Chiral EFT prediction for $nn \to ppee$ decay at leading order

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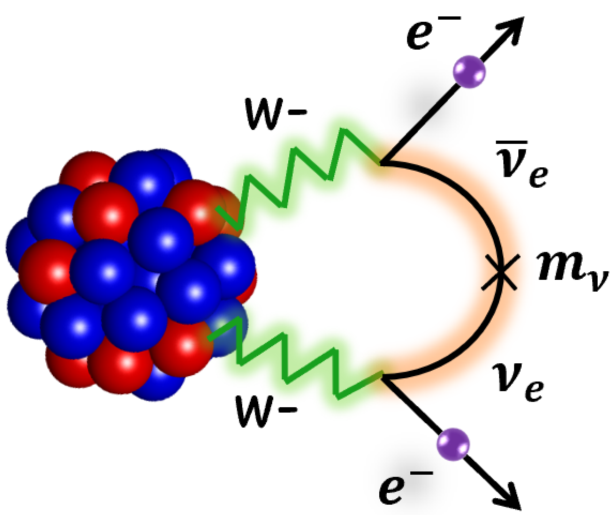
Y. L. Yang and P. W. Zhao, arXiv:2308.03356 (2023)

Outline

- $0\nu\beta\beta$ from light Majorana neutrino exchange
- The contact term at leading order
- A *Relativistic* EFT framework for $0\nu\beta\beta$
- Summary and outlook

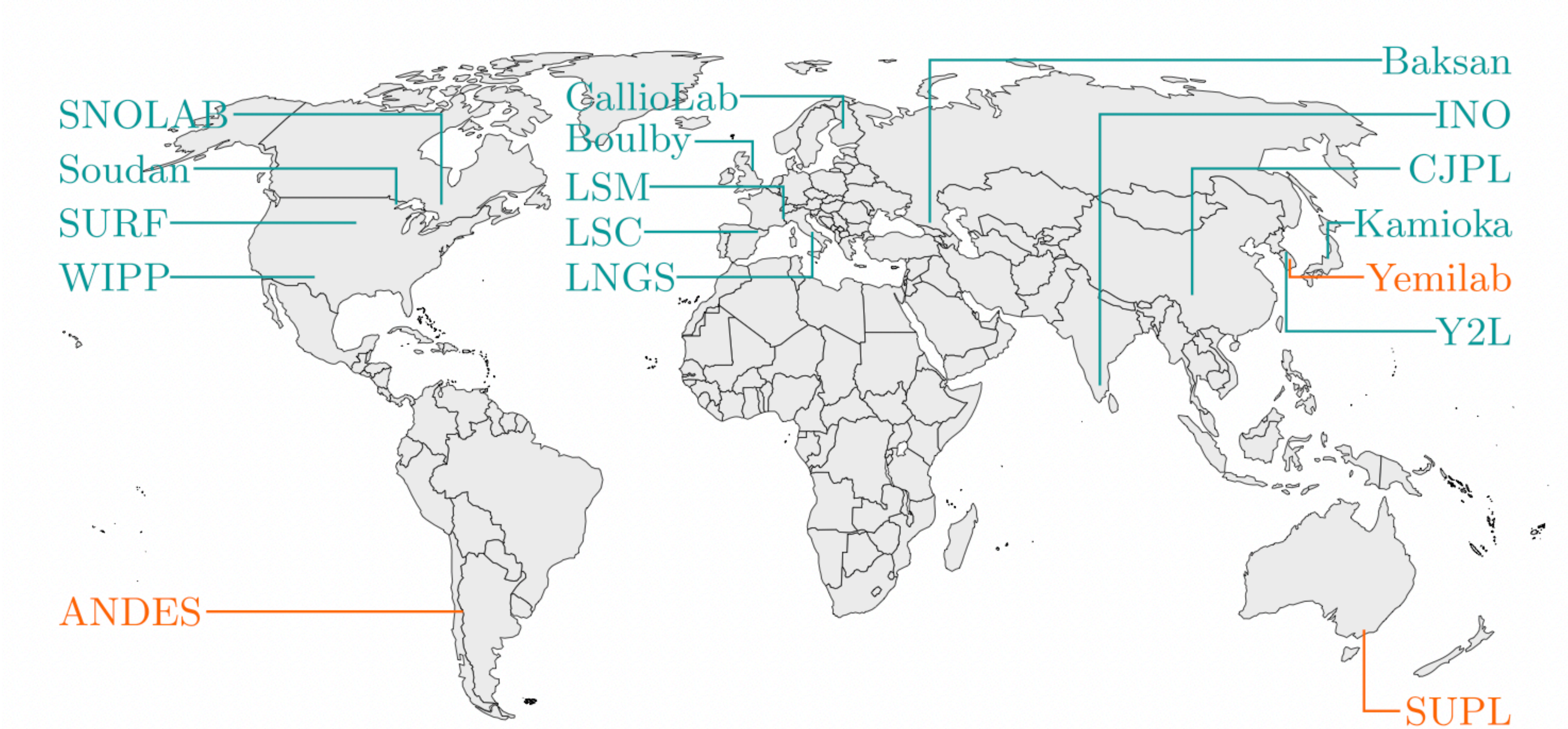
Neutrinoless double β decay

- Neutrinoless $\beta\beta$ decay ($0\nu\beta\beta$): $(A,Z) \rightarrow (A,Z+2) + e^- + e^-$
  - ✓ Lepton number violation
  - ✓ Majorana nature of neutrinos
  - ✓ Neutrino mass scale and hierarchy
  - ✓ matter-antimatter asymmetry



Avignone, Elliott, Engel, Rev. Mod. Phys. 80, 481 (2008)

- $0\nu\beta\beta$ search in worldwide experimental facilities



Agostini et al., Rev. Mod. Phys. 95, 025002 (2023)

Light Majorana ν exchange

Scale

Λ (>TeV)

Quarks

$$\mathscr{L}_{\rm BSM} = \mathscr{L}_{\rm SM} + \frac{C^{(5)}}{\Lambda} O^{(5)} + \cdots$$

"Standard model" EFT

Weinberg, PRL 43, 1566 (1979)

Λ_χ (~GeV)

Nucleons, Pions

$$\mathscr{L}_{\rm eff} = \mathscr{L}_{\pi,N} - \frac{1}{2} m_{\beta\beta} \nu_{eL}^T C \nu_{eL}$$

Light Majorana mass

$$m_{\beta\beta} = \sum_i U_{ei}^2 m_i$$

Nuclear matrix elements

Nuclei

$$[T_{1/2}^{0\nu}]^{-1} = |m_{\beta\beta}|^2 G^{0\nu} |M^{0\nu}|^2$$

$0\nu\beta\beta$ decay half-life

Nuclear matrix element

Nuclear matrix element is **crucial to interpreting experimental limits**, but so far suffers from **sizable uncertainties**.

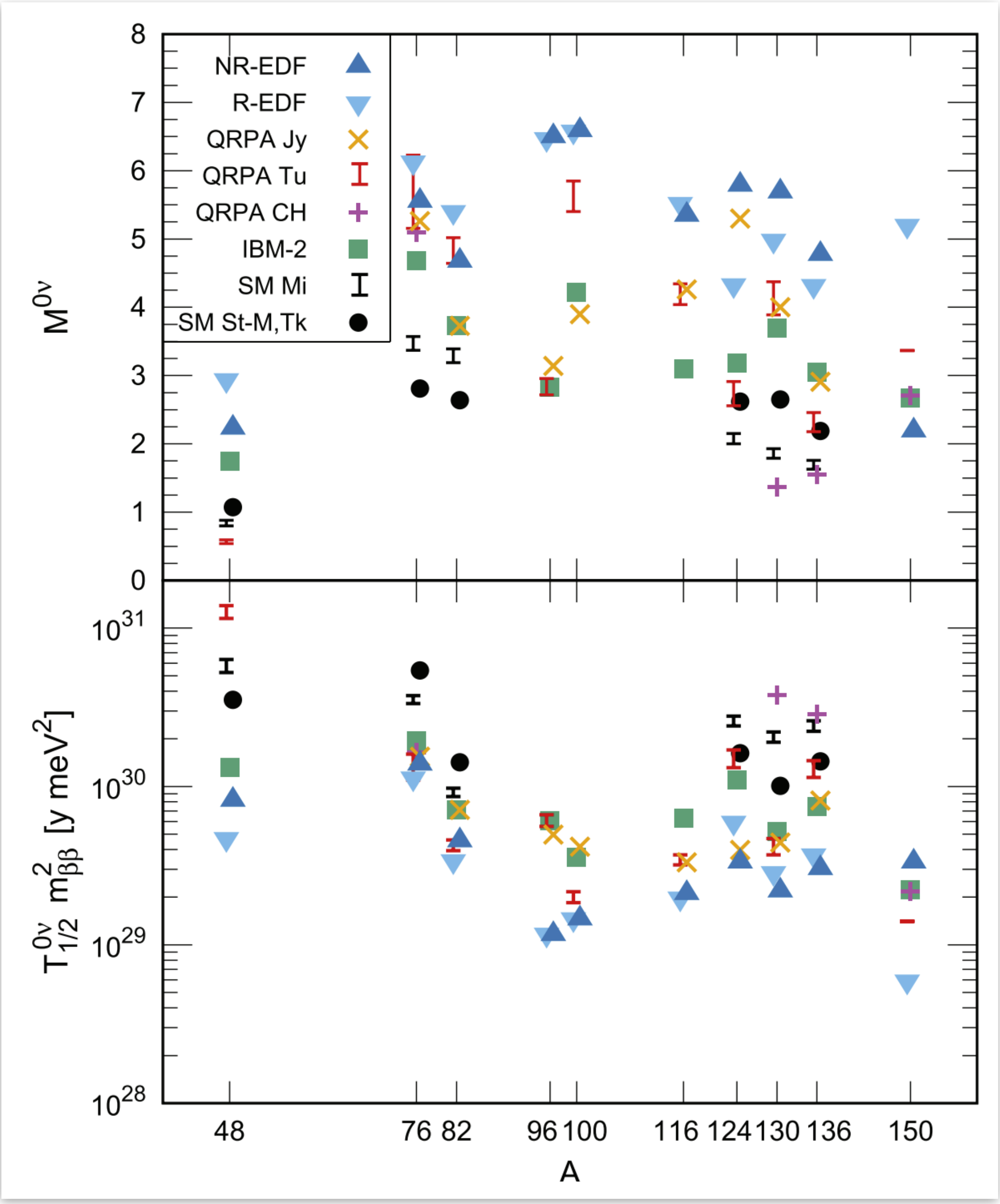
$$[T^{0\nu}_{1/2}]^{-1} = |m_{\beta\beta}|^2 G^{0\nu} \, |M^{0\nu}|^2$$

Nuclear matrix element

$$M^{0\nu} = \langle \Psi_f | \hat{O}^{0\nu} | \Psi_i \rangle$$

- Depending on **decay operator** $\hat{O}^{0\nu}$
- Differences between different **nuclear models** for $\Psi_{f,i}$

$\Rightarrow$ Sizable uncertainties



Engel and Menéndez, Rep. Prog. Phys. 80, 046301 (2017)

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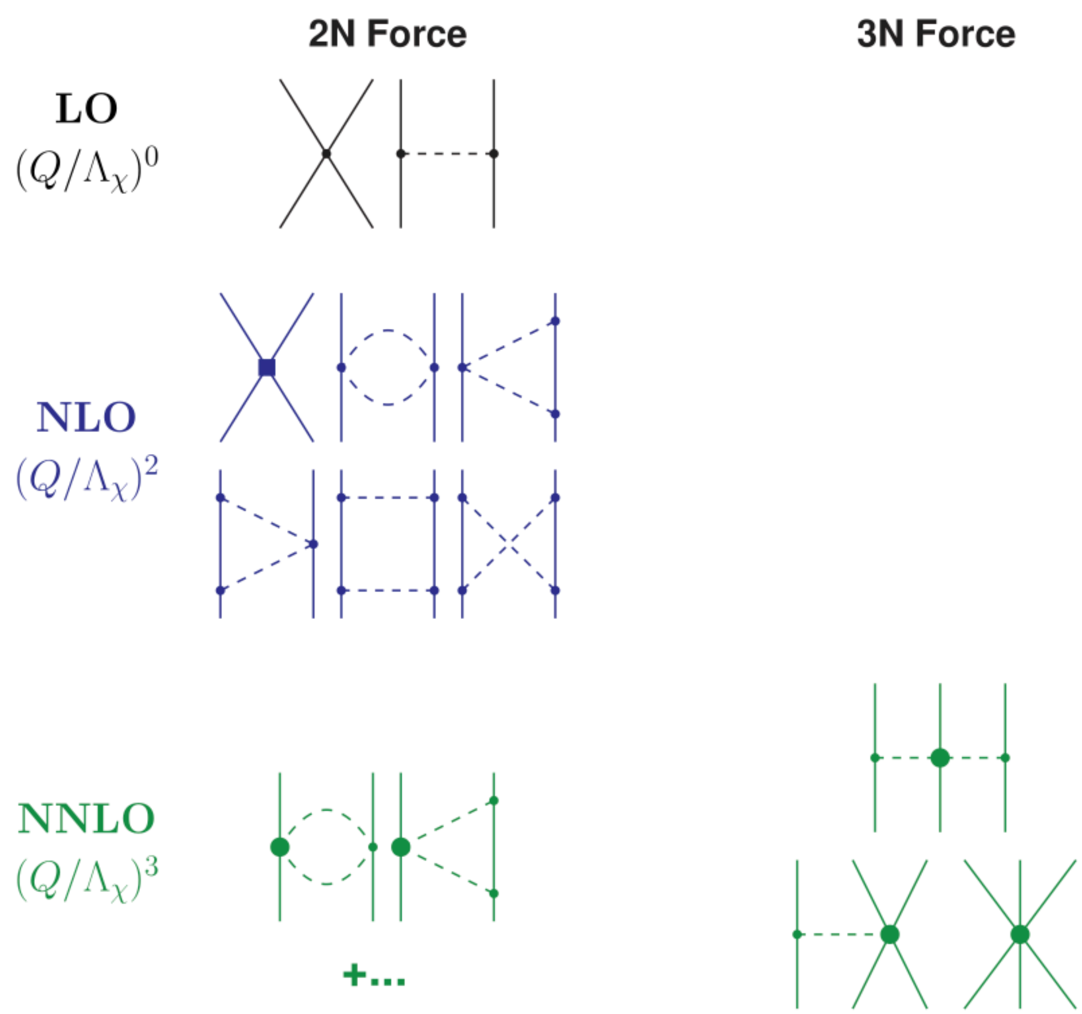
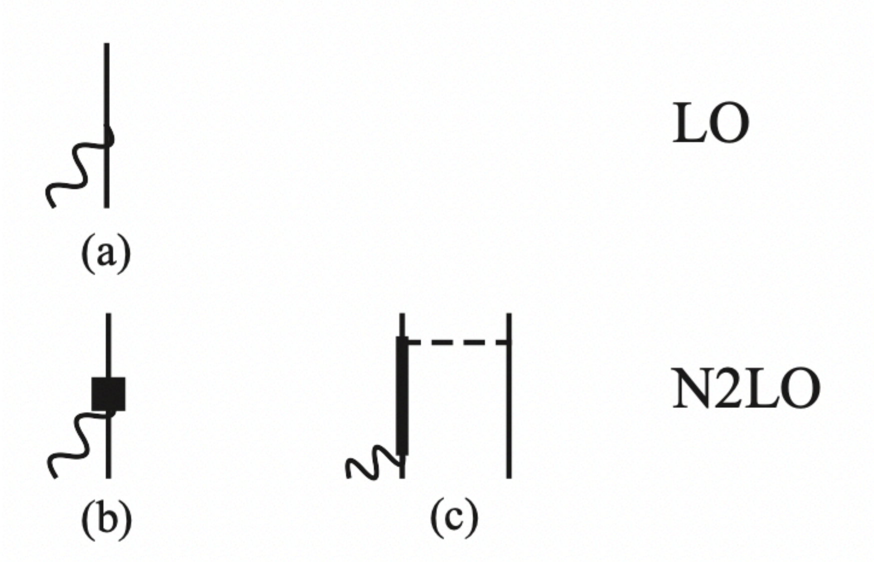
Chiral EFT for $0\nu\beta\beta$ decay

Chiral EFT can provide nuclear force and weak currents in a **consistent and systematically improvable** manner.

- Nuclear force



- Weak currents



Solid: nucleons

Dashed: pions

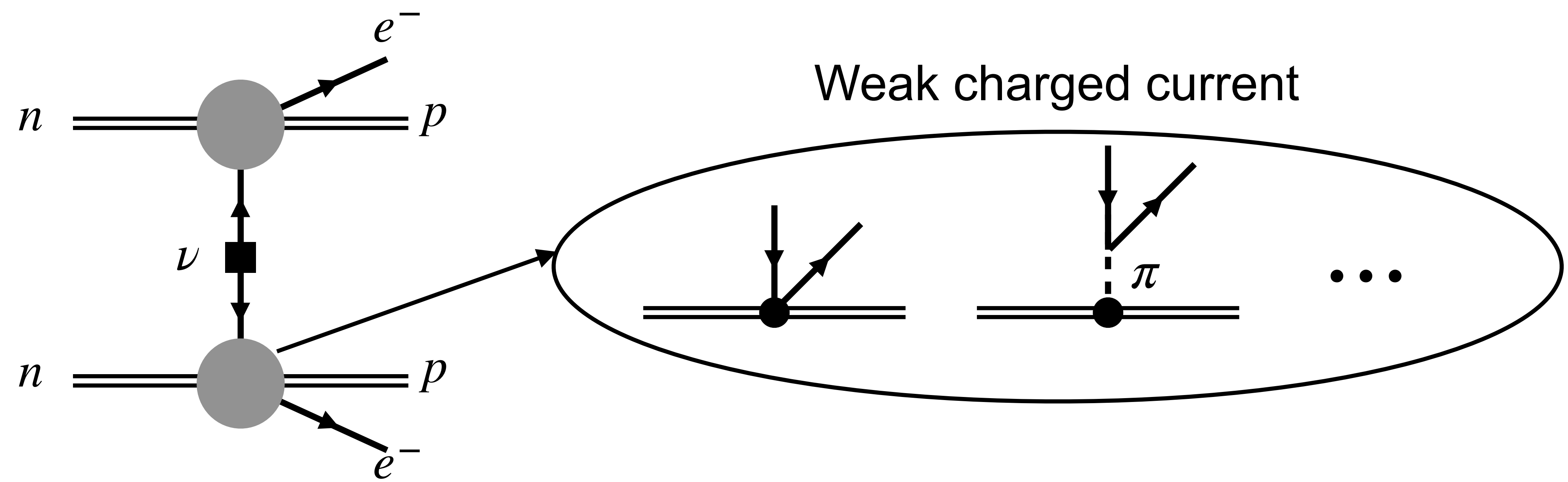
Wavy: external current

Machleidt and Entem, Phys. Rep. 503, 1 (2011)

King, Andreloi, Pastore, and Piarulli, Front. Phys. 8, 363 (2020)

Decay operator

- $nn \to ppee$ decay induced by the exchange of a Majorana neutrino



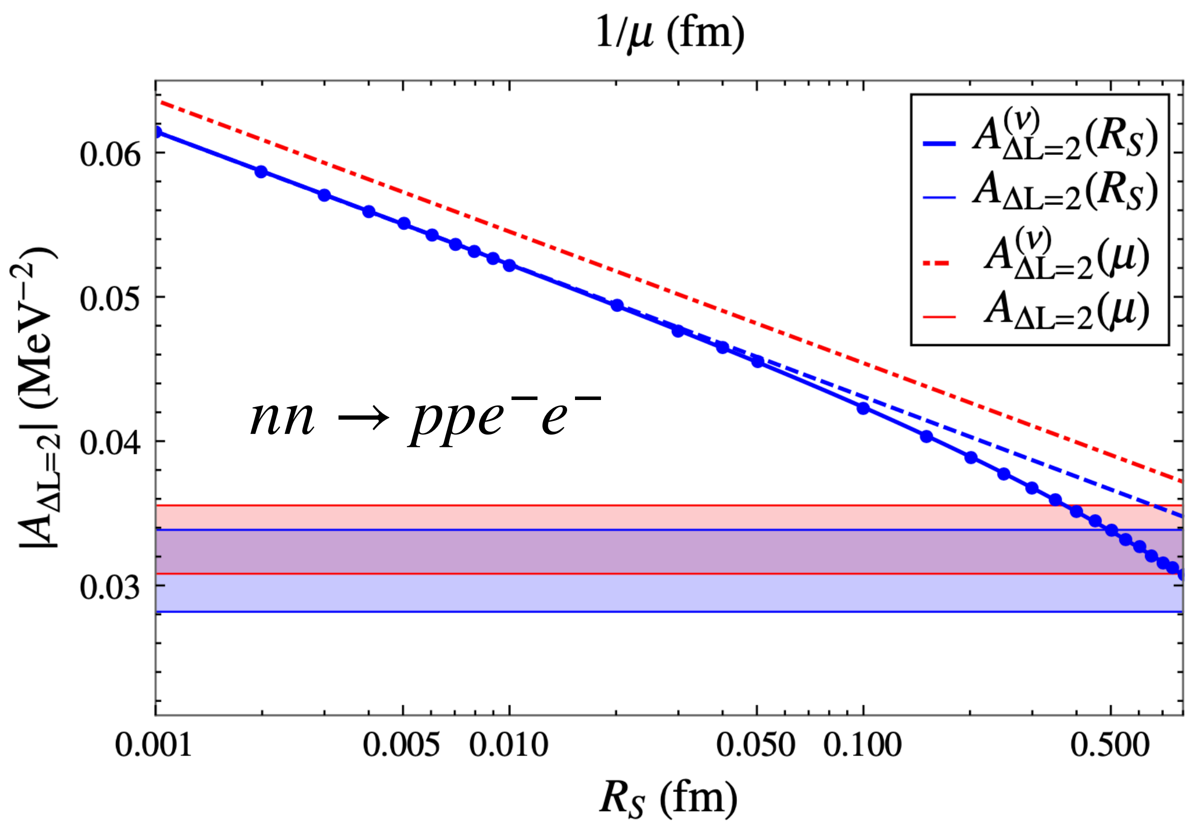
- Leading order (LO) decay operator:

$$O^{0\nu} = \tau_1^+ \tau_2^+ \frac{1}{\boldsymbol{q}^2} \left\{ g_V^2 - g_A^2 \left[\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 - \boldsymbol{\sigma}_1 \cdot \boldsymbol{q} \boldsymbol{\sigma}_2 \cdot \boldsymbol{q} \frac{2m_\pi^2 + \boldsymbol{q}^2}{(\boldsymbol{q}^2 + m_\pi^2)^2} \right] \right\}$$

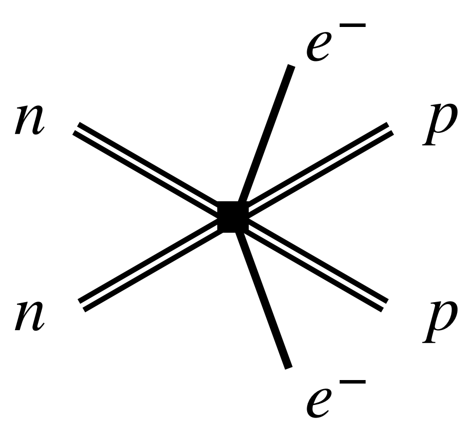
Vector coupling $g_V = 1$ Axial coupling $g_A = 1.27$

Contact term

For renormalization-group invariance, a **contact term with unknown size** is proposed to be **promoted to LO** in chiral EFT.



Cirigliano et. al., PRL 120, 202001 (2018)



$$V_{\nu,\mathrm{ct}} = -2 g_\nu^{\mathrm{NN}} \tau_1^+ \tau_2^+$$

Unknown LEC

- The size of contact term should be determined by:
  - Experimental data — Not available
  - Matching to lattice QCD results — Not available yet

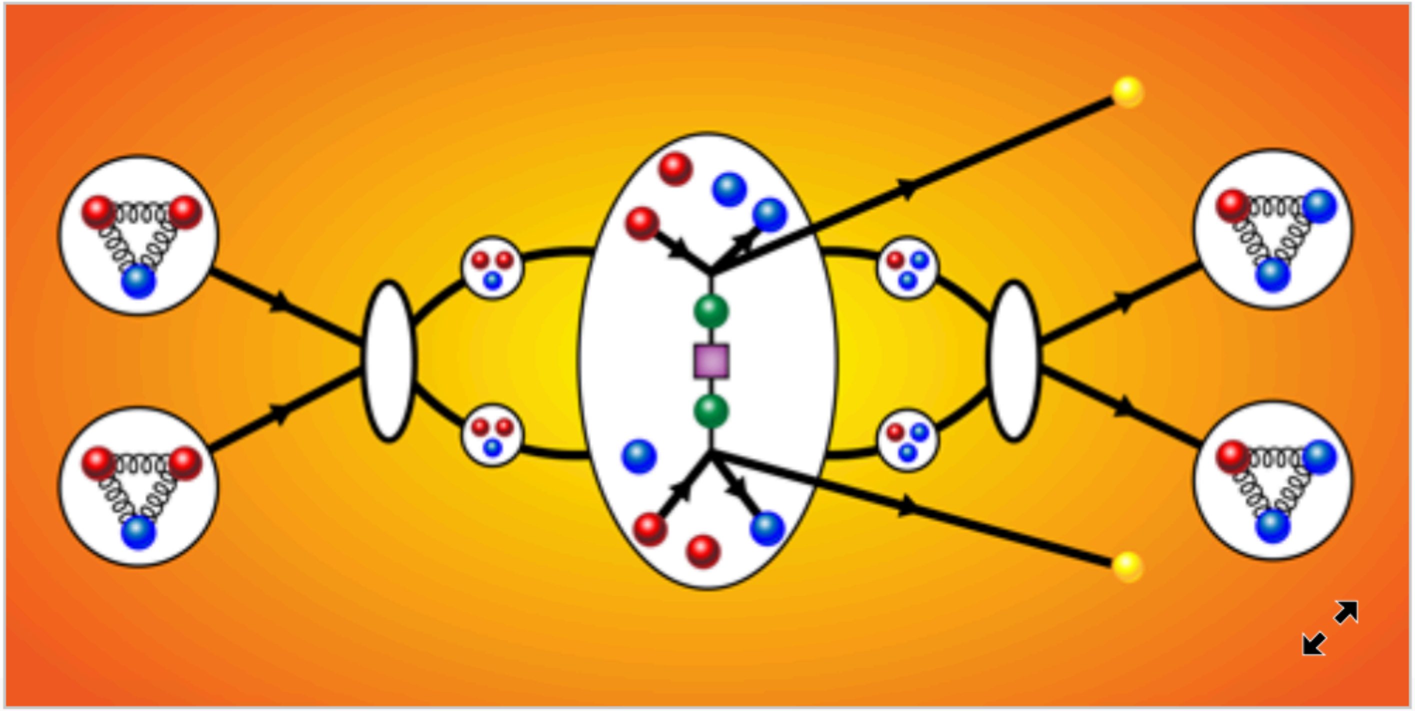
Contact term

SYNOPSIS

A Missing Piece in the Neutrinoless Beta-Decay Puzzle

May 16, 2018 • *Physics* 11, s58

The inclusion of short-range interactions in models of neutrinoless double-beta decay could impact the interpretation of experimental searches for the elusive decay.



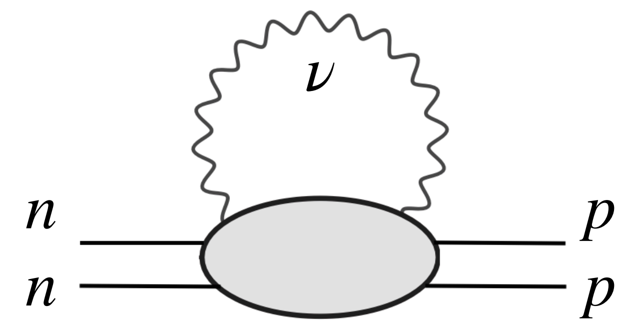
Model estimate of contact term

The contact term so far has to be fitted to a synthetic datum of $nn \to ppee$ amplitude from a **phenomenological model**.

Cirigliano et al., PRL 126, 172002 (2021)

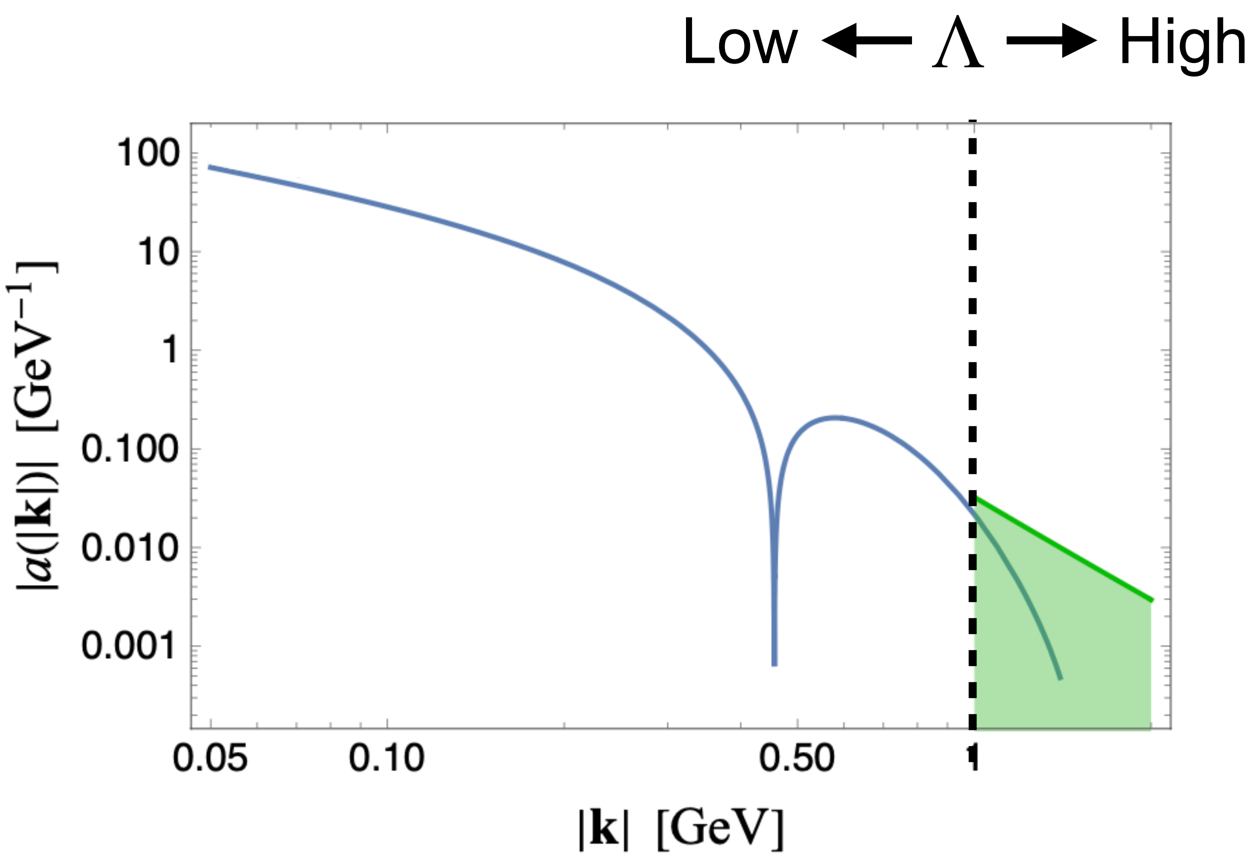
$$\mathcal{A}_\nu \propto \int \frac{d^4k}{(2\pi)^4}\frac{g_{\alpha\beta}}{k^2+i\epsilon}\int d^4x\, e^{ik\cdot x}\langle pp|T\{j_{\rm w}^\alpha(x)j_{\rm w}^\beta(0)\}|nn\rangle$$

$$= \int_0^\Lambda d|\mathbf{k}|\; a_<(|\mathbf{k}|) \;+\; \int_\Lambda^\infty d|\mathbf{k}|\; a_>(|\mathbf{k}|)\,,$$



- Model assumptions and inputs：

1. neglect inelastic intermediate states
2. Phenomenological off-shell NN amplitudes
3. Phenomenological weak form factors
4. Separation of low- and high-energy region
5. …



➡ $\tilde{\mathcal{C}}_1(\mu_\chi = M_\pi) \simeq 1.32(50)_{\rm inel}(20)_{V_S}(5)_{\rm par}$

Cirigliano et al., JHEP 05, 289 (2021)

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Relativistic effects in nuclear systems

The existing EFT studies on $0\nu\beta\beta$ decay are all non-relativistic.

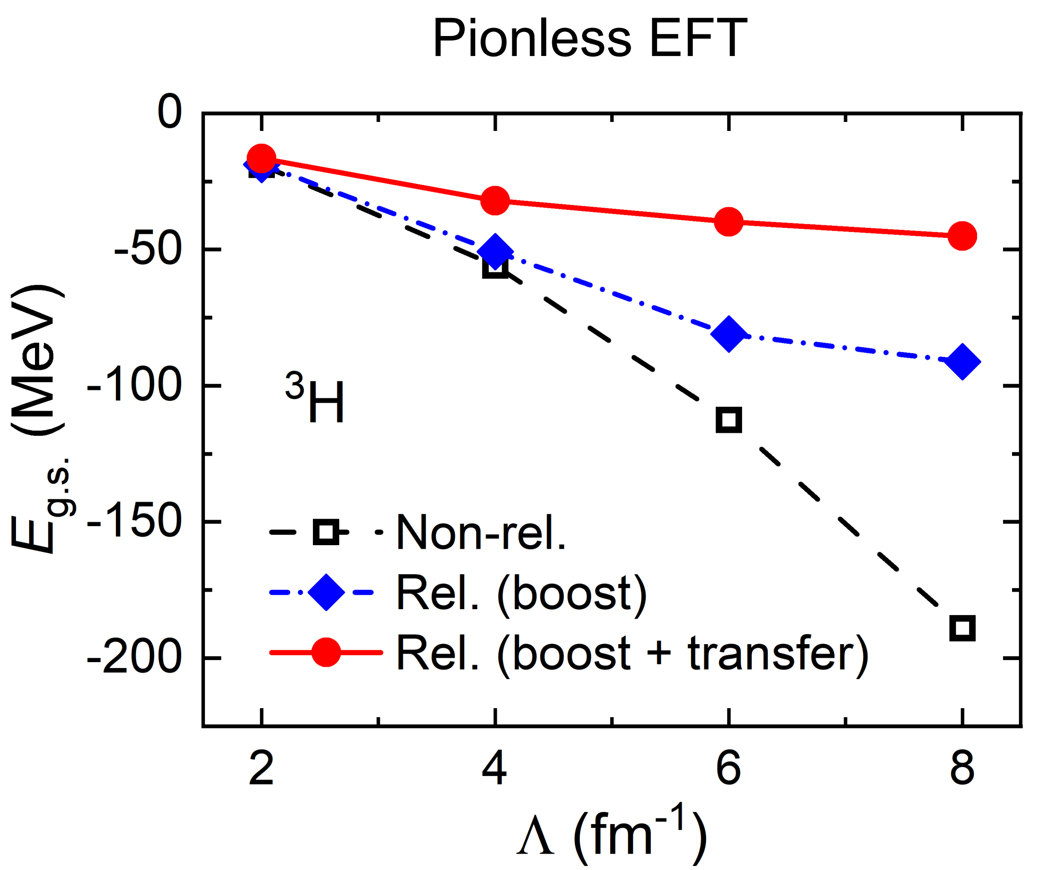
In the relativistic framework, do we need to promote a contact term to LO?

- Relativistic scattering equations have better ultraviolet (UV) behavior.
  - Nucleon-nucleon scattering phase shift
  - Binding energies of few-body nuclei

Epelbaum and Gegelia, PLB 716, 338 (2012)
Baru, Epelbaum, Gegelia, and Ren PLB 798, 134987 (2019)
Wang, Geng, and Long, Chin. Phys. C 45, 054101 (2021)

In relativistic framework, the three-body contact term is not needed for renormalization of few-body nuclei at leading order.

Y. L. Yang and P. W. Zhao, PLB 835, 137587 (2022)



Relativistic framework

- Manifestly Lorentz-invariant effective Lagrangian

Free — Strong — Weak

$$\mathscr{L}_{\Delta L=0} = \frac{1}{2}\partial_\mu \vec{\pi}\cdot\partial^\mu\vec{\pi} - \frac{1}{2}m_\pi^2\vec{\pi}^2 + \overline{\Psi}(\mathrm{i}\not{\partial} - M_N)\Psi + \frac{g_A}{2f_\pi}\overline{\Psi}\gamma^\mu\gamma_5\vec{\tau}\cdot\partial_\mu\vec{\pi}\Psi + \sum_\alpha C_\alpha(\overline{\Psi}\Gamma\Psi)^2$$
$$+\frac{1}{2}\mathrm{tr}(l_\mu\vec{\tau})\cdot\partial_\mu\vec{\pi} + \frac{1}{2}\overline{\Psi}\gamma^\mu l_\mu\Psi - \frac{g_A}{2}\overline{\Psi}\gamma^\mu\gamma_5 l_\mu\Psi + \cdots$$

Machleidt and Entem, Phys. Rep. 503, 1 (2011)

- Standard mechanism of $0\nu\beta\beta$: electron-neutrino Majorana mass

$$\mathscr{L}_{\Delta L=2} = -\frac{m_{\beta\beta}}{2}\nu_{eL}^T C\nu_{eL}, \quad C = \mathrm{i}\gamma_2\gamma_0$$

- Relativistic Kadyshevsky equation：

Kadyshevsky, NPB 6, 125 (1968)

$$T(\boldsymbol{p}',\boldsymbol{p};E) = V(\boldsymbol{p}',\boldsymbol{p}) + \int\frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi^3)}V(\boldsymbol{p}',\boldsymbol{k})\frac{M^2}{\omega_k^2}\frac{1}{E-2\omega_k+\mathrm{i}0^+}T(\boldsymbol{k},\boldsymbol{p};E) \qquad \omega_k = \sqrt{M^2+\boldsymbol{k}^2}$$

For the interaction $V(\boldsymbol{p}',\boldsymbol{p})$, (1) neglect anti-nucleon d.o.f (2) only include the leading term in Dirac spinor (3) neglect retardation effects.

Modified Weinberg's approach in NN scattering: Epelbaum and Gegelia, PLB 716, 338 (2012)

$nn \to ppee$ amplitude

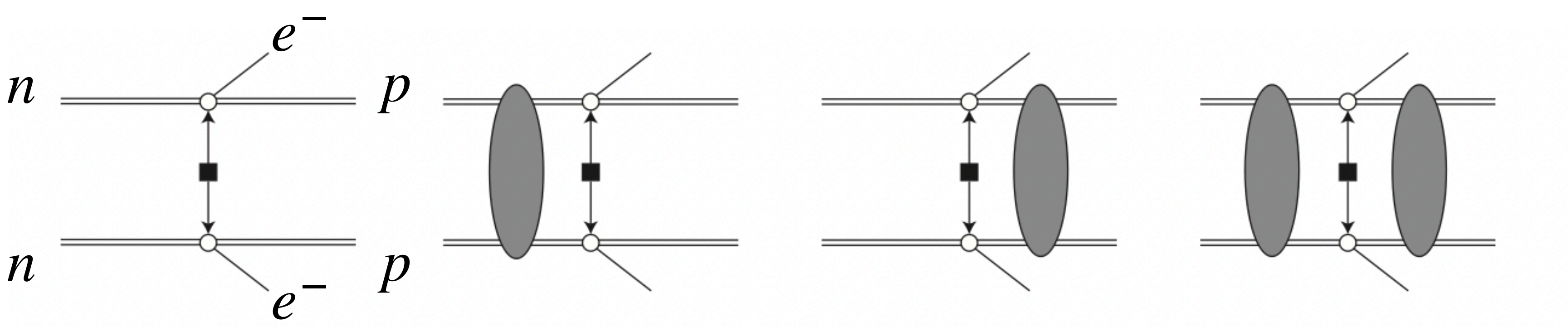
- We focus on $nn \to ppee$ in $^1S_0$ wave, as it is the only channel requiring a contact term in the nonrelativistic framework。

Cirigliano et al., PRL 120, 202001 (2018)
Cirigliano et al., PRC 97, 065501 (2018)

$$n(\boldsymbol{p})n(-\boldsymbol{p}) \to p(\boldsymbol{p}')p(-\boldsymbol{p}')e(\boldsymbol{0})e(\boldsymbol{0})$$

- The leading-order $nn \to ppee$ decay amplitude

$$\mathcal{A}_\nu^{\rm LO} = -\rho_{fi}(V_\nu + V_\nu G_0 T_s + T_s G_0 V_\nu + T_s G_0 V_\nu G_0 T_s)$$

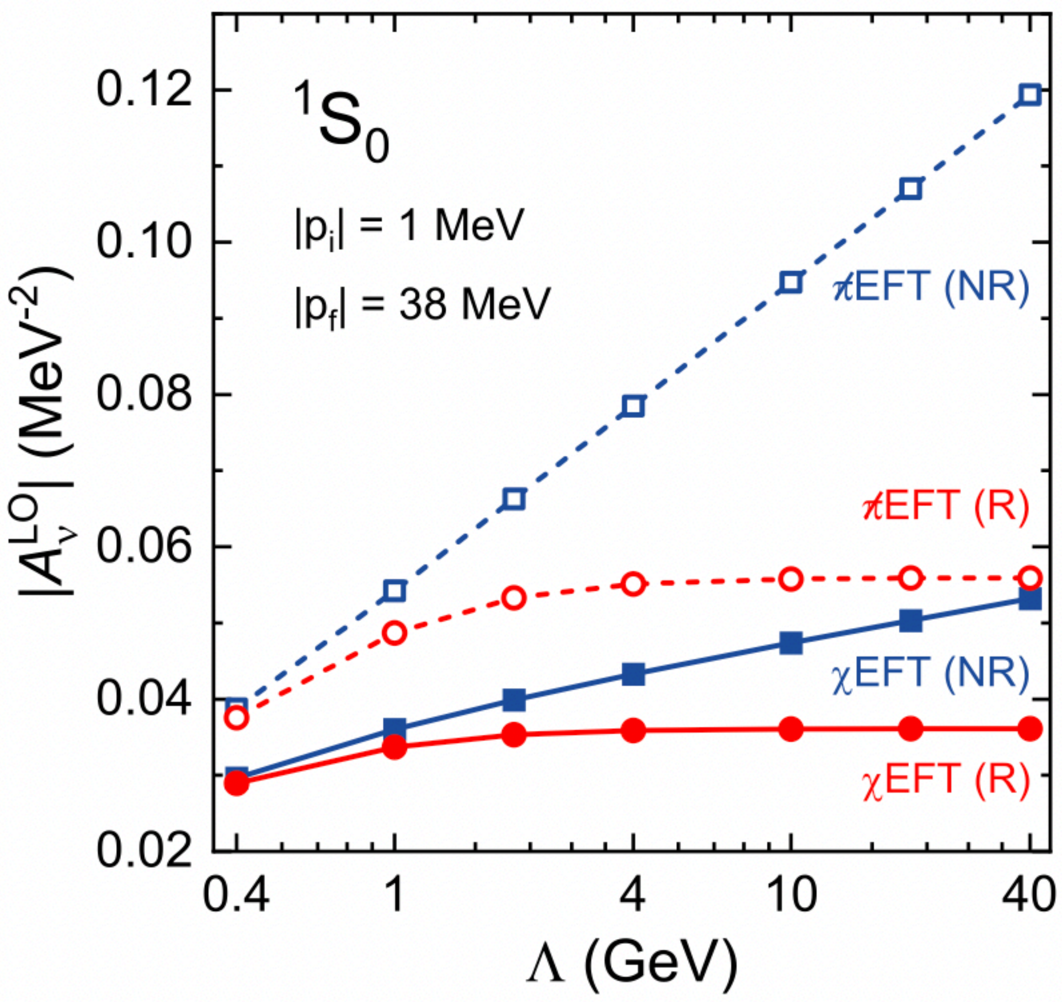


No Free Parameter at all!

Neutrino potential: $V_\nu = \frac{1}{\boldsymbol{q}^2}\left[1 - g_A^2 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + g_A^2 \boldsymbol{\sigma}_1 \cdot \boldsymbol{q} \boldsymbol{\sigma}_2 \cdot \boldsymbol{q} \frac{2m_\pi^2 + \boldsymbol{q}^2}{(\boldsymbol{q}^2 + m_\pi^2)^2}\right]$ $\quad g_A = 1.27$

Strong T-matrix: determined by $^1S_0$ scattering length $\quad a_{np} = -23.74$ fm

Renormalization without contact term



$\not\pi$EFT: pionless EFT

χEFT: chiral EFT

- Regularization scheme: $V_s(\boldsymbol{p}',\boldsymbol{p}) \to e^{-p'^4/\Lambda^4} V_s(\boldsymbol{p}',\boldsymbol{p}) e^{-p^4/\Lambda^4}$
- Nonrelativistic: Logarithmic divergent. Relativistic: Convergent.
- In relativistic framework, no need to introduce unknown contact terms!

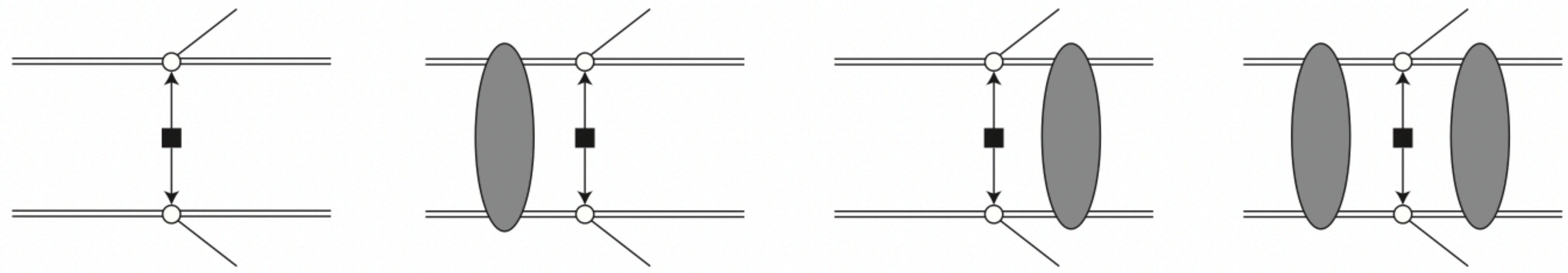
Y. L. Yang and P. W. Zhao, arXiv:2308.03356 (2023)

Analysis of UV divergence

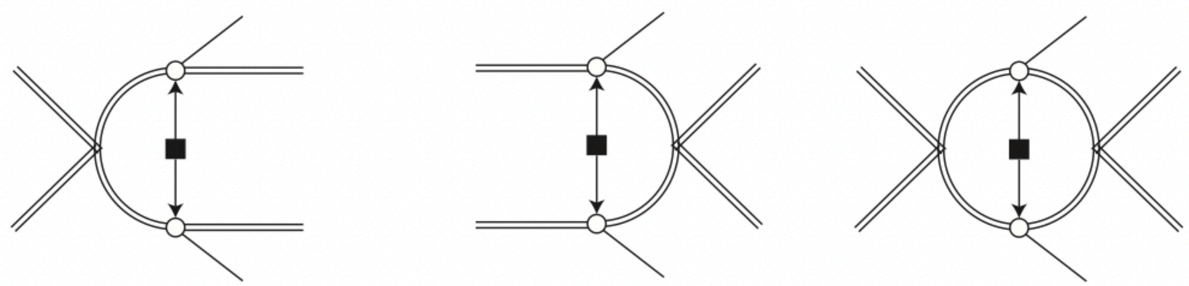
- Relativistic scattering equation has a milder ultraviolet (UV) behavior

	Relativistic		Nonrelativistic
Propagator:	$\frac{M^2}{\boldsymbol{k}^2+M^2}\frac{1}{E-2\sqrt{\boldsymbol{k}^2+M^2}+\mathrm{i}0^+}$	$\rightarrow$	$\frac{1}{E_{\mathrm{kin}}-\boldsymbol{k}^2/M+\mathrm{i}0^+}$
UV behavior ($k\sim\Lambda$):	$O(\Lambda^{-3})$		$O(\Lambda^{-2})$

- The degree of divergence of $nn\rightarrow ppee$ decay amplitude：

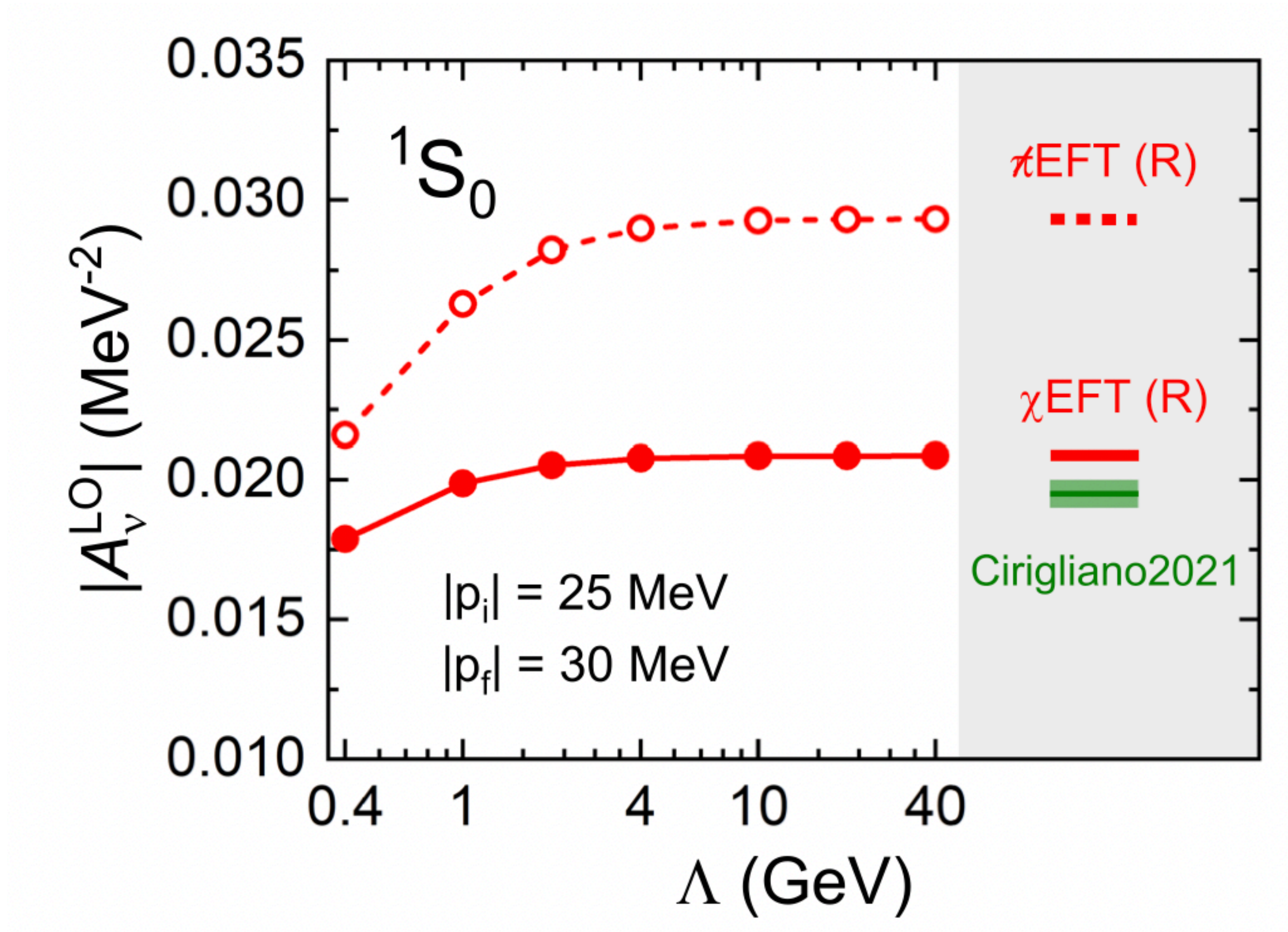


UV structure: –



					Renormalizable?
Relativistic:	–	$O(\Lambda^{-2})$	$O(\Lambda^{-2})$	$O(\Lambda^{-2})$	Yes
Nonrelativistic:	–	$O(\Lambda^{-1})$	$O(\Lambda^{-1})$	$O(\log\Lambda)$	No

Model-free prediction



This work： renormalized by relativity, no contact term

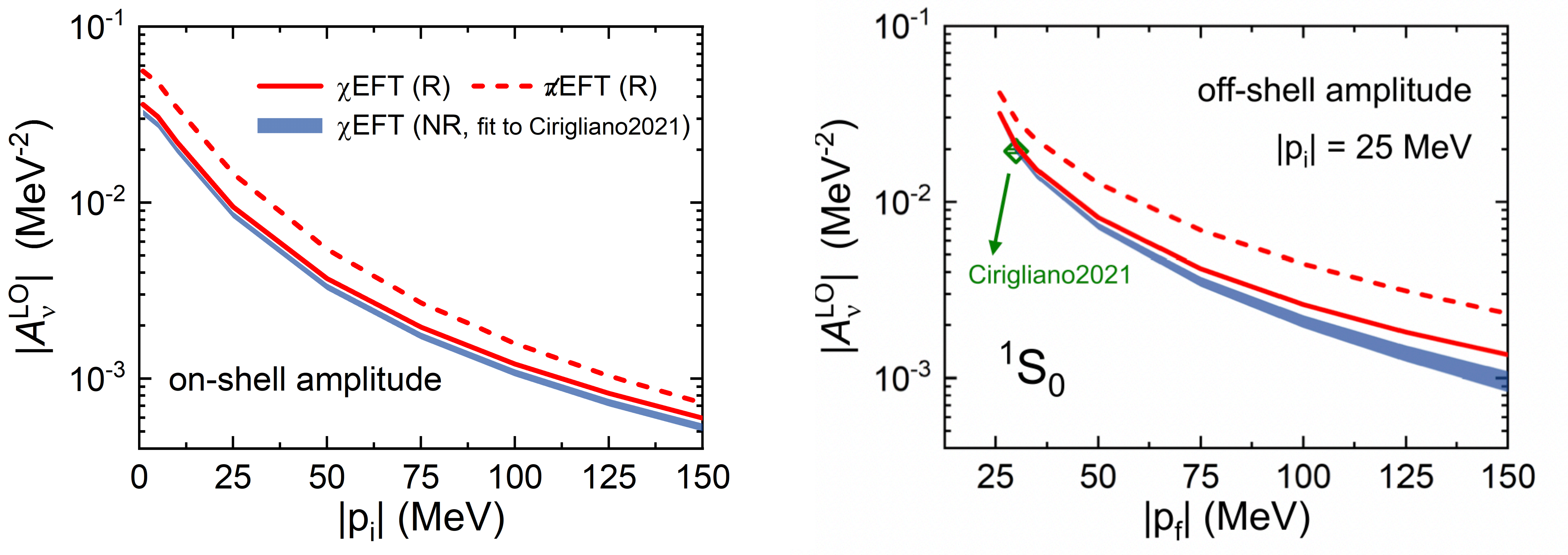
Cirigliano2021： renormalized by contact term, whose size estimated by model-dependent inputs

Cirigliano et al., PRL 126, 172002 (2021)

- Our prediction validates the previous model at the level of 10%.

Y. L. Yang and P. W. Zhao, arXiv:2308.03356 (2023)

Impact



- Relativistic v.s. Nonrelativistic (w/ contact term fit to Cirigliano2021)
- The present Relativistic framework predicts larger amplitudes.
  - 10%-20% for on-shell amplitude
  - 10%-40% for off-shell amplitude

Y. L. Yang and P. W. Zhao, arXiv:2308.03356 (2023)

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Summary

A **model-free** prediction of the $nn \to ppee$ decay amplitude is obtained by a **relativistic framework based on chiral EFT**…

✓ No contact term is needed for renormalization at LO.

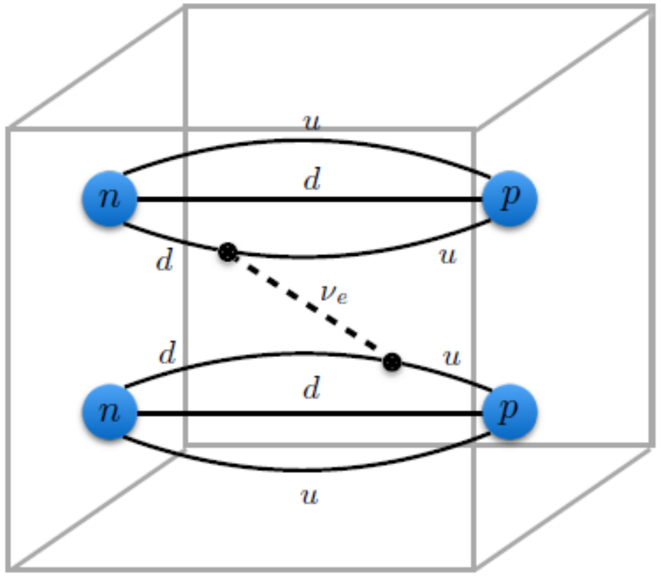
✓ LO Prediction: $\mathcal{A}_\nu^{\mathrm{LO}}(p_i = 25\ \mathrm{MeV}, p_f = 30\ \mathrm{MeV}) = -\,0.0209\ \mathrm{MeV}^{-2}$

✓ Validates the previous model estimation at 10% level.

✓ Predicts enhanced amplitudes compared to the nonrelativistic results fit to the previous pseudo datum for 10%-40%.

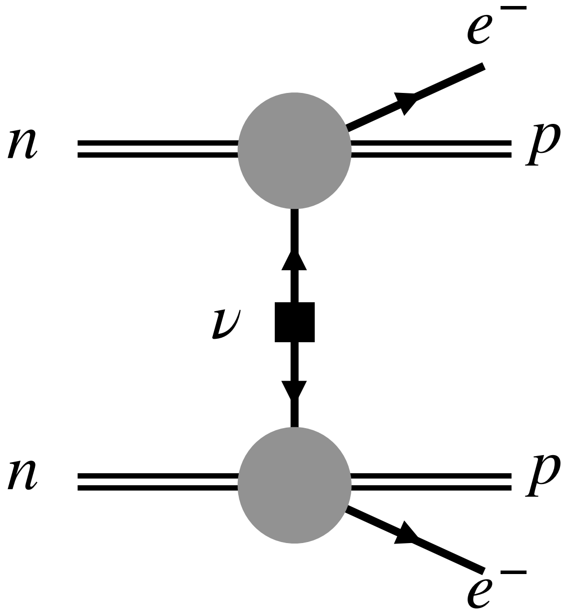
Outlooks

Benchmark with lattice QCD calculations of $0\nu\beta\beta$ decay will provide a stringent test on present framework



Lattice simulation of $0\nu\beta\beta$

Currently only possible at heavy quark (neutrino) masses



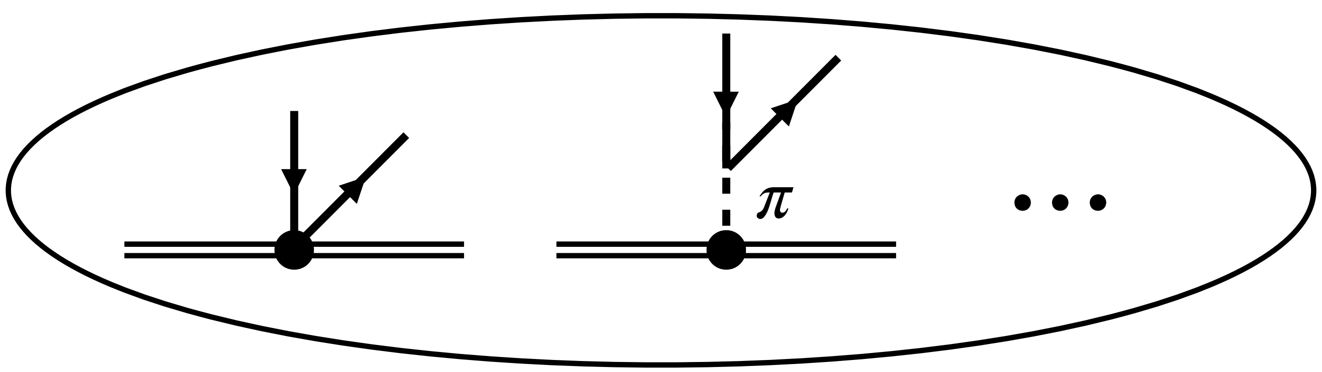
Rel.$\not\pi$EFT (χEFT) prediction

Fixed by $a_{np}(,\ g_A)$ at lattice-QCD conditions

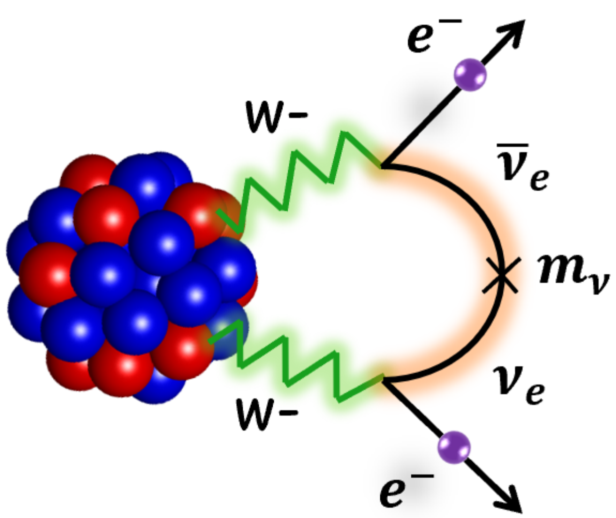
Outlooks

Relativistic calculations of $0\nu\beta\beta$ nuclear matrix elements with LO chiral decay operator

Weak charged current



Nuclear-structure calculations



$$\langle p' | J^\mu_{\rm LO}(x) | p \rangle = \mathrm{e}^{\mathrm{i}qx} \bar{u}(p') \left(g_V \gamma^\mu - g_A \gamma^\mu \gamma_5 + g_A \frac{2 m_N q^\mu}{m_\pi^2 + \boldsymbol{q}^2} \gamma_5 \right) u(p)$$

No need for contact term at LO!

Thank you for your attention!

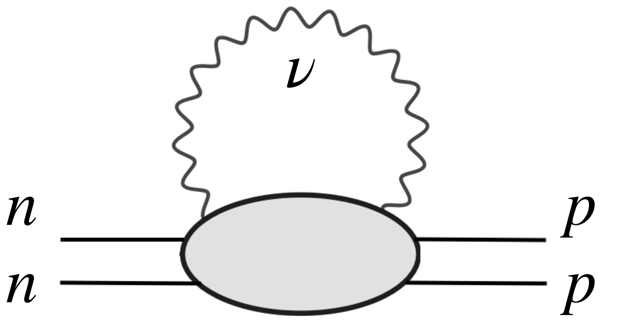
Previous model

- Integral representation

Cirigliano et al., PRL 126, 172002 (2021)

$$\mathcal{A}_\nu \propto \int \frac{d^4k}{(2\pi)^4} \frac{g_{\alpha\beta}}{k^2 + i\epsilon} \int d^4x \, e^{ik\cdot x} \langle pp | T\{ j_{\rm w}^\alpha(x) j_{\rm w}^\beta(0) \} | nn \rangle$$

$$= \int_0^\Lambda d|\mathbf{k}| \; a_<(|\mathbf{k}|) \; + \; \int_\Lambda^\infty d|\mathbf{k}| \; a_>(|\mathbf{k}|) \, ,$$



$$a_<(|\boldsymbol{k}|) = -\frac{r(|\boldsymbol{k}|)}{|\boldsymbol{k}|}\theta(|\boldsymbol{k}| - 2|\boldsymbol{p}|)\left[g_V^2(\boldsymbol{k}^2) + 2g_A^2(\boldsymbol{k}^2) + \frac{\boldsymbol{k}^2}{2M} g_M^2(\boldsymbol{k}^2)\right]$$

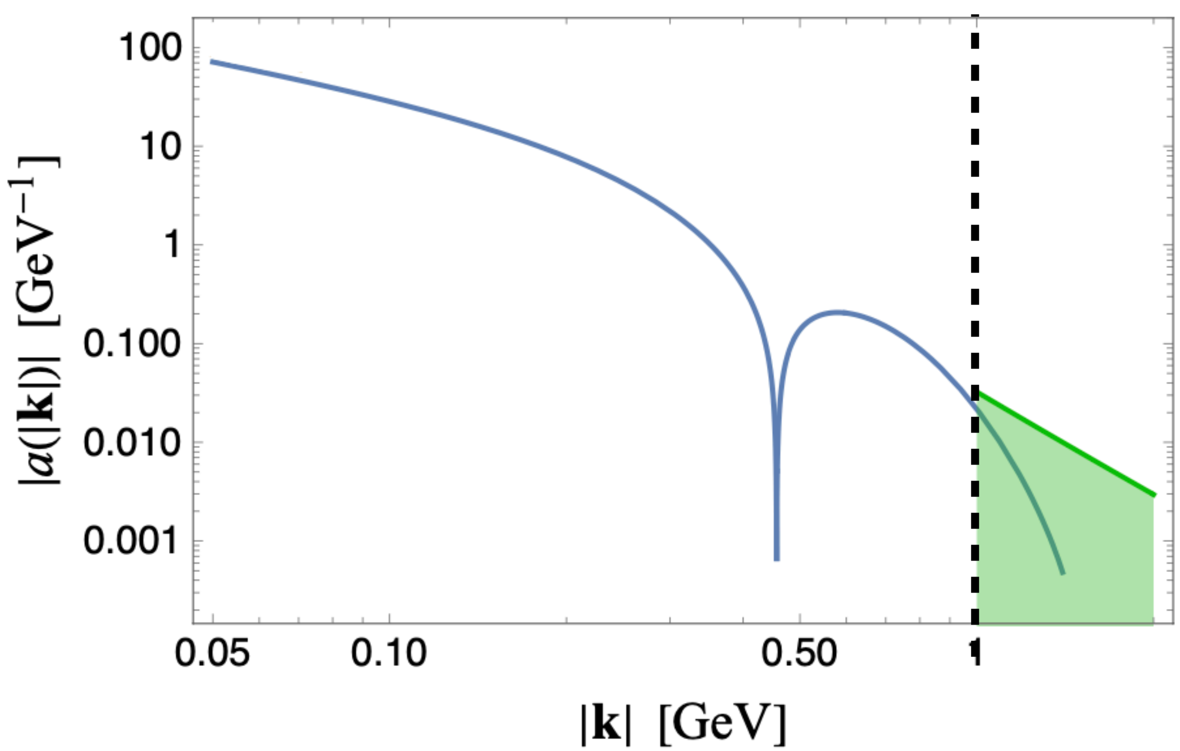
$$a_<(|\boldsymbol{k}|) = \frac{3\alpha_s(\mu)}{\pi} \bar{g}_1^{NN}(\mu) \frac{f_\pi^2}{|\boldsymbol{k}|^3}$$

- Model assumptions and inputs：

1. neglect inelastic intermediate states
2. Phenomenological off-shell NN amplitudes
3. Phenomenological weak form factors
4. Separation of low- and high-energy region
5. Unknown matrix element $\bar{g}_1^{NN}(\mu)$

→ $\tilde{\mathcal{C}}_1(\mu_\chi = M_\pi) \simeq 1.32(50)_{\rm inel}(20)_{V_S}(5)_{\rm par}$

Low ← Λ → High



Cirigliano et al., JHEP 05, 289 (2021)