

# QUARK MODEL WITH HIDDEN LOCAL SYMMETRY AND ITS APPLICATION TO THE HADRON SPECTRUM

## 具有隐藏定域对称性的夸克模型及其在强子谱 中的应用

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2306.03526  
2307.16280

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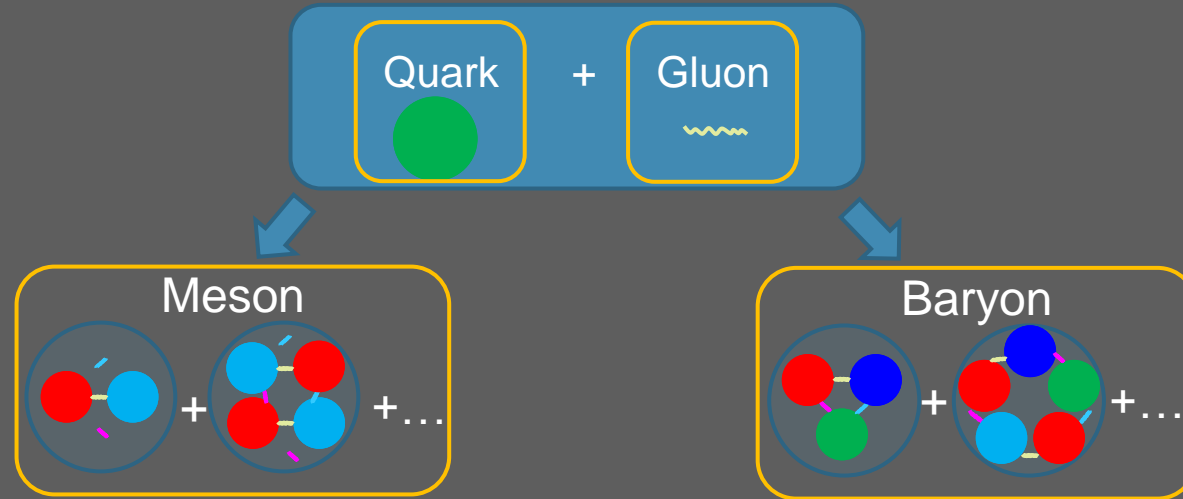
# Outline

## ➤ Introduction

- ⊙ Chiral quark model with HLS
- ⊙  $SU2$  ground states + excited states
- ⊙  $SU3$  ground states
- ⊙ Summary

# Introduction

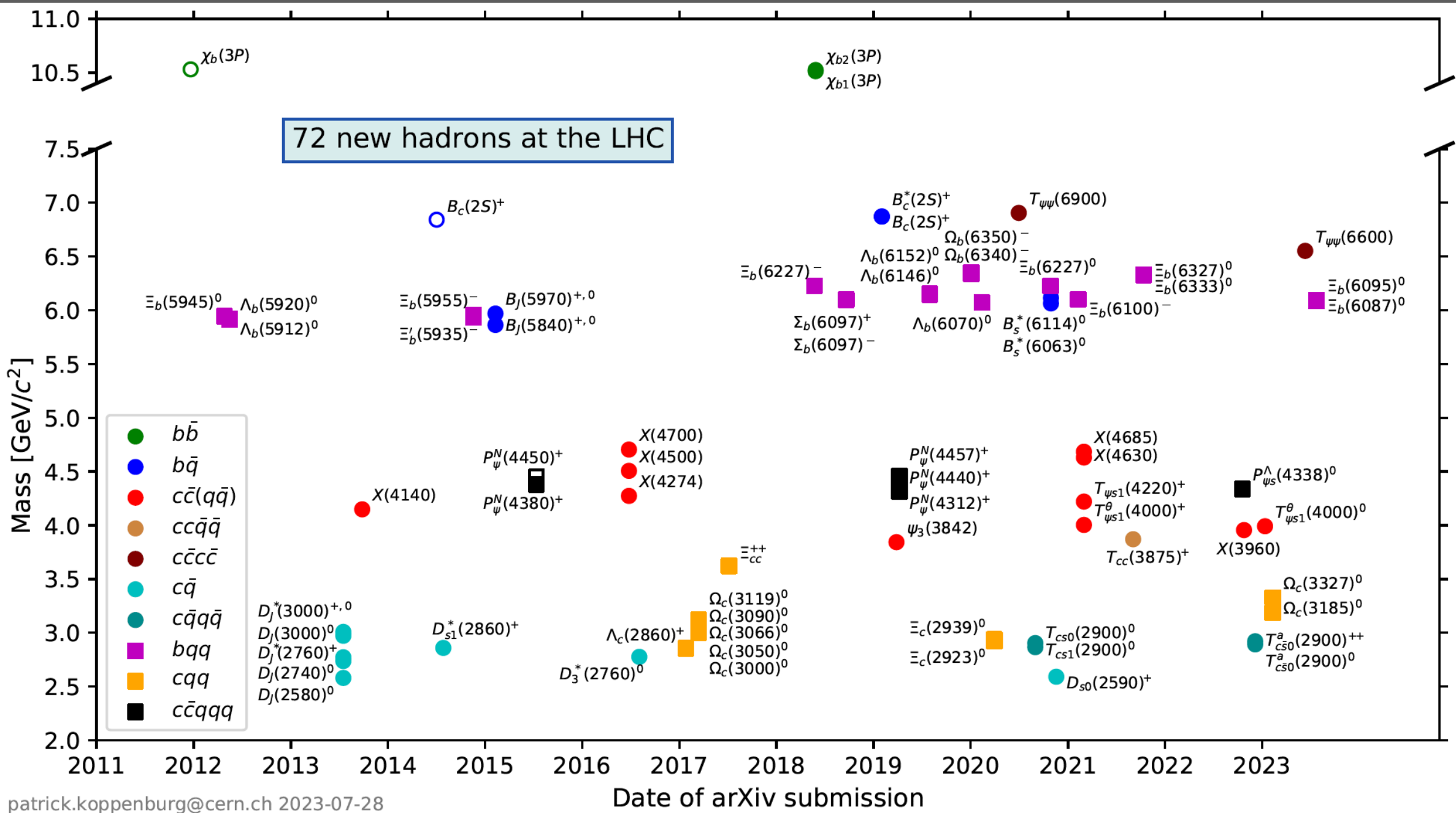
Hadron are made by quarks and gluons

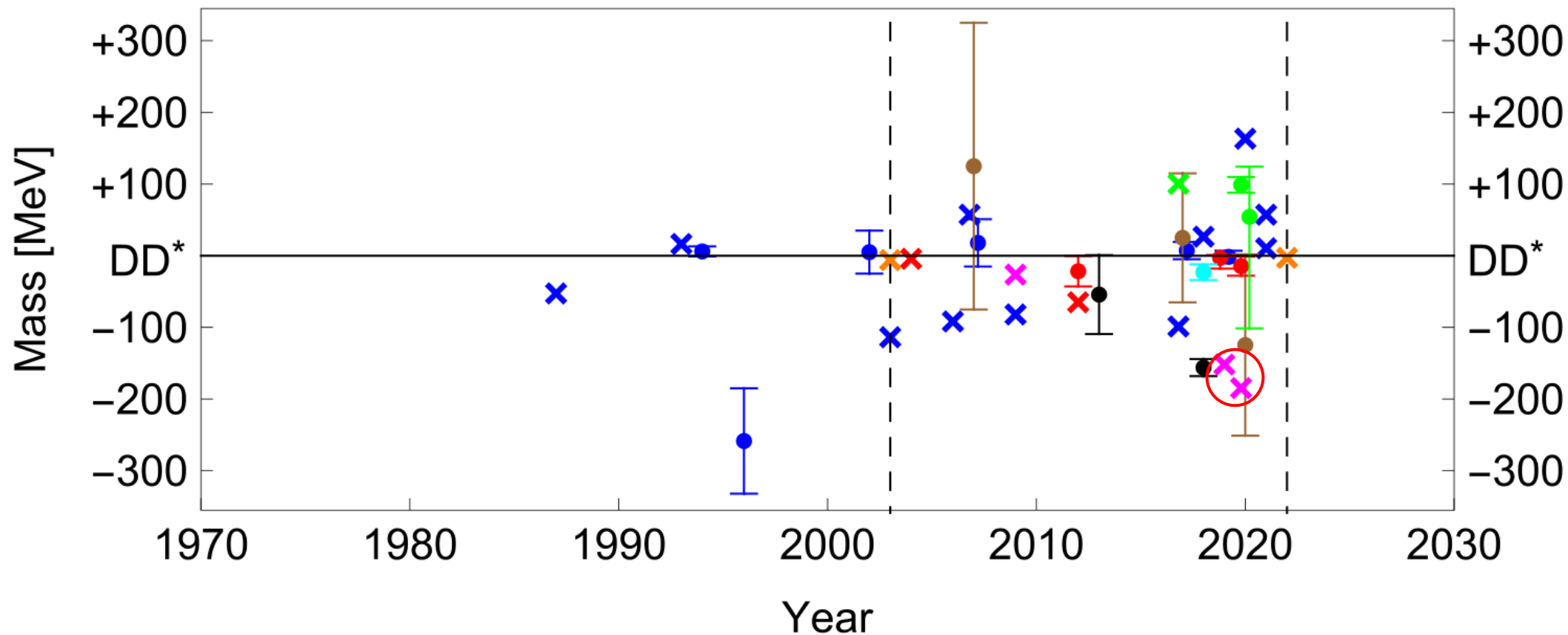


The dynamics of quarks and gluons are described by Quantum chromodynamics (QCD)

- QCD have two important features:
  - ◆ Color confinement
  - ◆ Asymptotic freedom
- In low energy region the perturbative calculation for QCD is impossible, alternatively:
  - ◆ Lattice QCD (non-perturbative calculation)
  - ◆ Effective models (chiral perturbation theory, quark model, etc...)







**Figure 42.** Theoretical predictions on the mass of the doubly charmed tetraquark state  $cc\bar{u}\bar{d}$  with  $(I)J^P = (0)1^+$ , with uncertainties (error bars) and without uncertainties (crosses), calculated based on the compact tetraquark picture through various quark models [454, 500–515] (blue), QCD sum rules [462, 516, 517] (brown), heavy quark symmetry [467–469] (green), and others [461, 466] (black), as well as those calculated through the hadronic molecular picture [477, 479, 518–520] (red), the quark model considering the mixture of the meson-meson and diquark–antidiquark structures [481–483] (magenta), and lattice QCD [499] (cyan). The two dashed lines with orange crosses denote the  $\chi_{c1}(3872)$  ( $X(3872)$ ) first observed by Belle in 2003 [30] and the  $T_{cc}^+$  recently observed by LHCb in 2021 [42, 43].

# Meson exchange in nuclear force:

- $\pi(138)$  **Long-ranged** tensor force
- $\sigma(500)$  **intermediate-ranged**, attractive central force plus LS force
- $\omega(782)$  **short-ranged**, repulsive central force plus strong LS force
- $\rho(770)$  **short-ranged** tensor force, opposite to pion

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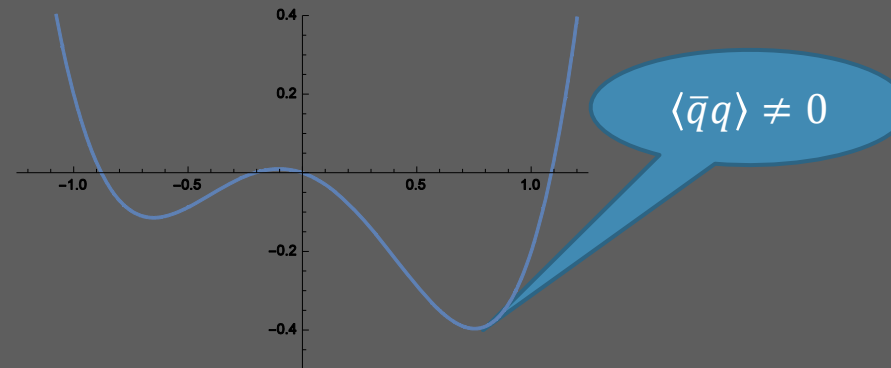
# The chiral symmetry

The chiral symmetry:



Spontaneously breaking of chiral symmetry:

$$\langle \bar{q}q \rangle = \langle \bar{q}_L q_R + \bar{q}_R q_L \rangle \neq 0$$



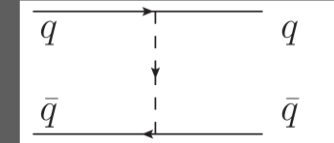
The effective theory based on chiral symmetry:

- Nonlinear sigma model
- Chiral perturbation theory

# Chiral quark model

## Naïve quark model:

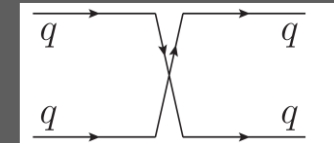
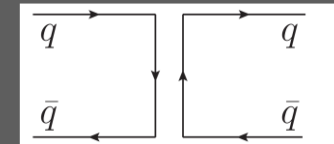
- Quark mass term
- Kinetic term
- Color confinement potential (CON)
- One gluon exchange (OGE)



- Gell-Mann, M., 1964, Phys. Lett. 8, 214.
- Zweig, G., 1964, CERN Reports No. 8182/TH. 401 and No. 8419/TH. 412).
- N. Isgur, G. Karl, Phys.Lett.B 72 (1977) 109.

## The Nambu–Goldstone boson exchange:

- Chiral symmetry is spontaneously broken
- Pseudoscalars ( $\pi$ ,  $K$ ,  $\eta$ ) are the Nambu–Goldstone (NG) bosons of chiral symmetry breaking
- Scalar meson  $\sigma$  as the chiral partner of NG bosons



- K. Shimizu, Phys. Lett. B 148, 418-422 (1984)
- L.Ya Glozman, Z. Papp, W. Plessas, Physics Letters B 381 (1996) 311-316
- Z.Y. Zhang, Y.W. Yu, P.N. Shen, L.R. Dai, A. Faessler, U. Straub, Nucl. Phys. A 625 (1997) 59.
- J. Vijande, F. Fernandez, A. Valcarce, J. Phys. G 31, 481(2005)

# The Hamiltonian

$$H = \sum_{i=1}^n \left( m_i + \frac{p_i^2}{2m_i} \right) - T_{cm} + \sum_{j>i=1}^n (V_{ij}^{CON} + V_{ij}^{OGE} + V_{ij}^{\pi} + V_{ij}^K + V_{ij}^{\eta} + V_{ij}^{\sigma})$$

J . Vijande, F . Fernandez, A . Valcarce, J. Phys. G 31, 481(2005)

$$V_{ij}^{CON} = (\boldsymbol{\lambda}_i^c \cdot \boldsymbol{\lambda}_j^c) [-a_c (1 - e^{-\mu_c r_{ij}}) + \Delta],$$

$$V_{ij}^{OGE} = \frac{1}{4} \alpha_s (\boldsymbol{\lambda}_i^c \cdot \boldsymbol{\lambda}_j^c) \left[ \frac{1}{r_{ij}} - \frac{1}{6m_i m_j} \frac{e^{-\frac{r_{ij}}{r_0(\mu_{ij})}}}{r_{ij} r_0^2(\mu_{ij})} \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \right],$$

$$V_{ij}^{\sigma} = -\frac{g_{ch}^2}{4\pi} \frac{\Lambda_{\sigma}^2}{\Lambda_{\sigma}^2 - m_{\sigma}^2} m_{\sigma} \left[ Y(m_{\sigma} r_{ij}) - \frac{\Lambda_{\sigma}}{m_{\sigma}} Y(\Lambda_{\sigma} r_{ij}) \right],$$

$$V_{ij}^{\pi} = \frac{g_{ch}^2}{4\pi} \frac{m_{\pi}^2}{12m_i m_j} \frac{\Lambda_{\pi}^2}{\Lambda_{\pi}^2 - m_{\pi}^2} m_{\pi} \left[ Y(m_{\pi} r_{ij}) - \frac{\Lambda_{\pi}^3}{m_{\pi}^3} Y(\Lambda_{\pi} r_{ij}) \right] \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \sum_{a=1}^3 \lambda_i^a \lambda_j^a,$$

$$V_{ij}^K = \frac{g_{ch}^2}{4\pi} \frac{m_K^2}{12m_i m_j} \frac{\Lambda_K^2}{\Lambda_K^2 - m_K^2} m_K \left[ Y(m_K r_{ij}) - \frac{\Lambda_K^3}{m_K^3} Y(\Lambda_K r_{ij}) \right] \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \sum_{a=4}^7 \lambda_i^a \lambda_j^a,$$

$$V_{ij}^{\eta} = \frac{g_{ch}^2}{4\pi} \frac{m_{\eta}^2}{12m_i m_j} \frac{\Lambda_{\eta}^2}{\Lambda_{\eta}^2 - m_{\eta}^2} m_{\eta} \left[ Y(m_{\eta} r_{ij}) - \frac{\Lambda_{\eta}^3}{m_{\eta}^3} Y(\Lambda_{\eta} r_{ij}) \right] \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j (\lambda_i^8 \lambda_j^8 \cos \theta_p - \lambda_i^0 \lambda_j^0 \sin \theta_p).$$

# The Gaussian expansion method

$$\psi_{lm}(\mathbf{r}) = \sum_{n=1}^{n_{max}} c_n \psi_{nlm}^G(\mathbf{r}),$$

$$\psi_{nlm}^G(\mathbf{r}) = N_{nl} r^l e^{-\nu_n r^2} Y_{lm}(\hat{\mathbf{r}}),$$

$$N_{nl} = \left( \frac{2^{l+2} (2\nu_n)^{l+\frac{3}{2}}}{\sqrt{\pi} (2l+1)!!} \right)^{\frac{1}{2}},$$

$$\nu_n = \frac{1}{r_n^2}, r_n = r_{min} a^{n-1}, a = \left( \frac{r_{max}}{r_{min}} \right)^{\frac{1}{n_{max}-1}}.$$

E. Hiyama, Y. Kino, and M. Kamimura, Prog. Part. Nucl. Phys. 51 223 (2003).

# Incorporate the vector meson contribution

## The hidden local symmetry:

M. Bando, T. Kugo, K. Yamawaki.  
Phys.Rept. 164 (1988) 217-314  
M. Harada, K. Yamawaki.  
Phys.Rept. 381 (2003) 1-233

$$U = \xi_L^\dagger \xi_R = e^{2i \frac{\pi(x)}{f_\pi}}$$

$$\xi_{L,R} \rightarrow h(x) \xi_{L,R} \cdot g_{L,R}^\dagger$$

$$\xi_{L,R} = e^{i \frac{V(x)}{f_V}} e^{\mp i \frac{\pi(x)}{f_\pi}}$$

$$h(x)^\dagger h(x) = 1$$

$$h(x) \in H_{\text{local}}, \quad g_{L,R} \in G_{\text{global}}$$

- The transformation for U do not changes, which seems that the freedom of vector meson is “hidden”

$$[SU(N_f)_L \times SU(N_f)_R]_{\text{global}} \times [SU(N_f)_V]_{\text{local}} \rightarrow [SU(N_f)_V]_{\text{global}}$$

- Hidden local symmetry is an extension of chiral perturbation theory
- Hidden local symmetry is a systematic way to include pseudo-scalar mesons  $(\pi, K, \eta, \eta')$  and vector mesons  $(\rho, K^*, \omega, \phi)$

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# The Hamiltonian

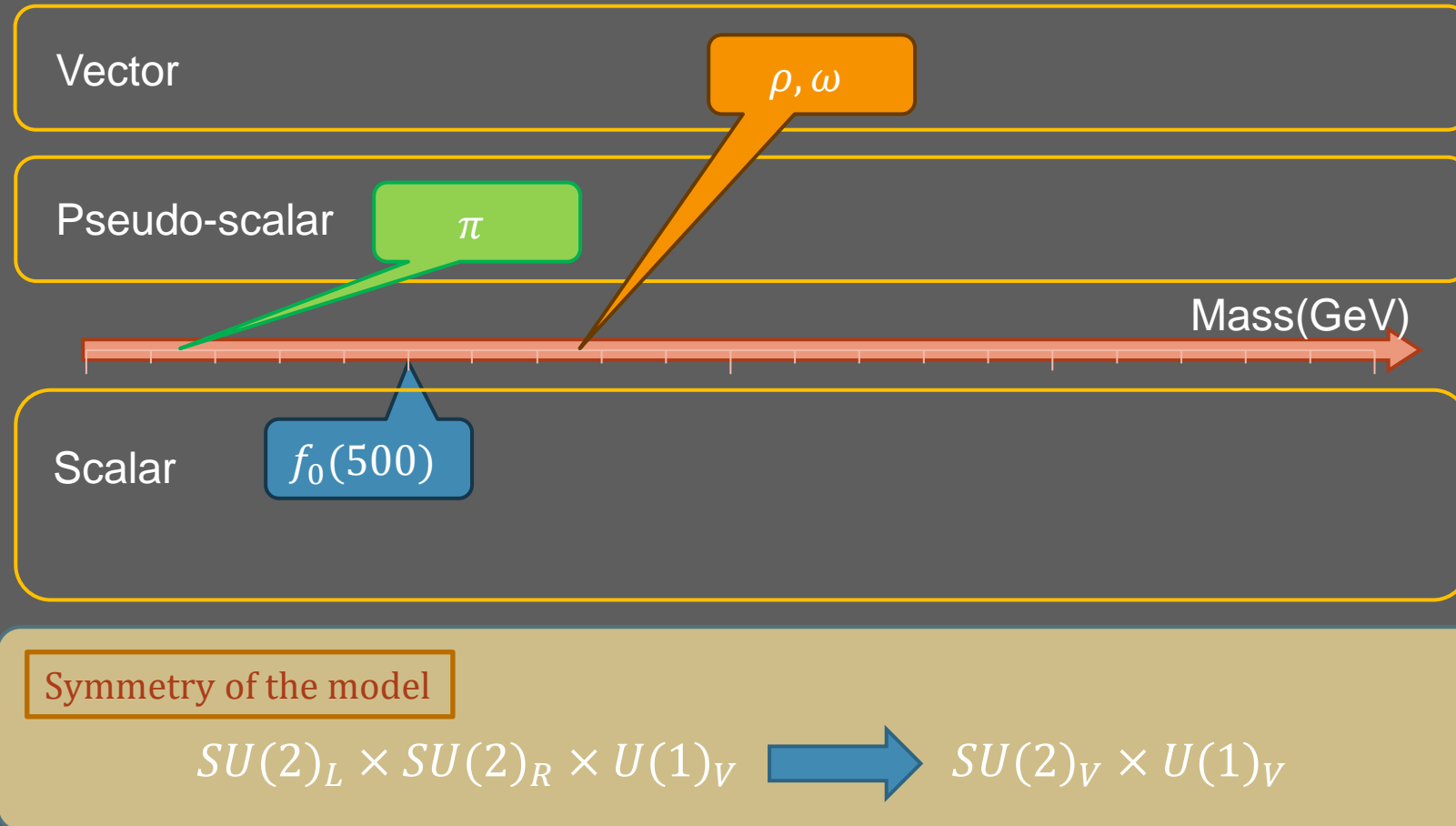
$$H = \sum_{i=1} \left( m_i + \frac{p_i^2}{2m_i} \right) - T_{CM} + \sum_{j>i=1} (V_{ij}^{\text{CON}} + V_{ij}^{\text{OGE}} + V_{ij}^{\sigma} + V_{ij}^{\pi} + V_{ij}^{\omega} + V_{ij}^{\rho})$$

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$$V_{ij}^v = \frac{\Lambda_v^2}{\Lambda_v^2 - m_v^2} \left\{ \frac{g_v^2}{4\pi} m_v \left[ Y(m_v r) - \left( \frac{\Lambda_v}{m_v} \right) Y(\Lambda_v r) \right] + \frac{m_v^3}{m_i m_j} \left( \frac{g_v(2f_v + g_v)}{16\pi} + \frac{\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j (f_v + g_v)^2}{6 \cdot 4\pi} \right) \times \left[ Y(m_v r) - \left( \frac{\Lambda_v}{m_v} \right)^3 Y(\Lambda_v r) \right] - \boldsymbol{S}_+ \cdot \boldsymbol{L} \frac{g_v(4f_v + 3g_v)}{8\pi} \frac{m_v^3}{m_i m_j} \times \left[ G(m_v r) - \left( \frac{\Lambda_v}{m_v} \right)^3 G(\Lambda_v r) \right] - \boldsymbol{S}_{ij} \frac{(f_v + g_v)^2}{4\pi} \frac{m_v^3}{12m_i m_j} \times \left[ H(m_v r) - \left( \frac{\Lambda_v}{m_v} \right)^3 H(\Lambda_v r) \right] \right\}$$



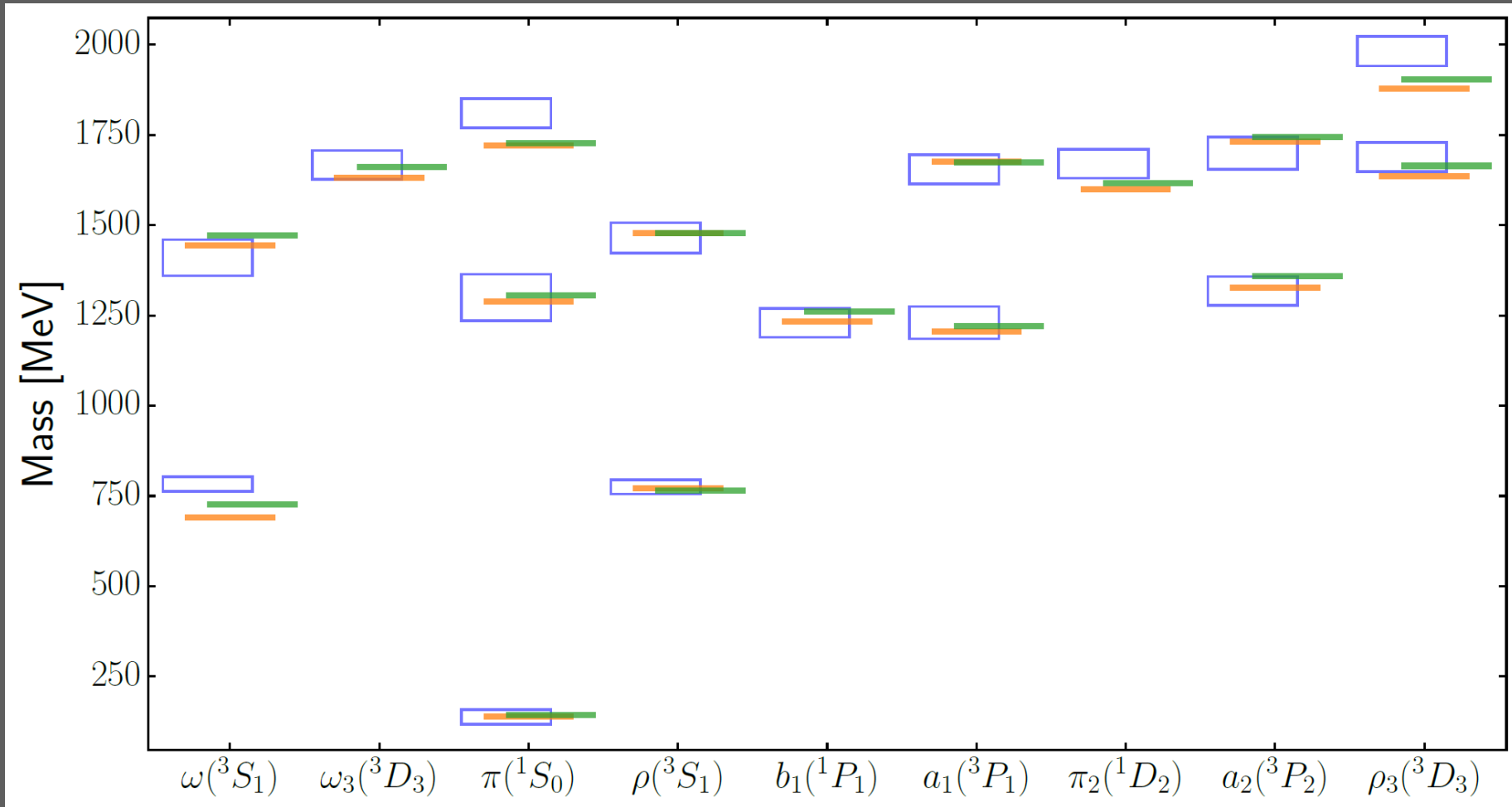
# The exchanged mesons in $SU2$ model

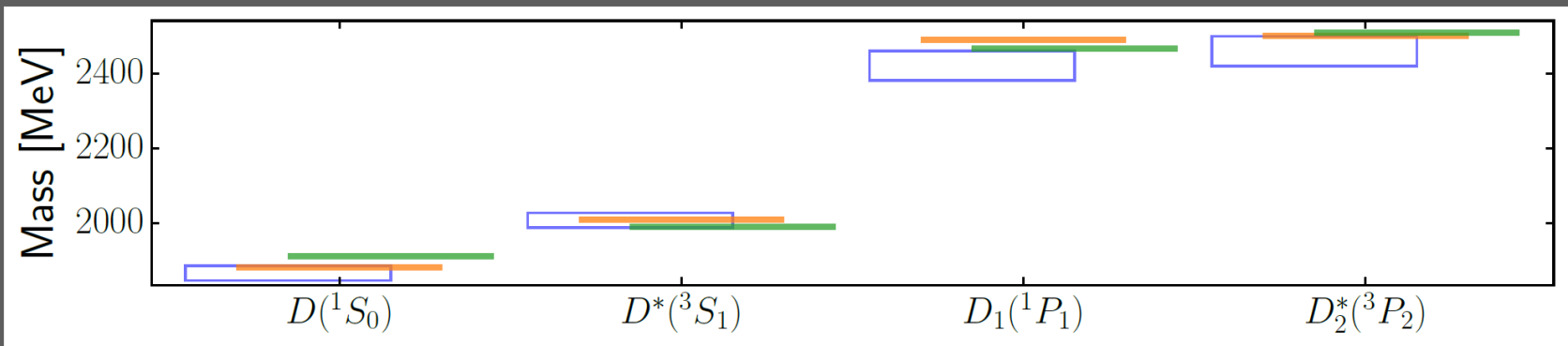
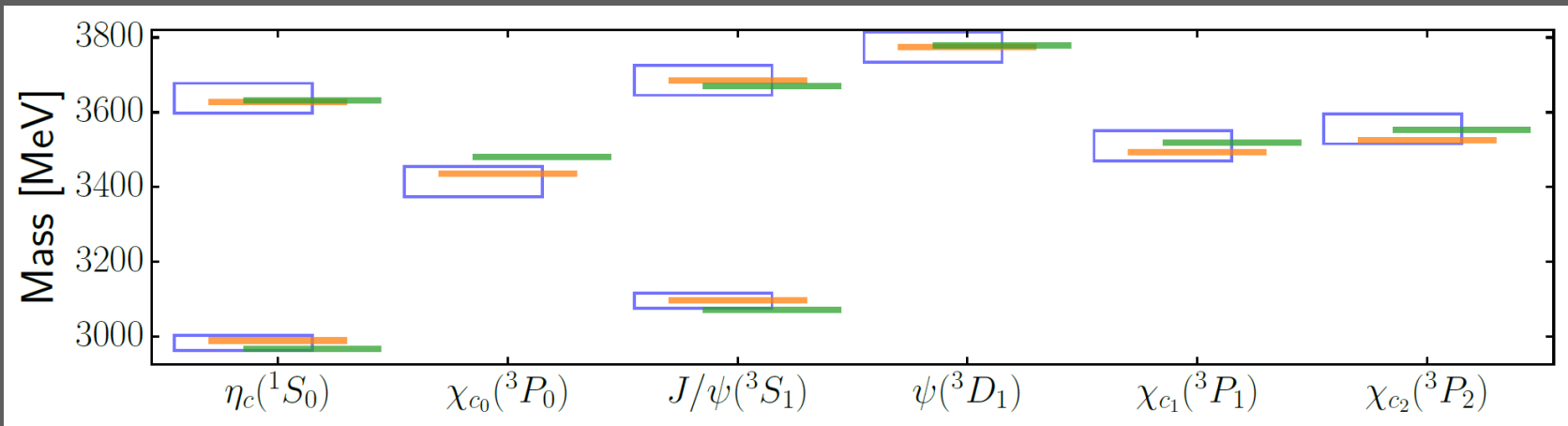


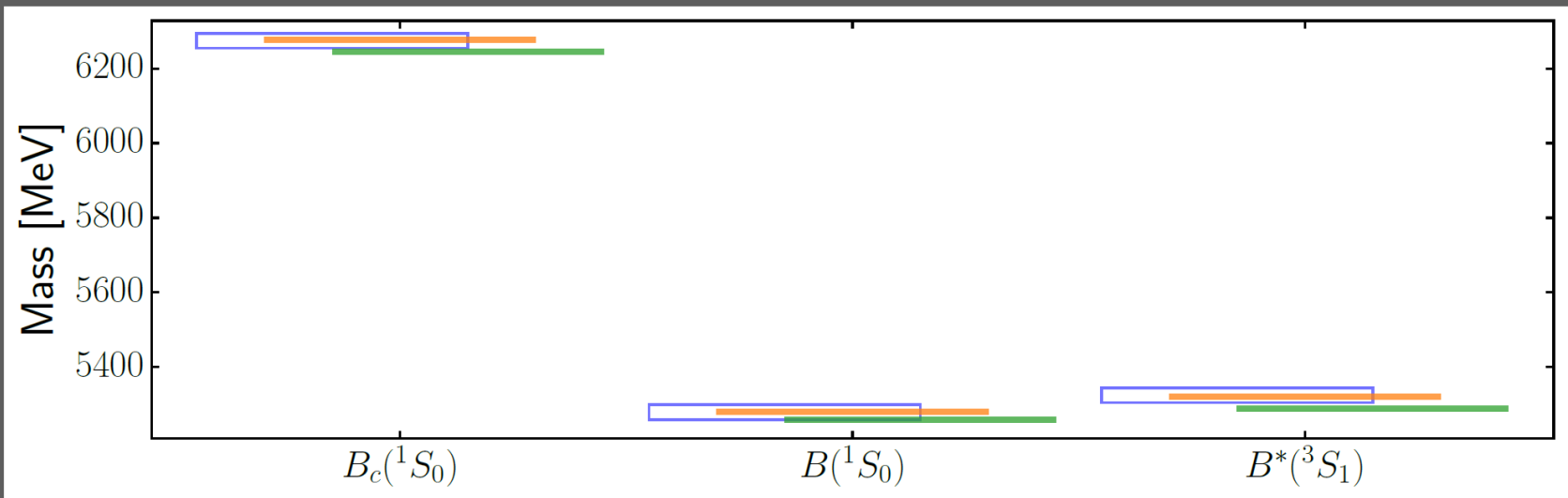
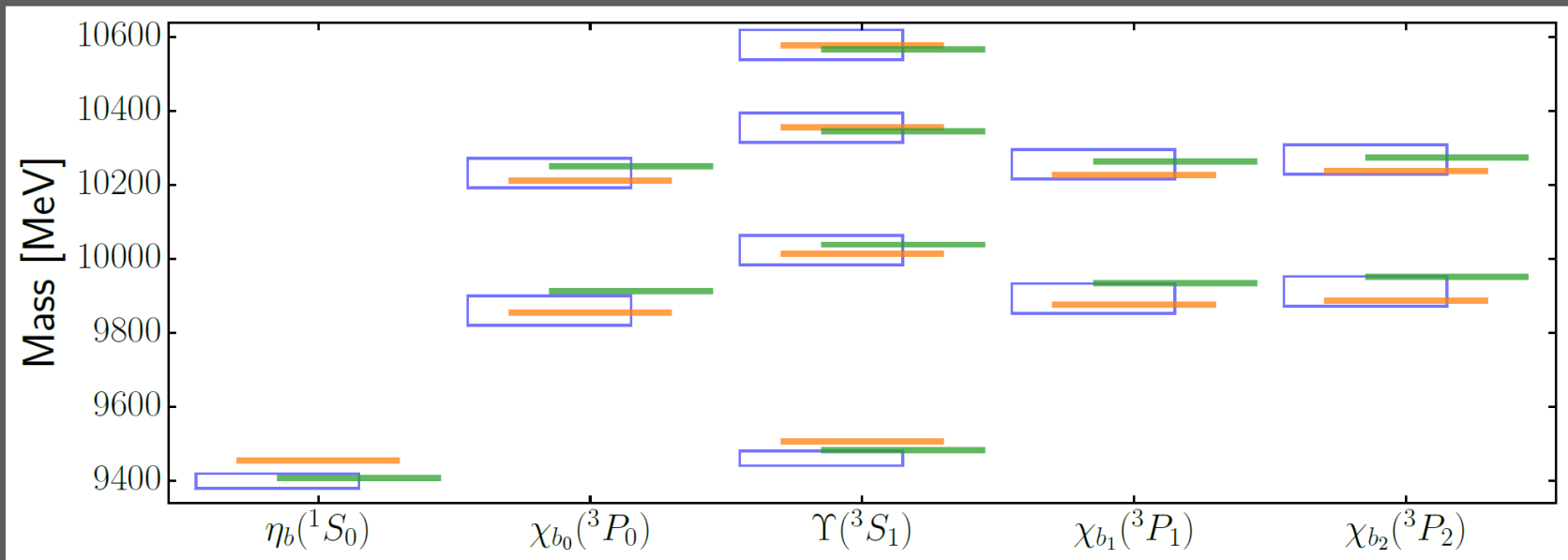
# Wave functions

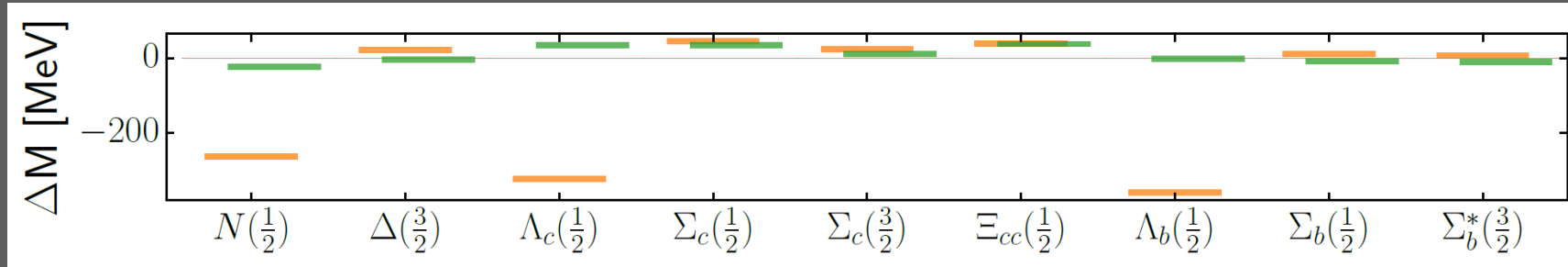
- Orbital (SO(3)):  $(\psi_L)$
- Spin (SU(2)):  $(\chi_S^\sigma)$
- Flavor (SU(2)):  $(\chi_I^f)$
- Color (SU(3)):  $(\chi^c)$

$$\Psi_{JM_J IM_I}^{ijk} = \mathcal{A} \left[ [\psi_L \chi_S^{\sigma i}]_{JM_J} \chi_I^{fj} \chi_k^c \right]$$









	1	$\tau_i \tau_j$	$\sigma_i \sigma_j$	$\sigma_i \sigma_j \cdot \tau_i \tau_j$
$\sigma$	-/-			
$\pi$				+/-
$a_0$		-/+		
OGE	-/-		+/+	
CON	+/+			
$\omega$ (This work)	+/-		+/-	
$\rho$ (This work)		+/+		+/+

$qq/q\bar{q}$

Channel	$E_B$		Channel	$E_B$	
$[DD^*]_{1\otimes 1}$	6.2	39%	$[BB^*]_{1\otimes 1}$	0.5	12%
$[D^*D]_{1\otimes 1}$	6.2	39%	$[B^*B]_{1\otimes 1}$	0.5	12%
$[D^*D^*]_{1\otimes 1}$	83.1	5%	$[B^*B^*]_{1\otimes 1}$	31	32%
$[DD^*]_{8\otimes 8}$	383.1	0%	$[BB^*]_{8\otimes 8}$	253.6	0%
$[D^*D]_{8\otimes 8}$	383.1	0%	$[B^*B]_{8\otimes 8}$	253.6	0%
$[D^*D^*]_{8\otimes 8}$	337.3	0%	$[B^*B^*]_{8\otimes 8}$	233.3	0%
$[(cc)(\bar{q}\bar{q})^*]_{6\otimes \bar{6}}$	337.5	0%	$[(bb)(\bar{q}\bar{q})^*]_{6\otimes \bar{6}}$	233.7	0%
$[(cc)^*(\bar{q}\bar{q})]_{\bar{3}\otimes 3}$	120.3	17%	$[(bb)^*(\bar{q}\bar{q})]_{\bar{3}\otimes 3}$	-37.8	44%
Mixed	-4.9		Mixed	-88.2	

	$r_{cc}$	$r_{\bar{q}c}$	$r_{\bar{q}\bar{q}}$	$r_{\bar{q}b}$	$r_{bb}$
$T_{cc}$	1.56	1.24	1.70		
$T_{bb}$			0.75	0.65	0.37

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- **$SU3$  ground states**
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# The Hamiltonian

$$H = \sum_{i=1} \left( m_i + \frac{p_i^2}{2m_i} \right) - T_{CM} + \sum_{j>i=1} \left( V_{ij}^{\text{CON}} + V_{ij}^{\text{OGE}} \right. \\ \left. + V_{ij}^{\bar{\sigma}} + V_{ij}^{\eta} + V_{ij}^{\eta'} + V_{ij}^{\pi} + V_{ij}^K + V_{ij}^{\omega} + V_{ij}^{\phi} + V_{ij}^{\rho} + V_{ij}^{K^*} \right)$$

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Bing-Song Zou, 2307.16280

$$V_{ij}^{\bar{\sigma}} = V_{ij}^{s=\bar{\sigma}, g_s=g_{\bar{\sigma}q}} \lambda_i^q \lambda_j^q + V_{ij}^{s=\bar{\sigma}, g_s=g_{\bar{\sigma}s}} \lambda_i^s \lambda_j^s,$$

$$V_{ij}^{\eta} = V_{ij}^{p=\eta, g_p=g_{\eta q}} \lambda_i^q \lambda_j^q + V_{ij}^{p=\eta, g_p=g_{\eta s}} \lambda_i^s \lambda_j^s,$$

$$V_{ij}^{\eta'} = V_{ij}^{p=\eta', g_p=g_{\eta'q}} \lambda_i^q \lambda_j^q + V_{ij}^{p=\eta', g_p=g_{\eta's}} \lambda_i^s \lambda_j^s,$$

$$\lambda^q = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda^s = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$V_{ij}^{\pi} = V_{ij}^{p=\pi, g_p=g_{\pi}} \sum_{a=1}^3 \lambda_i^a \lambda_j^a,$$

$$V_{ij}^K = V_{ij}^{p=K, g_p=g_K} \sum_{a=4}^7 \lambda_i^a \lambda_j^a,$$

$$V_{ij}^{\omega} = V_{ij}^{v=\omega, g_v=g_{\omega q}} \lambda_i^q \lambda_j^q + V_{ij}^{v=\omega, g_v=g_{\omega s}} \lambda_i^s \lambda_j^s,$$

$$V_{ij}^{\phi} = V_{ij}^{v=\phi, g_v=g_{\phi q}} \lambda_i^q \lambda_j^q + V_{ij}^{v=\phi, g_v=g_{\phi s}} \lambda_i^s \lambda_j^s,$$

$$V_{ij}^{\rho} = V_{ij}^{v=\rho, g_v=g_{\rho}} \sum_{a=1}^3 \lambda_i^a \lambda_j^a,$$

$$V_{ij}^{K^*} = V_{ij}^{v=K^*, g_v=g_{K^*}} \sum_{a=4}^7 \lambda_i^a \lambda_j^a$$

The coupling have relations based on  $SU_3$  flavor symmetry:

$$g_{\eta s} = g_{\eta q} - \sqrt{3} \cos \theta_p g_{\pi}$$

$$g_{\eta'q} = -\cot \theta_p g_{\eta q} + \frac{1}{\sqrt{3} \sin \theta_p} g_{\pi}$$

$$g_{\eta's} = -\cot \theta_p g_{\eta q} + \frac{\cos \theta_p \cot \theta_p - 2 \sin \theta_p}{\sqrt{3}} g_{\pi}$$

$$g_{\pi} = g_K$$

$$g_{\omega s} = g_{\omega q} - g_{\rho}$$

$$g_{\phi q} = -\sqrt{\frac{1}{2}} (g_{\omega q} - g_{\rho})$$

$$g_{\phi s} = -\sqrt{\frac{1}{2}} (g_{\omega q} + g_{\rho})$$

$$g_{\rho} = g_{K^*}$$

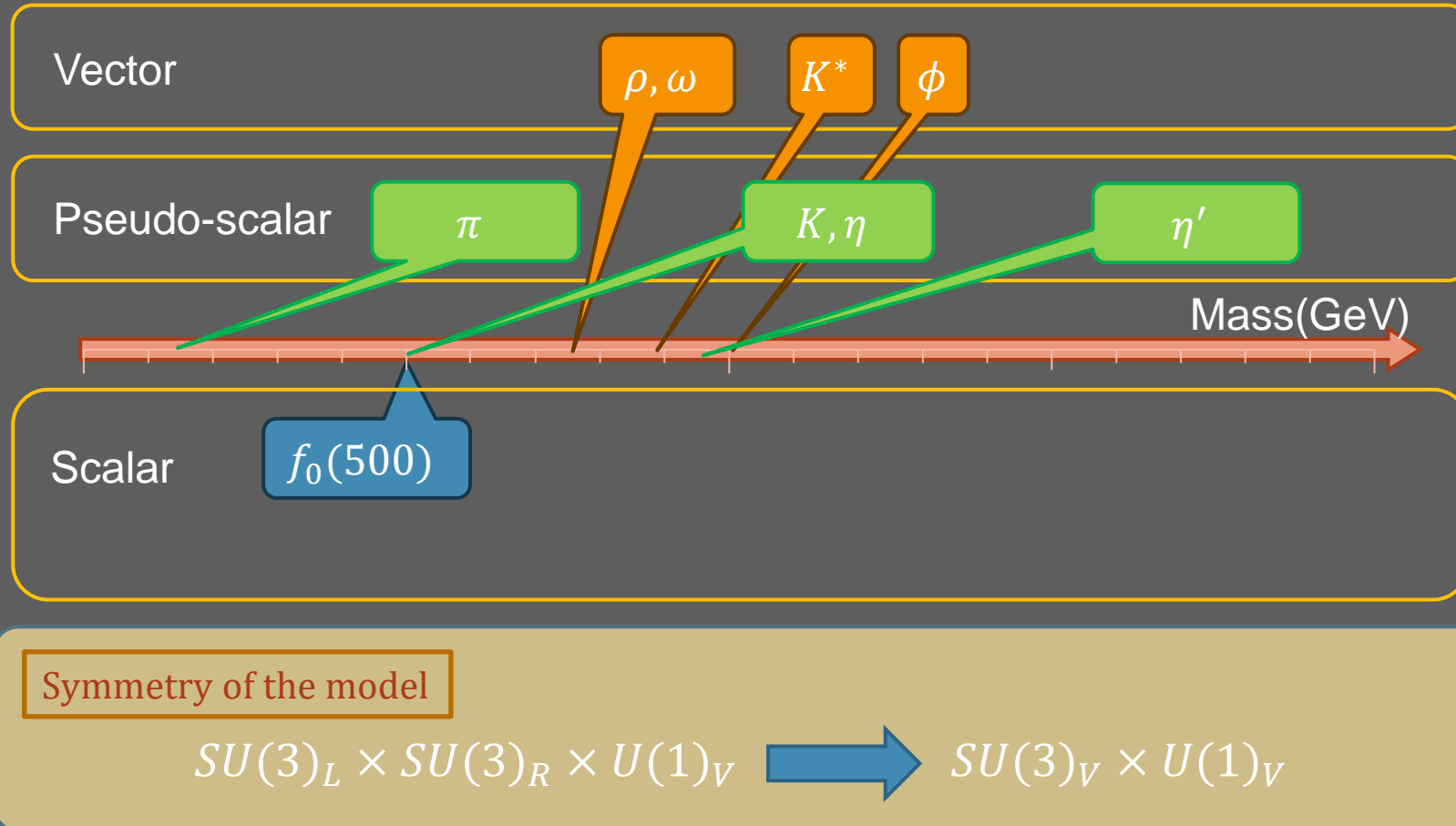
$$f_{\omega s} = f_{\omega q} - f_{\rho}$$

$$f_{\phi q} = -\sqrt{\frac{1}{2}} (f_{\omega q} - f_{\rho})$$

$$f_{\phi s} = -\sqrt{\frac{1}{2}} (f_{\omega q} + f_{\rho})$$

$$f_{\rho} = f_{K^*}$$

# The exchanged mesons in $SU3$ model



# Wave functions

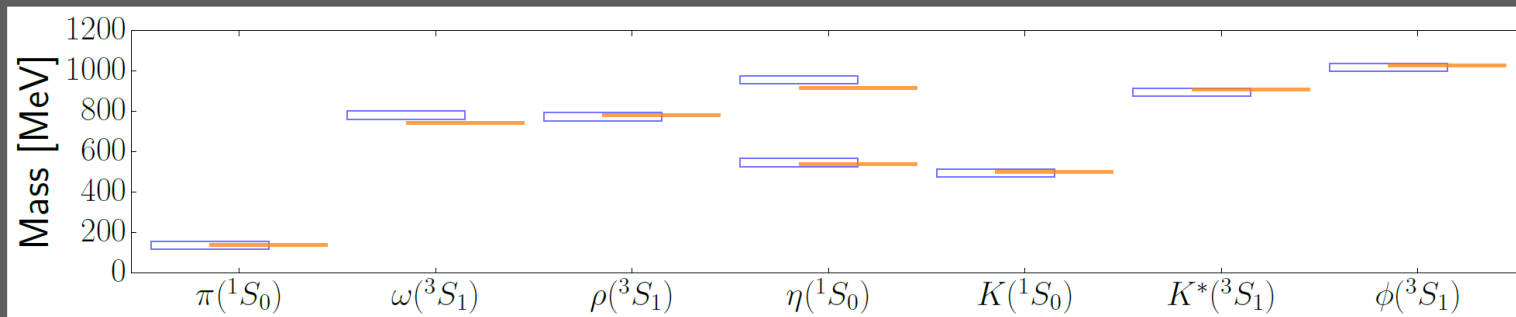
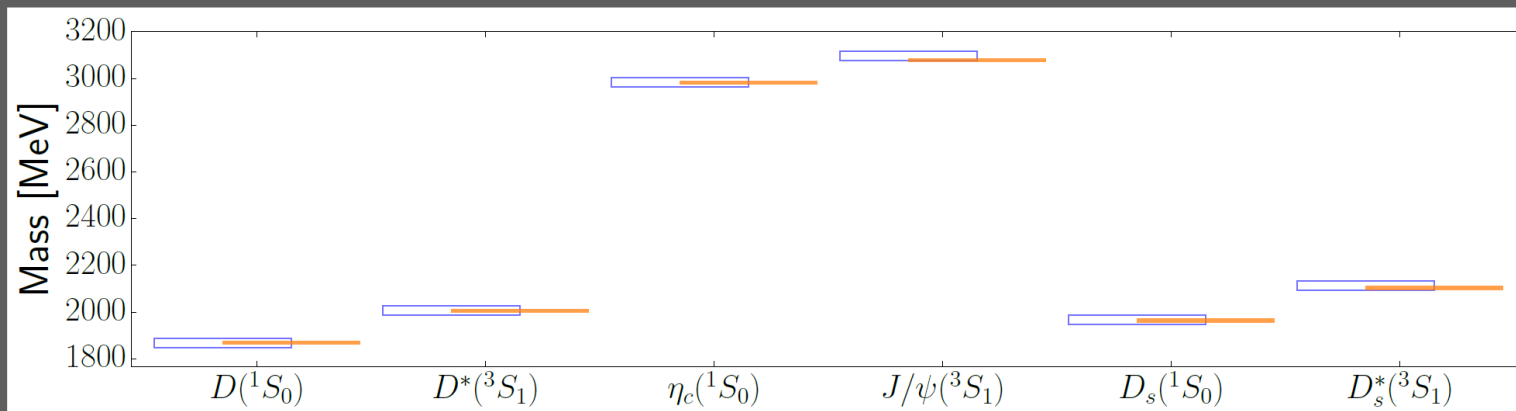
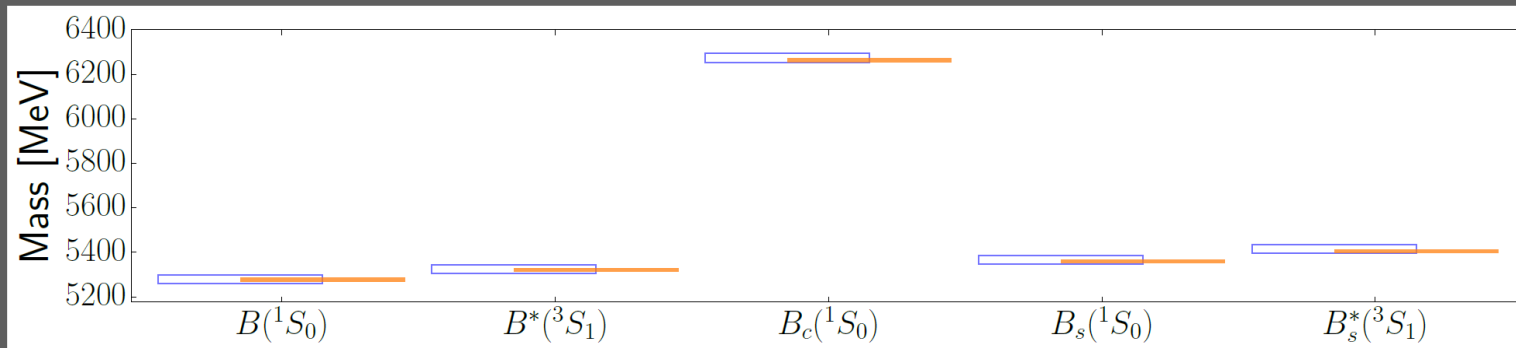
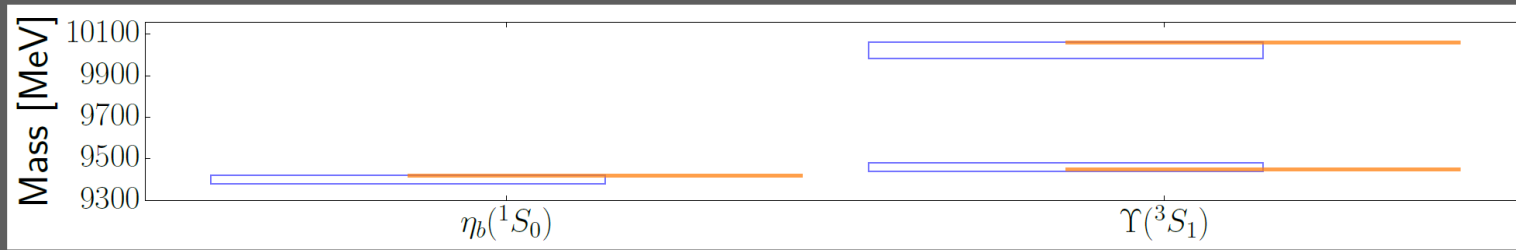
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- Color (SU(3)):  $(\chi^c)$

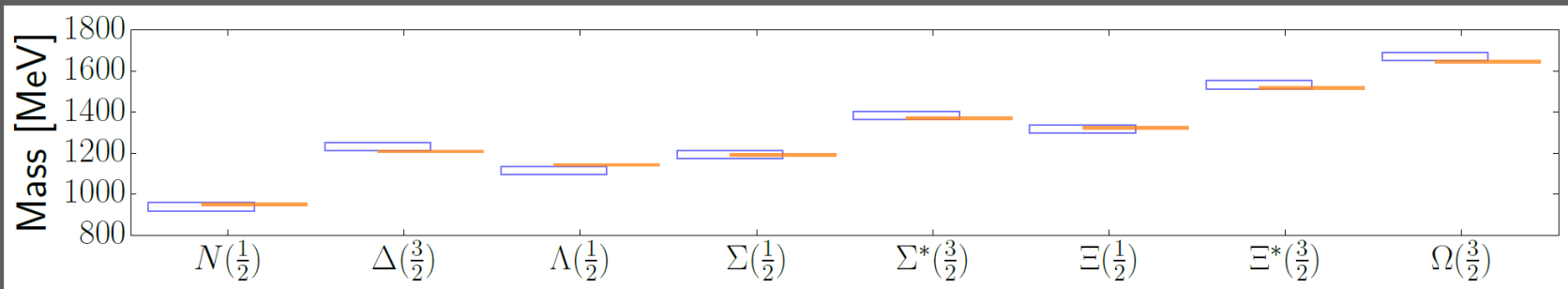
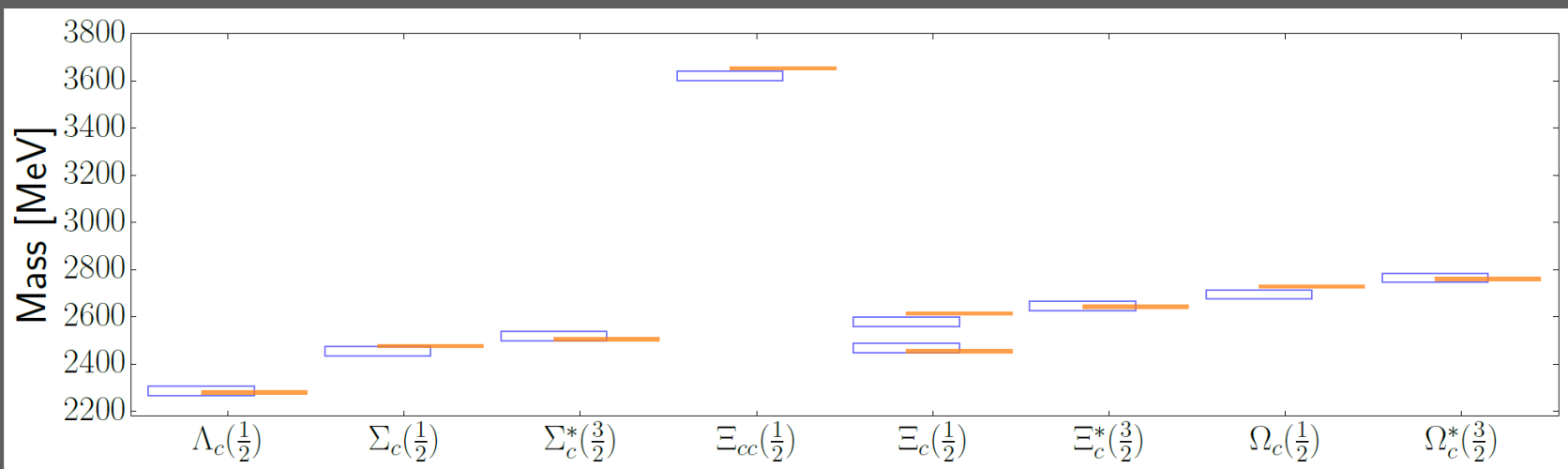
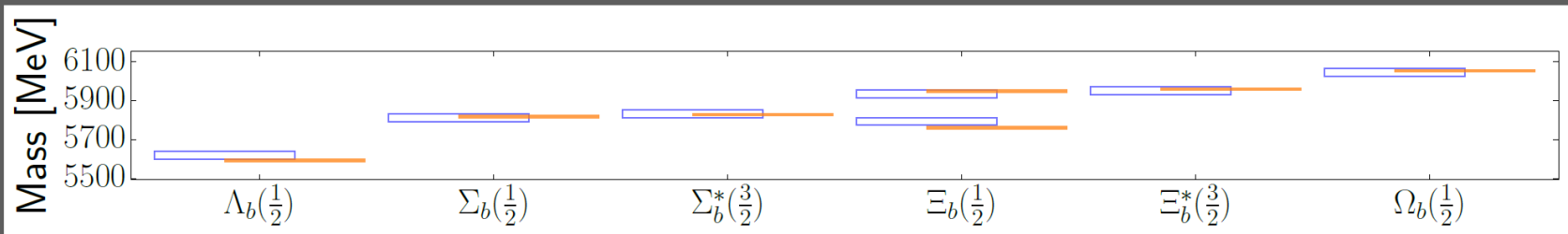
$$\Psi_{JM_J IM_I}^{ijk} = \mathcal{A} \left[ [\psi_L \chi_S^{\sigma i}]_{JM_J} \chi_I^{fj} \chi_k^c \right]$$

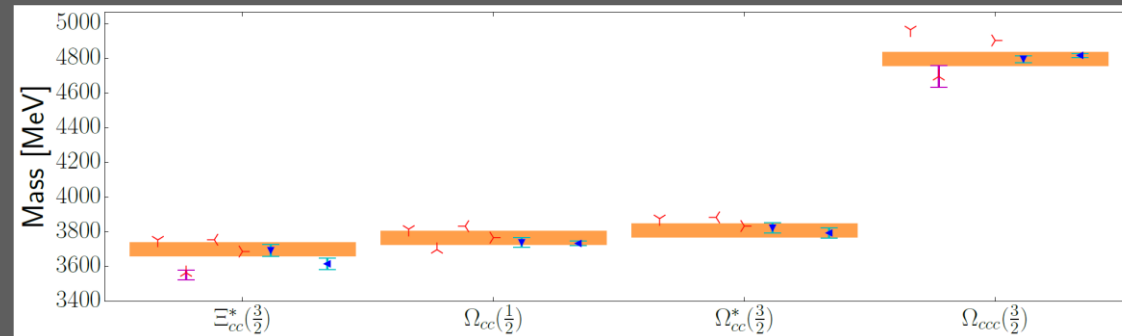
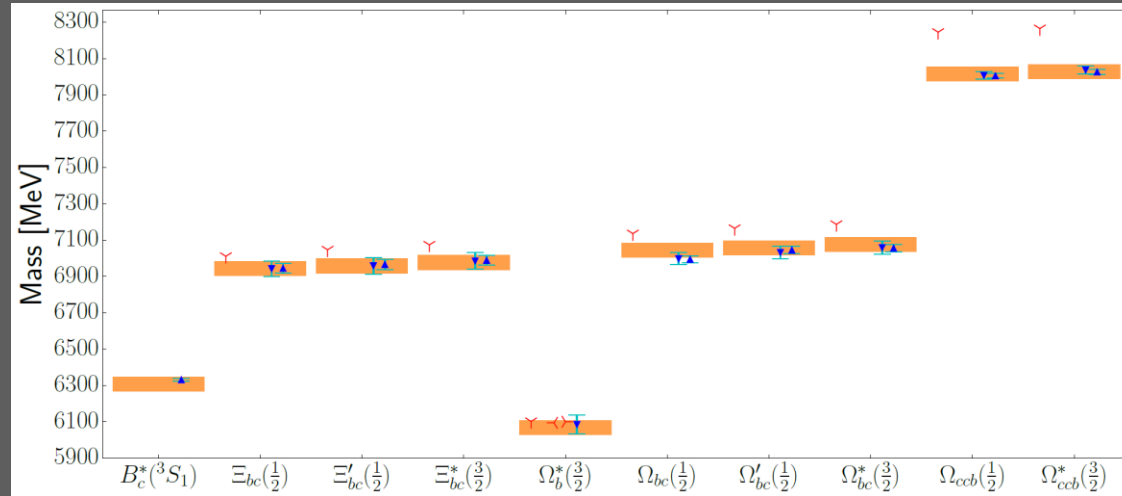
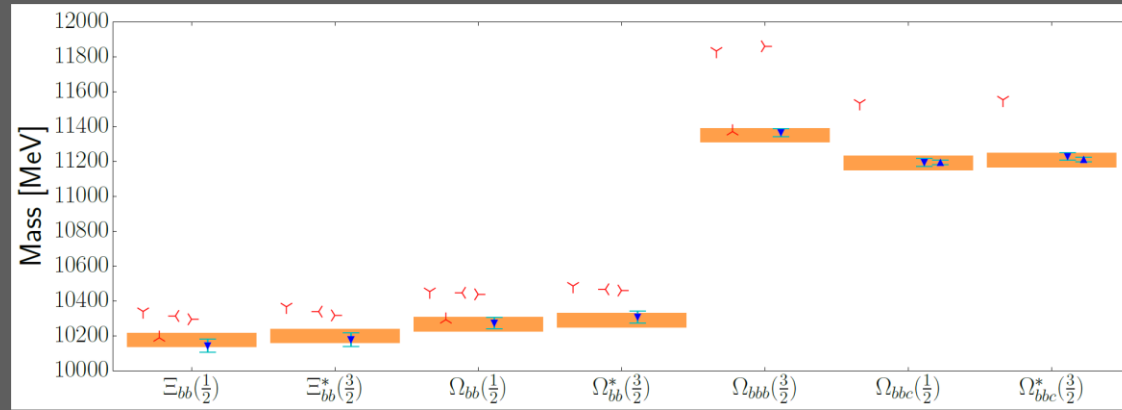
	1	$\lambda_i \lambda_j$	$\sigma_i \sigma_j$	$\sigma_i \sigma_j \cdot \lambda_i \lambda_j$
$\bar{\sigma}$	-/-			
$\eta/\eta'$			+/+	
$\pi/K$				+/-
$\omega/\phi$	+/-		+/-	
$\rho/K^*$		+/+		+/+
OGE	-/-		+/+	
CON	+/+			

	No vector J. Phys. G 31, 481(2005)	su2 vector 2306.03526	su3 vector 2307.16280
$\alpha_s(qq)$	0.536	0.880	0.456
$\alpha_s(qs)$	0.479		0.426
$\alpha_s(qc)$	0.426	0.774	0.363
$\alpha_s(qb)$	0.409	0.749	0.339
$\alpha_s(ss)$	0.419		0.388
$\alpha_s(sc)$	0.360		0.308
$\alpha_s(sb)$	0.340		0.279
$\alpha_s(cc)$	0.288	0.510	0.205
$\alpha_s(cb)$	0.260	0.447	0.168
$\alpha_s(bb)$	0.223	0.366	0.128

$qq/q\bar{q}$







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- Quark model with hidden local symmetry gives a systematic description from light meson/baryon to heavy meson/baryon
- Quark model with hidden local symmetry gives correct interaction between  $qq$  and  $q\bar{q}$ , thus its easy to extend the model to describe multiquark states, e.g., tetraquark, pentaquark, ...
- Quark model with hidden local symmetry gives size messages and percentage of components, which could help us to identify the particles observed from experiments

Thank you for your attention!