PHYSICAL REVIEW D **82,** 014008 (2010)

Hard-scattering approach to strongly hindered magnetic-dipole transitions in quarkonium

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(Received 9 February 2009; published 19 July 2010)

For a class of hindered magnetic-dipole ($M1$) transition processes, such as $\Upsilon(3S) \to \eta_b + \gamma$ (the discovery channel of the η_b meson), the emitted photon is rather energetic so that the traditional approaches based on multipole expansion may be invalidated. We propose that a ''hard-scattering'' picture, somewhat analogous to the pion electromagnetic form factor at large momentum transfer, may be more plausible to describe such types of transition processes. We work out a simple factorization formula at lowest order in the strong coupling constant, which involves convolution of the Schrödinger wave functions of quarkonia with a perturbatively calculable part induced by exchange of one semihard gluon between quark and antiquark. This formula, without any freely adjustable parameters, is found to agree with the measured rate of $\Upsilon(3S) \to \eta_b + \gamma$ rather well, and can also reasonably account for other recently measured hindered $M1$ transition rates. The branching fractions of $\Upsilon(4S) \to \eta_b^{(\prime)} + \gamma$ are also predicted.

DOI: 10.1103/PhysRevD.82.014008 PACS numbers: 12.38.Bx, 12.39.Pn, 13.40.Gp

As a century-old subject, electromagnetic (EM) transitions have been extensively studied in the fields of atomic, nuclear, and elementary particle physics. EM transition is of considerable interest in heavy quarkonium physics from both experimental and theoretical aspects [1]. Experimentally, it proves to be a powerful tool to discover new quarkonium states that cannot be directly produced in e^+e^- annihilation into a virtual photon. A very recent example is that the long-sought bottomonium ground state, the η_b meson, was finally seen by the *BABAR* Collaboration in the magnetic dipole ($M1$) transition process $\Upsilon(3S) \to \eta_b \gamma$ [2]. Theoretically, it provides a useful means to probe the internal structure and the interplay between different dynamic scales in quarkonium.

The standard textbook treatment of EM transitions is based on the concept of *multipole expansion*,[1] by assuming the emitted photon to be *ultrasoft*, i.e. $k^\mu \sim mv^2$, where m is heavy quark mass and v denotes the typical velocity of the quark inside a quarkonium. Consequently, the long wavelength of the photon cannot resolve the geometrical details of quarkonium. Obviously, the multipole expansion method is valid provided that $kr \ll 1$, where $r \sim 1/mv$ is the typical radius of a quarkonium.

One of the great theoretical undertakings is to understand EM transitions in a situation where the multipole expansion may break down. It is difficult to find such a situation in an atomic system, since the typical atomic energy spacings are always of the order $mv^2 \sim m\alpha^2$ (where $\alpha \approx 1/137$ is the fine structure constant). By contrast, in the realm of QCD, the linearly rising interquark strong force can host rather highly excited quarkonium states; thus one may encounter EM transitions in quarkonium with an energetic photon. The aim of this work is to offer a new perspective to tackle such a situation. For definiteness, in this work we will concentrate on the *hindered* $M1$ transition (i.e., two quarkonium states with the same orbital angular momentum but with different spin and principal quantum numbers). Such study is of practical importance, because it will help one to better understand the process $\Upsilon(3S) \to \eta_b \gamma$, where the photon carries a momentum as large as 1 GeV and multipole expansion may cease to be a decent method.

One usually assumes that the $M1$ transition can proceed without gluon exchange between Q and $\bar{Q}$. In the nonrelativistic limit, the transition rate between two S-wave quarkonia is usually described by the well-known formula [1]:

$$\Gamma[n^3S_1 \to n'^1S_0 + \gamma] = \frac{4}{3}\alpha e_Q^2 \frac{k^3}{m^2} \left| \int_0^\infty dr\, r^2 R^*_{n'0}(r) j_0\!\left(\frac{kr}{2}\right) R_{n0}(r) \right|^2, \quad (1)$$

where e_Q is the fractional electric charge of Q, k is the photon momentum viewed in the rest frame of the n^3S_1 state, and $R_{nl}(r)$ stands for the radial Schrödinger wave function of quarkonium of the principal quantum number n and orbital angular momentum l. The spherical Bessel function $j_0(\frac{kr}{2})$ [$j_0(x) \equiv \frac{\sin x}{x}$] takes into account the so-called finite-size effect (equivalently, resumming multipole-expanded magnetic amplitude to all orders). When k is expected to be ultrasoft, it is then legitimate to

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[1]The literal meaning of this term is to expand the electromagnetic field $A^\mu(t, \mathbf{R} \pm \frac{\mathbf{r}}{2})$ around $\mathbf{R}$, the center-of-mass coordinate of $Q\bar{Q}$ pair, in powers of the relative coordinate $\mathbf{r}$, and the expansion parameter is essentially kr, k denoting the photon momentum. Some authors prefer to dub it *long wavelength approximation*. These two terms are equivalent in this work.

expand this function, and the leading contribution to the hindered transition vanishes due to orthogonality of wave functions. For several observed hindered transition processes, Eq. (1) usually yields predictions a few times smaller than the measured values.

It is widely believed that hindered $M1$ transitions are very sensitive to the relativistic corrections to (1). Unfortunately, the way of implementing relativistic corrections seems to be rather model-dependent. For example, some authors proposed that, among the contributions from the relativistic corrections to the $M1$ transition, the hypothesized scalar part of the confinement potential may play an eminent role, as well as the *large* anomalous magnetic dipole moment that may be acquired by the bound quark due to some nonperturbative mechanism [1]. However, both of these suggestions seem not to be based on a firm and indisputable footing. As a matter of fact, a variety of quite different predictions to the transition rates of $\Upsilon(3S, 2S) \to \eta_b \gamma$ have been made by different authors over the years [3]. When confronting the recently established experimental results, however, most of them seem not to be favored.

Recently, relativistic corrections to the $M1$ transition have been readdressed from the angle of nonrelativistic effective field theories (EFT) [4], which allows one to critically examine the validity of some popular, yet maybe *ad hoc*, assumptions in many potential model approaches. However, it still remains a great challenge to accommodate the hindered $M1$ transition in this EFT framework. For instance, after including all types of conceivable relativistic corrections, the predicted rate for $\Upsilon(2S) \to \eta_b \gamma$ seems to be much larger than the measured one.

Impressive progress has been made in calculating the transition rate of $J/\psi \to \eta_c \gamma$ directly from lattice QCD simulation [5]. However, it is challenging to analyze very hindered EM transitions, since excited quarkonium states will be difficult to probe by lattice simulation. There are also attempts to model the coupled channel effects for $\psi' \to \eta_c \gamma$, but such framework seems not to be very predictive due to existence of several purely phenomenological parameters [6].

In view of shortcomings of the traditional approaches, we present a new attempt to analyze very hindered $M1$ transition processes typified by $\Upsilon(3S) \to \eta_b \gamma$. The key observation is very simple: in such situations, it is more appropriate to count the radiated photon as *semihard* [$k^\mu \sim mv$, often called *soft* in nonrelativistic QCD (NRQCD) terminology], rather than ultrasoft. As a consequence, we should and must give up the notion of multipole expansion. We further make a key assumption: the leading contribution to such a very hindered transition is described by Fig. 1. The underlying rationale is that, in order for the spectator antiquark to join the final quarkonium state with a significant probability, a semihard gluon must be exchanged between Q and $\bar{Q}$ to exert a kick on it.

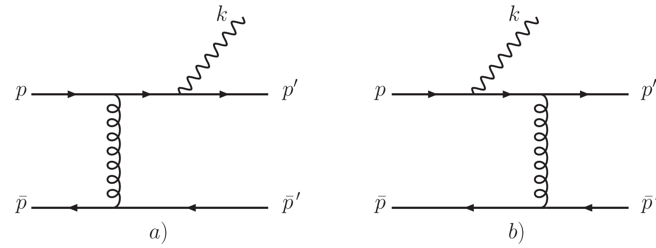

FIG. 1. Two of four lowest-order diagrams contributing to hindered $M1$ transition in our hard-scattering picture.

It may be worth digressing into pion EM form factor temporarily. At large momentum transfer, there exists a well known factorization theorem for this case [7]:

$$F_\pi(Q^2) = \int_0^1 \int_0^1 dxdy \phi_\pi(x) T(x, y, Q) \phi_\pi(y) + \cdots, \tag{2}$$

where ϕ_π implies the nonperturbative light-cone distribution amplitude of a pion, and T refers to the "hard-scattering" part, which can be computed in perturbation theory. The lowest-order contribution to T is also depicted by Fig. 1. It is generally believed that, at large Q^2, this hard-scattering picture is physically more plausible than the so-called Feynman mechanism (without exchange of a hard gluon).

We plan to derive a factorization formula analogous to (2). In our process, the analogous hard-scattering part is obtained by integrating out the semihard mode. We will assume this part is also perturbatively calculable, crucially because $mv \gg \Lambda_{\rm QCD}$, which seems legitimate for the Υ, presumably even for the ψ family. Quite naturally, we expect that the counterpart of $\phi_\pi(x)$ in our nonrelativistic problem will be the Schrödinger wave function of quarkonium.

In passing, we highlight the very different role played by the semihard mode in this work and in Ref. [4]. In the latter case, when the photon is treated as ultrasoft, the semihard mode can only appear in a loop. In contrast to the *potential* mode ($p^0 \sim mv^2$, $\mathbf{p} \sim mv$), it does not make a contribution when descending from NRQCD onto potential NRQCD [4]. According to our scheme, however, the semihard mode already makes a crucial contribution at tree level. It is the very mode that we attempt to integrate out perturbatively, in order to fulfill the intended factorization.

This said, let us turn to the derivation of the very hindered ${}^3S_1 \to {}^1S_0$ radiative transition rate. We will perform the calculation in a covariant fashion at the level of QCD. Since the *hard* ($p^\mu \sim m$) quanta decouple in this process, it is also feasible, perhaps more illuminating, to directly start from NRQCD. We first note that parity and Lorentz invariance constrain the transition amplitude to be the form

$$\mathcal{M}[n^3S_1(P) \to n'^1S_0(P') + \gamma(k)] = \mathcal{A}\epsilon_{\mu\nu\alpha\beta}P^\mu \varepsilon^\nu_{[n^3S_1]}k^\alpha \varepsilon^{*\beta}_\gamma, \tag{3}$$

where $\varepsilon_{[n^3S_1]}$ and ε_γ represent the polarization vectors of the initial quarkonium and the photon, respectively. At the rest frame of the initial state, as we will always work in, the Lorentz structure becomes $\varepsilon_{[n^3S_1]} \cdot \mathbf{k} \times \varepsilon^*_\gamma$, clearly corresponding to the $M1$ transition. The scalar coefficient $\mathcal{A}$ encodes all the nontrivial dynamics, and we will proceed to deduce its explicit form.

We begin with the parton process $Q(p)\bar{Q}(\bar{p}) \to Q(p')\bar{Q}(\bar{p}') + \gamma(k)$, as indicated in Fig. 1. We assign the momentum carried by each constituent as

$$p = \frac{P}{2} + q, \qquad \bar{p} = \frac{P}{2} - q; \qquad p' = \frac{P'}{2} + q', \qquad \bar{p}' = \frac{P'}{2} - q',$$

where q and q' are relative momenta inside each pair, which satisfies $P \cdot q = P' \cdot q' = 0$. The invariant masses of the pairs are $P^2 = 4E_q^2$ and $P'^2 = 4E_{q'}^2$, and the Lorentz scalars $E_q = \sqrt{m^2 - q^2}$, $E_{q'} = \sqrt{m^2 - q'^2}$, which guarantees that each (anti)quark stays on their mass shell. Note in the rest frame of $P^{(\prime)}$, $q^{(\prime)}$ becomes purely spacelike.

The quark propagator in Fig. 1(a) can be expanded:

$$\frac{1}{(p'+k)^2 - m^2} = \frac{1}{k \cdot P' + 2k \cdot q'} \approx \frac{1}{k \cdot P} + \frac{2\mathbf{k} \cdot \mathbf{q}'}{(k \cdot P)^2} + \cdots, \tag{4}$$

because $\mathbf{k} \cdot \mathbf{q}' \sim m^2 v^2 \ll k \cdot P' = k \cdot P \sim m^2 v$. We have neglected the small q'^0 component induced by the recoiling of P', as well as the Lorentz boost effect on $\mathbf{q}'$, which are higher order corrections. The quark propagator in Fig. 1(b) can be expanded in a similar fashion. Note this expansion is also legitimate when k is ultrasoft.

Figures 1(a) and 1(b) share a common gluon propagator:

$$\frac{1}{(\frac{k}{2} + q' - q)^2 + i\epsilon} \approx \frac{-1}{(\mathbf{q}' - \mathbf{q})^2 + \mathbf{k} \cdot (\mathbf{q}' - \mathbf{q}) - i\epsilon}. \tag{5}$$

Here we retain the $i\epsilon$ term explicitly, for the momentum integration to be properly evaluated. The two terms in the denominator are of comparable size, so (5) cannot be further expanded. If k is nevertheless counted as ultrasoft, the second term can be treated as a perturbation. Note our situation is in drastic contrast to the ordinary NRQCD calculation for hard exclusive processes. In that case, there is always a hard scale $\geq m$ in the propagators, so it is safe to neglect $q^{(\prime)}$ at the zeroth order of NRQCD expansion.

Having specified the concrete forms of the quark and gluon propagators, we then project the quark amplitude for $Q(p)\bar{Q}(\bar{p}) \to Q(p')\bar{Q}(\bar{p}') + \gamma(k)$ onto the corresponding color-singlet quarkonium Fock states, with the aid of the covariant spin projectors accurate to all orders in $q^{(\prime)}$ [8], and include the respective momentum–space wave function for each quarkonium state (e.g., see [9]). At the lowest order in q and q', we only need retain the first term in (4), and neglect all the occurrences of $q^{(\prime)}$ in the numerator of the amplitude. It turns out that Fig. 1(a) then exactly cancels against Fig. 1(b), thus rendering a net vanishing result at the lowest order in velocity expansion.[2]

To obtain a nonvanishing prediction, one must proceed to the first order in $q^{(\prime)}$ in the amplitude. To this level of accuracy, it is legitimate to set $E_q = E_{q'} \approx m$, since the induced error is of the quadratic order in $q^{(\prime)}$. It is a curious fact that, if one still keeps only the first term in the quark propagator (4), the $\mathcal{O}(q^{(\prime)})$ pieces from the spin projectors and the $\not{q}^{(\prime)}$ term from the quark propagator then make a nonzero contribution in an individual diagram, but their contributions again cancel upon summing Figs. 1(a) and 1(b). Therefore, the leading surviving $\mathcal{O}(q^{(\prime)})$ contribution can only be obtained by retaining the second term in the expanded quark propagator (4), while neglecting $q^{(\prime)}$ terms altogether elsewhere in the amplitude. After some efforts, we can read off the reduced amplitude,

$$\mathcal{A} = 2\frac{4ee_Q g_s^2 C_F}{(k \cdot P)^2} \iint \frac{d^3q}{(2\pi)^3}\frac{d^3q'}{(2\pi)^3}\phi^*_{n'0}(\mathbf{q}')T(\mathbf{q}' - \mathbf{q}) \times \phi_{n0}(\mathbf{q}), \tag{6}$$

where $C_F = \frac{4}{3}$, and the prefactor 2 indicates that two undrawn diagrams make equal contributions as Figs. 1(a) and 1(b) , owing to charge conjugation symmetry. $\phi_{n^{(\prime)}0}$ signifies the momentum-space Schrödinger wave function, and the hard-scattering kernel is

$$T(\mathbf{q}) = -\frac{\mathbf{k} \cdot \mathbf{q}}{\mathbf{q}^2 + \mathbf{k} \cdot \mathbf{q} - i\epsilon}. \tag{7}$$

Equation (6) is the desired factorization formula in momentum space.

It would be more convenient to work with the familiar spatial wave functions. Thanks to the fact that the hard-scattering part depends only on the difference between the relative momenta of two quarkonia, $\mathbf{q}' - \mathbf{q}$, and not on $\mathbf{q}$ or $\mathbf{q}'$ separately, upon Fourier transformations, one can arrive at a compact expression in the position space via contour integral:

$$\mathcal{A} = \frac{4ee_Q C_F \alpha_s}{M_n}\mathcal{E}_{nn'}, \tag{8a}$$

$$\mathcal{E}_{nn'} = \int_0^\infty dr r^2 R^*_{n'0}(r)\mathcal{T}(r)R_{n0}(r), \tag{8b}$$

[2] This is somewhat analogous to the hard exclusive process $\eta_b \to J/\psi J/\psi$, where the amplitude also vanishes at the lowest order in charm quark relative velocity [9].

where $R(r)$ appearing in the overlap integral $\mathcal{E}_{nn'}$ is the radial wave function. We have used the relation $k \cdot P = kM_n$, and M_n is the mass of the initial-state quarkonium. The dimensionless kernel $\mathcal{T}(r)$ is obtained by Fourier transforming $T(\mathbf{q})$ and integrating over solid angle:

$$\mathcal{T}(r) = \frac{e^{(i/2)kr}}{M_n r}\left[j_0\left(\frac{kr}{2}\right) - \frac{2}{kr}j_1\left(\frac{kr}{2}\right) + ij_1\left(\frac{kr}{2}\right)\right], \quad (9)$$

where $j_1(x) \equiv \frac{\sin x}{x^2} - \frac{\cos x}{x}$. It may be worth recalling that the above combination of spherical Bessel functions in the bracket resembles the conventional electric dipole ($E1$) transition formula with finite-size effect incorporated. Notice $\mathcal{T}(r)$ develops an imaginary part, since the exchanged semihard gluon can become on shell when $\mathbf{q} - \mathbf{q}' = \mathbf{k}$. However, we would like to stress that, the characteristic virtuality of the exchanged gluon in (5) should be of the order $m^2v^2 \gg \Lambda_{\rm QCD}^2$; thus the emergence of the imaginary part in the hard-scattering kernel should be viewed as an artifact due to ignoring the recoiling effect of $\mathbf{P}'$. If the effect of the imaginary part is insignificant with respect to that of the real part, we may feel such ignorance is tolerable, otherwise it will indicate a theoretical disaster. As we will see in later phenomenological analysis, the contamination of the imaginary part is indeed always negligible for a class of hindered $M1$ transitions in bottomonium and charmonium systems.

It is interesting to examine the asymptotic form of $\mathcal{T}(r)$ as $kr \ll 1$. Using $j_l(x) \sim \frac{x^l}{(2l+1)!!}$ at small x, one finds $\mathcal{T}(r) \to \frac{1}{3mr} + i\frac{k}{4m}$ as $kr \to 0$. It turns out that the real part might be identified with, up to a constant, the $\mathcal{O}(\alpha_s)$, matching coefficient $V_S^{[\sigma\cdot(r\times r\times B)]/m^2}$ in Sec. IIIC of [4] (note that there arises some subtle issue regarding gauge invariance). Since the imaginary part becomes r independent in the long wavelength limit, as expected, it does not contribute to the hindered $M1$ transition.

Finally, we can express the transition width as

$$\Gamma[n^3S_1 \to n'^1S_0 + \gamma] = \frac{k^3}{12\pi}|\mathcal{A}|^2 = \frac{16}{3}\alpha e_Q^2 \frac{k^3}{M_n^2} C_F^2 \alpha_s^2 |\mathcal{E}_{nn'}|^2, \quad (10)$$

where we have averaged upon the spin of the initial $^3S_1$ state and sum over two transverse polarizations of the photon.

Equation (10) is the key formula of this work, which looks quite simple. In evaluating the overlap integral $\mathcal{E}_{nn'}$, the input wave functions are obtained by solving the Schrödinger equation with the widely used Cornell potential model [10] and Buchmuller-Tye (BT) potential model [11]. Parameters in both potential models are tuned such that the $b\bar{b}$ and $c\bar{c}$ spectroscopy below open flavor threshold are successfully reproduced. The only freely adjustable parameter seems to be the strong coupling constant, $\alpha_s(\mu)$. However, the choice of the renormalization scale μ is by no means arbitrary. On physical ground, it should be fixed around the typical value of the quark 3-momentum in quarkonium, which is about 1.2 GeV for the $b\bar{b}$ system, and 0.9 GeV for the $c\bar{c}$ system [12]. Therefore, with α_s fixed, our formalism becomes rather predictive and readily falsifiable.

In Table I we have tabulated various predictions to hindered $M1$ transitions of $n^3S_1 \to n'^1S_0$. We also present the numerical results for $\mathcal{E}_{nn'}$, and reassuringly, the contribution from $\mathrm{Im}\mathcal{T}(r)$ is indeed insignificant. As one can tell, the agreement between our predictions, especially from the Cornell potential model, and the measurement for the transition rate of $\Upsilon(3S) \to \eta_b\gamma$, is strikingly successful. Curiously, for other hindered $M1$ transitions, where the photon is not that energetic so that the multipole expansion method may still apply, our formalism again appears to make a decent account of the measured transition rates, agreeing typically within 2–3σ. It seems fair to conclude that our simple factorization formula has passed quite nontrivial tests. Given the fact that there are almost no free parameters in (10), we feel encouraged that our formalism has captured at least some correct and relevant ingredients. We hope future measurements of $\Upsilon(4S) \to \eta_b\gamma$ can further test our mechanism.

It might be tempting to seek a simplified expression for the overlap integral $\mathcal{E}_{nn'}$, by exploiting some hierarchy between different $b\bar{b}$ energy levels. Higher radial excitation, for example, $\Upsilon(3S)$, is known to have a considerably larger radius than η_b. An intuitive guess is that $\mathcal{E}_{31}$ may not be necessarily sensitive to the full profile of $R_{30}(r)$, and instead may only be sensitive to its value at a small distance (about the radius of η_b). If this were true, one could pull $R_{30}(r)$ outside of the integral and approximate it by its value at the origin. The transition rate predicted this way turns out to be about two orders of magnitude greater than the measured one. If we play the same game for $R_{10}^*(r)$, the result would be about 10 times larger than the data. The failure of these approximations may be understood from the empirical fact that, in the Cornell or BT potential models, the average momentum of quark in different $b\bar{b}$ energy levels is more or less equal. As a result, there seems to be no ground to neglect $\mathbf{q}$ or $\mathbf{q}'$ in the hard-scattering kernel in (6).

For the $M1$ transition from n^1S_0 to n'^3S_1, one needs to multiply (10) by a statistical factor of 3. Various partial widths for $\eta_b(nS) \to \Upsilon\gamma$ are about 10 eV, and that for $\eta_c(2S) \to J/\psi\gamma$ is about 1 keV. These bottomonium transitions may be accessible in high-energy hadron collider experiments such as CERN Large Hadron Collider (LHC), and the BESIII program may provide a chance to look for this hindered charmonium transition.

As in any factorization framework, we expect that the factorization formula (6) is perturbatively improvable. It will be a major progress to calculate the next-to-leading

TABLE I. Measured and predicted branching fractions of various hindered $M1$ transition processes $n^3S_1 \to n'^1S_0 + \gamma$ for bottomonium and charmonium. The photon momentum k is determined by physical kinematics. The total widths of $\Upsilon(nS)$ and $\psi(2S)$ states, as well as all the quarkonium masses, are taken from PDG08 compilation [13], except η_b mass is taken to be 9389 MeV [2], and $\eta_b(2S)$ mass is taken as 9997 MeV [3]. For $\Upsilon(2S) \to \gamma\eta_b$, we use the preliminary *BABAR* result [14]; for $\psi(2S) \to \gamma\eta_c$, we quote the latest CLEO measurement [15] instead of the world average value given in [13]. We have taken $\alpha_s(\mu) = 0.43$ and 0.59 for $\mu = 1.2$ and 0.9 GeV, respectively.

Decay modes	k (MeV)	$\mathcal{B}$ (Exp.)	α_s	$\mathcal{E}_{nn'}(\times 10^{-2})$		$\mathcal{B}$ (Our predictions)	
				Cornell	BT	Cornell	BT
$\Upsilon(2S) \to \gamma\eta_b$	614	$(4.2 \pm 1.4) \times 10^{-4}$	0.43	$3.7e^{i2.0^\circ}$	$3.2e^{i2.7^\circ}$	1.4×10^{-4}	1.1×10^{-4}
$\Upsilon(3S) \to \gamma\eta_b$	921	$(4.8 \pm 1.3) \times 10^{-4}$	0.43	$2.7e^{i2.6^\circ}$	$2.3e^{i3.5^\circ}$	3.7×10^{-4}	2.8×10^{-4}
$\Upsilon(4S) \to \gamma\eta_b$	1123	...	0.43	$2.2e^{i2.8^\circ}$	$1.9e^{i3.7^\circ}$	4.3×10^{-7}	3.2×10^{-7}
$\Upsilon(4S) \to \gamma\eta_b(2S)$	566	...	0.43	$1.7e^{i2.2^\circ}$	$1.6e^{i2.7^\circ}$	3.2×10^{-8}	2.7×10^{-8}
$\psi(2S) \to \gamma\eta_c$	638	$(4.3 \pm 0.6) \times 10^{-3}$	0.59	$6.4e^{i9.7^\circ}$	$5.7e^{i12.9^\circ}$	2.7×10^{-3}	2.1×10^{-3}

order correction to the hard-scattering kernel. To achieve this, it might prove easier to reformulate our derivation in the context of NRQCD. It would also be interesting to implement relativistic corrections to (6).

Obviously, our strategy need not be confined to hindered $M1$ transitions only. It should be applicable whenever the radiated photon cannot be viewed as ultrasoft and multipole expansion breaks down. It will be interesting to work out the corresponding factorization formula for $E1$ transitions such as $\chi_{bJ}(2P) \to \Upsilon\gamma$. It would also be interesting to generalize this hard-scattering formalism to explore the hadronic transition processes such as $\Upsilon(3S, 4S) \to \Upsilon + \pi\pi$.

We thank Antonio Vairo, Wei Wang, and Yu-Ming Wang for useful discussions. This research was supported in part by the National Natural Science Foundation of China under Grants No. 10875130, No. 10605031, and No. 10935012.

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