Gravitational form factor

Mamiya Kawaguchi

Gravitational Form Factors (GFFs) are form factors associated with the QCD energy momentum tensor.

$\hat{T}^{a}_{\mu\nu}(x)$:Energy momentum tensor of gluon part and quark parts: $a = g,\ u,\ d,\ \ldots$

$$\langle p', s'|\hat{T}^{a}_{\mu\nu}(x)|p, s\rangle$$

- (Pseudo)scalar meson state
- Vector meson state
- Proton state

"Form factors" are related to "conserved physical quantities".

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Much like the electromagnetic form factors,
they contain a wealth of information about the structure of nucleons.

Electromagnetic form factors and axial form factor

em: $\partial_\mu J^\mu_{\rm em} = 0$	$\langle N'|J^\mu_{\bf em}|N\rangle$	$\longrightarrow$	$Q = 1.602176487(40)\times 10^{-19}{\rm C}$ $\mu = 2.792847356(23)\mu_N$
weak: PCAC	$\langle N'|J^\mu_{\bf weak}|N\rangle$	$\longrightarrow$	$g_A = 1.2694(28)$ $g_p = 8.06(55)$

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			$J = \frac{1}{2}$	
			$\boldsymbol{D} = \boldsymbol{?}$	

There is still a lack of understanding of D term.
→ What is the role of GFFs?
→ Are GFFs related to nonperturbative aspect like confinement property?

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$$\langle p', s' | \hat{T}^{a}_{\mu\nu}(x) | p, s \rangle$$

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GFFs of pion, rho meson, nucleon and delta baryon have been observed in lattice QCD simulation "Gluon gravitational structure of hadrons of different spin" *Phys.Rev.D* 105 (2022) 5, 054509

First attempt to extract unknown part of GFFs (D-term from) experimental data has been made in "The pressure distribution inside the proton," Nature 557 (2018) no.7705, 396-399.

GFFs have been extensively studied by

- Holographic QCD (arXiv:2206.06578, arXiv:2204.08857…)
- Skyrmion approach (arXiv:2304.05994…).

em:	$\partial_\mu J^{\mu}_{\text{em}} = 0$	$\langle N' \| J^{\mu}_{\text{em}} \| N \rangle$	$\longrightarrow$	$Q = 1.602176487(40) \times 10^{-19}\text{C}$ $\mu = 2.792847356(23)\mu_N$
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→ What is the role of GFFs?

→ Are GFFs related to nonperturbative aspect like confinement property?

- QCD energy momentum tensor

$$\hat{T}_{\mu\nu} = \sum_q \hat{T}^q_{\mu\nu} + \hat{T}^g_{\mu\nu} \qquad \eta^{\mu\nu} = \mathrm{diag}(1,-1,-1,-1)$$

Quark part: $T_q^{\mu\nu} = \frac{1}{4}\overline{\psi}_q\left(-i\overleftarrow{D}^{\mu}\gamma^{\nu} - i\overleftarrow{D}^{\nu}\gamma^{\mu} + i\overrightarrow{D}^{\mu}\gamma^{\nu} + i\overrightarrow{D}^{\nu}\gamma^{\mu}\right)\psi_q - g^{\mu\nu}\overline{\psi}_q\left(-\frac{i}{2}\overleftarrow{\not{D}} + \frac{i}{2}\overrightarrow{\not{D}} - m_q\right)\psi_q$

Gluon part: $T_g^{\mu\nu} = F^{a,\mu\eta}\, F^{a,}{}_{\eta}{}^{\nu} + \frac{1}{4}\, g^{\mu\nu} F^{a,\kappa\eta}\, F^{a,}{}_{\kappa\eta}$

- Energy momentum tensor is conserved: $\partial^{\mu}\hat{T}_{\mu\nu} = 0,$

- Conservation low of dilatation current and energy momentum tensor

Dilatation current: $j^{\mu} = x_{\nu}\hat{T}^{\mu\nu}$

Trace anomaly

Conservation low of dilatation current: $\partial_{\mu}j^{\mu} = \hat{T}_{\mu}^{\ \mu}$ $\qquad \hat{T}_{\mu}^{\ \mu} \equiv \frac{\beta(g)}{2g}\, F^{a,\mu\nu}F^{a,}{}_{\mu\nu} + (1+\gamma_m)\sum_q m_q\bar{\psi}_q\psi_q$

Energy momentum tensor form factor of proton (gravitational form factor of proton)

$$\langle p', s'|\hat{T}^a_{\mu\nu}(x)|p, s\rangle = \bar{u}'\left[A^a(t)\,\frac{\gamma_{\{\mu}P_{\nu\}}}{2} + B^a(t)\,\frac{i\,P_{\{\mu}\sigma_{\nu\}\rho}\Delta^\rho}{4m} + D^a(t)\,\frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{4m} + m\,\bar{c}^a(t)\,g_{\mu\nu}\right]u\,e^{i(p'-p)x}$$

Proton state

Proton mass

Energy momentum tensor of gluon part and quark parts: $a = g,\ u,\ d,\ \ldots$

In form factors, kinematic variables are introduced as $P = \frac{1}{2}(p' + p),\ \Delta = p' - p,\ t = \Delta^2$

Covariant normalization $\langle p'|\,p\rangle = 2p^0\,(2\pi)^3\delta^{(3)}(\boldsymbol{p'} - \boldsymbol{p})$

Normalization of spinors $\bar{u}(p, s)\,u(p, s) = 2\,m$

$a_{\{\mu}b_{\nu\}} = a_\mu b_\nu + a_\nu b_\mu$

$\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$

Using Gordon identity $2m\bar{u}'\gamma^\alpha u = \bar{u}'(2P^\alpha + i\sigma^{\alpha\kappa}\Delta_\kappa)u$
we can obtain an alternative decomposition

$$\langle p', s'|\hat{T}^a_{\mu\nu}(x)|p, s\rangle = \bar{u}'\left[A^a(t)\,\frac{P_\mu P_\nu}{m} + J^a(t)\,\frac{i\,P_{\{\mu}\sigma_{\nu\}\rho}\Delta^\rho}{2m} + D^a(t)\,\frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{4m} + m\,\bar{c}^a(t)g_{\mu\nu}\right]u\,e^{i(p'-p)x}$$

Gravitational form factor of proton

$$\langle p', s'|\hat{T}^a_{\mu\nu}(x)|p, s\rangle = \bar{u}'\left[A^a(t)\frac{\gamma_{\{\mu}P_{\nu\}}}{2} + B^a(t)\frac{i\,P_{\{\mu}\sigma_{\nu\}\rho}\Delta^\rho}{4m} + D^a(t)\frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{4m} + m\,\bar{c}^a(t)\,g_{\mu\nu}\right]u\,e^{i(p'-p)x}$$

or

$$A^a(t) + B^a(t) = 2\,J^a(t)$$

$$\langle p', s'|\hat{T}^a_{\mu\nu}(x)|p, s\rangle = \bar{u}'\left[A^a(t)\frac{P_\mu P_\nu}{m} + J^a(t)\frac{i\,P_{\{\mu}\sigma_{\nu\}\rho}\Delta^\rho}{2m} + D^a(t)\frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{4m} + m\,\bar{c}^a(t)\,g_{\mu\nu}\right]u\,e^{i(p'-p)x}$$

Energy momentum tensor of gluon part and quark parts: $a = g,\, u,\, d,\, \ldots$ $\hat{T}_{\mu\nu} = \sum_q \hat{T}^q_{\mu\nu} + \hat{T}^g_{\mu\nu}$

*Individual form factors (A^a, B^a, D^a and C^a) depend on renormalization scale.
*However, total form factors (A,B, D and C) are renormalization scale invariant.

Total form factors $A(t) \equiv \sum_a A^a(t)$ $B = \sum_a B^a$ $D = \sum_a D^a$ $\bar{c} = \sum_a \bar{c}^a$

Total energy momentum tensor (gluon part + quark part) satisfies the conservation low $\partial^\mu \hat{T}_{\mu\nu} = 0,$

To satisfy the conservation low, we have the constraint $\sum_a \bar{c}^a(t) = 0$

Gravitational form factor of proton

$$\langle p', s'|\hat{T}^a_{\mu\nu}(x)|p, s\rangle = \bar{u}'\left[A^a(t)\frac{\gamma_{\{\mu}P_{\nu\}}}{2} + B^a(t)\frac{i\,P_{\{\mu}\sigma_{\nu\}\rho}\Delta^\rho}{4m} + D^a(t)\frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{4m} + m\,\bar{c}^a(t)\,g_{\mu\nu}\right]u\,e^{i(p'-p)x}$$

or

$$A^a(t) + B^a(t) = 2\,J^a(t)$$

$$\langle p', s'|\hat{T}^a_{\mu\nu}(x)|p, s\rangle = \bar{u}'\left[A^a(t)\frac{P_\mu P_\nu}{m} + J^a(t)\frac{i\,P_{\{\mu}\sigma_{\nu\}\rho}\Delta^\rho}{2m} + D^a(t)\frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{4m} + m\,\bar{c}^a(t)g_{\mu\nu}\right]u\;e^{i(p'-p)x}$$

Is there any constraint for other form factors?

In the limit $\vec{p}, \vec{p'} \to 0$ (rest frame), only the 00-component remains $H = \int d^3x\,\hat{T}_{00}(x)$

Eigenvalue of Hamiltonian is mass of particle $H\,|\boldsymbol{p}\rangle = m\,|\boldsymbol{p}\rangle$

$$\langle\vec{p}|H|\vec{p}\rangle = 2m^2(2\pi)^3\delta^{(3)}(0) = 2m^2V_3$$

$$A(0) = 1$$

At zero momentum transfer limit: t=0 or $\Delta = p' - p = 0$

$$\langle p, s'|\sum_a \hat{T}^a_{\mu\nu}(x)|p, s\rangle = \bar{u}'\left[A(0)\frac{p_\mu p_\nu}{m}\right]u = 2A(0)p_\mu p_\nu$$

$$\langle p, s'|\sum_a \hat{T}^a_{00}(x)|p, s\rangle = 2A(0)(m^2 + \vec{p}^{\,2})$$

$\vec{p} \to 0$

$$\langle p, s'|\int d^3x \sum_a \hat{T}^a_{00}(x)|p, s\rangle = 2A(0)m^2V_3$$

$$\langle p', s'|\hat{T}^a_{\mu\nu}(x)|p, s\rangle = \bar{u}'\left[A^a(t)\frac{P_\mu P_\nu}{m} + J^a(t)\frac{i\,P_{\{\mu}\sigma_{\nu\}\rho}\Delta^\rho}{2m} + D^a(t)\frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{4m} + m\,\bar{c}^a(t)g_{\mu\nu}\right]u\ e^{i(p'-p)x}$$

$$A(0) = 1$$

Form factor "A" is related to energy conservation in rest frame (mass conservation.)

$$\langle p, s'|\int d^3x \sum_a \hat{T}^a_{00}(x)|p, s\rangle = 2A(0)m^2V_3$$

If $A(0) \neq 1$,
normalization for state maybe wrong.

What about J(0)?

$$J(t) = \frac{1}{2}\sum_a A^a(t) + \frac{1}{2}\sum_a B^a(t) = \frac{1}{2}A(t) + \frac{1}{2}B(t)$$

$$J(0) = \frac{1}{2} + \frac{1}{2}B(0)$$

J(0) corresponds to spin angular momentum.

B(0) is called "anomalous gravitomagnetic moment".
However, B(0) satisfies the following constraint:

$$B(0) = 0$$

This constraint was proven by classical and quantum field theories in various contexts.
→ Anomalous gravitomagnetic moment vanishes.

$$J(0) = \frac{1}{2}$$

$$\langle p', s' | \hat{T}^a_{\mu\nu}(x) | p, s \rangle = \bar{u}' \left[A^a(t) \frac{P_\mu P_\nu}{m} + J^a(t) \frac{i\, P_{\{\mu}\sigma_{\nu\}\rho}\Delta^\rho}{2m} + D^a(t) \frac{\Delta_\mu \Delta_\nu - g_{\mu\nu}\Delta^2}{4m} + m\, \bar{c}^a(t) g_{\mu\nu} \right] u\, e^{i(p'-p)x}$$

There exists constraints at zero-momentum transfer, which are related to mass and spin:

$A(0) = 1$ ⟷ Mass conservation

Correlation

$J(0) = \frac{1}{2} + \frac{1}{2}B(0)$

$B(0) = 0$ ⟷ Spin conservation

What about "D-term"?

*Note that the similarity to the D-term used in the terminology of supersymmetric theories is mostly accidental.

D term at zero-momentum transfer is unconstrained.

D-term is not related to "external properties" of a particle like mass and spin.

D-term is a "conserved charge", but it is unknown: we don't know the role and related obserbation...

We have

$\sum_a \bar{c}^a(t) = 0$

to satisfy conservation low:

$\partial^\mu \hat{T}_{\mu\nu} = 0,$

$$\langle p', s'| \sum_a \hat{T}^a_{ij}(x)|p, s\rangle = 2mED(t)\frac{\Delta^i\Delta^k - \delta^{ik}\vec{\Delta}^2}{4m^2}\delta_{ss'}$$

Breit frame

$p = (P - \frac{1}{2}\Delta)$ and $p' = (P + \frac{1}{2}\Delta)$

$P = (E, 0, 0, 0)$

$\Delta = (0, \boldsymbol{\Delta})$ $t = -\boldsymbol{\Delta}^2$

The ij components of EMT define the stress tensor:

$$T^{ij}(\boldsymbol{r}) = \left(\frac{r^i r^j}{r^2} - \frac{1}{3}\delta^{ij}\right)s(r) + \delta^{ij}\,p(r)$$

Stress tensor of quark+gluon can be decomposed into "shear forces s(r)" and "pressure $p(r)$"

$$D^a = -\frac{2}{5}m\int \mathrm{d}^3r\, T^a_{ij}(\boldsymbol{r})\left(r^i r^j - \frac{1}{3}r^2\delta^{ij}\right) = -\frac{4}{15}m\int \mathrm{d}^3r\, r^2 s^a(r)$$

D-term is related to the distribution of shear forces inside the nucleon, parametrized in the spherically symmetric case by the function s(x).

*The word `pressure' should not be taken literally in its usual sense in thermodynamics.
*Conjecture: Stable hadrons must have a negative D term

$$D < 0$$

If this is not in the case, the system would collapse.

D term may be related to confinement???

EM (weak) form factor v.s. gravitational form factors

em: $\partial_\mu J^\mu_{\rm em} = 0$	$\langle N'\vert J^\mu_{\rm em}\vert N\rangle$	$\longrightarrow$	$Q = 1.602176487(40)\times 10^{-19}\,{\rm C}$ $\mu = 2.792847356(23)\mu_N$
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EM (weak) form factor attracts much attention.

↓

Continual theoretical and experimental efforts more than 70 years.

We have not drawn attention to gravitational form factor.
But the situation has changed drastically in the past several years...

↓

- First attempt to extract the D-term from experimental data has been made in Nature 557 (2018) no.7705, 396-399.
- GFFs have been also evaluated in lattice QCD simulation *Phys.Rev.D* 105 (2022) 5, 054509.

⇒ There is still a lack of understanding of D term.
→ It would be worth studying it from various perspectives.

$$\sum A^a = A^q + A^g = 1 \qquad A^q = ?, \quad \boxed{A^g = ?} \qquad \sum J^a = J^q + J^g = \text{spin} \qquad J^q = ?, \quad \boxed{J^g = ?}$$

$$\sum D^a = D^q + D^g = ? \qquad D^q = ?, \quad \boxed{D^g = ?}$$

Parts of GFFs was observed in lattice QCD simulation.

Gluon gravitational structure of hadrons of different spin

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	tripole	z-expansion
$A_g^\pi(0)$	0.537(45)	0.544(46)
$D_g^\pi(0)$	−0.793(84)	−0.74(21)

	tripole	z-expansion
$A_g^\rho(0)$	0.485(41)	0.482(42)
$D_g^\rho(0)$	−1.16(14)	−0.81(38)
$J_g^\rho(0)$	0.491(42)	0.469(42)

	tripole	z-expansion
$A_g^N(0)$	0.429(39)	0.414(40)
$D_g^N(0)$	−1.93(53)	0.4(1.2)
$J_g^N(0)$	0.263(26)	0.211(57)

	tripole	z-expansion
$A_g^\Delta(0)$	0.393(36)	0.378(38)
$D_g^\Delta(0)$	−1.80(69)	0.9(1.5)
$J_g^\Delta(0)$	0.588(78)	0.41(18)

$$\sum A^a = A^q + A^g = 1$$

$$\sum D^a = D^q + D^g = ?$$

Comparison between lattice and hQCD

J/ψ near threshold in holographic QCD:
A and D gravitational form factors

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(Dated: April 20, 2022)

	$A(0)$	$D(0)$
Holographic QCD (*This work*)	0.430	−1.275 (or 0)
Holographic QCD (*tripole approx.*)	0.430	−1.275 (or 0)
Lattice QCD (*tripole fit*) [20]	0.429	−1.930
Lattice QCD (*dipole fit*) [29]	0.580	−10.000

Form factor "A" of hQCD is in good agreement with the lattice but the "D" term is not…

Whole D term has been evaluated
by the skyrmion approach

Gravitational form factors of nuclei in the Skyrme model

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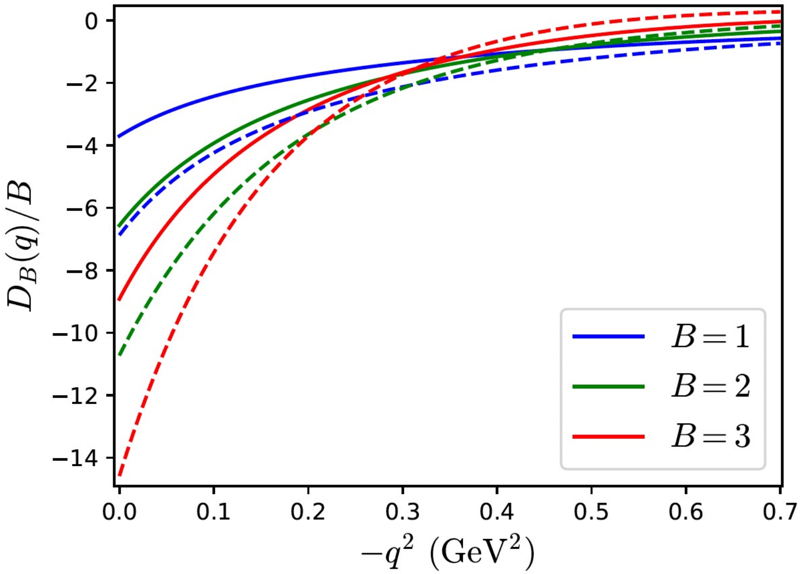

$$\sum D^a = D^q + D^g = ?$$

Multi skyrmion systems (A few body system of nucleons)

For single baryon, D(0)= -4

There would be no lattice data of whole D term.
→ skyrmion evaluation is benchmark value.

Summary

- Gravitational Form Factors (GFFs) have been extensively studied
  by lattice QCD, experimental physics and model approaches.

→ To clarify the confinment of hadron like proton. → GFF is hot topic (maybe).

→ But… the form factor of hadron is still unclear: what is the role/meaning of D-term?
→ It needs further analysis.

- GFFs of vector meson and pseudoscalar meson has been also discussed recently.

arXiv:2302.03383
Vector Meson Gravitational Form Factors and Generalized Parton Distributions
at finite temperature within the soft-wall AdS/QCD Model

→ GFFs of charm hadrons have not been discussed so far.
4-flavor hQCD approach may be worth a try.

- We can extend following skyrmion analysis (a few body system of baryon) to infinite system of baryon.

arXiv:2304.05994 "*Gravitational form factors of nuclei in the Skyrme model*"

→ Present Skyrme analysis is incomplete. It would be valuable to complement it.