Search for $\eta/\eta' \to l^+l^-\mu^+\mu^-$

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OUTLINE

Motivation

Data Sampel and MC Simulation

Event selection

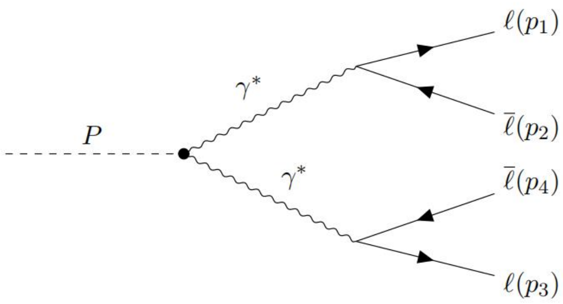
Analysis of $\eta\ /\eta' \rightarrow e^+e^-\mu^+\mu^-$、$\mu^+\mu^-\mu^+\mu^-$

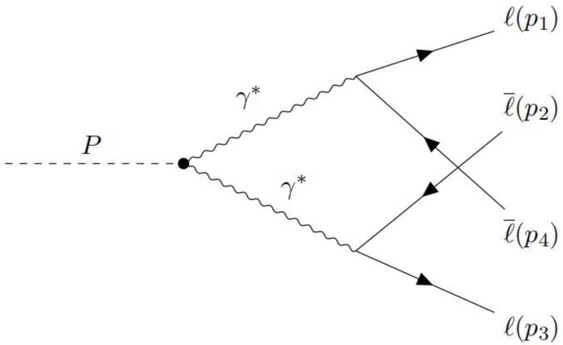
Systematic Uncertainty Study

Summary and Next

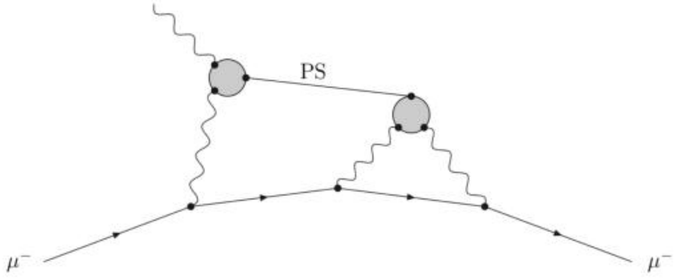
Motivation

- The double Dalitz decays $\eta/\eta' \rightarrow l^+ l^- \mu^+ \mu^- (l = e, \mu)$ proceeds through two virtual photons intermediate state with internal photon conversion to $\ell^+ \ell^-$pairs, which understand the pseudoscalar transition form factor.





- The knowledge of the η or η' coupling to virtual photons is important for the calculation of the anomalous magnetic moment of the muon, being pseudoscalar exchange the major contribution to the hadronic light-by-light scattering.



The dominant light-by-light contribution to the muon magnetic moment.

Motivation

- Theoretical predictions and previous experiment results of the branching fractions

Decay	VMD[1]	Data driven approach[2]	Experimental result[3]
$\eta \to e^+e^-\mu^+\mu^-$	2.154×10^{-6}	$(2.39 \pm 0.7) \times 10^{-6}$	$< 1.6 \times 10^{-4} (CL = 90\%)$
$\eta \to \mu^+\mu^-\mu^+\mu^-$	3.797×10^{-9}	$(3.98 \pm 0.15) \times 10^{-9}$	$(5.0 \pm 0.8) \times 10^{-9}$ [4]
$\eta \to e^+e^-e^+e^-$	2.668×10^{-5}	$(2.71 \pm 0.2) \times 10^{-5}$	$(2.40 \pm 0.22) \times 10^{-5}$ [5]
$\eta' \to e^+e^-\mu^+\mu^-$	7.968×10^{-7}	$(6.39 \pm 0.91) \times 10^{-7}$	not seen
$\eta' \to \mu^+\mu^-\mu^+\mu^-$	2.185×10^{-8}	$(1.69 \pm 0.36) \times 10^{-8}$	not seen
$\eta' \to e^+e^-e^+e^-$	2.317×10^{-6}	$(2.10 \pm 0.45) \times 10^{-6}$	$(4.5 \pm 1.0) \times 10^{-6}$ [6]

- BESIII experiment offers a unique possibility to investigate η or η' meson decays. Using $(10087 \pm 44) \times 10^6$ J/ψ events, we perform a search for $\eta/\eta' \to e^+e^-\mu^+\mu^-$、$\mu^+\mu^-\mu^+\mu^-$ via $J/\psi \to \gamma\eta/\gamma\eta'$.

[1] arXiv:1010.2378v1
[2] Chinese Physics C Vol. 42, No. 2 (2018) 023109
[3] Prog. Theor. Exp. Phys. 2020, 083C01 (2020)
[4] CMS-BPH-22-003, CERN-EP-2023-071
[5] Phys.Rev.D 105,112010(2022)
[6] Phys.Rev.D 105 (2022) 11, 112010

Data Sampel and MC Simulation

- BOSS version : 7.0.8
- Data sample : 09+12+18+19 J/ψ events
- Inclusive MC : 09+12 J/ψ events
- MC samples :

Signal

Background

Decay mode	Generator
$J/\psi \to \gamma\eta', \eta' \to e^+e^-\mu^+\mu^-$	HELAMP, DIY
$J/\psi \to \gamma\eta', \eta' \to \mu^+\mu^-\mu^+\mu^-$	HELAMP, DIY
$J/\psi \to \gamma\eta, \eta \to e^+e^-\mu^+\mu^-$	HELAMP, DIY
$J/\psi \to \gamma\eta, \eta \to \mu^+\mu^-\mu^+\mu^-$	HELAMP, DIY
$J/\psi \to \gamma\eta', \eta' \to \pi^+\pi^-e^+e^-$	HELAMP, DIY
$J/\psi \to \gamma\eta', \eta' \to \pi^+\pi^-\mu^+\mu^-$	HELAMP, DIY
$J/\psi \to \gamma\eta', \eta' \to \gamma\mu^+\mu^-$	HELAMP, DIY
$J/\psi \to \gamma\eta', \eta' \to e^+e^-$	HELAMP, DIY
$J/\psi \to \gamma\eta', \eta' \to \gamma\pi^+\pi^-$	HELAMP, DIY
$J/\psi \to \gamma\pi^+\pi^-\pi^+\pi^-$	PHSP
$J/\psi \to \gamma\eta', \eta' \to \pi^+\pi^-\eta, \eta \to \mu^+\mu^-$	HELAMP, DIY， PHSP
$J/\psi \to \gamma\eta, \eta \to \gamma\pi^+\pi^-$	HELAMP, DIY
$J/\psi \to \gamma\eta, \eta \to \pi^+\pi^-e^+e^-$	HELAMP, DIY
$J/\psi \to \gamma f_2(1270), \gamma f_2(1270) \to \pi^+\pi^-\pi^+\pi^-$	PHSP

Initial Event selection

Good charged track

- $|R_z| \le 10$ cm, $|R_{xy}| \le 1$ cm
- $|\cos\theta| \le 0.93$
- $N_{charge} = 0$, $N_{Good} = 4$
- $N_p = 2$, $N_m = 2$

Good photon

- $E_\gamma \ge 25$MeV, $|\cos\theta| \le 0.8$ (Barrel)
- $E_\gamma \ge 50$MeV, $0.86 < |\cos\theta| \le 0.92$ (Endcap)
- $0 \le$ TDC $_{\text{EMC}} \le 14$ ($\times$50ns)
- $N_\gamma \ge 1$

PID && Kinematic fit

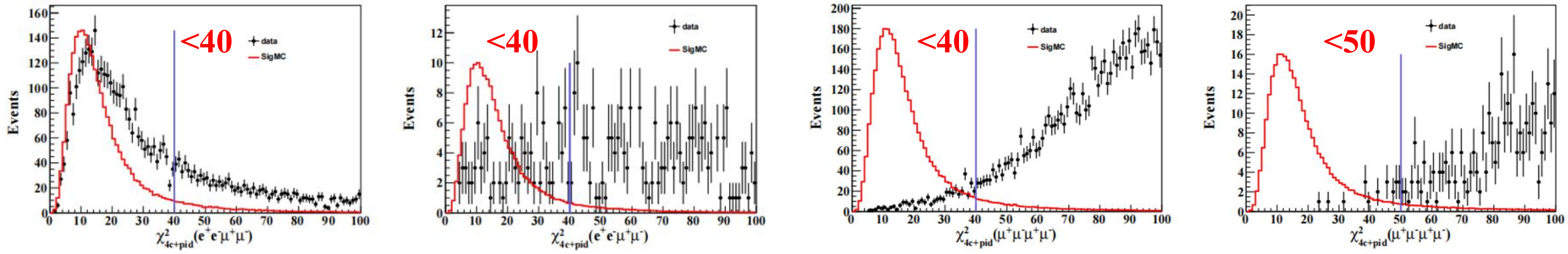
- Assuming the charged tracks to μ,the smallest $\chi^2_{PID+4C}(\gamma\mu^+\mu^-\mu^+\mu^-)$ is selected.

- Assuming the charged tracks to e, μ, π , define:

$$\chi^2_{PID+4C} = \chi^2_{4C} + \sum_{i=1}^{4} \chi^2_{PID}(i)$$

The hypothesis with the smallest $\chi^2_{PID+4C}(\gamma e^+e^-\mu^+\mu^-)$ is selected.

The distribusiton of $\chi^2_{PID+4C}(l^+l^-\mu^+\mu^-)$:

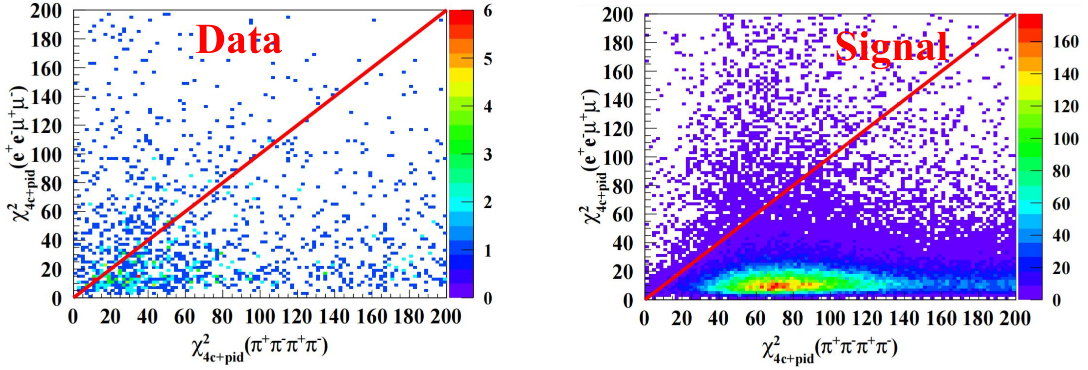


(a) $\eta' \to e^+e^-\mu^+\mu^-$ (b) $\eta \to e^+e^-\mu^+\mu^-$ (c) $\eta' \to \mu^+\mu^-\mu^+\mu^-$ (d) $\eta \to \mu^+\mu^-\mu^+\mu^-$

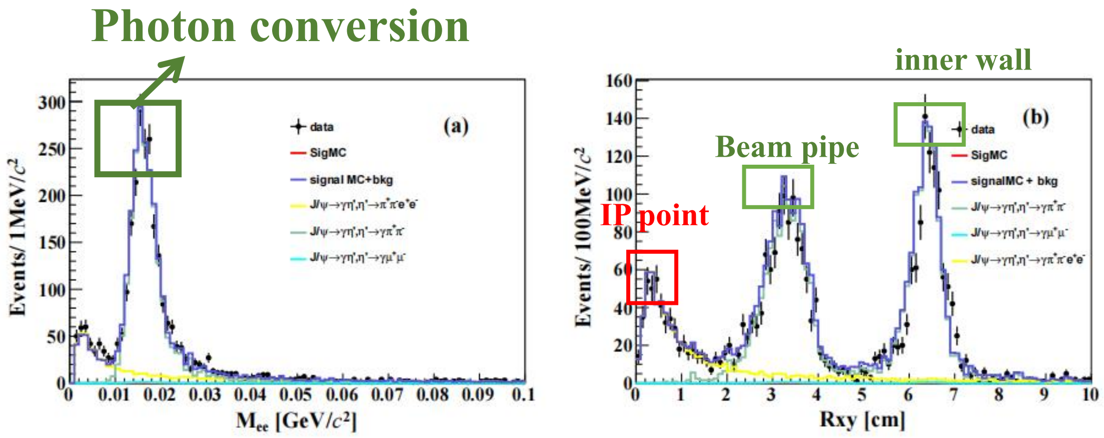
Analysis of $\eta' \to e^+e^-\mu^+\mu^-$

Futher Event Selection

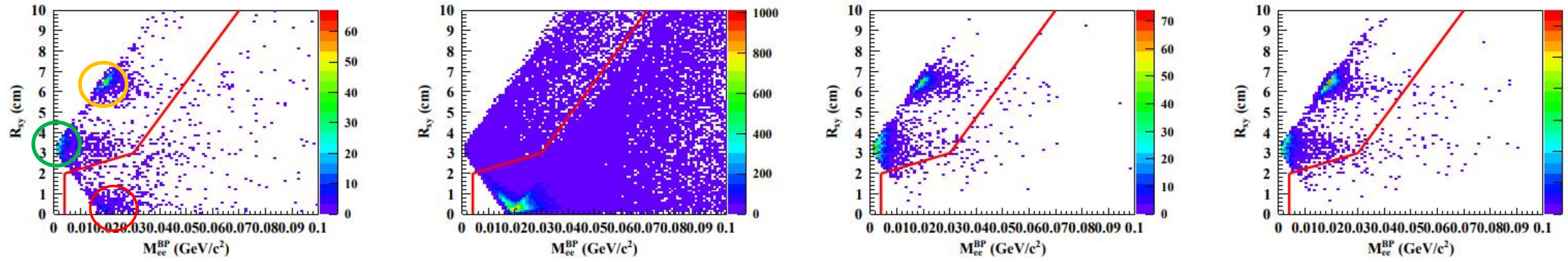
- $\chi^2_{PID+4C}(e^+e^-\mu^+\mu^-)$ versus $\chi^2_{PID+4C}(\pi^+\pi^-\pi^+\pi^-)$:



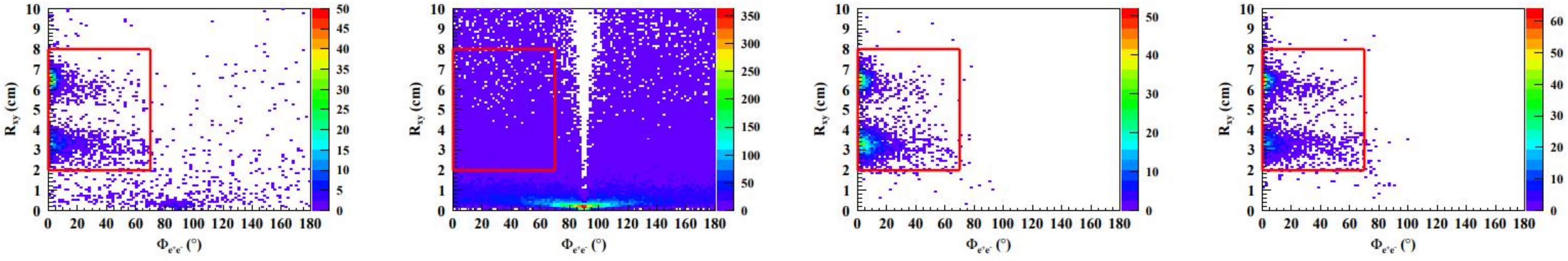
$\chi^2_{PID+4C}(e^+e^-\mu^+\mu^-) < \chi^2_{PID+4C}(\pi^+\pi^-\pi^+\pi^-)$



- Veto Photon Conversion : M_{ee}^{BP} versus R_{xy} and ϕ_{ee} versus R_{xy}



(a) Data (b) $\eta' \to e^+e^-\mu^+\mu^-$ (c) $\eta' \to \gamma\mu^+\mu^-$ (d) $\eta' \to \gamma\pi^+\pi^-$



(e) Data (f) $\eta' \to e^+e^-\mu^+\mu^-$ (g) $\eta' \to \gamma\mu^+\mu^-$ (h) $\eta' \to \gamma\pi^+\pi^-$

Cut	Range
M_{ee}^{BP} vs. R_{xy}	(0.004GeV/c2, 0cm) (0.004GeV/c2, 2cm) (0.03GeV/c2, 3cm) (0.07GeV/c^2, 10cm)
Φ_{ee} vs. R_{xy}	$2.0 < R_{xy} < 8$ && $0 < \Phi_{ee} < 70°$

Background Study

Topo

Table 1: Decay trees and their respective final states.

rowNo	decay tree	decay final state	iDcyTr	nEtr	nCEtr
1	$J/\psi \to \eta'\gamma, \eta' \to e^+e^-\pi^+\pi^-$	$e^+e^-\pi^+\pi^-\gamma$	0	481	481
2	$J/\psi \to \eta'\gamma, \eta' \to \pi^+\pi^-\gamma^F$	$\pi^+\pi^-\gamma^F\gamma$	1	43	524
3	$J/\psi \to \eta'\gamma, \eta' \to e^+e^-\pi^+\pi^-\gamma^f$	$e^+e^-\pi^+\pi^-\gamma\gamma^f$	2	12	536
4	$J/\psi \to \pi^0 h_1(1170), h_1(1170) \to \pi^0\rho^0, \rho^0 \to \pi^+\pi^-$	$\pi^0\pi^0\pi^+\pi^-$	3	1	537
5	$J/\psi \to e^+e^-\gamma^f$	$e^+e^-\gamma^f$	4	1	538
6	$J/\psi \to f_2(1270)\gamma, f_2(1270) \to \pi^+\pi^+\pi^-\pi^-$	$\pi^+\pi^+\pi^-\pi^-\gamma$	5	1	539

Backgrounds and Normalized Event Number

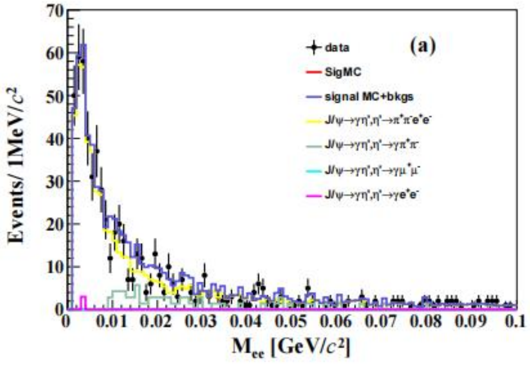
Decay mode	Normalized Event Number
$J/\psi \to \gamma\eta', \eta' \to \pi^+\pi^-e^+e^-$	3053± 132
$J/\psi \to \gamma\eta', \eta' \to \gamma\pi^+\pi^-$	296.8 ± 5.6
$J/\psi \to \gamma\eta', \eta' \to \gamma\mu^+\mu^-$	1.7 ± 0.4
$J/\psi \to \gamma\eta', \eta' \to \gamma e^+e^-$	0.83± 0.05
$J/\psi \to \gamma\pi^+\pi^-\pi^+\pi^-$	Free

Cut flow and Detection Efficiency

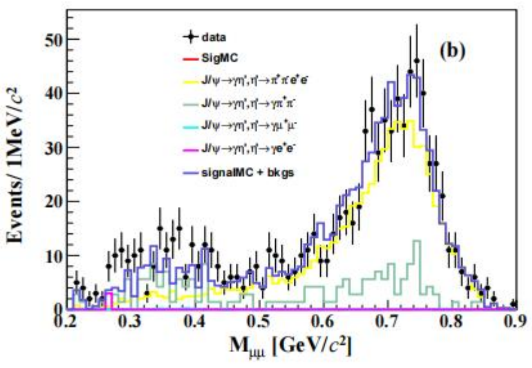
Cut	Efficiency
$M_{2e2\mu} \in (0.88,1.0)$	26.34%
$\chi^2_{\mathrm{PID+4C}}(e^+e^-\mu^+\mu^-)<40$	23.80%
M_{ee}^{BP} && R_{xy}	21.56%
ϕ_{ee} && R_{xy}	20.64%
$\chi^2_{\mathrm{PID+4C}}(e^+e^-\mu^+\mu^-) < \chi^2_{\mathrm{PID+4C}}(\pi^+\pi^-\pi^+\pi^-)$	20.54%

Data/MC Comparison

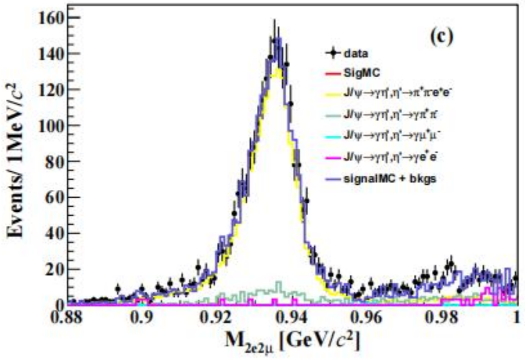
- The distribusiton of M_{2e} , $M_{2\mu}$, $M_{2e2\mu}$:



(a) The distribution of $M_{e^+e^-}$

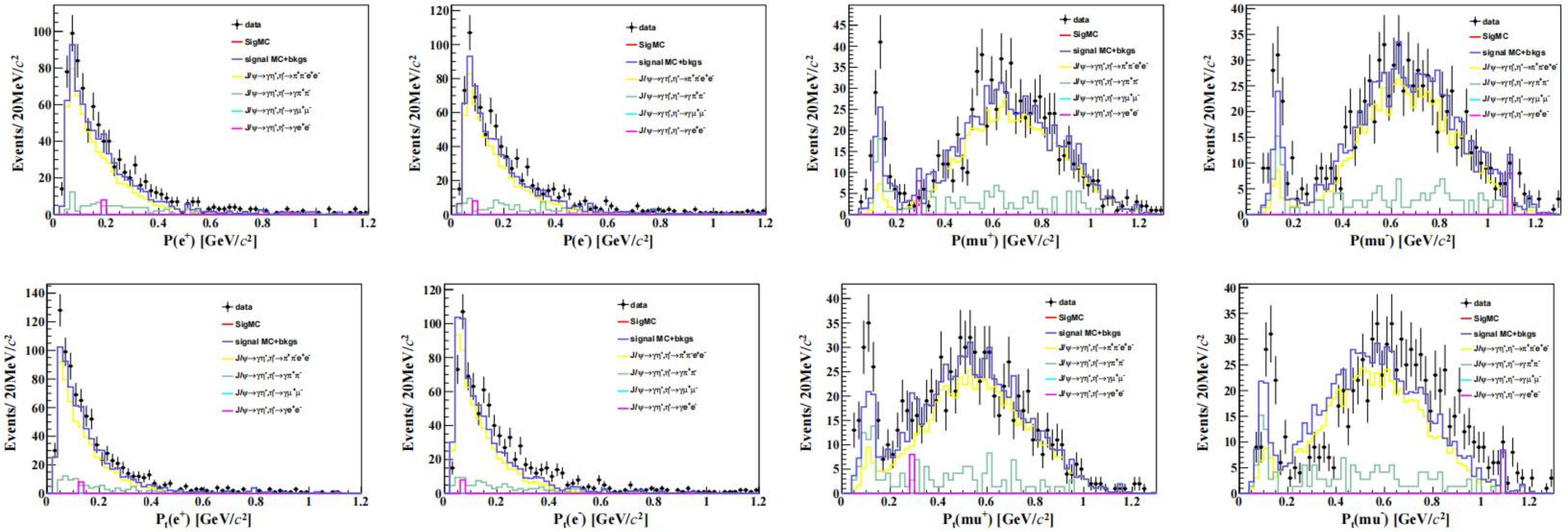


(b) The distribution of $M_{\mu^+\mu^-}$



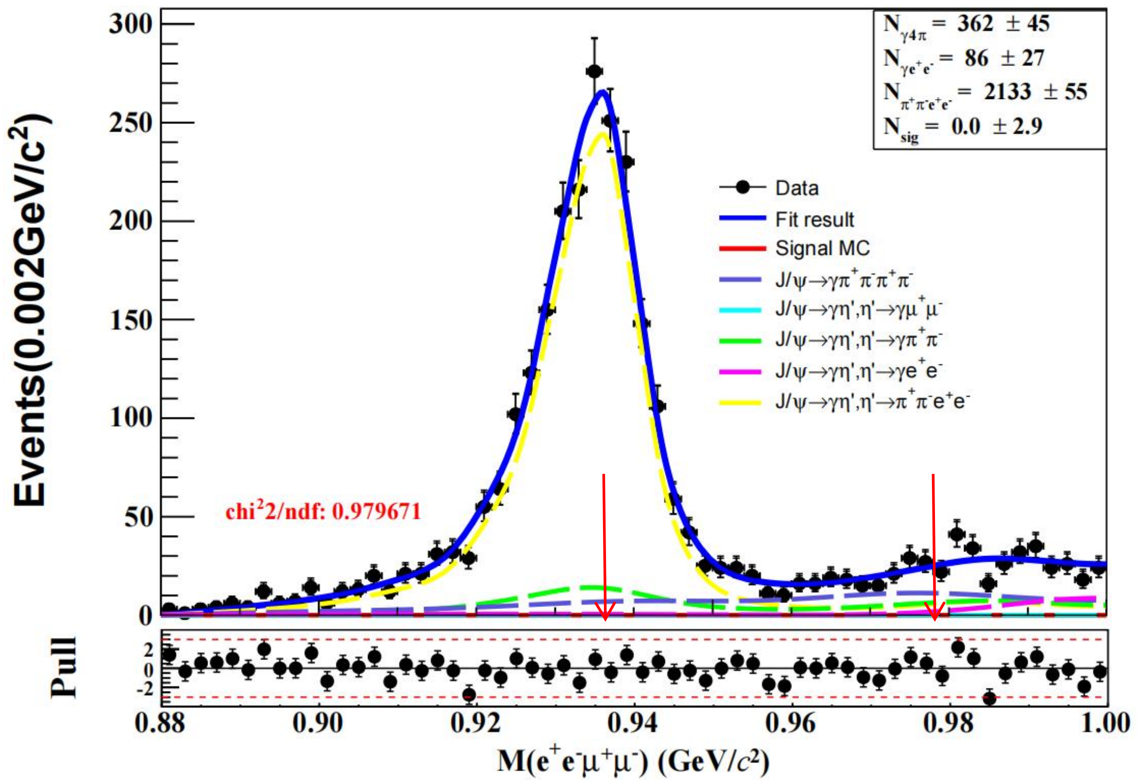
(c) The distribution of $M_{e^+e^-\mu^+\mu^-}$

- The distribusiton of P and P_t for e^+, e^-, μ^+, μ^-:



Fit result

■ Fit Model :



□ The fitting probability density function (PDF) can be written as :

MC shape + Background MC shape

□ Detection Efficiency:

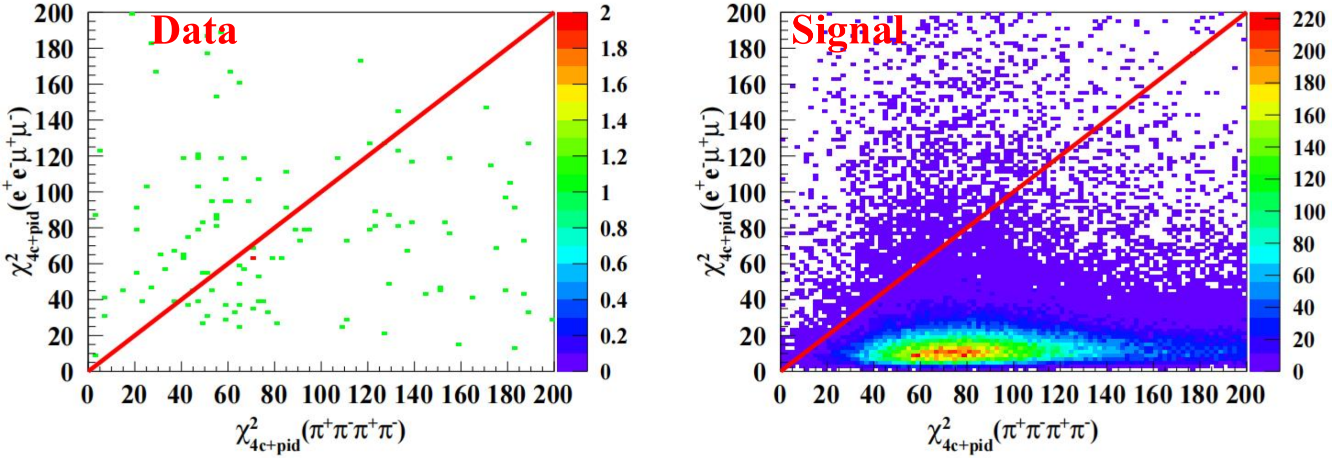
$$\varepsilon = \frac{92430}{450000} = (20.54 \pm 0.06)\%$$

□ No signal events are found within present precision.

Analysis of $\eta \to e^+e^-\mu^+\mu^-$

Futher Event Selection

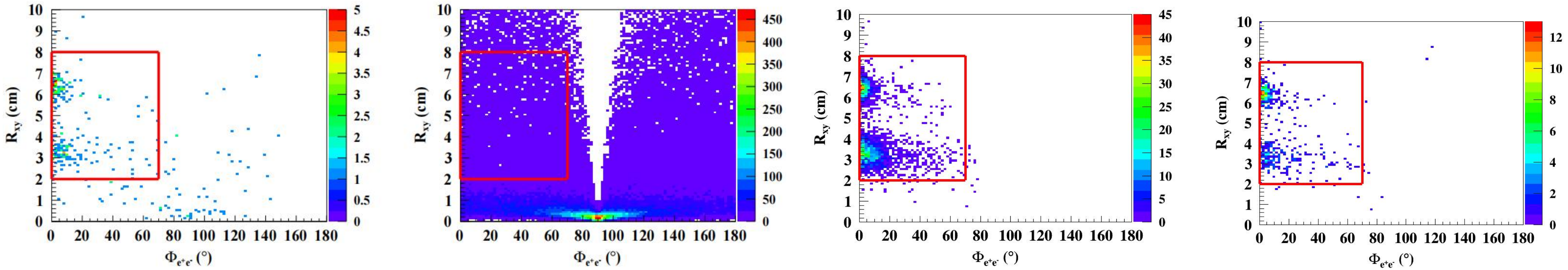
- $\chi^2_{PID+4C}(e^+e^-\mu^+\mu^-)$ versus $\chi^2_{PID+4C}(\pi^+\pi^-\pi^+\pi^-)$:



veto background $J/\psi \to \gamma\pi^+\pi^-\pi^+\pi^-$

Cut	Range
Φ_{ee} vs. R_{xy}	$2.0<R_{xy}<8$ && $0 < \Phi_{ee}< 70°$

- Veto Photon Conversion



(a) Data (b) MC simulation of $\eta \to e^+e^-\mu^+\mu^-$ (c) MC simulation of $\eta \to \gamma\mu^+\mu^-$ (d) MC simulation of $\eta \to \gamma\pi^+\pi^-$

Background Study

- Topo

Table 1: Decay trees and their respective final states.

rowNo	decay tree	decay final state	iDcyTr	nEtr	nCEtr
1	$J/\psi \to \eta\gamma, \eta \to e^+e^-\pi^+\pi^-$	$e^+e^-\pi^+\pi^-\gamma$	1	14	14
2	$J/\psi \to \eta\gamma, \eta \to \pi^0\pi^+\pi^-$	$\pi^0\pi^+\pi^-\gamma$	3	2	16
3	$J/\psi \to \eta\gamma, \eta \to \pi^+\pi^-\gamma^F$	$\pi^+\pi^-\gamma^F\gamma$	4	2	18
4	$J/\psi \to \eta\gamma, \eta \to \pi^0\pi^+\pi^-, \pi^0 \to e^+e^-\gamma^F$	$e^+e^-\pi^+\pi^-\gamma^F\gamma$	0	1	19
5	$J/\psi \to \eta\gamma, \eta \to e^+e^-\pi^+\pi^-\gamma^f$	$e^+e^-\pi^+\pi^-\gamma\gamma^f$	2	1	20

- Backgrounds and Normalized Event Number

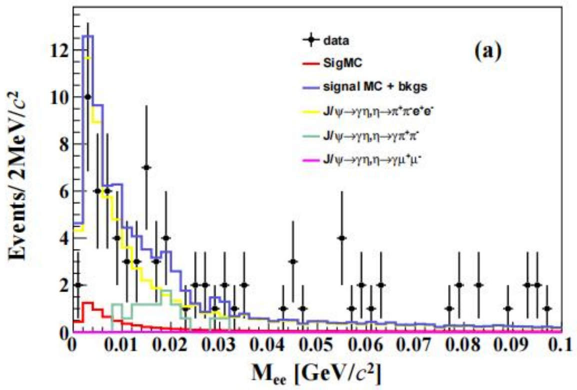
Decay mode	Normalized Event Number
$J/\psi \to \gamma\eta, \eta \to \pi^+\pi^-e^+e^-$	80.1 ± 3.5
$J/\psi \to \gamma\eta, \eta \to \gamma\pi^+\pi^-$	9.8 ± 0.2
$J/\psi \to \gamma\eta, \eta \to \gamma\mu^+\mu^-$	0.1 ± 0.02

- Cut flow and Detection Efficiency

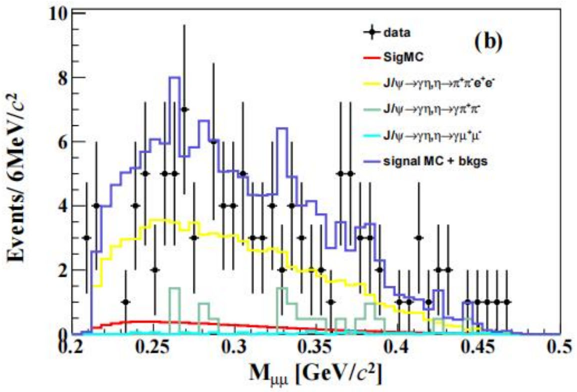
Cut	Efficiency
$M_{2e2\mu} \in (0.45,0.6)$	26.70%
$\chi^2_{\mathrm{PID+4C}}(e^+e^-\mu^+\mu^-)<40$	24.15%
ϕ_{ee} && R_{xy}	21.60%
$\chi^2_{\mathrm{PID+4C}}(e^+e^-\mu^+\mu^-) < \chi^2_{\mathrm{PID+4C}}(\pi^+\pi^-\pi^+\pi^-)$	21.50%

Data/MC Comparison

- The distribusiton of M_{2e} , $M_{2\mu}$:

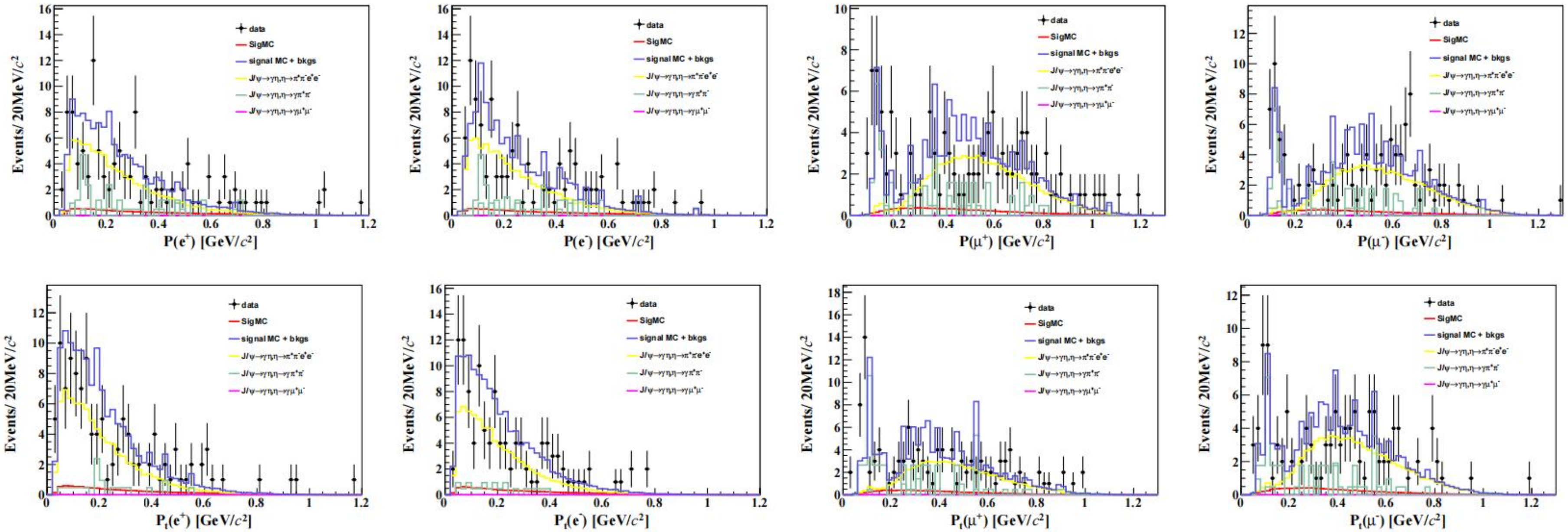


(a) The distribution of $M_{e^+e^-}$



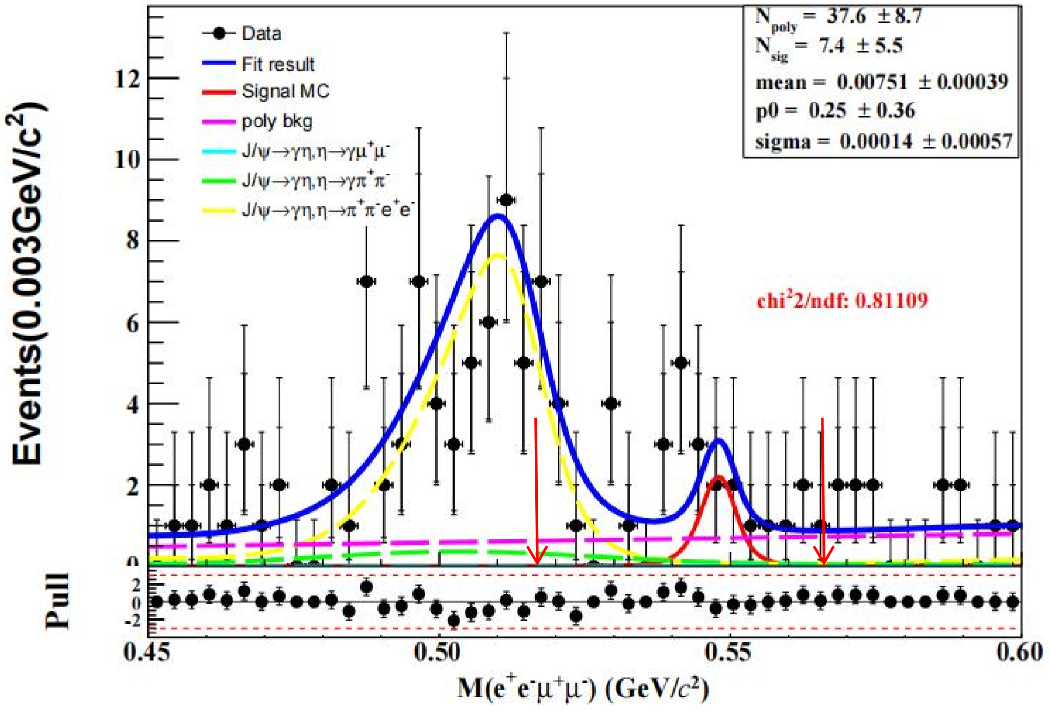
(b) The distribution of $M_{\mu^+\mu^-}$

- The distribusiton of P and P_t for e^+, e^-, μ^+, μ^-:



Fit result of $\eta \to e^+e^-\mu^+\mu^-$

- Fit Model :



- The fitting probability density function (PDF) can be written as :

  MC shape + Background MC shape +1-order poly

- Detection Efficiency:

$$\varepsilon = \frac{96750}{450000} = (21.50 \pm 0.06)\%$$

- Branching fraction: N_{sig}=**7.4±5.5**

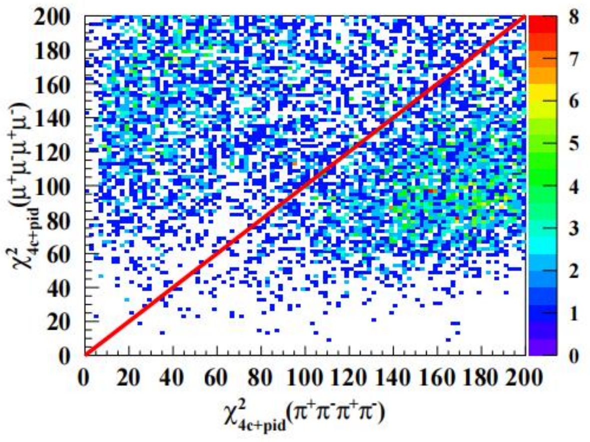
$$\mathcal{B}(\eta \to e^+e^-\mu^+\mu^-) = \frac{N_{sig}}{N_{J/\psi} \cdot \mathcal{B}(J/\psi \to \gamma\eta) \cdot \varepsilon}$$
$$= (3.17 \pm 2.14) \times 10^{-6}$$

significance is **2.27σ**

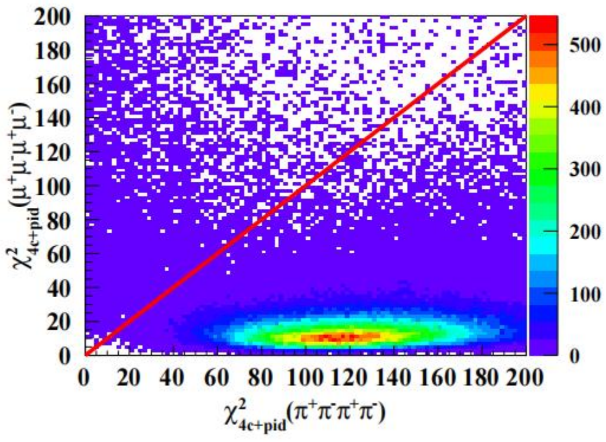
Analysis of $\eta' \to \mu^+\mu^-\mu^+\mu^-$

Futher Event Selection and Topo

- $\chi^2_{PID+4C}(\mu^+\mu^-\mu^+\mu^-)$ versus $\chi^2_{PID+4C}(\pi^+\pi^-\pi^+\pi^-)$:



(a) Data



(b) MC simulation of $\eta \to \mu^+\mu^-\mu^+\mu^-$

veto background $J/\psi \to \gamma\pi^+\pi^-\pi^+\pi^-$

- Backgrounds and Normalized Event Number

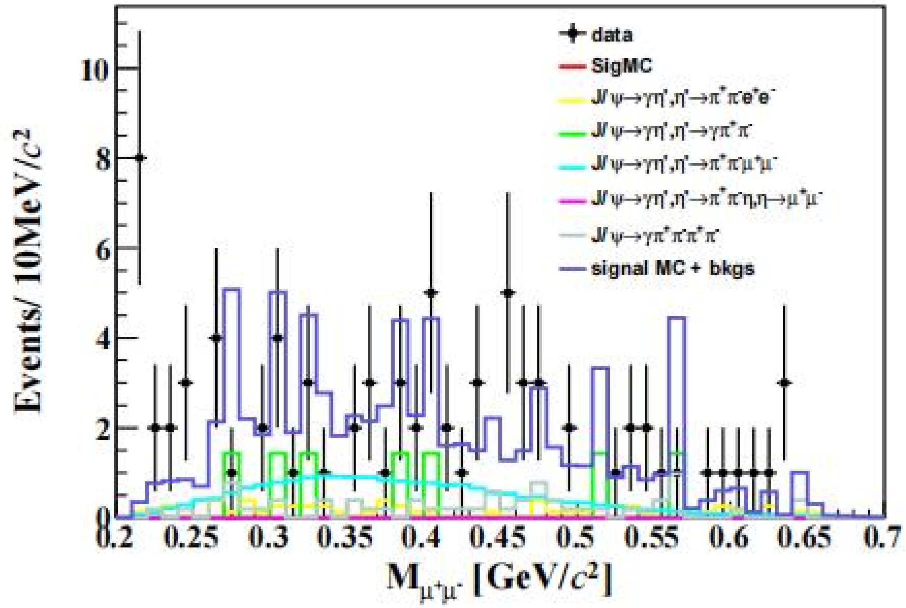
Decay mode	Normalized Event Number
$J/\psi \to \gamma\eta', \eta' \to \pi^+\pi^- e^+e^-$	30.7 ± 1.3
$J/\psi \to \gamma\eta', \eta' \to \gamma\pi^+\pi^-$	41.7 ± 0.8
$J/\psi \to \gamma\eta', \eta' \to \pi^+\pi^-\eta, \eta \to \mu^+\mu^-$	11.3 ± 1.6
$J/\psi \to \gamma\eta', \eta' \to \pi^+\pi^-\mu^+\mu^-$	220.6 ± 44.2
$J/\psi \to \gamma\pi^+\pi^-\pi^+\pi^-$	Free

- Cut flow and Detection Efficiency

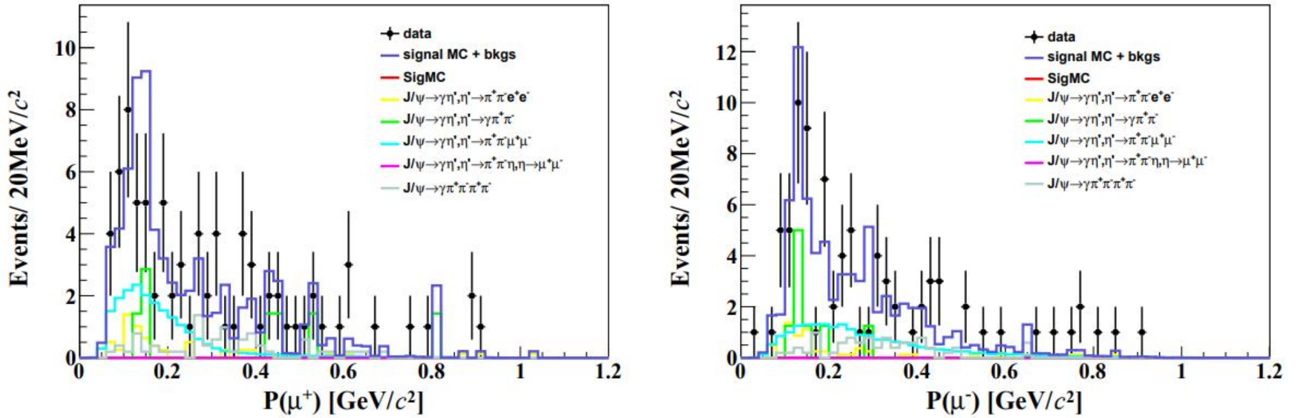
Cut	Efficiency
$M_{4\mu} \in (0.9, 1.0)$	48.86%
$\chi^2_{PID+4C}(\mu^+\mu^-\mu^+\mu^-)<40$	43.15%
$\chi^2_{PID+4C}(\mu^+\mu^-\mu^+\mu^-) < \chi^2_{PID+4C}(\pi^+\pi^-\pi^+\pi^-)$	42.91%

Data/MC Comparison

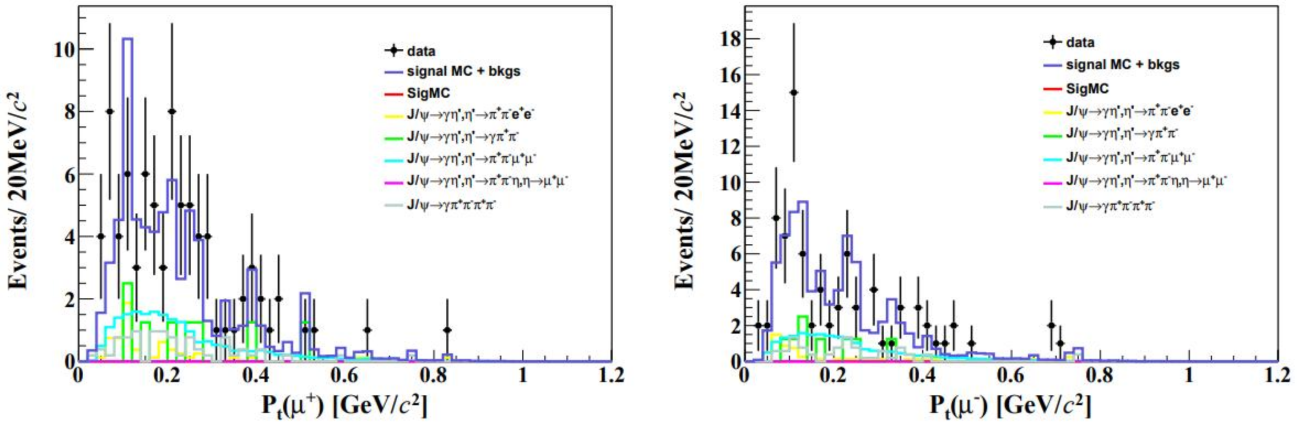
- The distribusiton of $M_{2\mu}$:



- The distribusiton of P and P_t for μ^+, μ^-:



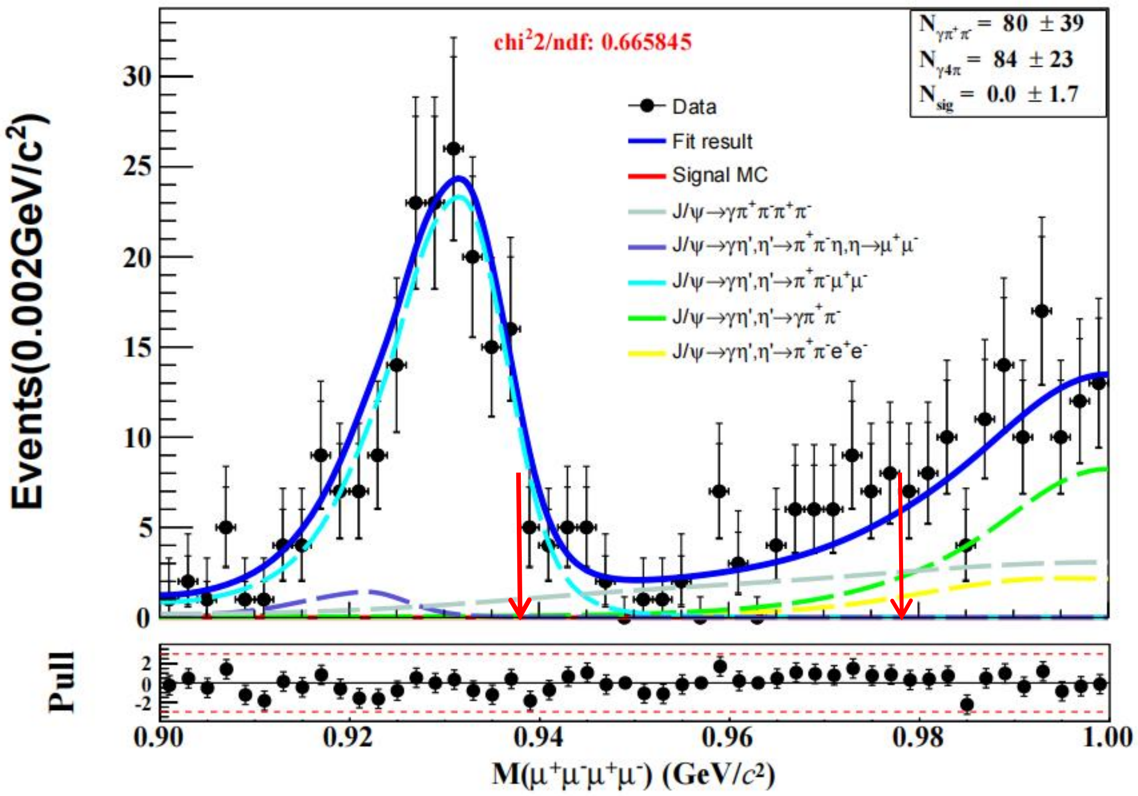
(a) The distribution of $P_{\mu+}, P_{\mu-}$



(b) The distribution of $P_{t\mu+}, P_{t\mu-}$

Fit result of $\eta' \to \mu^+\mu^-\mu^+\mu^-$

■ Fit Model :



- The fitting probability density function (PDF) can be written as :

  MC shape + background MC shape

- Detection Efficiency:

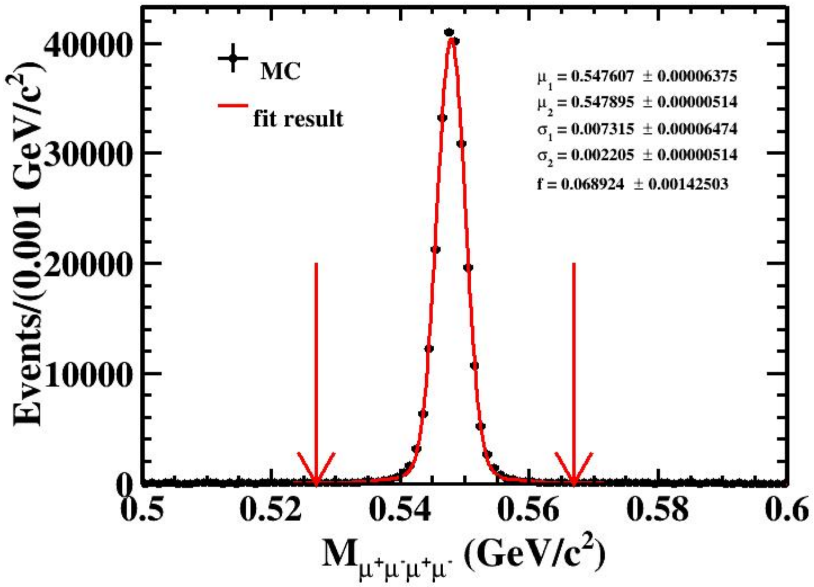
$$\varepsilon = \frac{193112}{450000} = (42.91 \pm 0.07)\%$$

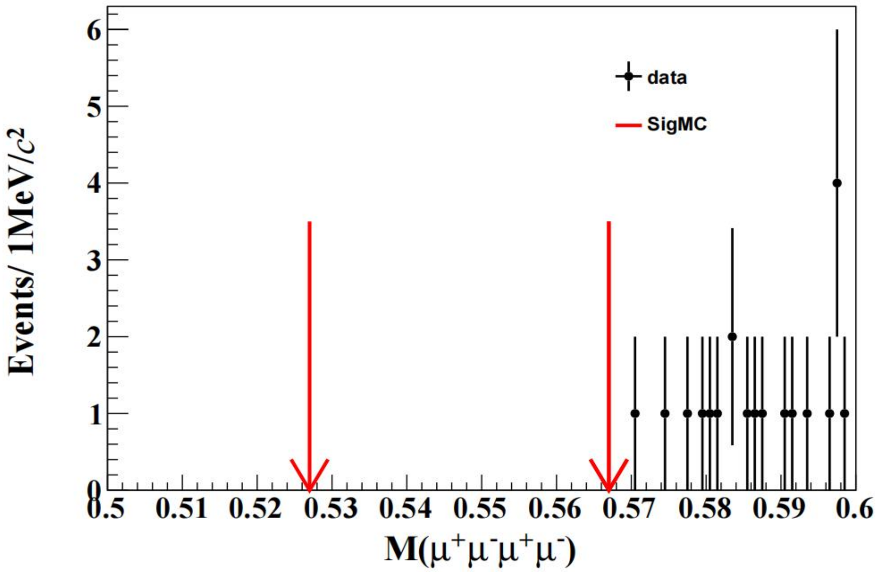
- No signal events are found within present precision.

Analysis of $\eta \to \mu^+\mu^-\mu^+\mu^-$

Result of $\eta \to \mu^+\mu^-\mu^+\mu^-$

- The distribusiton of $M_{\mu^+\mu^-\mu^+\mu^-}$





- Double Gaussian Function

$\mu = \mu_1 f + \mu_2(1-f)$

$\sigma^2 = \sigma_1^2 f + \sigma_2^2(1-f) + (\mu_1 - \mu_2)^2 f(1-f)$

- The signal window is determined by $[\mu - 3\sigma, \mu + 3\sigma]$, which is **[0.527 , 0.567]**

- $N_{\mathrm{sig}} = N_{\mathrm{bkg}} = N_{\mathrm{data}} = 0$

- Detection Efficiency:

$$\varepsilon = \frac{234540}{450000} = 52.12\%$$

Systematic Uncertainty Study

Systematic Uncertainty Study

- Number of J/ψ events： 0.44% is taken as uncertainty.

- Br($J/\psi \rightarrow \gamma\eta/\gamma\eta'$)： 1.66% is taken as uncertainty for $J/\psi \rightarrow \gamma\eta$， 1.33% is taken as uncertainty for $J/\psi \rightarrow \gamma\eta'$ by PDG[3].

- MDC Tracking： The control sample of $e^+e^- \rightarrow \gamma e^+e^-$ we uesd[7], the uncertainty is determined to be 1.0% per track of electron, there is no specific decay mode available for us to study the tracking of muon, we also ues 1.0% per track as the tracking uncertainty of muon. Then, 4.0% is taken as uncertainty.

- PID： The uncertainty is determined to be 1.0% per track, 4.0% is taken as uncertainty.

- Photon detection： The systematic uncertainty is studied using the control sample of $e^+e^- \rightarrow \gamma\mu^+\mu^-$[8], the result shows that the difference between data and MC is about 0.5%, then we take 0.5% as the systrmatic uncertainty for each photon.

[3] Prog. Theor. Exp. Phys. 2020, 083C01 (2020)
[7] Phys. Rev. Lett. 122, 121801 (2019)
[8] BAM- 00511 Phys.Rev.D 87 012002 (2013).

Systematic Uncertainty Study

Kinematic fit

- By correcting the track helix parameters to reduce the difference between data and MC simulation, we use the helix correction factors to analyze the uncertainty from the helix parameters for simulated charged tracks[9].
- The half difference between the efficiencies of the signal with and without correction is taken as the uncertainty.

[9] M. Ablikim et al., (BESIII Collaboration) Phys.Rev.D 87 012002 (2013)

$\eta' \to e^+e^-\mu^+\mu^-$	N_{total}	$N_{selected}$	Efficiency	Uncertainty
Before correction	450000	92433	20.54%	
After correction	450000	91539	20.34%	**0.49%**
$\eta \to e^+e^-\mu^+\mu^-$	N_{total}	$N_{selected}$	Efficiency	Uncertainty
Before correction	450000	96759	21.50%	
After correction	450000	95928	21.32%	**0.42%**
$\eta' \to \mu^+\mu^-\mu^+\mu^-$	N_{total}	$N_{selected}$	Efficiency	Uncertainty
Before correction	450000	193112	42.91%	
After correction	450000	191575	42.57%	**0.39%**
$\eta \to \mu^+\mu^-\mu^+\mu^-$	N_{total}	$N_{selected}$	Efficiency	Uncertainty
Before correction	450000	234552	52.12%	
After correction	450000	233850	51.97%	**0.14%**

Systematic Uncertainty Study

Generator Model

- The detection efficiency dependence on the genetator model is evaluated by replacing the c_3 introduced in the modified Vector Meson Dominance(VMD) model [6]. The maximum difference of the detection efficiency between hidden gauge model and modified VMD model is taken as the uncertainty due to the generator model.

  $c_3 = 1$ $c_3 = 0.927$ or 0.930

- The factor for $P \to l^+l^-l^+l^-$ can be constructed as :

$$VMD_1(s_{12}, s_{34}) = 1 - c_3 + c_3 \frac{m_V^2}{m_V^2 - s_{12} - im_V\Gamma(s_{12})} \frac{m_V^2}{m_V^2 - s_{34} - im_V\Gamma(s_{34})}$$

Generator model	$\eta' \to e^+e^-\mu^+\mu^-$	$\eta \to e^+e^-\mu^+\mu^-$	$\eta' \to \mu^+\mu^-\mu^+\mu^-$	$\eta \to \mu^+\mu^-\mu^+\mu^-$
Modified(c3=0.927) Efficiency	20.50%	21.50%	42.88%	52.2%
Modified(c3=0.930) Efficiency	20.51%	21.60%	42.87%	52.1%
Hidden(c3=1) Efficiency	20.54%	21.50%	42.91%	52.12%
Uncertainty	0.19%	0.47%	0.10%	0.15%

[6] T. Petri, arXiv:1010.2378[nucl-th].

Systematic Uncertainty Study

Photon conversion veto

- Control sample : $J/\psi \to \pi^+\pi^-\pi^0, \pi^0 \to \gamma e^+e^-$, by applying the $\eta/\eta' \to e^+e^-\mu^+\mu^-$ photon conversion selection conditions to the control sample.

Efficiency of veto photon conversion for $\eta' \to e^+e^-\mu^+\mu^-$

$\varepsilon_{\text{MC}} = 193189/219060 = 88.19\%$

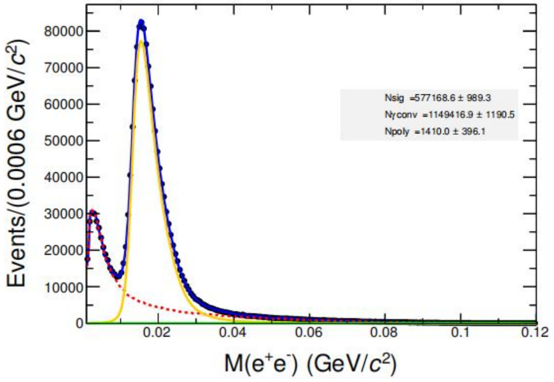
$\varepsilon_{\text{data}} = 502828.0/577168.6 = 87.12\%$ $\delta = \frac{\varepsilon_{\text{data}}}{\varepsilon_{\text{MC}}} - 1 =$ **1.21 %**

Efficiency of veto photon conversion for $\eta \to e^+e^-\mu^+\mu^-$

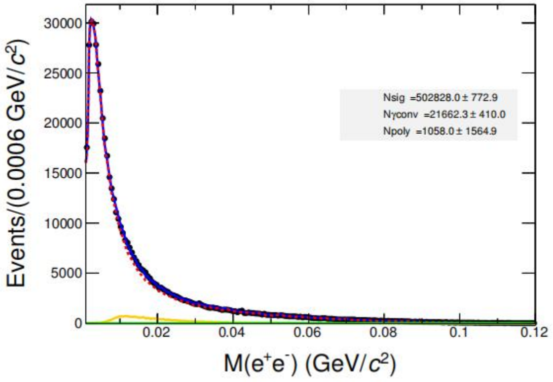
$\varepsilon_{\text{MC}} = 195744/219060 = 89.36\%$

$\varepsilon_{\text{data}} = 513468.9/578943.6 = 88.69\%$ $\delta = \frac{\varepsilon_{\text{data}}}{\varepsilon_{\text{MC}}} - 1 =$ **0.75%**

- $\eta' \to e^+e^-\mu^+\mu^-$

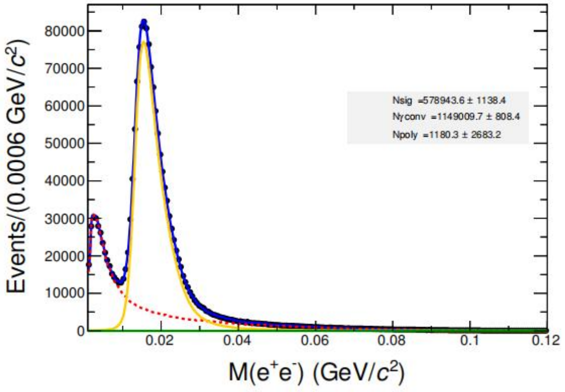


(a) Fit without gamma conversion requirements

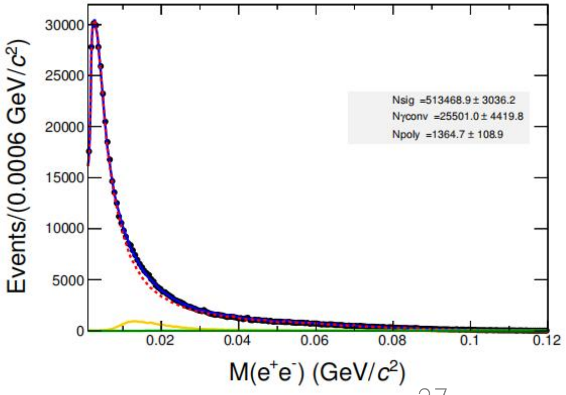


(b) Fit after gamma conversion requirements

- $\eta \to e^+e^-\mu^+\mu^-$



(a) Fit without gamma conversion requirements



(b) Fit after gamma conversion requirements

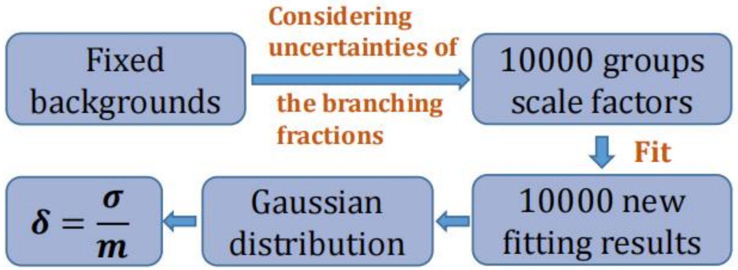
Systematic Uncertainty Study

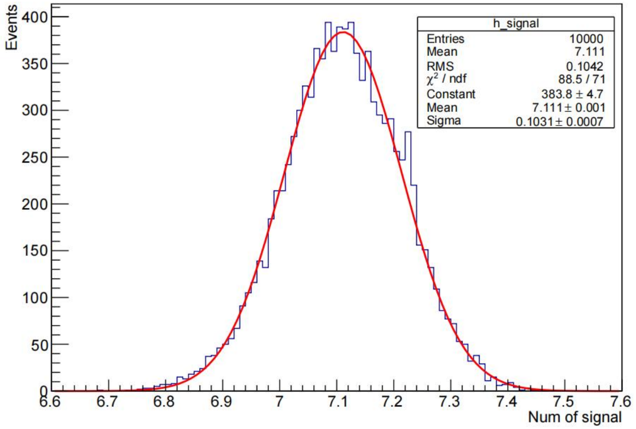
☐ Fit range for $\eta \to e^+e^-\mu^+\mu^-$

Varying the � range by 0.01GeV/c² and 0.02GeV/c², take the largest differences as the systematic uncertainties.

Fit range		N_{signal}	Efficiency	$\mathcal{B}(\times 10^{-6})$	Uncertainy	N^{UP}
Original	[0.45,0.6]	7.4±5.5	21.50%	(3.14436 ± 2.33761)	-	15
1	[0.44,0.61]	6.9±2.7	21.52%	(2.92918 ± 1.14723)	6.76%	15
2	[0.46,0.59]	6.8±3.8	21.48%	(2.89291 ± 1.61689)	8.11%	15
3	[0.45,0.61]	6.8±3.7	21.52%	(2.88673 ± 1.57145)	8.11%	16
4	[0.44,0.60]	7.4±3.7	21.50%	(3.14729 ± 1.57451)	0%	15
5	[0.45,0.59]	6.7±3.6	21.48%	(2.84957 ± 1.53184)	**9.46%**	15
6	[0.46,0.60]	7.4±6.7	21.50%	(3.14436 ± 2.84740)	0	15

☐ Background shape for $\eta \to e^+e^-\mu^+\mu^-$





$$\delta = \frac{\text{Sigma}}{\text{Mean}} = \frac{0.103083}{7.11104} = \mathbf{1.45\%}$$

Systematic Uncertainty Study

☐ Summary of the Systematic Uncertainty Study

Source(%)	$\eta' \to e^+e^-\mu^+\mu^-$	$\eta \to e^+e^-\mu^+\mu^-$	$\eta' \to \mu^+\mu^-\mu^+\mu^-$	$\eta \to \mu^+\mu^-\mu^+\mu^-$
N (J/ψ)	0.44	0.44	0.44	0.44
Br	1.33	1.66	1.33	1.66
MDC Tracking	4	4	4	4
PID	4	4	4	4
Photon detection	0.50	0.50	0.50	0.50
Kinematic fit	0.49	0.42	0.39	0.14
Photon Conversion Veto	1.21	0.75	-	-
Generator Model	0.19	0.47	0.1	0.15
Background Shape	-	1.45	-	-
Fit range	-	9.46	-	-
Total	**5.99**	**11.30**	**5.86**	**5.94**

Systematic Uncertainty Study

- In the upper limit calculation, these uncertainties are classified to 2 categories :

  The additive terms include: Fit range, signal shape, continuous background shape, peaking background shape and the number of peaking background events.

  The multiplicative terms include: Number of J/ψ events, photon detection efficiency, kinematic fit, generator model and the cited branching fraction, as shown in Table.

Multiplicative uncertainty(%) \ Decay	$\eta' \to e^+e^-\mu^+\mu^-$	$\eta \to e^+e^-\mu^+\mu^-$	$\eta' \to \mu^+\mu^-\mu^+\mu^-$
Total	5.99	6.01	5.86

- We perform multiple systematic tests, a series of alternative fits are performed by using different fitting model for $\eta'/\eta \to e^+e^-\mu^+\mu^-$ and $\eta' \to \mu^+\mu^-\mu^+\mu^-$ and select the maxium N^{UL} model.

Systematic Uncertainty Study

■ **Additive Systematic Uncertainty on upper limit for $\eta' \to e^+e^-\mu^+\mu^-$**

Source			N^{UL}(90%C.L.)
Fit range	original	[0.88,1.0]GeV/c^2	16
	1	[0.87,1.01]GeV/c^2	16
	2	[0.89,0.99]GeV/c^2	19
	3	[0.88,1.01]GeV/c^2	16
	4	[0.88,0.99]GeV/c^2	**22**
	5	[0.87,1.0]GeV/c^2	14
	6	[0.89,1.0]GeV/c^2	20
Signal shape	MC shape convolution Gaussian		16
Peaking bkg shape	MC shape convolution Gaussian		17
Continuous bkg shape	3_{nd} order polynomial		16
The number of peaking background events	305.682		15
	289.675		16

Systematic Uncertainty Study

- **Additive Systematic Uncertainty on upper limit for $\eta \rightarrow e^+e^-\mu^+\mu^-$**

Source			N^{UL}(90%C.L.)
Fit range	original	[0.45,0.6]GeV/c^2	15
	1	[0.44,0.61]GeV/c^2	15
	2	[0.46,0.59]GeV/c^2	15
	3	[0.45,0.61]GeV/c^2	16
	4	[0.44,0.60]GeV/c^2	15
	5	[0.45,0.59]GeV/c^2	15
	6	[0.46,0.60]GeV/c^2	15
Signal shape	MC shape convolution Gaussian		**17**
Peaking bkg shape	MC shape convolution Gaussian		15
Continuous bkg shape	2_{nd} order polynomial		15
The number of peaking background events	83.5		15
	75.4		16

Systematic Uncertainty Study

- **Additive Systematic Uncertainty on upper limit for $\eta' \to \mu^+\mu^-\mu^+\mu^-$**

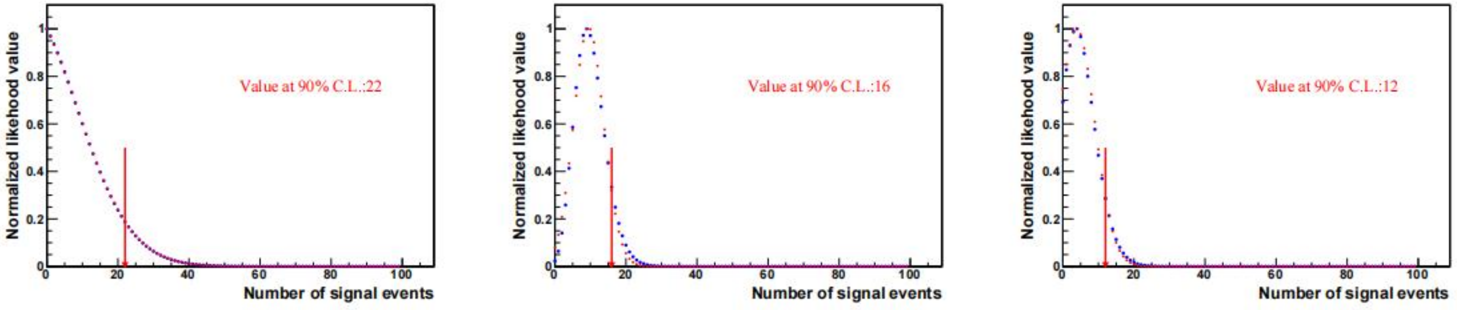
Source			N^{UL}(90%C.L.)
Fit range	original	[0.9,1.0]GeV/c^2	9
	1	[0.89,1.01]GeV/c^2	9
	2	[0.91,0.99]GeV/c^2	11
	3	[0.9,1.01]GeV/c^2	9
	4	[0.9,0.99]GeV/c^2	8
	5	[0.89,1.0]GeV/c^2	8
	6	[0.91,1.0]GeV/c^2	9
Signal shape	MC shape convolution Gaussian		10
Peaking bkg shape	MC shape convolution Gaussian		9
Continuous bkg shape	1_{nd} order polynomial		12
The number of peaking background events	311		10
	211		8

Upper limits

- **Determination of the branching fraction upper limits**
  - Then, the total multiplicative uncertainty is incorporated by convoluting a Gaussian in which the uncertainty is taken as a parameter [7]. The detail can be found in the following formula:

$$L'(N) = \int_0^1 L(\frac{\mathcal{S}}{\bar{\mathcal{S}}}N)exp\left[-\frac{(\mathcal{S}-\bar{\mathcal{S}})}{2\sigma_{\mathcal{S}}^2}\right] d\mathcal{S}$$

Finally, we obtain the upper limit of signal events with taking into account the systematic uncertainty.



(a) $\eta' \to e^+e^-\mu^+\mu^-$ (b) $\eta \to e^+e^-\mu^+\mu^-$ (c) $\eta' \to \mu^+\mu^-\mu^+\mu^-$

* Normalized likelihood distribution before (blue dots) and after (red dots) convolution

- Upper limits:

Decay mode	N^{UL}	$\mathcal{B}^{UL}$
$\eta \to e^+e^-\mu^+\mu^-$	22	$< (6.86)\times 10^{-6}$(CL = 90%)
$\eta' \to e^+e^-\mu^+\mu^-$	16	$< (2.04)\times 10^{-6}$(CL = 90%)
$\eta' \to \mu^+\mu^-\mu^+\mu^-$	12	$< (5.33)\times 10^{-7}$ (CL = 90%)

[7] arXiv:physics/0605236[hep-ex]

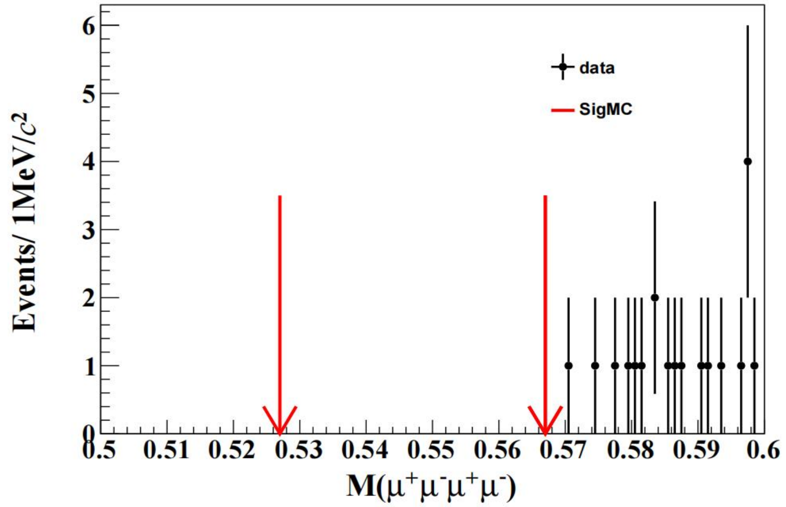
Upper limits

- **Determination of the branching fraction upper limit for $\eta \to \mu^+\mu^-\mu^+\mu^-$**

  - $N_{\mathrm{sig}} = N_{\mathrm{bkg}} = N_{\mathrm{data}} = 0$, $\varepsilon = 52.12\%$, $\delta_{\mathrm{sys}} = 5.94\%$

  - Using the **TROLKE**[8] program we have $N^{UL}=3.84$ at the 90% C.L.

    The upper limit is conservatively estimated by :



$$\mathcal{B}(\eta \to \mu^+\mu^-\mu^+\mu^-) < \frac{N_{UL}}{N_{J/\psi} \cdot \mathcal{B}(J/\psi \to \gamma\eta)} = 3.52 \times 10^{-7}$$

[8] arXiv:0403059[hep-ex].

Summary and Next

- Summary the Branching fraction and Upper Limit results :

Decay mode	$\varepsilon(\%)$	σ	$\mathcal{B}_{\text{this work}}$	VMD	Data driven approach	PDG
$\eta \to e^+e^-\mu^+\mu^-$	21.50	2.27	$(3.17 \pm 2.14 \pm 0.36) \times 10^{-6}$ $< (6.86) \times 10^{-6}(CL = 90\%)$	2.154×10^{-6}	$(2.39 \pm 0.7) \times 10^{-6}$	$<1.6\times10^{-4}$(CL=90%)
$\eta \to \mu^+\mu^-\mu^+\mu^-$	52.12	-	$<(3.52) \times 10^{-7}(CL = 90\%)$	3.797×10^{-9}	$(3.98 \pm 0.15) \times 10^{-9}$	$<3.6\times10^{-4}$(CL=90%)
$\eta' \to e^+e^-\mu^+\mu^-$	20.54	-	$< (2.04) \times 10^{-6}(CL = 90\%)$	7.968×10^{-7}	$(6.39 \pm 0.91) \times 10^{-7}$	-
$\eta' \to \mu^+\mu^-\mu^+\mu^-$	42.91	-	$<(5.33) \times 10^{-7}(CL = 90\%)$	2.185×10^{-8}	$(1.69 \pm 0.36) \times 10^{-8}$	-

- Link of memo: https://docbes3.ihep.ac.cn/DocDB/0012/001250/004/memo_v1.6.pdf

- Next to do:

1. Blind analysis
2. PS meeting

THANKS!☺

Event generators for eta/eta-prime rare decays into pi+ pi- l+ l- and e+ e- mu+ mu-

- **Abstract:** Study of the rare and forbidden decays of η/η' offers a sensitive probe to test fundamental symmetries of quantum chromodynamics and search for new physics beyond the Standard Model. To study the rare decays of η/η' to $\pi^+\pi^-e^+e^-$, $\pi^+\pi^-\mu^+\mu^-$ and $e^+e^-\mu^+\mu^-$ at the BESIII detector, we developed several event generators based on the vector meson dominant model with finite-width corrections and the pseudoscalar mesons mixing theory. The various distributions from event generators are in good agreement with the theoretical predictions, which indicates that the event generators work very well after implemention in the BESIII Monte Carlo simulation package. In the BESIII physics analysis, the performance of the event generators will be improved in accordance with the distributions of different variables of η/η' from data and the improvement on the theoretical calculations.

link: https://inspirehep.net/literature/1202555

■ $\eta' \rightarrow \pi^+\pi^-e^+e^-$

- The invariant decay amplitude[14]:

$$\overline{\left|\mathcal{A}_{\eta'\to\pi^+\pi^-l^+l^-}\right|^2}(s_{\pi\pi}, s_{ll}, \theta_\pi, \theta_1, \phi) = \frac{e^2}{8k^2}|M(s_{\pi\pi}, s_{ll})|^2 \times \lambda\left(m_{\eta'}^2, s_{\pi\pi}, s_{ll}\right) \times [1-\beta_1^2 \sin^2\theta_1 \sin^2\phi] s_{\pi\pi}\beta_\pi^2 \sin^2\theta_\pi$$

- Here the magnetic form factor $M(s_{\pi\pi}, s_{ll})$ contains the information of the decaying particle and the vector meson dominance input:

$$M(s_{\pi\pi}, s_{ll}) = \mathcal{M} \times VMD(s_{\pi\pi}, s_{ll})$$

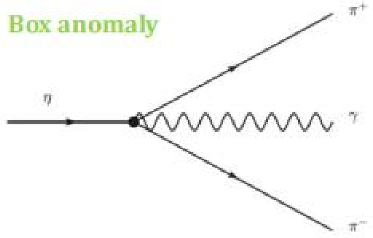
$$\mathcal{M} = \frac{e}{8\pi^2 f_\pi^3}\frac{1}{\sqrt{3}}\left(\frac{f_\pi}{f_8} sin\theta_{mix} + 2\sqrt{2}\frac{f_\pi}{f_0} cos\theta_{mix}\right)$$

$$\mathcal{M} = \begin{cases} \frac{e}{8\pi^2 f_\pi^3}\frac{1}{\sqrt{3}}\left(\frac{f_\pi}{f_8}\cos\theta_{mix} - 2\sqrt{2}\frac{f_\pi}{f_0}\sin\theta_{mix}\right) & P=\eta \\ \frac{e}{8\pi^2 f_\pi^3}\frac{1}{\sqrt{3}}\left(\frac{f_\pi}{f_8}\sin\theta_{mix} + 2\sqrt{2}\frac{f_\pi}{f_0}\cos\theta_{mix}\right) & P=\eta', \end{cases}$$

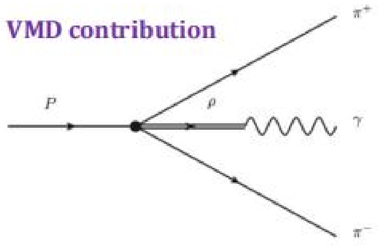
with the pion decay constant $f_\pi \approx 92.4$ MeV, the octet pseudoscalar decay constant $f_8 \approx 1.3 f_\pi$, the singlet pseudoscalar decay constant $f_0 \approx 1.04 f_\pi$ and the η–η' mixing angle $\theta_{mix} \approx -20^\circ$.

$$VMD(s_{\pi\pi}, s_{ll}) = 1 - \frac{3}{4}(c_1 - c_2 + c_3) + \frac{3}{4}(c_1 - c_2 - c_3)\frac{m_V^2}{m_V^2 - s_{ll} - i m_V \Gamma(s_{ll})} + \frac{3}{2} c_3 \frac{m_V^2}{m_V^2 - s_{ll} - i m_V \Gamma(s_{ll})}\frac{m_V^2}{m_V^2 - s_{\pi\pi} - i m_V \Gamma(s_{\pi\pi})}$$

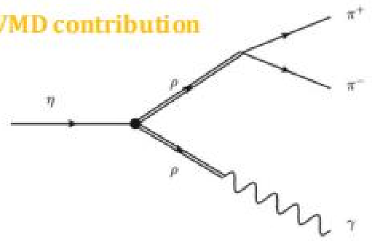
Box anomaly



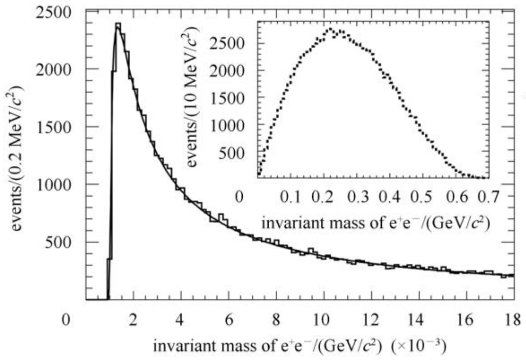
VMD contribution

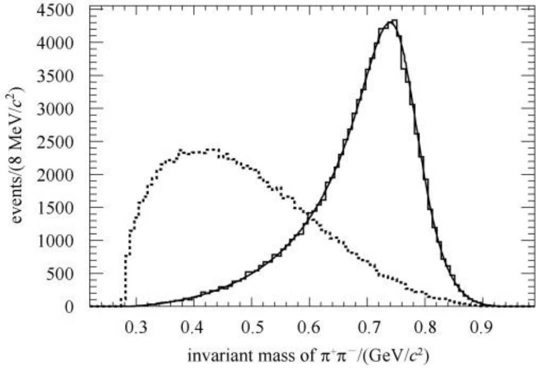


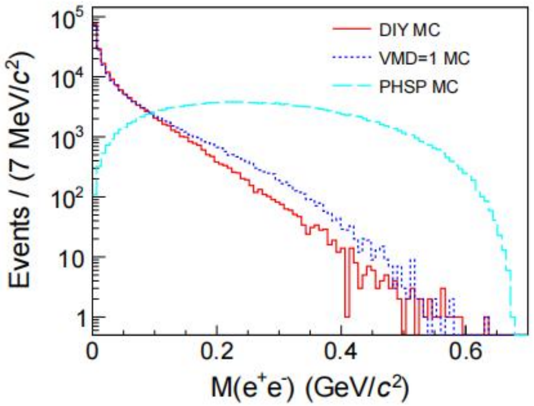
VMD contribution

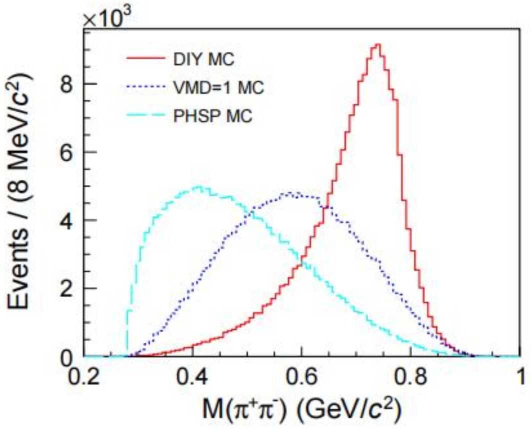


- $\eta' \to \pi^+\pi^-e^+e^-$

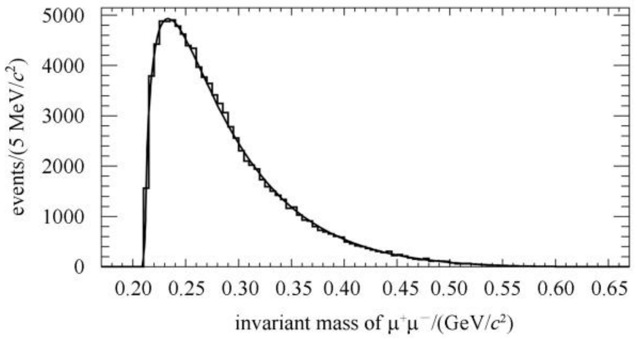


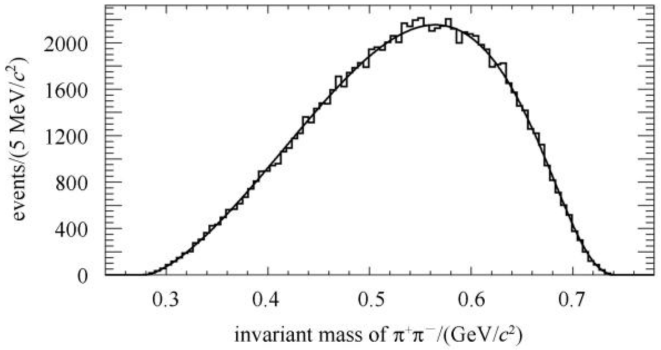


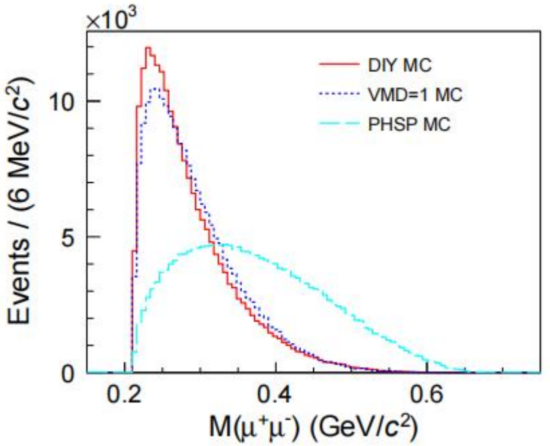


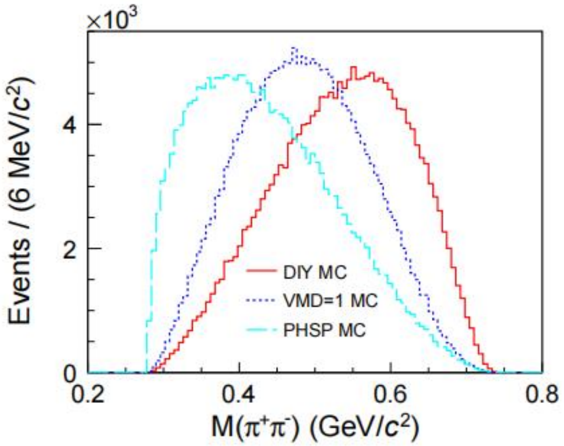


- $\eta' \to \pi^+\pi^-\mu^+\mu^-$









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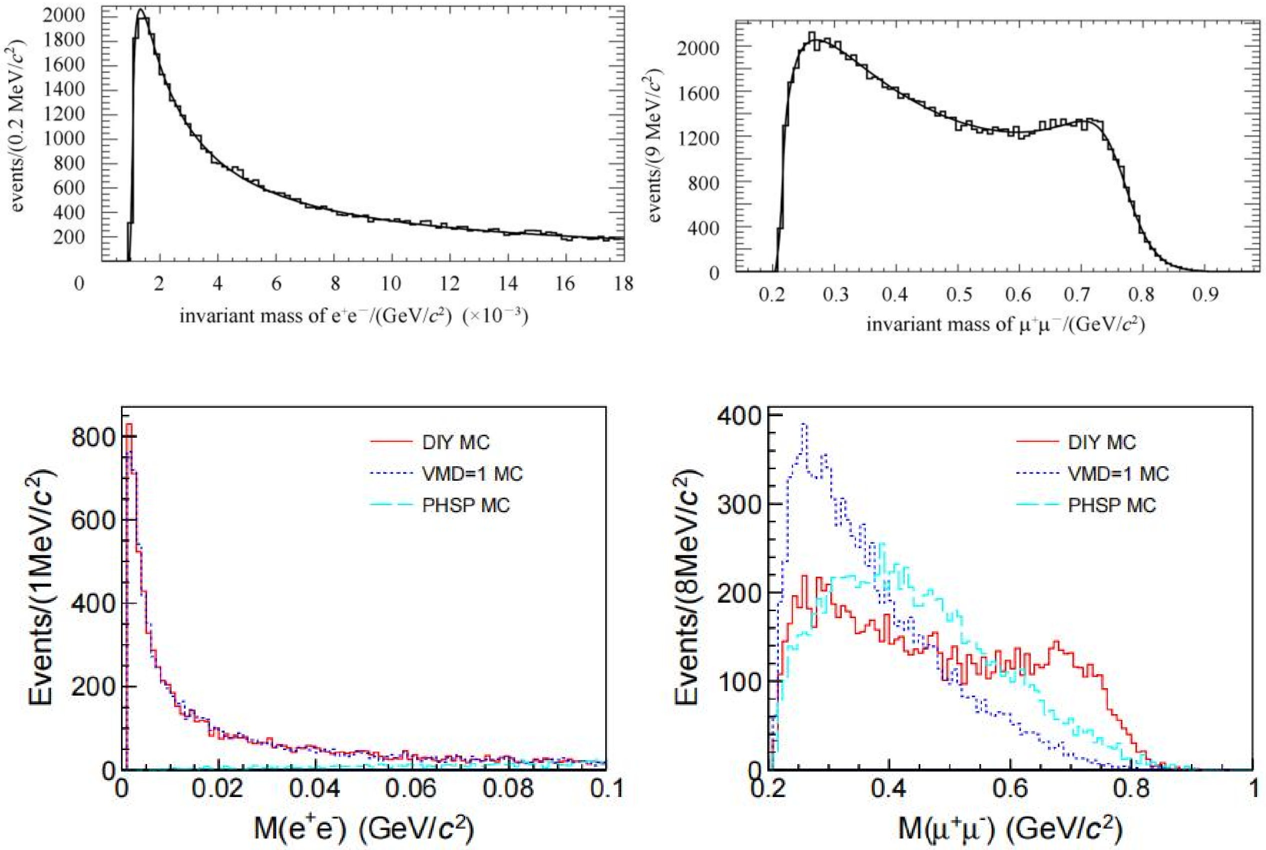
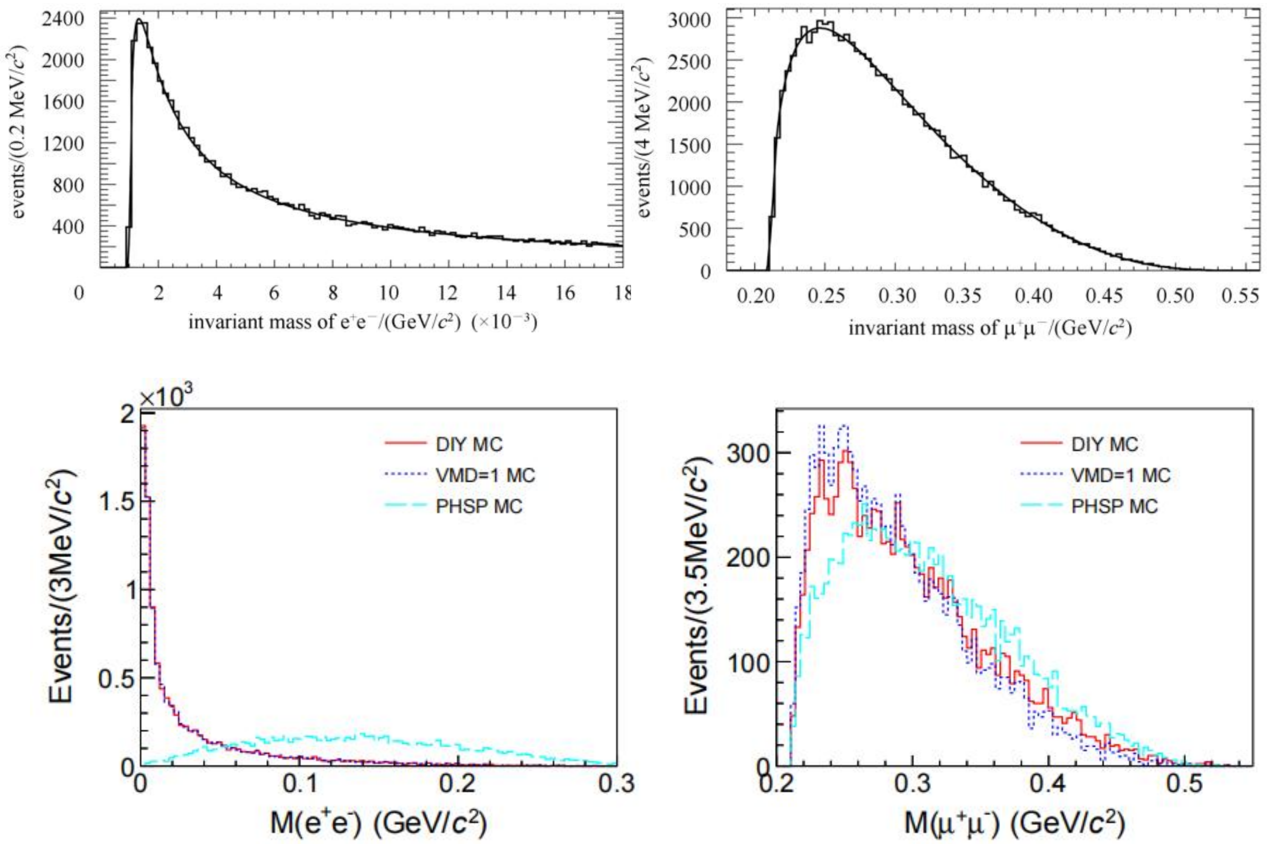
- $\eta' \to e^+e^-\mu^+\mu^-$



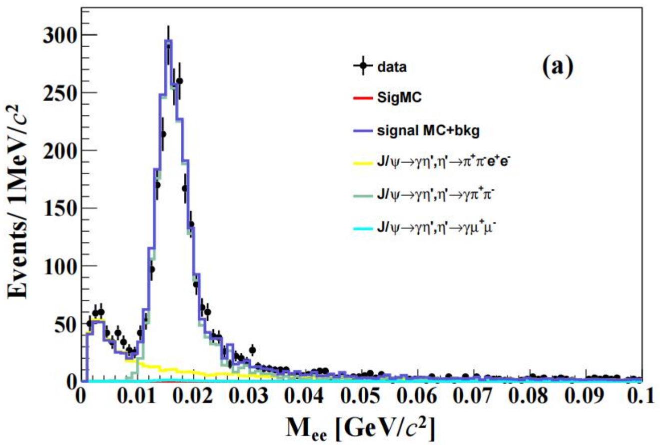
- $\eta \to e^+e^-\mu^+\mu^-$

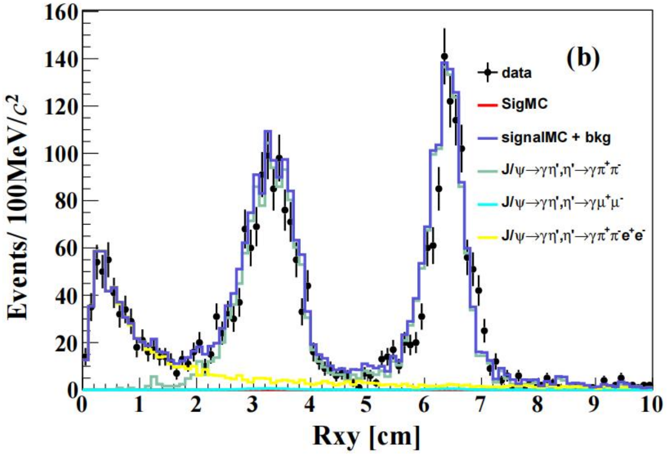


Thank you!

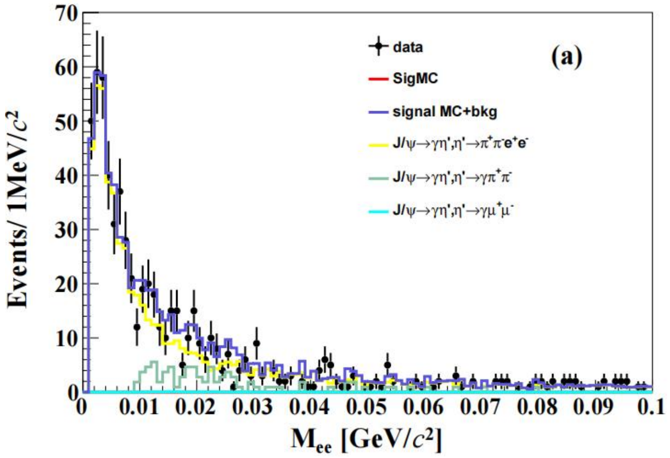
Back up

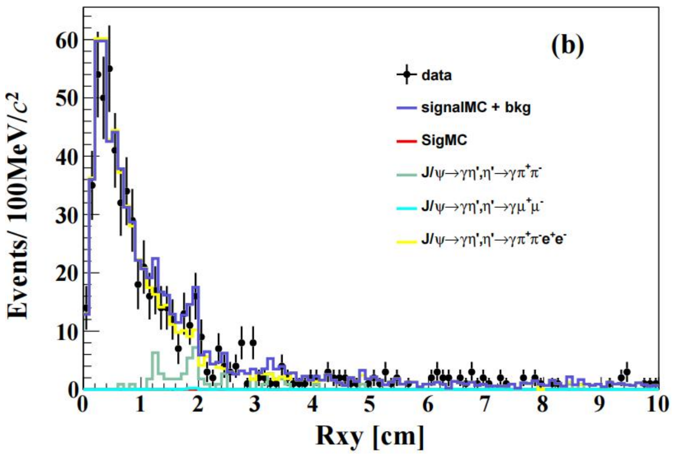
- The distribusiton of M_{2e} , R_{xy} before photon conversion veto for $\eta' \to e^+e^-\mu^+\mu^-$:





- The distribusiton of M_{2e} , R_{xy} after photon conversion veto for $\eta' \to e^+e^-\mu^+\mu^-$:

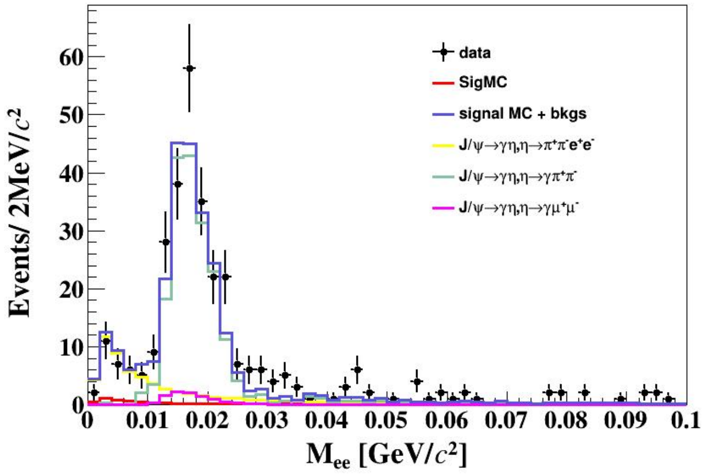


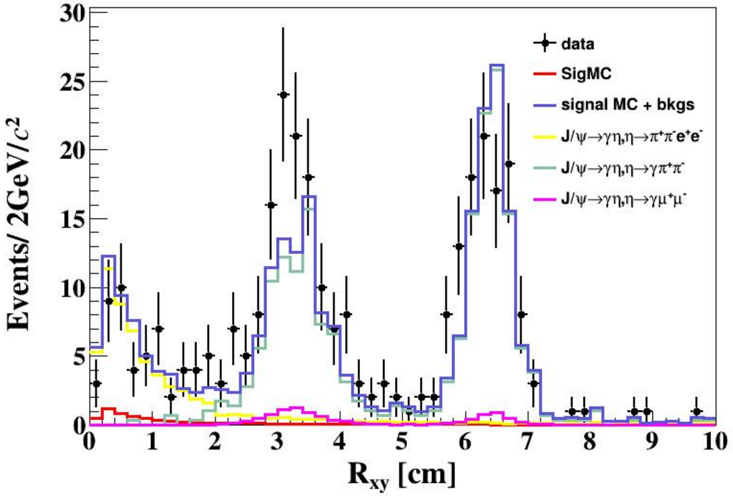


□ photon conversion backgrounds (gpipi,gmumu) have already suppesed.

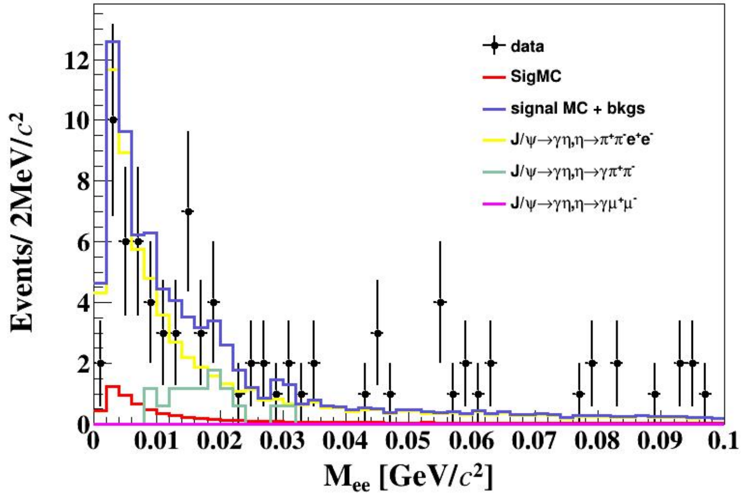
Back up

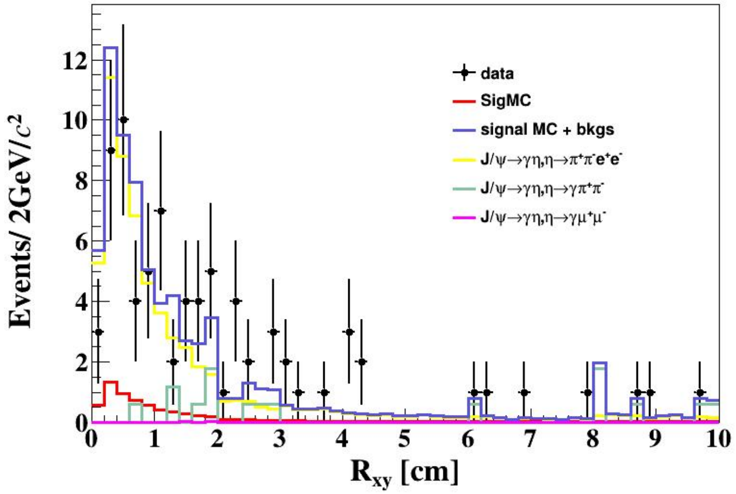
- The distribusiton of M_{2e} ， R_{xy} before photon conversion veto for $\eta \to e^+e^-\mu^+\mu^-$:





- The distribusiton of M_{2e} ， R_{xy} after photon conversion veto for $\eta \to e^+e^-\mu^+\mu^-$:





- photon conversion backgrounds (gpipi,gmumu) have already suppessed.

Back up

- systematic uncertainty of **Fit range** for $\eta \to e^+e^-\mu^+\mu^-$: