Effective field theory with resonant P-wave interaction

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Qingfeng Li, Songlin Lyu, Chen Ji, Bingwei Long, Phys. Rev. C 108, 024002

Nuclear Effective field theories(EFTs)

Energy Resolution

~1GeV Λ_χ	QCD	Quarks Gluons
~140MeV m_π	Chiral EFT	Pions Nucleons
	Pionless	Nucleons
~45MeV Q_D	Halo/Cluster	Nucleons Nucleus
~1MeV vibration state	EFT for nuclear vibrations deformed nuclei	

Degrees of freedom

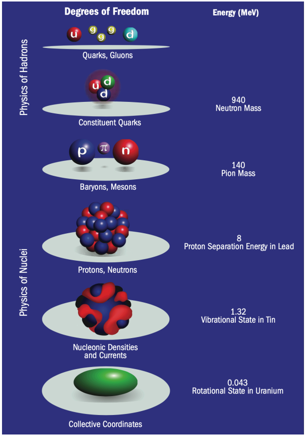


Fig.: Bertsch, Dean, Nazarewicz (2007)

Halo/Cluster nuclei

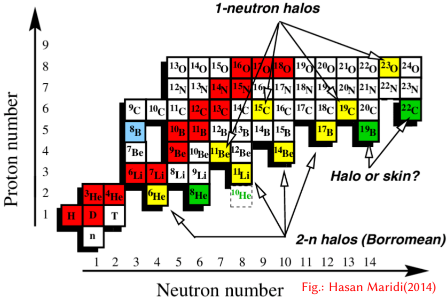
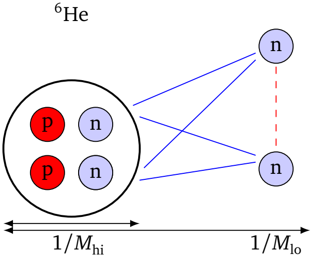


Fig.: Hasan Maridi(2014)

- Halo nuclei
  near driplines
  S-wave: $^{11}\mathrm{Li}(2n + {}^{9}\mathrm{Li})$
  P-wave: $^{6}\mathrm{He}(2n + {}^{4}\mathrm{He})$
- Cluster nuclei
  Hoyle state(3α cluster)

- Separation of scales
  M_{hi}: break clusters apart
  M_{lo}: remove halo nucleons
- Degrees of freedom
  core + valance nucleons
- Controlled expansion in $M_{\mathrm{lo}}/M_{\mathrm{hi}}$



EFT for P-wave resonance

- EFT for P-wave resonance is not easy
  amplitude with poles: nonpertubative

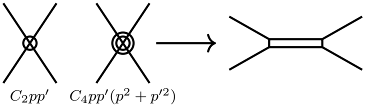
- Dimeron auxiliary field solution

  - energy dependent potential

$$\frac{pp'}{A + BE}$$

  - unphysical bound state pole
    state with negative probability

L+1 ERE parameters
at leading order



Wigner bound

Bertulani, Hammer, van Kolck NPA '02, Bedaque, Hammer, van Kolck PLB '03

- Energy-dependent formulations without auxiliary dimeron fields

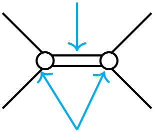
E. Epelbaum, et al., Few Body Syst. 62, 51 (2021)

Non-local P-wave EFT

- Non-local potential

$$V^{(0)}(p',p) = -\frac{2\pi}{\mu}\frac{\lambda p' p}{\sqrt{p'^2+2\mu\Delta}\sqrt{p^2+2\mu\Delta}}$$

  - static auxiliary field: momentum dependent
  - momentum-dependent form factor
  - two parameters at LO



- Not just another model
  - order-by-order convergence
  - recover effective range expansion(ERE)
  - no unphysical bound state
    crucial for many-body calculations

$$\mathcal{L}(x) = \frac{1}{2}n^\dagger\left(i\partial_0+\frac{\nabla^2}{2m_n}\right)n + \alpha^\dagger\left(i\partial_0+\frac{\nabla^2}{2m_\alpha}\right)\alpha + \frac{\mu}{\lambda}\Psi^\dagger\Psi$$
$$+\left[\Psi_a^\dagger(x)\int d^3r\,\mathcal{F}(r)\,n^T\left(x_0,\vec{x}+\frac{4}{5}\vec{r}\right)\vec{T}_a\cdot\hat{r}\,\alpha\left(x_0,\vec{x}-\frac{\vec{r}}{5}\right)+h.c.\right]$$

- Resummed-Range Effective Field Theory

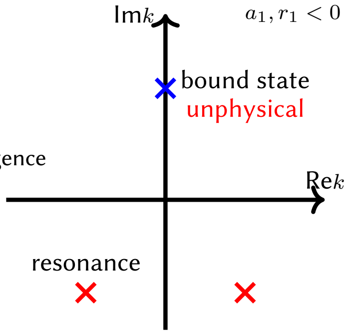
arXiv:2304.01165, V. S. Timoteo, U. van Kolck; arXiv:2308.01394, H. W. Griesshammer, U. van Kolck

Leading order

- Leading order(LO) two-body amplitude(ERE)

$$T^{(0)} = \frac{2\pi}{\mu} \frac{k^2}{-1/a_1 + r_1 k^2/2 - ik^3}$$

- Pole → bound state / resonance
- Energy-dependent form
  - unphysical bound state with negative norm
    violate unitarity
  - perturbative ik^3
    no spurious pole, smaller radius of convergence
- Momentum-dependent form
  a pole on the positive imaginary axis
  not a bound state



Bertulani, Hammer, van Kolck NPA '02; Bedaque, Hammer, van Kolck PLB '03

Higher orders

- NLO Potential

$$V_\lambda^{(1)}(p',p) = -\frac{2\pi}{\mu}\frac{p'p}{\sqrt{p'^2+2\mu\Delta^{(0)}}\sqrt{p^2+2\mu\Delta^{(0)}}} \times \left\{\lambda^{(1)} - \lambda^{(0)}\mu\Delta^{(1)}\left(\frac{1}{p'^2+2\mu\Delta^{(0)}}+\frac{1}{p^2+2\mu\Delta^{(0)}}\right)\right\}$$

$$V_{g_2}^{(1)}(p',p) = \frac{2\pi}{\mu}\,\frac{g_2}{2}\,\frac{p'p\,(p'^2+p^2)}{\sqrt{p'^2+\gamma^2}\sqrt{p^2+\gamma^2}}$$

- Higher orders as perturbations

$$T^{(1)} = (1+T^{(0)}G_0)V^{(1)}(G_0T^{(0)}+1)$$

- NLO: generalized shape parameter: $P_1 k^4$

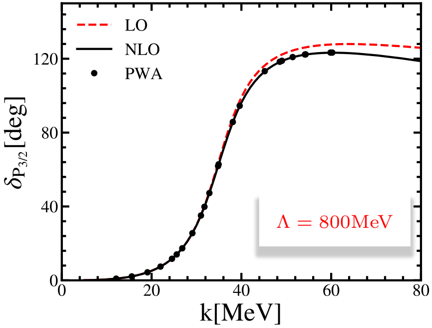
$$T^{(1)}(k) = -\frac{\mu}{2\pi}\frac{\left[T^{(0)}(k)\right]^2}{k^2}\left[\frac{\lambda_R^{(1)}}{\lambda_R^2}\gamma^2 - 2\mu\Delta^{(1)}\left(\lambda_R^{-1}+\frac{3}{2}\gamma\right) - \frac{g_{2R}}{\lambda_R}\gamma^5 + \left(\frac{\lambda_R^{(1)}}{\lambda_R^2} - \frac{g_{2R}}{\lambda_R}\gamma^2\left(\lambda_R^{-1}+\gamma\right)\right)k^2 - \frac{g_{2R}}{\lambda_R^2}k^4\right]$$

- Up to NNLO: no terms beyond ERE

- Renormalization verified analytically up to NNLO
  enough counterterms to absorb divergence

neutron-alpha scattering

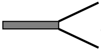
- $^5\mathrm{He}$: shallow P-wave resonances at $k_\pm = (\pm 34.5 - i6.2)$ MeV
  $n-\alpha$ scattering is dominated by the $P_{3/2}$ resonance state
- Fitting to empirical values of $n-\alpha$ elastic scattering phase shift



- Rapid order-by-order convergence
  sufficient accuracy achieved at NLO

n-n-alpha at leading order

- ^{6}He: Borromean states (none of the two-body subsystems are bound)
- Leading order
  - $n - \alpha$ P-wave interaction
  - $^1S_0$ nn interaction



$\sim iC_{\phi n}$

$$\mathcal{L}_{nn} = -C_{\phi n}\left(\phi^\dagger n_\delta S^\delta n_{-\delta} + h.c.\right) + \sigma\phi^\dagger\phi$$

  - $nn\alpha$ three-body interaction

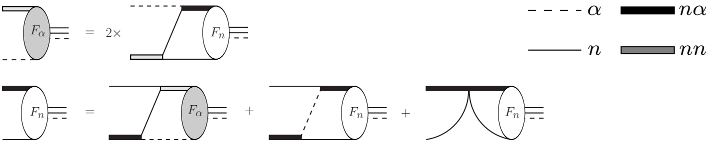
$\sim ih$

$$\mathcal{L}_{nn\alpha} = -h\left(G_0^{ab}T_a^{is}\Psi_b\overleftrightarrow{\partial_i}n_s\right)^\dagger\left(G_0^{cd}T_c^{jr}\Psi_d\overleftrightarrow{\partial_j}n_r\right)$$

- Three-body force at LO

  to eliminate the cutoff dependence

Faddeev equation



Building elements

- kernel functions

  propagators of the exchanged particles multiplied by vertices

  $X_{n\alpha}$: exchange neutron X_{nn}: exchange α

- dressed nn and $n\alpha$ propagators

Faddeev components:
F_α: α as spectator
F_n: neutron as spectator

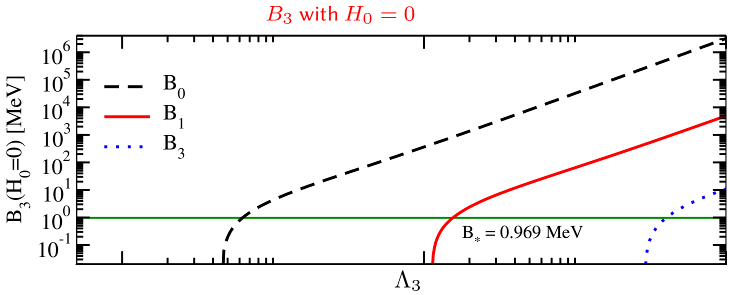
$$F_\alpha(q) = 8\pi \int_0^{\Lambda_3} q'^2 dq'\, X_{n\alpha}(q', q; B_3)\, D_{n\alpha}(\kappa_1)\, F_n(q'),$$

$$F_n(q) = 4\pi \int_0^{\Lambda_3} q'^2 dq'\, X_{n\alpha}(q, q'; B_3)\, D_{nn}(\kappa_0)\, F_\alpha(q')$$

$$+ 4\pi \int_0^{\Lambda_3} q'^2 dq' \left[X_{nn}(q, q'; B_3) + \frac{qq'}{\Lambda_3^4} H_0(\Lambda_3) \right] D_{n\alpha}(\kappa_1)\, F_n(q')$$

C. Ji, C. Elster, and D. R. Phillips, Phys. Rev. C 90, 044004 (2014)

n-n-alpha at leading order



- $H_0 = 0$: three-body system unbounded from below
- Three-body force at LO
  short-range force to prevent three-body system from collapsing
- H_0: fit to ^{6}He binding energy B_*
- Ready to many-body calculations
  ^{6}He structure properties: charge/matter radius, S_{2n}
  more valance neutrons

Summary and Outlook

- A momentum-dependent non-local potential to develop an EFT for shallow P-wave resonances
- $n - \alpha$ scattering and ^{6}He studied
- EFT and many-body methods
  - More valance neutrons
    Systematic calculation of neutron-rich Helium isotopes: $^{6-10}$He
    Halo EFT and shell model
    ^{8}He: related to four-neutron state experiment

M. Duer, et al., Nature 606 (2022) 678

- P-wave halo with different core: ^{11}Be($n - {}^{10}$Be), ^{8}Li($n - {}^{7}$Li)

Thanks ...

... to my collaborators:

李青峰, 计晨 (华中师范大学), 龙炳蔚

... and for your attention!

Backups

Power counting

- Non-local formula

$$\gamma \sim M_{\rm hi} \quad \lambda_R^{-1} \sim M_{\rm hi} \quad \eta = \gamma + \lambda_R^{-1} \sim M_{\rm lo}^2/M_{\rm hi}$$

$$1/a_1 = -\gamma^2\eta \sim M_{\rm lo}^2 M_{\rm hi} \quad r_1/2 = \lambda^{-1} \sim M_{\rm hi}$$

  one parameter fine-tuned

- ik^3 resummation

$$1/a_1 \sim M_{\rm lo}^3 \quad r_1 \sim M_{\rm lo}$$

  two-parameters fine-tuned

- Perturbative ik^3

$$1/a_1 \sim M_{\rm lo}^2 M_{\rm hi} \quad r_1 \sim M_{\rm hi}$$

  one parameter fine-tuned

Lagrangian

- Lagrangian

$$\mathcal{L}(x) = \frac{1}{2} n^\dagger \left(i\partial_0 + \frac{\nabla^2}{2m_n} \right) n + \alpha^\dagger \left(i\partial_0 + \frac{\nabla^2}{2m_\alpha} \right) \alpha + \frac{\mu}{\lambda} \Psi^\dagger \Psi$$
$$+ \left[\Psi_a^\dagger(x) \int d^3 r \mathcal{F}(r)\, n^T \left(x_0, \vec{x} + \frac{4}{5}\vec{r} \right) \vec{T}_a \cdot \hat{r}\, \alpha \left(x_0, \vec{x} - \frac{\vec{r}}{5} \right) + h.c. \right] + \cdots ,$$

$$\mathcal{L}^{(1)}(x) = \frac{g_2}{2\lambda} \left[\Psi_a^\dagger(x) \int d^3 r\, n^T \left(x_0, \vec{x} + \frac{4\vec{r}}{5} \right) \vec{T}_a \cdot \hat{r} \frac{d^2 \mathcal{F}(r)}{dr^2} \alpha \left(x_0, \vec{x} - \frac{\vec{r}}{5} \right) + h.c. \right] ,$$

$$\mathcal{F}(r) = \frac{d}{dr} \int \frac{d^3 p}{(2\pi)^3} \frac{e^{-i\vec{p}\cdot\vec{r}}}{\sqrt{\vec{p}^{\,2} + 2\mu\Delta}} .$$

- LS equation

$$T_l(p', p; k) = V_l(p', p) + \frac{\mu}{\pi^2} \int^\Lambda \mathrm{d}q\, q^2\, V_l(p', q) \frac{T_l(q, p; k)}{k^2 - q^2 + i0}$$

Poles

Type	η	Pole position $(k_\pm)$
bound / virtual	$\eta < 0$	$\frac{i}{2}\left[\lvert\eta\rvert \pm \sqrt{\lvert\eta\rvert(4\gamma + \lvert\eta\rvert)}\right]$
resonance	$0 < \eta$	$\frac{1}{2}\left[\pm\sqrt{\eta\left(3\gamma + \lvert\lambda_R^{-1}\rvert\right)} - i\eta\right]$
	$0 < \eta < 4\gamma$	$\frac{1}{2}\left(\pm\sqrt{4\eta\gamma - \eta^2} - i\eta\right)$
virtual	$4\gamma < \eta$	$-\frac{i}{2}\left(\eta \pm \sqrt{\eta^2 - 4\eta\gamma}\right)$

Table: Categories of pole positions of $\tau(k)$ according to the value of η.

$$T^{(0)}(p', p; k) = -\frac{2\pi}{\mu} \frac{p'}{\sqrt{p'^2 + \gamma^2}} \frac{ik - \gamma}{k^2 + i\eta k - \eta\gamma} \frac{p}{\sqrt{p^2 + \gamma^2}} \tag{1}$$

$$T(p', p; k) \to \frac{p'}{\sqrt{p'^2 + \gamma^2}} \frac{R(B)}{k^2/2\mu + B} \frac{p}{\sqrt{p^2 + \gamma^2}} + \text{finite terms} \tag{2}$$