Hadronic charmless B decays $B \to SA$

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- Study of $B_{u,d,s} \to K_0^*(1430)P\ (P = K\,,\pi)$ and $K_0^*(1430)V\ (V = K^*\,,\rho\,,\omega\,,\phi)$ decays within QCD factorization. Phys.Rev.D 105(2022)016002
- Study of the nonleptonic charmless $B \to SS(S = K_0^*(1430)\,,a_0(1450))$ decays with the QCD factorization approach. arXiv:2306.06492 [hep-ph]

探究标量介子内部结构提出两个方案 [1] [2]，对于重标量介子(大于$1GeV$):

S1: $q\bar{q}$激发态
S2: $q\bar{q}$基态

[1] R.L.Jaffe, Multi-quark hadrons.1.The phenomenology of (2 quark 2 anti-quark) mesons. Phys. Rev. D 15, 267(1977).
[2] M.G. Alford, R.L. Jaffe, Insight into the scalar mesons from a lattice calculation. Nucl. Phys. B 578, 367 - 382 (2000).

PHYSICAL REVIEW D **76,** 094002 (2007)

Branching ratios and *CP* asymmetries of $B \to a_1(1260)\pi$ and $a_1(1260)K$ decays

Kwei-Chou Yang

(a)

PHYSICAL REVIEW D **76,** 114020 (2007)

Hadronic charmless *B* decays $B \to AP$

Hai-Yang Cheng[1] and Kwei-Chou Yang[2]

(b)

PHYSICAL REVIEW D **76,** 094019 (2007)

Nonleptonic two-body *B* decays including axial-vector mesons in the final state

G. Calderón*

(c)

PHYSICAL REVIEW D **74,** 054035 (2006)

Nonleptonic *B* decays to axial-vector mesons and factorization

V. Laporta and G. Nardulli

(d)

PHYSICAL REVIEW D **96,** 113002 (2017)

Hadronic decays of $B \to a_1(1260)b_1(1235)$ in the perturbative QCD approach

Hao-Yang Jing and Xin Liu*

(e)

计算$B \to SA$衰变过程，对标量介子内部结构进一步的探究。

$n^{2s+1}\ell_J$	J^{PC}	$I=1$ $u\bar{d}, \bar{u}d, \frac{1}{\sqrt{2}}(d\bar{d}-u\bar{u})$	$I=\frac{1}{2}$ $u\bar{s}, d\bar{s}, \bar{d}s, -\bar{u}s,$	$I=0$ $c_1(d\bar{d}+u\bar{u})+c_2(s\bar{s})$	$I=0$
1^1P_1	1^{+-}	$b_1(1235)$	K_{1B}	$h_1(1380)$	$h_1(1170)$
1^3P_0	0^{++}	$a_0(1450)$	$K_0^*(1430)$		
1^3P_1	1^{++}	$a_1(1260)$	K_{1A}	$f_1(1420)$	$f_1(1285)$

Mixing angles

物理的质量本征态$K_1(1270)$, $K_1(1400)$不再是纯的$^3P_1$或$^1P_1$态，而应是$^3P_1$和$^1P_1$的混合态，也就是K_{1A} 和K_{1B}态的混合体，其混合形式可以写为：

$$\begin{pmatrix} K_1(1270) \\ K_1(1400) \end{pmatrix} = \begin{pmatrix} sin\theta_{K_1} & cos\theta_{K_1} \\ cos\theta_{K_1} & -sin\theta_{K_1} \end{pmatrix} \begin{pmatrix} K_{1A} \\ K_{1B} \end{pmatrix} \quad (1)$$

其中，θ_{K_1}为K_{1A} 和K_{1B}两态之间的混合角。混合角的确定在文献 [3] [4]中给出

$$\frac{\mathcal{B}(B \to K_1(1270)\gamma)}{\mathcal{B}(B \to K_1(1400)\gamma)} = \begin{cases} 10.1 \pm 6.2\ (280 \pm 200); & \text{for } \theta_{K_1} = -58^\circ(-37^\circ), \\ 0.02 \pm 0.02\ (0.05 \pm 0.04); & \text{for } \theta_{K_1} = +58^\circ(+37^\circ). \end{cases} \quad (2.2)$$

The Belle measurements $\mathcal{B}(B^+ \to K_1^+(1270)\gamma) = (4.3 \pm 0.9 \pm 0.9) \times 10^{-5}$ and $\mathcal{B}(B^+ \to K_1^+(1400)\gamma) < 1.5 \times 10^{-5}$ [25] clearly favor $\theta_{K_1} = -58^\circ$ over $\theta_{K_1} = 58^\circ$ and $\theta_{K_1} = -37^\circ$ over $\theta_{K_1} = 37^\circ$. In the ensuing discussions we will fix the sign of θ_{K_1} to be negative.

(f)

[3] H. Y . Cheng, Phys. Rev. D 67, 094007 (2003).

[4] H. Y . Cheng and K. C. Yang, Phys. Rev. D 76, 114020 (2007).

decay constants

$$\langle S(p)|\bar{q}_2\gamma_\mu q_1|0\rangle = f_S p_\mu, \qquad \langle S|\bar{q}_2 q_1|0\rangle = m_S\bar{f}_S, \tag{2}$$

$$\langle S(p)|\bar{q}_2\gamma_\mu q_1|0\rangle = \bar{f}_S\,\bar{\mu}_S^{-1}\,p_\mu, \tag{3}$$

$$\langle A(p,\lambda)|\bar{q}_2\gamma_\mu\gamma_5 q_1|0\rangle = if_A m_A \epsilon_\mu^*, \tag{4}$$

$$\bar{f}_S = \bar{\mu}_S f_S = \frac{\mu_S}{m_S} f_S = \frac{m_S}{m_q(\mu) - m_{\bar{q}}(\mu)} f_S, \qquad \text{with}\ \ \mu_S = \frac{m_S^2}{m_q(\mu) - m_{\bar{q}}(\mu)}, \tag{5}$$

$$\bar{f}_{\bar{S}} = \bar{f}_S, \qquad f_{\bar{S}} = -f_S, \qquad \mu_{\bar{S}} = -\mu_S \tag{6}$$

对于非奇异${}^1P_1$态，例如$b_1(1235)$，f_{b_1}消失并且$\mu_{b_1}(\mu_{{}^1P_1} = 1/a_0^{\parallel,{}^1P_1})$发散，但是$f_{b_1}\mu_{b_1}$是存在的。

Form factor

$$
\begin{aligned}
\langle S(p')|A_\mu|B(p)\rangle &= -i\left[\left(P_\mu - \frac{P\cdot q}{q^2}\,q_\mu\right)U_1(q^2) + \frac{P\cdot q}{q^2}q_\mu\,U_0(q^2)\right], \\
\langle A(p',\lambda)|A_\mu|B(p)\rangle &= i\frac{2}{m_B-m_A}\epsilon_{\mu\nu\alpha\beta}\epsilon^{*\nu}_{(\lambda)}p_B^\alpha p^\beta A^{BA}(q^2), \\
\langle A(p',\lambda)|V_\mu|B(p)\rangle &= -\{(m_B-m_A)\epsilon_\mu^{(\lambda)*}V_1^{BA}(q^2) \\
&\quad -(\epsilon^{(\lambda)*}\cdot p_B)(p_B+p)_\mu\frac{V_2^{BA}(q^2)}{m_B-m_A} \\
&\quad -2m_A\frac{\epsilon^*_{(\lambda)}p_B}{q^2}q^\mu[V_3^{BA}(q^2)-V_0^{BA}(q^2)]\}
\end{aligned}
\tag{7}
$$

where $q = p_B - p$, $V_3^{BA}(0) - V_0^{BA}(0)$

projection operator

$$M_B = -\frac{if_B}{4}[(\not{p}_B + m_B)\gamma_5]\Phi_B(\xi), \quad (8)$$

$$(M_{\parallel}^{A})_{\alpha\beta} = -\frac{if_A}{4}\left(\not{p}\gamma_5\Phi_A(x) + \frac{m_A f_A^{\perp}}{f_A}\gamma_5\frac{\not{k}_2\not{k}_1}{k_2\cdot k_1}\phi_A(x)\right)_{\alpha\beta}, \quad (9)$$

$$M_{\alpha\beta}^{S} = \frac{f_S}{4}\left(\not{p}\Phi_S(x) + \mu_S\frac{\not{k}_2\not{k}_1}{k_2\cdot k_1}\phi_S^s(x)\right)_{\alpha\beta}, \quad (10)$$

$$\bar{M}_{\alpha\beta}^{S} = \frac{\bar{f}_S}{4}\left(\not{p}\bar{\Phi}_S(x) + m_S\frac{\not{k}_2\not{k}_1}{k_2\cdot k_1}\phi_S^s(x)\right)_{\alpha\beta}, \quad (11)$$

LCDAs

$$\Phi_M(x,\mu) = 6x(1-x)\left[1+\sum_{n=1}^{\infty}\alpha_n^M(\mu)C_n^{3/2}(2x-1)\right],$$

$$\Phi_S(x,\mu) = 6x(1-x)\left[1+\sum_{m=1}^{\infty}\alpha_m^M(\mu)\,C_m^{3/2}(2x-1)\right],(\alpha_m^M=\bar{\mu}_S B_m^M(\mu)), \tag{12}$$

$$\bar{\Phi}_S(x,\mu) = 6x(1-x)\sum_{m=0}^{\infty}B_m^M(\mu)\,C_m^{3/2}(2x-1), \qquad \bar{\phi}_S^s(x) = \bar{f}_S, \tag{13}$$

$$\Phi_{\parallel}^{{}^1P_1}(x) = 6x\bar{x}\left[\alpha_0^{\parallel,{}^1P_1}+3\alpha_1^{\parallel,{}^1P_1}(2x-1)+\alpha_2^{\parallel,{}^1P_1}\frac{3}{2}[5(2x-1)^2-1]\right], \tag{14}$$

$$\Phi_{\perp}^{{}^1P_1}(x) = 6x\bar{x}\left[1+3\alpha_1^{\perp,{}^1P_1}(2x-1)+\alpha_2^{\perp,{}^1P_1}\frac{3}{2}[5(2x-1)^2-1]\right], \tag{15}$$

$$\phi_a^{{}^1P_1}(x) = 3\left[(2x-1)+\sum_{n=1}^{\infty}\alpha_n^{\perp,{}^1P_1}P_{n+1}(2x-1)\right], \tag{16}$$

$$\Phi_{\parallel}^{{}^3P_1}(x) = 6x\bar{x}\left[1+3\alpha_1^{\parallel,{}^3P_1}(2x-1)+\alpha_2^{\parallel,{}^3P_1}\frac{3}{2}[5(2x-1)^2-1]\right], \tag{17}$$

$$\Phi_{\perp}^{{}^3P_1}(x) = 6x\bar{x}\left[\alpha_0^{\perp,{}^3P_1}+3\alpha_1^{\perp,{}^3P_1}(2x-1)+\alpha_2^{\perp,{}^3P_1}\frac{3}{2}[5(2x-1)^2-1]\right], \tag{18}$$

$$\phi_a^{{}^3P_1}(x) = 3\left[\alpha_0^{\perp,{}^3P_1}(2x-1)+\sum_{n=1}^{\infty}\alpha_n^{\perp,{}^3P_1}P_{n+1}(2x-1)\right], \tag{19}$$

factorizable term

the explicit expression for factorizable term is given by

$$
\begin{aligned}
A_{SA} &\equiv \langle A|J^{\mu}|0\rangle\langle S|J'_{\mu}|\overline{B}\rangle = 2m_A(\epsilon^*_{(\lambda)}p_B)f_A U_1^{BS}(q^2) \\
&= 2f_A\, m_B p_c U_1^{BS}(q^2), \qquad (20)\\
\bar{A}_{AS} &\equiv \langle S|J^{\mu}|0\rangle\langle A|J'_{\mu}|\overline{B}\rangle = -2\bar{f}_S\, A_0^{BA}(q^2)m_A(\epsilon^*_{(\lambda)}p_B) \\
&= -2\bar{f}_S\, m_B p_c A_0^{BA}(q^2). \qquad (21)
\end{aligned}
$$

where p_c is the c.m. momentum.

文献分享

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Hadronic charmless B decays $B \to AP$

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The two-body hadronic decays of B mesons into pseudoscalar and axial-vector mesons are studied within the framework of QCD factorization. The light cone distribution amplitudes (LCDAs) for $^3P_1$ and $^1P_1$ axial-vector mesons have been evaluated using the QCD sum rule method. Owing to the G-parity, the chiral-even two-parton light cone distribution amplitudes of the $^3P_1$ ($^1P_1$) mesons are symmetric (antisymmetric) under the exchange of quark and antiquark momentum fractions in the SU(3) limit. For chiral-odd LCDAs, it is the other way around. The main results are the following: (i) The predicted rates for $a_1^{\pm}(1260)\pi^{\mp}$, $b_1^{\pm}(1235)\pi^{\mp}$, $b_1^0(1235)\pi^-$, $a_1^+K^-$, and $b_1^+K^-$ modes are in good agreement with the data. However, the naively expected ratios $\mathcal{B}(B^- \to a_1^0\pi^-)/\mathcal{B}(\bar{B}^0 \to a_1^+\pi^-) \lesssim 1$, $\mathcal{B}(B^- \to a_1^-\pi^0)/\mathcal{B}(\bar{B}^0 \to a_1^-\pi^+) \sim \frac{1}{2}$, and $\mathcal{B}(B^- \to b_1^0K^-)/\mathcal{B}(\bar{B}^0 \to b_1^+K^-) \sim \frac{1}{2}$ are not borne out by experiment. This should be clarified by the improved measurements of these decays. (ii) Since the $\bar{B} \to b_1 K$ decays receive sizable annihilation contributions, their rates are sensitive to the interference between penguin and annihilation terms. The measurement of $\mathcal{B}(\bar{B}^0 \to b_1^+K^-)$ implies a destructive interference which in turn indicates that the form factors for $B \to b_1$ and $B \to a_1$ transitions are of opposite signs. (iii) Sizable power corrections such as weak annihilation are needed to account for the observed rates of the penguin-dominated modes $K_1^-(1270)\pi^+$ and $K_1^-(1400)\pi^+$. (iv) The decays $B \to K_1\bar{K}$ with $K_1 = K_1(1270)$, $K_1(1400)$ are in general quite suppressed, of order 10^{-7}–10^{-8}, except for $\bar{B}^0 \to \bar{K}_1^0(1270)K^0$ which can have a branching ratio of order 2.3×10^{-6}. The decay modes $K_1^-K^+$ and $K_1^+K^-$ are of particular interest as they proceed only through weak annihilation. (v) The mixing-induced parameter S is predicted to be negative in the decays $B^0 \to a_1^{\pm}\pi^{\mp}$, while it is positive experimentally. This may call for a larger unitarity angle $\gamma \gtrsim 80°$. (vi) Branching ratios for the decays $B \to f_1\pi$, f_1K, $h_1\pi$ and h_1K with $f_1 = f_1(1285)$, $f_1(1420)$ and $h_1 = h_1(1170)$, $h_1(1380)$ are generally of order 10^{-6} except for the color-suppressed modes $f_1\pi^0$ and $h_1\pi^0$ which are suppressed by 1 to 2 orders of magnitude. Measurements of the ratios $\mathcal{B}(B^- \to h_1(1380)\pi^-)/\mathcal{B}(B^- \to h_1(1170)\pi^-)$ and $\mathcal{B}(\bar{B} \to f_1(1420)\bar{K})/\mathcal{B}(\bar{B} \to f_1(1285)\bar{K})$ will help determine the mixing angles $\theta_{^1P_1}$ and $\theta_{^3P_1}$, respectively.

文献分享

- 文献采用了QCDF方法对$B \to AP$衰变进行计算，文中使用的轴矢介子LCDAs是QCD SM方法得出；
- 分析得出：
  1、部分衰变道与实验符合得很好，希望未来能有更多的实验数据参考；
  2、$\bar{B} \to b_1 K$衰变有较大的湮灭图贡献；
  3、衰变$B \to K_1 \bar{K}(K_1 = K_1(1270), K_1 = K_1(1400))$受到较大的压低$(10^{-8} - 10^{-7})$，除了$\bar{B}^0 \to \bar{K}_1^0(1270)K^0$；
  4、衰变$B^0 \to a_1^{\pm}\pi^{\mp}$理论计算为负，而实验为正；可能需要更大的幺正角$\gamma \geq 80°$；

1、部分衰变道与实验符合得很好，希望未来能有更多的实验数据参考；

TABLE VI. Branching ratios (in units of 10^{-6}) for the decays $B \to a_1(1260)\pi$, $a_1(1260)K$, $b_1(1235)\pi$ and $b_1(1235)K$. The theoretical errors correspond to the uncertainties due to variation of (i) Gegenbauer moments, decay constants, (ii) quark masses, form factors, and (iii) λ_B, $\rho_{A,H}$, $\phi_{A,H}$, respectively. Other model predictions are also presented here for comparison. In [14], predictions are obtained for two different sets of form factors, denoted by I and II, respectively, corresponding to the mixing angles $\theta_{K_1} = 32°$ and 58° (see the text for more details).

Mode	CMV [15]	LNP(I)[14]	LNP(II)	This work	Expt. [2–7,10]
$\bar{B}^0 \to a_1^+ \pi^-$	74.3	4.7	11.8	$9.1^{+0.2+2.2+1.7}_{-0.2-1.8-1.1}$	12.2 ± 4.5 [a]
$\bar{B}^0 \to a_1^- \pi^+$	36.7	11.1	12.3	$23.4^{+2.3+6.2+1.9}_{-2.2-5.5-1.3}$	21.0 ± 5.4 [a]
$\bar{B}^0 \to a_1^{\pm} \pi^{\mp}$	111.0	15.8	24.1	$32.5^{+2.5+8.4+3.6}_{-2.4-7.3-2.4}$	31.7 ± 3.7 [b]
$B^- \to a_1^0 \pi^-$	43.2	3.9	8.8	$7.6^{+0.3+1.7+1.4}_{-0.3-1.3-1.0}$	$20.4 \pm 4.7 \pm 3.4$
$\bar{B}^0 \to a_1^0 \pi^0$	0.27	1.1	1.7	$0.9^{+0.1+0.3+0.7}_{-0.1-0.2-0.3}$	
$B^- \to a_1^- \pi^0$	13.6	4.8	10.6	$14.4^{+1.4+3.5+2.1}_{-1.3-3.2-1.9}$	$26.4 \pm 5.4 \pm 4.1$
$\bar{B}^0 \to a_1^+ K^-$	72.2	1.6	4.1	$18.3^{+1.0+14.2+21.1}_{-1.0-7.2-7.5}$	$16.3 \pm 2.9 \pm 2.3$
$\bar{B}^0 \to a_1^0 \bar{K}^0$	42.3	0.5	2.5	$6.9^{+0.3+6.1+9.5}_{-0.3-2.9-3.2}$	
$B^- \to a_1^- \bar{K}^0$	84.1	2.0	5.2	$21.6^{+1.2+16.5+23.6}_{-1.1-8.5-11.9}$	$34.9 \pm 5.0 \pm 4.4$
$B^- \to a_1^0 K^-$	43.4	1.4	2.8	$13.9^{+0.9+9.5+12.9}_{-0.9-5.1-4.9}$	
$\bar{B}^0 \to b_1^+ \pi^-$	36.2	6.9	0.7	$11.2^{+0.3+2.8+2.2}_{-0.3-2.4-1.9}$	
$\bar{B}^0 \to b_1^- \pi^+$	4.4	≈ 0	≈ 0	$0.3^{+0.1+0.1+0.3}_{-0.0-0.1-0.1}$	
$\bar{B}^0 \to b_1^{\pm} \pi^{\mp}$	40.6	6.9	0.7	$11.4^{+0.4+2.9+2.5}_{-0.3-2.5-2.0}$	$10.9 \pm 1.2 \pm 0.9$
$\bar{B}^0 \to b_1^0 \pi^0$	0.15	0.5	0.01	$1.1^{+0.2+0.1+0.2}_{-0.2-0.1-0.2}$	
$B^- \to b_1^- \pi^0$	4.2	4.8	0.5	$0.4^{+0.0+0.2+0.4}_{-0.0-0.1-0.2}$	
$B^- \to b_1^0 \pi^-$	18.6	4.5	0.4	$9.6^{+0.3+1.6+2.5}_{-0.3-1.6-1.5}$	$6.7 \pm 1.7 \pm 1.0$
$\bar{B}^0 \to b_1^+ K^-$	35.7	2.4	0.2	$12.1^{+1.0+9.7+12.3}_{-0.9-4.9-30.2}$	$7.4 \pm 1.0 \pm 1.0$
$\bar{B}^0 \to b_1^0 \bar{K}^0$	19.3	4.1	0.4	$7.3^{+0.5+5.4+6.7}_{-0.5-2.8-6.5}$	
$B^- \to b_1^- \bar{K}^0$	41.5	3.0	0.3	$14.0^{+1.3+11.5+13.9}_{-1.2-5.9-8.3}$	

2、$^1P_1$轴矢量介子的显着特征之一是其轴矢量衰变常数很小,在SU(3)极限内消失。观察$\Gamma(\bar{B}^0 \to b_1^- \pi^+) \ll \Gamma(\bar{B}^0 \to b_1^+ \pi^-)$;同时,$\Gamma(\bar{B}^0 \to a_1^- \pi^+) \gg \Gamma(\bar{B}^0 \to a_1^+ \pi^-)$,是因为$f_{a_1} \gg f_\pi$

3、对于企鹅图贡献为主的衰变过程($K_1^-(1270) and K_1^-(1400)$)相比于实验理论预言小得多，这种情况类似$B \to K^* \pi$需要较大的幂次修正；当前已有的观测结果对于混合角$\theta_{K_1}$更倾向于$-37°$

TABLE VII. Same as Table VI except for the decays $B \to K_1(1270)\pi$, $K_1(1270)K$, $K_1(1400)\pi$, and $K_1(1400)K$ for two different mixing angles $\theta_{K_1} = -37°$ and $-58°$ (in parentheses). In the framework of [14], only the $K_1^-(1400)\pi^0$ and $\bar{K}_1^0(1400)\pi^0$ modes depend on the mixing angle θ_{K_1}. Note that the results of [14,15] shown in the table are obtained for $\theta_{K_1} = 32°$ and $58°$ (in parentheses).

Mode	[15]	[14]	This work	Expt. [8]
$\bar{B}^0 \to K_1^-(1270)\pi^+$	4.3 (4.3)	7.6	$3.0^{+0.8+1.5+4.2}_{-0.6-0.9-1.4}$ $(2.7^{+0.6+1.3+4.4}_{-0.5-0.8-1.5})$	$12.0 \pm 3.1^{+9.3}_{-4.5} < 25.2$
$\bar{B}^0 \to \bar{K}_1^0(1270)\pi^0$	2.3 (2.1)	0.4	$1.0^{+0.0+0.6+1.7}_{-0.0-0.3-0.6}$ $(0.8^{+0.1+0.5+1.7}_{-0.1-0.3-0.6})$	
$B^- \to \bar{K}_1^0(1270)\pi^-$	4.7 (4.7)	5.8	$3.5^{+0.1+1.8+5.1}_{-0.1-1.1-1.9}$ $(3.0^{+0.2+0.1+2.7}_{-0.2-0.2-2.2})$	
$B^- \to K_1^-(1270)\pi^0$	2.5 (1.6)	4.9	$2.7^{+0.1+1.1+3.1}_{-0.1-0.7-1.0}$ $(2.5^{+0.1+1.0+3.2}_{-0.1-0.7-1.0})$	
$\bar{B}^0 \to K_1^-(1400)\pi^+$	2.3 (2.3)	4.0	$5.4^{+1.1+1.7+9.9}_{-1.0-1.3-2.8}$ $(2.2^{+1.1+0.7+2.6}_{-0.8-0.6-1.3})$	$16.7 \pm 2.6^{+3.5}_{-5.0} < 21.8$
$\bar{B}^0 \to \bar{K}_1^0(1400)\pi^0$	1.7 (1.6)	3.0 (1.7)	$2.9^{+0.3+0.7+5.5}_{-0.3-0.6-1.7}$ $(1.5^{+0.4+0.3+1.7}_{-0.3-0.3-0.9})$	
$B^- \to \bar{K}_1^0(1400)\pi^-$	2.5 (2.5)	3.0	$6.5^{+1.0+2.0+11.6}_{-0.9-1.6-3.6}$ $(2.8^{+1.0+0.9+3.0}_{-0.8-0.9-1.7})$	
$B^- \to K_1^-(1400)\pi^0$	0.7 (0.6)	1.0 (1.4)	$3.0^{+0.4+1.1+5.2}_{-0.4-0.7-1.3}$ $(1.0^{+0.4+0.4+1.2}_{-0.3-0.4-0.5})$	

4、时间相关的CP不对称性衰变$B^0 \to a_1^{\pm}\pi^{\mp}$理论计算为负，而实验为正；可能需要更大的幺正角$\gamma \geq 80°$；

TABLE IX. Direct CP asymmetries (in %) in the decays $B \to a_1(1260)\pi$, $a_1(1260)K$, $b_1(1235)\pi$ and $b_1(1235)K$. See Table VI for the explanation of theoretical errors. Experiments results are taken from [4,5,10,44].

Mode	Theory	Expt.	Mode	Theory	Expt.
$\bar{B}^0 \to a_1^+ \pi^-$	$-3.6^{+0.1+0.3+20.8}_{-0.1-0.5-20.2}$	$7 \pm 21 \pm 15$ [a]	$\bar{B}^0 \to a_1^+ K^-$	$2.6^{+0.0+0.7+10.1}_{-0.1-0.7-11.0}$	$-16 \pm 12 \pm 1$
$\bar{B}^0 \to a_1^- \pi^+$	$-1.9^{+0.0+0.0+14.6}_{-0.0-0.0-14.3}$	$15 \pm 15 \pm 7$ [a]	$\bar{B}^0 \to a_1^0 \bar{K}^0$	$-7.7^{+0.6+2.1+6.8}_{-0.6-2.2-7.0}$	
$\bar{B}^0 \to a_1^0 \pi^0$	$60.1^{+4.6+6.8+37.6}_{-4.9-8.3-60.7}$		$B^- \to a_1^- \bar{K}^0$	$0.8^{+0.0+0.1+0.6}_{-0.0-0.1-0.0}$	$12 \pm 11 \pm 2$
$B^- \to a_1^- \pi^0$	$0.5^{+0.3+0.6+12.0}_{-0.2-0.3-11.0}$		$B^- \to a_1^0 K^-$	$8.4^{+0.3+1.4+10.3}_{-0.3-1.6-12.0}$	
$B^- \to a_1^0 \pi^-$	$-4.3^{+0.3+1.4+14.1}_{-0.3-2.2-14.5}$				
$\bar{B}^0 \to b_1^+ \pi^-$	$-4.0^{+0.2+0.4+26.2}_{-0.0-0.6-25.5}$		$\bar{B}^0 \to b_1^+ K^-$	$5.5^{+0.2+1.2+47.2}_{-0.3-1.2-30.2}$	$-7 \pm 12 \pm 2$
$\bar{B}^0 \to b_1^- \pi^+$	$66.1^{+1.2+7.4+30.3}_{-1.4-4.8-96.6}$		$\bar{B}^0 \to b_1^0 \bar{K}^0$	$-8.6^{+0.8+3.3+8.3}_{-0.8-4.2-25.4}$	
$\bar{B}^0 \to b_1^0 \pi^0$	$53.4^{+6.4+9.0+5.2}_{-6.3-7.3-4.7}$		$B^- \to b_1^- \bar{K}^0$	$1.4^{+0.1+0.1+5.6}_{-0.1-0.1-0.1}$	
$B^- \to b_1^- \pi^0$	$-36.5^{+4.4+18.4+82.2}_{-4.3-17.7-59.6}$		$B^- \to b_1^0 K^-$	$18.7^{+1.6+7.8+57.7}_{-1.7-6.1-44.9}$	$-46 \pm 20 \pm 2$
$B^- \to b_1^0 \pi^-$	$0.9^{+0.6+2.3+18.0}_{-0.4-2.7-20.5}$	$5 \pm 16 \pm 2$			

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En_-^μ. To obtain the light-cone projection operator of the meson in the momentum space, we apply the following substitution in the calculation:

$$z^\mu \longrightarrow -i\frac{\partial}{\partial k_{1\mu}} \simeq -i\left(\frac{n_+^\mu}{2E}\frac{\partial}{\partial u} + \frac{\partial}{\partial k_{\perp\mu}}\right), \tag{2.26}$$

where terms of order $k_\perp^2$ have been omitted. Moreover, to perform the calculation in the momentum space, we need to express Eq. (2.24) in terms of z-independent variables P and $\epsilon^{*(\lambda)}$ instead of p and $\varepsilon^{*(\lambda)}$. Consequently, the light-cone projection operator of the meson in the momentum space, including twist-3 two-parton distribution amplitudes, reads

$$M_{\delta\alpha} = M_{\delta\alpha\parallel} + M_{\delta\alpha\perp}, \tag{2.27}$$

where $M_{\delta\alpha\parallel}$ and $M_{\delta\alpha\perp}$ are the longitudinal and transverse projectors, respectively.

For the vector meson, the longitudinal projector reads [34]

$$\begin{aligned} M_\parallel^V &= -i\frac{f_V}{4}\frac{m_V(\epsilon^*_{(\lambda)}n_+)}{2}\not{n}_-\Phi_\parallel(u) - i\frac{f_V^\perp m_V}{4}\frac{m_V(\epsilon^*_{(\lambda)}n_+)}{2E} \\ &\times\left\{-\frac{i}{2}\sigma_{\mu\nu}n_-^\mu n_+^\nu h_\parallel^{(t)}(u) - iE\int_0^u dv(\Phi_\perp(v)\right. \\ &\left.\left. - h_\parallel^{(t)}(v))\sigma_{\mu\nu}n_-^\mu\frac{\partial}{\partial k_{\perp\nu}} + \frac{h_\parallel^{\prime(s)}(u)}{2}\right\}\right|_{k=up} \\ &+ \mathcal{O}\left[\left(\frac{m_V}{E}\right)^2\right], \end{aligned} \tag{2.28}$$

and the transverse projector has the form

the vector meson. For the axial-vector meson, the longitudinal projector is given by

$$\begin{aligned} M_\parallel^A &= -i\frac{f_A}{4}\frac{m_A(\epsilon^*_{(\lambda)}n_+)}{2}\not{n}_-\gamma_5\Phi_\parallel(u) + i\frac{f_A^\perp m_A}{4}\frac{m_A(\epsilon^*_{(\lambda)}n_+)}{2E} \\ &\times\left\{-\frac{i}{2}\sigma_{\mu\nu}\gamma_5 n_-^\mu n_+^\nu h_\parallel^{(t)}(u) - iE\int_0^u dv(\Phi_\perp(v)\right. \\ &\left.\left. - h_\parallel^{(t)}(v))\sigma_{\mu\nu}\gamma_5 n_-^\mu\frac{\partial}{\partial k_{\perp\nu}} + \gamma_5\frac{h_\parallel^{\prime(p)}(u)}{2}\right\}\right|_{k=up} \\ &+ \mathcal{O}\left[\left(\frac{m_A}{E}\right)^2\right], \end{aligned} \tag{2.30}$$

and the transverse projector given by

$$\begin{aligned} M_\perp^A &= i\frac{f_A^\perp}{4}E\not{\epsilon}_\perp^{*(\lambda)}\not{n}_-\gamma_5\Phi_\perp(u) - i\frac{f_A m_A}{4}\left\{\not{\epsilon}_\perp^{*(\lambda)}\gamma_5 g_\perp^{(a)}(u)\right. \\ &- E\int_0^u dv(\Phi_\parallel(v) - g_\perp^{(a)}(v))\not{n}_-\gamma_5\epsilon_{\perp\mu}^{*(\lambda)}\frac{\partial}{\partial k_{\perp\mu}} \\ &+ i\varepsilon_{\mu\nu\rho\sigma}\gamma^\mu\epsilon_\perp^{*(\lambda)\nu}n_-^\rho\left[n_+^\sigma\frac{g_\perp^{\prime(v)}(u)}{8}\right. \\ &\left.\left.\left. - E\frac{g_\perp^{(v)}(u)}{4}\frac{\partial}{\partial k_{\perp\sigma}}\right]\right\}\right|_{k=up} + \mathcal{O}\left[\left(\frac{m_A}{E}\right)^2\right]. \end{aligned} \tag{2.31}$$

In the present study, we choose the coordinate systems in the Jackson convention; that is, in the $\bar{B}$ rest frame, one of the vector or axial-vector mesons is moving along the z axis of the coordinate system and the other along the $-z$ axis, while the x axes of both daughter particles are parallel [35]:

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[5] H. Y. Cheng and K. C. Yang, Phys. Rev. D 78, 094001(2008); Erratum, Phys. Rev. D 79, 039903 (2009).