Rigorous Roy equations meet precise lattice data: $\pi\pi$ scattering at unphysical pion masses

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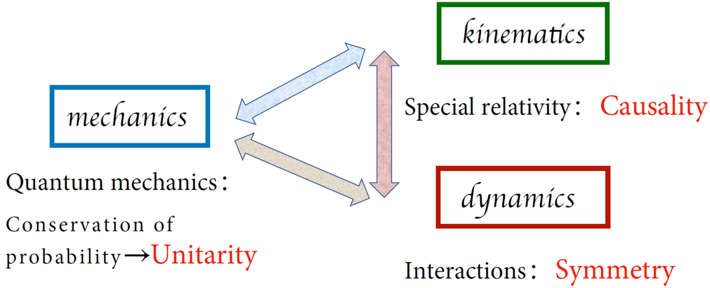
S-matrix theory

- 1943-1946: Heisenberg proposed the S-matrix theory as an alternative to field theory



- Avoid references to specific Hamiltonians and equations of motion
- It was primarily developed by G. Chew, S. Frautschi, S. Mandelstam, V. Gribov, and T. Regge...
  E.g. Regge theory, Veneziano model $\Longrightarrow$ String theory
- Dispersion relation (DR) + Optical theorem: retain analyticity and unitarity

★ **The pinnacle of the DRs are Roy and Roy-Steiner eqs. in the field of the hadron phenomenology**

A brief review of Roy & Roy-Steiner equations

- In 1971, Roy developed an exact integral equation based on axiomatic field theory [Roy, PLB (1971)]. Subsequently, Basdevant et al. and Pennington et al. realized the importance of Roy eq. and applied it to $\pi\pi$ data [Basdevant, et al., Nuovo Cim., PLB (1972), NPB (1973), Pennington and Protopopescu PRD (1973)]
- In 1973, Hite and Steiner analyzed previous dispersive approaches and proposed a new integral equations (Roy-Steiner eqs.), which can be applied to unequal mass scatterings such as $\pi N \to \pi N$, etc. [Hite and Steiner, Nuovo Cim. (1973)]

Shasanka Mohan Roy

Frank Steiner

★ **The last two decades are marked by a renaissance in Roy and Roy-Steiner equations caused by the development of χPT**

- Leutwyler et al. reanalyzed $\pi\pi$ Roy eq. using new data with chiral constrains [Colangelo, et al., NPB (2001), Ananthanarayan, et. al., Phys. Rept. (2001)] and demonstrate the existence of $\sigma/f_0(500)$–the lowest-lying resonance in QCD [Caprini, et al., PRL (2006)]
- Moussallam et al. analyzed πK low energy partial waves (PWs) using RS eq. [Buettiker, et al., EPJC (2004)] and found $\kappa/K_0^*(700)$ [Descotes-Genon, et al., EPJC (2006)]
- Hoferichter et al. given a RS eq. analyses of πN scattering [Ditsche, et al., JHEP (2012), Hoferichter, et al., Phys. Rept. (2016)], and applied it to extract πN σ term, $\sigma_{\pi N} = (59.1 \pm 3.5)$ MeV [Hoferichter, et al., PRL 115, 092301 (2015)] and χPT low energy constants [Hoferichter, et al., PRL 115, 192301 (2015)]; XHC, Z.-L. Qu and H.-Q. Zheng demonstrated the existence of $N^*(920)$ [XHC, et al., JHEP (2022)]
- Other processes: $\gamma\pi \to \pi\pi$ [Hannah, NPB (2001)], $\gamma\gamma \to \pi\pi$ [Hoferichter, et al., EPJC (2011)] and $\gamma^*\gamma^* \to \pi\pi$ [Hoferichter and Stoffer, JHEP (2019)], etc.;
  Theoretical improvements: (once-sub. DR) GKPY eqs.; high energy, $\pi\pi$ [Moussallam, EPJC (2011), Garcia-Martin, et al., PRL (2011), Caprini, et al., EPJC (2011)] and πK [Pelaez and Rodas, PRL (2020)]· · ·

Roy equations

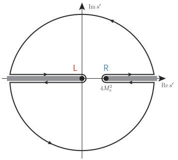
Roy eqs. $=$ Analyticity (Causality) $+$ Crossing symmetry $+$ Unitarity $+$ Froissart-Martin bound

- Fixed-t (twice sub.) DR for $\pi\pi$ scattering:

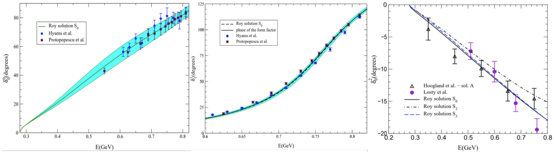
$$T(s,t,u)=\alpha(t)+s\beta(t)+\frac{s^2}{\pi}\int_{4m_\pi^2}^{\infty}\mathrm{d}s'\frac{\mathrm{Im}_s\,T(s',t,u')}{s'^2(s'-s)}+\frac{s^2}{\pi}\int_{-\infty}^{-t}\mathrm{d}s'\frac{\mathrm{Im}_u\,T(s',t,u')}{s'^2(s'-s)}$$

- Using crossing symmetry, (a) the contribution of the left-hand cuts are represented by the right-hand cuts; (b) $\alpha(t),\beta(t)$ are expressed by S wave scattering lengths a_0^0, a_0^2 and some DR integrals
- Partial wave expansion $\Longrightarrow$ Roy equations,

$$\mathrm{Re}\,t_J^I(s)=k_J^I(s)+\sum_{I'}\sum_{J'}\mathcal{P}\int_{4m_\pi^2}^{\infty}\mathrm{d}s'\underbrace{K_{JJ'}^{II'}(s',s)}_{\frac{1}{\pi}\frac{\delta_{JJ'}\delta_{II'}}{s'-s-i\epsilon}+\bar{K}_{JJ'}^{II'}(s,s')}\mathrm{Im}\,t_{J'}^{I'}(s')$$



[Colangelo, et al., NPB (2001), Ananthanarayan, et. al., Phys. Rept. (2001)]



- Phase shifts : $t_J^I(s)=\dfrac{\eta(s)e^{2i\delta_J^I(s)}-1}{2i\rho_{\pi\pi}(s)}$
- Scattering lengths: “free” parameters in $k_J^I(s)$

Lehmann-Martin ellipse

Jost-Lehmann-Dyson representation

A causal Green function satisfies a necessary and sufficient representation:

$$F(q) = \int \mathrm{d}^4 u \int_0^\infty \mathrm{d}\kappa^2 \epsilon\left(q_0 - u_0\right) \delta\left[(q-u)^2 - \kappa^2\right] \Phi\left(u, \kappa^2\right)$$

$\Phi(u, \kappa^2)$ is arbitrary but differs from 0 only in some domains Jost and Lehmann [1957] Dyson [1958].

- $F_R(q) = \frac{i}{2\pi} \int \mathrm{d}q_0' \frac{1}{q_0 - q_0' + i\epsilon} F\left(q'\right)\Big|_{q'=(q_0', q)} \overset{\text{“retarded” LSZ}}{\Longrightarrow} -iT = \frac{1}{2\pi} \int \mathrm{d}^4 u \, \mathrm{d}\kappa^2 \frac{\Phi\left(u, \kappa^2, p, k\right)}{\left((k'-p')/2 - u\right)^2 - \kappa^2}$
- PW unitarity $\left(\mathrm{Im}\, T_\ell = \frac{2q}{\sqrt{s}} \left|T_\ell\right|^2\right)$ + Analyticity of Legendre function $\left(\left(T_\ell\right)^{1/\ell} < \frac{1}{z_0 + \sqrt{z_0^2 - 1}}\right)$

Lehmann-Martin ellipse

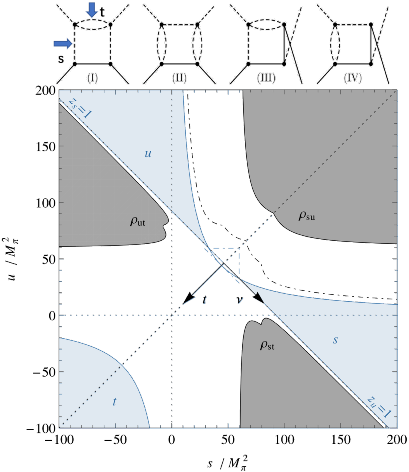
Any PWDR (or Im T_ℓ) can be shown to be valid in a finite region. The domain of validity relies on Lehmann-Martin ellipse Lehmann [1958].

★ Foci: $z = \pm 1$; Semi-major axis: $z_{\max} = 1 + \frac{2s}{\lambda_s} \mathcal{T}(s)$

Mandelstam Analyticity

Mandelstam Analyticity

The scattering amplitudes satisfy Mandelstam double spectral representation Mandelstam [1958, 1959].



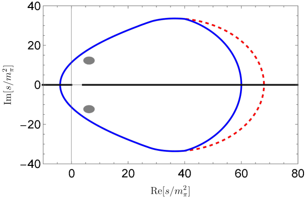
- Mandelstam double spectral representation:

$$T(s,t)=\frac{1}{\pi^2}\iint \mathrm{d}s'\mathrm{d}u'\,\frac{\rho_{su}(s',u')}{(s'-s)(u'-u)}+\frac{1}{\pi^2}\iint \mathrm{d}t'\mathrm{d}u'\,\frac{\rho_{tu}(t',u')}{(t'-t)(u'-u)}+\frac{1}{\pi^2}\iint \mathrm{d}s'\mathrm{d}t'\,\frac{\rho_{st}(s',t')}{(s'-s)(t'-t)}$$

- This concept can be justified in perturbation theory to all orders
- A rigorous proof from axiomatic field theory is absent

Validity (Analyticity) Domain

Lehmann-Martin ellipse + **Mandelstam analyticity** (**or analyticity from axiomatic field theory**) $\Longrightarrow$ Validity domain of $\pi\pi$ scattering

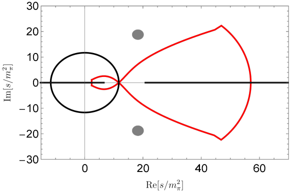


$$m_\sigma = 441^{+16}_{-8}\ \text{MeV}$$

$$\Gamma_\sigma = 544^{+18}_{-25}\ \text{MeV}$$

Caprini et al. [2006]

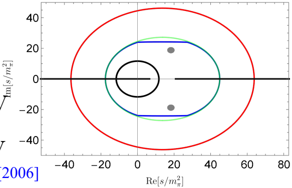
πK scattering: Roy-Steiner eqs. and hyperbolic DR



$$m_\kappa = 658 \pm 13\ \text{MeV}$$

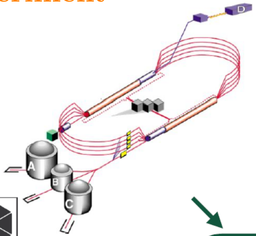
$$\Gamma_\kappa = 557 \pm 24\ \text{MeV}$$

Descotes-Genon and Moussallam [2006]

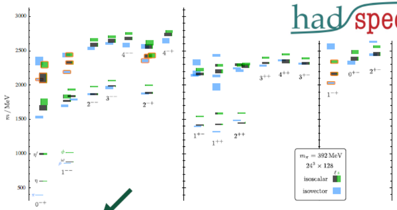


Why lattice QCD?

Experiment



Lattice QCD



Resonances manifest as amplitudes

Amplitude analyses

- Unitarity
- Analiticity
- Crossing

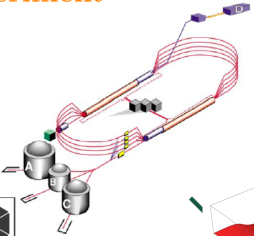
↔ **QCD**

↓

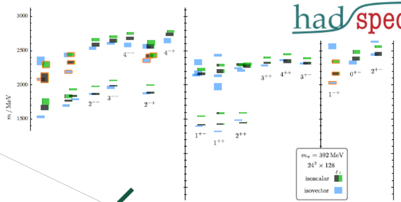
Observables

Why lattice QCD?

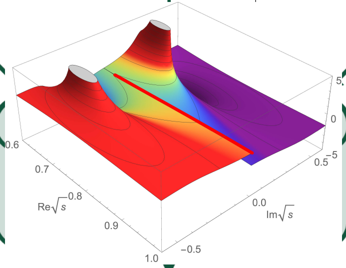
Experiment



Lattice QCD



Resonances manifest as amplitudes



QCD

Observables

Roy equation analyses of $\pi\pi$ scatterings at unphysical pion masses

XHC, Qu-Zhi Li, Zhi-Hui Guo and Han-Qing Zheng, Phys.Rev.D 108 (2023) 3, 034009

- The lowest resonance in QCD
- Extremely broad object at physical m_π
- Interesting "modeled" m_q dependence

★ ρ: Ordinary m_q dependence

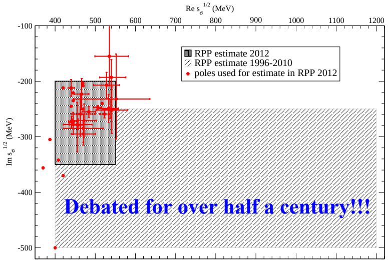
$$m_\rho \sim a + bm_q$$

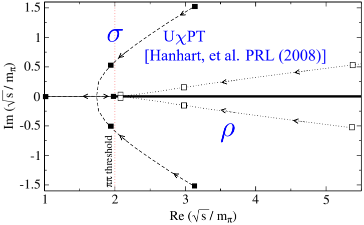
★ Sub-percent errors

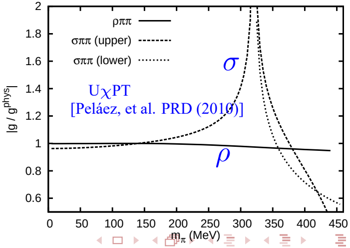
♦ $IJ = 20$: non-resonant channel
There is a virtual state pole! [Z.-Y. Zhou, et al. JHEP (2005)]

♦ Weak m_q dependence

PDG







σ as intermediate state: I

ELSEVIER Nuclear Physics B 578 (2000) 367–382
www.elsevier.nl/locate/npe

Insight into the scalar mesons from a lattice calculation

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Received 8 February 2000; accepted 8 March 2000

Abstract

We study the possibility that the light scalar mesons are $\bar{q}^2q^2$ states rather than $\bar{q}q$. We perform a lattice QCD calculation of pseudoscalar meson scattering amplitudes, ignoring quark loops and quark annihilation, and find indications that for sufficiently heavy quarks there is a stable four-quark bound state with $J^{PC}=0^{++}$ and non-exotic flavor quantum numbers.

PHYSICAL REVIEW D **70**, 034504 (2004)

Scalar mesons in lattice QCD

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[2]*Tokuyama Women's College, Shunan 745-8511, Japan*
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(Received 24 October 2003; published 16 August 2004)

We explore whether "$f_0(600)$ or σ" exists as a pole in QCD, by a full lattice QCD simulation on the $8^3 \times 16$ lattice using the plaquette action and Wilson fermions. It is shown that there exists a σ pole, whose mass is as low as that of the ρ, owing to the disconnected diagram of the meson propagator. A discussion is given on the physical content of the σ.

PHYSICAL REVIEW D **76**, 114505 (2007)

Lattice QCD study of the scalar mesons $a_0(1450)$ and $\sigma(600)$

N. Mathur,[1] A. Alexandru,[2] Y. Chen,[3] S. J. Dong,[2] T. Draper,[2] I. Horváth,[2] F. X. Lee,[4] K. F. Liu,[2] S. Tamhankar,[5] and J. B. Zhang[6]
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(Received 11 July 2006; revised manuscript received 30 July 2007; published 6 December 2007)

We study the a_0 and σ mesons with the overlap fermion in the chiral regime with the pion mass as low as 182 MeV in the quenched approximation. After the $\eta'\pi$ ghost states are separated, we find the a_0 mass with the $q\bar{q}$ interpolation field to be almost independent of the quark mass in the region below the strange quark mass. The chirally extrapolated results are consistent with $a_0(1450)$ being the $u\bar{d}$ meson and $K_0^*(1430)$ being the $u\bar{s}$ meson with calculated masses at 1.42 ± 0.13 GeV and 1.41 ± 0.12 GeV, respectively. We also calculate the scalar mesonium with a tetraquark interpolation field. In addition to the two-pion scattering states, we find a state at ~550 MeV. Through the study of volume dependence, we confirm that this state is a one-particle state, in contrast to the two-pion scattering states. This suggests that the observed state is a tetraquark mesonium which is quite possibly the $\sigma(600)$ meson.

The simulations performed before 2009 used
the quenched approximation

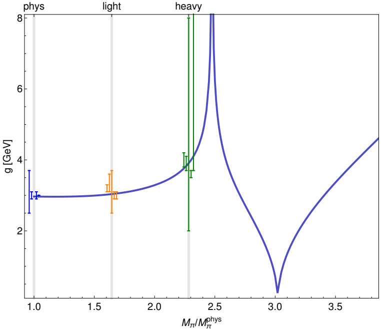
σ as intermediate state: II

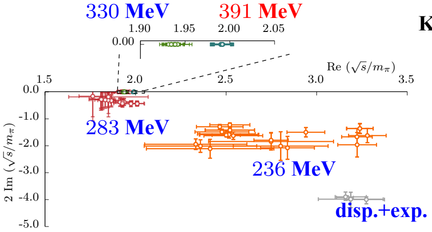
$N_f = 2$ [Guo, et al. PRD (2018)]

- $m_\pi \sim 224$ MeV $\Longrightarrow$ Broad resonance
- $m_\pi \sim 315$ MeV $\Longrightarrow$ Resonance

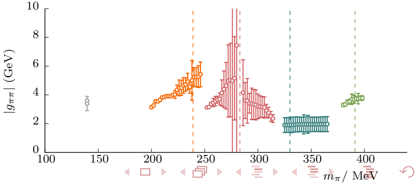
$N_f = 2 + 1$ [HadSpec. PRL,PRD... (2017-now)]

- ★ $m_\pi \sim 236$ MeV $\Longrightarrow$ Broad resonance
- ★ $m_\pi \sim 283$ MeV $\Longrightarrow$ Resonance v.s. Virtual state
- ★ $m_\pi \sim 330$ MeV $\Longrightarrow$ Extremely precise, shallow bound state
- ★ $m_\pi \sim 391$ MeV $\Longrightarrow$ Extremely precise, shallow bound state





K-matrix approaches



Extended Roy equation

- $\mathrm{Re}t_J^I(s) = k_J^I(s) + \sum_{I'=0}^{2}\sum_{J'=0}^{1} \mathcal{P}\int_{4m_\pi^2}^{s_m} \mathrm{d}s' K_{JJ'}^{II'}(s',s)\,\mathrm{Im}t_{J'}^{I'}(s') + d_J^I(s)$

$$k_0^0(s) = a_0^0 + \frac{s-4m_\pi^2}{12m_\pi^2}(2a_0^0-5a_0^2) + \frac{g_{\sigma\pi\pi}^2}{12}\left(\frac{16m_\pi^2(4s-s_\sigma)-4(2s-s_\sigma)(s+2s_\sigma)}{(4m_\pi^2-s_\sigma)\,s_\sigma(s_\sigma-s)} - \frac{8\mathbf{L}_\sigma}{4m_\pi^2-s}\right)$$

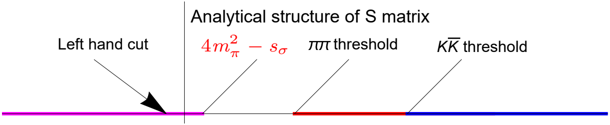
$$k_1^1(s) = 0 + \frac{s-4m_\pi^2}{72m_\pi^2}(2a_0^0-5a_0^2) + \frac{g_{\sigma\pi\pi}^2}{9}\left(-\frac{\left(4m_\pi^2-s\right)^2-48m_\pi^2 s_\sigma+12s^2}{(4m_\pi^2-s)(4m_\pi^2-s_\sigma)s_\sigma} + \frac{6\left(s+2s_\sigma-4m_\pi^2\right)\mathbf{L}_\sigma}{(4m_\pi^2-s)^2}\right)$$

$$k_0^2(s) = a_0^2 - \frac{s-4m_\pi^2}{24m_\pi^2}(2a_0^0-5a_0^2) - \frac{g_{\sigma\pi\pi}^2}{3}\left(\frac{4m_\pi^2+s-2s_\sigma}{s_\sigma(4m_\pi^2-s_\sigma)} + \frac{2\mathbf{L}_\sigma}{4m_\pi^2-s}\right)$$

★ The logarithm $L_\sigma = \ln\left(\frac{s+s_\sigma-4m_\pi^2}{s_\sigma}\right)$: left-hand cuts $(-\infty, 4m_\pi^2-s_\sigma]$ √ $(-\infty,0]$ ×

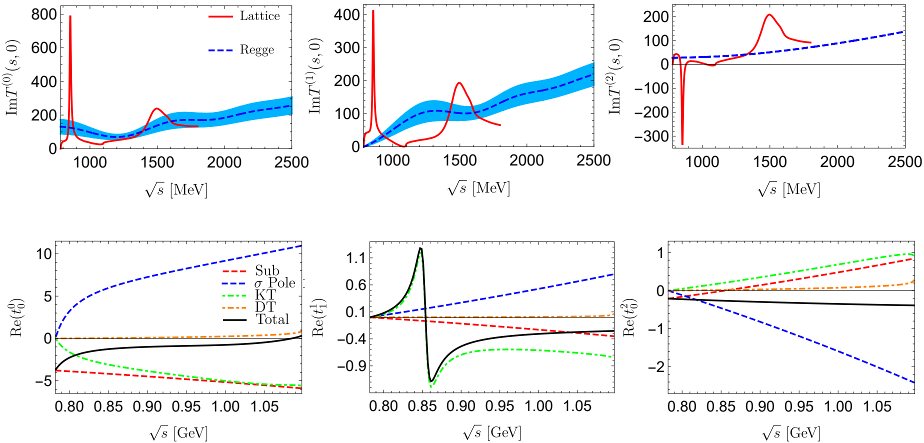
★ Correct threshold behavior: $\lim_{s\to 4m_\pi^2}(k_0^0,k_1^1,k_0^2)(s) = (a_0^0, 0, a_0^2)$

★ Four parameters : a_0^0, a_0^2, s_σ, $|g_{\sigma\pi\pi}|$



Numerical solution I: driving terms

- HadSpec LQCD data: S- and P-waves $\sqrt{s} \in (2m_K \sim 1.098\text{ GeV}, 1.8\text{ GeV})$; D-wave $\sqrt{s} \in (2m_\pi \sim 0.782\text{ GeV}, 1.8\text{ GeV})$
- Improved Veneziano model: S-,P- and D-Waves above $\sqrt{s} > 1.8$ GeV; Other high partial-waves



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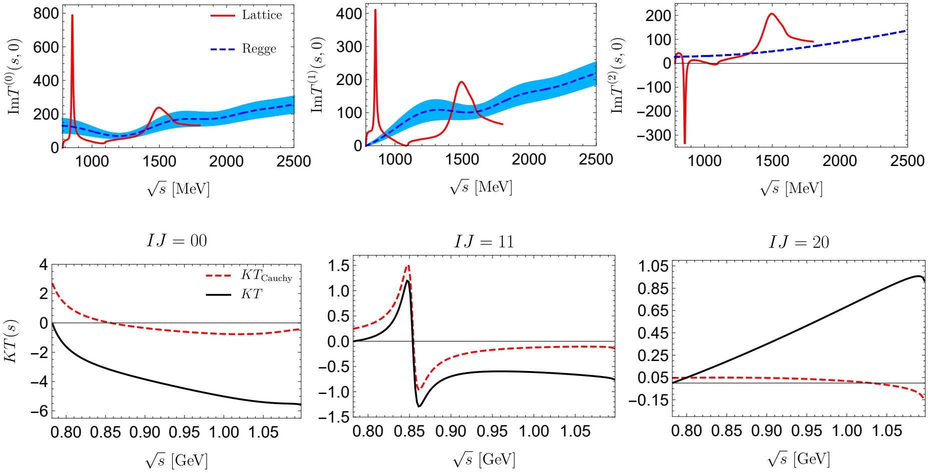
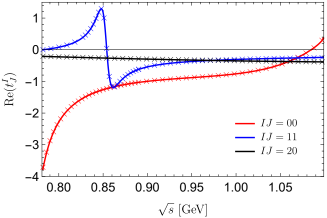
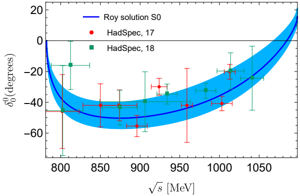
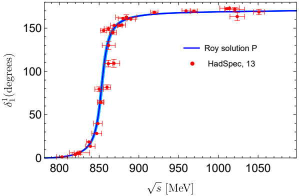


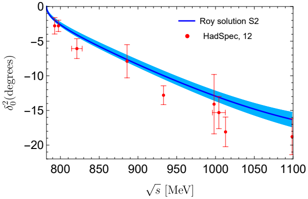
Numerical solution II: phase shifts



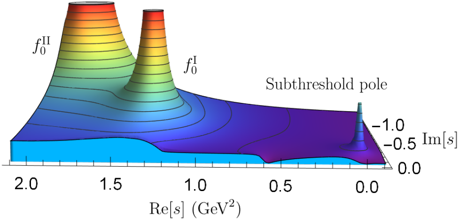
- Roy eqs.: $\mathrm{Re}(t^I_J) = \sum_{I',J'} \mathcal{F}[\mathrm{Im}(t^{I'}_{J'})]$
- $\hookrightarrow$ $a^0_0 = -(3.9^{+1.1}_{-1.2})$
  $a^2_0 = -(0.21^{+0.02}_{-0.03})$
  $\sqrt{s_\sigma} = 759^{+7}_{-16}$ MeV
  $|g_{\sigma\pi\pi}| = 493^{+27}_{-46}$ MeV



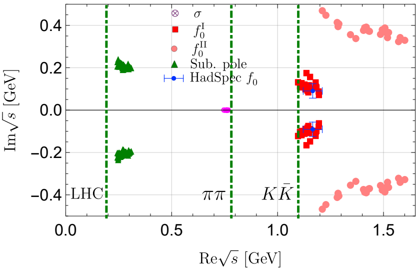


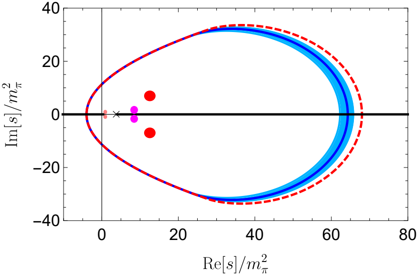


The singularities inside the validity domain



- 2nd Riemann sheet:
  $\sqrt{s_{\rm sub}} = (269^{+40}_{-25}) - i(211^{+26}_{-23})$ MeV !!!
  $\sqrt{s_{f_0^{I}}} = (1142^{+53}_{-46}) - i(112^{+59}_{-45})$ MeV
  $\sqrt{s_{f_0^{II}}} = (1434^{+167}_{-223}) - i(371^{+97}_{-49})$ MeV
- "$f_0(980)$" [HadSpec. PRD (2018)]:
  $(1166 \pm 45) - \frac{i}{2}(181 \pm 68)$ MeV $\surd$
- "$f_0(1370)$" ???



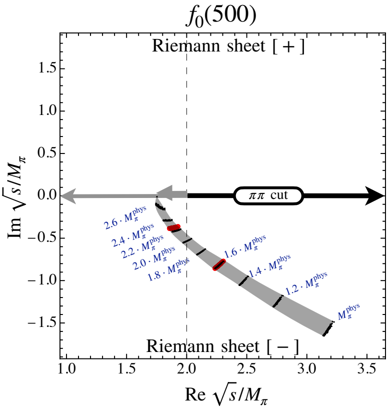


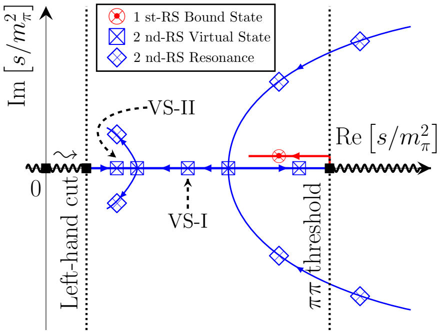
“σ” trajectory and the sub. pole

Controversy:

- **Only one σ** [Beveren and Rupp PRD (2023)...]
- **One “σ”+ one virtual state pole** [Gao et. al. PRD (2022)]

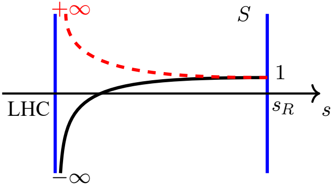
Where is the virtual state pole?

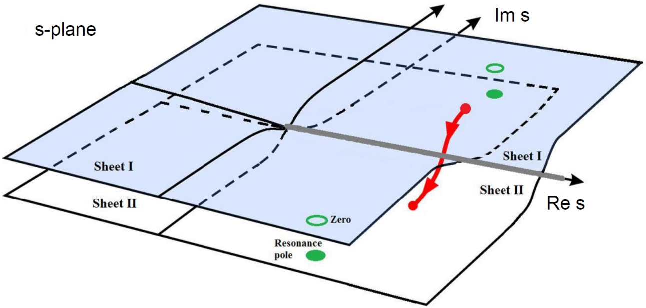




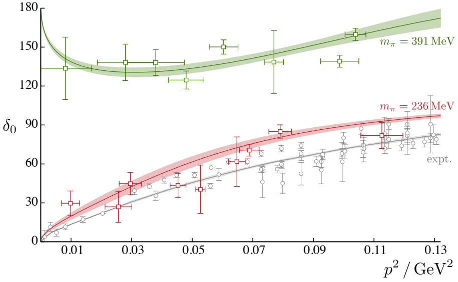
Virtual state pole mechanism

- The general phenomenon was firstly discussed in [Blankenbecler, Goldberger, MacDowell, and Treiman, Phys. Rev. (1961)] (rediscovered in $\pi\pi$ scatterings [Z.-Y Zhou, et al., JHEP (2005)], in πN scatterings [Q.-Z. Li and H.-Q. Zheng, CTP (2022)])
- $S^{\rm II} = 1/S^{\rm I}$, e.g., S-matrix zero in the 1-st sheet $\rightleftharpoons$ S-matrix pole (virtual state pole and resonance pole) in the 2-nd sheet (single channel case)



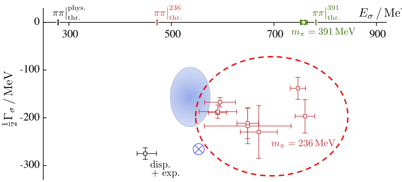


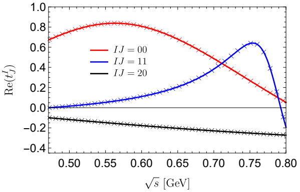
A preliminary attempt: $m_\pi \sim 236$ MeV

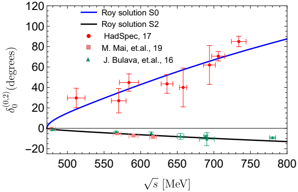


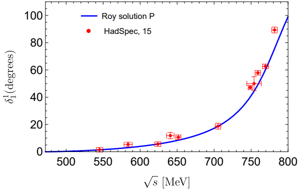
$IJ = 00$

- $p\cot\delta \sim \frac{1}{a}$: lower a $\longrightarrow$ broader σ
- Phase shifts are similar to the physical case
- Extremely "noisy" [HadSpec. PRL (2017)]





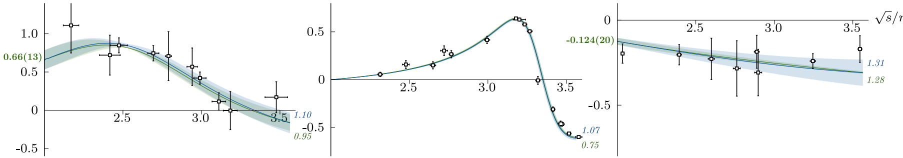




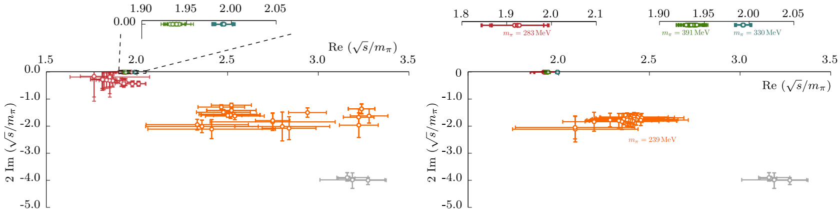
Other Roy equation analyses

A. Rodas et.al. [HadSpec. Collaboration] arXiv: 2304.03762

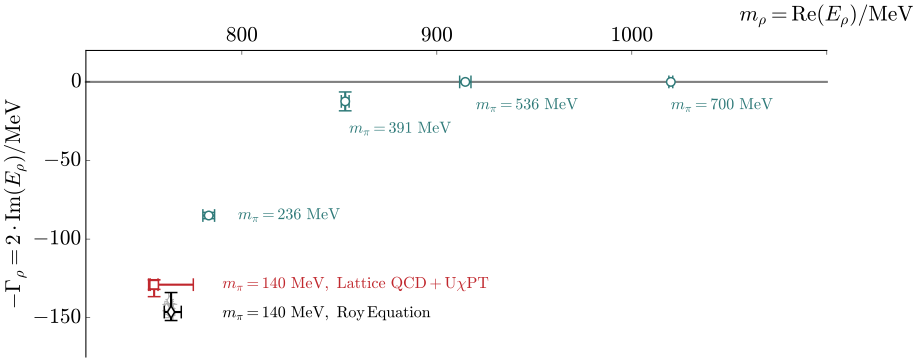
- Focus on two cases: $m_\pi \sim 283, 239$ MeV
- Use extrapolations of $IJ = 00$ elastic amplitude up to ~ 1.34 GeV , and for different parameterizations



- New σ pole positions via preliminary Roy equation analyses



$IJ = 11: \rho$

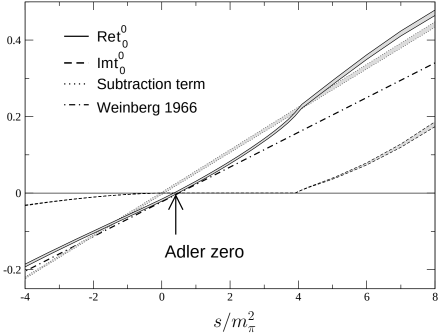


- K-Matrix: $m_\pi = 391$ MeV: $m_\rho = 854.1 \pm 1.1$ MeV, $\Gamma_\rho = 12.4 \pm 0.6$ MeV
  $m_\pi = 236$ MeV: $m_\rho = 783 \pm 2$ MeV, $\Gamma_\rho = 90 \pm 8$ MeV
- Roy Equation: $m_\pi = 391$ MeV: $m_\rho = 853.3^{+1.1}_{-1.1}$ MeV, $\Gamma_\rho = 13.4^{+0.4}_{-1.4}$ MeV
  $m_\pi = 236$ MeV: $m_\rho = 785$ MeV, $\Gamma_\rho = 86$ MeV

$IJ = 20$: Roy equation v.s. $\chi\text{PT}_{\text{NNLO}}$

	$m_\pi = 236$ MeV		$m_\pi = 391$ MeV	
	Roy equation	$\chi\text{PT}_{\text{NNLO}}$	Roy equation	$\chi\text{PT}_{\text{NNLO}}$
$\sqrt{s_{A,IJ=00}}$	162	140^{+46}_{-29}	$(206^{+29}_{-18}) \pm i(218^{+3}_{-18})$	225^{+131}_{-115}
$\sqrt{s_{A,IJ=20}}$	326	334^{+13}_{-16}	601^{+8}_{-17}	546^{+41}_{-73}
$\sqrt{s_{v,IJ=20}}$	117	167^{+8}_{-9}	435^{+4}_{-12}	410^{+30}_{-41}

[Caprini et. al. PRL (2006)]

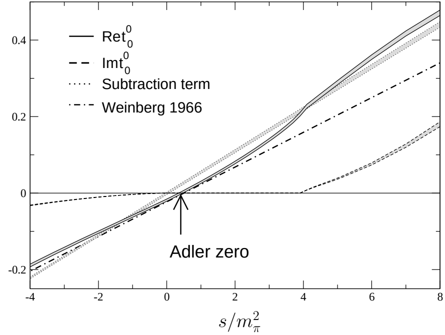


- Adler zero:
  special zero of amplitudes below the threshold
  Manifests chiral symmetry
  (even in unphysical region)!

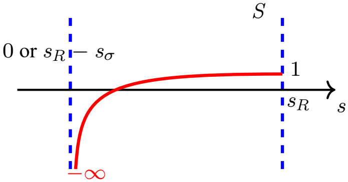
$IJ = 20$: Roy equation v.s. χPT$_{\rm NNLO}$

	$m_\pi = 236$ MeV		$m_\pi = 391$ MeV	
	Roy equation	χPT$_{\rm NNLO}$	Roy equation	χPT$_{\rm NNLO}$
$\sqrt{s_{A,IJ=00}}$	162	140^{+46}_{-29}	$(206^{+29}_{-18}) \pm i(218^{+3}_{-18})$	225^{+131}_{-115}
$\sqrt{s_{A,IJ=20}}$	326	334^{+13}_{-16}	601^{+8}_{-17}	546^{+41}_{-73}
$\sqrt{s_{v,IJ=20}}$	117	167^{+8}_{-9}	435^{+4}_{-12}	410^{+30}_{-41}

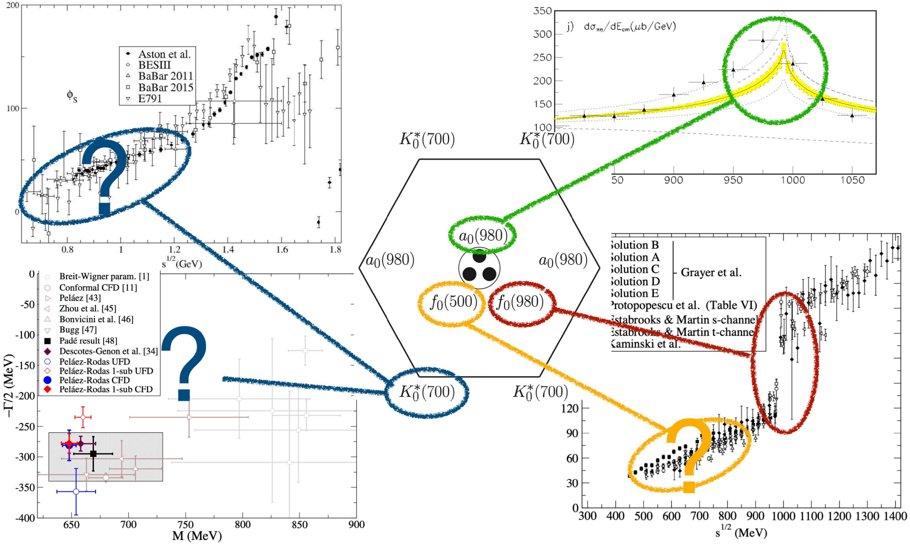
[Caprini et. al. PRL (2006)]



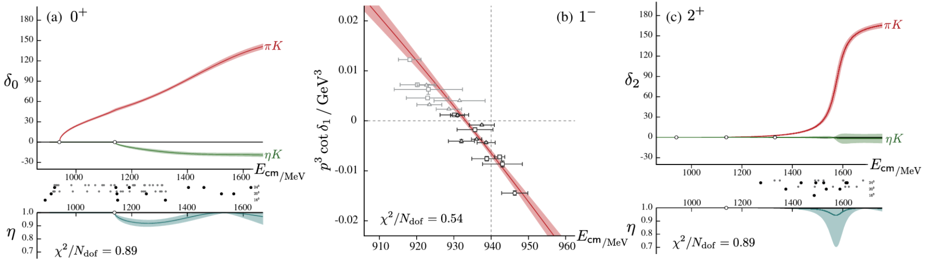
- Adler zero:
  special zero of amplitudes below the threshold
  Manifests chiral symmetry
  (even in unphysical region)!
- Virtual state pole in $IJ = 20$ channel:

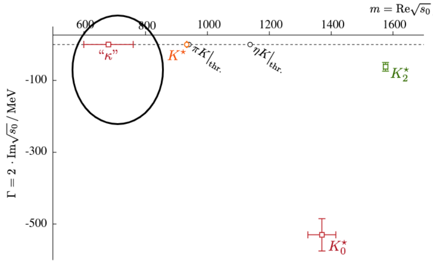


Light exotic states



Unphysical quark masses HadSpec. Collaboration, PRL(2014); PRD(2015); PRL(2019)





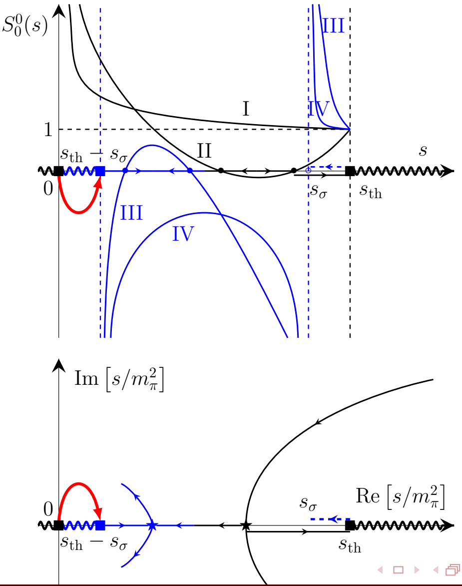
- $m_\pi \sim 390$ MeV, K-matrix fits:
  - $\kappa/K_0^*(700)$: **one virtual state???**
  - $K^*(892)$: **a shallow bound state**
  - $K_0^*(1430)$: $m\downarrow, \Gamma\uparrow$
  - $K_2^*(1430)$: $m\uparrow, \Gamma\downarrow$

Summary

- The unity of dispersive techniques and experimental data is powerful to investigate low energy hadron physics
  - Widely-used unitarization methods such as K-matrix, etc., are not good in light meson & baryon studies
  - (Covariant) Chiral pertubation theory obeys perturbative unitarity and analytic properties (correct LHC structure)
  - Dispersive approaches, Muskhelishvili-Omnès formalism, Roy and Roy-Steiner equations, etc. are necessary
- Lattice data+Roy equation (other dispersive methods)
  - σ found as a bound state, virtual state and resonance
  - Systematics $\Longrightarrow$ model independent!
  - **Generalize to πK scattering (in progress)**
  - **Meson-baryon (πN, KN, $\bar{K}N$...) scatterings?**

Thanks for your attention!

Trajectory


$S_0^0(s)$
1
I
II
III
IV
$s_{\rm th} - s_\sigma$
0
s_σ
$s_{\rm th}$
s
Im $[s/m_\pi^2]$
Re $[s/m_\pi^2]$

I. Caprini, G. Colangelo, and H. Leutwyler. Mass and width of the lowest resonance in QCD. *Phys. Rev. Lett.*, 96:132001, 2006. doi: 10.1103/PhysRevLett.96.132001.

S. Descotes-Genon and B. Moussallam. The K*0 (800) scalar resonance from Roy-Steiner representations of pi K scattering. *Eur. Phys. J. C*, 48:553, 2006. doi: 10.1140/epjc/s10052-006-0036-2.

H. Lehmann. Analytic properties of scattering amplitudes as functions of momentum transfer. *Nuovo Cim.*, 10(4):579–589, 1958. doi: 10.1007/bf02859794.

S. Mandelstam. Determination of the pion - nucleon scattering amplitude from dispersion relations and unitarity. General theory. *Phys. Rev.*, 112:1344–1360, 1958. doi: 10.1103/PhysRev.112.1344.

S. Mandelstam. Analytic properties of transition amplitudes in perturbation theory. *Phys. Rev.*, 115: 1741–1751, 1959. doi: 10.1103/PhysRev.115.1741.