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CP asymmetries in $\tau \to K_S \pi \nu_\tau$ decays

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based on:

Feng-Zhi Chen, Xin-Qiang Li, Shi-Can Peng, Ya-Dong Yang, Hong-Hao Zhang, JHEP 01 (2022) 108

Feng-Zhi Chen, Xin-Qiang Li, Ya-Dong Yang, JHEP 05 (2020) 151

Feng-Zhi Chen, Xin-Qiang Li, Ya-Dong Yang, Xin Zhang, PRD 100 (2019) 113006

Outline

- ☐ **Introduction**
- ☐ **CP asymmetries in $\tau \to K_S \pi \nu_\tau$ decays within the SM**
- ☐ **CP asymmetries in $\tau \to K_S \pi \nu_\tau$ decays within a general EFT**
- ☐ **Summary**

$$A_{\rm CP}^{\rm rate} \equiv \frac{\Gamma(\tau^+ \to [\pi^+\pi^-]_{``K_S"}\pi^+\bar{\nu}_\tau) - \Gamma(\tau^- \to [\pi^+\pi^-]_{``K_S"}\pi^-\nu_\tau)}{\Gamma(\tau^+ \to [\pi^+\pi^-]_{``K_S"}\pi^+\bar{\nu}_\tau) + \Gamma(\tau^- \to [\pi^+\pi^-]_{``K_S"}\pi^-\nu_\tau)}$$

BaBar Collaboration, PRD 85 (2012) 031102

- commonly discussed decay-rate asymmetry
- CP asymmetry in the angular distribution

$$A_{CP}^i = \frac{\int_{s_{1,i}}^{s_{2,i}} \int_{-1}^{1} \cos\alpha \left[\frac{d^2\Gamma(\tau^-\to K_S\pi^-\nu_\tau)}{ds\, d\cos\alpha} - \frac{d^2\Gamma(\tau^+\to K_S\pi^+\bar{\nu}_\tau)}{ds\, d\cos\alpha}\right] ds\, d\cos\alpha}{\frac{1}{2}\int_{s_{1,i}}^{s_{2,i}} \int_{-1}^{1} \left[\frac{d^2\Gamma(\tau^-\to K_S\pi^-\nu_\tau)}{ds\, d\cos\alpha} + \frac{d^2\Gamma(\tau^+\to K_S\pi^+\bar{\nu}_\tau)}{ds\, d\cos\alpha}\right] ds\, d\cos\alpha}$$

Belle Collaboration, PRL 107 (2011) 131801

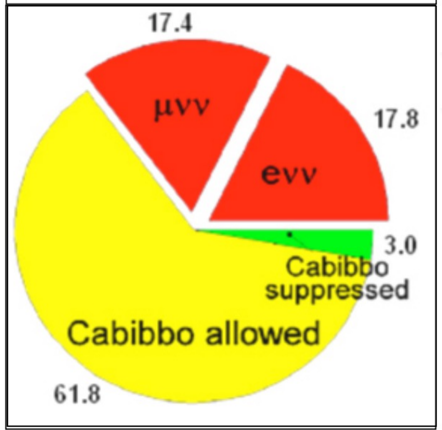
Tau lepton physics

- τ : **discovered in 1975 by Martin Perl *et al.* (SLAC-LBL)**
  - Mass: $m_\tau = 1776.86 \pm 0.12$ MeV;
  - Lifetime: $\tau_\tau = (2.903 \pm 0.005) \times 10^{-13}$s

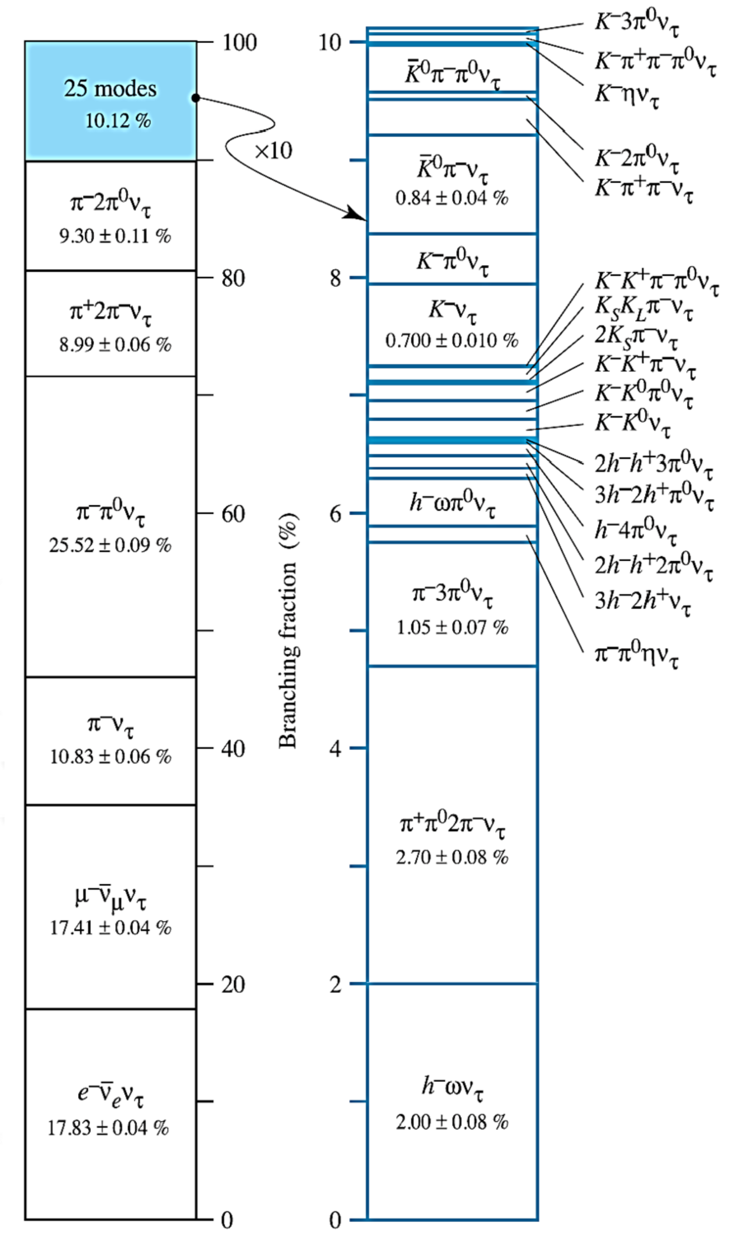
PDG 2022

- **In SM, tau decays via charged-current weak interaction:**
  - purely leptonic: $\tau \to \nu_\tau \ell \bar{\nu}_\ell$, $\tau \to \nu_\tau \ell \bar{\nu}_\ell \gamma$,
  - semi-leptonic: $\tau \to \nu_\tau \pi$, $\tau \to \nu_\tau K\pi$, …
  - rare and forbidden: LFV, LNV, BNV, …

Including QED & QCD corrections:



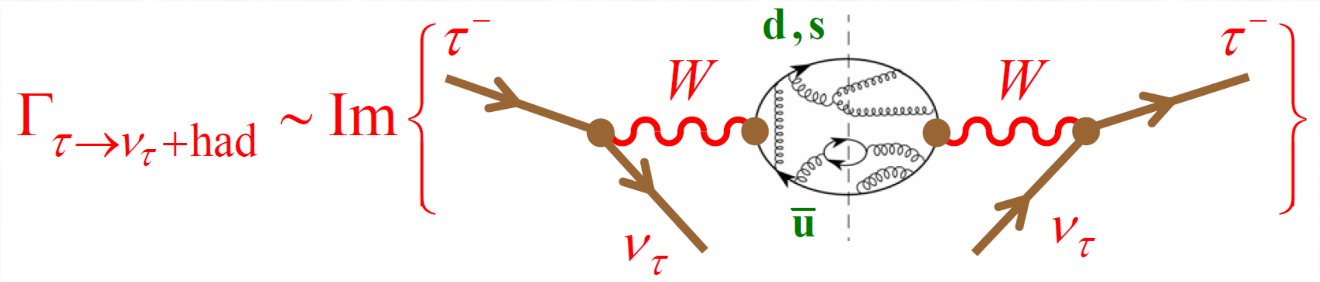
- **The only lepton heavy enough to decay into hadrons:** $\mathrm{Br} \simeq 66\%$!



- **Very rich phenomenology: see *biennial tau workshops!***

Semi-leptonic tau decays

- **Can be used to extract the fundamental SM parameters: $\alpha_s(m_\tau)$, m_s, $|V_{us}|$, …**

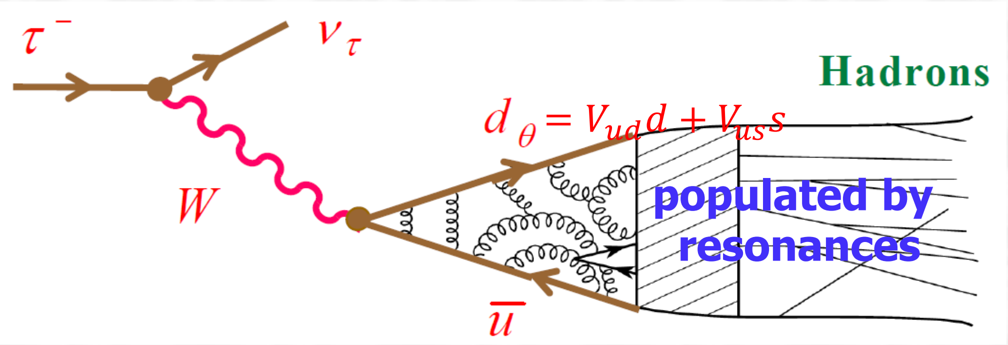


$\alpha_s(m_\tau) = 0.3235^{+0.0138}_{-0.0126}$

RGE 2203.08271

$\alpha_s(m_Z) = 0.1191 \pm 0.0016$

- **An ideal low-energy QCD-testing laboratory: how various QCD currents hadronized, further information about various hadronic resonance parameters (M_R, Γ_R), …**



Decay channel	Resonances
$\tau^- \to \pi^-\pi^0\nu_\tau$	$\rho(770), \rho(1450), \rho(1700)$
$\tau^- \to K^-K_S\nu_\tau$	$\rho(770), \rho(1450), \rho(1700)$
$\tau^- \to K_S\pi^-\nu_\tau$	$K^*(892), K^*(1410)$
$\tau^- \to K^-\eta^{(\prime)}\nu_\tau$	$K^*(1410)$
$\tau^- \to \pi^-\eta^{(\prime)}\nu_\tau$	$a_0(980)$

- **Offer several possibilities of studying the CPV effects** [I. I. Bigi, 1210.2968; 2111.08126]

Semi-leptonic tau decays

- **Two kinds of decay modes: strangeness-conserving & strangeness-changing processes**

Cabibbo-allowed decays ($\mathcal{B} \sim \cos^2\theta_c$)

$\mathcal{B}(S=0) = (61.85 \pm 0.11)\%$ (PDG)

Cabibbo-suppressed decays ($\mathcal{B} \sim \sin^2\theta_c$)

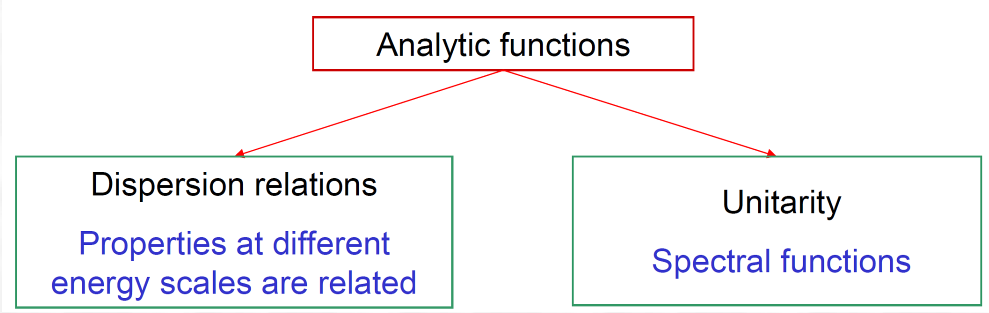
$\mathcal{B}(S=-1) = (2.88 \pm 0.05)\%$ (PDG)

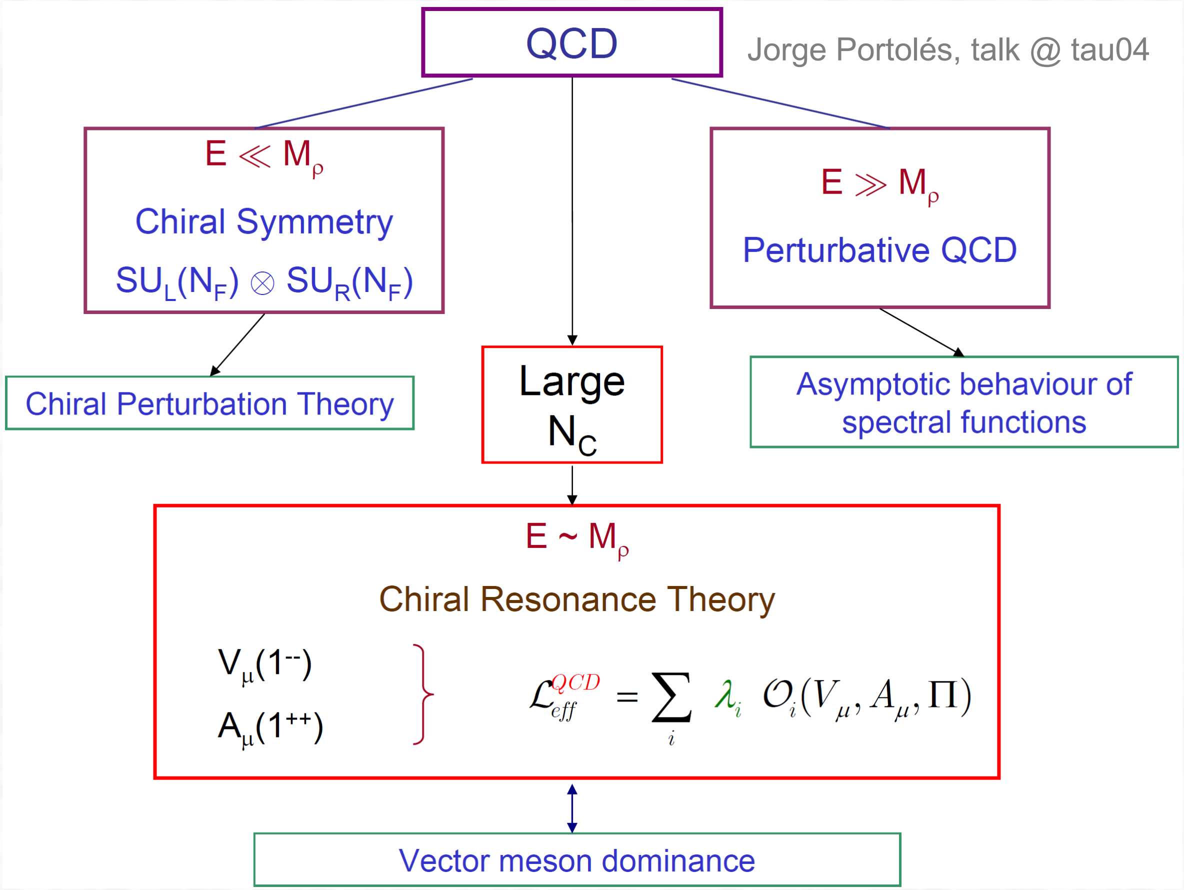
$$\mathcal{M}(\tau \to H\nu_\tau) = \frac{G_F}{\sqrt{2}} V_{CKM}[\bar{u}_{\nu_\tau}\gamma^\mu(1-\gamma_5)u_\tau]H_\mu$$

$$H_\mu = \langle H|(\mathcal{V}_\mu - \mathcal{A}_\mu)e^{i\mathcal{L}_{QCD}}|0\rangle$$

$$= \sum_i \underbrace{(\ldots\ldots\ldots\ldots)^i_\mu}_{\text{Lorentz structure}} \overbrace{F_i(q^2,\ldots)}^{\text{Form factor}}$$

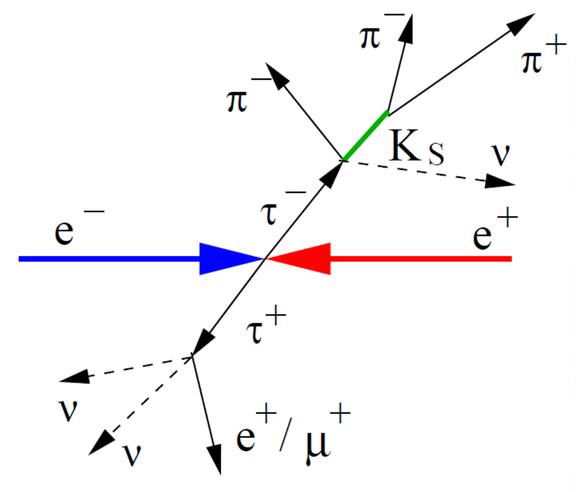
- **Main tasks: determine the hadronic FFs**





Why $\tau \to K_S\pi\nu_\tau$ decays

- **Have the largest Br among semi-lep. decays with 1 kaon** [D. Epifanov *et al.* [Belle], PLB 654 (2007) 65]



$$\mathrm{Br}(\tau \to K_S\pi\nu_\tau) = (0.404 \pm 0.002(\mathrm{stat.}) \pm 0.013(\mathrm{syst.}))\%$$

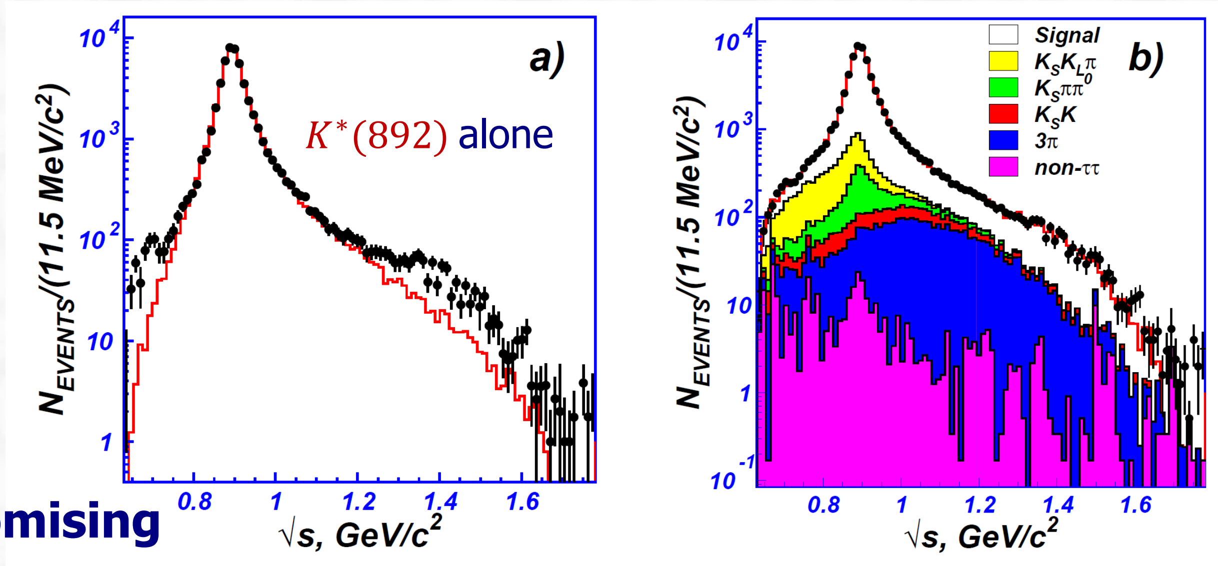
- The hadronic current parametrized by two form factors:

$$J^\mu = F_V(q^2)\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right)(q_1 - q_2)_\nu + F_S(q^2)q^\mu,\ q^\mu = q_1^\mu + q_2^\mu$$

- F_V: $K^*(892)^\pm, K^*(1410)^\pm, K^*(1680)^\pm$;
- F_S: $K^*(800)^\pm(\kappa), K^*(1430)^\pm$;

- The $K^*(892)$ alone not sufficient to describe the $K\pi$ spectrum

- Fitted result with $K^*(892) + K^*(800) + K^*(1410)$ model reproduces data well



- **Searches for CPV in $\tau \to K_S\pi\nu_\tau$ very promising**

Why $\tau \to K_S \pi \nu_\tau$ decays

- **Decay-rate asymmetry in $\tau \to K_S \pi \nu_\tau$ decays**

$$A_{\rm CP}^{\rm rate} \equiv \frac{\Gamma(\tau^+ \to [\pi^+\pi^-]_{``K_S"}\pi^+\bar{\nu}_\tau) - \Gamma(\tau^- \to [\pi^+\pi^-]_{``K_S"}\pi^-\nu_\tau)}{\Gamma(\tau^+ \to [\pi^+\pi^-]_{``K_S"}\pi^+\bar{\nu}_\tau) + \Gamma(\tau^- \to [\pi^+\pi^-]_{``K_S"}\pi^-\nu_\tau)}$$

2.8 σ 偏差

$$\begin{cases} A_{\rm CP}^{\rm Exp} = (-3.6 \pm 2.3 \pm 1.1) \times 10^{-3} \\ A_{\rm CP}^{\rm SM} = (3.6 \pm 0.1) \times 10^{-3} \end{cases}$$

BaBar Collaboration, PRD 85 (2012) 031102

I. Bigi and A. I. Sanda PLB 625 (2005) 47
Y. Grossman and Y. Nir, JHEP 04 (2012) 002

- **CP asymmetry in the angular distribution of $\tau \to K_S \pi \nu_\tau$ decays**

$$A_{CP}^i = \frac{\int_{s_{1,i}}^{s_{2,i}} \int_{-1}^{1} \cos\alpha \left[\frac{d^2\Gamma(\tau^- \to K_S\pi^-\nu_\tau)}{ds\, d\cos\alpha} - \frac{d^2\Gamma(\tau^+ \to K_S\pi^+\bar{\nu}_\tau)}{ds\, d\cos\alpha}\right] ds\, d\cos\alpha}{\frac{1}{2}\int_{s_{1,i}}^{s_{2,i}} \int_{-1}^{1} \left[\frac{d^2\Gamma(\tau^- \to K_S\pi^-\nu_\tau)}{ds\, d\cos\alpha} + \frac{d^2\Gamma(\tau^+ \to K_S\pi^+\bar{\nu}_\tau)}{ds\, d\cos\alpha}\right] ds\, d\cos\alpha}$$

Belle Collaboration, PRL 107 (2011) 131801

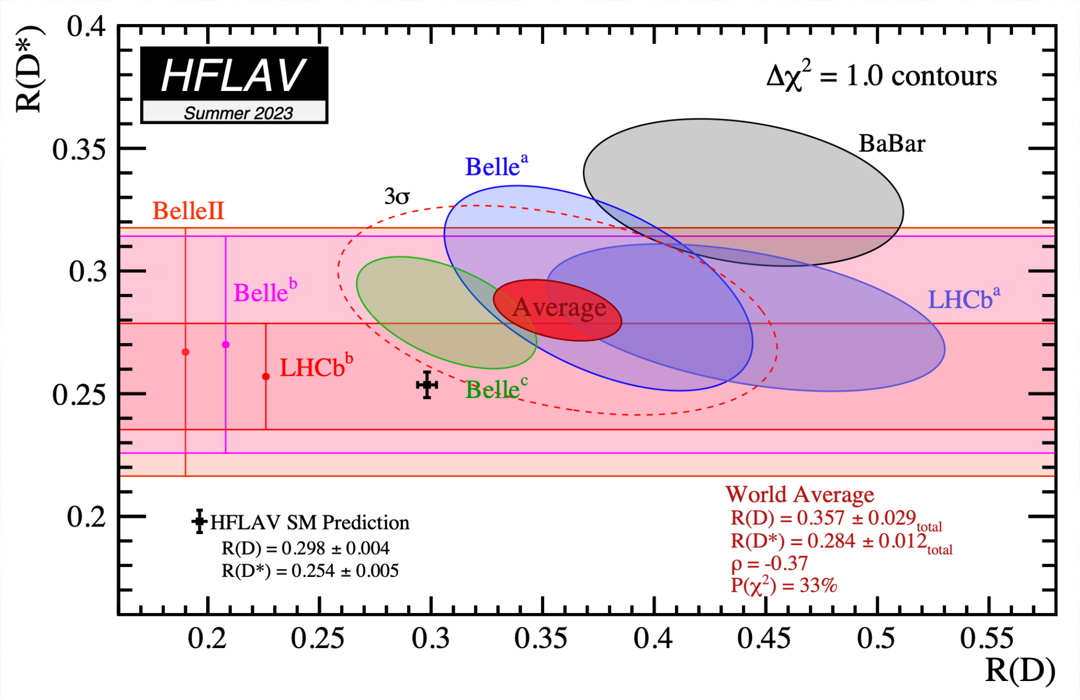
$\sqrt{s}$ [GeV]	$A^{CP}_{{\rm SM},i}$ [10^{-3}]	$A^{CP}_{{\rm exp},i}$ [10^{-3}]
$0.625 - 0.890$	0.39 ± 0.01	$7.9 \pm 3.0 \pm 2.8$
$0.890 - 1.110$	0.04 ± 0.01	$1.8 \pm 2.1 \pm 1.4$
$1.110 - 1.420$	0.12 ± 0.02	$-4.6 \pm 7.2 \pm 1.7$
$1.420 - 1.775$	0.27 ± 0.05	$-2.3 \pm 19.1 \pm 5.5$

compatible with zero with a sensitivity of $\mathcal{O}(10^{-3})$

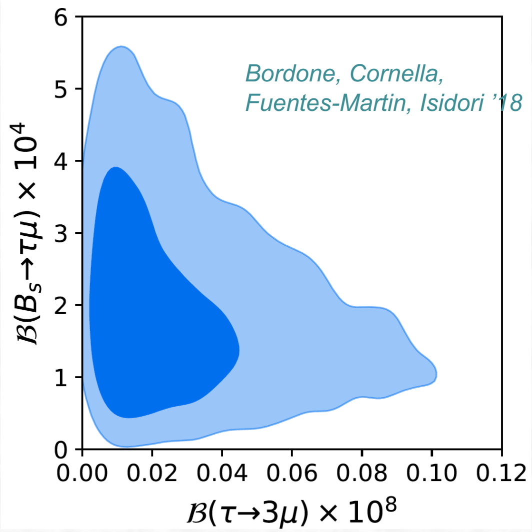
- **Can be used to test CPV mechanism and to probe BSM effects:** $H^\pm$, Leptoquark, ...

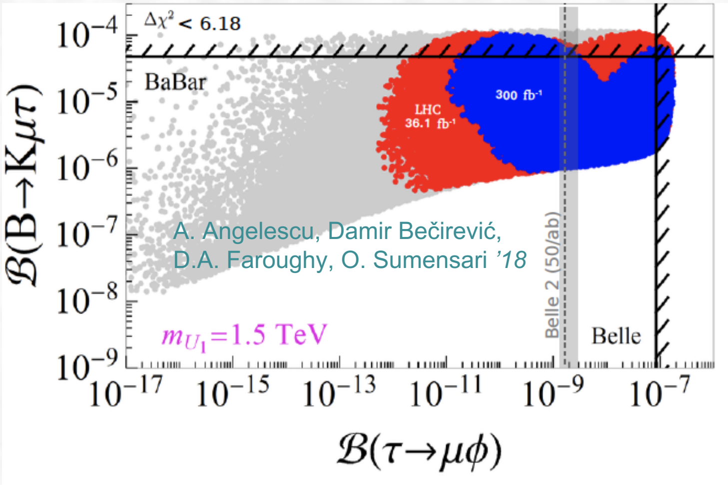
New physics associated with tau lepton

- **Current hints from $R(D^{(*)})$ anomalies indicate non-universal BSM physics**



~3.3σ





- These tau-related processes prove much information about the underlying BSM physics, very complementary!
- Very interesting to probe these relevant decay modes

Experimental facilities for tau physics

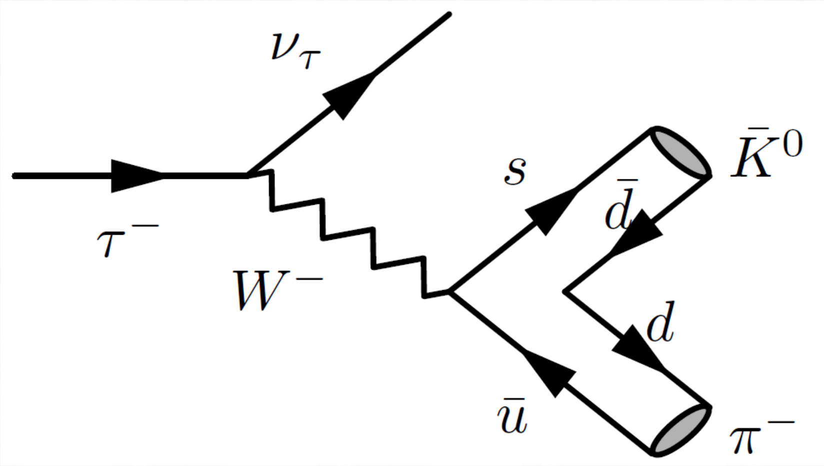
- Many dedicated facilities, with large tau samples [C. Z. Yuan, talk @ IAS Program on HEP 2021]

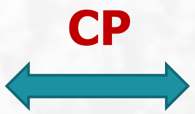
Experiment	Integrated luminosity (fb^{-1})	Cross section (nb)	Number of produced τ pairs (10^9)	Typical tag efficiency	Tagged τ pairs (10^9)	Fraction of Non-τ background
BESIII	50	0 ~ 3.6	~ 0.15	10%	0.015	<1%
BaBar+Belle	1,500	0.9	1.35	33%	0.45	8%
LEP (ALEPH, DELPHI, L3, OPAL)	0.20×4	1.5	0.0012	79% (ALEPH), 92% within $\|\cos\theta\|<0.90$	0.0007	1.2% (ALEPH)
STCF/SCT	10,000	2.5	25	10%=BESIII	1.5	<1%=BESIII
Belle II	50,000	0.9	45	33%=Belle	15	8%=Belle
CEPC	45,000	1.5	70	87% (↑10% over ALEPH)	60	<1.2%@ALEPH
FCC-ee	115,000	1.5	170	87% (↑10% over ALEPH)	150	<1.2%@ALEPH

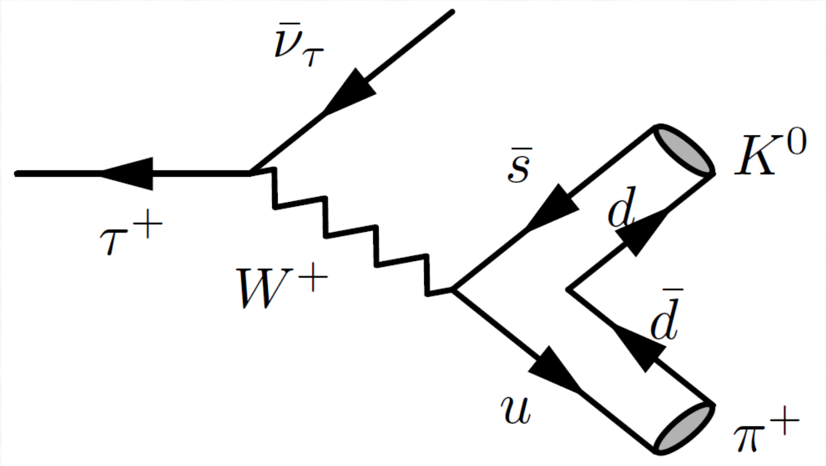
- Lots of tau physics programs with these large tau samples: see ***biennial tau workshops!***

$\tau \to K_S \pi \nu_\tau$ decays within the SM

- **Feynman diagrams at the tree level in weak interaction within the SM:**







- **According to the well-known ΔS = ΔQ rule, τ^- can only decay into $\bar{K}^0$, while τ^+ into K^0**

- **Within the SM, V_{us} is real (no weak phase) & the same strong phase between the two CP-related processes**

$$\boxed{\mathcal{A}(\tau^+ \to K^0 \pi^+ \bar{\nu}_\tau) = \mathcal{A}(\tau^- \to \bar{K}^0 \pi^- \nu_\tau)}$$

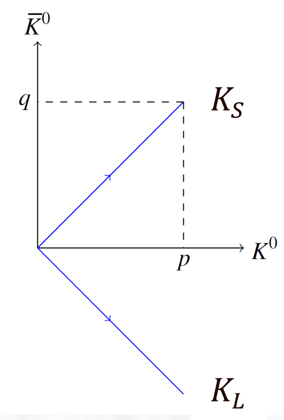
$\tau \to K_S \pi \nu_\tau$ decays within the SM

- **Caution: due to $K^0 - \bar{K}^0$ mixing, the exp. reconstructed kaons are the mass $(|K_S\rangle, |K_L\rangle)$ rather than the flavor $(|K^0\rangle, |\bar{K}^0\rangle)$ eigenstates**

$$|K_S^0\rangle = \frac{(1+\epsilon)|K^0\rangle + (1-\epsilon)|\bar{K}^0\rangle}{\sqrt{2(1+|\epsilon|^2)}}\,, \qquad |K^0\rangle = \frac{\sqrt{2(1+|\epsilon|^2)}}{2(1+\epsilon)}\left[|K_S^0\rangle + |K_L^0\rangle\right],$$

$$|K_L^0\rangle = \frac{(1+\epsilon)|K^0\rangle - (1-\epsilon)|\bar{K}^0\rangle}{\sqrt{2(1+|\epsilon|^2)}}\,, \qquad |\bar{K}^0\rangle = \frac{\sqrt{2(1+|\epsilon|^2)}}{2(1-\epsilon)}\left[|K_S^0\rangle - |K_L^0\rangle\right].$$

$\epsilon = 2.3\times 10^{-3}$ characterizes CPV in neutral kaon system



- **Once CPV in $K^0 - \bar{K}^0$ mixing included, non-zero CP asymmetries appear in the decays:**

- **When $\langle K_{S,L}|$ intermediate states involved, the so-called reciprocal basis more efficient**

$$\begin{pmatrix} |K_L\rangle \\ |K_S\rangle \end{pmatrix} = \begin{pmatrix} p & -q \\ p & q \end{pmatrix} \begin{pmatrix} |K^0\rangle \\ |\bar{K}^0\rangle \end{pmatrix} = \boldsymbol{X}^T \begin{pmatrix} |K^0\rangle \\ |\bar{K}^0\rangle \end{pmatrix}$$

$$\boldsymbol{X}^{-1}\boldsymbol{H}\boldsymbol{X} = \begin{pmatrix} \mu_L & 0 \\ 0 & \mu_S \end{pmatrix}, \qquad \boldsymbol{X} = \begin{pmatrix} p & p \\ -q & q \end{pmatrix}$$

$$\boxed{\exp(-i\boldsymbol{H}t) = e^{-i\mu_S t}|K_S\rangle\langle\tilde{K}_S| + e^{-i\mu_L t}|K_L\rangle\langle\tilde{K}_L|}$$

$$\langle\tilde{K}_{S,L}| = \frac{1}{2}\left(p^{-1}\langle K^0| \pm q^{-1}\langle\bar{K}^0|\right)$$

completeness orthornormality

$$\langle\tilde{K}_S|K_S\rangle = \langle\tilde{K}_L|K_L\rangle = 1\,, \quad \langle\tilde{K}_S|K_L\rangle = \langle\tilde{K}_L|K_S\rangle = 0\,,$$

$$|K_S\rangle\langle\tilde{K}_S| + |K_L\rangle\langle\tilde{K}_L| = 1$$

J. P. Silva, PRD 62 (2000) 116008

$\tau \to K_S \pi \nu_\tau$ decays within the SM

- **Experimentally, the K_S intermediate state is reconstructed via a $\pi^+\pi^-$ final state**

- **Complete time-dependent decay amplitudes of $\tau^\pm \to \pi^+\pi^-\pi^\pm\nu_\tau$ decays (omitting $\langle \pi\nu_\tau|$):**

$$\mathcal{A}(\tau^- \to K_{S,L} \to \pi^+\pi^-) = \langle \pi^+\pi^-|T|K_S\rangle e^{-i\mu_S t}\langle \tilde{K}_S|T|\tau^-\rangle + \langle \pi^+\pi^-|T|K_L\rangle e^{-i\mu_L t}\langle \tilde{K}_L|T|\tau^-\rangle$$
$$= \frac{1}{2q}\left[\langle \pi^+\pi^-|T|K_S\rangle e^{-i\mu_S t} - \langle \pi^+\pi^-|T|K_L\rangle e^{-i\mu_L t}\right]\langle \bar{K}^0|T|\tau^-\rangle\,,$$

ΔS = ΔQ rule

$$\mathcal{A}(\tau^+ \to K_{S,L} \to \pi^+\pi^-) = \langle \pi^+\pi^-|T|K_S\rangle e^{-i\mu_S t}\langle \tilde{K}_S|T|\tau^+\rangle + \langle \pi^+\pi^-|T|K_L\rangle e^{-i\mu_L t}\langle \tilde{K}_L|T|\tau^+\rangle$$
$$= \frac{1}{2p}\left[\langle \pi^+\pi^-|T|K_S\rangle e^{-i\mu_S t} + \langle \pi^+\pi^-|T|K_L\rangle e^{-i\mu_L t}\right]\langle K^0|T|\tau^+\rangle\,,$$

  - **the kaon decays are independent of the τ decays**

- **When the kaon decay time is long enough, the $\pi^+\pi^-$ final state can arise not only from K_S, but also from K_L;**

The interference effect between the K_S & K_L amplitudes important for CPV!

Time-dep. CPA in the angular distribution

□ Time-dependent, doubly differential decay widths:

$$\frac{d^2\Gamma(\tau^- \to K_{S,L}\pi^-\nu_\tau \to [\pi^+\pi^-]\pi^-\nu_\tau)}{ds\,d\cos\alpha} = \frac{d^2\Gamma(\tau^- \to \bar{K}^0\pi^-\nu_\tau)}{ds\,d\cos\alpha}\,\underline{\Gamma(\bar{K}^0(t) \to \pi^+\pi^-)}\,,$$

$$\frac{d^2\Gamma(\tau^+ \to K_{S,L}\pi^+\bar{\nu}_\tau \to [\pi^+\pi^-]\pi^+\bar{\nu}_\tau)}{ds\,d\cos\alpha} = \frac{d^2\Gamma(\tau^+ \to K^0\pi^+\bar{\nu}_\tau)}{ds\,d\cos\alpha}\,\underline{\Gamma(K^0(t) \to \pi^+\pi^-)}\,,$$

s: the $K\pi$ invariant mass squared;

α: the angle between the directions of K and τ seen in the $K\pi$ rest frame

□ Time-dependent $K(t) \to \pi^+\pi^-$ decay widths:

$$\Gamma(\bar{K}^0(t) \to \pi^+\pi^-) = \frac{|\langle\pi^+\pi^-|T|K_S\rangle|^2}{4|q|^2}\left[e^{-\Gamma_S t} + |\eta_{+-}|^2\,e^{-\Gamma_L t} - 2|\eta_{+-}|\,e^{-\Gamma t}\cos(\phi_{+-} - \Delta m t)\right]$$

$$\Gamma(K^0(t) \to \pi^+\pi^-) = \frac{|\langle\pi^+\pi^-|T|K_S\rangle|^2}{4|p|^2}\left[e^{-\Gamma_S t} + |\eta_{+-}|^2\,e^{-\Gamma_L t} + 2|\eta_{+-}|\,e^{-\Gamma t}\cos(\phi_{+-} - \Delta m t)\right]$$

$$\Delta m = M_L - M_S \qquad \Gamma = \frac{\Gamma_L + \Gamma_S}{2}$$

$$\eta_{+-} = \frac{\langle\pi^+\pi^-|T|K_L\rangle}{\langle\pi^+\pi^-|T|K_S\rangle}$$

$$|\eta_{+-}| = (2.232 \pm 0.011) \times 10^{-3}$$

$$\phi_{+-} = (43.51 \pm 0.05)^\circ$$

□ Time-dep. CP asymmetry in the angular distribution:

$(d\omega = ds\,d\cos\alpha)$

$$\boxed{A_i^{CP}(t_1,t_2) = \frac{\int_{s_{1,i}}^{s_{2,i}}\int_{-1}^{1}\cos\alpha\left[\frac{d\Gamma^{\tau^-}}{d\omega}\int_{t_1}^{t_2}F(t)\bar{\Gamma}_{\pi^+\pi^-}(t)\,dt - \frac{d\Gamma^{\tau^+}}{d\omega}\int_{t_1}^{t_2}F(t)\Gamma_{\pi^+\pi^-}(t)\,dt\right]d\omega}{\frac{1}{2}\int_{s_{1,i}}^{s_{2,i}}\int_{-1}^{1}\left[\frac{d\Gamma^{\tau^-}}{d\omega}\int_{t_1}^{t_2}F(t)\bar{\Gamma}_{\pi^+\pi^-}(t)\,dt + \frac{d\Gamma^{\tau^+}}{d\omega}\int_{t_1}^{t_2}F(t)\Gamma_{\pi^+\pi^-}(t)\,dt\right]d\omega}}$$

bin choice $[s_{1,i}, s_{2,i}]$, time interval $[t_1, t_2]$, exp.-dep. effects parametrized by $F(t)$

Time-dep. CPA in the angular distribution

- **The difference of the decay rates weighted by cosα:**

  α: the angle between the directions of K and τ seen in the $K\pi$ rest frame

$$A_i^{CP}(t_1,t_2) = \frac{\int_{s_{1,i}}^{s_{2,i}}\int_{-1}^{1}\cos\alpha\left[\frac{d\Gamma^{\tau^-}}{d\omega}\int_{t_1}^{t_2}F(t)\bar{\Gamma}_{\pi^+\pi^-}(t)\,dt - \frac{d\Gamma^{\tau^+}}{d\omega}\int_{t_1}^{t_2}F(t)\Gamma_{\pi^+\pi^-}(t)\,dt\right]d\omega}{\frac{1}{2}\int_{s_{1,i}}^{s_{2,i}}\int_{-1}^{1}\left[\frac{d\Gamma^{\tau^-}}{d\omega}\int_{t_1}^{t_2}F(t)\bar{\Gamma}_{\pi^+\pi^-}(t)\,dt + \frac{d\Gamma^{\tau^+}}{d\omega}\int_{t_1}^{t_2}F(t)\Gamma_{\pi^+\pi^-}(t)\,dt\right]d\omega}$$

$$= \frac{\left(\langle\cos\alpha\rangle_i^{\tau^-} + \langle\cos\alpha\rangle_i^{\tau^+}\right)A_K^{CP}(t_1,t_2) + \left(\langle\cos\alpha\rangle_i^{\tau^-} - \langle\cos\alpha\rangle_i^{\tau^+}\right)}{1 + A_K^{CP}(t_1,t_2)\cdot A_{\tau,i}^{CP}}$$

- **As the kaon decays independent of the τ decays, the 2nd line are obtained:**

$$\langle\cos\alpha\rangle_i^{\tau^-} + \langle\cos\alpha\rangle_i^{\tau^+} = \frac{\int_{s_{1,i}}^{s_{2,i}}\int_{-1}^{1}\cos\alpha\left[\frac{d\Gamma^{\tau^-}}{d\omega} + \frac{d\Gamma^{\tau^+}}{d\omega}\right]d\omega}{\frac{1}{2}\int_{s_{1,i}}^{s_{2,i}}\int_{-1}^{1}\left[\frac{d\Gamma^{\tau^-}}{d\omega} + \frac{d\Gamma^{\tau^+}}{d\omega}\right]d\omega}$$

$$\langle\cos\alpha\rangle_i^{\tau^-} - \langle\cos\alpha\rangle_i^{\tau^+} = \frac{\int_{s_{1,i}}^{s_{2,i}}\int_{-1}^{1}\cos\alpha\left[\frac{d\Gamma^{\tau^-}}{d\omega} - \frac{d\Gamma^{\tau^+}}{d\omega}\right]d\omega}{\frac{1}{2}\int_{s_{1,i}}^{s_{2,i}}\int_{-1}^{1}\left[\frac{d\Gamma^{\tau^-}}{d\omega} + \frac{d\Gamma^{\tau^+}}{d\omega}\right]d\omega}$$

$$A_K^{CP}(t_1,t_2) = \frac{\int_{t_1}^{t_2}F(t)\left[\bar{\Gamma}_{\pi^+\pi^-}(t) - \Gamma_{\pi^+\pi^-}(t)\right]dt}{\int_{t_1}^{t_2}F(t)\left[\bar{\Gamma}_{\pi^+\pi^-}(t) + \Gamma_{\pi^+\pi^-}(t)\right]dt},$$

$$A_{\tau,i}^{CP} = \frac{\int_{s_{1,i}}^{s_{2,i}}\int_{-1}^{1}\left[\frac{d\Gamma^{\tau^-}}{d\omega} - \frac{d\Gamma^{\tau^+}}{d\omega}\right]d\omega}{\int_{s_{1,i}}^{s_{2,i}}\int_{-1}^{1}\left[\frac{d\Gamma^{\tau^-}}{d\omega} + \frac{d\Gamma^{\tau^+}}{d\omega}\right]d\omega},$$

- **Within the SM:**

$$\mathcal{A}(\tau^+\to K^0\pi^+\bar{\nu}_\tau) = \mathcal{A}(\tau^-\to \bar{K}^0\pi^-\nu_\tau)$$

$$\frac{d\Gamma^{\tau^\pm}}{d\omega} = \frac{d^2\Gamma(\tau^\pm\to K^0(\bar{K}^0)\pi^\pm\bar{\nu}_\tau(\nu_\tau))}{ds\,d\cos\alpha}$$

$$\frac{d\Gamma^{\tau^+}}{d\omega} = \frac{d\Gamma^{\tau^-}}{d\omega} \Longrightarrow A_{\tau,i}^{CP} = 0\,,\quad \langle\cos\alpha\rangle_i^{\tau^-} = \langle\cos\alpha\rangle_i^{\tau^+}$$

$$\Downarrow$$

$$A_i^{CP}(t_1,t_2) = 2\,\langle\cos\alpha\rangle_i^{\tau^-}A_K^{CP}(t_1,t_2)$$

Time-dep. CPA in the angular distribution

◻ Results within the SM: $\boxed{A_i^{CP}(t_1,t_2) = 2\,\langle\cos\alpha\rangle_i^{\tau^-}\,A_K^{CP}(t_1,t_2)}$

$$|\eta_{+-}| \approx \frac{2\Re e(\epsilon_K)}{\sqrt{2}},\ \phi_{+-} \approx 45^\circ,$$

$$\Gamma \approx \frac{\Gamma_S}{2},\ \text{and}\ \Delta m \approx \frac{\Gamma_S}{2}$$

- $A_K^{CP}(t_1,t_2)$: CPV in $K^0 - \bar{K}^0$ mixing

$$A_K^{\rm CP}(t_1 \ll \Gamma_S^{-1}, \Gamma_S^{-1} \ll t_2 \ll \Gamma_L^{-1}) \approx -2{\rm Re}(\epsilon_K) = -(3.32 \pm 0.06)\times 10^{-3}$$

→ Thus easily reproduce Grossman-Nir result!

$$F(t) = \begin{cases} 1 & t_1 < t < t_2 \\ 0 & \text{otherwise.} \end{cases}$$

double-step form

Y. Grossman and Y. Nir, JHEP 04 (2012) 002

- $\langle\cos\alpha\rangle^{\tau^-} = \frac{2}{3}A_{\rm FB}^{\tau^-}$ L. Beldjoudi and T. N. Truong, PLB 351 (1995) 357368

$$A_{\rm FB}^{\tau^-}(s) = \frac{\int_0^1 \frac{d^2\Gamma^{\tau^-}}{ds\,d\cos\alpha}d\cos\alpha - \int_{-1}^0 \frac{d^2\Gamma^{\tau^-}}{ds\,d\cos\alpha}d\cos\alpha}{\int_0^1 \frac{d^2\Gamma^{\tau^-}}{ds\,d\cos\alpha}d\cos\alpha + \int_{-1}^0 \frac{d^2\Gamma^{\tau^-}}{ds\,d\cos\alpha}d\cos\alpha}$$

forward-backward asymmetry

→ Even within the SM, non-zero CPA in the angular distributions due to $K^0 - \bar{K}^0$ mixing

◻ Puzzle still exists:

2.8 σ 偏差

$$\begin{cases} A_{\rm CP}^{\rm Exp} = (-3.6 \pm 2.3 \pm 1.1)\times 10^{-3} \\ A_{\rm CP}^{\rm SM} = (3.6 \pm 0.1)\times 10^{-3} \end{cases}$$

using the efficiency function $F(t)$ provided by the BaBar collaboration!

[BaBar, PRD 85 (2012) 031102]

Time-dep. CPA in the angular distribution

- **Results within the SM:** $\boxed{A_i^{CP}(t_1,t_2) = 2\,\langle\cos\alpha\rangle_i^{\tau^-}\,A_K^{CP}(t_1,t_2)}$

$$A_K^{\rm CP}(t_1 \ll \Gamma_S^{-1}, \Gamma_S^{-1} \ll t_2 \ll \Gamma_L^{-1}) \approx -2{\rm Re}(\epsilon_K) = -(3.32 \pm 0.06)\times 10^{-3}$$

as input!

- **Let's now evaluate $\langle\cos\alpha\rangle_i^{\tau^\pm}$:**

$$\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}} V_{us}\,[\bar\tau\gamma_\mu(1-\gamma_5)\nu_\tau]\,[\bar u\gamma^\mu(1-\gamma_5)s] + {\rm h.c.}$$

$$\frac{d^2\Gamma^{\tau^-}}{ds\,d\cos\alpha} = \frac{G_F^2|F_+(0)V_{us}|^2 m_\tau^3 S_{\rm EW}}{512\pi^3 s^3}\left(1-\frac{s}{m_\tau^2}\right)^2 \lambda^{1/2}(s,M_K^2,M_\pi^2)$$
$$\times\left\{\underline{\left|\tilde F_+(s)\right|^2}\left(\frac{s}{m_\tau^2}+\left(1-\frac{s}{m_\tau^2}\right)\cos^2\alpha\right)\lambda(s,M_K^2,M_\pi^2) + \Delta_{K\pi}^2\underline{\left|\tilde F_0(s)\right|^2}\right.$$
$$\left. -\,2\Delta_{K\pi}\underline{\Re e\left[\tilde F_+(s)\tilde F_0^*(s)\right]}\lambda^{1/2}(s,M_K^2,M_\pi^2)\cos\alpha\right\},$$

normalized FFs

$$\boxed{\tilde F_{+,0}(s) = F_{+,0}(s)/F_+(0)}$$

- **Hadronic FFs:** $\langle \bar K^0(p_K)\pi^-(p_\pi)\,|\bar s\gamma^\mu u|\,0\rangle = \left[(p_K-p_\pi)^\mu - \frac{\Delta_{K\pi}}{s}q^\mu\right]F_+(s) + \frac{\Delta_{K\pi}}{s}q^\mu F_0(s)$

Time-dep. CPA in the angular distribution

- **For hadronic FFs, we adopt the state-of-the-art results:**

  The Breit-Wigner form violates Watson's theorem, and thus not physical and not applicable for CPV

- Vector form factor the thrice-subtracted dispersion representation

  D.R. Boito, R. Escribano and M. Jamin, Eur. Phys. J. C59 (2009) 821

$$F_+(s) = \exp\left\{\lambda'_+ \frac{s}{M^2_{\pi^-}} + \frac{1}{2}(\lambda''_+ - \lambda'^2_+)\frac{s^2}{M^4_{\pi^-}} + \frac{s^3}{\pi}\int_{s_{K\pi}}^{s_{cut}} ds' \frac{\delta_+(s')}{(s')^3(s'-s-i\epsilon)}\right\},$$

- Scalar form factor the coupled-channel dispersive representation

  M. Jamin, J.A. Oller and A. Pich, Nucl. Phys. B622 (2002) 279

$$F_0^1(s) = \frac{1}{\pi}\sum_{j=1}^{3}\int_{s_j}^{\infty} ds' \frac{\sigma_j(s')F_0^j(s')t_0^{1\to j}(s')^*}{s'-s-i\epsilon}, (1 \equiv K\pi,\ 2 \equiv K\eta,\ \text{and } 3 \equiv K\eta')$$

- **Angular observable:** differential decay width weighted by cosα

$$\langle\cos\alpha\rangle^{\tau^-}(s) = \frac{\int_{-1}^{1}\cos\alpha\left(\frac{d^2\Gamma^{\tau^-}}{ds\,d\cos\alpha}\right)d\cos\alpha}{\int_{-1}^{1}\left(\frac{d^2\Gamma^{\tau^-}}{ds\,d\cos\alpha}\right)d\cos\alpha}$$

$$= \frac{-2\Delta_{K\pi}\Re e[\tilde{F}_+(s)\tilde{F}_0^*(s)]\lambda^{1/2}(s, M_K^2, M_\pi^2)}{\left|\tilde{F}_+(s)\right|^2\left(1+\frac{2s}{m_\tau^2}\right)\lambda(s, M_K^2, M_\pi^2) + 3\Delta_{K\pi}^2\left|\tilde{F}_0(s)\right|^2}$$

Time-dep. CPA in the angular distribution

- **Results for $A_i^{CP}(t_1, t_2)$ in four mass bins:**

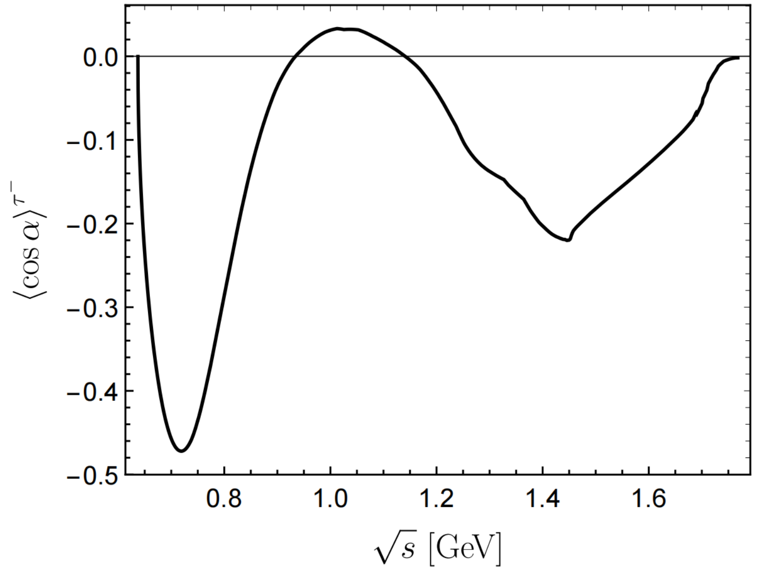
$\sqrt{s}$ [GeV]	$A_{\mathrm{SM},i}^{\mathrm{CP}}$ [10^{-3}]	$A_{\mathrm{exp},i}^{\mathrm{CP}}$ [10^{-3}]	n_i/N_s [%]
$0.625 - 0.890$	0.39 ± 0.01	$7.9 \pm 3.0 \pm 2.8$	36.53 ± 0.14
$0.890 - 1.110$	0.04 ± 0.01	$1.8 \pm 2.1 \pm 1.4$	57.85 ± 0.15
$1.110 - 1.420$	0.12 ± 0.02	$-4.6 \pm 7.2 \pm 1.7$	4.87 ± 0.04
$1.420 - 1.775$	0.27 ± 0.05	$-2.3 \pm 19.1 \pm 5.5$	0.75 ± 0.02

SM predictions still below Belle detection sensitivity of $\mathcal{O}(10^{-3})$, but expected to be detectable at Belle II, with $\sqrt{70}$ times more sensitive results!

- **Two more predictions:**

as large as SM prediction for the decay-rate asymmetry

$$A_i^{CP}(t_1, t_2) = \begin{cases} (3.06 \pm 0.06) \times 10^{-3}, & 0.70\,\mathrm{GeV} < \sqrt{s} < 0.75\,\mathrm{GeV} \\ (1.38 \pm 0.18) \times 10^{-3}, & 1.40\,\mathrm{GeV} < \sqrt{s} < 1.50\,\mathrm{GeV} \end{cases}$$



- **Can Belle II and STCF measure the CP-violating angular observables in such two mass intervals?**

$\tau^{\pm} \to K^0(\bar{K}^0)\pi^{\pm}\bar{\nu}_\tau(\nu_\tau)$ in a general EFT

- **When NP presents in tau decays:**

$$\mathcal{A}(\tau^+ \to K^0\pi^+\bar{\nu}_\tau) \neq \mathcal{A}(\tau^- \to \bar{K}^0\pi^-\nu_\tau)$$

$$\frac{d\Gamma^{\tau^+}}{d\omega} \neq \frac{d\Gamma^{\tau^-}}{d\omega} \Longrightarrow A^{CP}_{\tau,i} \neq 0\,, \quad \langle\cos\alpha\rangle_i^{\tau^-} \neq \langle\cos\alpha\rangle_i^{\tau^+}$$

- **CP-violating observables:**

$$\boxed{A_i^{\rm CP} \simeq \left(\langle\cos\alpha\rangle_i^{\tau^-} + \langle\cos\alpha\rangle_i^{\tau^+}\right) A_K^{\rm CP} + \left(\langle\cos\alpha\rangle_i^{\tau^-} - \langle\cos\alpha\rangle_i^{\tau^+}\right)}$$

- **The most general $SU(3)_C \otimes U(1)_{em}$-invariant low-energy effective Lagrangian:**

$$\mathcal{L}_{\rm eff} = -\frac{G_F V_{us}}{\sqrt{2}}\Big\{\bar{\tau}\gamma_\mu(1-\gamma_5)\nu_\tau \cdot \bar{u}\left[\gamma^\mu - (1-2\,\hat{\epsilon}_R)\gamma^\mu\gamma_5\right]s$$

$$+\,\bar{\tau}(1-\gamma_5)\nu_\tau \cdot \bar{u}\left[\hat{\epsilon}_S - \hat{\epsilon}_P\gamma_5\right]s + 2\,\hat{\epsilon}_T\,\bar{\tau}\sigma_{\mu\nu}(1-\gamma_5)\nu_\tau \cdot \bar{u}\sigma^{\mu\nu}s\Big\} + \text{h.c.}$$

- **Decay amplitude for $\tau^- \to \bar{K}^0\pi^-\nu_\tau$:**

scalar operator + tensor operator

$$\mathcal{M} = \mathcal{M}_V + \mathcal{M}_S + \mathcal{M}_T$$
$$= \frac{G_F V_{us}}{\sqrt{2}}\left[L_\mu H^\mu + \hat{\epsilon}_S^* LH + 2\hat{\epsilon}_T^* L_{\mu\nu}H^{\mu\nu}\right]$$

Tensor form factors

Leptonic currents:

$$
\begin{aligned}
L &= \bar{u}(p_{\nu_\tau})(1+\gamma_5)u(p_\tau)\,,\\
L_\mu &= \bar{u}(p_{\nu_\tau})\gamma_\mu(1-\gamma_5)u(p_\tau)\,,\\
L_{\mu\nu} &= \bar{u}(p_{\nu_\tau})\sigma_{\mu\nu}(1+\gamma_5)u(p_\tau)\,,
\end{aligned}
$$

Hadronic matrix elements:

$$
\begin{aligned}
H &= \langle \pi^- \overline{K}^0 \mid \bar{s}u \mid 0\rangle = F_S(s)\\
H^\mu &= \langle \pi^- \overline{K}^0 \mid \bar{s}\gamma^\mu u \mid 0\rangle = Q^\mu F_+(s) + \frac{\Delta_{K\pi}}{s} q^\mu F_0(s)\\
H^{\mu\nu} &= \langle \pi^- \overline{K}^0 \mid \bar{s}\sigma^{\mu\nu} u \mid 0\rangle = iF_T(s)\,(p_K^\mu p_\pi^\nu - p_\pi^\mu p_K^\nu)
\end{aligned}
$$

$K\pi$ tensor FF: due to lack of enough exp. data, the once-subtracted dispersion representation

$$
F_T(s) = F_T(0)\exp\left\{\frac{s}{\pi}\int_{s_{K\pi}}^{\infty} ds' \frac{\delta_T(s')}{s'(s'-s-i\epsilon)}\right\}
$$

- **for $F_T(0)$: obtained from the lowest chiral order of χPT with tensor source**

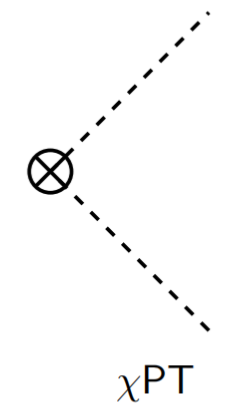
- $\mathcal{L}_4^{\chi\mathrm{PT}} = \Lambda_1 \langle t_+^{\mu\nu} f_{+\mu\nu}\rangle - i\Lambda_2 \langle t_+^{\mu\nu} u_\mu u_\nu\rangle + \Lambda_3 \langle t_+^{\mu\nu} t^{+\mu\nu}\rangle + \Lambda_4 \langle t_+^{\mu\nu}\rangle^2$

$\Downarrow$

O. Cata and V. Mateu, JHEP 09 (2007) 078

$$
\left\langle \bar{K}^0(p_K)\pi^-(p_\pi)\left|\frac{\delta L_4^{\chi\mathrm{PT}}}{\delta \bar{t}_{\mu\nu}}\right|0\right\rangle = i\frac{\Lambda_2}{F_\pi^2}(p_K^\mu p_\pi^\nu - p_K^\nu p_\pi^\mu) \Rightarrow
$$

$F_T(0) = \Lambda_2/F_\pi^2$, with $\Lambda_2 = (11.1 \pm 0.4)\mathrm{MeV}$



I. Baum *et al.*, PRD 84 (2011) 074503

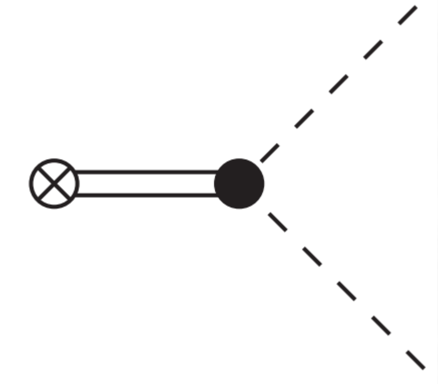
Tensor form factors

- **With $s \in [(M_K + M_\pi)^2, m_\tau^2]$, light resonances to FFs must be included to give s-dep. of FFs**

- **As spin-1 resonances described equivalently by vector or anti-symmetric tensor fields, resonance contributions to $F_+(s)$ & $F_T(s)$ are both dominated by $K^*(892)$ & $K^*(1410)$**

- **$F_T(s)$ obtained with $\mathrm{R}\chi\mathrm{T}$ including spin-1 resonances:**

  - $\mathcal{L}_6^{\mathrm{R}\chi\mathrm{T}} = \mathcal{L}_{kin}(\hat{V}_\mu) - \frac{1}{2\sqrt{2}}\left(f_V\langle \hat{V}_{\mu\nu} f_+^{\mu\nu}\rangle + ig_V\langle \hat{V}_{\mu\nu}[u^\mu, u^\nu]\rangle\right) - f_V^T\langle \hat{V}_{\mu\nu} t_+^{\mu\nu}\rangle$



Ecker, J. Gasser, H. Leutwyler, A. Pich, and E. de Rafael, PLB 223 (1989) 425

$$F_T(s) = \frac{\Lambda_2}{F_\pi^2}\left[1 + \frac{\sqrt{2}f_V^T g_V}{\Lambda_2}\frac{s}{M_{K^*}^2 - s} + \frac{\sqrt{2}f_V^{T\prime} g_V^\prime}{\Lambda_2}\frac{s}{M_{K^{*\prime}}^2 - s}\right]$$

$$= \frac{\Lambda_2}{F_\pi^2}\left[\frac{M_{K^*}^2 + \beta s}{M_{K^*}^2 - s} - \frac{\beta s}{M_{K^{*\prime}}^2 - s}\right]$$

$\xrightarrow{\text{energy-dep. width } \gamma_n(s)}$

$$\tilde{F}_T(s) = F_T(s)/F_T(0)$$

$$= \frac{m_{K^*}^2 - \kappa_{K^*}\tilde{H}_{K\pi}(0) + \beta s}{D(m_{K^*}, \gamma_{K^*})} - \frac{\beta s}{D(m_{K^{*\prime}}, \gamma_{K^{*\prime}})}$$

$\beta = \frac{\sqrt{2}f_V^T g_V}{\Lambda_2} - 1 \simeq \pm 0.75\gamma$: characterizes relative weight of the two resonances, and plays the same role as γ for $F_+(s)$

Feng-Zhi Chen, Xin-Qiang Li, Ya-Dong Yang, Xin Zhang, PRD 100 (2019) 113006

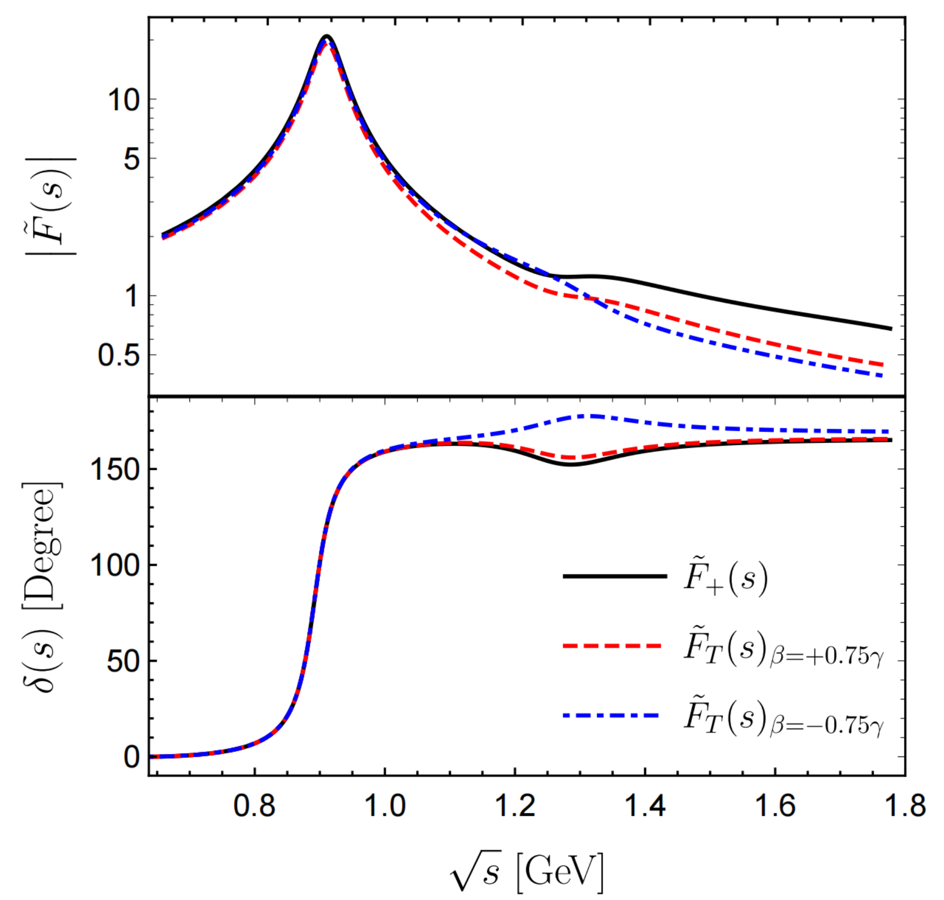
Tensor form factors

- Combining χPT @ low s + RχT @ intermediate s + asymptotic behaviors @ high s, we obtain the once-subtracted dispersion representation:

$$\boxed{F_T(s) = F_T(0)\exp\left\{\frac{s}{\pi}\int_{s_{K\pi}}^{\infty} ds' \frac{\delta_T(s')}{s'(s'-s-i\epsilon)}\right\}}$$

$$\delta_T(s) = \begin{cases} \arctan\left[\frac{\Im m \tilde{F}_T(s)}{\Re e \tilde{F}_T(s)}\right], & s_{K\pi} < s < s_{\text{cut}} \\ n_T\pi, & s \geq s_{\text{cut}} \end{cases}$$

asymptotic $1/s$ as dictated by pQCD



- in elastic region (below ~ 1.2 GeV), $\delta_T(s) = \delta_+(s)$ as required by Watson's theorem [K. M. Watson, Phys. Rev. 95 (1954) 228]

- in inelastic region (above ~ 1.2 GeV), $\delta_T(s)$ and $\delta_+(s)$ start to behave differently due to the different relative weights of the two resonances $K^*(892)$ & $K^*(1410)$

CP-violating observables in general EFT

- **Decay-rate asymmetry:**

  only vector-tensor interference as the only possible mechanism

$$A_{\rm CP}^{\rm rate}(\tau\to K\pi\nu_\tau)=\frac{\Gamma(\tau^+\to K^0\pi^+\bar\nu_\tau)-\Gamma(\tau^-\to \bar K^0\pi^-\nu_\tau)}{\Gamma(\tau^+\to K^0\pi^+\bar\nu_\tau)+\Gamma(\tau^-\to \bar K^0\pi^-\nu_\tau)}$$
$$=\frac{{\rm Im}[\hat\epsilon_T]\,G_F^2|V_{us}|^2S_{\rm EW}}{128\,\pi^3\,m_\tau^2\,\Gamma(\tau\to K_S\pi\nu_\tau)}\int_{s_{K\pi}}^{m_\tau^2}ds\left(1-\frac{m_\tau^2}{s}\right)^2\lambda^{\frac{3}{2}}\left(s,M_K^2,M_\pi^2\right)$$
$$\times\,\underline{|F_T(s)|\,|F_+(s)|\,\sin\left[\delta_T(s)-\delta_+(s)\right]}\,,$$

- **CPA in angular distribution:**

  both from scalar-vector and scalar-tensor interferences

$$A_{CP}^i\simeq\Delta_{K\pi}\,S_{\rm EW}\,\frac{N_s}{n_i}\int_{s_{1,i}}^{s_{2,i}}\left\{-\frac{{\rm Im}[\hat\epsilon_S]}{m_\tau(m_s-m_u)}\,\underline{{\rm Im}\left[F_+(s)F_0^*(s)\right]}-\frac{2{\rm Im}[\hat\epsilon_T]}{m_\tau}\,\underline{{\rm Im}\left[F_T(s)F_0^*(s)\right]}\right.$$
$$\left.+\left[\left(\frac{1}{s}+\frac{{\rm Re}[\hat\epsilon_S]}{m_\tau(m_s-m_u)}\right)\underline{{\rm Re}\left[F_+(s)F_0^*(s)\right]}-\frac{2{\rm Re}[\hat\epsilon_T]}{m_\tau}\,\underline{{\rm Re}[F_T(s)F_0^*(s)]}\right]A_K^{CP}\right\}C(s)\,ds\,.$$

- **Constraints on ${\rm Re}[\hat\epsilon_{S,T}]$: more stringent from decay rates of various exclusive τ decays**

$${\rm Re}[\hat\epsilon_S]=(0.8^{+0.8}_{-0.9}\pm0.3)\%,\ {\rm Re}[\hat\epsilon_T]=(0.9\pm0.7\pm0.4)\%$$

S. Gonzàlez-Solís, A. Miranda, J. Rendón and P. Roig, Phys. Lett. B 804 (2020) 135371

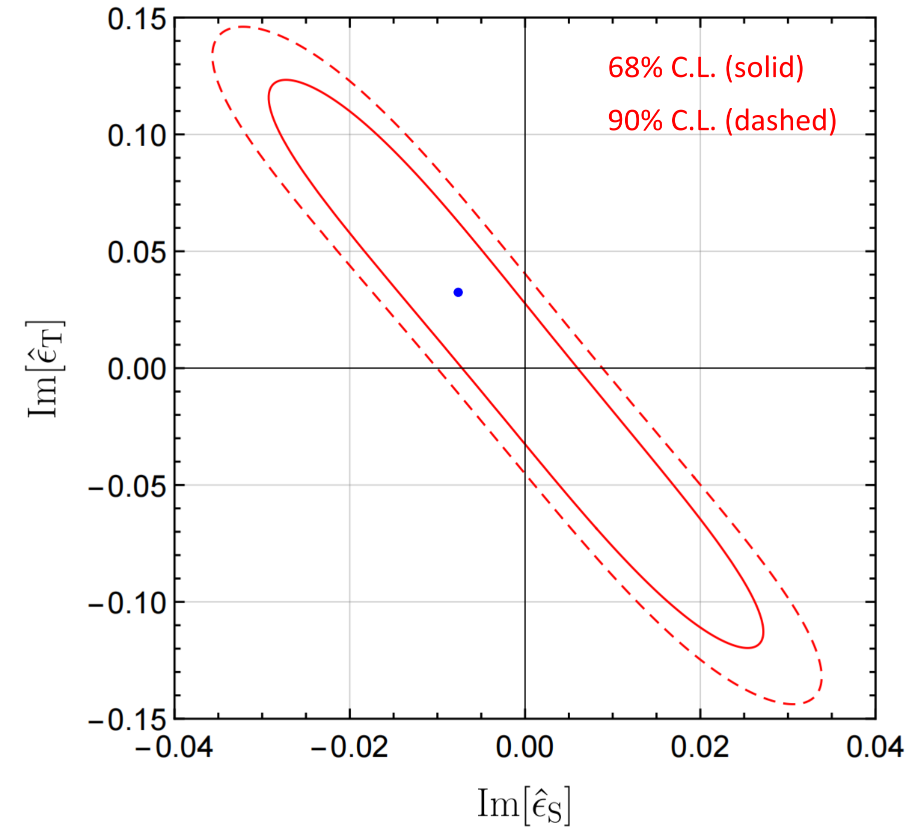
- **Constraints on ${\rm Im}[\hat\epsilon_{S,T}]$: more sensitive to these CP-violating observables**

CP-violating observables in general EFT

- **Fit results on $\mathrm{Im}[\hat{\epsilon}_{S,T}]$ from $\mathcal{B}^{\tau^-}_{\mathrm{exp}}$ and four bins of $A^{CP}_{\mathrm{exp},i}$:**

$$\chi^2 = \sum_{i=1}^{4}\left(\frac{A^{CP}_{\mathrm{exp},i} - A^{CP}_{\mathrm{th},i}}{\sigma_i}\right)^2 + \left(\frac{\mathcal{B}^{\tau^-}_{\mathrm{exp}} - \mathcal{B}^{\tau^-}_{\mathrm{th}}}{\sigma_{\mathcal{B}}}\right)^2$$

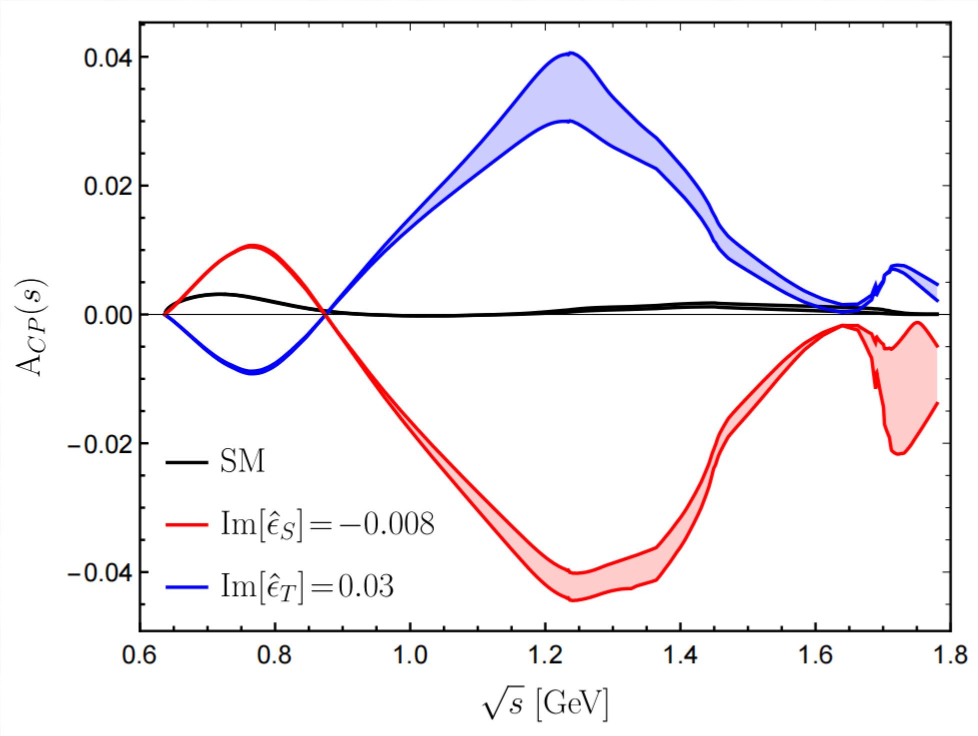
$$\chi^2_{\mathrm{min}} = 4.20, \quad \mathrm{Im}[\hat{\epsilon}_S] = -0.008 \pm 0.027, \quad \mathrm{Im}[\hat{\epsilon}_T] = 0.03 \pm 0.12$$



- remarkably negative correlation between $\mathrm{Im}[\hat{\epsilon}_S]$ & $\mathrm{Im}[\hat{\epsilon}_T]$, as both vector & tensor FFs dominated by $K^*(892)$ and $K^*(1410)$, and thus have almost the same phases, especially in elastic region
- bound on $\mathrm{Im}[\hat{\epsilon}_S]$ consistent with $|\mathrm{Im}(\eta_S)| < 0.026$ @ 90% C.L. obtained by Belle [PRL 107 (2011) 131801]
- upper bound on $\mathrm{Im}[\hat{\epsilon}_T]$ only of $\mathcal{O}(10^{-1})$, much weaker than $2|\mathrm{Im}[\hat{\epsilon}_T]| \leq 10^{-5}$ from neutron EDM & $D^0 - \bar{D}^0$ mixing [V. Cirigliano *et al.*, PRL 120 (2018) 141803]

CP-violating observables in general EFT

- A_i^{CP} **in the presence of non-standard scalar & tensor interactions**



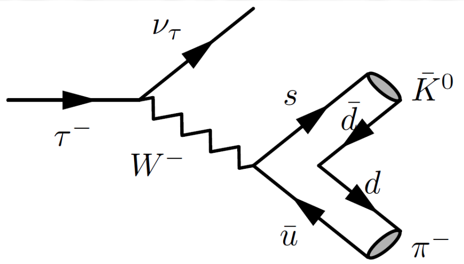
- with best-fit values of $\mathrm{Im}[\hat{\epsilon}_S]$ and $\mathrm{Im}[\hat{\epsilon}_T]$, the CPA distributions have almost the same magnitude but opposite in sign in whole $K\pi$ invariant-mass region
- the maximum absolute values are reached at around $\sqrt{s} = 1.2\ \mathrm{GeV}$ for both cases
- the non-standard scalar & tensor contributions about one order of magnitude larger than SM prediction

- **We strongly suggest to make more precise measurement of CP asymmetry in the angular distributions, especially at Belle II & SCTF, ...**

Belle-II, *PTEP* **2019** (2019) 123C01; H. Sang, X. Shi, X. Zhou, X. Kang and J. Liu, *CPC* **45** (2021) 053003

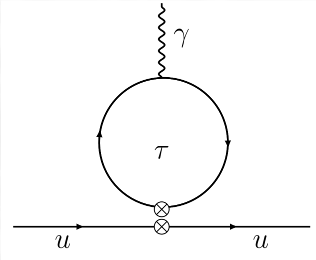
Summary

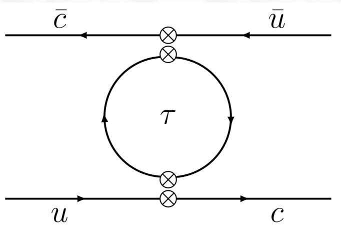
- **With large τ samples from exp. facilities, precision τ-lepton physics very promising**
- **Semi-leptonic τ decays: an ideal laboratory for low-energy QCD-testing & BSM probes**
- **$\tau \to K_S \pi \nu_\tau$ decays: large branching ratio & very promising CPV observables**
  - within the SM, there exist both decay-rate asymmetry & CP asymmetry in angular distribution due to CPV in $K^0 - \bar{K}^0$ mixing, with results of $\mathcal{O}(10^{-3})$ and detectable @ Belle II & STCF, …
  - With a general EFT, only vector-tensor interference produces a direct decay-rate asymmetry, while both scalar-vector & scalar-tensor interferences possible for CPA in angular distribution
- **Bounds on $\mathrm{Im}[\hat{\epsilon}_{S,T}]$:** $\mathrm{Im}[\hat{\epsilon}_S] = -0.008 \pm 0.027, \quad \mathrm{Im}[\hat{\epsilon}_S] \in [-3.1, 1.6]\times 10^{-4}$ @ 2σ



$\mathrm{Im}[\hat{\epsilon}_T] = 0.03 \pm 0.12, \qquad |\mathrm{Im}[\hat{\epsilon}_T]| \lesssim 4\times 10^{-6}$

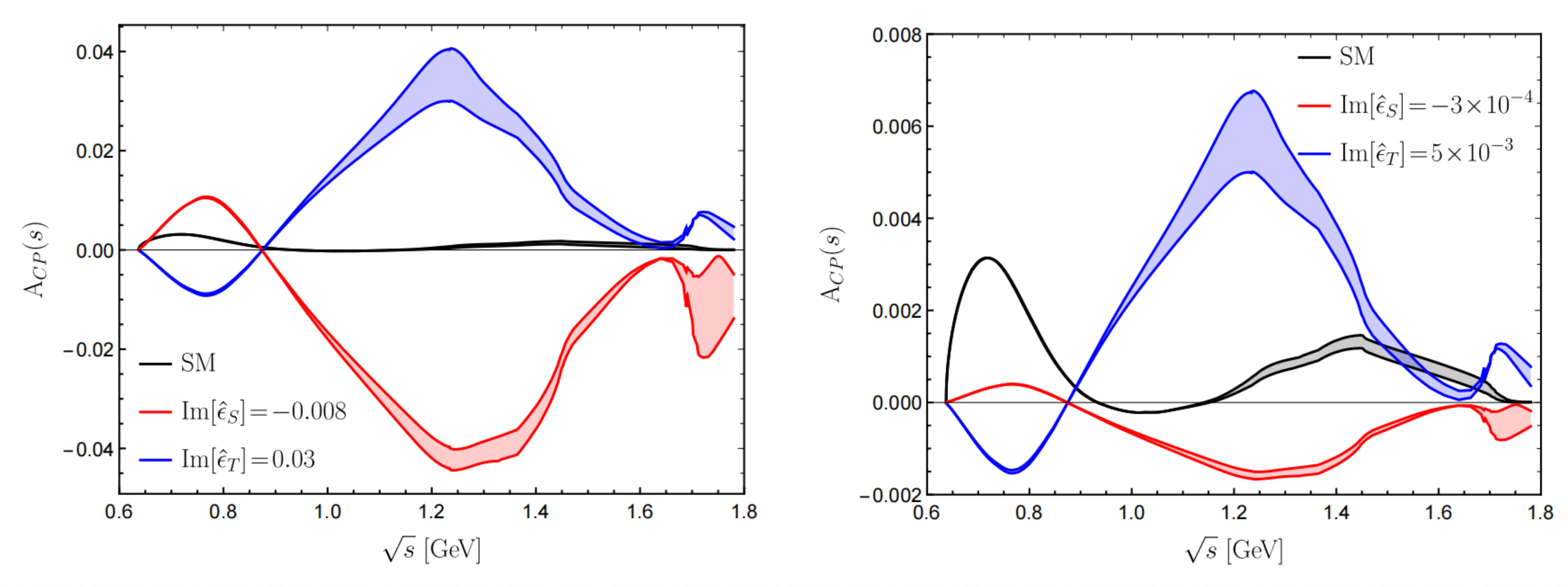
τ decays give much less stringent than from d_n & $D^0 - \bar{D}^0$ mixing





Summary

- With other bounds considered, 2.8σ **deviation** for A_{CP}^{rate} not easily explained by heavy NP
- Predictions for **CP asymmetry in the angular distribution** in three different cases:



sensitive to the non-standard scalar & tensor interactions!

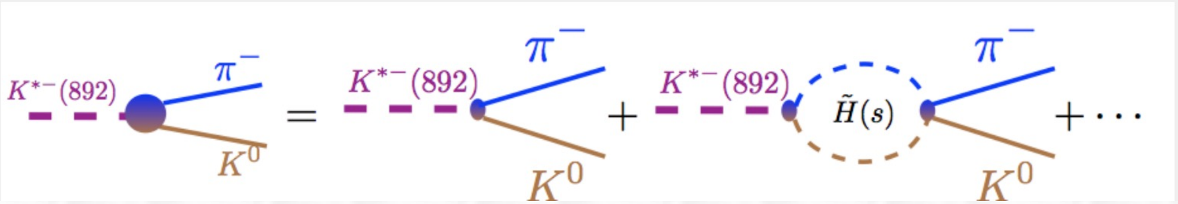
- We strongly suggest to measure these observables more precisely @ Belle II & STCF!

Thank you for your attention!

Backup

Vector & scalar FFs for $\tau \to K_S \pi \nu_\tau$ decays

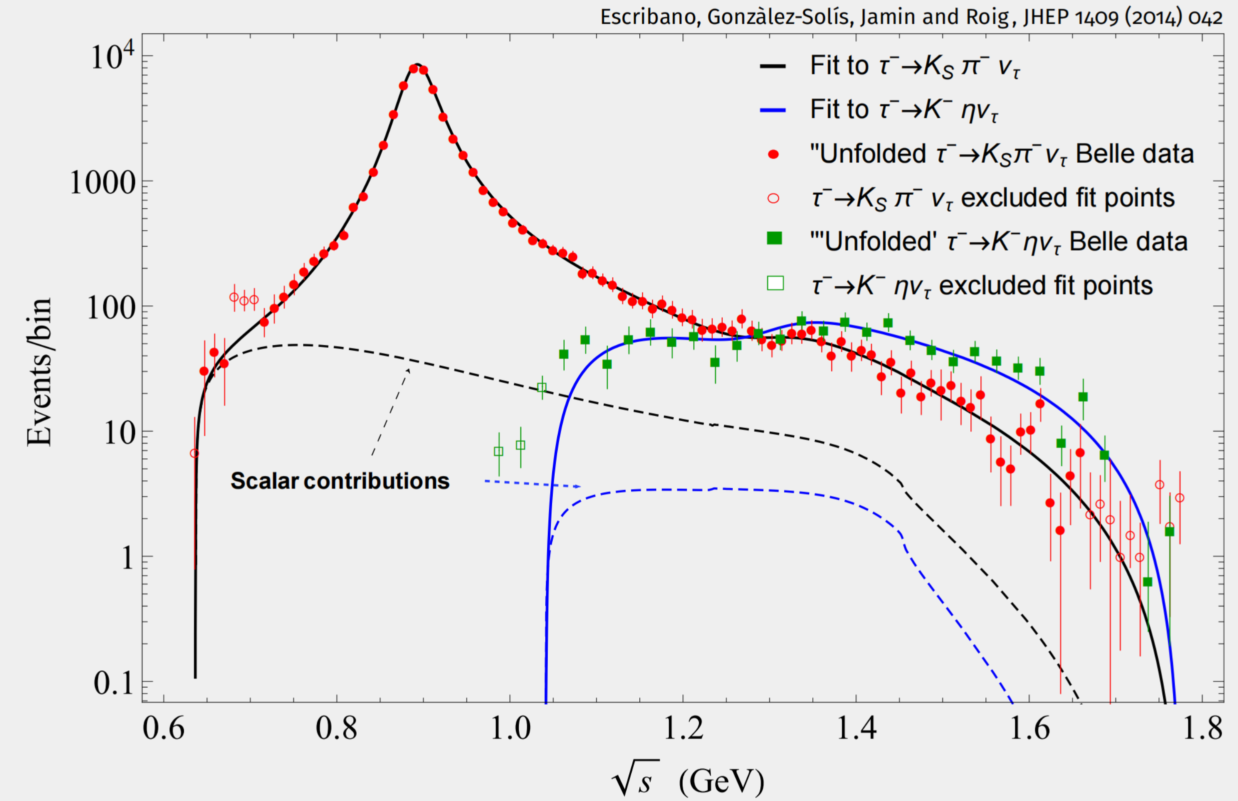
□ Form factors with $\chi\mathrm{PT} + \mathrm{R}\chi\mathrm{T}$ including the two resonances $K^*(892)$ and $K^*(1410)$:



$$\widetilde{F}_V^{K\pi}(s) = \frac{m_{K^*}^2 - \kappa_{K^*}\widetilde{H}_{K\pi}(0) + \gamma s}{D(m_{K^*}, \gamma_{K^*})} - \frac{\gamma s}{D(m_{K^{*\prime}}, \gamma_{K^{*\prime}})},$$

$$D(m_n, \gamma_n) \equiv m_n^2 - s - \kappa_n \mathrm{Re}\left[H_{K\pi}(s)\right] - i m_n \Gamma_n(s),$$

$$\kappa_n = \frac{192\pi F_K F_\pi}{\sigma_{K\pi}(m_{K^*}^2)} \frac{\gamma_{K^*}}{m_{K^*}}, \quad \Gamma_n(s) = \Gamma_n \frac{s}{m_n^2} \frac{\sigma_{K\pi}^3(s)}{\sigma_{K\pi}^3(m_n^2)},$$



Fitted value	Reference Fit
$\bar{B}_{K\pi}(\%)$	0.404 ± 0.012
$(B^{th}_{K\pi})(\%)$	(0.402)
M_{K^*}	892.03 ± 0.19
Γ_{K^*}	46.18 ± 0.42
$M_{K^{*\prime}}$	1305^{+15}_{-18}
$\Gamma_{K^{*\prime}}$	168^{+52}_{-44}
$\gamma_{K\pi} \times 10^2$	$= \gamma_{K\eta}$
$\lambda'_{K\pi} \times 10^3$	23.9 ± 0.7
$\lambda''_{K\pi} \times 10^4$	11.8 ± 0.2
$\bar{B}_{K\eta} \times 10^4$	1.58 ± 0.10
$(B^{th}_{K\eta}) \times 10^4$	(1.45)
$\gamma_{K\eta} \times 10^2$	$-3.4^{+1.0}_{-1.3}$
$\lambda'_{K\eta} \times 10^3$	20.9 ± 1.5
$\lambda''_{K\eta} \times 10^4$	11.1 ± 0.4
χ^2/n.d.f.	$108.1/105 \sim 1.03$

gives a good description of the exp. data