Low-Energy-Constant relations in baryon ChPT from chiral quark model

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J. Jiang, S.Z. Jiang, S.Y. Li, Y.R. Liu, Z.G. Si, H.Q. Wang, Eur. Phys. J. C 83, 296 (2023)

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Contents

LECs in baryon ChPT and chiral quark model

- Various systems and constructed chiral Lagrangians:

Pseudoscalar mesons $q\bar{q}$: $\mathcal{O}(p^8)$

Light quark baryons qqq: $\mathcal{O}(p^4)$

$$\begin{cases} SU(2) : \mathcal{L}_{\pi NN} \\ SU(3) : \mathcal{L}_{MBB} \\ SU(2) : \mathcal{L}_{\pi\Delta\Delta} , \mathcal{L}_{\pi N\Delta} \\ SU(3) : \mathcal{L}_{MTT} , \mathcal{L}_{MBT} \end{cases}$$

B: octet
T: decuplet

Heavy quark hadrons $Q\bar{q}$, Qqq, QQq: $\mathcal{O}(p^4)$

LECs in baryon ChPT and chiral quark model

- Increasing number of LECs in the SU(3) light hadron case

Pseudoscalar mesons:

$\mathcal{L}_2$	$\mathcal{L}_4$	$\mathcal{L}_6$	$\mathcal{L}_8$
2	12	94	1254

Light quark baryons

$\mathcal{L}_{MBB}$: 2, 16, 78, 540

S.Z. Jiang et al., PRD 95, 014012 (2017)

$\mathcal{L}_{MTT}$: 1, 13, 55, 548

$\mathcal{L}_{MBT}$: 1, 5, 67, 611

S.Z. Jiang et al., PRD 97, 054031 (2018)

LECs in baryon ChPT and chiral quark model

- LEC → hadron masses, hadron-hadron scattering, phase shifts, ...
- Difficult to determine LECs from QCD
- Fitting experimental data, lattice QCD, resonance saturation, quark model, ...
- Here: focus on LEC relations using chiral quark model (χQM)
- Fields between Λ_{QCD} and $\Lambda_{\chi SB}$ in χQM: quarks, gluons, and goldstone bosons [A.Manohar, H.Georgi, Nucl.Phys.B 234, 189 (1984)]

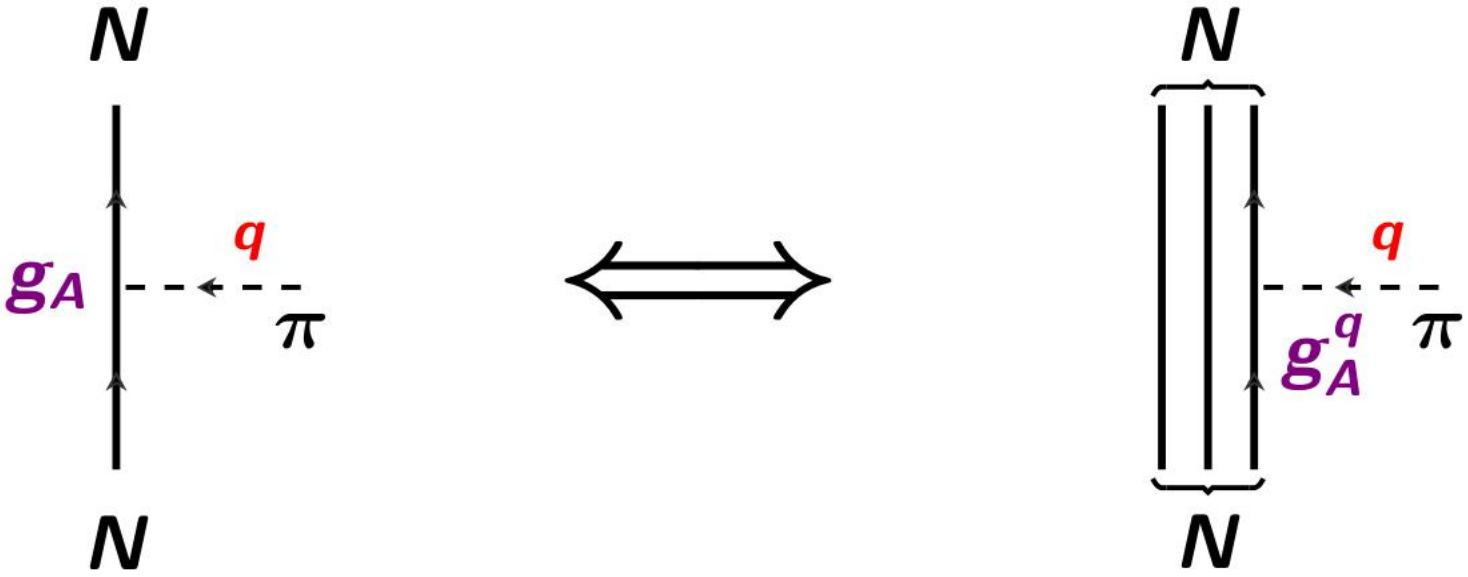
$$\mathcal{L}_{\chi QM} = \sum_{q=u,d} \bar{\psi}_q (i\not{D} + \not{V})\psi_q + g_A^q \bar{\psi}_q \not{A}\,\gamma_5 \psi_q + \frac{1}{4} F_\pi{}^2 \langle u^\mu u_\mu \rangle - \frac{1}{4}\langle \mathcal{G}_{\mu\nu}\mathcal{G}^{\mu\nu}\rangle + \cdots$$

$$A_\mu = \frac{i}{2}(u^\dagger \partial_\mu u - u\partial_\mu u^\dagger), \quad V_\mu = \frac{1}{2}(u^\dagger \partial_\mu u + u\partial_\mu u^\dagger), \quad u = exp(\frac{i\phi}{2F_\pi}), \quad \phi = \sum_{i=1}^{3} \tau_i \pi_i \text{ or } \sum_{i=1}^{8} \lambda_i \pi_i$$

LECs in baryon ChPT and chiral quark model

- Coupling constant in χQM

SU(2) LO Lagrangian



$$\left.\begin{array}{l}\langle t\rangle_{\text{hadron}} = \frac{g_A}{2} q_3 \langle p_\uparrow | \sigma_3 \tau_3 | p_\uparrow \rangle = \frac{g_A}{2} q_3 \\ \langle t\rangle_{\text{quark}} = \frac{g_A^q}{2} q_3 \langle p_\uparrow | \sum_{i=1}^{3} \sigma_3^{(i)} \tau_3^{(i)} | p_\uparrow \rangle = \frac{g_A^q}{2} q_3 \; \frac{5}{3}\end{array}\right\} \Rightarrow g_A = \frac{5}{3} g_A^q$$

$\mathcal{O}(p^1)$ 阶 χPT 和 χQM 的拉氏量

J. Gasser, Nucl. Phys. B, 307, 779 (1988)

$SU(2)_{\chi PT/\chi QM}$	$SU(3)_{\chi QM}$	$SU(3)_{\chi PT}$	
g_A/g_A^q	g_A^q	$D+F$	$D-F$
$\frac{1}{2}\bar{\psi}u^\mu\gamma_\mu\gamma_5\psi$	$\frac{1}{2}\bar{\Psi}u^\mu\gamma_\mu\gamma_5\Psi$	$\frac{1}{2}\langle\bar{B}u^\mu\gamma_\mu\gamma_5 B\rangle$	$\frac{1}{2}\langle\bar{B}\gamma_\mu\gamma_5 Bu^\mu\rangle$

$$SU(2)_{\chi PT} \xleftarrow{\text{w.f.}} SU(2)_{\chi QM} \longleftrightarrow SU(3)_{\chi QM} \xrightarrow{\text{w.f.}} SU(3)_{\chi PT}$$

$g_A = \frac{5}{3}g_A^q$ $\quad$ g_A^q $\quad$ g_A^q

$$D = g_A^q = \frac{3}{5}g_A, \quad F = \frac{2}{3}g_A^q = \frac{2}{5}g_A.$$

LECs in baryon ChPT and chiral quark model

- Extend relation determination to higher chiral orders
- Purpose: constrain SU(3) from SU(2)
- Treating χQM as quark-level descriptions of ChPT

手征夸克模型方法 $\Rightarrow$ 不同体系拉氏量低能常数间关系

八重态重子	十重态重子
SU(2) $\mathcal{L}_{\pi NN} \Longleftrightarrow \mathcal{L}_{\pi qq}$	SU(3) $\mathcal{L}_{MTT}$、$\mathcal{L}_{MBT}$
SU(3) $\mathcal{L}_{MBB} \Longleftrightarrow \mathcal{L}_{Mqq}$	SU(2) $\mathcal{L}_{\pi\Delta\Delta}$、$\mathcal{L}_{\pi N\Delta}$
χQM $\mathcal{L}_{Mqq} \Longleftrightarrow \mathcal{L}_{\pi qq}$	$\mathcal{L}_{\pi\Delta\Delta} \Longleftrightarrow \mathcal{L}_{\pi qq}$
χPT $\mathcal{L}_{MBB} \Longleftrightarrow \mathcal{L}_{\pi NN}$	$\mathcal{L}_{\pi N\Delta} \Longleftrightarrow \mathcal{L}_{\pi qq}$

- Final: LEC relations between hadron-level Lagrangians

$$\mathcal{O}(p^1) \Rightarrow \mathcal{O}(p^3)$$

- Need high-order Lagrangians in χQM (bridging role)
- Results of QM symmetry: SU(2), SU(3), and different representations for quarks and baryons
- A systematic approach to get LEC relations
- Grouping is needed according to operator structures

LECs in baryon ChPT and chiral quark model

$\mathcal{O}(p^2)$ 阶 χPT 和 χQM 的拉氏量

N. Fettes, U.G. Meissner, M. Mojzis, Annl. Phys. 283, 273 (2000)
J.A. Oller, M. Verbeni, J. Prades, JHEP 09, 079 (2006)

$SU(2)_{\chi PT/\chi QM}$		$SU(3)_{\chi QM}$		$SU(3)_{\chi PT}$			
	α_i/β_i		c_i		d_i		d_i
1	$\bar{\psi}\langle u^\mu u_\mu\rangle\psi$	1	$\bar{\Psi}\langle u^\mu u_\mu\rangle\Psi$	1	$\langle\bar{B}u^\mu u_\mu B\rangle$	3	$\langle\bar{B}Bu^\mu u_\mu\rangle$
		2	$\bar{\Psi}u^\mu u_\mu\Psi$	2	$\langle\bar{B}u^\mu Bu_\mu\rangle$	4	$\langle\bar{B}B\rangle\langle u^\mu u_\mu\rangle$
2	$i\bar{\psi}u^\mu u^\nu\sigma_{\mu\nu}\psi$	3	$i\bar{\Psi}u^\mu u^\nu\sigma_{\mu\nu}\Psi$	5	$i\langle\bar{B}u^\mu u^\nu\sigma_{\mu\nu}B\rangle$	7	$i\langle\bar{B}u^\mu\rangle\langle u^\nu\sigma_{\mu\nu}B\rangle$
				6	$i\langle\bar{B}\sigma_{\mu\nu}Bu^\mu u^\nu\rangle$		
3	$\bar{\psi}\langle u^\mu u^\nu\rangle D_{\mu\nu}\psi$	4	$\bar{\Psi}\langle u^\mu u^\nu\rangle D_{\mu\nu}\Psi$	8	$\langle\bar{B}u^\mu u^\nu D_{\mu\nu}B\rangle$	10	$\langle\bar{B}D_{\mu\nu}Bu^\mu u^\nu\rangle$
		5	$\bar{\Psi}u^\mu u^\nu D_{\mu\nu}\Psi$	9	$\langle\bar{B}u^\mu D_{\mu\nu}Bu^\nu\rangle$	11	$\langle\bar{B}D_{\mu\nu}B\rangle\langle u^\mu u^\nu\rangle$
4	$\bar{\psi}f_+^{\mu\nu}\sigma_{\mu\nu}\psi$	6	$\bar{\Psi}f_+^{\mu\nu}\sigma_{\mu\nu}\Psi$	12	$\langle\bar{B}f_+^{\mu\nu}\sigma_{\mu\nu}B\rangle$	13	$\langle\bar{B}\sigma_{\mu\nu}Bf_+^{\mu\nu}\rangle$
5	$\bar{\psi}\langle f_+^{\mu\nu}\rangle\sigma_{\mu\nu}\psi \to 0$						
6	$\bar{\psi}\tilde{\chi}_+\psi$	7	$\bar{\Psi}\tilde{\chi}_+\Psi$	14	$\langle\bar{B}\tilde{\chi}_+B\rangle$	16	$\langle\bar{B}B\rangle\langle\chi_+\rangle$
7	$\bar{\psi}\langle\chi_+\rangle\psi$	8	$\bar{\Psi}\langle\chi_+\rangle\Psi$	15	$\langle\bar{B}B\tilde{\chi}_+\rangle$		

LECs in baryon ChPT and chiral quark model

$\mathcal{O}(p^3)$ 阶 χPT 和 χQM 的拉氏量第 1 到第 4 组

N. Fettes, U.G. Meissner, M. Mojzis, Annl. Phys. 283, 273 (2000)
J.A. Oller, M. Verbeni, J. Prades, JHEP 09, 079 (2006)
Jiang et al., Phys. Rev. D, 106, 054023 (2022)

	$SU(2)_{\chi PT/\chi QM}$ α_i/β_i		$SU(3)_{\chi QM}$ c_i		$SU(3)_{\chi PT}$ d_i		d_i
1	$\bar\psi\langle u^\mu u_\mu\rangle u^\nu\gamma_\nu\gamma_5\psi$	1	$\bar\Psi\langle u^\mu u_\mu\rangle u^\nu\gamma_\nu\gamma_5\Psi$	1	$[\langle\bar B u^\mu u_\mu u^\nu\gamma_\nu\gamma_5 B\rangle]_+$	7	$\langle\bar B\gamma_\nu\gamma_5 B u^\mu u^\nu u_\mu\rangle$
2	$\bar\psi\langle u^\mu u^\nu\rangle u_\mu\gamma_\nu\gamma_5\psi$	2	$\bar\Psi\langle u^\mu u^\nu\rangle u_\mu\gamma_\nu\gamma_5\Psi$	2	$\langle\bar B u^\mu u^\nu u_\mu\gamma_\nu\gamma_5 B\rangle$	8	$[\langle\bar B\gamma_\nu\gamma_5 B u^\nu u^\mu u_\mu\rangle]_+$
		3	$\bar\Psi\langle u^\mu u_\mu u^\nu\rangle\gamma_\nu\gamma_5\Psi$	3	$\langle\bar B u^\mu u_\mu\gamma_\nu\gamma_5 B u^\nu\rangle$	9	$\langle\bar B\gamma_\nu\gamma_5 B u^\nu\rangle\langle u^\mu u_\mu\rangle$
		4	$[\bar\Psi u^\mu u_\mu u^\nu\gamma_\nu\gamma_5\Psi]_+$	4	$[\langle\bar B u^\nu u^\mu\gamma_\nu\gamma_5 B u_\mu\rangle]_+$	10	$\langle\bar B\gamma_\nu\gamma_5 B\rangle\langle u^\nu u^\mu u_\mu\rangle$
				5	$\langle\bar B u^\nu\gamma_\nu\gamma_5 B u^\mu u_\mu\rangle$	11	$[\langle\bar B u^\mu u_\mu\rangle\langle u^\nu\gamma_\nu\gamma_5 B\rangle]_+$
				6	$[\langle\bar B u_\mu\gamma_\nu\gamma_5 B u^\mu u^\nu\rangle]_+$		
3	$\varepsilon_{\mu\nu\lambda\rho}\bar\psi\langle u^\mu u^\nu u^\lambda\rangle D^\rho\psi$	5	$\epsilon_{\mu\nu\lambda\rho}\bar\Psi\langle u^\mu u^\nu u^\lambda\rangle D^\rho\Psi$	12	$\varepsilon_{\mu\nu\lambda\rho}\langle\bar B u^\mu u^\nu u^\lambda D^\rho B\rangle$	15	$\varepsilon_{\mu\nu\lambda\rho}\langle\bar B D^\rho B u^\mu u^\nu u^\lambda\rangle$
		6	$\epsilon_{\mu\nu\lambda\rho}\bar\Psi u^\mu u^\nu u^\lambda D^\rho\Psi$	13	$\varepsilon_{\mu\nu\lambda\rho}\langle\bar B u^\mu u^\nu D^\rho B u^\lambda\rangle$	16	$\varepsilon_{\mu\nu\lambda\rho}\langle\bar B D^\rho B\rangle\langle u^\mu u^\nu u^\lambda\rangle$
				14	$\varepsilon_{\mu\nu\lambda\rho}\langle\bar B u^\lambda D^\rho B u^\mu u^\nu\rangle$		
4	$\bar\psi\langle u^\mu u^\nu\rangle u^\lambda\gamma_\mu\gamma_5 D_{\nu\lambda}\psi$	7	$\bar\Psi\langle u^\mu u^\nu\rangle u^\lambda\gamma_\mu\gamma_5 D_{\nu\lambda}\Psi$	17	$[\langle\bar B u^\mu u^\nu u^\lambda\gamma_\mu\gamma_5 D_{\nu\lambda}B\rangle]_+$	23	$[\langle\bar B\gamma_\mu\gamma_5 D_{\nu\lambda}B u^\mu u^\nu u^\lambda\rangle]_+$
5	$\bar\psi\langle u^\mu u^\nu\rangle u^\lambda\gamma_\lambda\gamma_5 D_{\mu\nu}\psi$	8	$\bar\Psi\langle u^\nu u^\lambda\rangle u^\mu\gamma_\mu\gamma_5 D_{\nu\lambda}\Psi$	18	$\langle\bar B u^\nu u^\mu u^\lambda\gamma_\mu\gamma_5 D_{\nu\lambda}B\rangle$	24	$\langle\bar B\gamma_\mu\gamma_5 D_{\nu\lambda}B u^\nu u^\mu u^\lambda\rangle$
		9	$\bar\Psi\langle u^\mu u^\nu u^\lambda\rangle\gamma_\mu\gamma_5 D_{\nu\lambda}\Psi$	19	$\langle\bar B u^\nu u^\lambda\gamma_\mu\gamma_5 D_{\nu\lambda}B u^\mu\rangle$	25	$\langle\bar B\gamma_\mu\gamma_5 D_{\nu\lambda}B\rangle\langle u^\mu u^\nu u^\lambda\rangle$
		10	$[\bar\Psi u^\mu u^\nu u^\lambda\gamma_\mu\gamma_5 D_{\nu\lambda}\Psi]_+$	20	$[\langle\bar B u^\mu u^\nu\gamma_\mu\gamma_5 D_{\nu\lambda}B u^\lambda\rangle]_+$	26	$\langle\bar B\gamma_\mu\gamma_5 D_{\nu\lambda}B u^\mu\rangle\langle u^\nu u^\lambda\rangle$
				21	$[\langle\bar B u^\lambda\gamma_\mu\gamma_5 D_{\nu\lambda}B u^\mu u^\nu\rangle]_+$	27	$[\langle\bar B u^\nu u^\lambda\rangle\langle u^\mu\gamma_\mu\gamma_5 D_{\nu\lambda}B\rangle]_+$
				22	$\langle\bar B u^\mu\gamma_\mu\gamma_5 D_{\nu\lambda}B u^\nu u^\lambda\rangle$		
6	$[\bar\psi u_\mu h^{\mu\nu}D_\nu\psi]_+$	11	$[\bar\Psi u_\mu h^{\mu\nu}D_\nu\Psi]_+$	28	$[\langle\bar B u_\mu h^{\mu\nu}D_\nu B\rangle]_+$	30	$[\langle\bar B h^{\mu\nu}\rangle\langle u_\mu D_\nu B\rangle]_+$
				29	$[\langle\bar B D_\nu B u_\mu h^{\mu\nu}\rangle]_+$		

LECs in baryon ChPT and chiral quark model

$\mathcal{O}(p^3)$ 阶 χPT 和 χQM 的拉氏量第 5 到第 11 组

N. Fettes, U.G. Meissner, M. Mojzis, Annl. Phys. 283, 273 (2000)
J.A. Oller, M. Verbeni, J. Prades, JHEP 09, 079 (2006)
Jiang et al., Phys. Rev. D, 106, 054023 (2022)

	$SU(2)_{\chi PT/\chi QM}$ α_i/β_i		$SU(3)_{\chi QM}$ c_i		$SU(3)_{\chi PT}$ d_i		d_i
7	$[\bar{\psi} u^\mu h^{\nu\lambda} D_{\mu\nu\lambda} \psi]_+$	12	$[\bar{\Psi} u^\mu h^{\nu\lambda} D_{\mu\nu\lambda} \Psi]_+$	31	$[\langle \bar{B} u^\mu h^{\nu\lambda} D_{\mu\nu\lambda} B\rangle]_+$	33	$[\langle \bar{B} h^{\mu\nu}\rangle\langle u^\lambda D_{\mu\nu\lambda} B\rangle]_+$
				32	$[\langle \bar{B} D_{\mu\nu\lambda} B u^\mu h^{\nu\lambda}\rangle]_+$		
8	$i\bar{\psi}\langle u^\mu h^{\nu\lambda}\rangle \sigma_{\mu\nu} D_\lambda \psi$	13	$i\bar{\Psi}\langle u^\mu h^{\nu\lambda}\rangle \sigma_{\mu\nu} D_\lambda \Psi$	34	$[i\langle \bar{B} u^\mu h^{\nu\lambda} \sigma_{\mu\nu} D_\lambda B\rangle]_+$	36	$[i\langle \bar{B} \sigma_{\mu\nu} D_\lambda B u^\mu h^{\nu\lambda}\rangle]_+$
		14	$[i\bar{\Psi} u^\mu h^{\nu\lambda} \sigma_{\mu\nu} D_\lambda \Psi]_+$	35	$i\langle \bar{B} u^\mu \sigma_{\mu\nu} D_\lambda B h^{\nu\lambda}\rangle$	37	$i\langle \bar{B} \sigma_{\mu\nu} D_\lambda B\rangle\langle u^\mu h^{\nu\lambda}\rangle$
9	$[i\bar{\psi} f_+^{\mu\nu} u_\mu \gamma_\nu \gamma_5 \psi]_+$	15	$[i\bar{\Psi} f_+^{\mu\nu} u_\mu \gamma_\nu \gamma_5 \Psi]_+$	38	$[i\langle \bar{B} f_+^{\mu\nu} u_\mu \gamma_\nu \gamma_5 B\rangle]_+$	40	$[i\langle \bar{B} u_\mu\rangle\langle f_+^{\mu\nu} \gamma_\nu \gamma_5 B\rangle]_+$
				39	$[i\langle \bar{B} \gamma_\nu \gamma_5 B f_+^{\mu\nu} u_\mu\rangle]_+$		
10	$i\varepsilon_{\mu\nu\lambda\rho} \bar{\psi}\langle f_+^{\mu\nu}\rangle u^\lambda D^\rho \psi \to 0$	16	$i\epsilon_{\mu\nu\lambda\rho} \bar{\Psi}\langle f_+^{\mu\nu} u^\lambda\rangle D^\rho \Psi$	41	$[i\varepsilon_{\mu\nu\lambda\rho}\langle \bar{B} f_+^{\mu\nu} u^\lambda D^\rho B\rangle]_+$	44	$[i\varepsilon_{\mu\nu\lambda\rho}\langle \bar{B} D^\rho B f_+^{\mu\nu} u^\lambda\rangle]_+$
11	$i\varepsilon_{\mu\nu\lambda\rho} \bar{\psi}\langle f_+^{\mu\nu} u^\lambda\rangle D^\rho \psi$	17	$[i\epsilon_{\mu\nu\lambda\rho} \bar{\Psi} f_+^{\mu\nu} u^\lambda D_\rho \Psi]_+$	42	$i\varepsilon_{\mu\nu\lambda\rho}\langle \bar{B} f_+^{\mu\nu} D^\rho B u^\lambda\rangle$	45	$i\varepsilon_{\mu\nu\lambda\rho}\langle \bar{B} D^\rho B\rangle\langle f_+^{\mu\nu} u^\lambda\rangle$
				43	$i\varepsilon_{\mu\nu\lambda\rho}\langle \bar{B} u^\lambda D^\rho B f_+^{\mu\nu}\rangle$		
12	$i\bar{\psi}\nabla_\mu f_+^{\mu\nu} D_\nu \psi$	18	$i\bar{\Psi}\nabla_\mu f_+^{\mu\nu} D_\nu \Psi$	46	$i\langle \bar{B}\nabla_\mu f_+^{\mu\nu} D_\nu B\rangle$	47	$i\langle \bar{B} D_\nu B \nabla_\mu f_+^{\mu\nu}\rangle$
13	$i\bar{\psi}\langle\nabla_\mu f_+^{\mu\nu}\rangle D_\nu \psi \to 0$						
14	$[i\bar{\psi} f_+^{\mu\nu} u^\lambda \gamma_\mu \gamma_5 D_{\nu\lambda} \psi]_+$	19	$[i\bar{\Psi} f_+^{\mu\nu} u^\lambda \gamma_\mu \gamma_5 D_{\nu\lambda} \Psi]_+$	48	$[i\langle \bar{B} f_+^{\mu\nu} u^\lambda \gamma_\mu \gamma_5 D_{\nu\lambda} B\rangle]_+$	50	$[i\langle \bar{B} u^\lambda\rangle\langle f_+^{\mu\nu} \gamma_\mu \gamma_5 D_{\nu\lambda} B\rangle]_+$
				49	$[i\langle \bar{B} \gamma_\mu \gamma_5 D_{\nu\lambda} B f_+^{\mu\nu} u^\lambda\rangle]_+$		
15	$[\bar{\psi} u_\mu f_-^{\mu\nu} D_\nu \psi]_+$	20	$[\bar{\Psi} u_\mu f_-^{\mu\nu} D_\nu \Psi]_+$	51	$[\langle \bar{B} u_\mu f_-^{\mu\nu} D_\nu B\rangle]_+$	53	$[\langle \bar{B} f_-^{\mu\nu}\rangle\langle u_\mu D_\nu B\rangle]_+$
				52	$[\langle \bar{B} D_\nu B u_\mu f_-^{\mu\nu}\rangle]_+$		

LECs in baryon ChPT and chiral quark model

$\mathcal{O}(p^3)$ 阶 χPT 和 χQM 的拉氏量第 12 到第 16 组

N. Fettes, U.G. Meissner, M. Mojzis, Annl. Phys. 283, 273 (2000)
J.A. Oller, M. Verbeni, J. Prades, JHEP 09, 079 (2006)
Jiang et al., Phys. Rev. D, 106, 054023 (2022)

$SU(2)_{\chi PT/\chi QM}$		$SU(3)_{\chi QM}$		$SU(3)_{\chi PT}$			
	α_i/β_i		c_i		d_i		d_i
16	$i\bar\psi\langle u^\mu f_-^{\nu\lambda}\rangle\sigma_{\mu\nu}D_\lambda\psi$	21	$i\bar\Psi\langle u^\mu f_-^{\nu\lambda}\rangle\sigma_{\mu\nu}D_\lambda\Psi$	54	$[i\langle\bar B u^\mu f_-^{\nu\lambda}\sigma_{\mu\nu}D_\lambda B\rangle]_+$	59	$i\langle\bar B u^\mu\sigma_{\nu\lambda}D_\mu B f_-^{\nu\lambda}\rangle$
17	$i\bar\psi\langle u^\mu f_-^{\nu\lambda}\rangle\sigma_{\nu\lambda}D_\mu\psi$	22	$i\bar\Psi\langle u^\mu f_-^{\nu\lambda}\rangle\sigma_{\nu\lambda}D_\mu\Psi$	55	$[i\langle\bar B u^\mu f_-^{\nu\lambda}\sigma_{\nu\lambda}D_\mu B\rangle]_+$	60	$[i\langle\bar B\sigma_{\mu\nu}D_\lambda B u^\mu f_-^{\nu\lambda}\rangle]_+$
		23	$[i\bar\Psi u^\mu f_-^{\nu\lambda}\sigma_{\mu\nu}D_\lambda\Psi]_+$	56	$i\langle\bar B f_-^{\nu\lambda}\sigma_{\nu\lambda}D_\mu B u^\mu\rangle$	61	$[i\langle\bar B\sigma_{\nu\lambda}D_\mu B u^\mu f_-^{\nu\lambda}\rangle]_+$
		24	$[i\bar\Psi u^\mu f_-^{\nu\lambda}\sigma_{\nu\lambda}D_\mu\Psi]_+$	57	$i\langle\bar B f_-^{\nu\lambda}\sigma_{\mu\nu}D_\lambda B u^\mu\rangle$	62	$i\langle\bar B\sigma_{\mu\nu}D_\lambda B\rangle\langle u^\mu f_-^{\nu\lambda}\rangle$
				58	$i\langle\bar B u^\mu\sigma_{\mu\nu}D_\lambda B f_-^{\nu\lambda}\rangle$	63	$i\langle\bar B\sigma_{\nu\lambda}D_\mu B\rangle\langle u^\mu f_-^{\nu\lambda}\rangle$
18	$\bar\psi\nabla_\mu f_-^{\mu\nu}\gamma_\nu\gamma_5\psi$	25	$\bar\Psi\nabla_\mu f_-^{\mu\nu}\gamma_\nu\gamma_5\Psi$	64	$\langle\bar B\nabla_\mu f_-^{\mu\nu}\gamma_\nu\gamma_5 B\rangle$	65	$\langle\bar B\gamma_\mu\gamma_5 B\nabla_\nu f_-^{\mu\nu}\rangle$
19	$\bar\psi\langle u^\mu\tilde\chi_+\rangle\gamma_\mu\gamma_5\psi$	26	$\bar\Psi\langle u^\mu\tilde\chi_+\rangle\gamma_\mu\gamma_5\Psi$	66	$[\langle\bar B u^\mu\tilde\chi_+\gamma_\mu\gamma_5 B\rangle]_+$	70	$\langle\bar B\gamma_\mu\gamma_5 B\rangle\langle u^\mu\tilde\chi_+\rangle$
20	$\bar\psi\langle\chi_+\rangle u^\mu\gamma_\mu\gamma_5\psi$	27	$\bar\Psi\langle\chi_+\rangle u^\mu\gamma_\mu\gamma_5\Psi$	67	$\langle\bar B\tilde\chi_+\gamma_\mu\gamma_5 B u^\mu\rangle$	71	$\langle\bar B u^\mu\gamma_\mu\gamma_5 B\rangle\langle\chi_+\rangle$
		28	$[\bar\Psi u^\mu\tilde\chi_+\gamma_\mu\gamma_5\Psi]_+$	68	$\langle\bar B u^\mu\gamma_\mu\gamma_5 B\tilde\chi_+\rangle$	72	$\langle\bar B\gamma_\mu\gamma_5 B u^\mu\rangle\langle\chi_+\rangle$
				69	$[\langle\bar B\gamma_\mu\gamma_5 B u^\mu\tilde\chi_+\rangle]_+$		
21	$i\bar\psi\tilde\chi_-^\mu\gamma_\mu\gamma_5\psi$	29	$i\bar\Psi\tilde\chi_-^\mu\gamma_\mu\gamma_5\Psi$	73	$i\langle\bar B\tilde\chi_-^\mu\gamma_\mu\gamma_5 B\rangle$	75	$i\langle\bar B\gamma_\mu\gamma_5 B\rangle\langle\chi_-^\mu\rangle$
22	$i\bar\psi\langle\chi_-^\mu\rangle\gamma_\mu\gamma_5\psi$	30	$i\bar\Psi\langle\chi_-^\mu\rangle\gamma_\mu\gamma_5\Psi$	74	$i\langle\bar B\gamma_\mu\gamma_5 B\tilde\chi_-^\mu\rangle$		
23	$[i\bar\psi u^\mu\tilde\chi_- D_\mu\psi]_+$	31	$[i\bar\Psi u^\mu\tilde\chi_- D_\mu\Psi]_+$	76	$[i\langle\bar B u^\mu\tilde\chi_- D_\mu B\rangle]_+$	78	$[i\langle\bar B\tilde\chi_-\rangle\langle u^\mu D_\mu B\rangle]_+$
				77	$[i\langle\bar B D_\mu B u^\mu\tilde\chi_-\rangle]_+$		

LEC relations in baryon ChPT

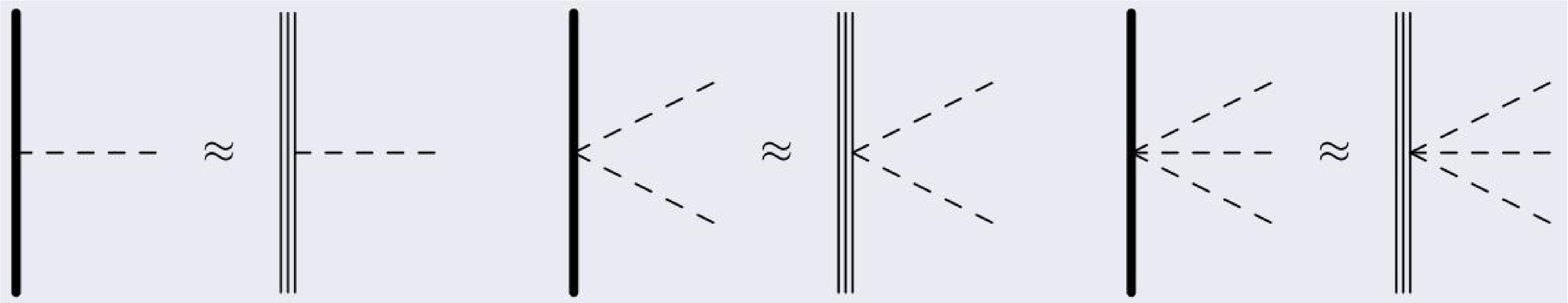
- Nonrelativistic reduction and adopted approximation

$$\bar{\psi}\psi \rightarrow \psi_H^\dagger \psi_H$$

$$\bar{\psi}\gamma^\mu\gamma_5\psi \approx 2\bar{\psi}_v S^\mu \psi_v \rightarrow \begin{cases} 0, & (\mu = 0) \\ \psi_H^\dagger \sigma^k \psi_H, & (\mu = k) \end{cases}$$

$$\bar{\psi}\sigma^{\mu\nu}\psi \approx 2\epsilon^{\mu\nu\alpha\beta} v_\alpha \bar{\psi}_v S_\beta \psi_v \rightarrow \begin{cases} \epsilon^{ijk}\psi_H^\dagger \sigma^k \psi_H, & (\mu = i, \nu = j) \\ 0, & (other) \end{cases}$$

Pauli-Lubanski 自旋矢量 : $S^\mu = \frac{i}{2}\gamma_5\sigma^{\mu\nu}v_\nu, \psi_v = \frac{1+\slashed{v}}{2}\psi$



two-quark spectator

LEC relations in baryon ChPT

- Structure correspondences

D. Drechsel, Nuovo Cimento A 76, 388 (1983)

$SU(2)$ 情形

$$1 \to \alpha_i = 3\beta_i, \quad \sigma \to \alpha_i = \beta_i, \quad \tau \to \alpha_i = \beta_i, \quad \tau \otimes \sigma \to \alpha_i = \frac{5}{3}\beta_i.$$

SU(3) 情形

$$\begin{aligned}
\langle \bar{B}B \rangle \to \bar{\Psi}\Psi \quad &\Rightarrow \quad (\text{若干 } d \text{ 的线性组合}) = 3(\text{若干 } c \text{ 的线性组合}),\\
\langle \bar{B}\lambda B \rangle \to \bar{\Psi}\lambda\Psi \quad &\Rightarrow \quad (\text{若干 } d \text{ 的线性组合}) = (\text{若干 } c \text{ 的线性组合}),\\
\langle \bar{B}\sigma B \rangle \to \bar{\Psi}\sigma\Psi \quad &\Rightarrow \quad (\text{若干 } d \text{ 的线性组合}) = (\text{若干 } c \text{ 的线性组合}),\\
\langle \bar{B}\lambda\sigma B \rangle \to \bar{\Psi}\lambda\sigma\Psi \quad &\Rightarrow \quad (\text{若干 } d \text{ 的线性组合}) = \frac{5}{3}(\text{若干 } c \text{ 的线性组合}),\\
\langle \bar{B}\sigma B\lambda \rangle \to \bar{\Psi}\lambda\sigma\Psi \quad &\Rightarrow \quad (\text{若干 } d \text{ 的线性组合}) = \frac{1}{3}(\text{若干 } c \text{ 的线性组合}),\\
\langle \bar{B}B\lambda \rangle \to \bar{\Psi}\lambda\Psi \quad &\Rightarrow \quad (\text{若干 } d \text{ 的线性组合}) = -(\text{若干 } c \text{ 的线性组合}).
\end{aligned}$$

八重态重子自旋-味道波函数

$$p_\uparrow = \frac{1}{3\sqrt{2}}[udu \otimes (2\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow - \uparrow\uparrow\downarrow) + duu \otimes (2\downarrow\uparrow\uparrow - \uparrow\downarrow\uparrow - \uparrow\uparrow\downarrow) - uud \otimes (\uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow - 2\uparrow\uparrow\downarrow)],$$

$$n_\uparrow = \frac{1}{3\sqrt{2}}[udd \otimes (\uparrow\downarrow\uparrow - 2\downarrow\uparrow\uparrow + \uparrow\uparrow\downarrow) + dud \otimes (\downarrow\uparrow\uparrow - 2\uparrow\downarrow\uparrow + \uparrow\uparrow\downarrow) + ddu \otimes (\uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow - 2\uparrow\uparrow\downarrow)],$$

$$\boldsymbol{\Sigma}^+_\uparrow = -\frac{1}{3\sqrt{2}}[usu \otimes (2\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow - \uparrow\uparrow\downarrow) + suu \otimes (2\downarrow\uparrow\uparrow - \uparrow\downarrow\uparrow - \uparrow\uparrow\downarrow) - uus \otimes (\uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow - 2\uparrow\uparrow\downarrow)],$$

$$\Sigma^-_\uparrow = \frac{1}{3\sqrt{2}}[dsd \otimes (2\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow - \uparrow\uparrow\downarrow) + sdd \otimes (2\downarrow\uparrow\uparrow - \uparrow\downarrow\uparrow - \uparrow\uparrow\downarrow) - dds \otimes (\uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow - 2\uparrow\uparrow\downarrow)],$$

$$\boldsymbol{\Xi}^0_\uparrow = -\frac{1}{3\sqrt{2}}[uss \otimes (\uparrow\downarrow\uparrow - 2\downarrow\uparrow\uparrow + \uparrow\uparrow\downarrow) + sus \otimes (\downarrow\uparrow\uparrow - 2\uparrow\downarrow\uparrow + \uparrow\uparrow\downarrow) + ssu \otimes (\uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow - 2\uparrow\uparrow\downarrow)],$$

$$\Xi^-_\uparrow = \frac{1}{3\sqrt{2}}[sds \otimes (\downarrow\uparrow\uparrow + \uparrow\uparrow\downarrow - 2\uparrow\downarrow\uparrow) + dss \otimes (\uparrow\downarrow\uparrow + \uparrow\uparrow\downarrow - 2\downarrow\uparrow\uparrow) + ssd \otimes (\uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow - 2\uparrow\uparrow\downarrow)],$$

$$\Sigma^0_\uparrow = \frac{1}{6}[(dsu + usd) \otimes (2\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow - \uparrow\uparrow\downarrow) + (sdu + sud) \otimes (2\downarrow\uparrow\uparrow - \uparrow\downarrow\uparrow - \uparrow\uparrow\downarrow)$$
$$+(dus + uds) \otimes (-\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow + 2\uparrow\uparrow\downarrow)],$$

$$\boldsymbol{\Lambda}_\uparrow = -\frac{1}{2\sqrt{3}}[(dsu - usd) \otimes (\downarrow\uparrow\uparrow - \uparrow\uparrow\downarrow) + (sdu - sud) \otimes (\uparrow\downarrow\uparrow - \uparrow\uparrow\downarrow) + (uds - dus) \otimes (\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow)].$$

LEC relations in baryon ChPT: ChPT$\Leftrightarrow$ χQM

	Group	$SU(2)_{\chi PT} \Leftrightarrow SU(2)_{\chi QM}$
$\mathcal{O}(p^1)$	1	$g_A = \frac{5}{3} g_A^q$.
$\mathcal{O}(p^2)$	1	$\alpha_1 = 3\beta_1$;
	2	$\alpha_2 = \frac{5}{3}\beta_2$;
	3	$\alpha_3 = 3\beta_3$;
	4	$\alpha_4 = \frac{5}{3}\beta_4$;
	5	$\alpha_6 = \beta_6,\ \alpha_7 = 3\beta_7$.
$\mathcal{O}(p^3)$	1	$\alpha_1 = \frac{5}{3}\beta_1,\ \alpha_2 = \frac{5}{3}\beta_2$;
	2	$\alpha_3 = 3\beta_3$;
	3	$\alpha_4 = \frac{5}{3}\beta_4,\ \alpha_5 = \frac{5}{3}\beta_5$;
	4	$\alpha_6 = \beta_6$;
	5	$\alpha_7 = \beta_7$;
	6	$\alpha_8 = \beta_8$;
	7	$\alpha_9 = \frac{5}{3}\beta_9$;
	8	$\alpha_{11} = 3\beta_{11}$;
	9	$\alpha_{12} = \beta_{12}$;
	10	$\alpha_{14} = \frac{5}{3}\beta_{14}$;
	11	$\alpha_{15} = \beta_{15}$;
	12	$\alpha_{16} = \beta_{16},\ \alpha_{17} = \beta_{17}$;
	13	$\alpha_{18} = \frac{5}{3}\beta_{18}$;
	14	$\alpha_{19} = \beta_{19},\ \alpha_{20} = \frac{5}{3}\beta_{20}$;
	15	$\alpha_{21} = \frac{5}{3}\beta_{21},\ \alpha_{22} = \beta_{22}$;
	16	$\alpha_{23} = \beta_{23}$.

	Group	$SU(3)_{\chi PT} \Leftrightarrow SU(3)_{\chi QM}$
$\mathcal{O}(p^1)$	1	$D = g_A^q,\ F = \frac{2}{3} g_A^q$.
$\mathcal{O}(p^2)$	1	$d_1 = -d_3 = c_2,\ d_2 = 0,\ d_4 = 3c_1 + c_2$;
	2	$d_5 = 5d_6 = \frac{5}{3}c_3,\ d_7 = 0$;
	3	$d_8 = -d_{10} = c_5,\ d_9 = 0,\ d_{11} = 3c_4 + c_5$;
	4	$d_{12} = 5d_{13} = \frac{5}{3}c_6$;
	5	$d_{14} = -d_{15} = c_7,\ d_{16} = 3c_8$.
$\mathcal{O}(p^3)$	1	$d_1 = \frac{5}{6}(2c_1 + c_2 + 2c_4),\ d_2 = 5d_7 = \frac{5}{3}c_2,\ d_3 = d_5 = -\frac{5}{4}d_9 = -d_{11} = \frac{5}{6}(2c_1 - c_2)$, $d_4 = d_6 = 0,\ d_8 = \frac{1}{6}(10c_1 - 3c_2 + 2c_4),\ d_{10} = \frac{1}{6}(-10c_1 - 7c_2 + 6c_3 - 4c_4)$;
	2	$d_{12} = -d_{15} = c_6,\ d_{13} = d_{14} = 0,\ d_{16} = 3c_5 + c_6$;
	3	$d_{17} = \frac{5}{6}(c_7 + 2c_8 + 2c_{10}),\ d_{18} = 5d_{24} = \frac{5}{3}c_7,\ d_{19} = d_{22} = -\frac{5}{4}d_{26} = -d_{27} = \frac{5}{6}(-c_7 + 2c_8)$, $d_{20} = d_{21} = 0,\ d_{23} = \frac{1}{6}(-3c_7 + 10c_8 + 2c_{10}),\ d_{25} = \frac{1}{6}(-7c_7 - 10c_8 + 6c_9 - 4c_{10})$;
	4	$d_{28} = -d_{29} = c_{11},\ d_{30} = 0$;
	5	$d_{31} = -d_{32} = c_{12},\ d_{33} = 0$;
	6	$d_{34} = 5d_{36} = \frac{5}{3}c_{14},\ d_{35} = 0,\ d_{37} = c_{13} - \frac{2}{3}c_{14}$;
	7	$d_{38} = 5d_{39} = \frac{5}{3}c_{15},\ d_{40} = 0$;
	8	$d_{41} = -d_{44} = c_{17},\ d_{42} = d_{43} = 0,\ d_{45} = 3c_{16} + 2c_{17}$;
	9	$d_{46} = -d_{47} = c_{18}$;
	10	$d_{48} = 5d_{49} = \frac{5}{3}c_{19},\ d_{50} = 0$;
	11	$d_{51} = -d_{52} = c_{20},\ d_{53} = 0$;
	12	$d_{54} = 5d_{60} = \frac{5}{3}c_{23},\ d_{55} = 5d_{61} = \frac{5}{3}c_{24},\ d_{56} = d_{57} = d_{58} = d_{59} = 0$, $d_{62} = c_{21} - \frac{2}{3}c_{23},\ d_{63} = c_{22} - \frac{2}{3}c_{24}$;
	13	$d_{64} = 5d_{65} = \frac{5}{3}c_{25}$;
	14	$d_{66} = 5d_{69} = \frac{5}{3}c_{28},\ d_{67} = d_{68} = 0,\ d_{70} = c_{26} - \frac{2}{3}c_{28},\ d_{71} = 5d_{72} = \frac{5}{3}c_{27}$;
	15	$d_{73} = 5d_{74} = \frac{5}{3}c_{29},\ d_{75} = c_{30}$;
	16	$d_{76} = -d_{77} = c_{31},\ d_{78} = 0$.

LEC relations in baryon ChPT: χQM ⇔ χQM, ChPT⇔ChPT

nc.: SU(2) gives no constraint

	Group	$SU(3)_{\chi QM} \Leftrightarrow SU(2)_{\chi QM}$
$\mathcal{O}(p^1)$	1	$g_A^q = g_A^q$.
$\mathcal{O}(p^2)$	1	$\beta_1 = c_1 + \frac{1}{2}c_2$;
	2	$\beta_2 = c_3$;
	3	$\beta_3 = c_4 + \frac{1}{2}c_5$;
	4	$\beta_4 = c_6$;
$\mathcal{O}(p^3)$	1	$\beta_1 = c_1 + c_4,\ \beta_2 = c_2,\ c_3\ (nc.)$;
	2	$\beta_3 = c_5 + \frac{1}{2}c_6$;
	3	$\beta_4 = c_7,\ \beta_5 = c_8 + c_{10},\ c_9\ (nc.)$;
	4	$\beta_6 = c_{11}$;
	5	$\beta_7 = c_{12}$;
	6	$\beta_8 = c_{13} + c_{14}$;
	7	$\beta_9 = c_{15}$;
	8	$\beta_{11} = c_{16} + c_{17}$;
	9	$\beta_{12} = c_{18}$;
	10	$\beta_{14} = c_{19}$;
	11	$\beta_{15} = c_{20}$;
	12	$\beta_{16} = c_{21} + c_{23},\ \beta_{17} = c_{22} + c_{24}$;
	13	$\beta_{18} = c_{25}$;
	14	$\beta_{19} = c_{26} + c_{28},\ \beta_{20} = c_{27} + \frac{1}{3}c_{28}$;
	15	$\beta_{21} = c_{29},\ \beta_{22} = \frac{1}{6}c_{29} + c_{30}$;
	16	$\beta_{23} = c_{31}$.

$c_3\,\bar{\Psi}\langle u^\mu u_\mu u^\nu\rangle\gamma_\nu\gamma_5\Psi$

$c_9\,\bar{\Psi}\langle u^\mu u^\nu u^\lambda\rangle\gamma_\mu\gamma_5 D_{\nu\lambda}\Psi$

	Group	With χQM	Without χQM
$\mathcal{O}(p^1)$	1	$D = \frac{3}{5}g_A,\ F = \frac{2}{5}g_A$.	$D + F = g_A$.
$\mathcal{O}(p^2)$	1	$\alpha_1 = \frac{1}{2}d_1 + d_4 = -\frac{1}{2}d_3 + d_4,\ d_2 = 0$;	$\alpha_1 = \frac{1}{2}d_1 + d_4$;
	2	$\alpha_2 = d_5 = 5d_6,\ d_7 = 0$;	$\alpha_2 = d_5$;
	3	$\alpha_3 = \frac{1}{2}d_8 + d_{11} = -\frac{1}{2}d_{10} + d_{11},\ d_9 = 0$;	$\alpha_3 = \frac{1}{2}d_8 + d_{11}$;
	4	$\alpha_4 = d_{12} = 5d_{13}$;	$\alpha_4 = d_{12}$;
	5	$\alpha_6 = d_{14} = -d_{15},\ \alpha_7 = \frac{1}{2}d_{14} + d_{16} = -\frac{1}{2}d_{15} + d_{16}$.	$\alpha_6 = d_{14},\ \alpha_7 = \frac{1}{2}d_{14} + d_{16} - \frac{1}{3}(d_{14} + d_{15})$.
$\mathcal{O}(p^3)$	1	$\alpha_1 + \frac{1}{2}\alpha_2 = d_1 = 5d_8 - 4d_3,\ d_3 = d_5 = -\frac{5}{4}d_9 = -d_{11},$ $\alpha_2 = d_2 = 5d_7,\ d_4 = d_6 = 0,\ d_{10}$ (nc.);	$\alpha_1 = d_1 - \frac{1}{2}d_2,\ \alpha_2 = d_2$;
	2	$\alpha_3 = \frac{1}{2}d_{12} + d_{16} = -\frac{1}{2}d_{15} + d_{16},\ d_{13} = d_{14} = 0$;	$\alpha_3 = \frac{1}{2}d_{12} - d_{16}$;
	3	$\alpha_4 = d_{18} = 5d_{24},\ \alpha_5 + \frac{1}{2}\alpha_4 = d_{17} = 5d_{23} - 4d_{19},$ $d_{19} = d_{22} = -\frac{5}{4}d_{26} = -d_{27},\ d_{20} = d_{21} = 0,\ d_{25}\ (nc.)$;	$\alpha_4 = d_{18},\ \alpha_5 = d_{17} - \frac{1}{2}d_{18}$;
		$\alpha_6 = d_{28} = -d_{29},\ d_{30} = 0$;	$\alpha_6 = d_{28}$;
	5	$\alpha_7 = d_{31} = -d_{32},\ d_{33} = 0$;	$\alpha_7 = d_{31}$;
		$\alpha_8 = d_{34} + d_{37} = 5d_{36} + d_{37},\ d_{35} = 0$;	$\alpha_8 = d_{34} + d_{37}$;
	7	$\alpha_9 = d_{38} = 5d_{39},\ d_{40} = 0$;	$\alpha_9 = d_{38}$;
	8	$\alpha_{11} = d_{41} + d_{45} = -d_{44} + d_{45},\ d_{42} = d_{43} = 0$;	$\alpha_{11} = d_{41} + d_{45}$;
	9	$\alpha_{12} = d_{46} = -d_{47}$;	$\alpha_{12} = d_{46}$;
	10	$\alpha_{14} = d_{48} = 5d_{49},\ d_{50} = 0$;	$\alpha_{14} = d_{48}$;
	11	$\alpha_{15} = d_{51} = -d_{52},\ d_{53} = 0$;	$\alpha_{15} = d_{51}$;
	12	$\alpha_{16} = d_{54} + d_{62} = 5d_{60} + d_{62},\ \alpha_{17} = d_{55} + d_{63} = 5d_{61} + d_{63},$ $d_{56} = d_{57} = d_{58} = d_{59} = 0$;	$\alpha_{16} = d_{54} + d_{62},\ \alpha_{17} = d_{55} + d_{63}$;
	13	$\alpha_{18} = d_{64} = 5d_{65}$;	$\alpha_{18} = d_{64}$;
	14	$\alpha_{19} = d_{66} + d_{70} = 5d_{69} + d_{70},\ d_{67} = d_{68} = 0,$ $\alpha_{20} = \frac{1}{3}d_{66} + d_{71} = \frac{5}{3}d_{69} + d_{71} = \frac{1}{3}d_{66} + 5d_{72}$;	$\alpha_{19} = d_{66} + d_{70},\ \alpha_{20} = \frac{1}{3}d_{66} - \frac{1}{3}d_{68} + d_{71}$;
	15	$\alpha_{21} = d_{73} = 5d_{74},\ \alpha_{22} = \frac{1}{10}d_{73} + d_{75} = \frac{1}{2}d_{74} + d_{75}$;	$\alpha_{21} = d_{73},\ \alpha_{22} = \frac{1}{6}d_{73} - \frac{1}{3}d_{74} + d_{75}$;
	16	$\alpha_{23} = d_{76} = -d_{77},\ d_{78} = 0$.	$\alpha_{23} = d_{76}$.

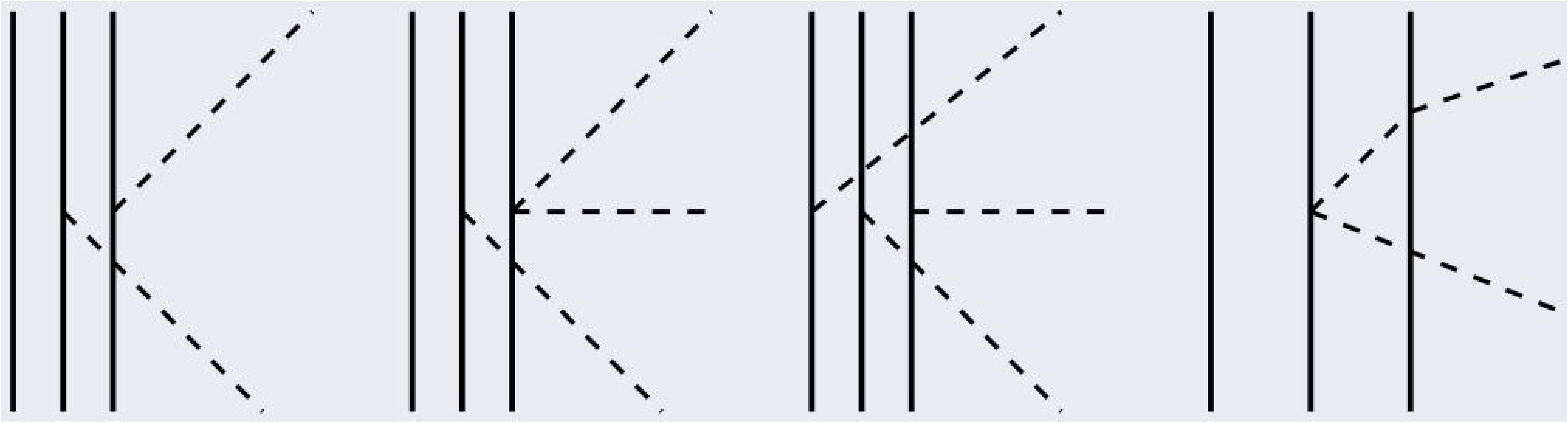
LEC relations in baryon ChPT

$$\left\{ \begin{array}{c} \bar{p}\pi^0 p \\ \bar{n}\pi^0 n \end{array} \right\} \Rightarrow D + F = \frac{5}{3} g_A^q$$

$$\left\{ \begin{array}{c} \bar{\Sigma}^+\pi^0\Sigma^+ \\ \bar{\Sigma}^-\pi^0\Sigma^- \end{array} \right\} \left\{ \begin{array}{c} \bar{\Sigma}^0\pi^+\Sigma^- \\ \bar{\Sigma}^-\pi^-\Sigma^0 \end{array} \right\} \Rightarrow F = \frac{2}{3} g_A^q$$

← s quark as spectator

Numerical analysis: corrections become important for high-order LEC relations



The case including $\Delta(1232)$

$$(i\partial\!\!\!/ - m_\Delta)\Psi_\mu = 0,$$
$$\gamma^\mu \Psi_\mu = 0,$$
$$\partial^\mu \Psi_\mu = 0,$$

- Rarita-Schwinger field

$$\Psi_\mu(x) = \sum_{s_\Delta} \int \frac{d^3p\, M_\Delta}{(2\pi)^3 E} \left[b(\boldsymbol{p}, s_\Delta) u_\mu(\boldsymbol{p}, s_\Delta) e^{-ip\cdot x} + d^\dagger(\boldsymbol{p}, s_\Delta) v_\mu(\boldsymbol{p}, s_\Delta) e^{ip\cdot x} \right].$$

其中 $u_\mu(\boldsymbol{p}, s_\Delta)$ 是 Rarita-Schwinger 旋量，自旋-1 矢量和自旋-1/2 的旋量的耦合

$$u_\mu(\boldsymbol{p}, s_\Delta) = \sum_{\lambda, s} \langle 1\lambda \frac{1}{2} s | \frac{3}{2} s_\Delta \rangle e_\mu(\boldsymbol{p}, \lambda) u(\boldsymbol{p}, s)$$

$$e^\mu(\boldsymbol{p}, \lambda) = \left(\frac{\hat{e}_\lambda \cdot \boldsymbol{p}}{M_\Delta},\ \hat{e}_\lambda + \frac{\boldsymbol{p}(\hat{e}_\lambda \cdot \boldsymbol{p})}{M_\Delta(p_0 + M_\Delta)} \right),\quad u(\boldsymbol{p}, s) = \sqrt{\frac{E + M_\Delta}{2M_\Delta}} \begin{pmatrix} \chi_s \\ \frac{\boldsymbol{\sigma}\cdot\boldsymbol{p}}{E+m}\chi_s \end{pmatrix}$$

其中单位矢量 $\hat{e}_\lambda$, $\lambda = 0, \pm 1$ 的球面表示 $\hat{e}_+ = -\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \\ 0 \end{pmatrix}$ $\hat{e}_0 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ $\hat{e}_- = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \\ 0 \end{pmatrix}$

Hemmert et al., J. Phys. G 24, 1831 (1998)

The case including $\Delta(1232)$

自旋-3/2 的 Δ 场的全对称张量形式

$$T^{111} = \Delta^{++}, \quad T^{222} = \Delta^{-},$$
$$T^{112} = T^{121} = T^{211} = \frac{\Delta^{+}}{\sqrt{3}},$$
$$T^{122} = T^{212} = T^{221} = \frac{\Delta^{0}}{\sqrt{3}}.$$

$\mathcal{O}(p^1)$ 阶拉氏量和 LEC 关系

Hemmert et al., J. Phys. G 24, 1831 (1998)

$\mathcal{L}_{\pi NN/\pi qq}$	$\mathcal{L}_{\pi\Delta\Delta}$	$\mathcal{L}_{\pi N\Delta}$
g_A/g_A^q	$e_i^{(1)}$	$f_i^{(1)}$
$\frac{1}{2}\bar{\psi}u^{\mu}\gamma_{\mu}\gamma_5\psi$	$\bar{T}^{abc,\lambda}u^{ad,\mu}\gamma_{\mu}\gamma_5 T_{\lambda}^{bcd}$	$\epsilon^{ab}\bar{N}^c u^{ad,\mu}T_{\mu}^{bcd} + h.c.$

$\mathcal{L}_{\pi NN}$ ← w.f. — $\mathcal{L}_{\pi qq}$ → w.f. → $\mathcal{L}_{\pi\Delta\Delta}$; w.f. → $\mathcal{L}_{\pi N\Delta}$

$$g_A = \frac{5}{3}g_A^q \quad \leftarrow \quad g_A^q$$

$$\mathcal{L}_{\pi\Delta\Delta}: \quad e_1^{(1)} = -\frac{3}{2}g_A^q$$

$$\mathcal{L}_{\pi N\Delta}: \quad f_1^{(1)} = k_1 g_A^q$$

Introduced multiplication factor

The case including $\Delta(1232)$

非相对论约化

$$\bar{B}B' \to B_H^\dagger B_H',$$
$$\bar{B}\gamma^\mu\gamma_5 B' \to B_H^\dagger \sigma^k B_H' \quad (\mu = k = 1,2,3),$$
$$\bar{B}\sigma^{\mu\nu} B' \to \epsilon^{ijk} B_H^\dagger \sigma^k B_H' \quad (\mu = i, \nu = j).$$

Define transition spin $S_{t\mu}$:

$$T_\mu^{abc} \equiv S_{t\mu} T^{abc} = \sqrt{\frac{1}{3}(\frac{9}{2} - a - b - c)^2 + \frac{1}{4}}\,\Phi_\mu^{abc} = \sqrt{\frac{1}{3}(\frac{9}{2} - a - b - c)^2 + \frac{1}{4}}\,S_{t\mu}\Phi^{abc}$$

取静态极限，$S_t^\mu = (0, \vec{S}_t)$，Δ 重子自旋波函数的第三分量如下

$$\Phi_{s_z=3/2} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix},\ \Phi_{s_z=1/2} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix},\ \Phi_{s_z=-1/2} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix},\ \Phi_{s_z=-3/2} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}.$$

使用 RS 旋量表达式 $\Phi_\mu(s_z) = \sum_{\lambda,s}\langle 1\lambda\frac{1}{2}s|\frac{3}{2}s_z\rangle e_\mu(\lambda)u(s) =$

$$S_t^3 = \begin{pmatrix} 0 & \sqrt{\frac{2}{3}} & 0 & 0 \\ 0 & 0 & \sqrt{\frac{2}{3}} & 0 \end{pmatrix}.$$

The case including $\Delta(1232)$

$\sigma^{\mu}_{RS} \equiv -\bar{S}_{t\rho}\gamma^{\mu}\gamma^{5}S^{\rho}_{t} \rightarrow \vec{\sigma}_{RS} = (S^{\dagger}_{t})^{j}\vec{\sigma}(S_{t})^{j}$: spin operator of RS field

$$\sigma^{3}_{RS} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 \\ 0 & 0 & -\frac{1}{3} & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}.$$

$\mathcal{L}_{\pi NN/\pi qq}$	$\mathcal{L}_{\pi\Delta\Delta}$	$\mathcal{L}_{\pi N\Delta}$
$\frac{1}{2}(g_A/g^q_A)\bar{\psi}u^{\mu}\gamma_{\mu}\gamma_5\psi$	$-e^{(1)}_1 T^{abc,\dagger}u^{ad,\mu}\sigma_{RS,\mu}T^{bcd}$	$f^{(1)}_1\epsilon^{ab}\bar{N}^{c}u^{ad,\mu}S_{t,\mu}T^{bcd} + h.c.$

$$k_1 S^{\mu}_{t} \rightarrow \gamma^{\mu}\gamma^{5},$$
$$\sigma^{\mu}_{RS} \equiv -\bar{S}_{t\rho}\gamma^{\mu}\gamma^{5}S^{\rho}_{t} \rightarrow \gamma^{\mu}\gamma^{5}.$$

$\Rightarrow$

$$f^{(1)}_1 = k_1 g^q_A,$$
$$e^{(1)}_1 = -\frac{3}{2}g^q_A.$$

$k_1 = ?$

The case including $\Delta(1232)$

$\mathcal{L}_{\pi NN/\pi qq}$	$\mathcal{L}_{\pi\Delta\Delta}$	$\mathcal{L}_{\pi N\Delta}$
$\frac{1}{2}(g_A/g_A^q)\bar{\psi}u^\mu\gamma_\mu\gamma_5\psi$	$-e_1^{(1)}T^{abc,\dagger}u^{ad,\mu}\sigma_{RS,\mu}T^{bcd}$	$f_1^{(1)}\epsilon^{ab}\bar{N}^c u^{ad,\mu}S_{t,\mu}T^{bcd}+h.c.$

Value of k_1:

$$g_{\pi N\Delta}=\sqrt{2}f_1^{(1)}、f_1^{(1)}=k_1g_A^q、g_A=\frac{5}{3}g_A^q\Rightarrow g_{\pi N\Delta}=\frac{3\sqrt{2}}{5}k_1g_A$$

- 当不引入 k_1 时，$g_{\pi N\Delta}^{QM}/g_{\pi NN}^{QM}\approx 1.7$，与实验数据 $g_{\pi N\Delta}^{expt}/g_{\pi NN}^{expt}\approx 2.21$ 不一致

Hemmert et al., J. Phys. G 24, 1831 (1998)

- 当 $k_1=\frac{5}{4}$ 时，$g_{\pi N\Delta}^{QM}/g_{\pi NN}^{QM}\approx 2.13$，与实验相符

Bernard et al., Int. J. Mod. Phys. E 4, 193 (1995)

- 考虑到 S_t^3 中的数值，我们取 $k_1=\sqrt{\frac{3}{2}}\approx 1.225$

$$S_t^3=\begin{pmatrix}0 & \sqrt{\frac{2}{3}} & 0 & 0\\ 0 & 0 & \sqrt{\frac{2}{3}} & 0\end{pmatrix}$$

$$f_1^{(1)}=k_1g_A^q,\quad e_1^{(1)}=-\frac{3}{2}g_A^q.$$

The case including $\Delta(1232)$

$\mathcal{L}_{\pi NN/\pi qq}$	$\mathcal{L}_{\pi\Delta\Delta}$	$\mathcal{L}_{\pi N\Delta}$
$\frac{1}{2}(g_A/g_A^q)\bar{\psi}u^{\mu}\gamma_{\mu}\gamma_5\psi$	$-e_1^{(1)}T^{abc,\dagger}u^{ad,\mu}\sigma_{RS,\mu}T^{bcd}$	$f_1^{(1)}\epsilon^{ab}\bar{N}^c u^{ad,\mu}S_{t,\mu}T^{bcd}+h.c.$

Phenomenological perspective

- J=1/2 case operator correspondence $\vec{\sigma}_{hadron} \leftrightarrow \vec{\sigma}_{quark}$

$|\frac{1}{2},\frac{1}{2}\rangle_{hadron} = \sigma^z_{hadron}|\frac{1}{2},\frac{1}{2}\rangle_{hadron}$: σ^z_{hadron} (σ^z_{quark}) transits $|\frac{1}{2},\frac{1}{2}\rangle$ to the same state.

- Present operator correspondence $\sqrt{\frac{3}{2}}\vec{S}_t \rightarrow \vec{\sigma}_{quark}$

$|\frac{1}{2},\frac{1}{2}\rangle_{hadron} = \sqrt{\frac{3}{2}}S^z_{t,hadron}|\frac{3}{2},\frac{1}{2}\rangle_{hadron}$: $\sqrt{\frac{3}{2}}S^z_t$ =1 transits $|\frac{3}{2},\frac{1}{2}\rangle$ to $|\frac{1}{2},\frac{1}{2}\rangle$.

- Present operator correspondence $\vec{\sigma}_{RS} \rightarrow \vec{\sigma}_{quark}$

$|\frac{3}{2},\frac{3}{2}\rangle = \sigma^z_{RS}|\frac{3}{2},\frac{3}{2}\rangle$: σ^z_{RS} transits $|\frac{3}{2},\frac{3}{2}\rangle$ to the same state.

The case including $\Delta(1232)$

$\mathcal{O}(p^1)$ 和 $\mathcal{O}(p^2)$ 阶的结构对应关系

$$
\begin{aligned}
k_1 S_t^{\mu} &\rightarrow \gamma^{\mu}\gamma^5, \\
k_2\sigma_t^{\mu\nu} \equiv ik_2(\gamma^{\mu}\gamma^5 S_t^{\nu} - \gamma^{\nu}\gamma^5 S_t^{\mu}) = k_2\epsilon^{\mu\nu\rho\lambda}\gamma_{\rho}S_{t\lambda} &\rightarrow \sigma^{\mu\nu}; \\
-\bar{S}_{t\rho}S_t^{\rho} = 1 &\rightarrow 1, \\
\sigma_{RS}^{\mu} \equiv -\bar{S}_{t\rho}\gamma^{\mu}\gamma^5 S_t^{\rho} &\rightarrow \gamma^{\mu}\gamma^5, \\
\sigma_{RS}^{\mu\nu} \equiv -i(\bar{S}_t^{\mu}S_t^{\nu} - \bar{S}_t^{\nu}S_t^{\mu}) = -\bar{S}_{t\rho}\sigma^{\mu\nu}S_t^{\rho} &\rightarrow \sigma^{\mu\nu}.
\end{aligned}
$$

$\mathcal{O}(p^3)$ 阶的结构对应关系

$$
\begin{aligned}
k_3\sigma^{\mu\nu}S_t^{\lambda} &\rightarrow i\left(g^{\nu\lambda}\gamma^{\mu}\gamma^5 - g^{\mu\lambda}\gamma^{\nu}\gamma^5 - i\varepsilon^{\mu\nu\lambda\rho}\gamma_{\rho}\right) \\
&\quad \doteq i\left(g^{\nu\lambda}\gamma^{\mu}\gamma^5 - g^{\mu\lambda}\gamma^{\nu}\gamma^5 + \frac{1}{m_{\Delta}}\varepsilon^{\mu\nu\lambda\rho}D_{\rho}\right); \\
k_4(\gamma^{\mu}\gamma_5 S_t^{\nu} + \gamma^{\nu}\gamma_5 S_t^{\mu}) &\rightarrow g^{\mu\nu}; \\
-k_5(\bar{S}_t^{\mu}S_t^{\nu} + \bar{S}_t^{\nu}S_t^{\mu}) &\rightarrow g^{\mu\nu}, \\
-k_6(\bar{S}_t^{\mu}\gamma^{\lambda}\gamma_5 S_t^{\nu} + \bar{S}_t^{\nu}\gamma^{\lambda}\gamma_5 S_t^{\mu}) &\rightarrow (g^{\lambda\mu}\gamma^{\nu}\gamma_5 + g^{\lambda\nu}\gamma^{\mu}\gamma_5 - g^{\mu\nu}\gamma^{\lambda}\gamma^5), \\
-k_7(\bar{S}_t^{\mu}\gamma^{\lambda}\gamma_5 S_t^{\nu} - \bar{S}_t^{\nu}\gamma^{\lambda}\gamma_5 S_t^{\mu}) &\rightarrow i\varepsilon^{\mu\nu\lambda\rho}\gamma_{\rho} \doteq -\frac{1}{m_{\Delta}}\varepsilon^{\mu\nu\lambda\rho}D_{\rho}.
\end{aligned}
$$

The case including $\Delta(1232)$

$\mathcal{O}(p^2)$ 阶拉氏量

	$\mathcal{L}_{\pi NN/\pi qq}$ $\alpha_i^{(2)}/\beta_i^{(2)}$		$\mathcal{L}_{\pi\Delta\Delta}$ $\hat{e}_i^{(2)}$		$\mathcal{L}_{\pi N\Delta}$ $\hat{f}_i^{(2)}$
1	$\bar{\psi}\langle u^\mu u_\mu\rangle\psi$	1	$\bar{T}^{abc,\lambda}\langle u^\mu u_\mu\rangle T_\lambda^{abc}$		
		2	$\bar{T}^{abc,\lambda}u^{ag,\mu}u_\mu^{be}T_\lambda^{cge}$		
		3	$\bar{T}_\mu^{abc}\langle u^\mu u^\nu\rangle T_\nu^{abc}$		
		4	$\bar{T}_\mu^{abc}u^{ag,\mu}u^{be,\nu}T_\nu^{cge}$	1	$[\epsilon^{bc}\bar{N}^a u^{ag,\mu}u^{be,\nu}(\gamma_\mu\gamma_5 T_\nu^{cge}+\gamma_\nu\gamma_5 T_\mu^{cge})]_+$
2	$i\bar{\psi}u^\mu u^\nu\sigma_{\mu\nu}\psi$	5	$i\bar{T}^{abc,\lambda}(u^\mu u^\nu)^{ae}\sigma_{\mu\nu}T_\lambda^{bce}$	2	$[\epsilon^{ab}\bar{N}^c(u^\mu u^\nu)^{ae}(\gamma_\mu\gamma_5 T_\nu^{bce}-\gamma_\nu\gamma_5 T_\mu^{bce})]_+$
3	$\bar{\psi}\langle u^\mu u^\nu\rangle D_{\mu\nu}\psi$	6	$\bar{T}^{abc,\lambda}\langle u^\mu u^\nu\rangle D_{\mu\nu}T_\lambda^{abc}$		
		7	$\bar{T}^{abc,\lambda}u^{ag,\mu}u^{be,\nu}D_{\mu\nu}T_\lambda^{cge}$		
4	$\bar{\psi}\tilde{f}_+^{\mu\nu}\sigma_{\mu\nu}\psi$	8	$\bar{T}^{abc,\lambda}(\tilde{f}_+^{\mu\nu})^{ae}\sigma_{\mu\nu}T_\lambda^{bce}$	3	$[i\epsilon^{ab}\bar{N}^c(\tilde{f}_+^{\mu\nu})^{ae}(\gamma_\mu\gamma_5 T_\nu^{bce}-\gamma_\nu\gamma_5 T_\mu^{bce})]_+$
5	$\bar{\psi}\langle f_+^{\mu\nu}\rangle\sigma_{\mu\nu}\psi$	9	$\bar{T}^{abc,\lambda}\langle f_+^{\mu\nu}\rangle\sigma_{\mu\nu}T_\lambda^{abc}$		
6	$\bar{\psi}\tilde{\chi}_+\psi$	10	$\bar{T}^{abc,\lambda}\tilde{\chi}_+^{ae}T_\lambda^{bce}$		
7	$\bar{\psi}\langle\chi_+\rangle\psi$	11	$\bar{T}^{abc,\lambda}\langle\chi_+\rangle T_\lambda^{abc}$		

The case including $\Delta(1232)$

$\mathcal{O}(p^3)$ 阶拉氏量第 1 到第 5 组

	$\mathcal{L}_{\pi NN/\pi qq}$		$\mathcal{L}_{\pi\Delta\Delta}$		$\mathcal{L}_{\pi N\Delta}$
	$\alpha_i^{(3)}/\beta_i^{(3)}$		$\hat{e}_i^{(3)}$		$\hat{f}_i^{(3)}$
1	$\bar{\psi}\langle u^\mu u_\mu\rangle u^\nu\gamma_\nu\gamma_5\psi$	1	$\bar{T}^{abc,\lambda}\langle u^\mu u_\mu\rangle u^{ae,\nu}\gamma_\nu\gamma_5 T_\lambda^{bce}$	1	$[\epsilon^{bc}\bar{N}^d\langle u^\mu u_\mu\rangle u^{be,\nu}T_\nu^{cde}]_+$
2	$\bar{\psi}\langle u^\mu u^\nu\rangle u_\mu\gamma_\nu\gamma_5\psi$	2	$\bar{T}^{abc,\lambda}\langle u^\mu u^\nu\rangle u_\mu^{ae}\gamma_\nu\gamma_5 T_\lambda^{bce}$	2	$[\epsilon^{bc}\bar{N}^d\langle u^\mu u^\nu\rangle u_\mu^{be}T_\nu^{cde}]_+$
3	$\varepsilon_{\mu\nu\lambda\rho}\bar{\psi}\langle u^\mu u^\nu u^\lambda\rangle D^\rho\psi$	3	$\varepsilon_{\mu\nu\lambda\rho}\bar{T}^{abc,\eta}\langle u^\mu u^\nu u^\lambda\rangle D^\rho T_\eta^{abc}$		
		4	$\bar{T}^{abc,\lambda}u^{ad,\mu}u_\mu^{be}u^{cf,\nu}\gamma_\nu\gamma_5 T_\lambda^{def}$	3	$[\epsilon^{bc}\bar{N}^d(u^\mu u^\nu)^{be}u_\mu^{df}T_\nu^{cef}]_+$
		5	$\bar{T}_\rho^{abc}u^{ad,\rho}u^{be,\nu}u^{cf,\lambda}\gamma_\nu\gamma_5 T_\lambda^{def}$	4	$[i\epsilon^{bc}\bar{N}^d(u^\mu u^\nu)^{be}u^{df,\lambda}\sigma_{\mu\nu}T_\lambda^{cef}]_+$
		6	$\bar{T}_\rho^{abc}\langle u^\rho u^\lambda\rangle u^{ae,\nu}\gamma_\nu\gamma_5 T_\lambda^{bce}$	5	$[i\epsilon^{bc}\bar{N}^d\langle u^\mu u^\lambda\rangle u^{be,\nu}\sigma_{\mu\nu}T_\lambda^{cde}]_+$
		7	$[\bar{T}_\rho^{abc}\langle u^\rho u^\nu\rangle u^{ae,\lambda}\gamma_\nu\gamma_5 T_\lambda^{bce}]_+$		
		8	$\bar{T}_\rho^{abc}[u^\rho, u^\lambda]^{ad}u^{be,\nu}\gamma_\nu\gamma_5 T_\lambda^{cde}$		
4	$\bar{\psi}\langle u^\mu u^\nu\rangle u^\lambda\gamma_\mu\gamma_5 D_{\nu\lambda}\psi$	9	$\bar{T}^{abc,\rho}\langle u^\mu u^\nu\rangle u^{ae,\lambda}\gamma_\mu\gamma_5 D_{\nu\lambda}T_\rho^{bce}$	6	$[\epsilon^{bc}\bar{N}^d\langle u^\mu u^\nu\rangle u^{be,\lambda}D_{\nu\lambda}T_\mu^{cde}]_+$
5	$\bar{\psi}\langle u^\mu u^\nu\rangle u^\lambda\gamma_\lambda\gamma_5 D_{\mu\nu}\psi$	10	$\bar{T}^{abc,\rho}\langle u^\mu u^\nu\rangle u^{ae,\lambda}\gamma_\lambda\gamma_5 D_{\mu\nu}T_\rho^{bce}$	7	$[\epsilon^{bc}\bar{N}^d\langle u^\mu u^\nu\rangle u^{be,\lambda}D_{\mu\nu}T_\lambda^{cde}]_+$
		11	$\bar{T}^{abc,\rho}u^{ad,\mu}u^{be,\nu}u^{cf,\lambda}\gamma_\mu\gamma_5 D_{\nu\lambda}T_\rho^{def}$	8	$[\epsilon^{bc}\bar{N}^d(u^\mu u^\nu)^{be}u^{df,\lambda}D_{\mu\lambda}T_\nu^{cef}]_+$
6	$[\bar{\psi}u_\mu h^{\mu\nu}D_\nu\psi]_+$	12	$[\bar{T}^{abc,\rho}(u_\mu h^{\mu\nu})^{ae}D_\nu T_\rho^{bce}]_+$		
7	$[\bar{\psi}u^\mu h^{\nu\lambda}D_{\mu\nu\lambda}\psi]_+$	13	$[\bar{T}^{abc,\rho}(u^\mu h^{\nu\lambda})^{ae}D_{\mu\nu\lambda}T_\rho^{bce}]_+$		
8	$i\bar{\psi}\langle u^\mu h^{\nu\lambda}\rangle\sigma_{\mu\nu}D_\lambda\psi$	14	$i\bar{T}^{abc,\rho}\langle u^\mu h^{\nu\lambda}\rangle\sigma_{\mu\nu}D_\lambda T_\rho^{abc}$		
				9	$[\epsilon^{bc}\bar{N}^d(u^\mu h^{\nu\lambda})^{be}D_\lambda(\gamma_\mu\gamma_5 T_\nu^{cde}+\gamma_\nu\gamma_5 T_\mu^{cde})]_+$
				10	$[\epsilon^{bc}\bar{N}^d u^{be,\mu}h^{df,\nu\lambda}\gamma_\mu\gamma_5 D_\lambda T_\nu^{cef}]_+$
9	$i\bar{\psi}[\tilde{f}_+^{\mu\nu}, u_\mu]\gamma_\nu\gamma_5\psi$	15	$i\bar{T}^{abc,\rho}[\tilde{f}_+^{\mu\nu}, u_\mu]^{ae}\gamma_\nu\gamma_5 T_\rho^{bce}$	11	$[i\epsilon^{bc}\bar{N}^d[\tilde{f}_+^{\mu\nu}, u_\mu]^{be}T_\nu^{cde}]_+$
10	$i\varepsilon_{\mu\nu\lambda\rho}\bar{\psi}\langle f_+^{\mu\nu}\rangle u^\lambda D^\rho\psi$	16	$i\varepsilon_{\mu\nu\lambda\rho}\bar{T}^{abc,\eta}\langle f_+^{\mu\nu}\rangle u^{ae,\lambda}D^\rho T_\eta^{bce}$		
11	$i\varepsilon_{\mu\nu\lambda\rho}\bar{\psi}\langle\tilde{f}_+^{\mu\nu}u^\lambda\rangle D^\rho\psi$	17	$i\varepsilon_{\mu\nu\lambda\rho}\bar{T}^{abc,\eta}\langle\tilde{f}_+^{\mu\nu}u^\lambda\rangle D^\rho T_\eta^{abc}$		
		18	$i\bar{T}_\mu^{abc}\langle\tilde{f}_+^{\mu\nu}u^\lambda\rangle\gamma_\lambda\gamma_5 T_\nu^{abc}$	12	$[i\epsilon^{bc}\bar{N}^d\tilde{f}_+^{be,\mu\nu}u_\mu^{df}T_\nu^{cef}]_+$
		19	$[i\bar{T}_\lambda^{abc}[\tilde{f}_+^{\mu\nu}, u^\lambda]^{ae}\gamma_\mu\gamma_5 T_\nu^{bce}]_+$	13	$f_{27}^{(3)}:[i\epsilon^{bc}\bar{N}^d\langle f_+^{\mu\nu}\rangle u_\mu^{be}T_\nu^{cde}]_+$
		20	$i\varepsilon_{\mu\nu\lambda\rho}\bar{T}^{abc,\eta}\tilde{f}_+^{ad,\mu\nu}u^{be,\lambda}D^\rho T_\eta^{cde}$	14	$[\epsilon^{bc}\bar{N}^d\langle f_+^{\mu\nu}\rangle u^{be,\lambda}\sigma_{\mu\nu}T_\lambda^{cde}]_+$
		21	$i\bar{T}_\mu^{abc}\tilde{f}_+^{ad,\mu\nu}u^{be,\lambda}\gamma_\lambda\gamma_5 T_\nu^{cde}$	15	$[\epsilon^{bc}\bar{N}^d\tilde{f}_+^{be,\mu\nu}u^{df,\lambda}\sigma_{\mu\nu}T_\lambda^{cef}]_+$
		22	$i\bar{T}_\mu^{abc}\langle f_+^{\mu\nu}\rangle u^{ae,\lambda}\gamma_\lambda\gamma_5 T_\nu^{bce}$	16	$[\epsilon^{bc}\bar{N}^d[\tilde{f}_+^{\mu\nu}, u^\lambda]^{be}\sigma_{\mu\nu}T_\lambda^{cde}]_+$
12	$i\bar{\psi}\nabla_\mu\tilde{f}_+^{\mu\nu}D_\nu\psi$	23	$i\bar{T}^{abc,\rho}(\nabla_\mu\tilde{f}_+^{\mu\nu})^{ae}D_\nu T_\rho^{bce}$		
13	$i\bar{\psi}\langle\nabla_\mu f_+^{\mu\nu}\rangle D_\nu\psi$	24	$i\bar{T}^{abc,\rho}\langle\nabla_\mu f_+^{\mu\nu}\rangle D_\nu T_\rho^{abc}$		

The case including $\Delta(1232)$

$\mathcal{O}(p^3)$ 阶拉氏量第 6 到第 11 组

	$\mathcal{L}_{\pi NN/\pi qq}$ $\alpha_i^{(3)}/\beta_i^{(3)}$		$\mathcal{L}_{\pi\Delta\Delta}$ $\hat{e}_i^{(3)}$		$\mathcal{L}_{\pi N\Delta}$ $\hat{f}_i^{(3)}$
14	$[i\bar{\psi}\tilde{f}_+^{\mu\nu}u^\lambda\gamma_\mu\gamma_5 D_{\nu\lambda}\psi]_+$	25	$[i\bar{T}^{abc,\rho}(\tilde{f}_+^{\mu\nu}u^\lambda)^{ae}\gamma_\mu\gamma_5 D_{\nu\lambda}T_\rho^{bce}]_+$	17	$[i\epsilon^{bc}\bar{N}^d[\tilde{f}_+^{\mu\nu},u^\lambda]^{be}D_{\nu\lambda}T_\mu^{cde}]_+$
				18	$[i\epsilon^{bc}\bar{N}^d\tilde{f}_+^{be,\mu\nu}u^{df,\lambda}D_{\nu\lambda}T_\mu^{cef}]_+$
				19	$[i\epsilon^{bc}\bar{N}^d\langle f_+^{\mu\nu}\rangle u^{be,\lambda}D_{\nu\lambda}T_\mu^{cde}]_+$
15	$[\bar{\psi}u_\mu f_-^{\mu\nu}D_\nu\psi]_+$	26	$[\bar{T}^{abc,\rho}(u_\mu f_-^{\mu\nu})^{ae}D_\nu T_\rho^{bce}]_+$		
16	$i\bar{\psi}\langle u^\mu f_-^{\nu\lambda}\rangle\sigma_{\mu\nu}D_\lambda\psi$	27	$i\bar{T}^{abc,\rho}\langle u^\mu f_-^{\nu\lambda}\rangle\sigma_{\mu\nu}D_\lambda T_\rho^{abc}$		
17	$i\bar{\psi}\langle u^\mu f_-^{\nu\lambda}\rangle\sigma_{\nu\lambda}D_\mu\psi$	28	$i\bar{T}^{abc,\rho}\langle u^\mu f_-^{\nu\lambda}\rangle\sigma_{\nu\lambda}D_\mu T_\rho^{abc}$		
		29	$[\bar{T}_\mu^{abc}[u^\mu,f_-^{\nu\lambda}]^{ae}D_\lambda T_\nu^{bce}]_+$	20	$[\epsilon^{bc}\bar{N}^d(u^\mu f_-^{\nu\lambda})^{be}D_\lambda(\gamma_\mu\gamma_5 T_\nu^{cde}-\gamma_\nu\gamma_5 T_\mu^{cde})]_+$
		30	$i\bar{T}^{abc,\rho}u^{ad,\mu}f_-^{be,\nu\lambda}\sigma_{\mu\nu}D_\lambda T_\rho^{cde}$	21	$[\epsilon^{bc}\bar{N}^d(u^\mu f_-^{\nu\lambda})^{be}D_\mu(\gamma_\nu\gamma_5 T_\lambda^{cde}-\gamma_\lambda\gamma_5 T_\nu^{cde})]_+$
		31	$i\bar{T}^{abc,\rho}u^{ad,\mu}f_-^{be,\nu\lambda}\sigma_{\nu\lambda}D_\mu T_\rho^{cde}$	22	$[\epsilon^{bc}\bar{N}^d(u^\mu f_-^{\nu\lambda})^{be}D_\lambda(\gamma_\mu\gamma_5 T_\nu^{cde}+\gamma_\nu\gamma_5 T_\mu^{cde})]_+$
				23	$[\epsilon^{bc}\bar{N}^d u^{be,\mu}f_-^{df,\nu\lambda}D_\lambda(\gamma_\mu\gamma_5 T_\nu^{cef}-\gamma_\nu\gamma_5 T_\mu^{cef})]_+$
				24	$[\epsilon^{bc}\bar{N}^d u^{be,\mu}f_-^{df,\nu\lambda}D_\mu(\gamma_\nu\gamma_5 T_\lambda^{cef}-\gamma_\lambda\gamma_5 T_\nu^{cef})]_+$
				25	$[\epsilon^{bc}\bar{N}^d u^{be,\mu}f_-^{df,\nu\lambda}D_\lambda(\gamma_\mu\gamma_5 T_\nu^{cef}+\gamma_\nu\gamma_5 T_\mu^{cef})]_+$
				26	$[\varepsilon_{\mu\nu\lambda\rho}\epsilon^{bc}\bar{N}^d(u^\mu f_-^{\nu\lambda})^{be}T^{cde,\rho}]_+$
				27	$[\varepsilon_{\mu\nu\lambda\rho}\epsilon^{bc}\bar{N}^d u^{be,\mu}f_-^{df,\nu\lambda}T^{cef,\rho}]_+$
18	$\bar{\psi}\nabla_\mu f_-^{\mu\nu}\gamma_\nu\gamma_5\psi$	32	$\bar{T}^{abc,\rho}(\nabla_\mu f_-^{\mu\nu})^{ae}\gamma_\nu\gamma_5 T_\rho^{bce}$	28	$[\epsilon^{bc}\bar{N}^d(\nabla_\mu f_-^{\mu\nu})^{be}T_\nu^{cde}]_+$
				29	$[i\epsilon^{bc}\bar{N}^d(\nabla^\mu f_-^{\nu\lambda})^{be}\sigma_{\mu\nu}T_\lambda^{cde}]_+$
19	$\bar{\psi}\langle u^\mu\tilde{\chi}_+\rangle\gamma_\mu\gamma_5\psi$	33	$\bar{T}^{abc,\rho}\langle u^\mu\tilde{\chi}_+\rangle\gamma_\mu\gamma_5 T_\rho^{abc}$		
20	$\bar{\psi}\langle\chi_+\rangle u^\mu\gamma_\mu\gamma_5\psi$	34	$\bar{T}^{abc,\rho}\langle\chi_+\rangle u^{ae,\mu}\gamma_\mu\gamma_5 T_\rho^{abc}$	30	$[\epsilon^{bc}\bar{N}^d\langle\chi_+\rangle u^{be,\mu}T_\mu^{cde}]_+$
		35	$\bar{T}^{abc,\rho}u^{ad,\mu}\tilde{\chi}_+^{be}\gamma_\mu\gamma_5 T_\rho^{cde}$	31	$[\epsilon^{bc}\bar{N}^d(u^\mu\tilde{\chi}_+)^{be}T_\mu^{cde}]_+$
				32	$[\epsilon^{bc}\bar{N}^d u^{be,\mu}\tilde{\chi}_+^{df}T_\mu^{cef}]_+$
21	$i\bar{\psi}\tilde{\chi}_-^\mu\gamma_\mu\gamma_5\psi$	36	$i\bar{T}^{abc\rho}(\tilde{\chi}_-^\mu)^{ae}\gamma_\mu\gamma_5 T_\rho^{bce}$	33	$[i\epsilon^{bc}\bar{N}^d(\tilde{\chi}_-^\mu)^{be}T_\mu^{cde}]_+$
22	$i\bar{\psi}\langle\chi_-^\mu\rangle\gamma_\mu\gamma_5\psi$	37	$i\bar{T}^{abc,\rho}\langle\chi_-^\mu\rangle\gamma_\mu\gamma_5 T_\rho^{abc}$		
23	$[i\bar{\psi}u^\mu\tilde{\chi}_- D_\mu\psi]_+$	38	$[i\bar{T}^{abc,\rho}(u^\mu\tilde{\chi}_-)^{ae}D_\mu T_\rho^{bce}]_+$		

The case including $\Delta(1232)$

强子与夸克层次 LEC 关系

Chiral order	Group	$\mathcal{L}_{\pi\Delta\Delta} \Leftrightarrow \mathcal{L}_{\pi qq}$	$\mathcal{L}_{\pi N\Delta} \Leftrightarrow \mathcal{L}_{\pi qq}$
$\mathcal{O}(p^1)$	1	$e_1^{(1)} = -\frac{3}{2}g_A^q.$	$f_1^{(1)} = k_1 g_A^q.$
$\mathcal{O}(p^2)$	1	$\hat{e}_1^{(2)} = -3\beta_1^{(2)},\ \hat{e}_{2,4}^{(2)} = 0,\ \hat{e}_3^{(2)} = -6k_5\beta_1^{(2)};$	$\hat{f}_1^{(2)} = 0.$
	2	$\hat{e}_5^{(2)} = -3\beta_2^{(2)};$	$\hat{f}_2^{(2)} = 2k_2\beta_2^{(2)}$
	3	$\hat{e}_6^{(2)} = -3\beta_3^{(2)},\ \hat{e}_7^{(2)} = 0;$	
	4	$\hat{e}_8^{(2)} = -3\beta_4^{(2)},\ \hat{e}_9^{(2)} = -3\beta_5^{(2)};$	$\hat{f}_3^{(2)} = 2k_2\beta_4^{(2)}.$
	5	$\hat{e}_{10}^{(2)} = -3\beta_6^{(2)},\ \hat{e}_{11}^{(2)} = -3\beta_7^{(2)}.$	
$\mathcal{O}(p^3)$	1	$\hat{e}_1^{(3)} = -3\beta_1^{(3)},\ \hat{e}_2^{(3)} = -3\beta_2^{(3)},\ \hat{e}_3^{(3)} = -3\beta_3^{(3)},\ \hat{e}_{4,5,8}^{(3)} = 0,$	$\hat{f}_1^{(3)} = 2k_1\beta_1^{(3)},\ \hat{f}_2^{(3)} = 2k_1\beta_2^{(3)},$
		$\hat{e}_6^{(3)} = -3k_6\beta_2^{(3)},\ \hat{e}_7^{(3)} = -3k_6(\beta_1^{(3)} + \frac{1}{2}\beta_2^{(3)});$	$\hat{f}_{3,4}^{(3)} = 0,\ \hat{f}_5^{(3)} = k_3(\beta_1^{(3)} - \beta_2^{(3)});$
	2	$\hat{e}_9^{(3)} = -3\beta_4^{(3)},\ \hat{e}_{10}^{(3)} = -3\beta_5^{(3)},\ \hat{e}_{11}^{(3)} = 0;$	$\hat{f}_6^{(3)} = 2k_1\beta_4^{(3)},\ \hat{f}_7^{(3)} = 2k_1\beta_5^{(3)},\ \hat{f}_8^{(3)} = 0;$
	3	$\hat{e}_{12}^{(3)} = -3\beta_6^{(3)};$	
	4	$\hat{e}_{13}^{(3)} = -3\beta_7^{(3)};$	
	5	$\hat{e}_{14}^{(3)} = -3\beta_8^{(3)};$	$\hat{f}_9^{(3)} = 4k_4\beta_6^{(3)},\ \hat{f}_{10}^{(3)} = 0;$
	6	$\hat{e}_{15}^{(3)} = -3\beta_9^{(3)},\ \hat{e}_{16}^{(3)} = -3\beta_{10}^{(3)},\ \hat{e}_{17}^{(3)} = -3\beta_{11}^{(3)},$	$\hat{f}_{11}^{(3)} = 2k_1\beta_9^{(3)},\ \hat{f}_{12,13,15}^{(3)} = 0,$
		$\hat{e}_{18}^{(3)} = 6k_7m_\Delta\beta_{11}^{(3)},\ \hat{e}_{19}^{(3)} = -\frac{3}{2}k_6\beta_9^{(3)},$	$\hat{f}_{14}^{(3)} = 2k_3m_\Delta\beta_{10}^{(3)},\ \hat{f}_{16}^{(3)} = -k_3\beta_9^{(3)};$
		$\hat{e}_{20,21}^{(3)} = 0,\ \hat{e}_{22}^{(3)} = 6k_7m_\Delta\beta_{10}^{(3)};$	
	7	$\hat{e}_{23}^{(3)} = -3\beta_{12}^{(3)},\ \hat{e}_{24}^{(3)} = -3\beta_{13}^{(3)};$	
	8	$\hat{e}_{25}^{(3)} = -3\beta_{14}^{(3)};$	$\hat{f}_{17}^{(3)} = 2k_1\beta_{14}^{(3)},\ \hat{f}_{18,19}^{(3)} = 0;$
	9	$\hat{e}_{26}^{(3)} = -3\beta_{15}^{(3)},\ \hat{e}_{27}^{(3)} = -3\beta_{16}^{(3)},$	$\hat{f}_{20,21,23-27}^{(3)} = 0,\ \hat{f}_{22}^{(3)} = 4k_4\beta_{15}^{(3)};$
		$\hat{e}_{28}^{(3)} = -3\beta_{17}^{(3)},\ \hat{e}_{29}^{(3)} = -3k_5\beta_{15}^{(3)},\ \hat{e}_{30,31}^{(3)} = 0;$	
	10	$\hat{e}_{32}^{(3)} = -3\beta_{18}^{(3)};$	$\hat{f}_{28}^{(3)} = 2k_1\beta_{18}^{(3)},\ \hat{f}_{29}^{(3)} = 2k_3\beta_{18}^{(3)};$
	11	$\hat{e}_{33}^{(3)} = -3\beta_{19}^{(3)},\ \hat{e}_{34}^{(3)} = -3\beta_{20}^{(3)},\ \hat{e}_{35}^{(3)} = 0;$	$\hat{f}_{30}^{(3)} = 2k_1\beta_{20}^{(3)},\ \hat{f}_{31,32}^{(3)} = 0;$
	12	$\hat{e}_{36}^{(3)} = -3\beta_{21}^{(3)},\ \hat{e}_{37}^{(3)} = -3\beta_{22}^{(3)};$	$\hat{f}_{33}^{(3)} = 2k_1\beta_{21}^{(3)}$
	13	$\hat{e}_{38}^{(3)} = -3\beta_{23}^{(3)}.$	

Summary

- With structure correspondences, obtain LEC relations up to $\mathcal{O}(\boldsymbol{p}^3)$ in baryon ChPT using χQM.

- Find several structure correspondences for the case including $\boldsymbol{\Delta}(\mathbf{1232})$ and multiplication factors ($\boldsymbol{k}_{\boldsymbol{i}}$) are introduced.

- Ongoing work: heavy quark hadron case and further studies of $\boldsymbol{k}_{\boldsymbol{i}}$.

Thanks for your attention!