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Method of region

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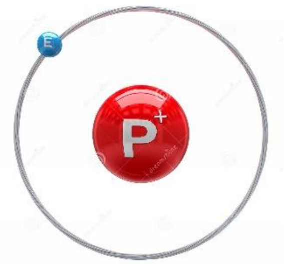
时间：2023年6月20日

回答上次组会遗留问题。
粲偶素视为非相对论系统。

氢原子是最简单的含多标度的非相对论性束缚态



基态束缚能

$$E_{binding} \sim mc^2\alpha^2$$

$$< T > = -\frac{1}{2} < V > \sim mv^2$$ 维里定理

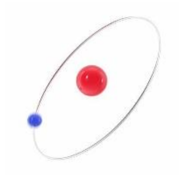
因此 $\frac{v}{c} \sim \alpha = \frac{1}{137} \ll 1$

电子的特征动量(玻尔动量)： $p = \frac{\hbar}{m\, a_0} \sim mv \sim mc\alpha$

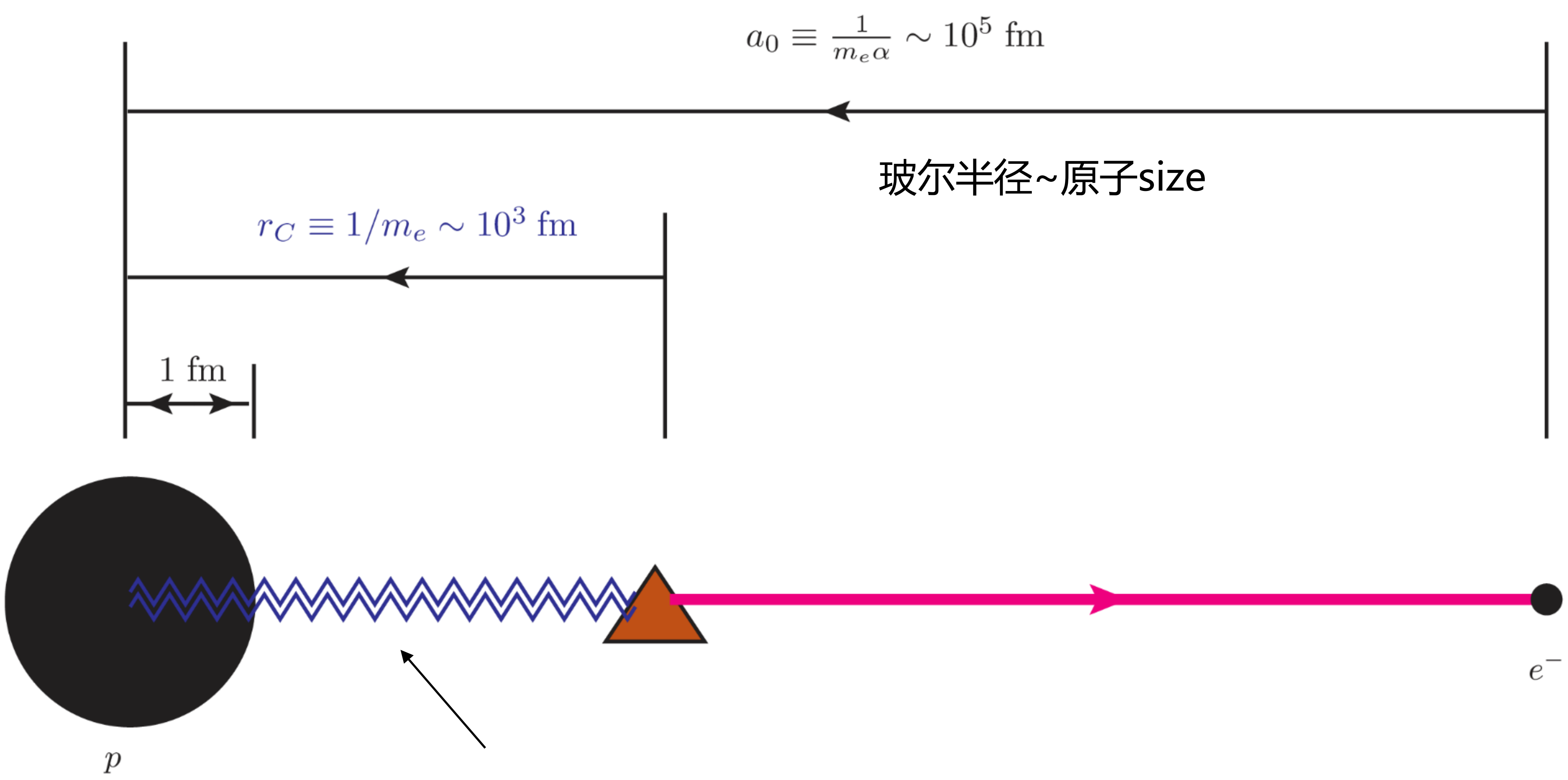
氢原子含有三个分得很开的特征能标：

$$\boldsymbol{m} \gg \boldsymbol{mv} \gg \boldsymbol{mv^2}$$

MeV ***keV*** ***eV***



氢原子的三个特征长度



e-p距离小于电子康普顿波长时，薛定谔方程失效!

Three characterstic lengths scales $r_{\text{proton}} \ll r_C \ll a_0$

实验证据：为什么重夸克偶素是非相对论性束缚态

重夸克质量：$\mathbf{M_c = 1.5\ GeV,\ M_b = 4.18\ GeV \gg \Lambda_{QCD}}$

QCD特征能标

1. **重夸克偶素质量大约是两倍重夸克质量**：

束缚能约为几百MeV, 远小于重夸克质量

2. 基态粲偶素质量劈裂： $M_{J/\psi(^3S_1)} - M_{\eta_c(^1S_0)} = 113\ MeV;$

基态底偶素质量劈裂： $M_{\Upsilon(^3S_1)} - M_{\eta_b(^1S_0)} = 62\ MeV;$

P波态$\chi_{c0,1,2}$之间质量劈裂: $96, 46\ MeV$, $\chi_{b0,1,2}$ 之间质量劈裂: $33, 20 MeV$

远小于典型的束缚能

实验表明**自旋依赖**的夸克-反夸克相互作用被严重压低。
因此主导的binding force是自旋无关(spin-independent)的！

(类比于氢原子中主导的是自旋无关的Coulomb势)

Rule of thumb: 估算重夸克偶素的三个特征能标

第一径向激发和基态的质量劈裂： $M_{\psi(2S)} - M_{J/\psi}\ , M_{\Upsilon(2S)} - M_{\Upsilon} = 600\ MeV$

第一轨道激发和基态的质量劈裂： $M_{\chi_{cJ}(^3P_J)} - M_{J/\psi}\ , M_{\chi_{bJ}(^3P_J)} - M_{\Upsilon} = 400\ MeV$

取中心值作为估计， $\boldsymbol{mv^2 = 500\ MeV}$

因此 $\boldsymbol{v^2 = \frac{500}{1500} = \frac{1}{3}}$ **for charmonium**

$\boldsymbol{v^2 = \frac{500}{4700} = \frac{1}{9}}$ **for bottomonium**

几何平均

	$c\bar{c}$	$b\bar{b}$	$t\bar{t}$
M	1.5 GeV	4.7 GeV	180 GeV
Mv	0.9 GeV	1.5 GeV	16 GeV
Mv^2	0.5 GeV	0.5 GeV	1.5 GeV

$\boldsymbol{m \gg mv \gg mv^2}$

Thus v can acts expansion parameter

Method of region

- 一、动量积分区域展开思想
- 二、二维标量粒子的自能修正
- 三、无质量粒子的顶点修正
- 四、具有相等非零质量的单圈费曼图

主要思想

1、先将整个积分动量分成不同的能量区域；

2、在分割的区域内进行幂次展开；

3、最后将各个部分的贡献求和起来。

方便理解SECT 和 NRQCD 等有效理论

(i) divide the integration domain into regions of small and large momenta and use leading approximations for the integrand (i.e. take the zero-order terms in Taylor series in the small parameters) (see (1.8));
(ii) extend the integration to *all* loop momenta in the expanded contribution of every region.

In this book, the strategy of expansion by regions will be formulated for

- arbitrary order of expansion,
- a general diagram,
- any limit.

Beneke M, Smirnov V A. Asymptotic expansion of Feynman integrals near threshold[J]. Nucl. Phys. B, 1998, 522: 321-344. arXiv: hep-ph/9711391. DOI: 10.1016/S0550-3213(98)00138-2.

Semenova T Y, Smirnov A V, Smirnov V A. On the status of expansion by regions[J]. Eur. Phys. J. C, 2019, 79(2): 136. arXiv: 1809.04325 [hep-th]. DOI: 10.1140/epjc/s10052-019-6653-3.

Smirnov V A. Asymptotic expansions in limits of large momenta and masses[J]. Commun. Math. Phys., 1990, 134: 109-137. DOI: 10.1007/BF02102092.

Introduction to Soft-Collinear Effective Theory
Thomas Becher (U. Bern, AEC), Alessandro Broggio (PSI, Villigen), Andrea Ferroglia (New York City Coll. Tech. and CUNY, Graduate School - U. Ctr.) (Oct 7, 2014)
Published in: *Lect.Notes Phys.* 896 (2015) pp.1-206 • e-Print: 1410.1892 [hep-ph]

目录

二维标量粒子的自能修正

假设在二维空间中，可以存在两个可以发生相互作用的标量粒子，他们的自能图修正为：

$$I=\int_0^{\infty}dk\frac{k}{(k^2+m^2)(k^2+M^2)}=\frac{ln(M/m)}{M^2-m^2}$$

在$m\ll M$的前提下，可将振幅进行幂次（$0<\lambda=\frac{m^2}{M^2}<1$）展开：

$$I=\frac{\ln\left(\frac{M}{m}\right)}{M^2}\left(1+\frac{m^2}{M^2}+\frac{m^4}{M^4}+\cdots\right)=\frac{\ln\left(\frac{M}{m}\right)}{M^2}(1+\lambda+\lambda^2+\cdots)$$

引入新标度Λ：

$$I=\underbrace{\int_0^{\Lambda}dk\frac{k}{(k^2+m^2)(k^2+M^2)}}_{I_{(I)}}+\underbrace{\int_{\Lambda}^{\infty}dk\frac{k}{(k^2+m^2)(k^2+M^2)}}_{I_{(II)}}$$

二维标量粒子的自能修正

低能区域 $[0,\Lambda]$：$k \sim m \ll M$

$$I_{(I)} = \int_0^\Lambda dk \frac{k}{(k^2+m^2)(k^2+M^2)}$$

$$= \int_0^\Lambda dk \frac{k}{(k^2+m^2)M^2}\left(1-\frac{k^2}{M^2}+\frac{k^4}{M^4}+\cdots\right)$$

高能区域 $[\Lambda,\infty]$：$m \ll k \sim M$

$$I_{(II)} = \int_\Lambda^\infty dk \frac{k}{(k^2+m^2)(k^2+M^2)}$$

$$= \int_\Lambda^\infty dk \frac{k}{k^2(k^2+M^2)}\left(1-\frac{m^2}{k^2}+\frac{m^4}{k^4}+\cdots\right)$$

将低能区贡献的前两项进行积分:

$$I_{(I)} \approx \frac{M^2+m^2}{2M^4} ln\left(1+\frac{\Lambda^2}{m^2}\right) - \frac{\Lambda^2}{2M^2}$$

$$= -\frac{1}{M^2} ln\left(\frac{m}{\Lambda}\right) - \frac{\Lambda^2}{2M^4} + \mathcal{O}\left(\frac{\Lambda^4}{M^6}, \frac{m^2}{M^4} ln\left(\frac{\Lambda}{m}\right)\right)$$

将高能区贡献的第一项进行积分:

$$I_{(II)} \approx \frac{1}{2M^2} ln\left(1+\frac{M^2}{\Lambda^2}\right) = \frac{1}{M^2} ln\left(\frac{M}{\Lambda}\right) + \frac{\Lambda^2}{2M^2} + \mathcal{O}\left(\frac{\Lambda^4}{M^6}, ln\left(\frac{M}{\Lambda}\right)\right)$$

二维标量粒子的自能修正

将两个区域的贡献加到一起

$$I = I_{(I)} + I_{(II)} = \frac{ln(M/m)}{M^2} + \mathcal{O}(\frac{m^2}{M^4} ln(\frac{M}{m}))$$

以维数正规化的方法再次说明 method of region

$$I = \int_0^\infty dk\, k^{-\varepsilon} \frac{k}{(k^2 + m^2)(k^2 + M^2)}$$

仅考虑动量大小部分，没有考虑 d 维角度的影响，利用 Γ 函数的定义：

$$I = \frac{1}{2}\Gamma(1 - \frac{\varepsilon}{2})\Gamma(\frac{\varepsilon}{2}) \frac{m^{-\varepsilon} - M^{-\varepsilon}}{M^2 - m^2}$$

二维标量粒子的自能修正

低能区域 $k \sim m \ll M$，展开式可通过取 $\varepsilon > 0$ 正规化 UV

$$I_{(I)} = \int_0^\infty dk\, k^{-\varepsilon} \frac{k}{(k^2+m^2)M^2}\left(1 - \frac{k^2}{M^2} + \frac{k^4}{M^4} + \cdots\right)$$

高能区域 $m \ll k \sim M$，展开式可通过取 $\varepsilon < 0$ 正规化 IR

$$I_{(II)} = \int_0^\infty dk\, k^{-\varepsilon} \frac{k}{k^2(k^2+M^2)}\left(1 - \frac{m^2}{k^2} + \frac{m^4}{k^4} + \cdots\right)$$

在领头阶，第一项积分分别为：

$$I_{(I)} = \frac{m^{-\varepsilon}}{2M^2}\Gamma\left(1-\frac{\varepsilon}{2}\right)\Gamma\left(\frac{\varepsilon}{2}\right) = \frac{1}{M^2}\left(\frac{1}{\varepsilon} - \ln m + \mathcal{O}(\varepsilon)\right)$$

$$I_{(II)} = -\frac{M^{-\varepsilon}}{2M^2}\Gamma\left(1-\frac{\varepsilon}{2}\right)\Gamma\left(\frac{\varepsilon}{2}\right) = \frac{1}{M^2}\left(-\frac{1}{\varepsilon} + \ln M + \mathcal{O}(\varepsilon)\right)$$

$$I = I_{(I)} + I_{(II)} = \frac{1}{2}\Gamma\left(1-\frac{\varepsilon}{2}\right)\Gamma\left(\frac{\varepsilon}{2}\right)\frac{m^{-\varepsilon} - M^{-\varepsilon}}{M^2 - m^2} = \frac{\ln(M/m)}{M^2}$$

二维标量粒子的自能修正

证明低能区域与高能区域无重合:

$$I^{\Lambda}{}_{(I)} = \int_0^{\Lambda} dk \frac{k^{1-\varepsilon}}{(k^2+m^2)M^2}(1-\frac{k^2}{M^2}+\frac{k^4}{M^4}+\cdots)$$

$$= \left[\int_0^{\infty} dk - \int_{\Lambda}^{\infty} dk\right] \frac{k^{1-\varepsilon}}{(k^2+m^2)M^2}(1-\frac{k^2}{M^2}+\frac{k^4}{M^4}+\cdots)$$

$$= I_{(I)} - R_{(I)}$$

有相同的第一项 $I_{(I)}$,和与截断参数依赖的 $R_{(I)}$，当 $k \gg \Lambda \gg m^2$ 时:

$$R_{(I)} = \int_{\Lambda}^{\infty} dk \frac{k^{1-\varepsilon}}{(k^2+m^2)M^2}(1-\frac{k^2}{M^2}+\cdots)$$

$$= \int_{\Lambda}^{\infty} dk \frac{k^{1-\varepsilon}}{k^2M^2}(1-\frac{m^2}{k^2}-\frac{k^2}{M^2}+\cdots)$$

二维标量粒子的自能修正

同样的，在高能区贡献引入一个低能截断能标 Λ，可以类似的得到：

$$R_{(II)} = \int_0^{\Lambda} dk\, k^{-\varepsilon} \frac{k}{k^2(k^2+M^2)}\left(1-\frac{m^2}{k^2}+\cdots\right)$$

$$= \int_0^{\Lambda} dk\, k^{-\varepsilon} \frac{k}{k^2 M^2}\left(1-\frac{m^2}{k^2}-\frac{k^2}{M^2}+\cdots\right)$$

$$R_{(I)} + R_{(II)} = \int_0^{\infty} dk\, k^{-\varepsilon} \frac{k}{k^2 M^2}\left(1-\frac{m^2}{k^2}-\frac{k^2}{M^2}+\cdots\right)$$

R不依赖截断参数，无标度积分为0，因此可证高能区域与低能区域无重叠。

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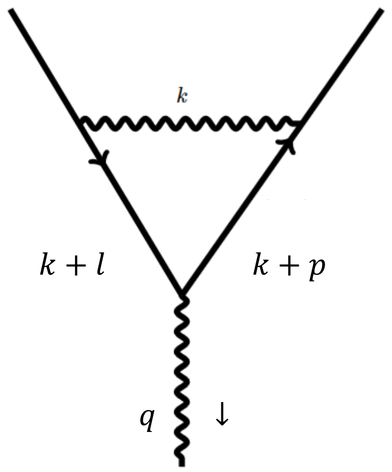
无质量粒子的顶点修正

method of region 步骤

i)找出所有给出领头阶贡献的动量区域
ii)将被积分式进行幂次展开，然后在全空间积分；
iii)将不同积分结果加起来得到完整的积分结果。

$$I = i\pi^{d/2}\mu^{4-d}\int d^d k \frac{1}{(k^2+i\epsilon)[(k+l)^2+i\epsilon][(k+p)^2+i\epsilon]}$$



(a) 标量量子电动力学中的单圈图顶点修正

引入以下的记号： $L^2 \equiv -l^2 - i\epsilon\,, P^2 \equiv -p^2 - i\epsilon\,, Q^2 \equiv -(l-p)^2 - i\epsilon\,,$

在 $L^2 \sim P^2 \ll Q^2$ **计算上式积分。为了方便计算，引入两个类光矢量：**

$$n_+^\mu = (1,0,0,1), \qquad n_-^\mu = (1,0,0,-1)$$

满足正交归一化条件：

$$n_+^2 = n_-^2 = 0, \qquad n_+\cdot n_- = 2$$

任意一个矢量可以投影到 n_+ , n_- 方向，以及垂直方向：

$$p^\mu = (n_+\cdot p)\frac{n_-^\mu}{2} + (n_-\cdot p)\frac{n_+^\mu}{2} + p_\perp^\mu \equiv p_+^\mu + p_-^\mu + p_\perp^\mu$$

方便标记，引入：

$$p^\mu = (n_+\cdot p, \quad n_-\cdot p, \quad p_\perp^\mu)$$

进行光锥投影可以将动量等矢量各分量的不同幂次展示出来：

$$p^2 = (n_+\cdot p)(n_-\cdot p) + p_\perp^2$$

$$p\cdot q = (p_+\cdot q_-) + (p_-\cdot q_+) + p_\perp q_\perp$$

引入幂次展开参数：

$$\lambda^2 \sim \frac{P^2}{Q^2} \sim \frac{L^2}{Q^2} \qquad p^2 \sim l^2 \sim \lambda^2 Q^2$$

可以选择动量方向，使得 $p^\mu \approx Qn_+^\mu/2$, $l^\mu \approx Qn_-^\mu/2$ 这样动量 p, l 可以分解为

$$p^\mu \sim (\lambda^2, 1, \lambda)Q\,, \qquad l^\mu \sim (1, \lambda^2, \lambda)Q$$

hard : $k^\mu \sim (1, 1, 1)Q$
soft : $k^\mu \sim (\lambda^2, \lambda^2, \lambda^2)Q$
collinear (p): $k^\mu \sim (\lambda^2, 1, \lambda)Q$
collinear (l): $k^\mu \sim (1, \lambda^2, \lambda)Q$

当被积动量属于 hard region 时，显然 $k^2 \sim \lambda^0 Q^2$ **，另外两个传播子可展开为：**

$$(k+l)^2 = k^2 + 2(\underbrace{k_+ \cdot l_-}_{\mathcal{O}(1)} + \underbrace{k_- \cdot l_+}_{\mathcal{O}(\lambda^2)} + \underbrace{k_\perp \cdot l_\perp}_{\mathcal{O}(1)}) + \underbrace{l^2}_{\mathcal{O}(\lambda)}\ \underbrace{}_{\mathcal{O}(\lambda^2)}$$

$$= k^2 + 2\, k_- \cdot l_+ + \mathcal{O}(\lambda)$$

$$(k+p)^2 = k^2 + 2\, k_+ \cdot p_- + \mathcal{O}(\lambda)$$

该区域对积分贡献为：

$$I = i\pi^{d/2}\mu^{4-d} \int d^d k \frac{1}{(k^2+i\epsilon)(k^2+2\, k_- \cdot l_+ + i\epsilon)(k^2 + k_+ \cdot p_- + i\epsilon)}$$

积分结果为：

$$I_h = \frac{\Gamma(1+\varepsilon)}{2 l_+ \cdot p_-} \frac{\Gamma(-\varepsilon)}{\Gamma(1-2\varepsilon)} \left(\frac{\mu^2}{2\, l_+ \cdot p_-}\right)^{\varepsilon}$$

$$= \frac{\Gamma(1+\varepsilon)}{Q^2} \left(\frac{1}{\varepsilon^2} + \frac{1}{\varepsilon} ln \frac{\mu^2}{Q^2} + \frac{1}{2} ln^2 \frac{\mu^2}{Q^2} - \frac{\pi^2}{6}\right) + \mathcal{O}(\varepsilon)$$

(其中极点来自IR region)

与 p 平行的 region，$k^\mu \sim (\lambda^2, 1, \lambda)Q$， $k^2 \sim \lambda^2 Q^2$

$$(k+l)^2 = 2\,k_- \cdot l_+ + \mathcal{O}(\lambda^2)\,, \qquad (k+p)^2 = \mathcal{O}(\lambda^2)$$

该区域对积分贡献为：

$$I_c = i\pi^{d/2}\mu^{4-d}\int d^d\,k \frac{1}{(k^2+i\epsilon)(2\,k_-\cdot l_+ + i\epsilon)[(k+p)^2+i\epsilon]}$$

$$= -\frac{\Gamma(1+\varepsilon)}{2l_+\cdot p_-}\frac{\Gamma^2(-\varepsilon)}{\Gamma(1-2\varepsilon)}\left(\frac{\mu^2}{P^2}\right)^\varepsilon$$

$$= \frac{\Gamma(1+\varepsilon)}{Q^2}\left(-\frac{1}{\varepsilon^2} - \frac{1}{\varepsilon} ln\frac{\mu^2}{P^2} - \frac{1}{2} ln^2\frac{\mu^2}{P^2} + \frac{\pi^2}{6}\right) + \mathcal{O}(\varepsilon)$$

整个积分结果正比于$P^{-2\varepsilon}$。与 l 平行的 region 给出的贡献可由上式将 P^2 替换成 L^2

对于 soft region，所有的动量分量都是 λ^2

$$k^2 = \mathcal{O}(\lambda^4)\,, \qquad (k+l)^2 = 2\,k_-\cdot l_+ + l^2 + \mathcal{O}(\lambda^3)\,, \qquad (k+p)^2 = 2\,k_+\cdot p_- + p^2 + \mathcal{O}(\lambda^3)$$

$$I_s = i\pi^{d/2}\mu^{4-d}\int d^d\,k \frac{1}{(k^2+i\epsilon)(2\,k_-\cdot l_+ + l^2 + i\epsilon)[2\,k_+\cdot p_- + p^2 + i\epsilon]}$$

$$= -\frac{\Gamma(1+\varepsilon)}{2l_+\cdot p_-}\Gamma(\varepsilon)\Gamma(-\varepsilon)(\frac{2l_+\cdot p_-\mu^2}{L^2P^2})^{\varepsilon}$$

$$= \frac{\Gamma(1+\varepsilon)}{Q^2}(\frac{1}{\varepsilon^2}+\frac{1}{\varepsilon}ln\frac{\mu^2Q^2}{L^2P^2}+\frac{1}{2}ln^2\frac{\mu^2Q^2}{L^2P^2}+\frac{\pi^2}{6})+\mathcal{O}(\varepsilon)$$

(其中极点来自UV region)

$$I_h = \frac{\Gamma(1+\varepsilon)}{Q^2}(\frac{1}{\varepsilon^2}+\frac{1}{\varepsilon}ln\frac{\mu^2}{Q^2}+\frac{1}{2}ln^2\frac{\mu^2}{Q^2}-\frac{\pi^2}{6})+\mathcal{O}(\varepsilon)$$

$$I_c = \frac{\Gamma(1+\varepsilon)}{Q^2}(-\frac{1}{\varepsilon^2}-\frac{1}{\varepsilon}ln\frac{\mu^2}{P^2}-\frac{1}{2}ln^2\frac{\mu^2}{P^2}+\frac{\pi^2}{6})+\mathcal{O}(\varepsilon)$$

$$I_{\bar{c}} = \frac{\Gamma(1+\varepsilon)}{Q^2}(-\frac{1}{\varepsilon^2}-\frac{1}{\varepsilon}ln\frac{\mu^2}{L^2}-\frac{1}{2}ln^2\frac{\mu^2}{L^2}+\frac{\pi^2}{6})+\mathcal{O}(\varepsilon)$$

$$I_s = \frac{\Gamma(1+\varepsilon)}{Q^2}(\frac{1}{\varepsilon^2}+\frac{1}{\varepsilon}ln\frac{\mu^2Q^2}{L^2P^2}+\frac{1}{2}ln^2\frac{\mu^2Q^2}{L^2P^2}+\frac{\pi^2}{6})+\mathcal{O}(\varepsilon)$$

$$I_h + I_c + I_{\bar{c}} + I_s = \frac{1}{Q^2}(ln\frac{Q^2}{L^2}ln\frac{Q^2}{P^2}+\frac{\pi^2}{3})+\mathcal{O}(\lambda)$$

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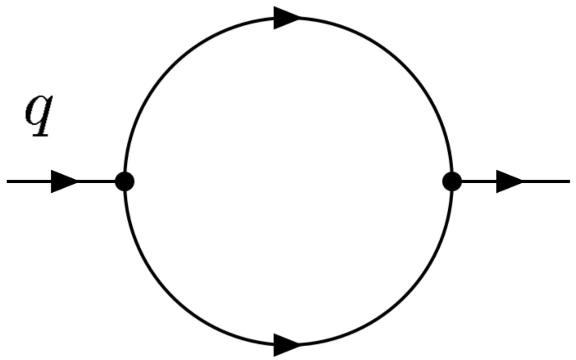
具有相等非零质量的单圈费曼图

该图贡献的费曼传播子积分形式为：

$$F(q^2, y: d) = \int \frac{d^d k}{[(\frac{q}{2}+k)^2 - m^2][(\frac{q}{2}-k)^2 - m^2]}$$



在NRQCD框架下：

hard : $(k^0, \boldsymbol{k}) \sim (m, m)$

soft : $(k^0, \boldsymbol{k}) \sim (mv, mv)$

potential : $(k^0, \boldsymbol{k}) \sim (mv^2, mv)$

altrasoft : $(k^0, \boldsymbol{k}) \sim (mv^2, mv^2)$

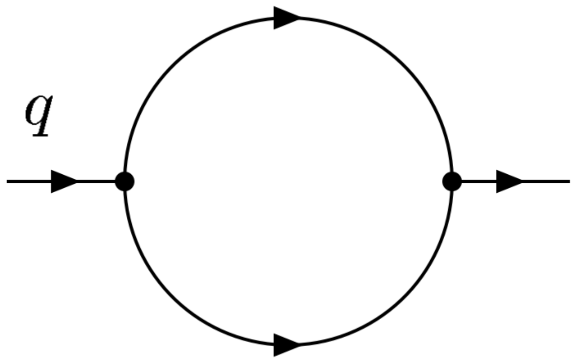
Vladimir A. Smirnov: Applied Asymptotic Expansions in Momenta and Masses, STMP 177, 1–15 (2002)

具有相等非零质量的单圈费曼图

该图贡献的费曼传播子积分形式为：

$$F(q^2, y: d) \equiv \int \frac{d^d k}{[(\frac{q}{2}+k)^2 - m^2][(\frac{q}{2}-k)^2 - m^2]}$$

$$= \int \frac{d^d k}{(k^2 + q\cdot k + \frac{q^2}{4} - m^2)(k^2 - q\cdot k + \frac{q^2}{4} - m^2)}$$



小量展开参数定义为 $y = m^2 - \frac{q^2}{4}$, 以 $\lambda \sim \frac{\sqrt{y}}{q} \ll 1$ 作为*展开参数*

hard $\sim$ $(k^0, \boldsymbol{k}) \sim (q, q)$

soft : $(k^0, \boldsymbol{k}) \sim (\sqrt{y}, \sqrt{y}) \sim (\lambda q, \lambda q)$

potential : $(k^0, \boldsymbol{k}) \sim (\frac{y}{\sqrt{q^2}}, \sqrt{y}) \sim (\lambda^2 q, \lambda q)$

altrasoft : $(k^0, \boldsymbol{k}) \sim (\frac{y}{\sqrt{q^2}}, \frac{y}{\sqrt{q^2}}) \sim (\lambda^2 q, \lambda^2 q)$

$$F^{(h)} = \int d^d k \frac{1}{[k^2 + q \cdot k - y][k^2 - q \cdot k - y]}$$

$$= \int \frac{d^d k}{[k^2 + q \cdot k][k^2 - q \cdot k]} + \cdots$$

$$= i\pi^{d/2} \left(\frac{4}{q^2}\right)^{\varepsilon} \sum_{n=0}^{\infty} \frac{\Gamma(n+\varepsilon)}{n!\,(1 - 2\varepsilon - 2n)} \left(\frac{-4y}{q^2}\right)^2$$

$$F^{(s)} = \int d^d k \frac{1}{[q \cdot k + i0][-q \cdot k + i0]} + \cdots$$

$$= -\int d^d k \frac{1}{[q \cdot k + i0][-q \cdot k - i0]} + \cdots = 0$$

$$F^{(us)} = \int d^d k \frac{1}{[q \cdot k - y + i0][q \cdot k + y - i0]} + \cdots = 0$$

Keeping in mind the non-relativistic flavour of the problem. q= $\{q_0\ ,\mathbf{0}\}$

$$F^{(p)}=\int\frac{dk_0d^{d-1}\boldsymbol{k}}{[\boldsymbol{k}^2-k_0^2+q_0\cdot k_0+y-i0][\boldsymbol{k}^2-k_0^2-q_0\cdot k_0+y-i0]}$$

$$=\int\frac{dk_0d^{d-1}\boldsymbol{k}}{[\boldsymbol{k}^2+q_0\cdot k_0+y-i0][\boldsymbol{k}^2-q_0\cdot k_0+y-i0]}+\cdots$$

$$=\frac{i\pi}{\sqrt{q^2}}\int\frac{d^{d-1}\boldsymbol{k}}{(\boldsymbol{k}^2+y)}=i\pi^{d/2}\Gamma(\varepsilon-1/2)\sqrt{\frac{\pi y}{q^2}}y^{-\varepsilon}$$

$$F^{(p)}=i\pi^{d/2}\Gamma(\varepsilon-1/2)\sqrt{\frac{\pi y}{q^2}}y^{-\varepsilon}$$

$$F(q^2, y:\ d) = i\pi^{d/2}\Gamma(\varepsilon) y^{-\varepsilon}\ {}_2F_1\left(\frac{1}{2}, \varepsilon; \frac{3}{2}; -\frac{q^2}{4y}\right)$$

$$\begin{aligned}{}_2F_1(a,b;c;z) &= \frac{\Gamma(c)\Gamma(b-a)}{\Gamma(b)\Gamma(c-a)}(-z)^{-a}\ {}_2F_1\left(a, 1-c+a; 1-b+a; \frac{1}{z}\right) \\ &+ \frac{\Gamma(c)\Gamma(a-b)}{\Gamma(a)\Gamma(c-b)}(-z)^{-b}\ {}_2F_1\left(b, 1-c+b; 1-a+b; \frac{1}{z}\right)\end{aligned}$$

$$F(q^2, y; d) = i\pi^{d/2}\Gamma(\varepsilon) y^{-\varepsilon} F_1\left(\frac{1}{2}, \varepsilon; \frac{3}{2}; \frac{-q^2}{4y}\right)$$

$$= i\pi^{d/2}\Gamma(\varepsilon) y^{-\varepsilon}\left[\frac{\Gamma(\frac{3}{2})\Gamma(\varepsilon-\frac{1}{2})}{\Gamma(\varepsilon)\Gamma(\frac{3}{2}-\frac{1}{2})}\sqrt{\frac{4y}{q^2}}\ {}_2F_1\left(\frac{1}{2}, 0, \frac{3}{2}+\varepsilon; -\frac{4y}{q^2}\right)\right.$$

$$\left.+ \frac{\Gamma(\frac{3}{2})\Gamma(\frac{1}{2}-\varepsilon)}{\Gamma(\frac{1}{2})\Gamma(\frac{3}{2}-\varepsilon)}\left(\frac{4y}{q^2}\right)^{\varepsilon}{}_2F_1\left(\varepsilon, \varepsilon-\frac{1}{2}, \frac{1}{2}+\varepsilon; -\frac{4y}{q^2}\right)\right]$$

$$= i\pi^{d/2}\Gamma(\varepsilon) y^{-\varepsilon}\left[\frac{\frac{\sqrt{\pi}}{2}\Gamma(\varepsilon-\frac{1}{2})}{\Gamma(\varepsilon)\Gamma(1)}\sqrt{\frac{4y}{q^2}}\cdot 1\right.$$

$$\left.+ \frac{\frac{\sqrt{\pi}}{2}\cdot\Gamma(\frac{1}{2}-\varepsilon)}{\sqrt{\pi}\cdot(\frac{1}{2}-\varepsilon)\Gamma(\frac{1}{2}-\varepsilon)}\left(\frac{4y}{q^2}\right)^{\varepsilon}\sum_{n=0}^{\infty}\frac{(\varepsilon)_n(\varepsilon-\frac{1}{2})_n}{(\varepsilon+\frac{1}{2})_n}\left(\frac{-4y}{q^2}\right)^n\right]$$

$$\Gamma(\varepsilon+\tfrac{1}{2}) = \Gamma(\varepsilon+\tfrac{1}{2}+\tfrac{1}{2}-\tfrac{1}{2}) = \Gamma(\varepsilon-\tfrac{1}{2}+1) = (\varepsilon-\tfrac{1}{2})\Gamma(\varepsilon-\tfrac{1}{2})$$

$$\Gamma(\varepsilon+\tfrac{1}{2}+n) = \Gamma(\varepsilon+\tfrac{1}{2}+\tfrac{1}{2}-\tfrac{1}{2}+n) = \Gamma(\varepsilon+n-\tfrac{1}{2}+1) = (\varepsilon+n-\tfrac{1}{2})\Gamma(\varepsilon+n-\tfrac{1}{2})$$

$$= i\pi^{d/2}\Gamma(\varepsilon) y^{-\varepsilon}\left[\frac{\Gamma(\varepsilon-\frac{1}{2})}{\Gamma(\varepsilon)}\sqrt{\frac{\pi y}{q^2}} + \frac{1}{2(\frac{1}{2}-\varepsilon)}\sum_{n=0}^{\infty}\frac{\Gamma(\varepsilon+n)}{\Gamma(\varepsilon)}\cdot\frac{\Gamma(\varepsilon-\frac{1}{2}+n)}{\Gamma(\varepsilon-\frac{1}{2})}\cdot\frac{\Gamma(\varepsilon+\frac{1}{2})}{\Gamma(\varepsilon+\frac{1}{2}+n)}\cdot\frac{1}{n!}\cdot\left(-\frac{4y}{q^2}\right)^n\right]$$

$$= i\pi^{d/2} y^{-\varepsilon}\Gamma(\varepsilon-\tfrac{1}{2})\sqrt{\frac{\pi y}{q^2}} + i\pi^{d/2}\left(\frac{4}{q^2}\right)^{\varepsilon}\sum_{n}^{\infty}\frac{1}{n!}\left(-\frac{4y}{q^2}\right)^n\cdot\frac{\Gamma(\varepsilon+n)}{(1-2\varepsilon)}\cdot\frac{\Gamma(\varepsilon-\frac{1}{2}+n)}{\Gamma(\varepsilon-\frac{1}{2})}\cdot\frac{(\varepsilon-\frac{1}{2})\Gamma(\varepsilon-\frac{1}{2})}{(\varepsilon+n-\frac{1}{2})\Gamma(\varepsilon+n-\frac{1}{2})}$$

$$= \underbrace{i\pi^{d/2} y^{-\varepsilon}\Gamma(\varepsilon-\tfrac{1}{2})\sqrt{\frac{\pi y}{q^2}}}_{\text{"potential region"}} + \underbrace{i\pi^{d/2}\left(\frac{4}{q^2}\right)^{\varepsilon}\sum_{n=0}^{\infty}\frac{\Gamma(n+\varepsilon)}{n!(1-2\varepsilon-2n)}\left(-\frac{4y}{q^2}\right)^n}_{\text{"hard region"}}$$

优点：1、简化计算

2、分离积分中的各种标度，方便研究某一特定问题。

3、对物理理解更为清晰。例如直观的得到一个圈积分的紫外或者红外发散行为。

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PHYSICAL REVIEW D **82,** 014008 (2010)

Hard-scattering approach to strongly hindered magnetic-dipole transitions in quarkonium

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(Received 9 February 2009; published 19 July 2010)

For a class of hindered magnetic-dipole ($M1$) transition processes, such as $\Upsilon(3S) \to \eta_b + \gamma$ (the discovery channel of the η_b meson), the emitted photon is rather energetic so that the traditional approaches based on multipole expansion may be invalidated. We propose that a "hard-scattering" picture, somewhat analogous to the pion electromagnetic form factor at large momentum transfer, may be more plausible to describe such types of transition processes. We work out a simple factorization formula at lowest order in the strong coupling constant, which involves convolution of the Schrödinger wave functions of quarkonia with a perturbatively calculable part induced by exchange of one semihard gluon between quark and antiquark. This formula, without any freely adjustable parameters, is found to agree with the measured rate of $\Upsilon(3S) \to \eta_b + \gamma$ rather well, and can also reasonably account for other recently measured hindered $M1$ transition rates. The branching fractions of $\Upsilon(4S) \to \eta_b^{(\prime)} + \gamma$ are also predicted.

DOI: 10.1103/PhysRevD.82.014008 PACS numbers: 12.38.Bx, 12.39.Pn, 13.40.Gp

重夸克偶素的电磁跃迁

从NRQCD的拉氏量出发

$$\mathcal{L}_{\mathrm{NRQCD}} = \psi^\dagger \left\{ iD_0 + \frac{\mathbf{D}^2}{2m} + c_F\, g \frac{\boldsymbol{\sigma}\cdot\mathbf{B}}{2m} + c_D\, g \frac{[\mathbf{D}\cdot,\mathbf{E}]}{8m^2} + ic_S\, g \frac{\boldsymbol{\sigma}\cdot[\mathbf{D}\times,\mathbf{E}]}{8m^2} + \dots \right\}\psi$$

$\boldsymbol{D} = \nabla - ig_s\boldsymbol{A}_a t^a + ie\boldsymbol{A}^{em}$ (规范群为$SU(3)\times U(1)$)

其中E和B分别表示(色)电场和(色)磁场的场强

费米子和电磁场的相互作用项描述：

$$\mathbf{j}\cdot\mathbf{A}_{\mathrm{em}} = e_Q\psi^\dagger \left\{ \frac{\{\mathbf{D}\cdot,\mathbf{A}_{\mathrm{em}}\}}{2m} + (1+\kappa_Q)\frac{\boldsymbol{\sigma}\cdot\mathbf{B}_{\mathrm{em}}}{2m} + \dots \right\}\psi$$

其中$\mathbf{j}$是费米子流，$\mathbf{A}_{em}$是电磁场，κ_Q是夸克的反常磁矩。

$$\mathcal{M}(i\to f) = [\mathbf{M}^{(1)}(i\to f) + \mathbf{M}^{(2)}(i\to f)]\cdot\epsilon_\gamma(k)$$

$M^{(1)}$为光子从质量为m_1，电荷为e_1的夸克Q_1发射出的振幅，$M^{(2)}$为光子从质量为m_2电荷为 $-e_2$ 的夸克$\overline{Q_2}$发射出的振幅，

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重夸克偶素的电磁跃迁

电偶极跃迁E1的选择定则:

- 自旋不变 $\Delta S = 0$
- 轨道角动量$\Delta L = \pm 1$
- 总角动量$\Delta J = 0, \pm 1$
- P宇称改变

例子：$\chi_{c1}(^3P_1) \to J/\psi(^3S_1)\gamma$

磁偶极跃迁M1的选择定则:

- 自旋改变 $\Delta S = 1$
- 轨道角动量$\Delta L = 0$
- 总角动量$\Delta J = 0, \pm 1$
- P宇称不变

例子：$J/\psi(^3S_1) \to \eta_c(^1S_0)\gamma$

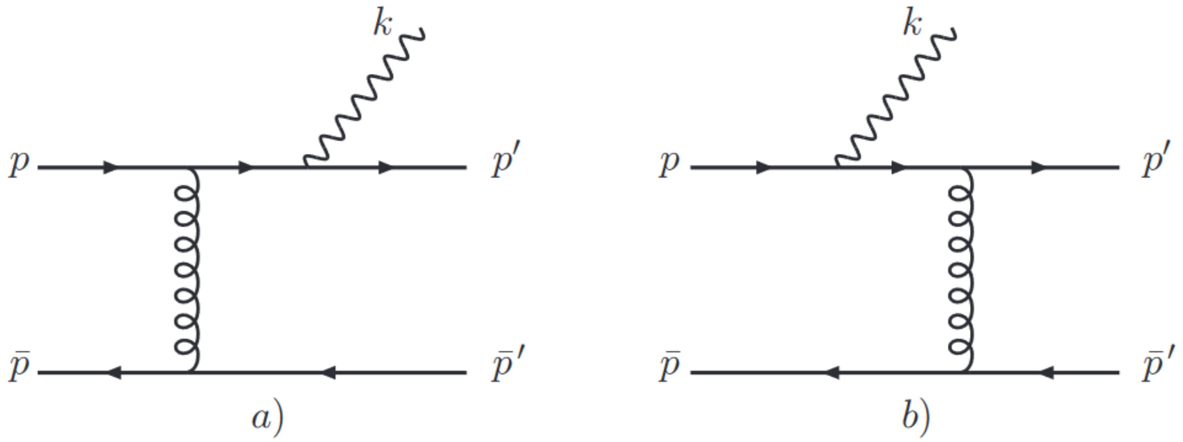
传统算电磁跃迁用的多级矩展开中：$k \sim mv^2$，　　$kr \sim v \ll 1$,

$\Upsilon(3S) \to \eta_b + \gamma$　　光子的动量：$k = 920MeV$

典型半径为: $r \sim \frac{1}{mv} \sim (833\mathrm{MeV})^{-1}$,　　$kr \sim 1$

在$\Upsilon(3S) \to \eta_b + \gamma$过程中，$kr \sim 1$，多极矩展开在这里明显就失效。

该过程 k 的动量不应视为处于$altrasoft\ region$



四个费曼图中的两个，其他两个可由费米子线反转得到。

TABLE I. Measured and predicted branching fractions of various hindered $M1$ transition processes $n^3S_1 \to n'^1S_0 + \gamma$ for bottomonium and charmonium. The photon momentum k is determined by physical kinematics. The total widths of $\Upsilon(nS)$ and $\psi(2S)$ states, as well as all the quarkonium masses, are taken from PDG08 compilation [13], except η_b mass is taken to be 9389 MeV [2], and $\eta_b(2S)$ mass is taken as 9997 MeV [3]. For $\Upsilon(2S) \to \gamma\eta_b$, we use the preliminary *BABAR* result [14]; for $\psi(2S) \to \gamma\eta_c$, we quote the latest CLEO measurement [15] instead of the world average value given in [13]. We have taken $\alpha_s(\mu) = 0.43$ and 0.59 for $\mu = 1.2$ and 0.9 GeV, respectively.

Decay modes	k (MeV)	$\mathcal{B}$ (Exp.)	α_s	$\mathcal{E}_{nn'}(\times 10^{-2})$		$\mathcal{B}$ (Our predictions)	
				Cornell	BT	Cornell	BT
$\Upsilon(2S) \to \gamma\eta_b$	614	$(4.2 \pm 1.4) \times 10^{-4}$	0.43	$3.7e^{i2.0^\circ}$	$3.2e^{i2.7^\circ}$	1.4×10^{-4}	1.1×10^{-4}
$\Upsilon(3S) \to \gamma\eta_b$	921	$(4.8 \pm 1.3) \times 10^{-4}$	0.43	$2.7e^{i2.6^\circ}$	$2.3e^{i3.5^\circ}$	3.7×10^{-4}	2.8×10^{-4}
$\Upsilon(4S) \to \gamma\eta_b$	1123	$\cdots$	0.43	$2.2e^{i2.8^\circ}$	$1.9e^{i3.7^\circ}$	4.3×10^{-7}	3.2×10^{-7}
$\Upsilon(4S) \to \gamma\eta_b(2S)$	566	$\cdots$	0.43	$1.7e^{i2.2^\circ}$	$1.6e^{i2.7^\circ}$	3.2×10^{-8}	2.7×10^{-8}
$\psi(2S) \to \gamma\eta_c$	638	$(4.3 \pm 0.6) \times 10^{-3}$	0.59	$6.4e^{i9.7^\circ}$	$5.7e^{i12.9^\circ}$	2.7×10^{-3}	2.1×10^{-3}

order correction to the hard-scattering kernel. To achieve this, it might prove easier to reformulate our derivation in the context of NRQCD. It would also be interesting to implement relativistic corrections to (6).

Obviously, our strategy need not be confined to hindered $M1$ transitions only. It should be applicable whenever the radiated photon cannot be viewed as ultrasoft and multipole expansion breaks down. It will be interesting to work out the corresponding factorization formula for $E1$ transitions such as $\chi_{bJ}(2P) \to \Upsilon\gamma$. It would also be interesting to generalize this hard-scattering formalism to explore the hadronic transition processes such as $\Upsilon(3S, 4S) \to \Upsilon + \pi\pi$.

We thank Antonio Vairo, Wei Wang, and Yu-Ming Wang for useful discussions. This research was supported in part by the National Natural Science Foundation of China under Grants No. 10875130, No. 10605031, and No. 10935012.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THANK YOU

静电势的多极矩展开
(multipole expansion)

Localized electric charge distribution with density $\rho(\vec{r})$.

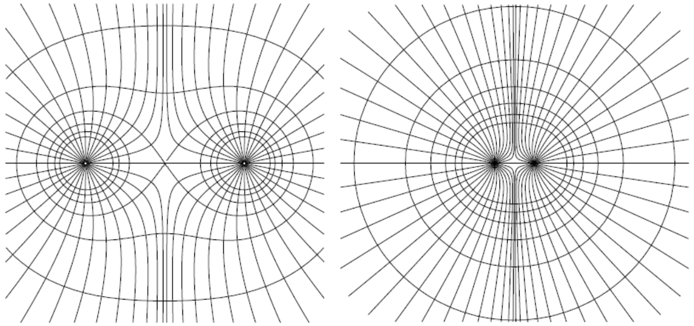
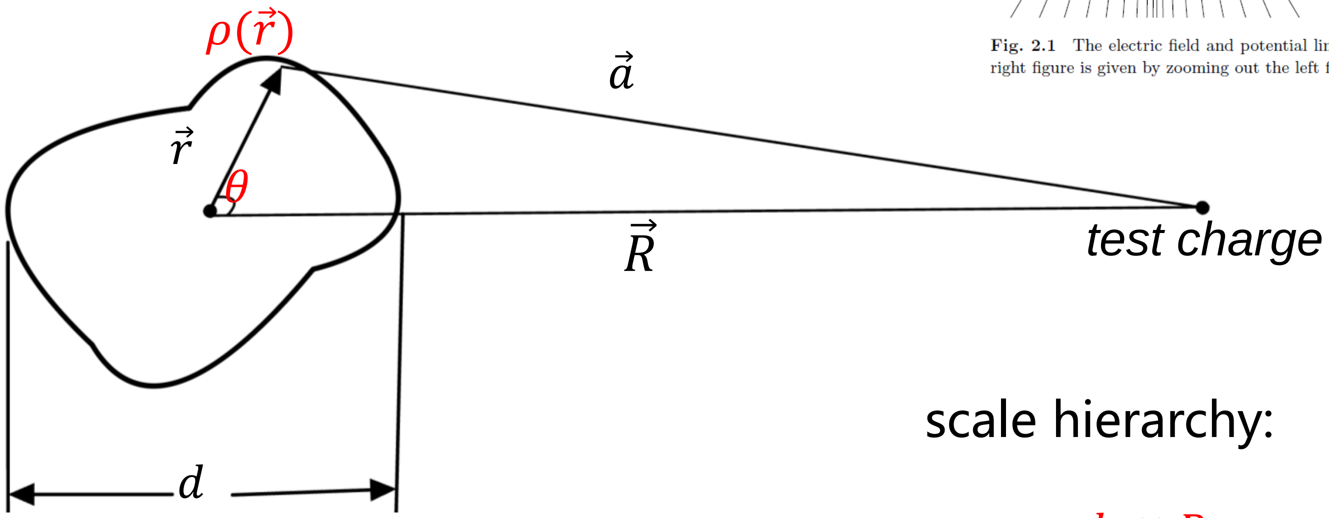


Fig. 2.1 The electric field and potential lines for two point charges of the same sign. The right figure is given by zooming out the left figure.



源的尺寸为d

scale hierarchy:

$d \ll R$

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静电势的多极矩展开
(multipole expansion)

$$V = \int d^3\,\vec{r}\,\frac{\rho(\vec{r})}{|\vec{a}|} = \int d^3 r\,\frac{\rho(\vec{r})}{\sqrt{R^2 + 2Rr cos\theta + R^2}}$$

非常复杂!

但利用$|\vec{r}| \sim d \ll R$， 做Taylor expansion in $\frac{r}{R}$.

$$V \approx \sum_{n=0}^{\infty} \frac{1}{R^{n+1}} \int d^3 r\, r^n \rho(\vec{r}) P_n(cos\theta)$$

$$= \frac{q}{R} + \frac{p}{R^2} + \frac{Q}{R^3} + \cdots$$

mono pole/charge　dipole　quadrupole

级数收敛很快

Wilson系数

Long-distance physics only sensitive to bulk properties :　q, p, Q, ...