Exploring the string axiverse and parity violation in gravity with gravitational waves

DAISKE YOSHIDA

Physics Department, Kobe University,
*Kobe, 657-8501, Japan**

JIRO SODA

Physics Department, Kobe University,
Kobe, 657-8501, Japan†

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Model

The model we consider is the dynamical Chern-Simons gravity coupled with the axion.

$$S = S_{\rm EH} + S_{\rm CS} + S_{\Phi} \ ,$$

Einstein-Hilbert action: $$S_{\rm EH} \equiv \kappa \int_{\mathcal{V}} dx^4 \sqrt{-g} R.$$

The Chern-Simons term: $$S_{\rm CS} = \frac{1}{4}\alpha \int_{\mathcal{V}} dx^4 \sqrt{-g}\, \Phi \tilde{R} R$$ $$\tilde{R}R \equiv \frac{1}{2}\epsilon^{\rho\sigma\alpha\beta} R^{\mu\nu}{}_{\alpha\beta} R_{\nu\mu\rho\sigma}$$

it is convenient to express the coupling constant as $$\alpha = \sqrt{\frac{\kappa}{2}}\ell^2$$

The action of the axion field $$S_{\Phi} \equiv -\frac{1}{2}\int_{\mathcal{V}} dx^4 \sqrt{-g}\left[g^{\mu\nu}\left(\nabla_\mu \Phi\right)\left(\nabla_\nu \Phi\right) + 2V(\Phi)\right]$$ $$V(\Phi) \equiv \frac{1}{2} m^2 \Phi^2$$

equation of motion

The equation of motion for the axion field is the modified Klein-Gordon equation

$$\Box\Phi - \frac{dV(\Phi)}{d\Phi} = -\frac{\alpha}{4}\tilde{R}R$$

for homogeneous background spacetime $ds^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = -dt^2 + a^2(t)\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j$

$$\Phi'' + 2\frac{a'}{a}\Phi' + a^2m^2\Phi = 0$$

if ignore the cosmic expansion $\Phi = \Phi_0 \cos(m\eta)$

where Φ_0 is determined by the energy density of the axion field

$$\Phi_0 = \frac{\sqrt{2\rho}}{m} \cong 2.1 \times 10^7 \text{ eV} \times \left(\frac{10^{-10} \text{ eV}}{m}\right)\sqrt{\frac{\rho}{0.3 \text{ GeV/cm}^3}}$$

equation of motion

The GWs can be described by the metric

$$ds^2 \simeq -d\eta^2 + \delta_{ij}dx^i dx^j + h_{ij}dx^i dx^j$$

two linear polarization modes

$$h_{ij}(\eta, \boldsymbol{k}) = h_+(\eta, \boldsymbol{k})\, e^+_{ij}(\boldsymbol{n}) + h_\times(\eta, \boldsymbol{k})\, e^\times_{ij}(\boldsymbol{n})$$

where the polarization tensors $e^+_{ij}(\boldsymbol{n}) \equiv u_i u_j - v_i v_j \quad \text{and} \quad e^\times_{ij}(\boldsymbol{n}) \equiv u_i v_j + v_i u_j .$ u,v is two orthogonal unit vectors

the circular polarization modes are defined by

$$h_{\rm R} \equiv \frac{1}{\sqrt{2}}(h_+ - ih_\times)\ , \quad h_{\rm L} \equiv \frac{1}{\sqrt{2}}(h_+ + ih_\times)$$

the gravitational wave equations can be diagonalized as

$$h''_{\rm A} + \frac{\epsilon_{\rm A}\delta\ \cos(m\eta)}{1+\epsilon_{\rm A}\frac{k}{m}\delta\ \sin(m\eta)} k\, h'_{\rm A} + k^2 h_{\rm A} = 0 \qquad \delta \equiv \frac{\alpha}{\kappa} m^2 \Phi_0 \qquad \epsilon_{\rm A} \equiv \begin{cases} 1 & {\rm A} = {\rm R}\ , \\ -1 & {\rm A} = {\rm L}\ . \end{cases}$$

Resonant amplification of GWs

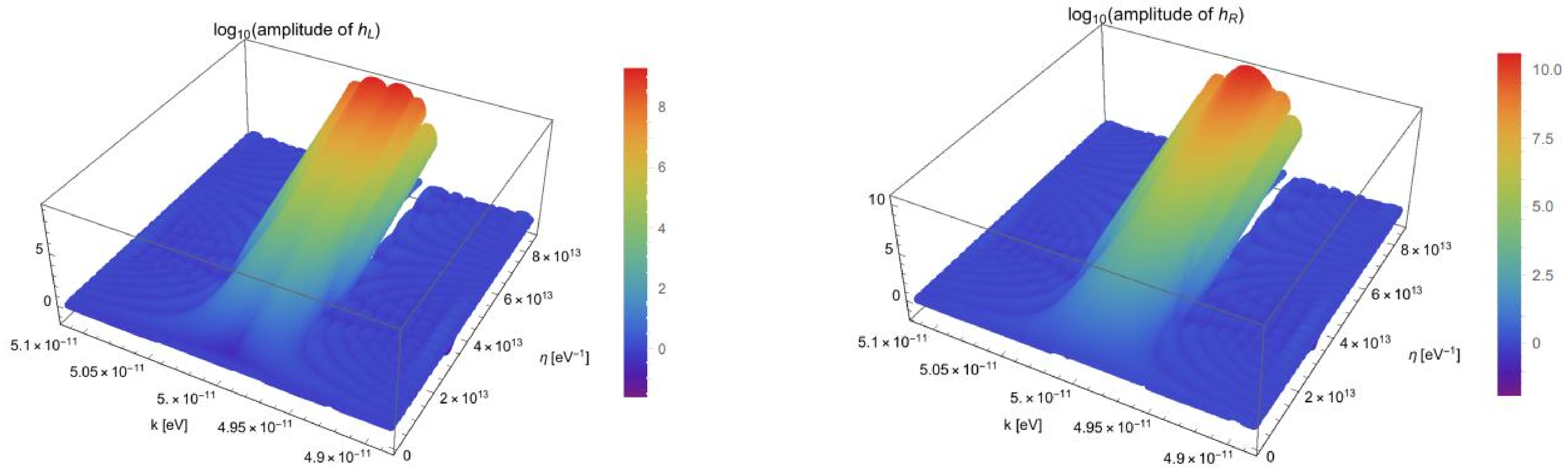


Fig. 1. The growth of the amplitude of GWs is plotted for $\ell = 10^8\,\mathrm{km}$, $m = 10^{-10}\,\mathrm{eV}$, $\rho = 0.3 \times 10^6\,\mathrm{GeV/cm^3}$.

Resonant amplification of GWs

the first resonance wave-number k_r is given by $k_r = m/2$.

$$f_{\rm r} = \frac{k_{\rm r}}{2\pi} \simeq \ 1.2 \times 10^4 \ {\rm Hz} \ \left(\frac{m}{10^{-10} \ {\rm eV}}\right)$$

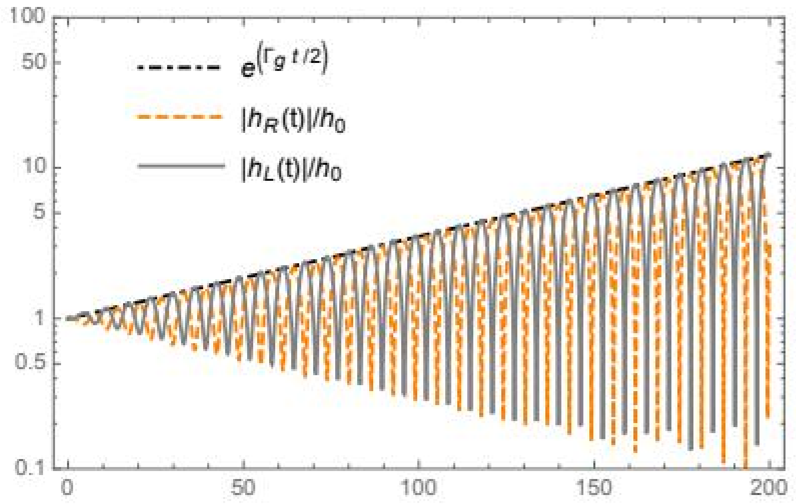
the width of the resonance as

$$\frac{m}{2} - \frac{m}{8}\delta \lesssim k_{\rm r} \lesssim \frac{m}{2} + \frac{m}{8}\delta.$$

h_A grows exponentially $\quad |h_A| \sim e^{\Gamma t}$

the growth rate

$$\begin{aligned}\Gamma &= \frac{m\delta}{8} \\ &= 2.8 \times 10^{-16}{\rm eV} \left(\frac{m}{10^{-10}{\rm eV}}\right)^2 \left(\frac{\ell}{10^8{\rm km}}\right)^2 \sqrt{\frac{\rho}{0.3{\rm GeV/cm^3}}}.\end{aligned}$$



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Resonant amplification of GWs

If the distance from the GWs source to the earth is 10 kpc, the amplitude of GWs can be significantly enhanced.

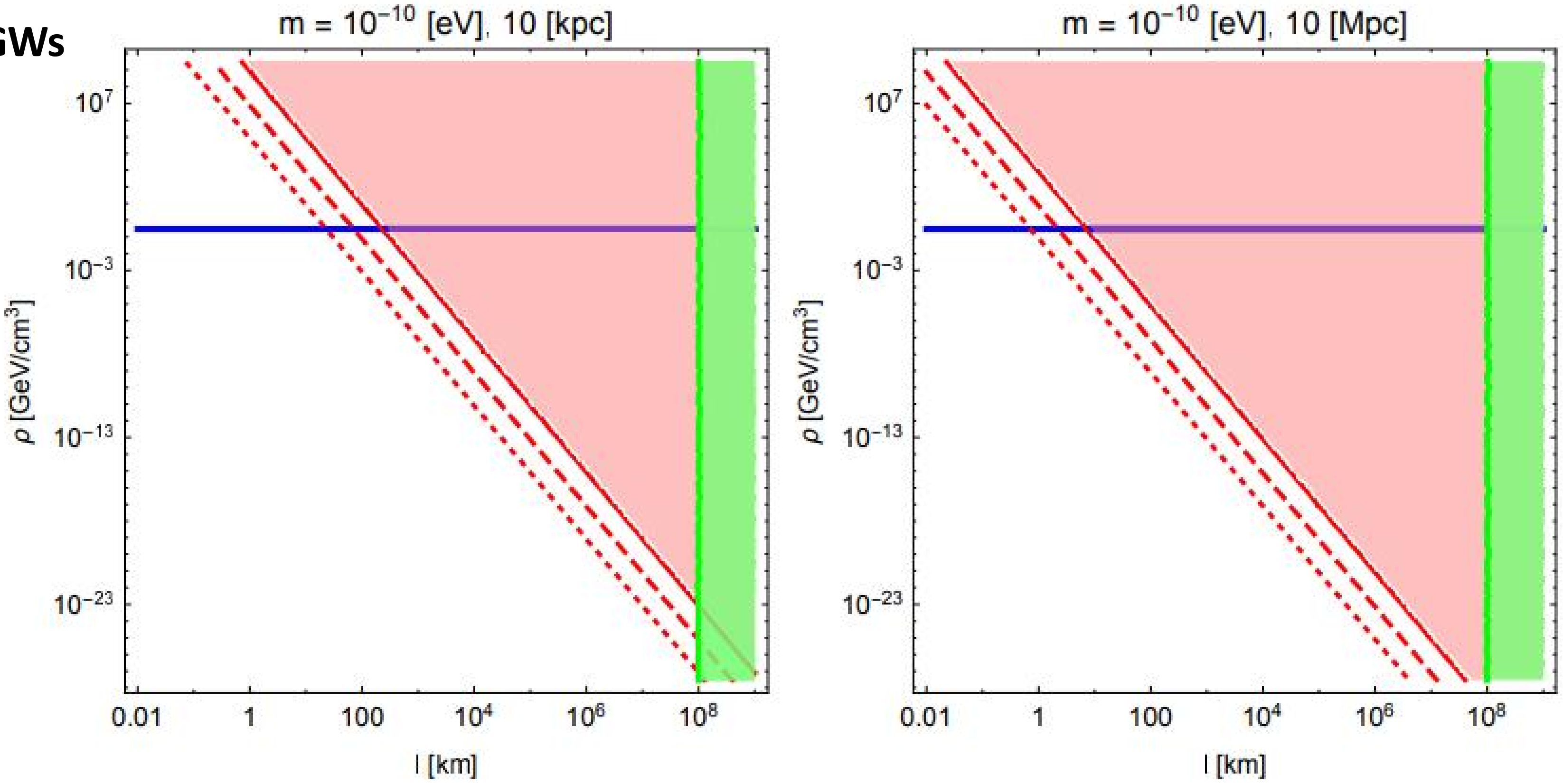

Fig. 3. The constraint on the coupling constant l and the density of the axion dark matter ρ for $m = 10^{-10}$ eV is shown. The left one shows the excluded region by the gravitaitonal wave travelling 10 kpc in the axion dark matter, and the right one shows the excluded region for 10 Mpc propagation in the axion dark matter. The blue line show the local dark matter density 0.3 GeV/cm^3. The green region is excluded by the observation.[29] The red line represents the ten times enhancement of gravitational waves. The red dashed line represents the 0.1 times enhancement. The red dotted line represents the 0.01 times enhancement. Since these sinatures have never been observed, the upper parameter regions of these lines are excluded.

Resonant amplification of GWs

the growth rate depends on the chirality

$$\mathrm{parity}(\eta) \equiv \frac{|h_{\mathrm{R}}|^2 - |h_{\mathrm{L}}|^2}{|h_{\mathrm{R}}|^2 + |h_{\mathrm{L}}|^2}.$$

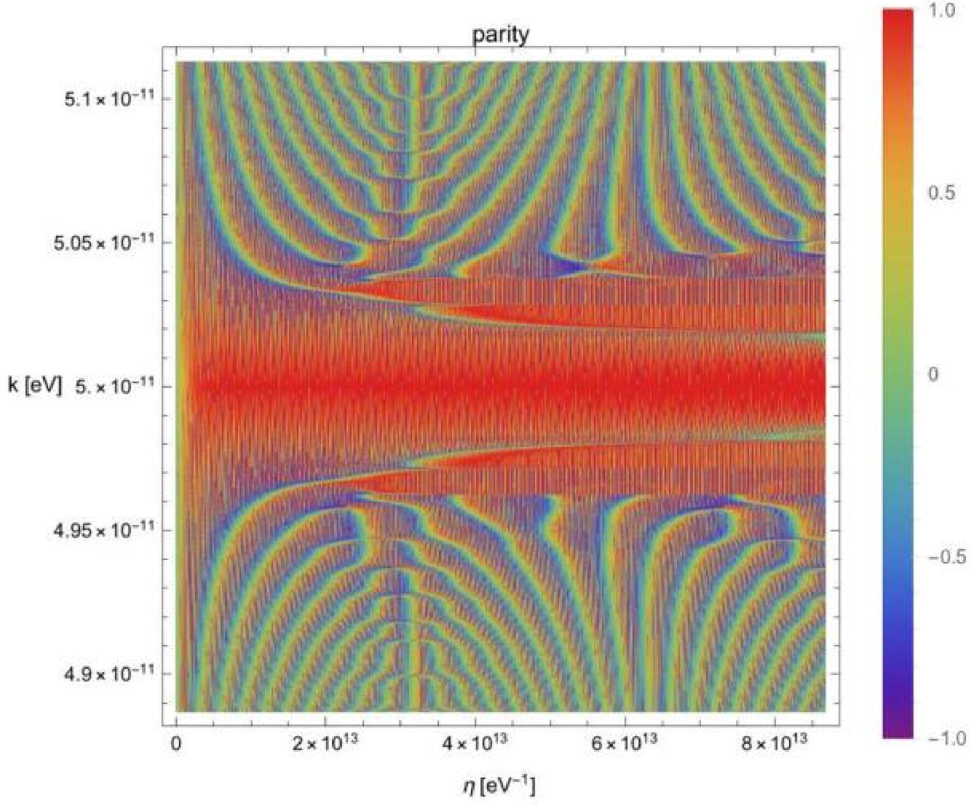

Fig. 2. The growth of the parity-violation is plotted for $\ell = 10^8\,\mathrm{km}$, $m = 10^{-10}\,\mathrm{eV}$, $\rho = 0.3 \times 10^6\,\mathrm{GeV/cm^3}$.

Conclusion

The axion coherent oscillation induces the parametric resonance of GWs
due to the Chern-Simons term resulting in the circular polarization of GWs.

The observation of GWs can strongly constrain the coupling
constant of Chern-Simons term and/or the abundance of the light axions.