Study of near-threshold exotic state from coupled-channel effect

Zhi Yang (杨智)

University of Electronic Science and
Technology of China, Chengdu (电子科技大学，成都)

Based on Phys.Rev.Lett. 128,112001(2022); JHEP01(2023)058; arXiv: 2306.12406
In collaboration with 王广娟, 吴佳俊, Makoto Oka, 朱世琳

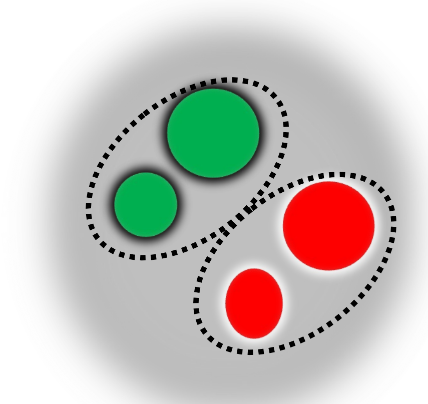
第八届手征有效场论研讨会 开封 2023/10/29

Content

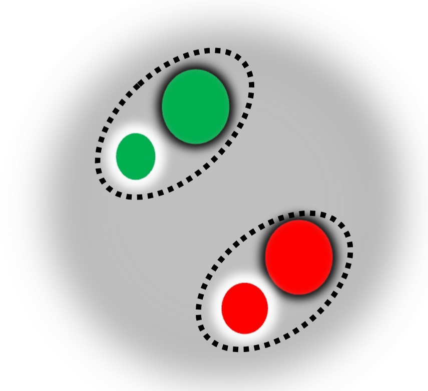
- ❖ $D_{s0}(2317)$ and $D_{s1}(2460)$

- ❖ $X(3872)$

Meson

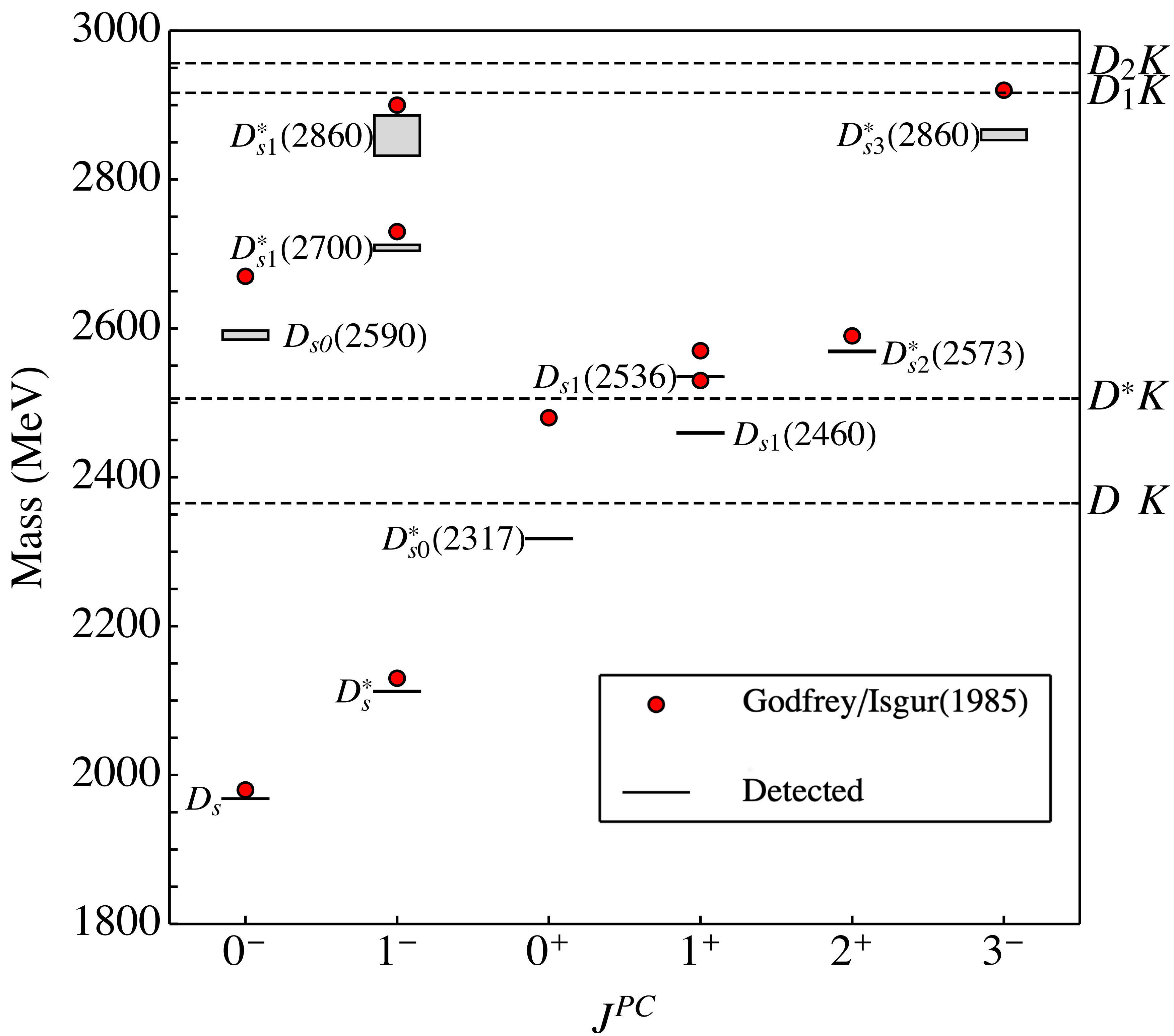


Compact multiquark



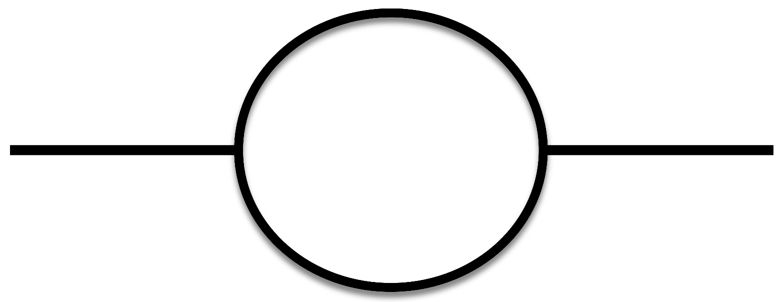
Hadronic molecule

$D_{s0}(2317)$ and $D_{s1}(2460)$ in quark model



The relativized quark model Godfrey, Isgur, **Phys. Rev. D 32,189 (1985)**

Coupled-channel effect



1. Yu. S. Kalashnikova, Phys.Rev.D 72, 034010 (2005)

    ☞ Charmonium

2. F.-K. Guo, S. Krewald, and U.-G. Meißner, Phys.Lett.B 665,157 (2008)
    Z.-Y. Zhou and Z. Xiao, Phys. Rev. D 84, 034023 (2011)

    ☞ Charmed and charmed-strange spectra

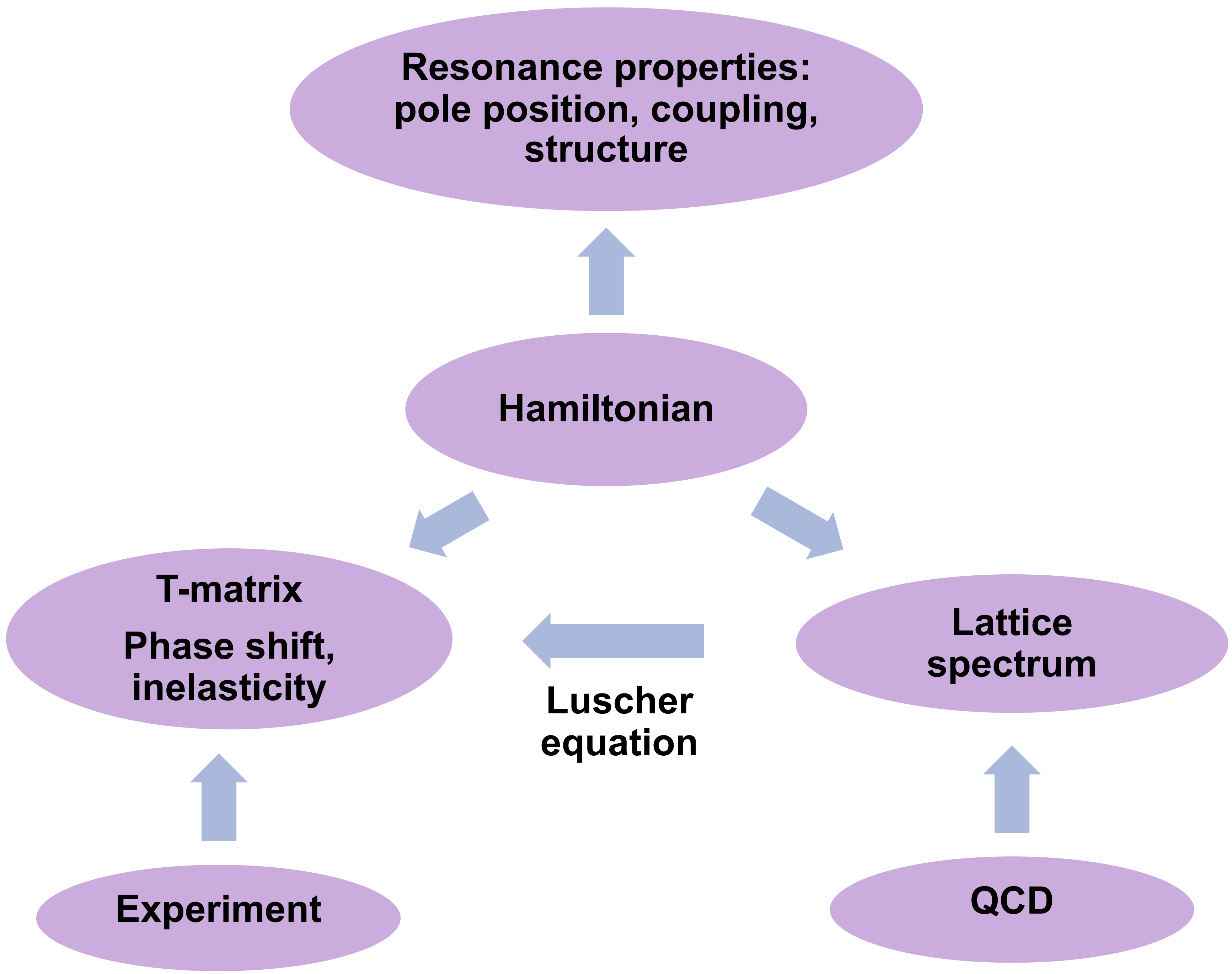
3. Y. Lu, M. N. Anwar, B. S. Zou, Phys.Rev.D 94, 034021 (2016)

    ☞ Bottomonium

……

- **Coupled-channel effect due to hadron loop could cause sizable mass shift on the state in quark model.**

Hamiltonian effective field theory



Hamiltonian effective field theory

1. **Finite-volume matrix Hamiltonian model for a Δ→Nπ system**
   J.M.M. Hall, A.C.-P. Hsu, D.B. Leinweber, A.W.Thomas, R.D. Young **Phys.Rev. D87 (2013) no.9, 094510**
2. **Finite-volume Hamiltonian method for coupled-channels interactions in lattice QCD**
   Jia-Jun Wu, T.-S.H.Lee, A.W.Thomas, R.D. Young **Phys.Rev. C90 (2014) no.5, 055206**
3. **Hamiltonian effective field theory study of the N∗(1535) resonance in lattice QCD**
   Z.-W. Liu, W. Kamleh, Derek B. Leinweber, Finn M. Stokes, Anthony W. Thomas, J.-J. Wu **Phys.Rev.Lett. 116 (2016) no.8, 082004**
4. **Lattice QCD Evidence that the Λ(1405) Resonance is an Antikaon-Nucleon Molecule**
   J.M.M. Hall, W. Kamleh, D. B. Leinweber, B.J. Menadue, B.J. Owen, A.W.Thomas, R.D. Young **Phys.Rev.Lett. 114 (2015), 132002**
5. **Hamiltonian effective field theory study of the N∗(1440) resonance in lattice QCD**
   Z.-W. Liu, Waseem Kamleh, Derek B. Leinweber, Finn M. Stokes, Anthony W. Thomas, J.-J. Wu **Phys.Rev. D95 (2017) no.3, 034034**
6. **Structure of the Λ(1405) from Hamiltonian effective field theory**
   Zhan-Wei Liu, Jonathan M.M. Hall, Derek B. Leinweber, Anthony W. Thomas, Jia-Jun Wu **Phys.Rev. D95 (2017) no.1, 014506**
7. **Nucleon resonance structure in the finite volume of lattice QCD**
   Jia-jun Wu, H. Kamano, T.-S.H.Lee , Derek B. Leinweber, Anthony W. Thomas **Phys.Rev. D95 (2017) no.11, 114507**
8. **Structure of the Roper Resonance from Lattice QCD Constraints**
   Jia-jun Wu, Derek B. Leinweber, Zhan-wei Liu, Anthony W. Thomas **Phys.Rev. D97 (2018) no.9, 094509**
9. **Kaonic Hydrogen and Deuterium in Hamiltonian Effective Field Theory**
   Zhan-wei Liu, Jia-jun Wu, Derek B. Leinweber, Anthony W. Thomas **Phys.Lett.B 808(2020),135652**
10. **Partial Wave Mixing in Hamiltonian Effective Field Theory**
    Yan Li, Jia-jun Wu, Curtis D. Abell, Derek B. Leinweber, Anthony W. Thomas **Phys.Rev. D101(2020) no.11,114501**
11. **Hamiltonian effective field theory in elongated or moving finite volume**
    Yan Li, Jia-jun Wu, Derek B. Leinweber, Anthony W. Thomas **Phys.Rev. D103(2021) no.9, 094518**

Hamiltonian framework

The Hamiltonian reads

$$H = H_0 + H_I,$$

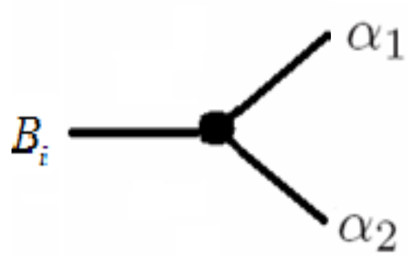
where the non-interacting one is

$$H_0 = \sum_B |B\rangle m_B \langle B| + \sum_\alpha \int d^3\vec{k} |\alpha(\vec{k})\rangle E_\alpha(\vec{k}) \langle \alpha(\vec{k})|.$$

And the interacting one includes two parts

$$H_I = g + v$$

bare state core -> channel：



$$g = \sum_{\alpha, B} \int d^3\vec{k} \left\{ |\alpha(\vec{k})\rangle g_{\alpha B}(|\vec{k}|) \langle B| + h.c. \right\}$$

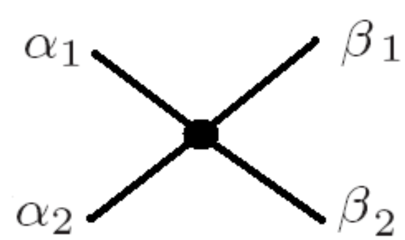
Quark pair creation model (QPC):

$$g_{\alpha B}(|\vec{k}|) = \gamma I_{\alpha B}(|\vec{k}|) e^{-\frac{\vec{k}^2}{2\Lambda'^2}}$$

truncate the hard vertices given by usual QPC

P. G. Ortega, et al,
Phys. Rev. D 94, 074037 (2016)

channel -> channel：



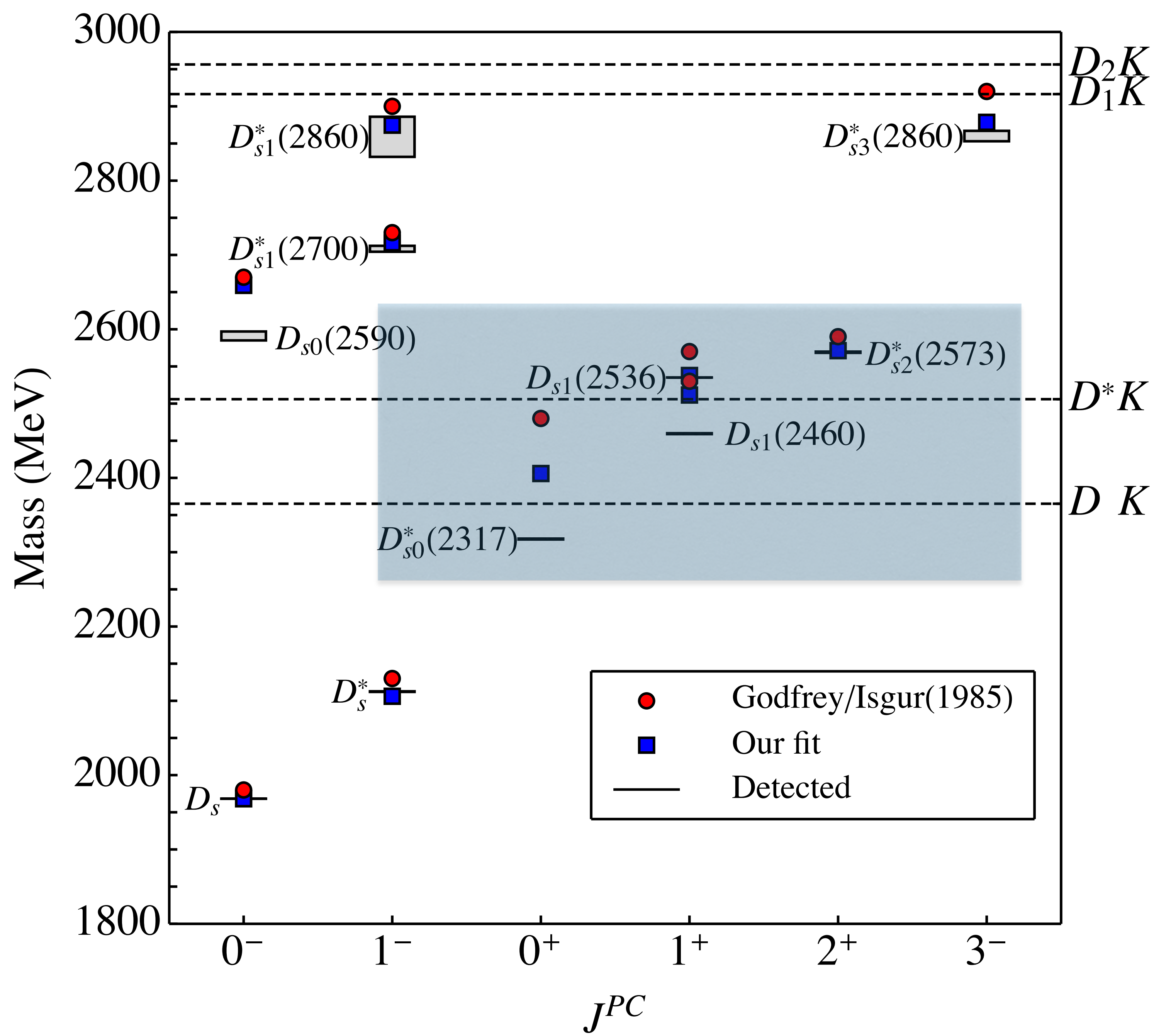
$$v = \sum_{\alpha, \beta} \int d^3\vec{k} d^3\vec{k}' |\alpha(\vec{k})\rangle V^L_{\alpha, \beta}(|\vec{k}|, |\vec{k}'|) \langle \beta(\vec{k}')|$$

Effective Lagrangian: (exchanging ρ/ω)

$$\mathcal{L} = \mathcal{L}_{PPV} + \mathcal{L}_{VVV}$$
$$= i g_v \mathrm{Tr}(\partial^\mu P [P, V_\mu]) + i g_v \mathrm{Tr}(\partial^\mu V^\nu [V_\mu, V_\nu])$$

Form factor: $\left(\frac{\Lambda^2}{\Lambda^2 + p_f^2}\right)^2 \left(\frac{\Lambda^2}{\Lambda^2 + p_i^2}\right)^2$

D_s mesons in quark model



- Fit the updated masses of low-lying states away from thresholds
- Our fit is more consistent with observation.

Four near-threshold D_s states in quark model

Transformation from physical bases to the heavy quark limit bases.

	$c\bar{s}$ cores		channel	
	$B(\vert^{2S+1}L_J\rangle)$	B(mass)	α	L
$D_{s0}^*(2317)$	$\vert^3P_0\rangle$	2405.9	DK	S
$D_{s1}^*(2460)$	$0.68\vert^1P_1\rangle - 0.74\vert^3P_1\rangle$ $= -0.99\phi_s + 0.13\phi_d$	2511.5	D^*K	S, D
$D_{s1}^*(2536)$	$-0.74\vert^1P_1\rangle - 0.68\vert^3P_1\rangle$ $= -0.13\phi_s - 0.99\phi_d$	2537.8	D^*K	S, D
$D_{s2}^*(2573)$	$\vert^3P_2\rangle$	2571.2	DK, D^*K	D

$$\phi_s = \left|\tfrac{1}{2}_l \otimes \tfrac{1}{2}_h\right\rangle$$

$$\phi_d = \left|\tfrac{3}{2}_l \otimes \tfrac{1}{2}_h\right\rangle$$

GI results

$J^P = 1^+$	$B(\vert^{2S+1}L_J\rangle)$	B(mass)	α	L
$D_{s1}^*(2460)$	$-0.97\vert^1P_1\rangle + 0.24\vert^3P_1\rangle$ $= 0.76\phi_s - 0.65\phi_d$	2549.7	D^*K	S, D
$D_{s1}^*(2536)$	$-0.24\vert^1P_1\rangle - 0.97\vert^3P_1\rangle$ $= -0.65\phi_s - 0.76\phi_d$	2559.46	D^*K	S, D

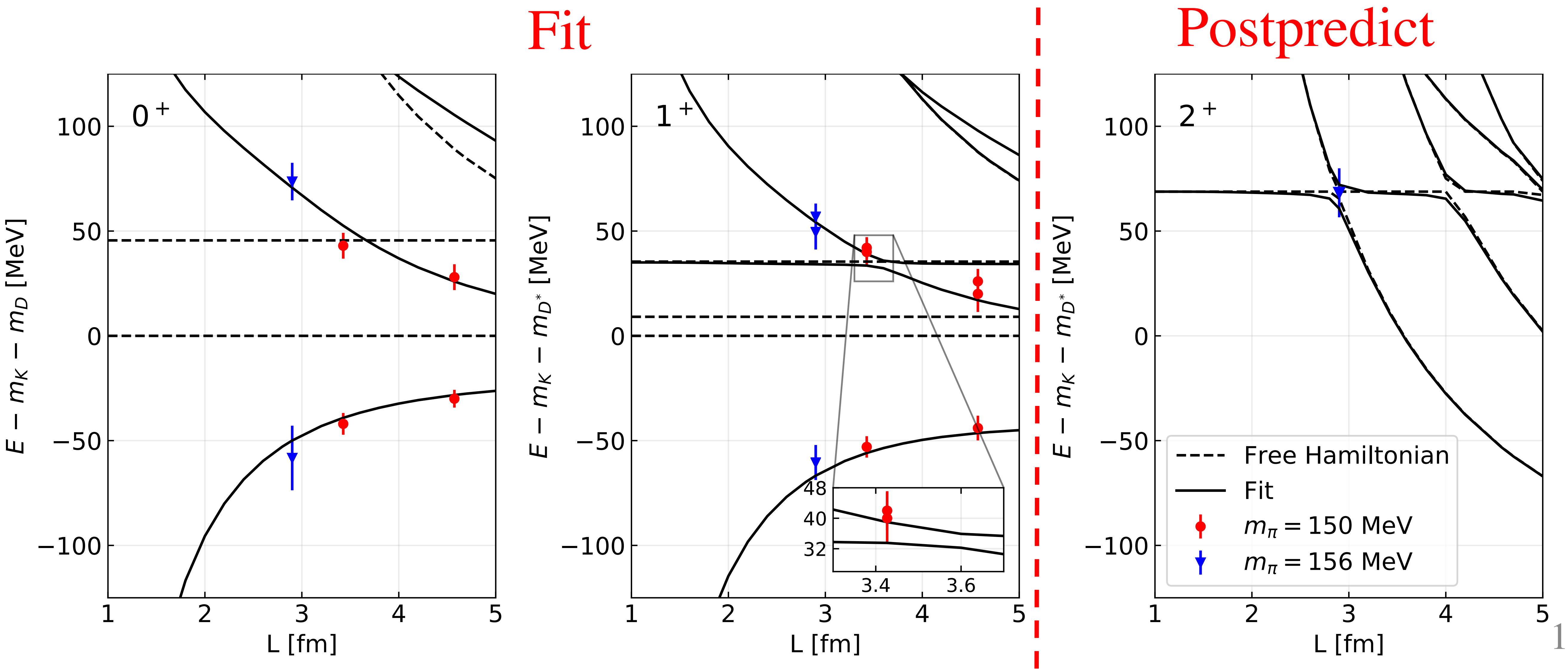
- Ds(2317) and Ds(2460) are much heavier than detected.
- The bare 1^+ states are almost purely given by the states with heavy-quark spin bases.

Fit the lattice data： $D_s(2317,2460,2536)$

$$(H_0+H_{\rm I})|\Psi\rangle = E|\Psi\rangle$$

Eigenvalues ⟷ Lattice levels

Lattice data from: C. B. Lang et al., **Phys. Rev. D 90, 034510 (2014);**
G. S. Bali et al., **Phys. Rev. D 96, 074501 (2017)**



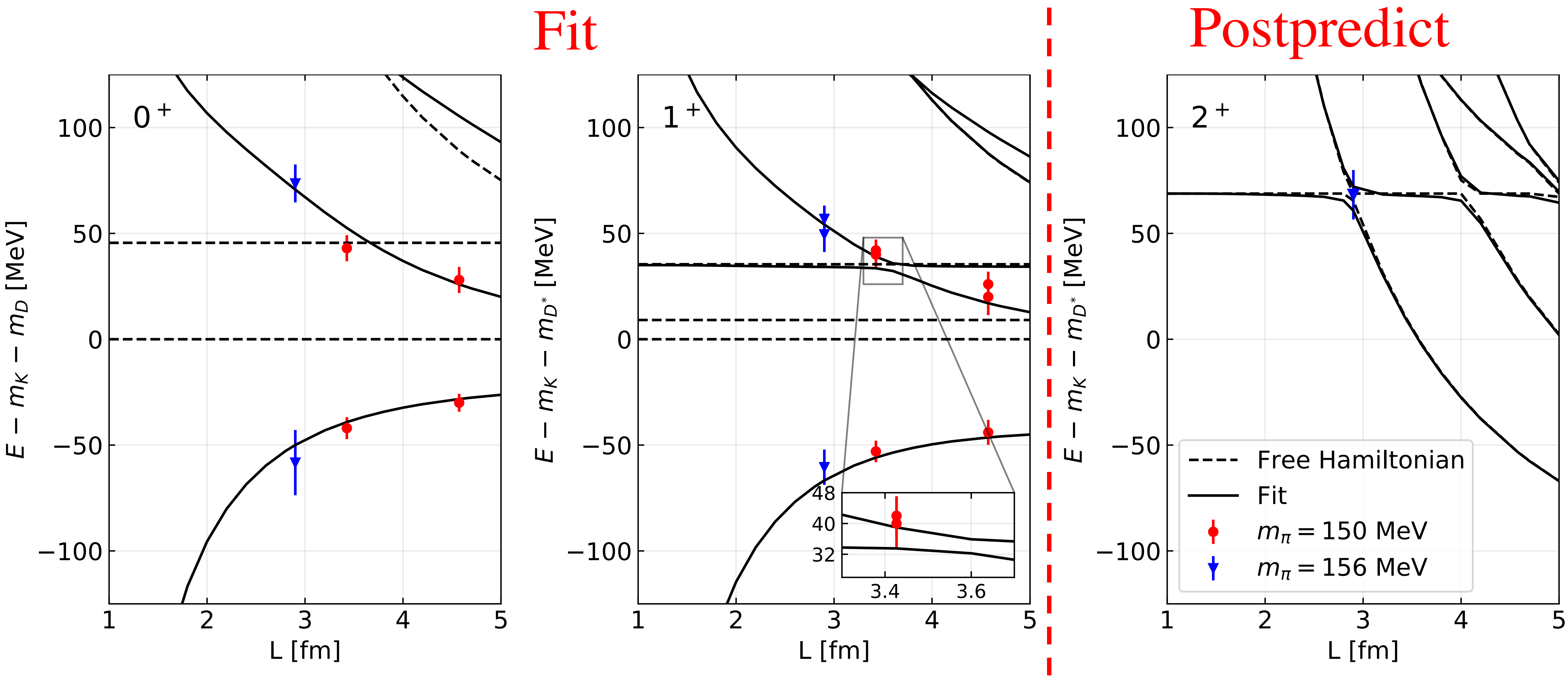
Fit the lattice data： $D_s(2317,2460,2536)$

- With fixed $\Lambda = 1.0$ GeV, $\chi^2/\text{dof} = 0.95$

$$g_c = 4.2^{+2.2}_{-3.1}\ , \Lambda' = 0.323^{+0.033}_{-0.031}\ \text{GeV},$$

$$\gamma = 10.3^{+1.1}_{-1.0}$$

Lattice data from: C. B. Lang et al., **Phys. Rev. D 90, 034510 (2014);**
G. S. Bali et al., **Phys. Rev. D 96, 074501 (2017)**



Component and pole mass

- Component

$$(H_0+H_\mathrm{I})|\Psi\rangle = E|\Psi\rangle$$

$$|\Psi_E\rangle = C_0|B> + \sum_{\vec{k}_n=\frac{2\pi}{L}\vec{n}} C_E(\vec{k}_n)|\alpha(\vec{k}_n)\rangle$$

Eigenvector → Component

- Pole mass

In the infinite volume, the scattering T-matrix reads

$$T_{\alpha,\,\beta}(k,k';E) = \mathcal{V}_{\alpha,\,\beta}(k,k';E) + \sum_{\alpha'}\int q^2 dq \frac{\mathcal{V}_{\alpha,\,\alpha'}(k,q;E)T_{\alpha,\,\beta}(q,k';E)}{E-E_{\alpha'}(q)+i\epsilon}$$

where the effective potential reads

$$\mathcal{V}_{\alpha,\,\beta}(k,k';E) = \sum_B \frac{g_{\alpha\,B}(k)g^*_{\beta\,B}(k')}{E-m_B} + V^L_{\alpha,\beta}(k,k').$$

T-matrix → Pole mass

Component and pole mass

state	L=4.57 fm	Pole mass at $L \to \infty$

	$P(c\bar{s})[\%]$	ours	exp
$D_{s0}^*(2317)$	$32.0^{+5.2}_{-3.9}$	$2338.9^{+2.1}_{-2.7}$	2317.8 ± 0.5
$D_{s1}^*(2460)$	$52.4^{+5.1}_{-3.8}$	$2459.4^{+2.9}_{-3.0}$	2459.5 ± 0.6
$D_{s1}^*(2536)$	$98.2^{+0.1}_{-0.2}$	$2536.6^{+0.3}_{-0.5}$	2535.11 ± 0.06
$D_{s2}^*(2573)$	$95.9^{+1.0}_{-1.5}$	$2570.2^{+0.4}_{-0.8}$	2569.1 ± 0.8

$D_{s0}(2317), D_{s1}(2460)$

- Bare $c\bar{s}$ has strong coupling to S-wave $D^{(*)}K$ channels, and significant mass shift.
- Both the bare $c\bar{s}$ core and molecular components are significant and essential.

$D_{s1}(2536), D_{s2}(2573)$

- Coupling to D-wave $D^{(*)}K$ channels can be neglected.
- Mainly pure $c\bar{s}$.

Component and pole mass

state	L=4.57 fm	Pole mass at $L \to \infty$

	$P(c\bar{s})[\%]$	ours	exp
$D_{s0}^*(2317)$	$32.0^{+5.2}_{-3.9}$	$2338.9^{+2.1}_{-2.7}$	2317.8 ± 0.5
$D_{s1}^*(2460)$	$52.4^{+5.1}_{-3.8}$	$2459.4^{+2.9}_{-3.0}$	2459.5 ± 0.6
$D_{s1}^*(2536)$	$98.2^{+0.1}_{-0.2}$	$2536.6^{+0.3}_{-0.5}$	2535.11 ± 0.06
$D_{s2}^*(2573)$	$95.9^{+1.0}_{-1.5}$	$2570.2^{+0.4}_{-0.8}$	2569.1 ± 0.8

A. M. Torres, E. Oset, S. Prelovsek, and A. Ramos **JHEP 05, 153 (2015)**

$$P(KD) = 72 \pm 13 \pm 5\ \%,\ \text{for the } D_{s0}^*(2317)$$

$$P(KD^*) = 57 \pm 21 \pm 6\ \%,\ \text{for the } D_{s1}(2460)$$

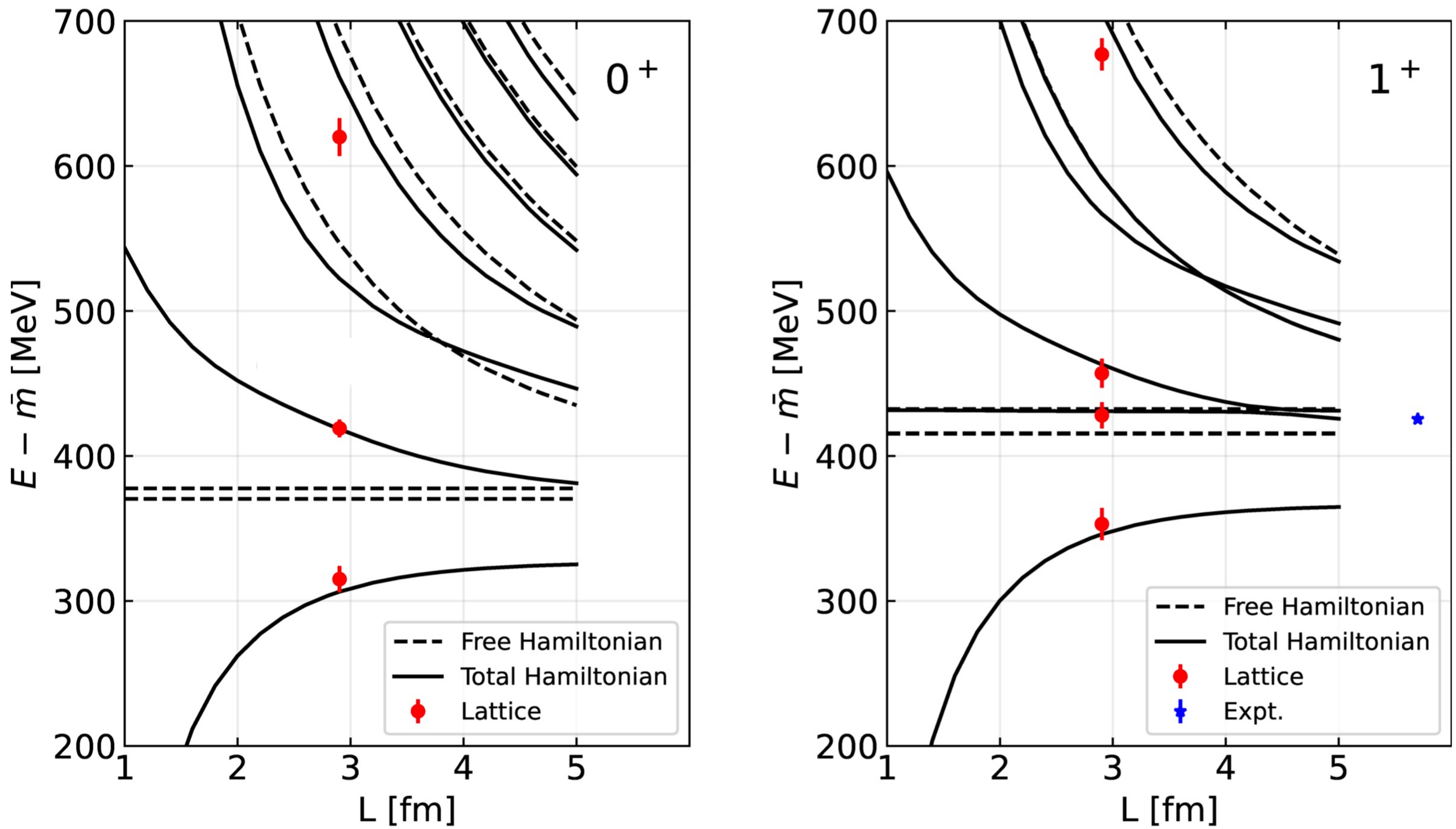
L.M. Liu, K. Orginos, F.-K. Guo, C. Hanhart, Ulf-G. Meissner **Phys.Rev.D 87 (2013) 1, 014508**

$$P(KD) = [0.68, 0.73],\ \text{for the } D_{s0}^*(2317)$$

B_s energy levels

- The heavy quark symmetry seems to be a good symmetry here.
- Use the same parameters as D_s .

Postprediction, not a fit !



$\bar{m} = \frac{1}{4}(m_{B_s} + 3m_{B_s^*})$

Lattice data from: C. B. Lang et al., **Phys. Lett. B 750, 17 (2015)**

Content

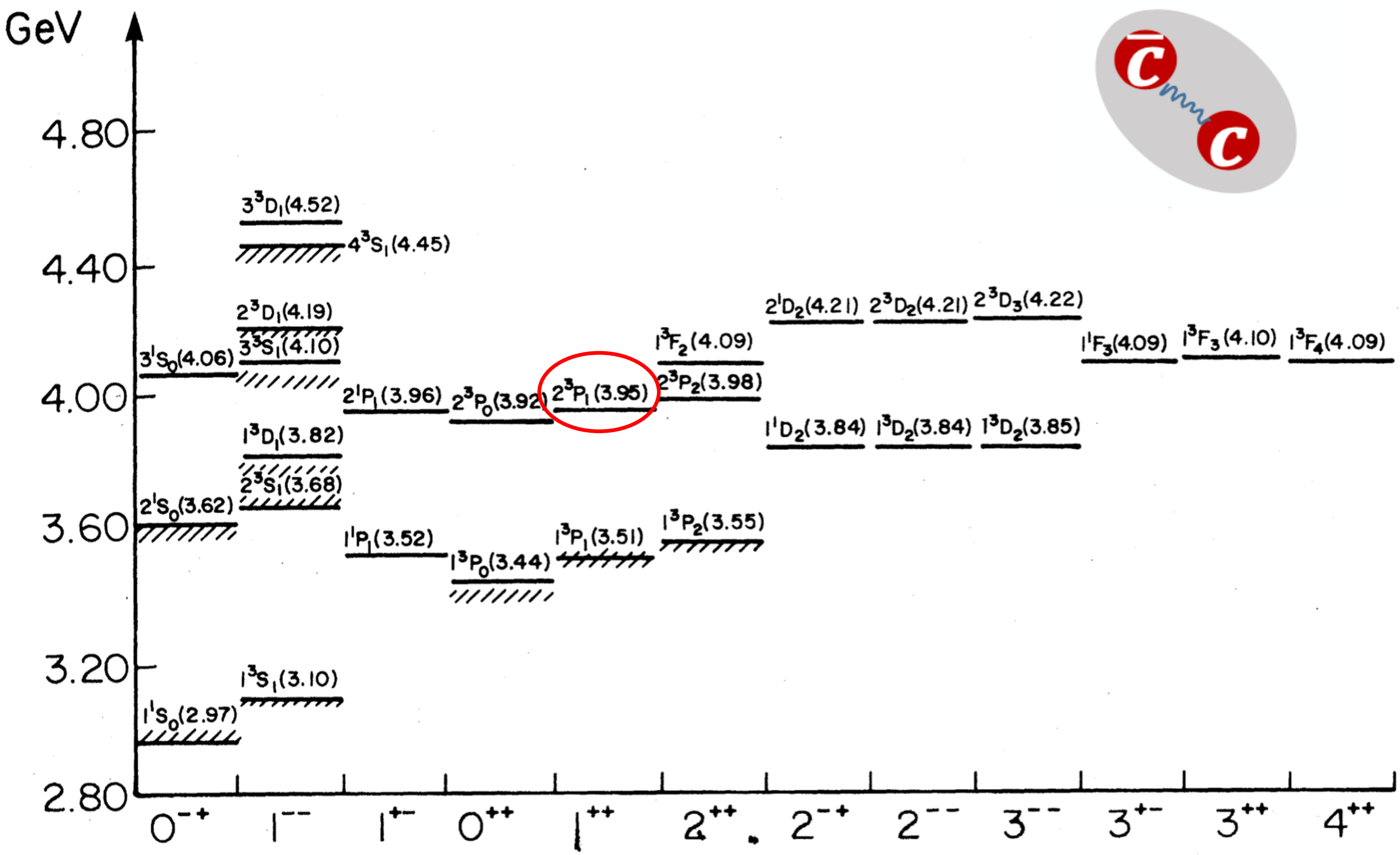
- ❖ $D_{s0}(2317)$ and $D_{s1}(2460)$
- ❖ $X(3872)$

GI quark model

Mesons in a Relativized Quark Model with Chromodynamics

S. Godfrey (Toronto U.), Nathan Isgur (Toronto U.) (1985)

Published in: *Phys.Rev.D* 32 (1985) 189-231



X(3872)

Experiment	Mass [MeV]	Width [MeV]
Belle [63]	$3872 \pm 0.6 \pm 0.5$	< 2.3
Belle [75]	–	–
Belle [76]	$3875.4 \pm 0.7^{+0.4}_{-1.7} \pm 0.9$	–
Belle [77]	$3871.46 \pm 0.37 \pm 0.07$	–
Belle [78]	$3872.9^{+0.6+0.4}_{-0.4-0.5}$	$3.9^{+2.8+0.2}_{-1.4-1.1}$
Belle [79]	–	–
Belle [80]	$3871.84 \pm 0.27 \pm 0.19$	< 1.2
CDF [67]	$3871.3 \pm 0.7 \pm 0.4$	–
CDF [81]	–	–
CDF [82]	–	–
CDF [83]	$3871.61 \pm 0.16 \pm 0.19$	–
DØ [68]	$3871.8 \pm 3.1 \pm 3.0$	–
BaBar [84]	3873.4 ± 1.4	–
BaBar [85]	$3871.3 \pm 0.6 \pm 0.1$	< 4.1
	$3868.6 \pm 1.2 \pm 0.2$	–
BaBar [86]	–	–
BaBar [87]	$3875.1^{+0.7}_{-0.5} \pm 0.5$	$3.0^{+1.9}_{-1.4} \pm 0.9$
BaBar [88]	$3871.4 \pm 0.6 \pm 0.1$	< 3.3
	$3868.7 \pm 1.5 \pm 0.4$	–
BaBar [89]	–	–
BaBar [90]	$3873.0^{+1.8}_{-1.6} \pm 1.3$	–
LHCb [91]	$3871.95 \pm 0.48 \pm 0.12$	–
LHCb [70]	–	–
LHCb [92]	–	–
CMS [73]	–	–
BESIII [93]	$3871.9 \pm 0.7 \pm 0.2$	< 2.4

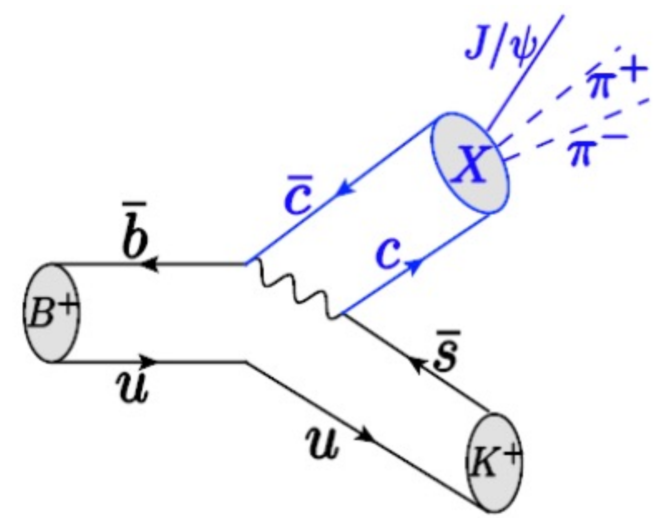
Phys. Rept. 639 (2016) 1-121

Observation of a narrow charmonium-like state in exclusive $B^{\pm} \to K^{\pm}\pi^{+}\pi^{-}J/\psi$ decays

Belle Collaboration • S.K. Choi (Gyeongsang Natl. U.) et al. (Sep, 2003)

Published in: *Phys.Rev.Lett.* 91 (2003) 262001 • e-Print: hep-ex/0309032 [hep-ex]

pdf links DOI cite claim 2,295 citations



- The $D\bar{D}^{*}/D^{*}\bar{D}$ molecular state.

  Swanson, Wong, Guo, liu,....

Close to $D^0\bar{D}^{*0}/D^{*0}\bar{D}^{0}$ thresholds

$$\delta m = m_{D^0\bar{D}^{*0}} - m_{X(3872)}$$
$$= 0.00 \pm 0.18 \text{ MeV}$$

PDG 22

Theoretical interpretation of X(3872)

Where is the $\chi_{c1}(2P)$ in quark model?

- The mixing of the $\bar{c}c$ core with $D\bar{D}^*/D^*\bar{D}$ component.

Chao, H. Q. Zheng, Yu. S. Kalashnikova, P. G. Ortega...

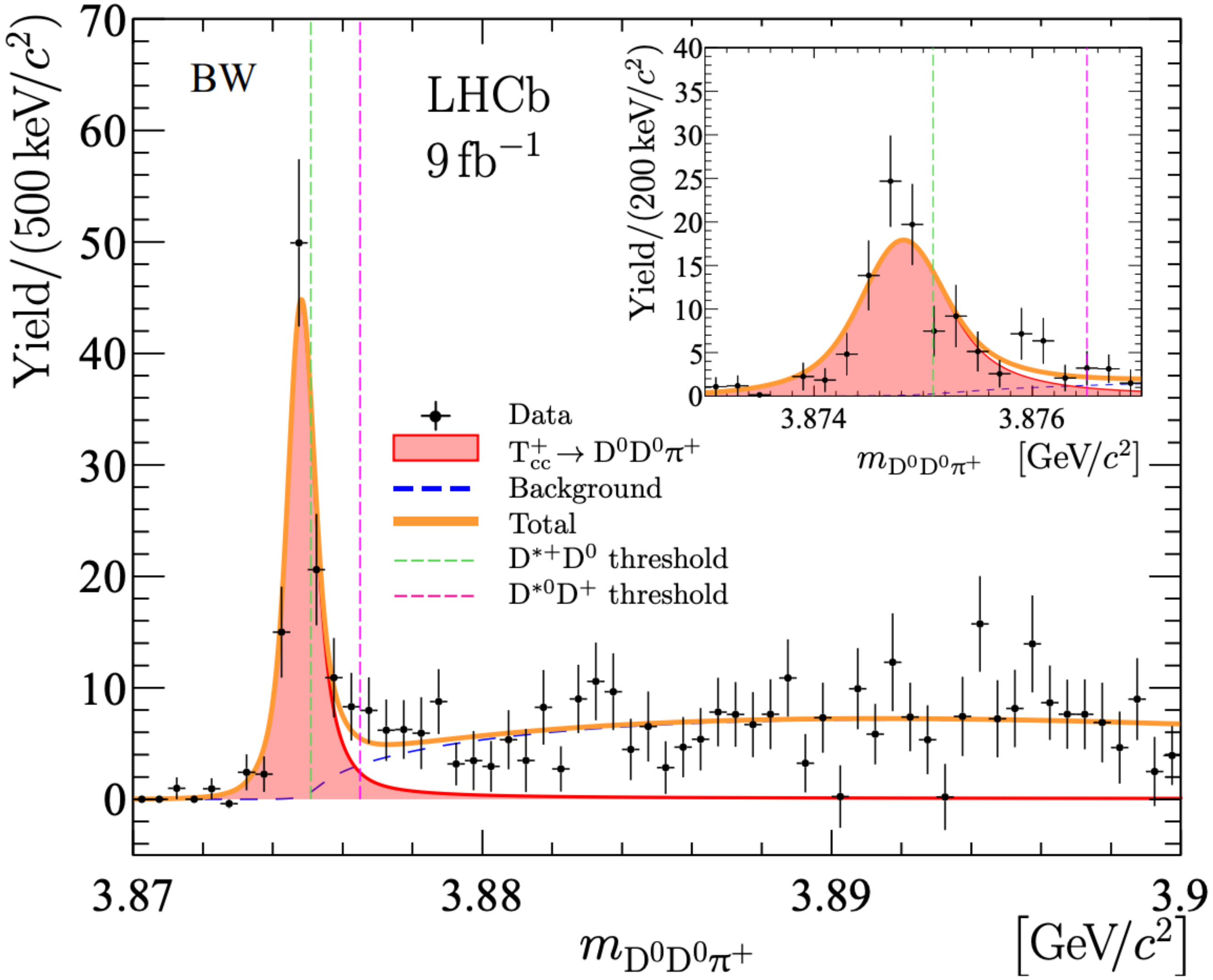
Close to charmonium $\chi_{c1}(2P)$: m=3953.5 MeV

$$\delta m = m_{\chi_{c1}(2P)} - m_{X(3872)} = 81.35\ \text{MeV}$$

→ *Complicated coupled-channel effect:* $\bar{c}c$ & $D\bar{D}^*/D^*\bar{D}$

Phys. Rev. D 32, 189 (1985)

How to determine the component in the X(3872): from Tcc



- $D^0D^0\pi^+$ channel
- Close to $D^{*+}D^0$ thresholds:

Conventional Breit-Wigner: assumed $J^P = 1^+$.

$$\delta m_{BW} = m_{T_{cc}} - m_{D^{*+}D^0}$$
$$= -273 \pm 61 \text{ keV}$$
$$\Gamma_{BW} = 410 \pm 165 \text{keV}$$

EPS-HEP conference, Ivan Polyakov's talk,29/07/2021; Nature Physics,22'

Unitarized Breit-Wigner:

$$\delta m_U = m_{T_{cc}} - m_{D^{*+}D^0}$$
$$= -361 \pm 40 \text{ keV}$$
$$\Gamma_U = 47.8 \pm 1.9 \text{ keV}$$

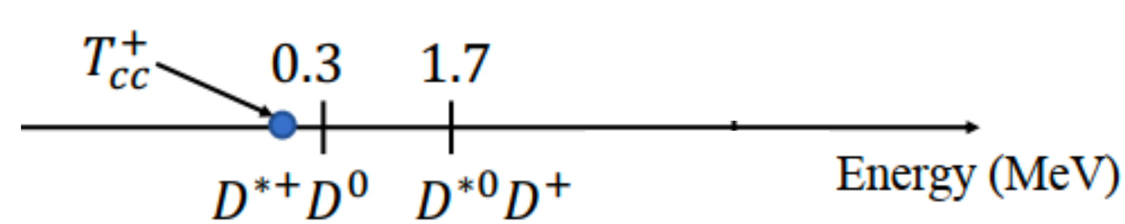
LHCb, Nature Commun. 13 (2022) 1, 3351

❖ Quark content: $cc\bar{u}\bar{d}$

❖ ***Only the D^*D coupled channel effect***

C-parity

$\overline{D^*}D$ / $\overline{D}D^*$ interaction



One-boson-exchange model

DD^*

$$H_a^{(Q)} = \frac{1+\not v}{2}\left[P_a^{*\mu}\gamma_\mu - P_a\gamma_5\right]$$

$$\bar{H}_a^{(Q)} \equiv \gamma_0 H^{(Q)\dagger}\gamma_0 = \left[P_a^{*\dagger\mu}\gamma_\mu + P_a^\dagger\gamma_5\right]\frac{1+\not v}{2}$$

$$P = \left(D^0, D^+, D_s^+\right) \& P^* = \left(D^{*0}, D^{*+}, D_s^{*+}\right)$$

$$\mathcal{L}_{MH^{(Q)}H^{(Q)}} = ig\,\mathrm{Tr}\left[H_b^{(Q)}\gamma_\mu\gamma_5 A_{ba}^\mu \bar{H}_a^{(Q)}\right]$$

$$\mathcal{L}_{VH^{(Q)}H^{(Q)}} = i\beta\,\mathrm{Tr}\left[H_b^{(Q)} v_\mu \left(V_{ba}^\mu - \rho_{ba}^\mu\right)\bar{H}_a^{(Q)}\right] + i\lambda\,\mathrm{Tr}\left[H_b^{(Q)}\sigma_{\mu\nu}F^{\mu\nu}(\rho)_{ba}\bar{H}_a^{(Q)}\right]$$

$D\bar{D}^*$

$$H_a^{(\bar{Q})} \equiv C\left(\mathcal{C}H_a^{(Q)}\mathcal{C}^{-1}\right)^T C^{-1} = \left[P_{a\mu}^{(\bar{Q})*}\gamma^\mu - P_a^{(\bar{Q})}\gamma_5\right]\frac{1-\not v}{2}$$

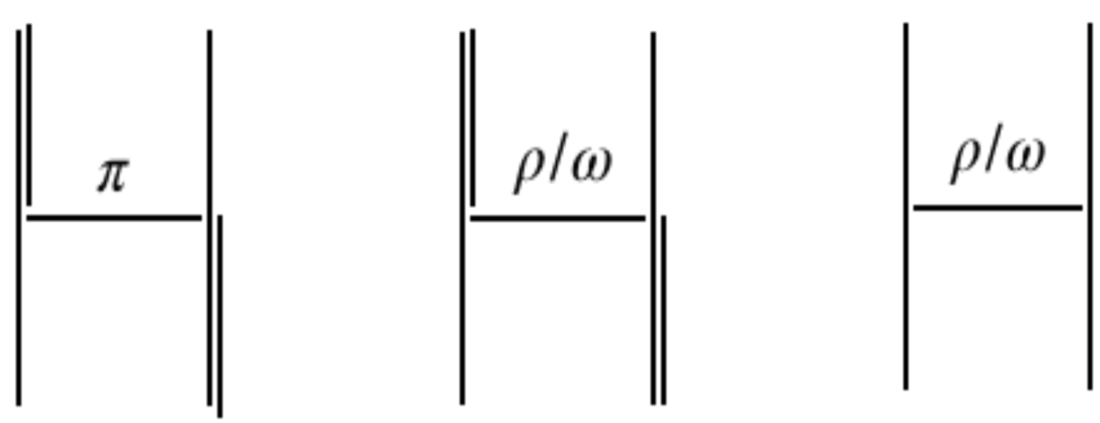
$$\bar{H}_a^{(\bar{Q})} \equiv \gamma_0 H_a^{(\bar{Q})\dagger}\gamma_0 = \frac{1-\not v}{2}\left[P_{a\mu}^{(\bar{Q})*\dagger}\gamma^\mu + P_a^{(\bar{Q})\dagger}\gamma_5\right]$$

$$\tilde{P} = \left(\bar{D}^0, D^-, D_s^-\right) \& \tilde{P}^* = \left(\bar{D}^{*0}, D^{*-}, D_s^{*-}\right)$$

$$\mathcal{L}_{MH^{(\bar{Q})}H^{(\bar{Q})}} = ig\,\mathrm{Tr}\left[\bar{H}_a^{(\bar{Q})}\gamma_\mu\gamma_5 A_{ab}^\mu H_b^{(\bar{Q})}\right]$$

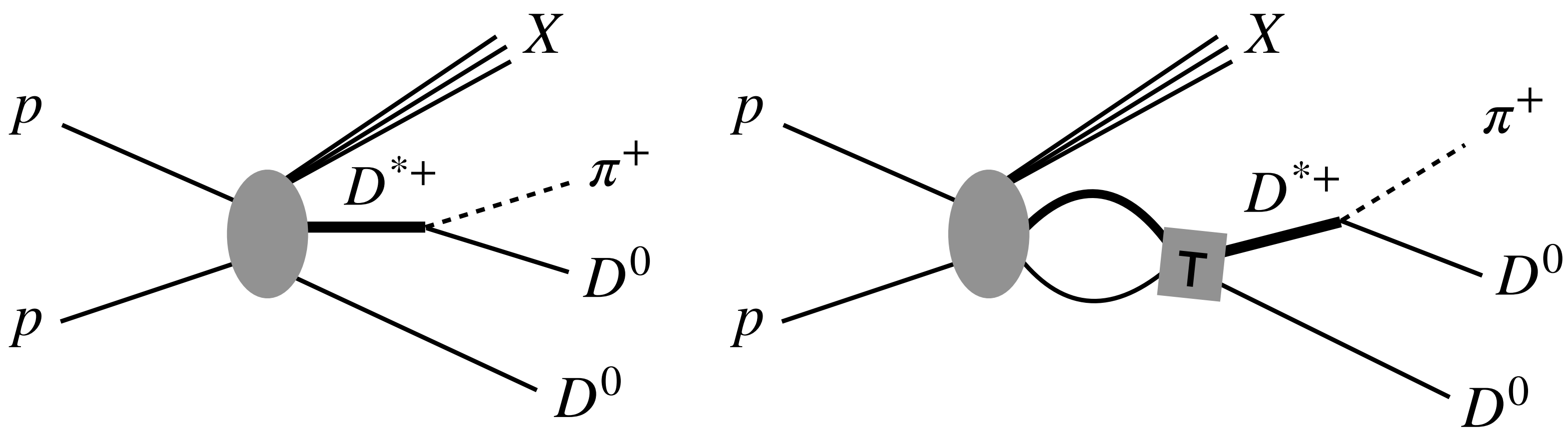
$$\mathcal{L}_{VH^{(\bar{Q})}H^{(\bar{Q})}} = -i\beta\,\mathrm{Tr}\left[\bar{H}_a^{(\bar{Q})} v_\mu \left(V_{ab}^\mu - \rho_{ab}^\mu\right) H_b^{(\bar{Q})}\right] + i\lambda\,\mathrm{Tr}\left[\bar{H}_a^{(\bar{Q})}\sigma_{\mu\nu}F_{ab}'^{\mu\nu}(\rho) H_b^{(\bar{Q})}\right]$$

- $g = 0.57$ is determined by the strong decays $D^* \to D\pi$.
- undetermined λ & β.



The inclusive production of the T_{cc}

$pp \to D^0(p_{D_1})D^0(p_{D_2})\pi^+(p_\pi)X$, X denotes all the other produced particles



The amplitude of the process

$$i\mathcal{M}_{pp\to DD\pi X} = \mathcal{A}^{\mu}_{pp\to DD^*X}\left\{g_{\mu\alpha} - \frac{i}{(2\pi)^4}\int d^4q_{D^*}G_{D^*\,\mu\nu}(q_{D^*})G_D(p_{D_1}+p_{D_2}+p_\pi-q_{D^*})T^{\nu}_{\alpha}(q_{D^*},p_{D_1}+p_\pi)\right\}$$
$$\times G^{\alpha\beta}_{D^*}(p_{D_2}+p_\pi)(g\,p_{\pi,\beta}) + (p_{D_1}\to p_{D_2}),$$

The iso-vector and iso-scalar assignment for the $\mathcal{A}$ with the production amplitudes satisfying

$$\mathcal{A}^{\mu}_{pp\to D^+D^{0*}X} = \pm\mathcal{A}^{\mu}_{pp\to D^0D^{*+}X}$$

- We can only find a satisfactory fit to the experimental data only in the iso-scalar case.

T-matrix

The T-matrix can be solved from the Lippmann-Schwinger equation

$$T(\vec{k}_{D^*},\vec{k}'_{D^*};E)=\mathcal{V}(\vec{k}_{D^*},\vec{k}'_{D^*};E)+\int d\,\vec{q}\,\frac{\mathcal{V}(\vec{k}_{D^*},\vec{q};E)T(\vec{q},\vec{k}'_{D^*};E)}{E-\sqrt{m_D^2+q^2}-\sqrt{m_{D^*}^2+q^2}+i\epsilon}$$

The effective potential is obtained with light-meson exchange potentials

$$\mathcal{V}=\left(V_\pi+V^t_{\rho/\omega}+V^u_{\rho/\omega}\right)\left(\frac{\Lambda^2}{\Lambda^2+p_f^2}\right)^2\left(\frac{\Lambda^2}{\Lambda^2+p_i^2}\right)^2$$

with

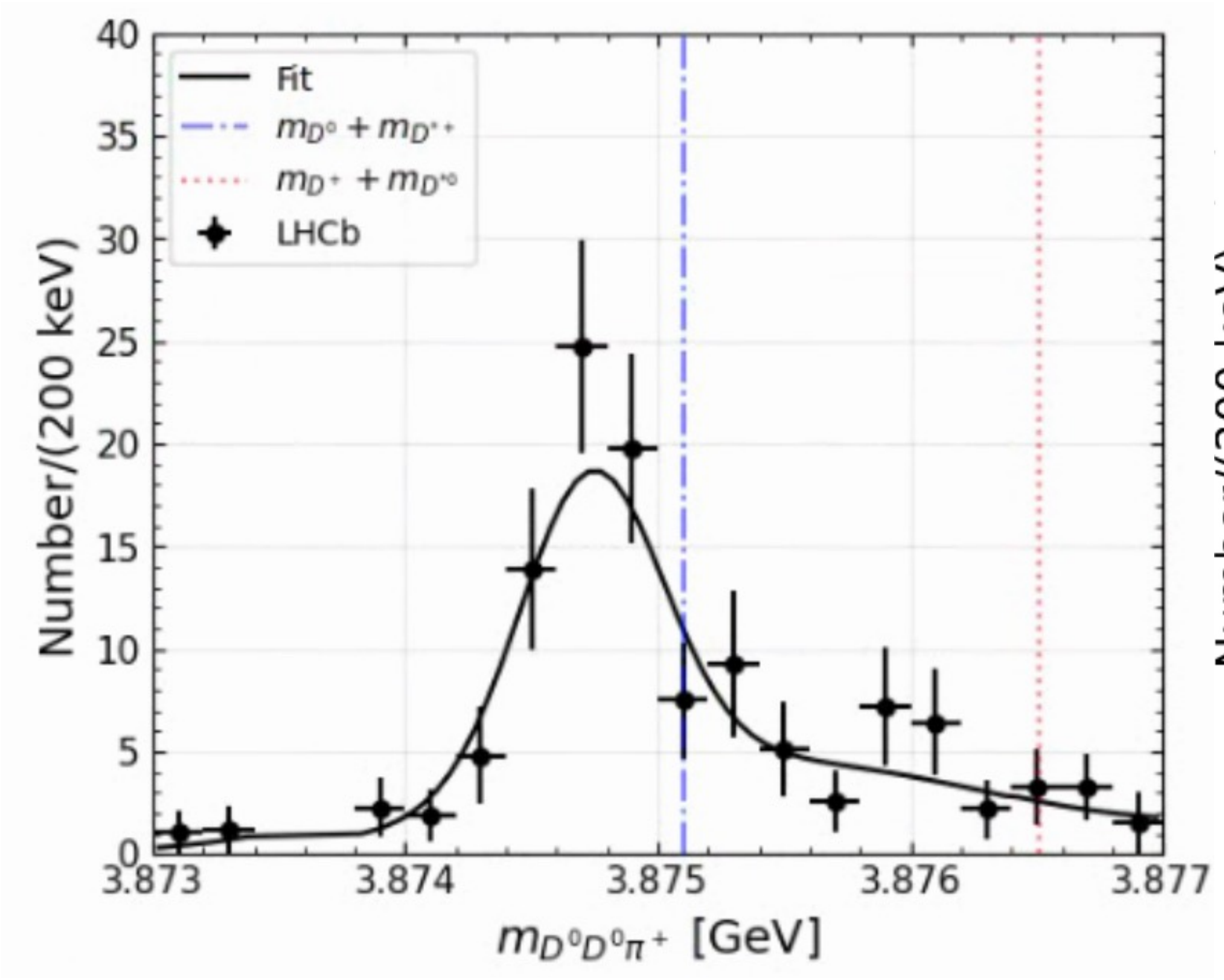
$$V_\pi=\frac{g^2}{f_\pi^2}\frac{(q\cdot\epsilon_\lambda)(q\cdot\epsilon^\dagger_{\lambda'})}{q^2-m_\pi^2},$$

$$\mathcal{V}^u_{\rho/\omega}=-2\lambda^2 g_V^2\,\frac{(\epsilon^\dagger_{\lambda'}\cdot q)(\epsilon_\lambda\cdot q)-q^2(\epsilon_\lambda\cdot\epsilon^\dagger_{\lambda'})}{q^2-m^2_{\rho/\omega}},$$

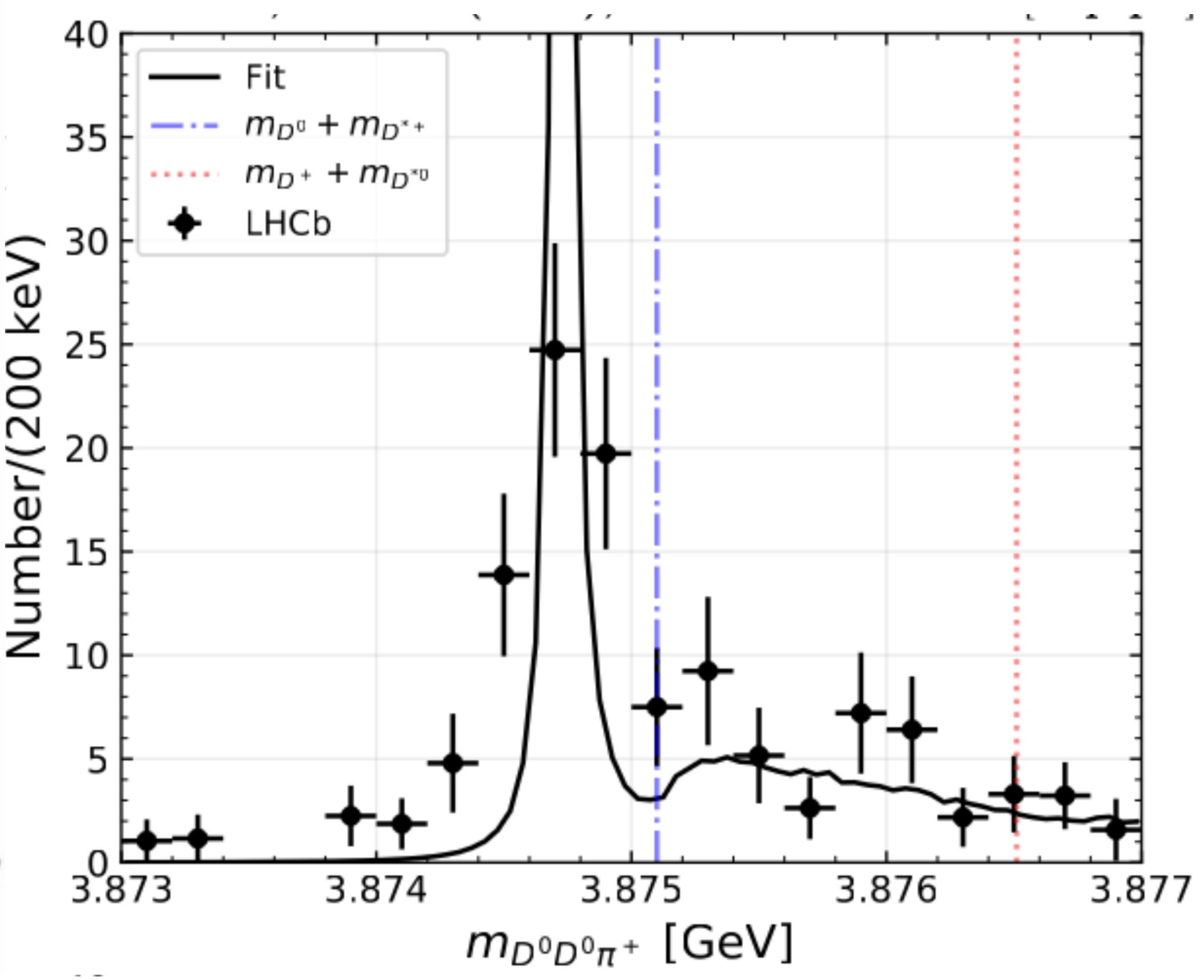
$$V^t_{\rho/\omega}=\frac{\beta^2 g_V^2}{2}\frac{(\epsilon_\lambda\cdot\epsilon^\dagger_{\lambda'})}{q^2-m^2_{\rho/\omega}}.$$

Fitting result

$\Lambda = 0.8$ GeV, χ^2/dof =0.76

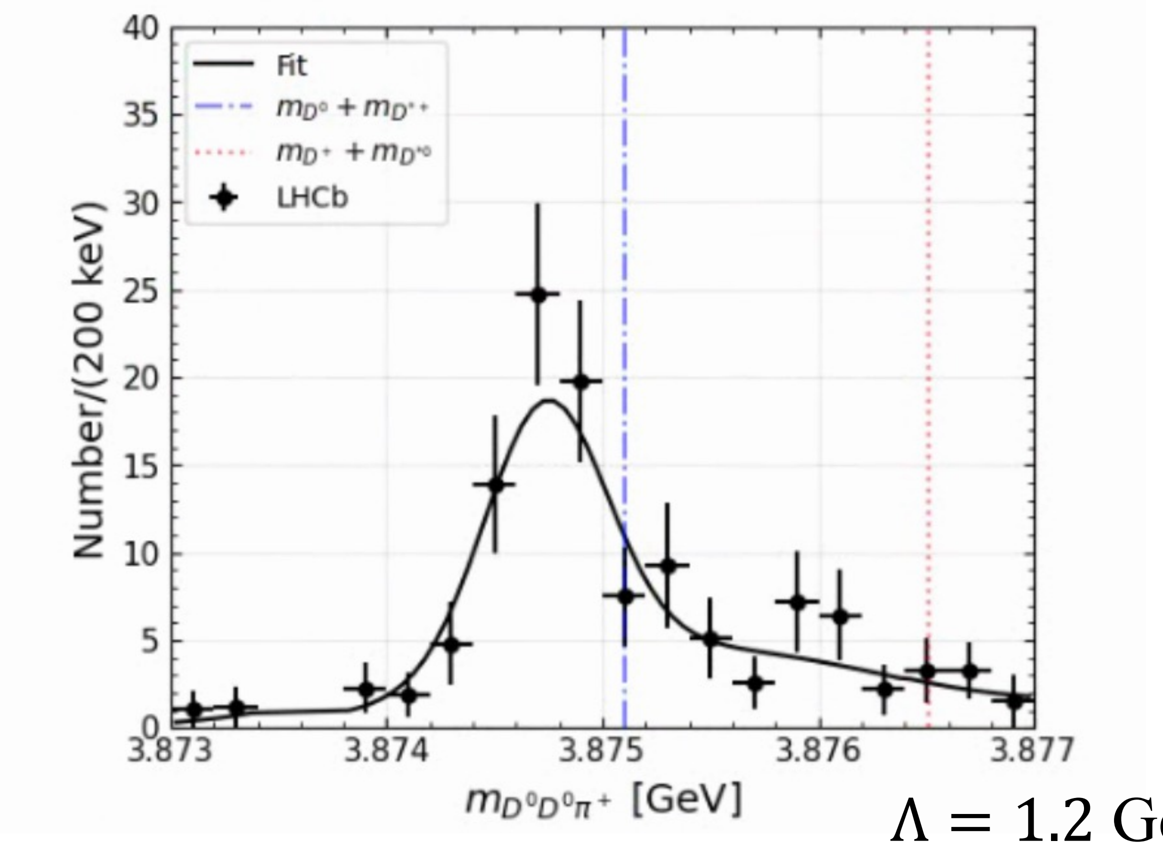


Without resolution function

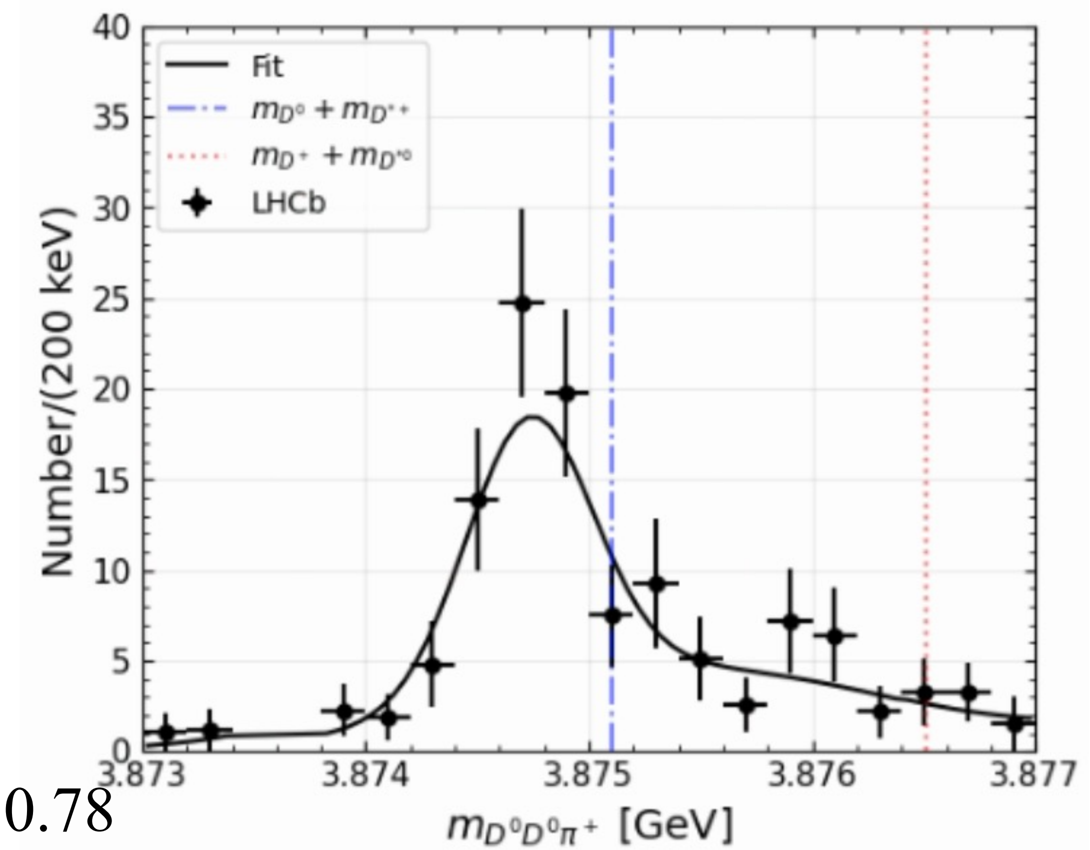


Fitting result

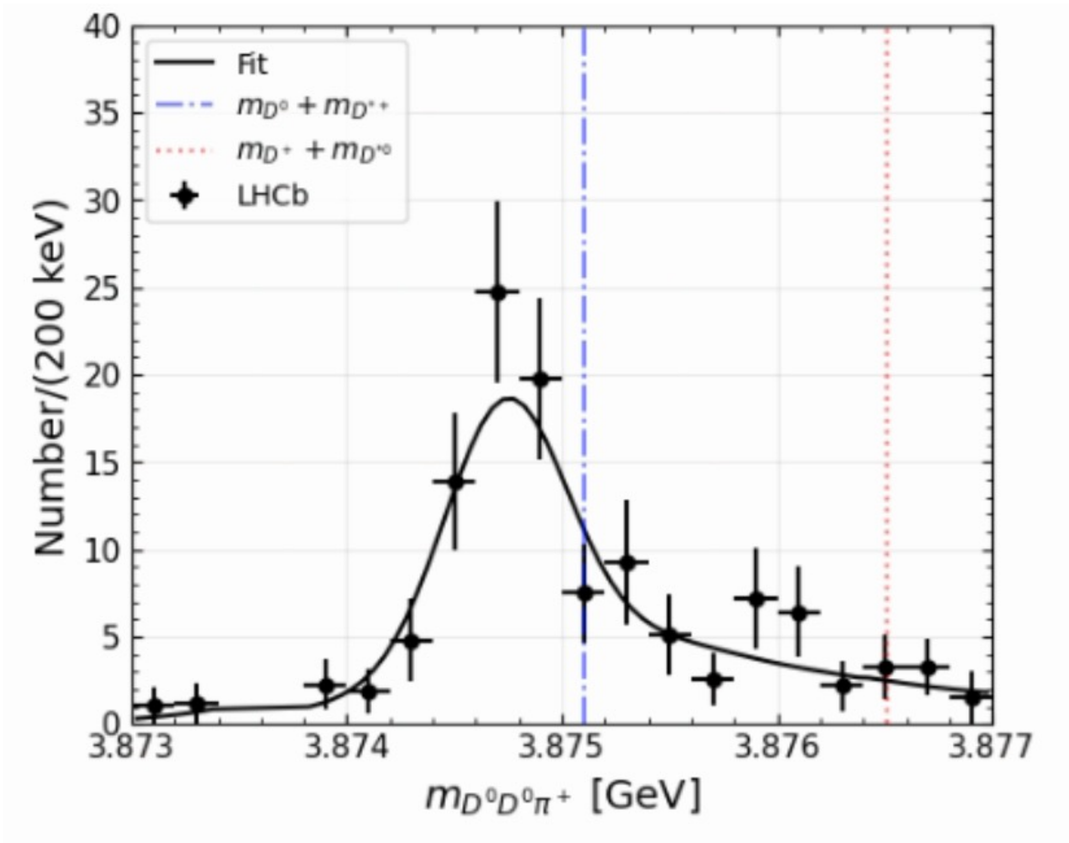
$\Lambda = 0.8$ GeV, χ^2/dof =0.76



$\Lambda = 1.0$ GeV, χ^2/dof =0.76



$\Lambda = 1.2$ GeV, χ^2/dof =0.78



Parameters

- Parameters consistent with those in one-boson-exchange model

Parameters	Λ(fixed)	λ	β
Best fit	0.8 GeV	0.890 ± 0.20	0.810 ± 0.11
Best fit	1 GeV	0.683 ± 0.025	0.687 ± 0.017
Best fit	1.2 GeV	0.587 ± 0.027	0.550 ± 0.027
Ref. [1]	1.17 GeV	0.56	0.9

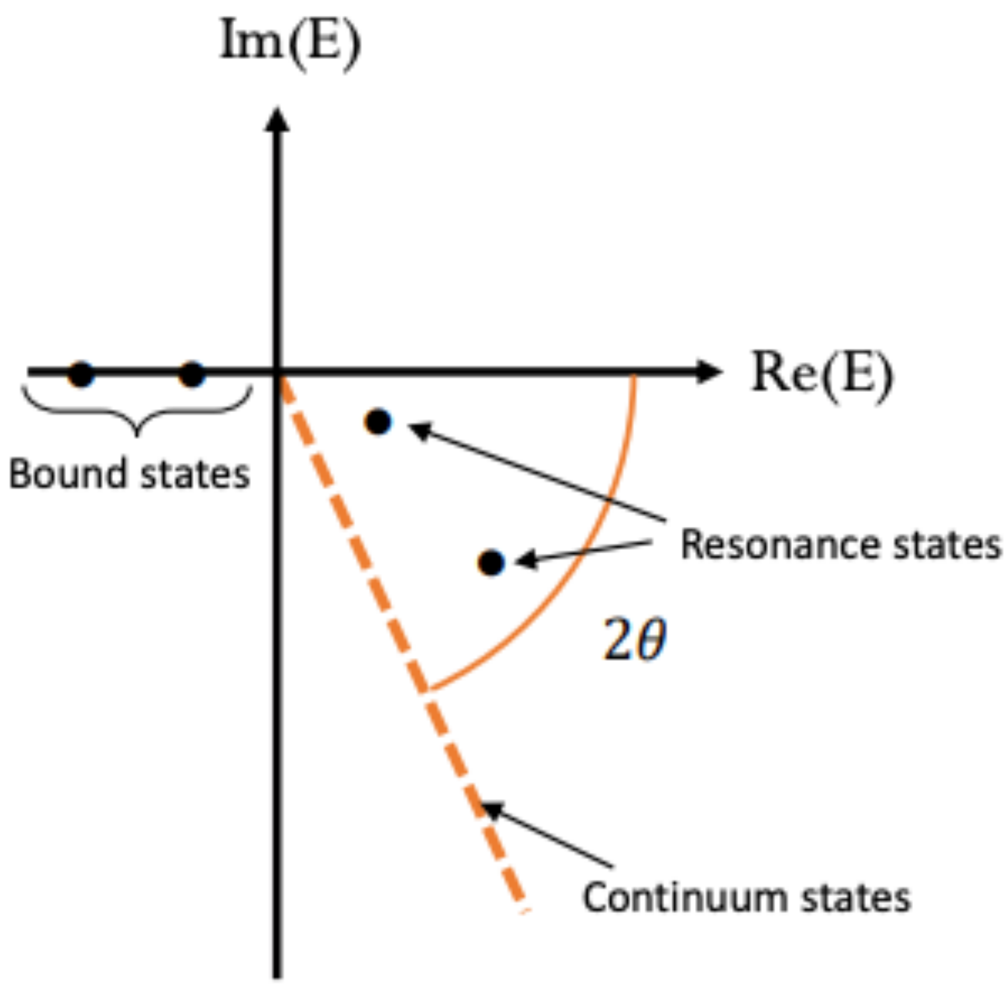
[1] Cheng, et al. Phys. Rev. D 106,016012 (2022).

Complex scaling method

The radius and momentum will rotate with an angle θ:

$$\boldsymbol{r} \to \boldsymbol{r}e^{i\theta}, \quad \boldsymbol{q} \to \boldsymbol{q}e^{-i\theta}$$

$$H_\theta \Phi_\theta = E_\theta \Phi_\theta, \qquad H_\theta = H(\mathbf{r}_\theta, \mathbf{q}_\theta) = \frac{q^2}{2u}e^{-2i\theta} + V(\mathbf{r}e^{i\theta}, \mathbf{q}e^{-i\theta})$$



S.Aoyama et al. PTP. 116, 1 (2006).
T. Myo et al. PPNP. 79, 1 (2014)
N. Moiseyev, Physics reports 302, 212 (1998)

With the varying θ:

- the scattering states will rotate with 2θ
- while the bound and resonant states will stay stable

Results with $\Lambda = 0.8$ GeV

- Only one pole appears—bound states

  $m_{T_{cc}}$=3874.7 MeV, $\Delta E = -387.7$ keV
  $\Gamma_{T_{cc}} = 67.3$ keV

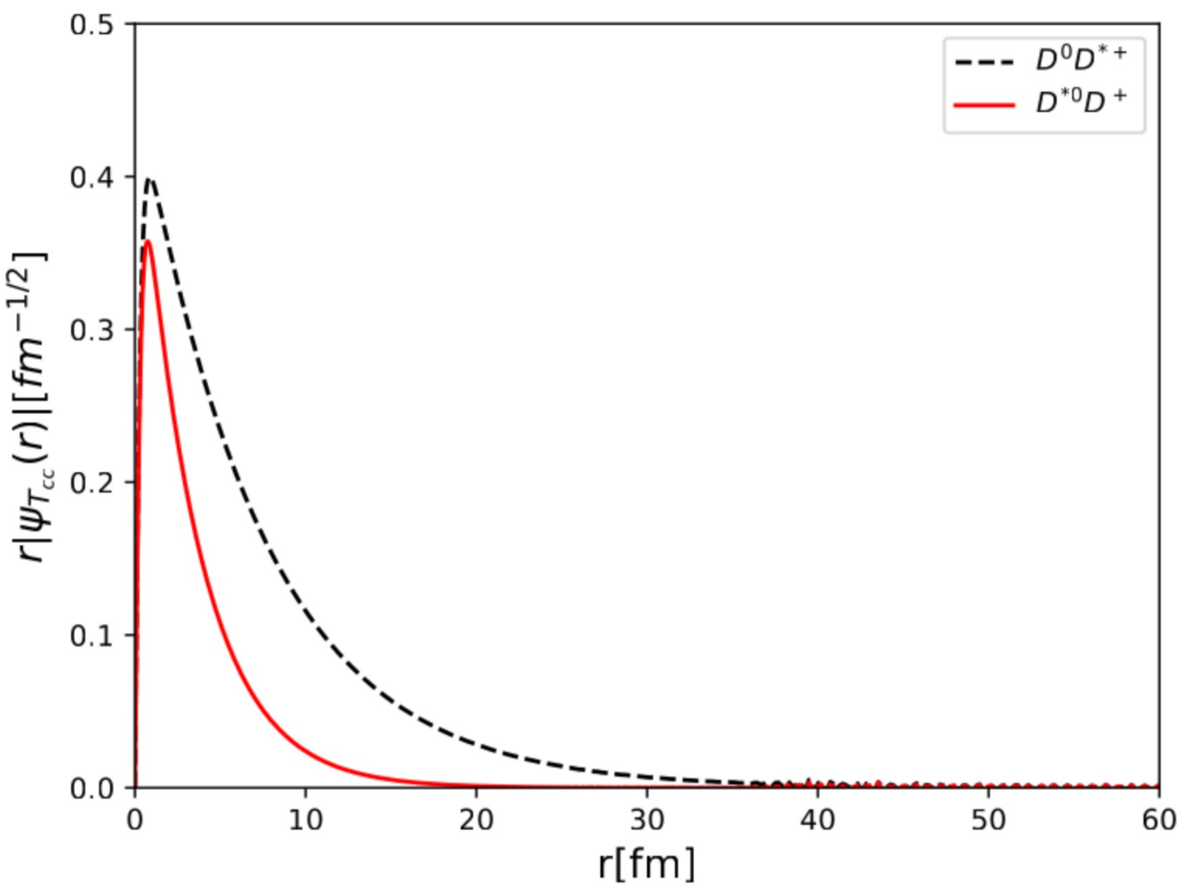
- $\sqrt{\langle r^2 \rangle} = 4.8\ fm$

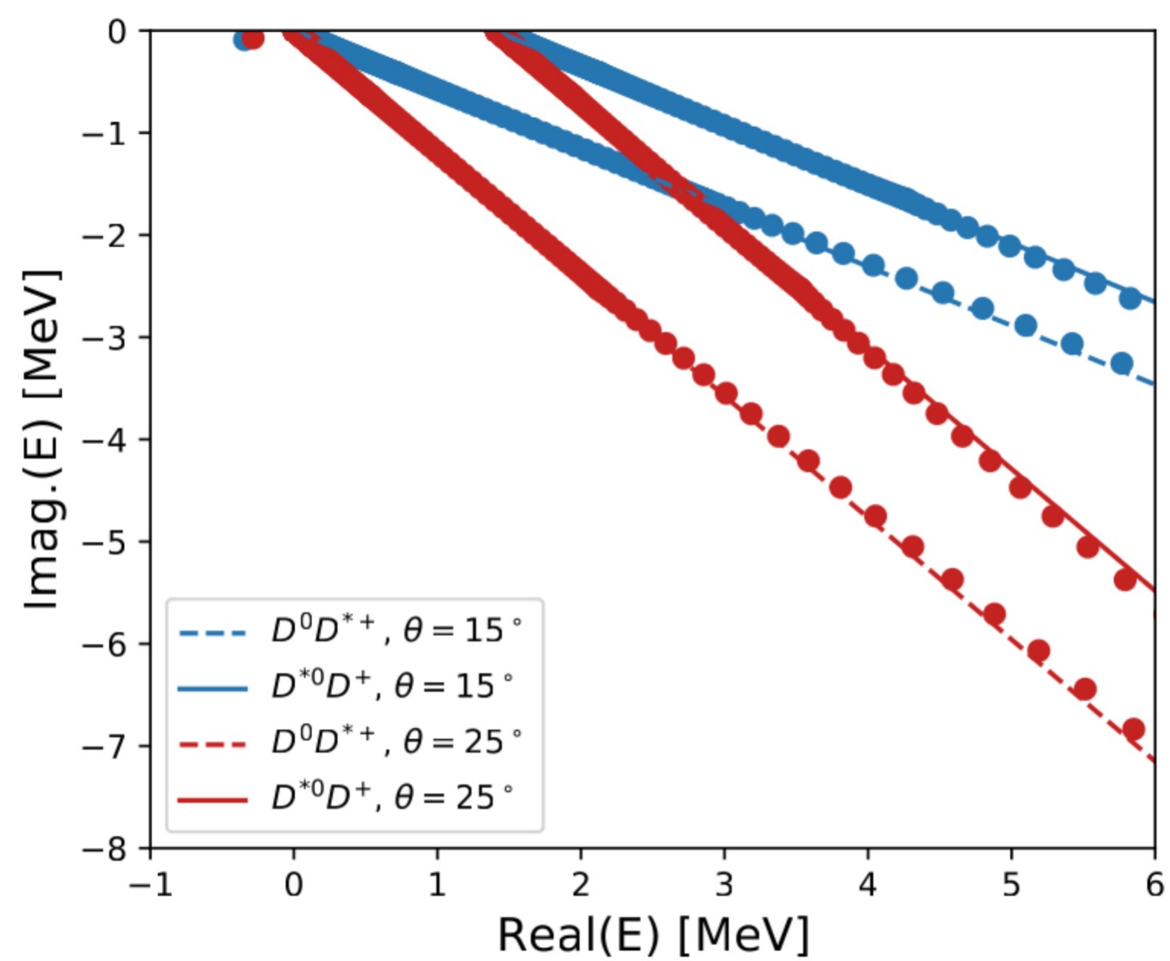
- 70.1% $D^{*+}D^0$, 30% D^+D^{*0} ⟺ 95.8%, $DD^*(I=0)$ 4.2% $DD^*(I=1)$

$$[I=0] = \frac{1}{\sqrt{2}}(D^{*+}D^0 - D^{*0}D^+)$$

$$[I=1] = \frac{1}{\sqrt{2}}(D^{*+}D^0 + D^{*0}D^+)$$

Mass differences of $D^{*+}D^0$ and D^+D^{*0}





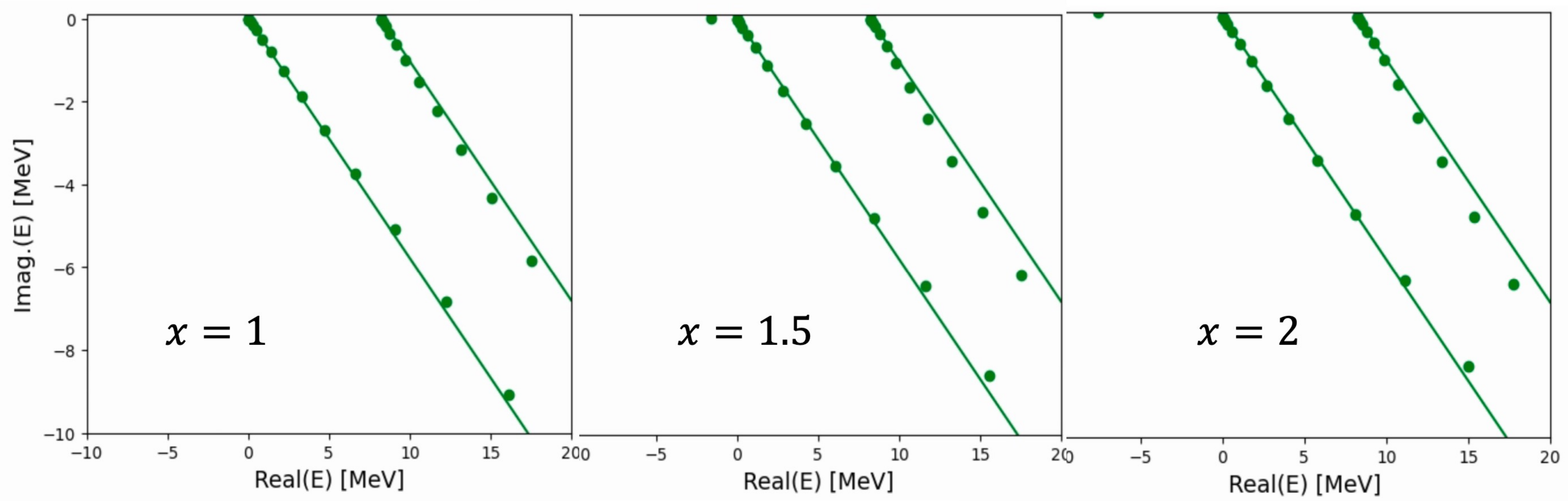
Results with three Λ

Λ (GeV)	BE (keV)	Γ (keV)	$\sqrt{\langle r^2 \rangle}$	$I=0$	$I=1$	$P(D^0 D^{*+})$	$P(D^+ D^{*0})$	$\frac{\mathrm{Res}(D^0 D^{*+})}{\mathrm{Res}(D^+ D^{*0})}$
0.8	-387.7	67.3	4.8 fm	95.8%	4.2%	70.0%	30.0%	$-1.063 + 0.001I$
1.0	-393.0	70.4	4.7 fm	95.8%	4.2%	70.0%	30.0%	$-1.055 + 0.001I$
1.2	-391.6	72.7	4.7 fm	95.7%	4.3%	70.3%	29.7%	$-1.052 + 0.001I$

- The conclusion remains the same using the three different cutoff values.
- The binding energy of the bound state is around $\Delta E \sim -390\mathrm{keV}$, which is consistent with that of the measurement $\left(\Delta E_{\mathrm{exp}} = -360(40)\mathrm{keV}\right)$. LHCb, Nature Commun. 13 (2022) 1, 3351

Direct application to $D\bar{D}^*$: $X(3872)$

- Without the $c\bar{c}$ core, there are no bound states.
- $V'_{D\bar{D}^*} = x * V_{D\bar{D}^*}$



$D\bar{D}^*$ interaction is attractive but not strong enough to form a bound state. ➡ ***Inclusion of $c\bar{c}$ core***

$X(3872) : D\bar{D}^* + c\bar{c}$

- The $D\bar{D}^*$ system with quantum number $I(J^{PC}) = 0(1^{++})$ can couple with the $\chi_{c1}(2P)$.
- The coupled channel effect between them can be described by the quark-pair-creation model:

$$g_{D\bar{D}^*,c\bar{c}}\left(\left|\vec{k}_{D\bar{D}^*}\right|\right) = \gamma I_{D\bar{D}^*,c\bar{c}}\left(\left|\vec{k}_{D\bar{D}^*}\right|\right)$$

where $\vec{k}_{D\bar{D}^*}$ is the relative momentum in the $D\bar{D}^*$ channel.

$I_{D\bar{D}^*,cc}\left(\left|\vec{k}_{D\bar{D}^*}\right|\right)$ is the overlap of the meson wave functions $\leftarrow$ GI quark model

- γ is determined to reproduce the $\psi(3770)$:

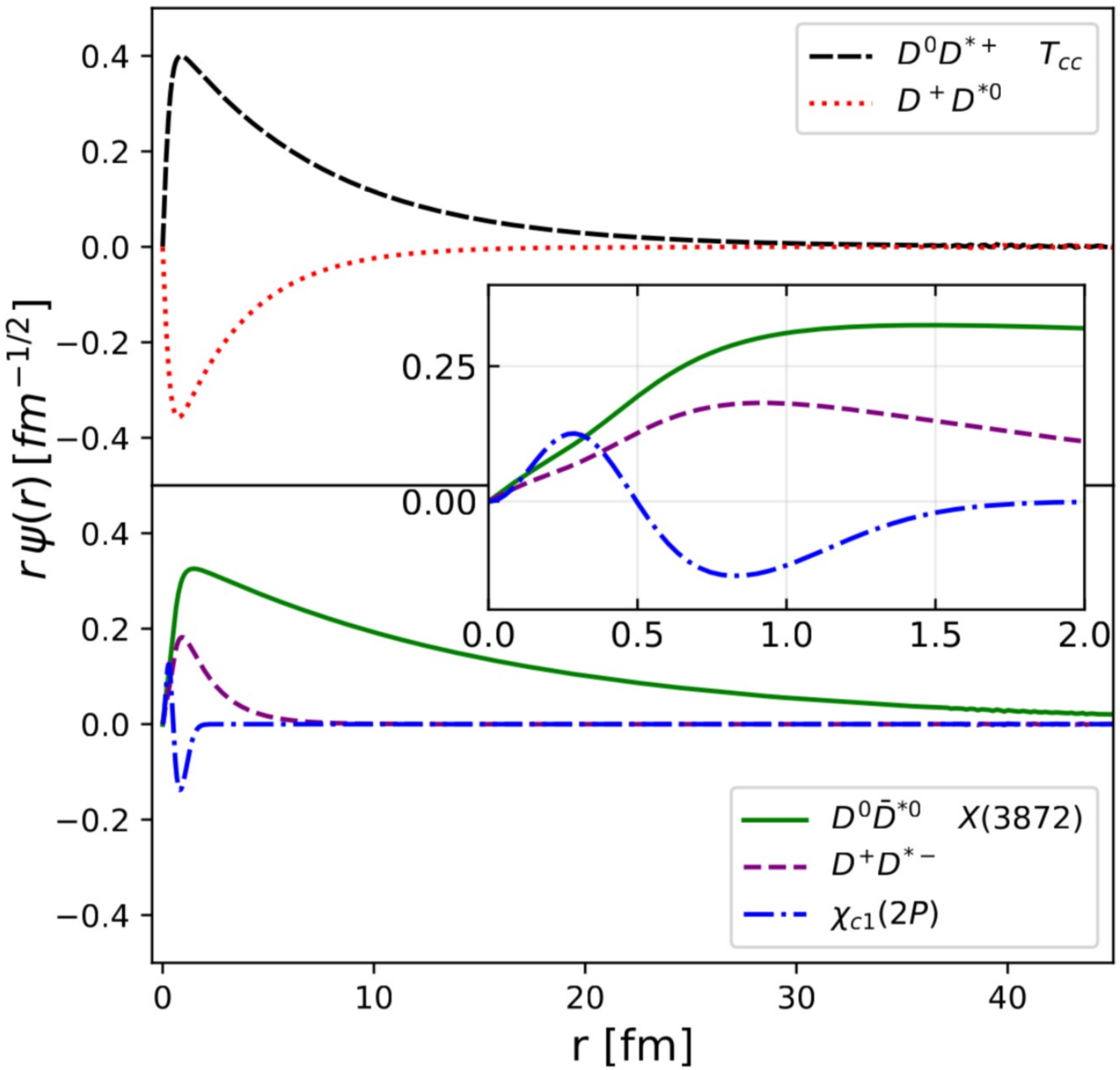
$$\gamma = 4.69$$

- The the X(3872) can be obtained:

$X(3872)$	BE (keV)	Γ (keV)	$\sqrt{\langle r^2\rangle}$	$I=0$	$I=1$	$P(D^0\bar{D}^{*0})$	$P(D^+D^{*-})$	$P(c\bar{c})$
	-80.4	32.5	11.2 fm	71.9%	28.1%	94.0%	4.8%	1.2%

Direct application to $D\bar{D}^*$: X(3872)

Wave functions of Tcc and X(3872)



- Long tails for the radius distribution.
- $X(3872)$ has a even longer tails than T_{cc}

✓ $r < 2$ fm, $c\bar{c} + \bar{D}D^*$ are important.

✓ $r < 0.5$ fm, $c\bar{c}$ core dominates.

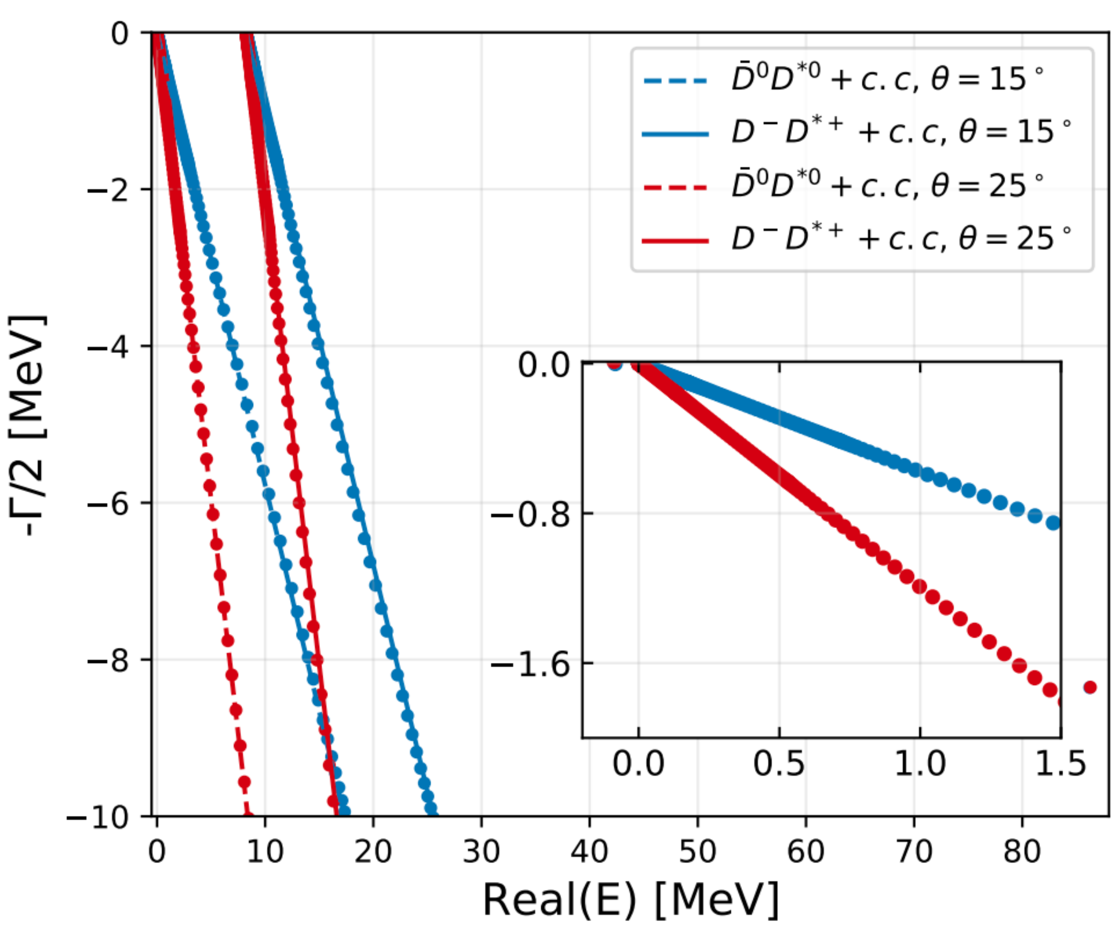
✓ $D\bar{D}^*$ plays the dominant role in the long-distance region, which contributes to $\sqrt{\langle r^2 \rangle}$.

Direct application to $D\bar{D}^*$: Candidate for X(3940)?

- Besides the $X(3872)$, we also find a signal of the resonant state $\chi_{c1}(2P)$ with

$$M = 3957.9\text{MeV},\ \Gamma = 16.7\text{MeV},$$

which might be related to the $X(3940)$ observed in the $D\bar{D}^*$ channel.



$X(3940)$ MASS	3942 ± 9 MeV
$X(3940)$ WIDTH	37^{+27}_{-17} MeV

$X(3940)$ Decay Modes

	Mode	Fraction (Γ_i/Γ)
Γ_1	$D\bar{D}^*$ + c.c.	seen

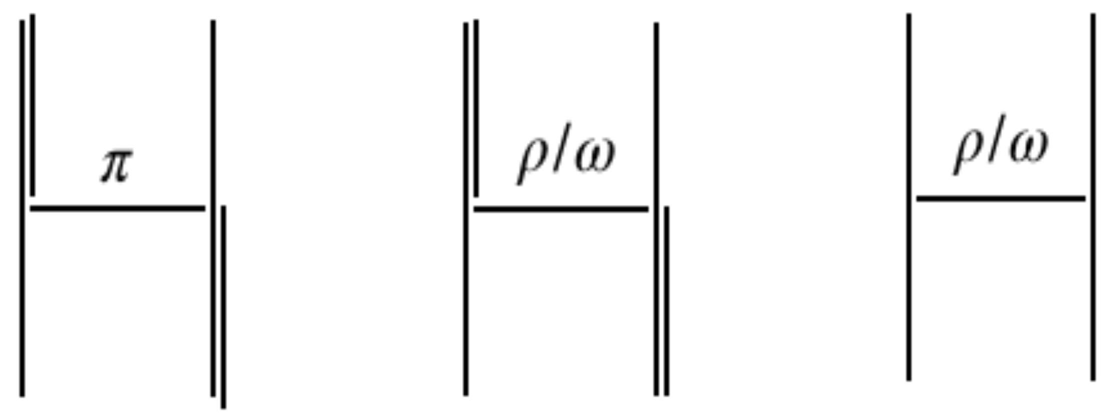
Summary

- A new framework connecting quark model and lattice QCD is constructed to study the components and pole masses of the physical $D_s(2317)$, $D_s(2460)$, $D_s(2536)$ and $D_s(2573)$.

- Short-range interactions and structures of $X(3872)$ should be studied by considered the $c\bar{c}$ core.

Thank you !

Backup: One-boson-exchange model



	π	$\rho/\omega,\ u$	$\rho/\omega,\ t$
$V_{DD*}^{I=0}$	$\frac{3}{2}V_\pi$	$\frac{3}{2}V_\rho^u - \frac{1}{2}V_\omega^u$	$-\frac{3}{2}V_\rho^t + \frac{1}{2}V_\omega^t$
$V_{DD*}^{I=1}$	$\frac{1}{2}V_\pi$	$\frac{1}{2}V_\rho^u + \frac{1}{2}V_\omega^u$	$\frac{1}{2}V_\rho^t + \frac{1}{2}V_\omega^t$
$V_{X(3872)}^{I=0,C=+}$	$\frac{3}{2}V_\pi$	$-\frac{3}{2}V_\rho^u - \frac{1}{2}V_\omega^u$	$-\frac{3}{2}V_\rho^t - \frac{1}{2}V_\omega^t$
$V_{X'}^{I=1,C=+}$	$-\frac{1}{2}V_\pi$	$\frac{1}{2}V_\rho^u - \frac{1}{2}V_\omega^u$	$\frac{1}{2}V_\rho^t - \frac{1}{2}V_\omega^t$

- The π interactions for $DD^*(I=0, T_{cc})$ are the same with those of $D\bar{D}^*(I=0, C=+)(X(3872))$
- The long-range meson-meson interactions for $T_{cc}, X(3872)$ are related to each other.