The electromagnetic form factors of baryon from chiral EFT

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2023 年 10 月 30 日

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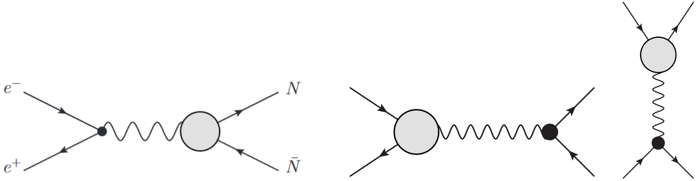
The electromagnetic form factor

The electromagnetic form factor of nucleon can be extracted by following processes



For $\gamma \bar{N} N$ vertex are related with Dirac form factors $F_{1,2}$

$$-ie\bar{u}(p)\left[\gamma^\mu F_1 + \frac{i}{2M_N}\sigma^{\mu\nu}(p'-p)_\nu F_2\right]v(p')$$

and

$$G_E = F_1 + \frac{s}{4M_N^2}F_2, \quad G_M = F_1 + F_2$$

The oscillation behavior of EMFFs

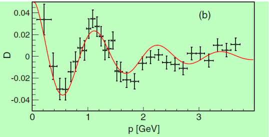
$$G^N_{\rm osc}(q^2) = |G^N| - G^N_D$$

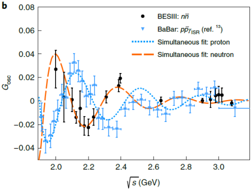
$G_D(q^2) \equiv G_D$ dipole formula

- BaBar first found the oscillation behavior of EMFFs of proton [Phys.Rev.Lett,2015,114];

- There is a phase difference between the oscillation of EMFFs of proton and neutron is found by BESIII [Nature Phys.,2021, 17];

- How to explain this oscillation behavior?





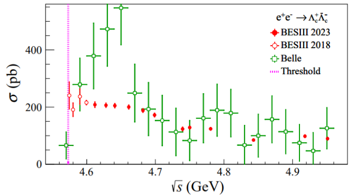
The oscillation behavior of EMFFs

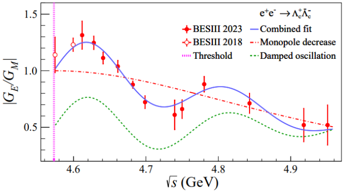
For Λ_c EMFFs

- Only threshold enhancement is exhibited for BESIII latest measurement [BESIII Collaboration, arXiv:2307.07316v1].

- The $|G_E/G_M|$ exhibit significant oscillation behavior .

- Dose the separated EMFFs of Λ_c and effective FFs $G_{\rm eff}$ also exhibit oscillation behavior similar to nucleon?





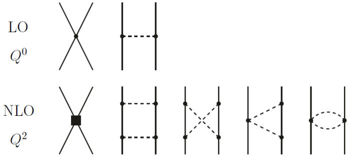
The scattering of $N\bar{N} \to N\bar{N}$

- One-photon exchange: $J = 1$
- Partial wave: $^3S_1$ and $^3D_1$, $L = 0, 2$

Lippmann-Schwinger equation

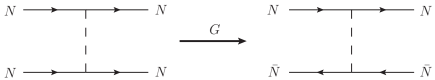
$$\begin{aligned} T_{L''L'}(p'', p'; E_k) =& V_{L''L'}(p'', p') \\ &+ \sum_L \int \frac{dpp^2}{(2\pi)^3} V_{L''L}(p'', p) \frac{1}{2E_k - 2E_p + i0^+} T_{LL'}(p, p'; E_k) \end{aligned}$$

$V_{L''L'}(p'', p')$ is $SU(3)$ interaction potential. The potential are cut off with a regulator function, $f_R(\Lambda) = \exp[-(p^6 + p'^6)/\Lambda^6]$.
Feynman diagram



The scattering potential

- The meson exchange potential (OBE and TBE): performing G-parity transformation on NN potential [Nucl.Phys.A,2013,915]



$$V_{\bar{N}N}^{\mathrm{OBE}} = (-1)^{I} V_{NN}^{\mathrm{OBE}}, \quad V_{\bar{N}N}^{\mathrm{TBE}} = (-1)^{I_1+I_2} V_{NN}^{\mathrm{TBE}}$$

- The contact term and annihilation potential for $^1S_0$ partial wave

$$\begin{aligned} V(^3S_1) =& \tilde{C}_{^3S_1} + C_{^3S_1}(p^2 + p'^2), \\ V^{\mathrm{ann}}(^3S_1) =& -i(\tilde{C}^a_{^3S_1} + C^a_{^3S_1} p^2)(\tilde{C}^a_{^3S_1} + C^a_{^3S_1} p'^2) \end{aligned}$$

DWBA method

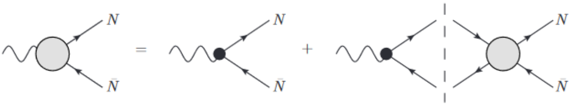
The $\bar{N}N\gamma$ vertex: the distorted wave Born approximation (DWBA) method

$$f_{L'}(k;E_k) = f^0_{L'}(k) + \sum_L \int_0^\infty \frac{dpp^2}{(2\pi)^3} f^0_L(p) \frac{1}{2E_k - 2E_p + i0^+} T_{LL'}(p,k;E_k),$$

f^0_L bare $\gamma\bar{N}N$ vertices and parameterized as

$$f^0_0(p) = G^0_{\mathrm{M}} + \frac{M_N}{2\sqrt{M_N^2+p^2}} G^0_{\mathrm{E}}, \qquad f^0_2(p) = \frac{1}{\sqrt{2}}\left(G^0_{\mathrm{M}} - \frac{M_N}{\sqrt{M_N^2+p^2}} G^0_{\mathrm{E}}\right)$$

The G^0_{M} and G^0_{E} are assumed to be energy independent, and they are taken constants and determined by fitting data.
The Feynman diagram describing DWBA method



The amplitude and cross section

The partial wave amplitude of $e^+e^- \to \bar{N}N$

$$F_{LL'}^{\bar{N}N,e^+e^-} = -\frac{4\alpha}{9} f_L^{\bar{N}N} f_{L'}^{e^+e^-}$$

$f_{L'}^{e^+e^-}$ is γe^+e^- vertex

$$f_0^{e^+e^-} = 1 + \frac{m_e}{\sqrt{s}}, \quad f_2^{e^+e^-} = \frac{1}{\sqrt{2}}\left[1 - \frac{2m_e}{\sqrt{s}}\right]$$

The cross section of $e^+e^- \to \bar{N}N$

$$\sigma_{e^+e^- \to \bar{N}N} = \frac{3\pi\beta}{s} C(s) \left(|F_{00}^{\bar{N}N,e^+e^-}|^2 + |F_{20}^{\bar{N}N,e^+e^-}|^2 + |F_{02}^{\bar{N}N,e^+e^-}|^2 + |F_{22}^{\bar{N}N,e^+e^-}|^2 \right)$$

The cross section of $\bar{N}N \to e^+e^-$

$$\sigma_{\bar{N}N \to e^+e^-} = \frac{k_e^2}{k_N^2} \sigma_{e^+e^- \to \bar{N}N}$$

The electromagnetic form factor

The relation between vertices $f_L^{\bar{N}N}$ and EMFFs G_M and G_E

$$f_0^{\bar{N}N} = G_{\mathrm{M}} + \frac{M_N}{\sqrt{s}} G_{\mathrm{E}}, \quad f_2^{\bar{N}N} = \frac{1}{2}\left(G_M - \frac{2M_N}{\sqrt{s}} G_E\right)$$

The effective form factor

$$|G_{\mathrm{eff}}(s)| = \sqrt{\frac{\sigma_{e^+e^- \to \bar{N}N}(s)}{\frac{4\pi\alpha^2\beta}{3s} C(s)[1 + \frac{2M_N^2}{s}]}}$$

The uncertainty

The uncertainty $\Delta X^{\rm NLO}(k)$ to the NLO prediction $X^{\rm NLO}(k)$ of a given observable $X(k)$

$$\Delta X^{\rm NLO}(k) = \max\left(Q^3 \times |X^{\rm LO}(k)|, Q \times |X^{\rm LO}(k) - X^{\rm NLO}(k)|\right)$$

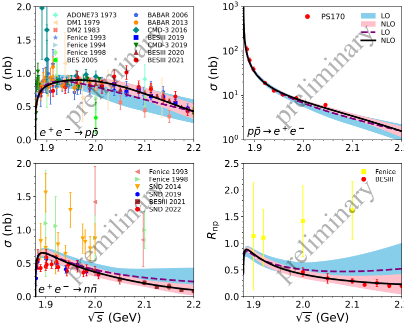
the expansion parameter Q

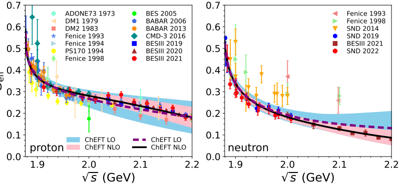
$$Q = \max\left(\frac{k}{\Lambda_b}, \frac{M_\pi}{\Lambda_b}\right)$$

k the momentum in the center of mass frame, and Λ_b the breakdown scale that depends on cutoff Λ. Here we take Λ_b=900 MeV.

Cross section and effective form factors

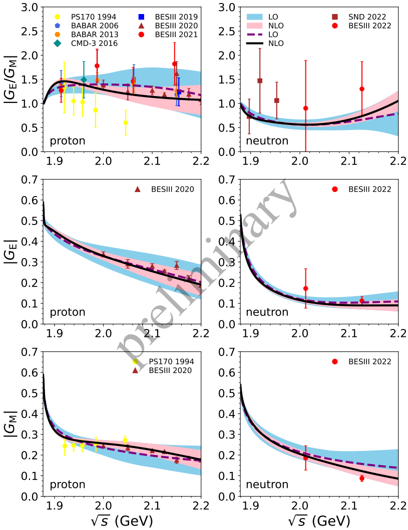
The cross section of the processes $e^+e^- \to \bar{p}p/\bar{n}n$, $\bar{p}p \to e^+e^-$, and the ratio of cross section R_{np}, and the effective form factor of proton and neutron





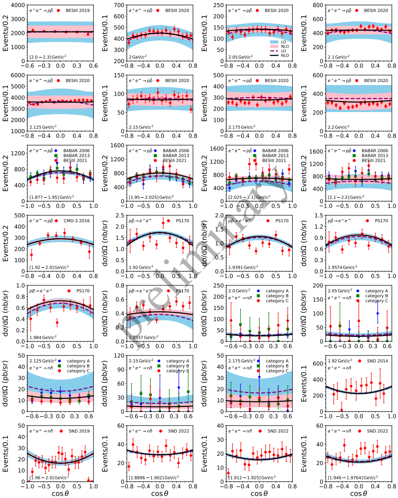
EMFFs

The results of $|G_M|$, $|G_E|$ and $|G_E/G_M|$



Differential cross section

The differential cross section of the processes $e^+e^- \to p\bar{p}/n\bar{n}$ and $p\bar{p} \to e^+e^-$



The oscillation of form factor

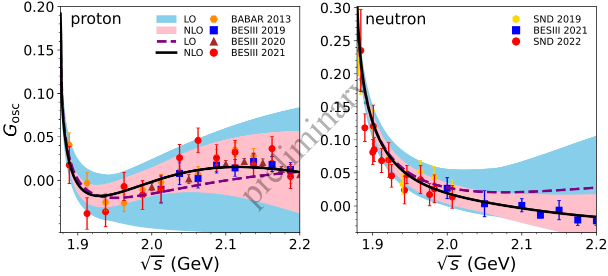
The oscillation behavior of form factor of neutron and proton

[Phy.Rev.Lett.,2015,114,232301; Nature Phys.,2021,17,1200-1204]

$$G_{\rm osc}(s) = |G_{\rm eff}| - G_D(s), \quad G_D^p(s) = \frac{\mathcal{A}_p}{(1+s/m_a^2)[1-s/q_0^2]^2}, \quad G_D^n(s) = \frac{\mathcal{A}_n}{[1-s/q_0^2]^2}$$

$\mathcal{A}_p$=7.7, $\mathcal{A}_n = 3.5 \pm 0.1$, m_a^2=14.8 $(\mathrm{GeV}/c)^2$ and q_0^2=0.71 $(\mathrm{GeV}/c)^2$.

The comparison between the G_{osc} results of χEFT and experiment



Fractional oscillation

We try to use fractional oscillation to describe the oscillation behavior of form factor of neutron and proton

$$G_{\rm osc}^{N}(\tilde{p}) = G_{{\rm osc},1}^{0,N}(0)\tilde{E}_{\alpha_1^N,1}(-\omega_1^2\tilde{p}^{\alpha_1^N}) + G_{{\rm osc},2}^{0,N}(0)\tilde{E}_{\alpha_2^N,1}(-\omega_2^2(\tilde{p}+p_0^N)^{\alpha_2^N})$$

p_0^N describes the 'phase delay' , $p_0^p = 0$ and $p_0^n \neq 0$.

The Mittag-Leffler function

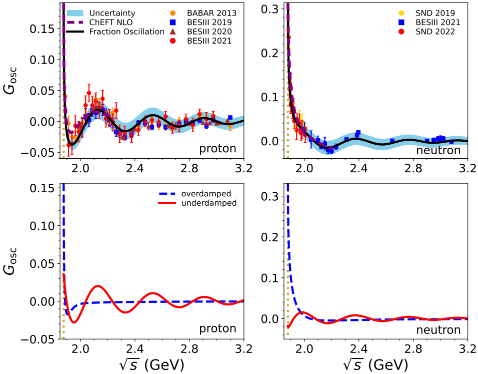
$$\tilde{E}_{\alpha,\beta}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\alpha k + \beta)}$$

The equations of motions of the fractional oscillators

$$\begin{aligned} G_{\rm osc}^{N}(\tilde{p}) =& G_{{\rm osc},1}^{N}(\tilde{p}) + G_{{\rm osc},2}^{N}(\tilde{p}), \\ G_{{\rm osc},j}^{N}(\tilde{p}) =& G_{{\rm osc},j}^{0,N} - \frac{\omega_j^2}{\Gamma(\alpha_j^N)} \int_0^{\tilde{p}+p_0^N} (\tilde{p}+p_0^N - t)^{\alpha_j^N - 1} G_{{\rm osc},j}^{N}(t)dt \end{aligned}$$

Fractional oscillation

The comparision between the fractional oscillations with data, and the oscillation behavior is different near the threshold and at high energy.



Parameters	proton	neutron
$\beta(10^{-2})$	6.020±0.034	17.453±0.023
α_1	1.263±0.002	1.060±0.001
α_2	1.880±0.001	1.880±0.001
$\omega_1(10^{-2})$	5.371±0.015	5.371±0.015
$\omega_2(10^{-3})$	7.472±0.022	7.472±0.022
p_0 (MeV)	0	1035.93±2.44

Fractional oscillation

- The diffusion solution($\alpha^N = 1, \tilde{E}_{1,1}(z) = e^z$) of a multi-particle system is caused by nonuniform distribution.
- A wave usually moves with a constant period in a uniform medium.

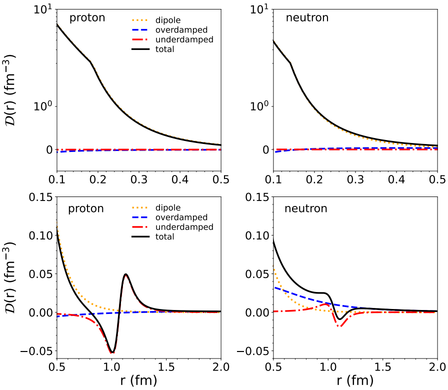
The fractional oscillator reveal the distributions of the higher order polarized electric charges for the nucleons

- The ‘quadrupole’ contribution from the underdamped oscillation, also called uniform distributions.
- the ‘octupole’ contribution from the overdamped oscillation, called nonuniform distributions.

Fractional oscillation

the Fourier transformation on the EMFFs

$$\mathcal{D}_{\rm eff}^N(r) = \frac{1}{(2\pi)^3}\int d^3\vec{r}\, G_{\rm eff}^N(\tilde{p})\ \exp(-i\vec{\tilde{p}}\cdot\vec{r})$$

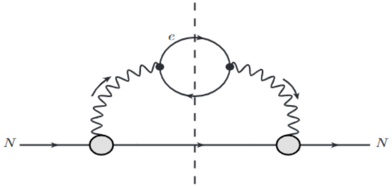


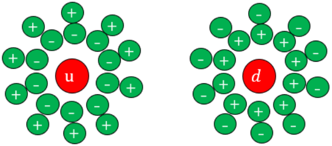
Fractional oscillation

- The Fourier transformation on time-like EMFFs represents the distributions of the polarized electric charges generated by hard photons for the nucleon.
- the distributions of polarized electric charges of the proton would climb to a positive peak and then fall to a negative trough. However, the opposite situation occurs for the neutron.

$$p_{N,in} = (m_N, 0)\,,\; p_{N,out} = (-E_N, -\vec{\tilde{p}}\,)\,,\; p_{\gamma^*,out} = (\frac{s}{2m_N}, \vec{\tilde{p}}\,)\,,$$

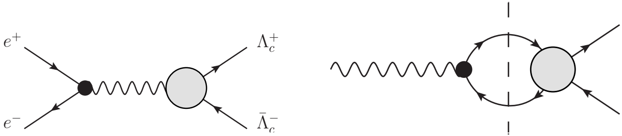
The phase difference: The difference of the vacuum polarization of u and d quark in proton (uud) and neutron (udd)





$e^+e^- \to \Lambda_c^+\bar{\Lambda}_c^-$

Feynman diagram of the process $e^+e^- \to \Lambda_c^+\bar{\Lambda}_c^-$ [L.Y.Dai,Phys.Rev.D 96,116001(2017)]



$\gamma\Lambda_c^+\bar{\Lambda}_c^-$ vertex is obtained by DWBA method

$$f_{L'}(k;E_k) = f_{L'}^0(k) + \sum_L \int_0^\infty \frac{dpp^2}{(2\pi)^3} f_L^0(p) \frac{1}{2E_k - 2E_p + i0^+} T_{LL'}(p,k;E_k),$$

$T_{LL'}$ the amplitude of the $\Lambda_c^+\bar{\Lambda}_c^-$ scattering

$\Lambda_c^+\bar{\Lambda}_c^-$ scattering

Lippmann-Schwinger equation

$$
\begin{aligned}
T_{L''L'}(p'',p';E_k) =& V_{L''L'}(p'',p') \\
&+ \sum_L \int \frac{dpp^2}{(2\pi)^3} V_{L''L}(p'',p) \frac{1}{2E_k - 2E_p + i0^+} T_{LL'}(p,p';E_k)
\end{aligned}
$$

$$
\begin{aligned}
V(^3S_1)(p',p) =& \tilde{C}_{^3S_1} + C_{^3S_1}(p'^2 + p^2) \\
& - i(\tilde{C}^a_{^3S_1} + C^a_{^3S_1} p'^2)(\tilde{C}^a_{^3S_1} + C^a_{^3S_1} p^2) \,, \\
V(^3D_1 -^3 S_1)(p',p) =& C_{\epsilon_1} p'^2 - i C^a_{\epsilon_1} p'^2 (\tilde{C}^a_{^3S_1} + C^a_{^3S_1} p^2) \,, \\
V(^3S_1 -^3 D_1)(p',p) =& C_{\epsilon_1} p^2 - i C^a_{\epsilon_1} p^2 (\tilde{C}^a_{^3S_1} + C^a_{^3S_1} p'^2) \,, \\
V(^3D_1)(p',p) =& 0 \,,
\end{aligned}
$$

- There is no contribution from one-pion exchange because $\Lambda_c^+(\bar{\Lambda}_c^-)$ has isospin $I = 0$
- Two-pion exchange contributions involving intermediate $\Sigma_c\bar{\Sigma}_c$ states arise at NLO; there is rather mass difference $M_{\Sigma_c} - M_{\Lambda_c}$ ≈167 MeV.

Cross section and EMFFs

The cross section

$$F_{LL'}^{\Lambda_c^+\bar{\Lambda}_c^-, e^+e^-} = -\frac{4\alpha}{9} f_L^{\Lambda_c^+\bar{\Lambda}_c^-} f_{L'}^{e^+e^-}$$

$$\sigma_{e^+e^-\to\Lambda_c^+\bar{\Lambda}_c^-} = \frac{3\pi\beta}{s} C(s) \Big(|F_{00}^{\Lambda_c^+\bar{\Lambda}_c^-, e^+e^-}|^2 + |F_{20}^{\Lambda_c^+\bar{\Lambda}_c^-, e^+e^-}|^2 + |F_{02}^{\Lambda_c^+\bar{\Lambda}_c^-, e^+e^-}|^2 + |F_{22}^{\Lambda_c^+\bar{\Lambda}_c^-, e^+e^-} \Big)$$

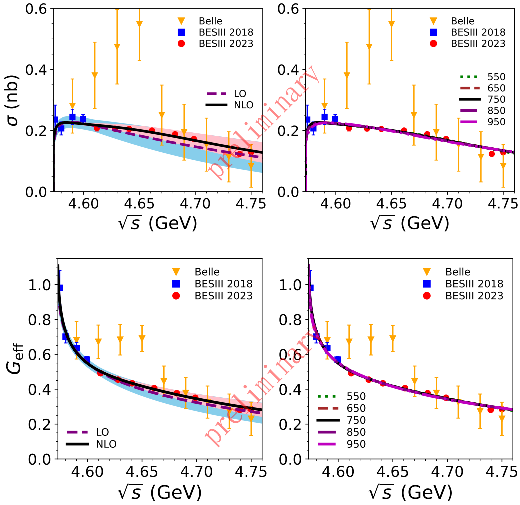
EMFFs and the effective form factor

$$f_0^{\Lambda_c^+\bar{\Lambda}_c^-} = G_{\mathrm{M}} + \frac{M_{\Lambda_c}}{\sqrt{s}} G_{\mathrm{E}}, \quad f_2^{\Lambda_c^+\bar{\Lambda}_c^-} = \frac{1}{2}\left(G_M - \frac{2M_{\Lambda_c}}{\sqrt{s}} G_E \right)$$

$$|G_{\mathrm{eff}}(s)| = \sqrt{\frac{\sigma_{e^+e^-\to\Lambda_c^+\bar{\Lambda}_c^-}(s)}{\frac{4\pi\alpha^2\beta}{3s} C(s)[1 + \frac{2M_B^2}{s}]}}$$

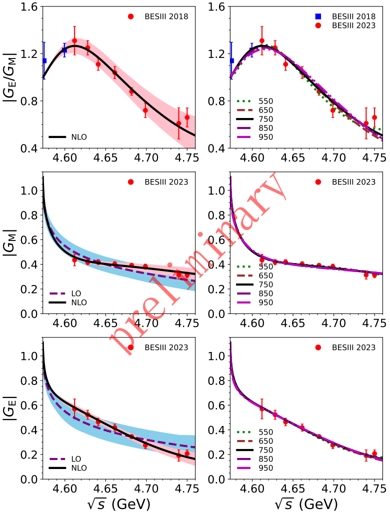
Results

The cross section and effective form factor



Results

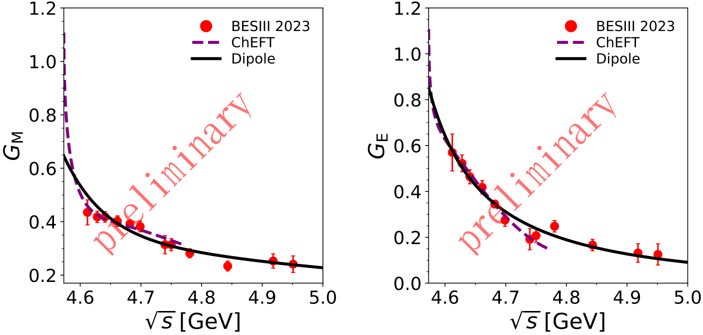
The separated EMFFs



Oscillation of the separated EMFFs

The subtracted EMFFs

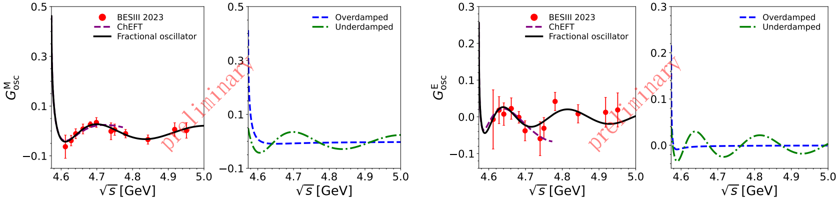
$$G_{\mathrm{osc}}^{\mathrm{M/E}}(s) = |G_{\mathrm{M/E}}| - G_D^{\mathrm{M/E}}(s)$$



Oscillation of the separated EMFFs

The fractional oscillation

$$G_{\mathrm{osc}}^{M/E}(\tilde{p}) = G_{\mathrm{osc},1}^{0,M/E}(0)\tilde{E}_{\alpha_1^{M/E},1}(-\omega_{1,M/E}^2\tilde{p}^{\alpha_1^{M/E}}) + G_{\mathrm{osc},2}^{0,M/E}(0)\tilde{E}_{\alpha_2^{M/E},1}(-\omega_{2,M/E}^2\tilde{p}^{\alpha_2^{M/E}})$$



Summary

- For the processes $e^+e^- \to N\bar{N}$, the $\bar{N}N$ scattering are considered in the FSI;

- Using Lippmann-Schwinger equation and DWBA method to constructed the amplitude of the processes $e^+e^- \to N\bar{N}$ based on $SU(3)$ ChEFT;

- Using fractional oscillation to describe the oscillation behavior of form factor of nucleon;

- The fractional oscillation and the origin of phase difference can be explained by polarized electric charges;

- The process $e^+e^- \to \Lambda_c^+\bar{\Lambda}_c^-$ are calculated through the same method as $e^+e^- \to N\bar{N}$ based on $SU(2)$ ChEFT, the separated EMFFs are extracted and they have oscillation behavior that can be descired by fractional oscillation.

Thank you!