Spin entanglement in neutron-proton scattering

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Quantum entanglement

Q: What are **entangled states**?
A: First define what states are **not** entangled (aka **separable**), and the complementary set contains all the **entangled** states.

◦ **Separable** (pure) state: If an N-partite state ($N = 2$, bipartite state) can be decomposed into the tensor product of quantum states of its subunits

$$|\Phi\rangle = |\phi_1\rangle \otimes |\phi_2\rangle \otimes \cdots \otimes |\phi_N\rangle \,,$$

it is a **separable** state.

◦ **Entangled** state: If an N-partite state is not **separable**, it is an **entangled** state.

◦ Example: Bell states

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle), \quad |\Phi^-\rangle = \frac{1}{\sqrt{2}}(|\uparrow\uparrow\rangle - |\downarrow\downarrow\rangle),$$

$$|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle), \quad |\Psi^-\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle).$$

○ The term **"entanglement"** was introduced by **Schrödinger**, Math. Proc. Cambridge Philos. Soc. **31**, 555 (1935), as a response to **E**instein, **P**oldosky, and **R**osen, Phys. Rev. **47**, 777 (1935).

○ **At the very beginning of that paper**:

"When two systems, of which we know the states by their respective representatives, enter into temporary physical interaction due to known forces between them, and when after a time of mutual influence the systems separate again, then they can no longer be described in the same way as before, viz. by endowing each of them which a representative of its own. I would not call that one but rather the characteristic trait of quantum mechanics, the one that enforce its entire departure from classical lines of thought."

○ **entanglement generation by nuclear forces?**

Spin entanglement in neutron-proton scattering

○ Lamehi-Rachti & Mittig, PRD (1976): test Bell's inequality in $p+p$ scattering, less known in nuclear physics.

○ **Beane, Kaplan, Klco & Savage, PRL (2019)**: **the *S*-wave approximation**, followed up by Low & Mehen (2021), etc.

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Entanglement Suppression and Emergent Symmetries of Strong Interactions

Silas R. Beane, David B. Kaplan, Natalie Klco, and Martin J. Savage

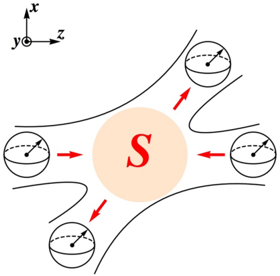
Phys. Rev. Lett. **122**, 102001 – Published 14 March 2019

○ **DB**, Phys. Lett. B **845**, 138162 (2023): **beyond the *S*-wave approximation**, submitted on May 4, 2023, discussed in this talk.

○ See also G. A. Miller, Phys. Rev. C **108**, L031002 (2023): similar work, submitted on June 5, 2023.

■ **Basic picture**: neutron-proton scattering (distinguishable **qubits**)

○ **Qubit**: quantum mechanical two-level system



Given $|\mathrm{in}\rangle = |\boldsymbol{p}\rangle\,|\chi_{\mathrm{in}}\rangle$, $|\mathrm{out}\rangle$ is given by

$$|\mathrm{out}\rangle = \int \mathrm{d}^2\hat{p}'\, |\boldsymbol{p}'\rangle\, \hat{\boldsymbol{S}}(\boldsymbol{p}',\boldsymbol{p})\,|\chi_{\mathrm{in}}\rangle\,,$$

$$\hat{\boldsymbol{S}}(\boldsymbol{p}',\boldsymbol{p}) \equiv \sum_{\substack{m_1' m_2' \\ m_1 m_2}} \hat{S}_{m_1' m_2' m_1 m_2}(\boldsymbol{p}',\boldsymbol{p})\,|m_1' m_2'\rangle\langle m_1 m_2|\,,$$

with $\langle \boldsymbol{p}' m_1' m_2' | S | \boldsymbol{p} m_1 m_2\rangle = \frac{\delta(E_{p'} - E_p)}{\mu p}$
$\times \hat{S}_{m_1' m_2 m_1 m_2}(\boldsymbol{p}',\boldsymbol{p})$.

■ **Our goal:** work out spin entanglement with the *exact* neutron-proton *S*-matrix and analyze its properties from different perspectives.

○ As $\hat{\boldsymbol{S}}(\boldsymbol{p}',\boldsymbol{p}) = \delta^2(\hat{p}' - \hat{p})\mathbf{1}_4 + i\frac{p}{2\pi}\boldsymbol{M}(\boldsymbol{p}',\boldsymbol{p})$, the spin component of the out state at a *specific* $\boldsymbol{p}' \neq \boldsymbol{p}$ is given by

$$|\mathrm{out}\rangle = \int \mathrm{d}^2\hat{p}'\, |\boldsymbol{p}'\rangle\, \hat{\boldsymbol{S}}(\boldsymbol{p}',\boldsymbol{p})\,|\chi_{\mathrm{in}}\rangle \;\Rightarrow\; |\chi_{\mathrm{out}}\rangle = \boldsymbol{M}(\boldsymbol{p}',\boldsymbol{p})\,|\chi_{\mathrm{in}}\rangle\,.$$

Neutron-proton spin amplitude:

The spin amplitude $\boldsymbol{M}(\boldsymbol{p}', \boldsymbol{p})$, as a 4×4 matrix, can be expanded in terms of $\{\sigma_\mu \otimes \sigma_\nu\}$, with $\sigma_\mu = (\mathbf{1}_2, \sigma_x, \sigma_y, \sigma_z)$.

○ First pioneered by **Wolfenstein** in 1950s.

○ Later on, similar parameterizations proposed by other groups, e.g., **Saclay**, Hoshizaki, Helicity, Singlet-Triplet.

Saclay amplitude system: (invariant under parity, time reversal, isospin, ...)

$$
\begin{aligned}
\boldsymbol{M}(\boldsymbol{p}', \boldsymbol{p}) = & \frac{1}{2}\Big\{(a+b) + (a-b)(\boldsymbol{\sigma}_1 \cdot \boldsymbol{n})(\boldsymbol{\sigma}_2 \cdot \boldsymbol{n}) \\
& + (c+d)(\boldsymbol{\sigma}_1 \cdot \boldsymbol{m})(\boldsymbol{\sigma}_2 \cdot \boldsymbol{m}) + (c-d)(\boldsymbol{\sigma}_1 \cdot \boldsymbol{l})(\boldsymbol{\sigma}_2 \cdot \boldsymbol{l}) \\
& + (e+f)\boldsymbol{\sigma}_1 \cdot \boldsymbol{n} + (e-f)\boldsymbol{\sigma}_2 \cdot \boldsymbol{n}\Big\},
\end{aligned}
$$

where $\boldsymbol{l}$, $\boldsymbol{m}$, $\boldsymbol{n}$ are the unit vectors

$$
\boldsymbol{l} = \frac{\boldsymbol{p}' + \boldsymbol{p}}{|\boldsymbol{p}' + \boldsymbol{p}|}, \quad \boldsymbol{m} = \frac{\boldsymbol{p}' - \boldsymbol{p}}{|\boldsymbol{p}' - \boldsymbol{p}|}, \quad \boldsymbol{n} = \frac{\boldsymbol{p} \times \boldsymbol{p}'}{|\boldsymbol{p} \times \boldsymbol{p}'|},
$$

and a, b, c, d, e, f are the six Saclay amplitudes depending on the relative momentum p and the angle θ between $\boldsymbol{p}$ and $\boldsymbol{p}'$.

Entanglement power and concurrence

■ **Entanglement power:** quantify entanglement generation capability of a quantum operator

$$\mathcal{E}(\boldsymbol{M}) = \int \frac{d\Omega_1}{4\pi} \int \frac{d\Omega_2}{4\pi} \left(1 - \mathrm{Tr}_1(\rho_1^2)\right),$$

$$|\chi_{\rm in}\rangle = [\cos(\theta_1/2)\,|\uparrow\rangle_1 + \exp(i\phi_1)\sin(\theta_1/2)\,|\downarrow\rangle_1] \otimes [\cos(\theta_2/2)\,|\uparrow\rangle_2 + \exp(i\phi_2)\sin(\theta_2/2)\,|\downarrow\rangle_2]\,,$$

$$|\chi_{\rm out}\rangle = \boldsymbol{M}(\boldsymbol{p}', \boldsymbol{p})\,|\chi_{\rm in}\rangle\,,$$

$$\rho_{12} = |\chi_{\rm out}\rangle\langle\chi_{\rm out}|\,/\,\langle\chi_{\rm out}|\chi_{\rm out}\rangle\,, \quad \rho_1 = \mathrm{Tr}_2(\rho_{12}),$$

i.e., averaging the 2-entropy of $|\chi_{\rm out}\rangle$ over all the possible $|\chi_{\rm in}\rangle$.

P. Zanardi, Phys. Rev. A **63**, 040304(R) (2001)

■ **Concurrence:** quantify spin entanglement of a specific $|\chi_{\rm out}\rangle$.

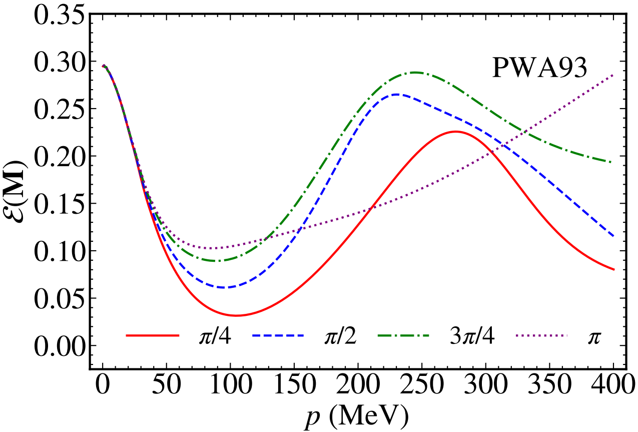
○ With $|\chi_{\rm out}\rangle = \alpha\,|00\rangle + \beta\,|01\rangle + \gamma\,|10\rangle + \delta\,|11\rangle$, the concurrence is given by

$$\Delta(p, \theta) = 2|\alpha\delta - \beta\gamma|,$$

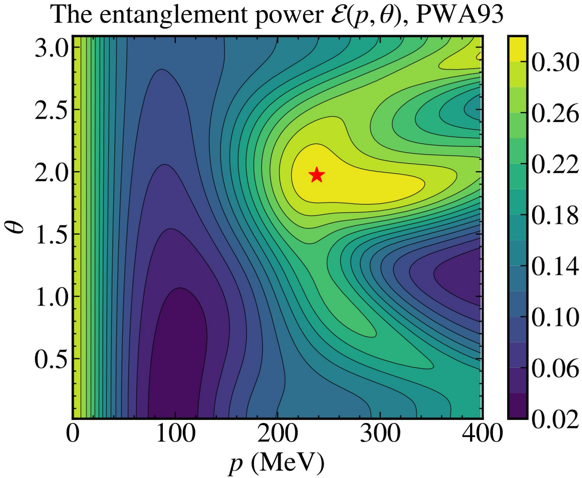
satisfying $0 \leq \Delta(p,\theta) \leq 1$, with $\Delta(p,\theta) = 0$ and $\Delta(p,\theta) = 1$ being separable and maximally entangled states.

$\mathcal{E}(\boldsymbol{M})$ as a function of the relative momentum p at $\theta = \pi/4$, $\pi/2$, $3\pi/4$, and π, taking the PWA93 amplitude and phase-shift data from the Nijmegen group as inputs.

V. G. J. Stoks *et al.*, Phys. Rev. C **48**, 792 (1993).

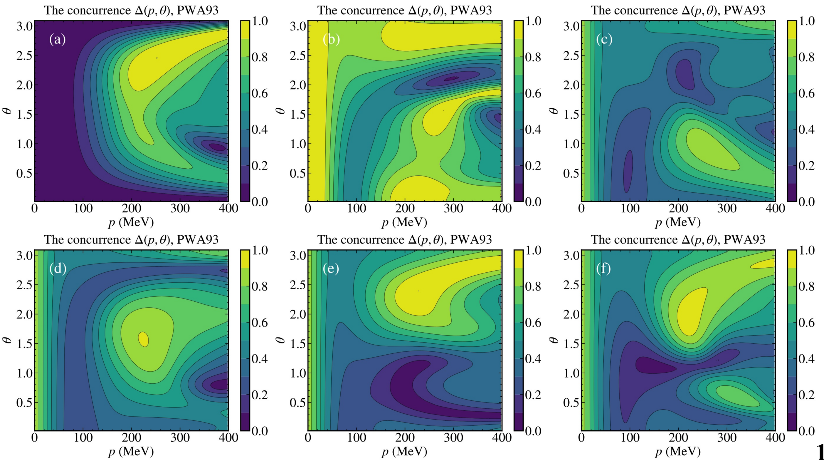


Dependence of $\mathcal{E}(\boldsymbol{M})$ on p and θ. $\mathcal{E}(\boldsymbol{M})$ takes the maximal value at $(p, \theta) \approx (238\ \mathrm{MeV}, 113^\circ)$.



The dependence of the out-state concurrence $\Delta(p, \theta)$ on the relative momentum p and the scattering angle θ, when taking different initial spin orientations.

- (a) $|\uparrow\uparrow\rangle$, (b) $|\uparrow\downarrow\rangle$, (c) $|\uparrow\rangle \otimes \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)$, (d) $|\uparrow\rangle \otimes \frac{1}{\sqrt{2}}(|\uparrow\rangle - |\downarrow\rangle)$, (e) $|\uparrow\rangle \otimes \frac{1}{\sqrt{2}}(|\uparrow\rangle + i\,|\downarrow\rangle)$, (f) $|\uparrow\rangle \otimes \frac{1}{\sqrt{2}}(|\uparrow\rangle - i\,|\downarrow\rangle)$



Symmetry emergence and entanglement extrema

- At low energies with $p \ll M_\pi$, the $n+p$ scattering can be described by **pionless effective field theory** ($\not\pi$EFT) at leading order (LO)

$$\mathcal{L}_{\text{LO}} = \mathcal{L}_{\text{kin}} + \mathcal{L}_{\text{int}},$$

$$\mathcal{L}_{\text{kin}} = N^\dagger \left(i\partial_t + \frac{\boldsymbol{\nabla}^2}{2M_N} \right) N,$$

$$\mathcal{L}_{\text{int}} = -\frac{C_S}{2}(N^\dagger N)(N^\dagger N) - \frac{C_T}{2}(N^\dagger \boldsymbol{\sigma} N)\cdot(N^\dagger \boldsymbol{\sigma} N),$$

with $N = (n_\uparrow, n_\downarrow, p_\uparrow, p_\downarrow)^T$ being the nucleon field, and C_S, C_T being the low-energy constants.

- The neutron-proton S-matrix is given by

$$\hat{\boldsymbol{S}}(p) = \delta^2(\hat{p}' - \hat{p})\hat{\mathbf{1}}_4 + i\frac{p}{2\pi}\boldsymbol{M}(p),$$

$$\boldsymbol{M}(p) = \frac{1}{8ip}\left[(3e^{2i\delta_1} + e^{2i\delta_0} - 4)\hat{\mathbf{1}}_4 + (e^{2i\delta_1} - e^{2i\delta_0})\hat{\boldsymbol{\sigma}}\cdot\hat{\boldsymbol{\sigma}}\right],$$

with δ_0 and δ_1 being the $^1S_0$ and $^3S_1$ phase shift.

■ Effective range expansion

In the low-energy limit $p \to 0$,

$$e^{2i\delta_{0,1}} = \frac{1 + i\tan\delta_{0,1}}{1 - i\tan\delta_{0,1}} = \frac{1 - ipa_{0,1}}{1 + ipa_{0,1}},$$

with a_0 and a_1 being $^1S_0$ and $^3S_1$ scattering lengths.

■ Wigner symmetry

$N = (n_\uparrow, n_\downarrow, p_\uparrow, p_\downarrow)^T$ is the fundamental representation of SU(4)

$$N \to \mathrm{SU}(4)N.$$

It becomes the exact symmetry as $C_T \to 0$,

$$\mathcal{L}_{\mathrm{int}} = -\frac{C_S}{2}(N^\dagger N)(N^\dagger N).$$

Correspondingly, the scattering lengths satisfy $a_0 = a_1$.

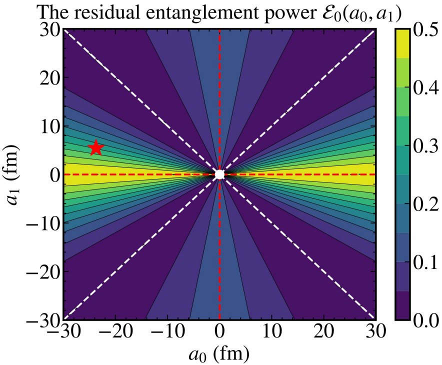
■ Schrödinger symmetry (nonrelativistic conformal symmetry)

When $(|a_0|, |a_1|) = (0,0), (0,\infty), (\infty,0)$ and (∞,∞), $\not{\pi}$EFT has Schrödinger symmetry at LO.

■ Spin entanglement *vs* symmetry emergence

○ First pioneered by S. R. Beane *et al.*, Phys. Rev. Lett. **122**, 102001 (2019).

○ I study the connection between **the residual entanglement power $\mathcal{E}_0(a_0, a_1)$** and symmetry emergence.



- **Symmetry emergence** appears when $a_0 = a_1 \rightarrow$ Wigner symmetry and $(|a_0|, |a_1|) = (0, 0), (0, \infty), (\infty, 0), (\infty, \infty) \rightarrow$ Schrödinger symmetry.
- **Entanglement extrema** appear at $|a_0| = |a_1|$, $|a_0| = 0$ **and** $|a_1| = 0 \Rightarrow$ The appearance of **entanglement extrema** provides the necessary condition for symmetry emergence in the neutron-proton system.

Summary

■ **Spin entanglement in neutron-proton scattering**

○ go beyond the S-wave approximation: strong dependence on relative momentum, scattering angle, and initial spin configuration;

○ entanglement extrema as necessary conditions for emergent low-energy symmetries in the neutron-proton system.

○ see **DB**, 2308.12327 for preliminary results towards experimental determination of spin entanglement of nucleon pairs.