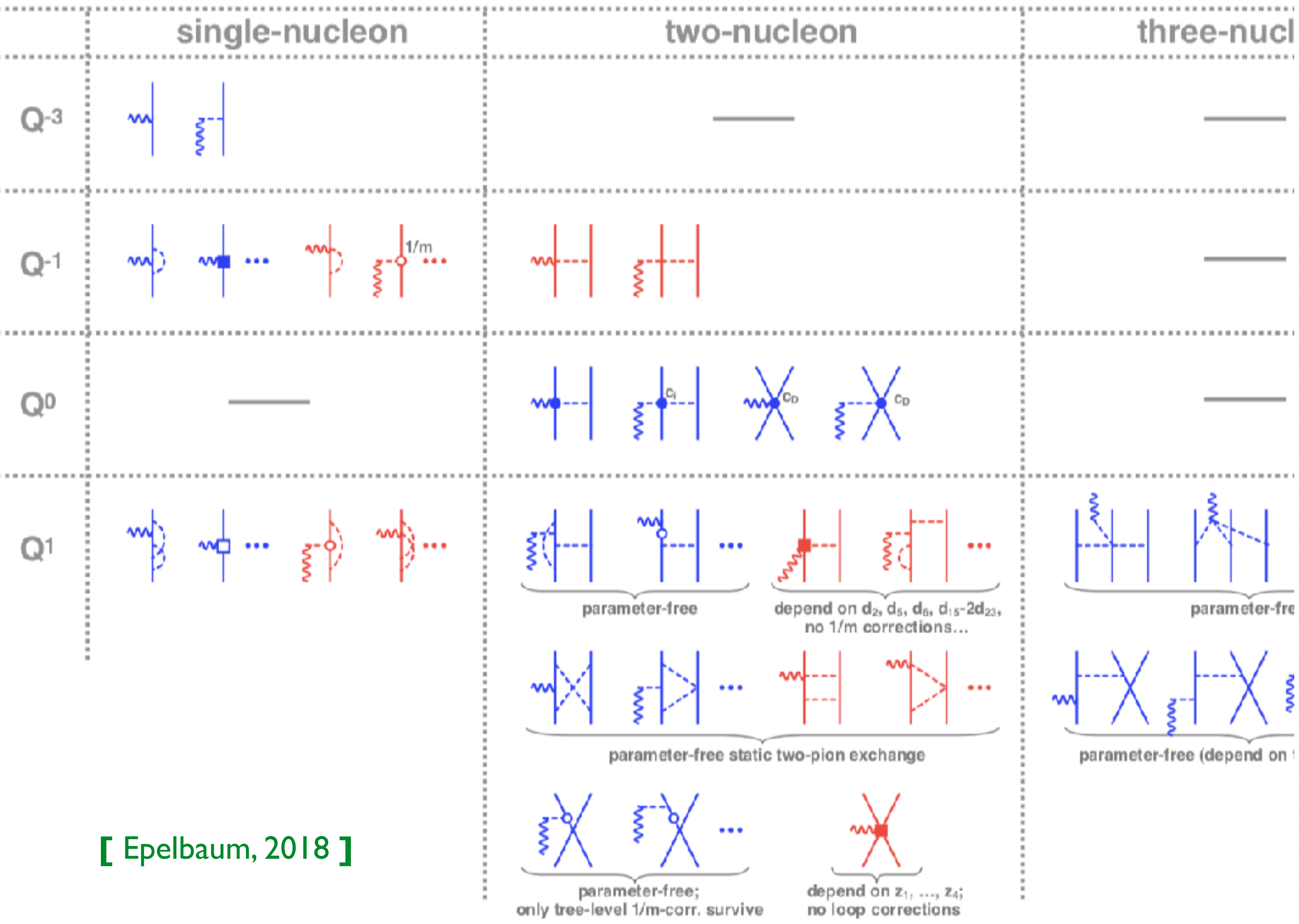
[Sun, Wang, **Yu**, in préparation]

$$N(x) = e^{-iMv\cdot x}\left[\underbrace{e^{iMv\cdot x}P_v^+N(x)}_{\equiv\mathcal{N}(x)} + \underbrace{e^{iMv\cdot x}P_v^-N(x)}_{\equiv\mathcal{H}(x)}\right]$$

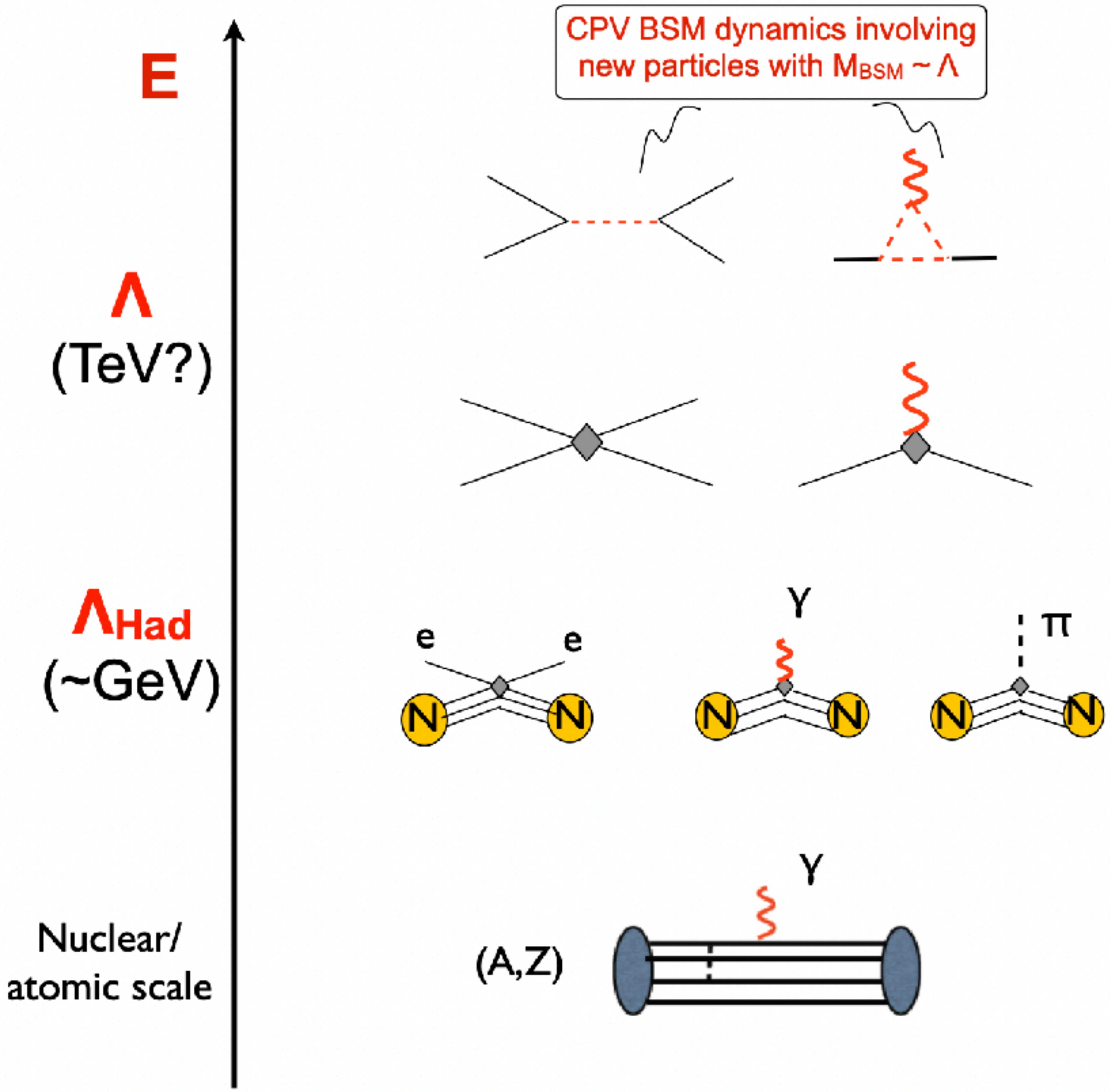
[Li, Wang, **Yu**, in préparation]

Nuclear Weak Currents

Explore the nuclear weak currents (EDM, 0vbb, etc) in chiral EFT

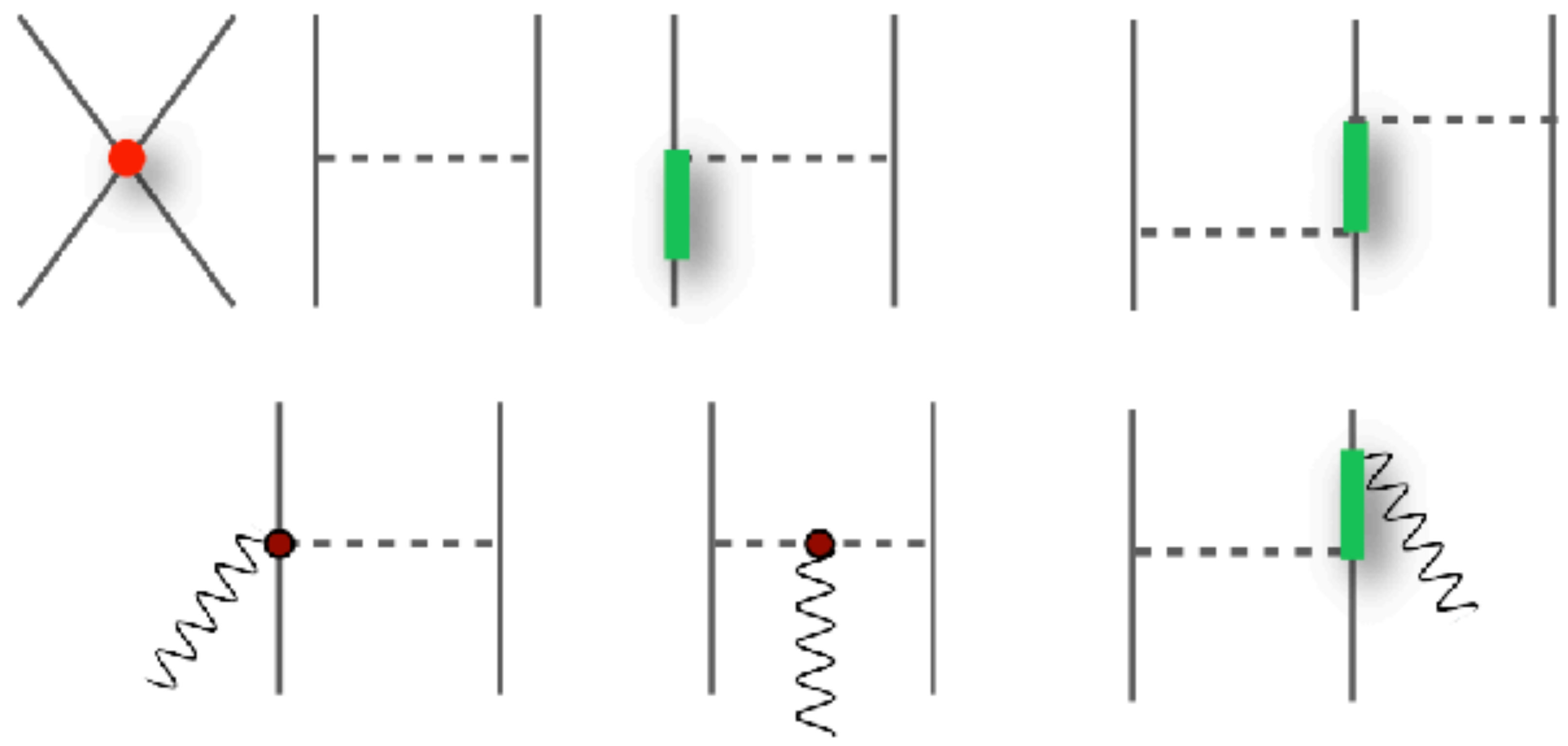


[Epelbaum, 2018]



Ab initio nuclear structure

Effective Hamiltonians and consistent currents

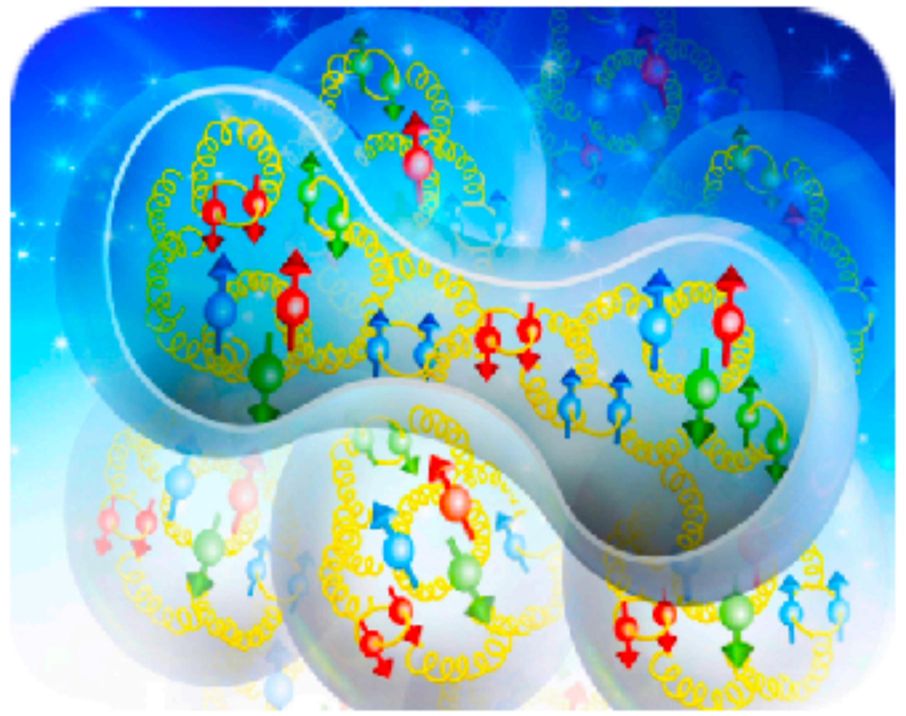


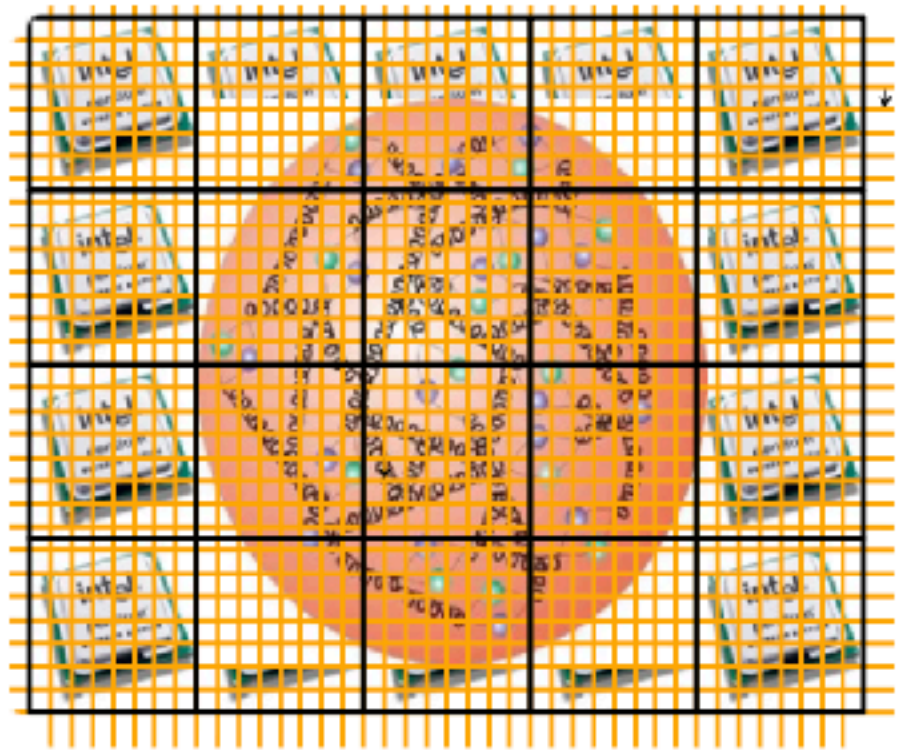
Accurate nuclear many-body methods

$$H|\Psi_n\rangle = E_n|\Psi_n\rangle$$

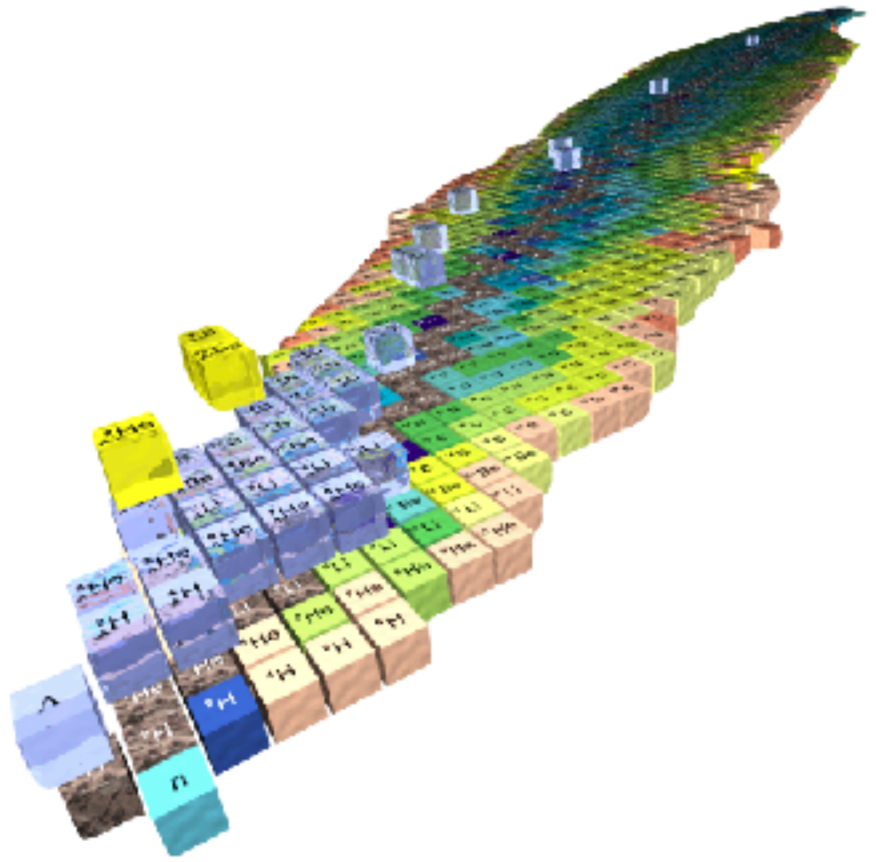
$$J_{mn} = \langle\Psi_m|J|\Psi_n\rangle$$

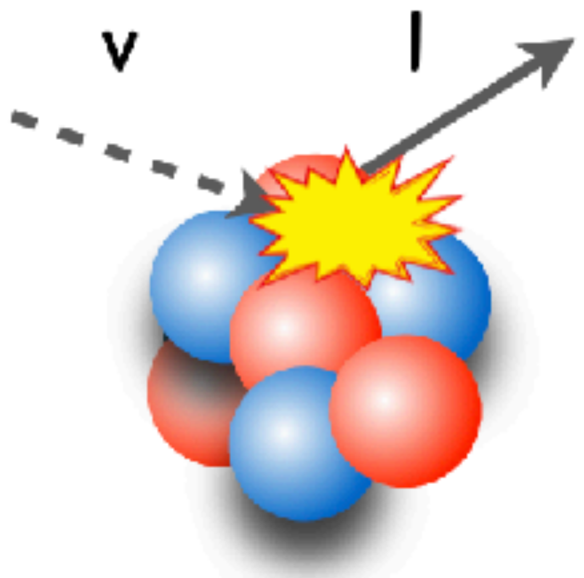
Quantum Chromodynamcs





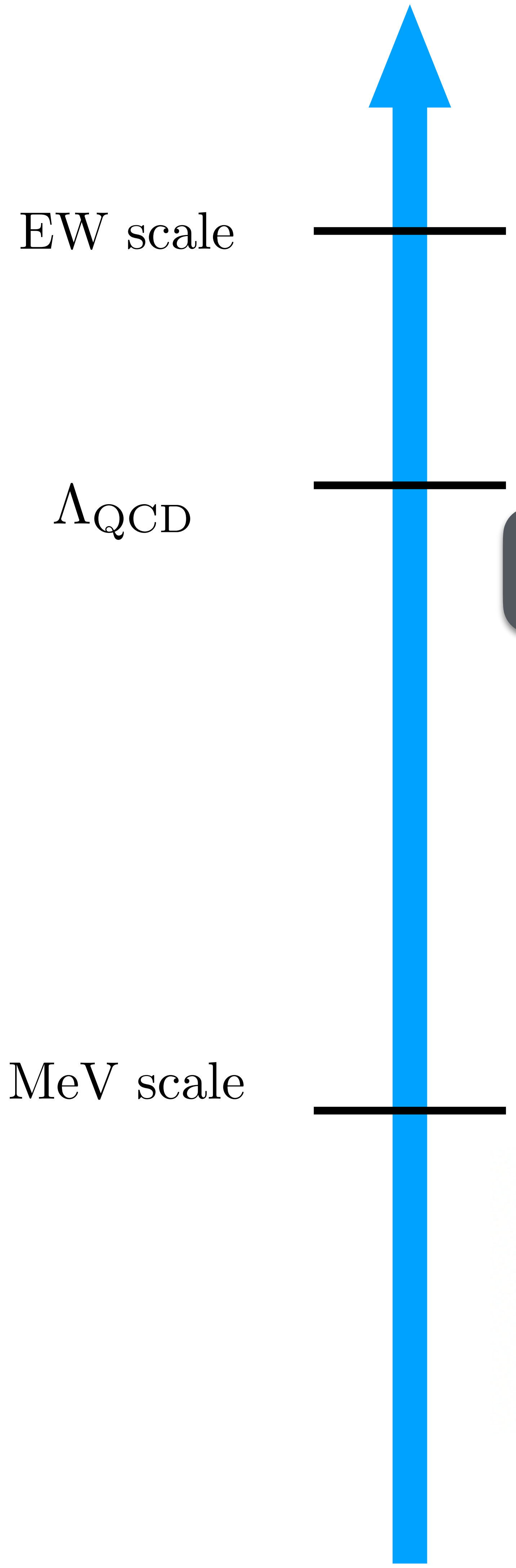
Nuclei and electroweak interactions





Neutrinoless double beta decay

Involving in meson, meson-baryon, baryon-baryon Lagrangian with weak currents



Standard model effective field theory up to dim-9

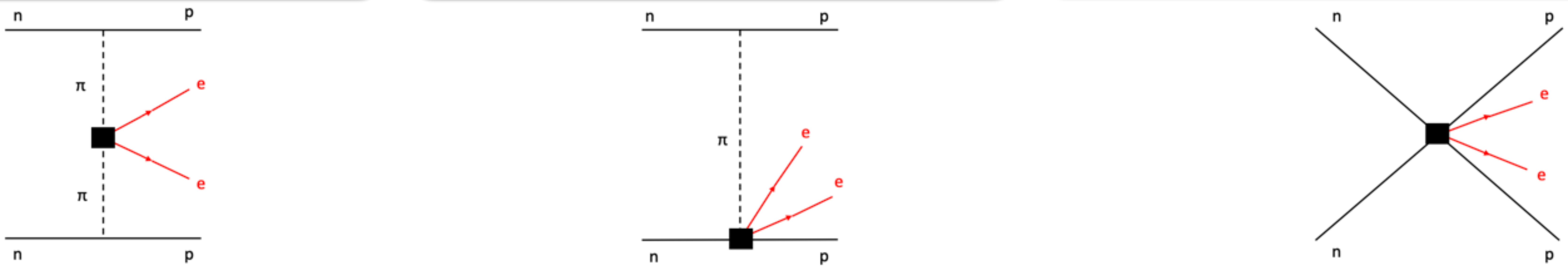
Low energy effective field theory up to dim-9

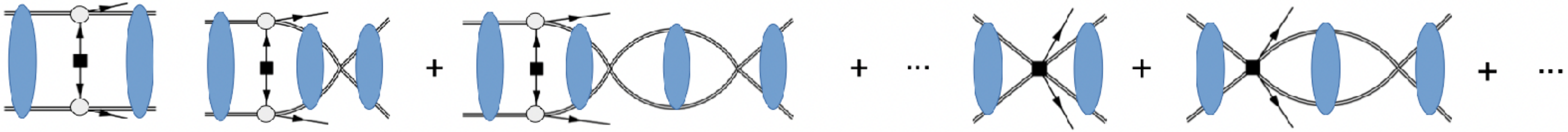
[Sun, Wang, **Yu**, in préparation]

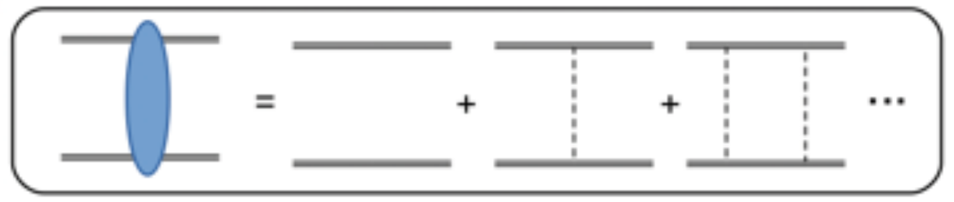
Pure meson with lepton source

Meson-nucleon with lepton source

Nucleon-nucleon with lepton source



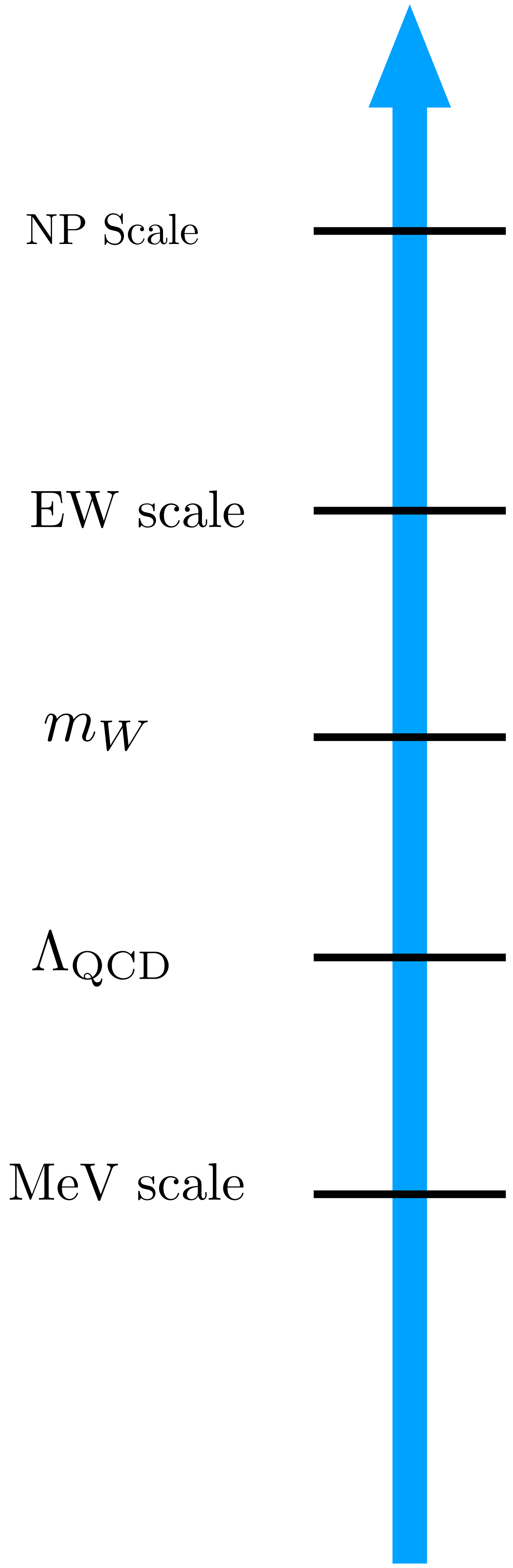




[Cirigliano, Dekens, de Vries, Graesser, Mereghetti,etc 2019]

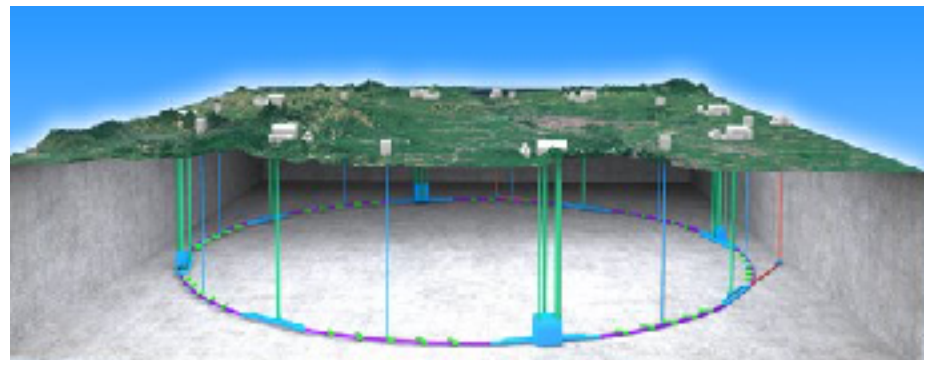
Low energy probe of high energy physics

Intensity frontier: weak currents of nuclear processes



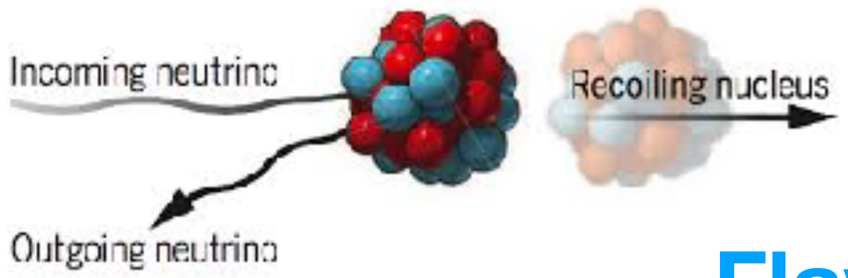
Energy frontier

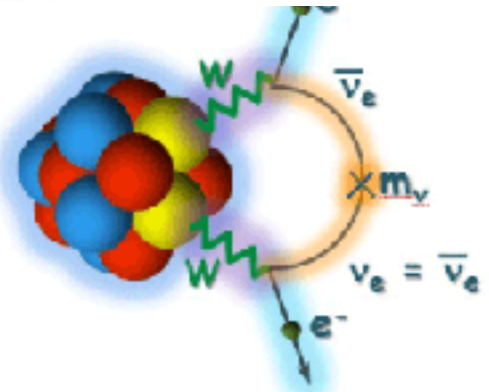
high energy, high cost!

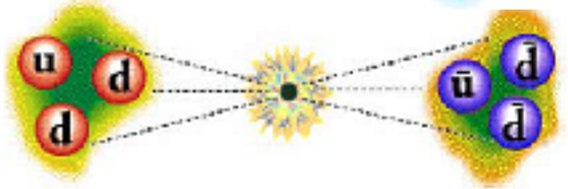


Intensity frontier

high intensity, low cost!



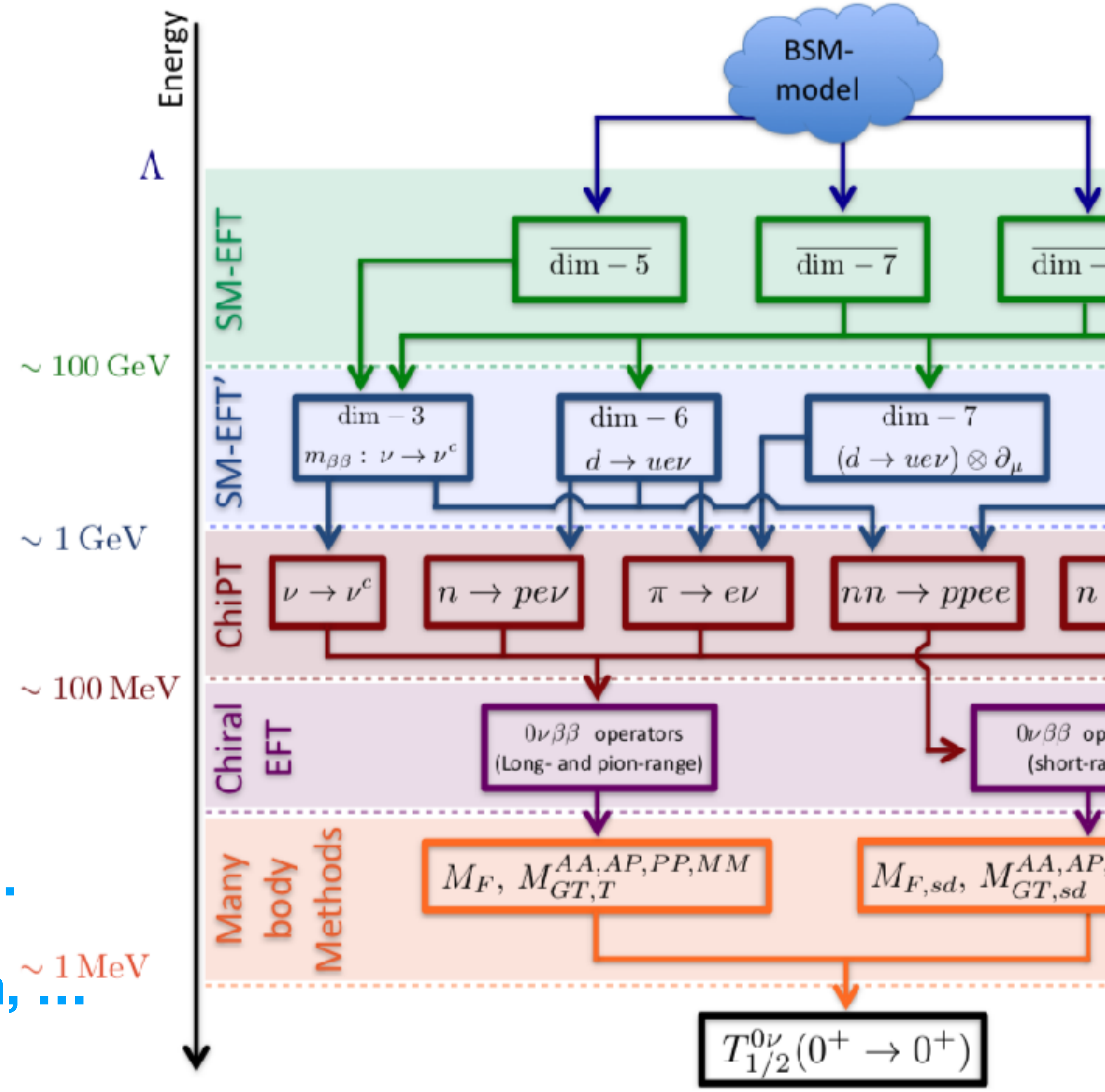




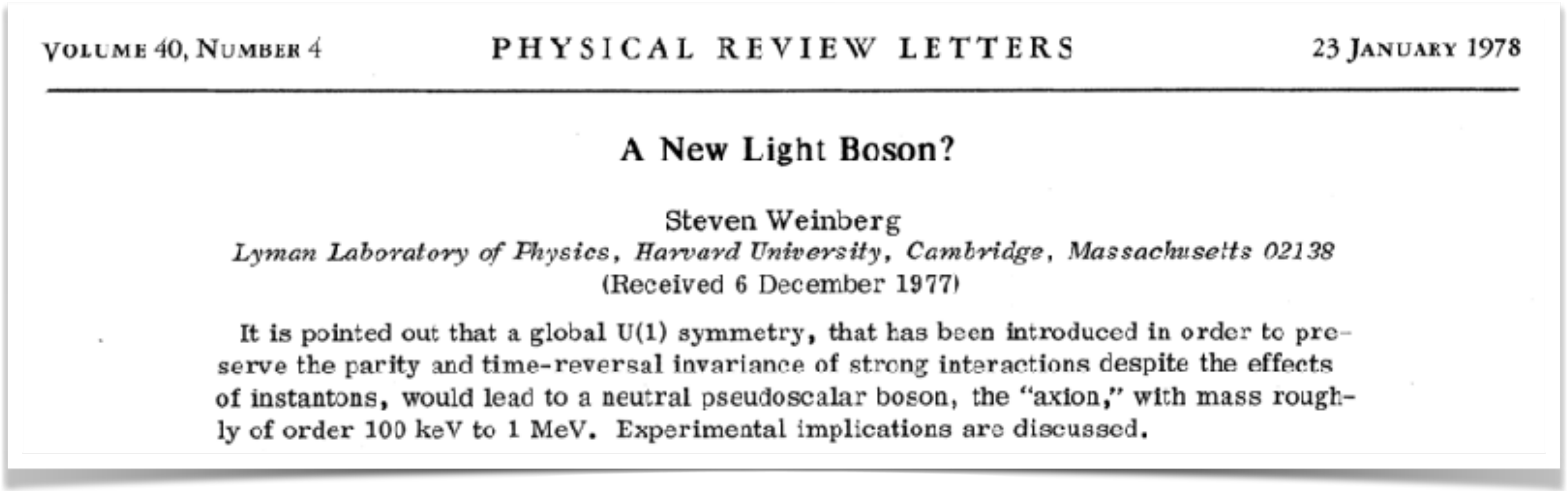
Flavor physics: pion, kaon, bottom, …

Rare process: cLFV, mu-e conversion, …

Neutrino physics: NSI, CEvNS, 0vbb, …



Jiang-Hao Yu (ITP-CAS)


VOLUME 40, NUMBER 4 PHYSICAL REVIEW LETTERS 23 JANUARY 1978

A New Light Boson?

Steven Weinberg
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138
(Received 6 December 1977)

It is pointed out that a global U(1) symmetry, that has been introduced in order to preserve the parity and time-reversal invariance of strong interactions despite the effects of instantons, would lead to a neutral pseudoscalar boson, the "axion," with mass roughly of order 100 keV to 1 MeV. Experimental implications are discussed.

Axion Effective Field Theory

[Huayang Song, Hao Sun, **J.H.Yu**, 2305.16770]

[Huayang Song, Hao Sun, **J.H.Yu**, 2306.05999]

QCD Theta Term

Gauge invariant degenerate QCD vacuum (instanton) induces theta term in Lagrangian

Chiral fermion

$$\mathcal{L} = -\frac{1}{4}G^{a\mu\nu}G^a_{\mu\nu} + \theta\frac{g_s^2}{32\pi^2}G^{a\mu\nu}\tilde{G}^a_{\mu\nu} + \sum_j \bar{\psi}_j\left(i\bar{D} - M_j e^{i\theta_j^\rho\gamma_5}\right)\psi_j$$

$$U(1)_A : q_i \to e^{-i\alpha\gamma_5} q_i$$

$$q_{Li} \to e^{+i\alpha} q_{Li}, q_{Ri} \to e^{-i\alpha} q_{Ri}$$

$$\delta\mathscr{L} = \alpha\frac{Ng^2}{16\pi^2}\mathrm{Tr}\left(G^{\mu\nu}\tilde{G}^{\mu\nu}\right)$$

$$M \to e^{-2i\alpha} M$$

$$\arg\det M \to \arg\det M - 2\alpha N$$

Rephasing invariant

$$\bar{\theta} \to \bar{\theta}' = \theta + 2\alpha N + \arg\det M - 2\alpha N = \theta + \arg\det M = \bar{\theta}$$

If $\det M = 0$, $\arg\det M$ would not be a physical parameter anymore

Instanton changes chirality of fermion zero mode (anomaly)

$$\int d^4x \frac{g^2}{16\pi^2}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma} = \int d^4x \sum_n \bar{\phi}_n\gamma^5\phi_n = n_R - n_L$$

Generate t'Hooft vertex in Lagrangian

$$\mathcal{L} + \frac{\theta}{16\pi^2}\mathrm{Tr}(F_{\mu\nu} \star F_{\mu\nu}) \to \mathcal{L} + e^{-8\pi^2/g^2} e^{i\theta} \det \psi_L(x)\bar{\psi}_R(x) + e^{-8\pi^2/g^2} e^{-i\theta} \det \psi_R(x)\bar{\psi}_L(x)$$

Chiral condensate $\langle\bar{\psi}_-\psi_+\rangle_{\nu=1} = \det{}'(i\not{D})\,\bar{\phi}_0\phi_0$

Strong CP problem and Axion

[Weinberg 1933 - 2021]

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$$\mathscr{L} = f_\pi^2 \,\mathrm{Tr}\, \partial_\mu U \partial^\mu U^\dagger + a f_\pi^3 \,\mathrm{Tr}\, MU + b f_\pi^4 \det U + h.c. \qquad u \to e^{i\alpha} u, \qquad d \to e^{i\alpha} d, \qquad \theta \to \theta - 2\alpha.$$

$$V = -a f_\pi^3 \,\mathrm{Tr}\left(\begin{pmatrix} m_u e^{i\theta_u} & 0 \\ 0 & m_d e^{i\theta_d} \end{pmatrix} U\right) + h.c. = -2a f_\pi^3 \left[m_u \cos(\phi + \frac{\bar{\theta}}{2}) + m_d \cos(\phi - \frac{\bar{\theta}}{2}) \right]$$

$$\mathscr{L} = -m_N N U^\dagger N^c - c_1 N M N^c - c_2 N U^\dagger M^\dagger U^\dagger N^c - \frac{i}{2}(g_A - 1)\left[N^\dagger \sigma^\mu U \partial_\mu U^\dagger N + N^{c,\dagger} \sigma^\mu U^\dagger \partial_\mu U N^c \right]$$

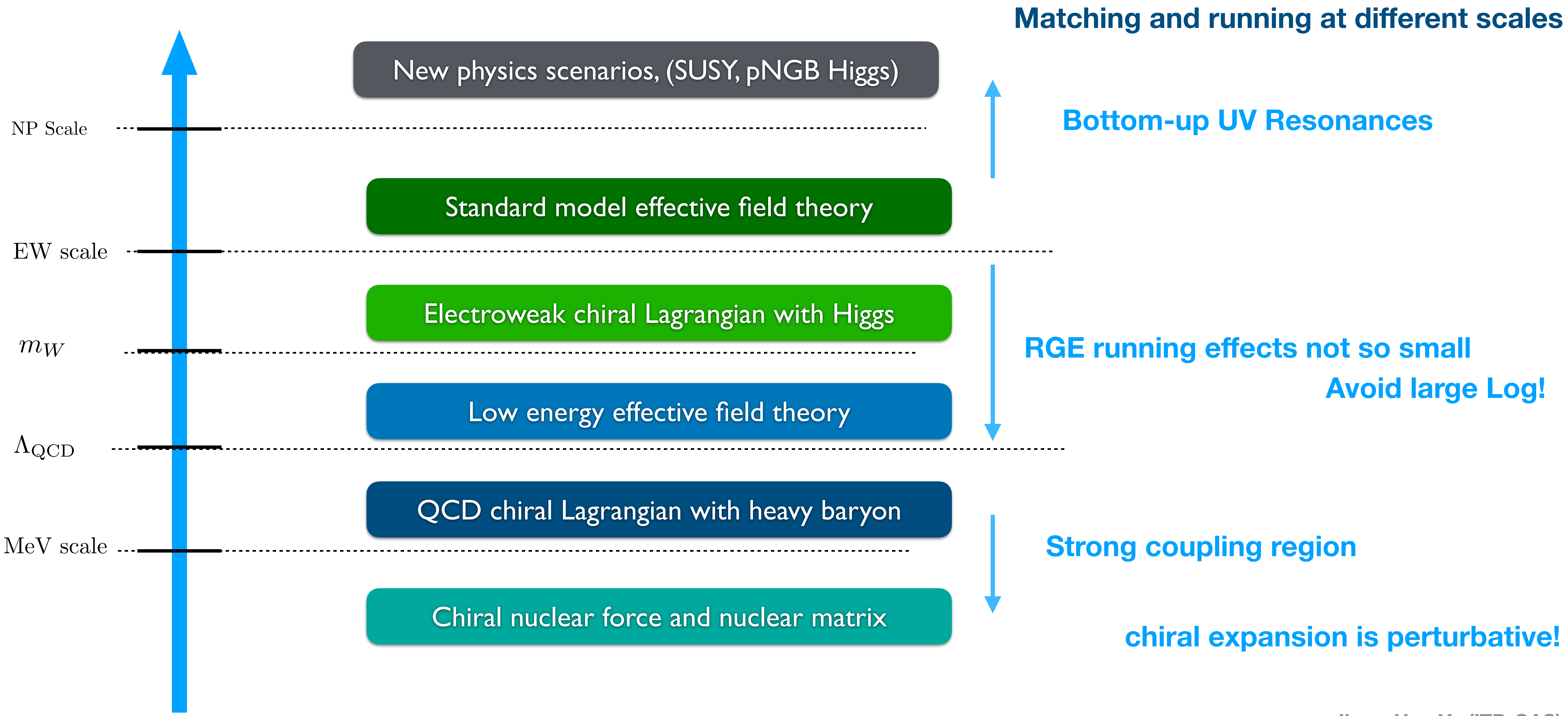
$$\mathscr{L} = -\bar{\theta} \frac{c+\mu}{f_\pi} \pi^a N \tau^a N^c - i \frac{g_A m_N}{f_\pi} \pi^a N \tau^a N^c, \qquad \mu = \frac{m_u m_d}{m_u + m_d}.$$

Stay Tuned!

Summary

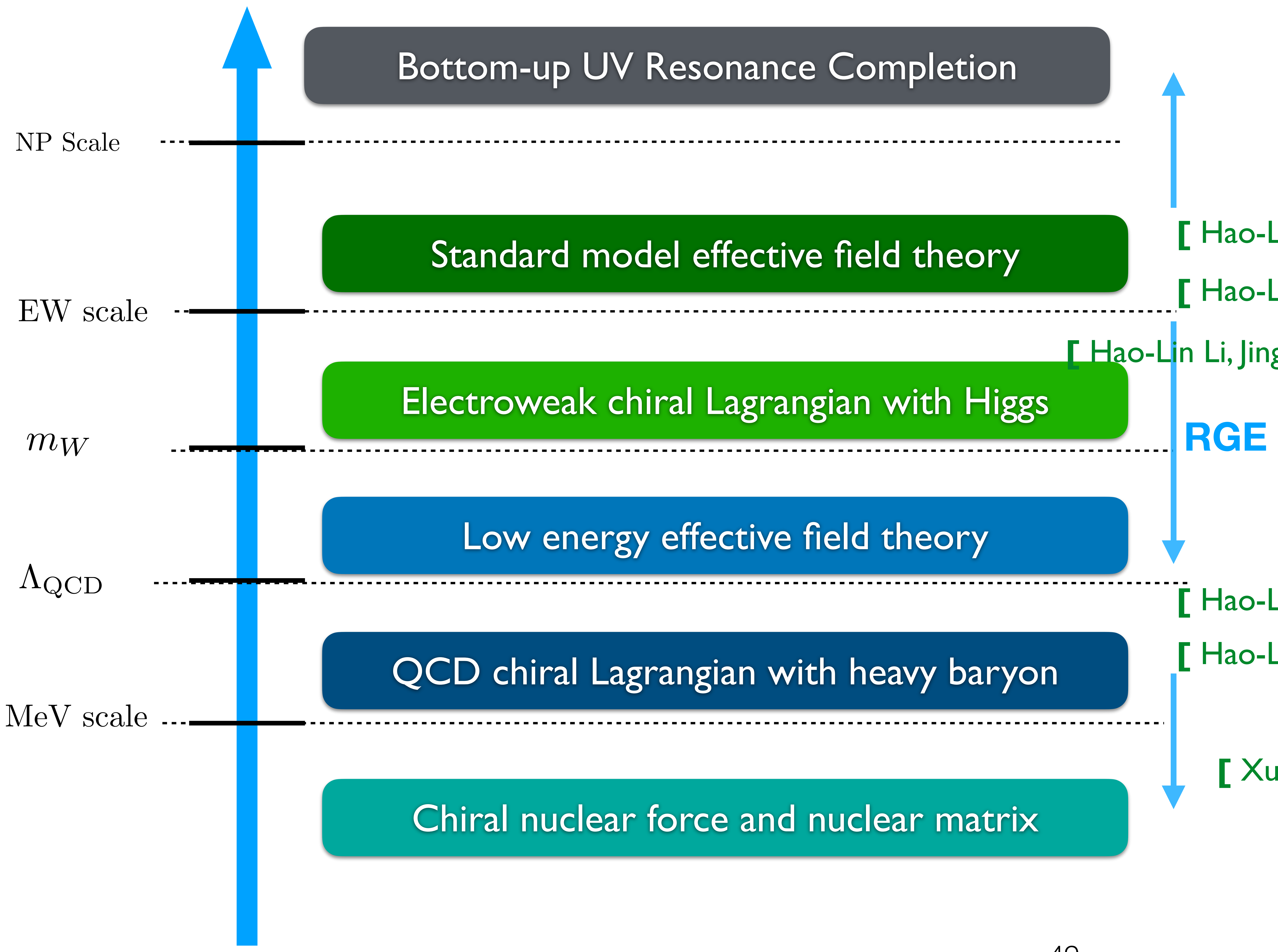
Summary

- Revisit tower of EFTs based on Young tensor, J-basis, and Adler zero/spurion techniques



Tower of effective field theories

Five years (2019 - 2023) on reorganizing effective field theories among several scales



[Hao-Lin Li, Yu-Han Ni, Ming-Lei Xiao, **J.H.Yu**, 2204.03660]

[Xu-Xiang Li, Zhe Ren, **J.H.Yu,** 2307.10380]

[Hao-Lin Li, Yu-Han Ni, Ming-Lei Xiao, **J.H.Yu**, 2306.15933]

[Yong Du, Xu-Xiang Li, **J.H.Yu,** 2201.04646]

[Hao-Lin Li, Zhe Ren, Ming-Lei Xiao, **J.H.Yu**, Yu-Hui Zheng, 2201.04639]

[Hao-Lin Li, Zhe Ren, Ming-Lei Xiao, **J.H.Yu**, Yu-Hui Zheng, 2007.07899]

[Hao-Lin Li, Jing Shu, Zhe Ren, Ming-Lei Xiao, **J.H.Yu**, Yu-Hui Zheng, 2005.00008]

[Hao Sun, Ming-Lei Xiao, **J.H.Yu**, 2206.07722]

[Hao Sun, Ming-Lei Xiao, **J.H.Yu**, 2210.14939]

[Hao Sun, Yi-Ning Wang, **J.H.Yu**, 2211.11598]

[Hao-Lin Li, Zhe Ren, Ming-Lei Xiao, **J.H.Yu**, Yu-Hui Zheng, 2012.09188]

[Hao-Lin Li, Zhe Ren, Ming-Lei Xiao, **J.H.Yu**, Yu-Hui Zheng, 2105.09329]

[Hao Sun, Yi-Ning Wang, **J.H.Yu**, in préparation]

[Xuan-He Li, Chuan-Qiang Song, Hao Sun, **J.H.Yu**, in préparation]

[Yong-Kang Li, Yi-Ning Wang, **J.H.Yu**, in préparation]

Thanks for your attention!