Hadronic mass spectrum in AdS/QCD with Deep Learning

A early proposal

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Outline

Hadrons in AdS/QCD

Motivation

- Hadrons are bounded states of quarks and gluons.
- Hadrons are direct evidence of confinement.
- Hadron masses are organized in taxonomic structures called *Regge trajectories*.
- Hadronic spectroscopy is non-perturbative.
- Non-holo. approaches to hadron spectroscopy:
  - Relativistic and non-relativistic potential models.
  - Bethe-Salpeter equation.
  - Light-cone QCD.
  - Lattice QCD.

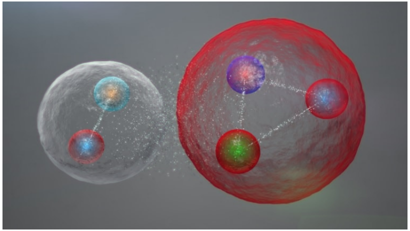

Figure: Cartoon describing mesons and baryons

Hadrons in AdS/QCD

- Gauge/gravity duality: hadrons are nonperturbative boundary objects dual to bulk fields.

  $$\mathcal{O}\,|0\rangle = \alpha\,|p\rangle$$

  with operator creating hadrons $\mathcal{O}$ defined as

  $$\begin{aligned}\mathcal{O} &= f(q, \bar{q}, G_{\mu\nu}, D_\mu) \\ \mathbf{dim}\,\mathcal{O} &= \Delta_0 + L + \gamma\end{aligned}$$

- Field Operator duality:

  $$\mathbf{dim}\,\mathcal{O} \Leftrightarrow \dim\psi(z,q) \equiv \Delta$$

  Bulk field: p-form in AdS_5
  $A_p(z,q) = A_p(q)\,\psi(z,q)$ with mass M_5.

  $$\psi(z,q)\big|_{z\to 0} = \mathcal{C}\,z^{\Delta - p}$$

  See Polchinski 2002.

Hadrons in AdS/QCD

- Hadronic identity:
  From p-form EOMs:

$$
\begin{aligned}
\tau = \Delta - p &= \frac{1-\beta}{2} + \frac{1}{2}\sqrt{(1-\beta)^2 + 4\, M_{d+2}^2\, R^2}, \\
\beta &= -3 + 2\, p
\end{aligned}
$$

  We get the *master formula* for defining hadrons in AdS/QCD:

$$
M_5^2\, R^2 = (\Delta - p)\,(\Delta + p - 4) \tag{1}
$$

- *Observations:*
  - Hadron spin S is equivalent to p-form index p.
  - OPE twist $\tau = \mathbf{dim}\,\mathcal{O} - S$. See Wise's lecutures, 1999.
  - Holographic hadrons usually written in S-wave, i.e., $L = 0$. (See Branz et al., 2010).
  - For fermionic expression, a similar procedure is to get a fermionic bulk mass expression.
  - Similar expressions for LFHQCD (See Brodsky 2009).

Hadronic Identity

- Hadrons in AdS/QCD are normalizable modes labeled by their bulk mass.
- From OPE and QCD sum rules, we can write a general expression (Vega, 2010)

$$
\begin{aligned}
M_5^2\,R^2 &= (\Delta_0 + L + \gamma - p)\,(\Delta_0 + L + \gamma + p - 4) \\
\Delta_0 &\rightarrow \text{info. from hadron constituents.} \\
L &\rightarrow \text{angular momentum.} \\
\gamma &\rightarrow \text{anomalous dimension.}
\end{aligned}
$$

- *Observations:* Hadrons are not univocally defined.

Hadronic Identity

Scalar hadrons		**Vector hadrons**		**Spin 1/2 hadrons**	
$(nQ)(mG)$	Δ_0	$(nQ)(mG)$	Δ_0	$(nQ)(mG)$	Δ_0
(2Q)	3	(2Q)	3	(3Q)	9/2
(2G)	4	(2Q)(1G)	5	(1Q)(3G) or (3Q)(1G)	13/2
(2Q)(1G)	5	(4Q) or (3G)	6	(5Q)	15/2
(4Q)	6	(2Q)(2G)	7	(3Q)(2G)	17/2
(2Q)(2G)	7	(4Q)(1G)	8	(5Q)(1G)	19/2
(4Q)(1G) or (4G)	8	(6Q) or (2Q)(3G)	9	(3Q)(3G) or (7Q)	21/2
(6Q) or (2Q)(3G)	9	(5G) or (4Q)(2G)	10	(5Q)(2G)	23/2

Table: Possible hadronic states composed by n quarks (or antiquarks) and m gluons and their conformal dimensions. To know the corresponding value of the associated bulk mass, solve the equation of motion for the bulk fields, and compute the low z limit to see how the fields scale. This scaling is the conformal dimension defined in terms of the bulk mass. Invert the relation, and you will find the expected $M_{d+2} = M_{d+2}(\Delta)$ relation.

Hadron mass spectrum in AdS/QCD

AdS/QCD *a la* bottom-up in a nutshell

- **Motivation:** confinement.
  - Dilaton field $\Phi(z)$ in the bulk action.
  - Geometry deformation $h(z)$, i.e., AdS warp factor $A(z) = \log(R/z) + h(z)$, $h(z \to 0) \to 0$.

  Both induce bounded spectra in AdS space.
- How? Using the Schrodinger-like potential:

$$V(z) = \frac{1}{4} B'(z)^2 - \frac{1}{2} B''(z) + \frac{M_5^2 \, R^2}{z^2} e^{h(z)}, \tag{2}$$

  with $B(z) = \Phi(z) + \beta \left[\log \frac{R}{z} + \frac{1}{2} h(z)\right]$.
- For linear Regge trajectories (KKSS for example):

$$V(z) = \frac{A}{z^2} + B\, z^2 + C + \text{corrections}$$

- Holographic hadron masses (M_n^2) are eigenvalues of $V(z)$ in a Schrodinger-like potential.

Hadron Mass spectrum

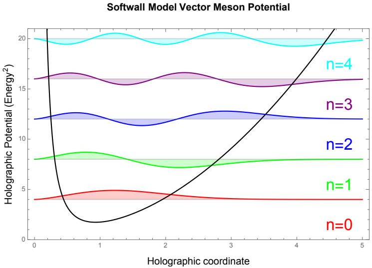

Figure: Holographic potential .

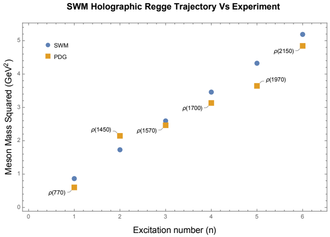

Figure: ρ meson Regge Trajectory.

- Regge trajectories emerge as eigenvalue spectrum of $V(z)$.

- Linearity is controlled by the high-z behavior of $V(z)$. **Example:**
  - Hardwall model (see Boschi-Filho, 2002): $M_n^2 \propto n^2$.
  - Softwall model (see KKSS, 2006): $M_n^2 \propto n$

Dilaton Engineering

Dilaton/deformation Reconstruction

Main idea

Given a phenomenological structure for $V(z)$, it is possible to infer $\Phi(z)$ or $h(z)$.

$$V(z) = \frac{1}{4}\left[\Phi'(z)^2 + \beta^2\, h'(z)^2\right] + \frac{1}{2}\beta\, h'(z)\,\Phi'(z) - \frac{1}{2}\left[\Phi''(z) + \beta\, h''(z)\right] - \frac{\beta}{2\,z}\left[\beta\, h'(z) + \Phi'(z)\right] + \frac{4\,M_5^2\,R^2\,e^{2\,h(z)} - 2\,\beta + \beta^2}{4\,z^2} \quad (3)$$

- Low z region of the potential is dominated by the AdS warp factor.
- High z region is dominated by $\Phi(z)$ or $h(z)$.
- WKB analysis from the eigenvalue spectrum can be used to define a phenomenological potential. (Schrödinger inverse problem.)

WKB analysis

- From a eigenvalue spectrum $M_n^2(n)$, we can calculate the potential at high z (turning point):

$$z(V) = 2\int_0^V \frac{d\,M^2}{\frac{\partial\,M^2}{\partial\,n}\sqrt{V - M^2}}.$$

- Compute the dilaton and deformation as

$$V\,(z)|_{z\to\infty} = \frac{1}{4}\left[\Phi'(z)^2 + \beta^2\,h'(z)^2\right] + \frac{1}{2}\beta\,h'(z)\,\Phi'(z) - \frac{1}{2}\left[\Phi''(z) + \beta\,h''(z)\right] - \frac{\beta}{2\,z}\left[\beta\,h'(z)\right.$$

- Build the full potential by adding the low-z part.
- Compute the holographic eigenspectrum.

Example: Softwall model

- Let us consider linear Regge Trajectories: $M_n^2 = a(n+b)$.
- WKB:

$$z(V) = 2\int_0^V \frac{d\,M^2}{a\sqrt{V-M^2}} = \frac{4}{a}V^{1/2} \Rightarrow V(z) = \frac{a^2}{16}\,z^2$$

- Solve for the dilaton field $\Phi(z)$ with $\beta = -1$ (vector mesons) and $h(z) = 0$,

$$C(a)\,z^2 = \frac{1}{4}\Phi'(z)^2 - \frac{1}{2}\Phi''(z) + \frac{1}{2}\Phi'(z) \Rightarrow \Phi(z) = c_1 - 2\log\left[\cosh\left(\frac{a}{2}z^2 - 2\,c_2\right)\right]$$

- Fix $c_1 = 0$ and $c_2 = \infty$, after some algebra:

$$\Phi(z) = -2\log\left[\cosh\frac{a}{8}z^2 - \sinh\frac{a}{8}z^2\right] = 2\log\left(e^{-\frac{a}{8}z^2}\right) = \frac{a}{4}z^2 \equiv \kappa^2\,z^2$$

- For vector mesons $a = 4\,\kappa^2$:

$$M_n^2 = a(n+b) \Rightarrow M_n^2 = 4\,\kappa^2(n+b)$$

Deep Learning for Hadronic spectrum

Motivation

- Hadrons are not univocally defined in AdS/QCD. How can be extended the model?
- *Can hadronic inner structure information be captured in other bulk structures?*

Idea

Try to solve $V(z)$ for different dilatons and deformations to match non-$q\,\bar{q}$ masses, starting from a proposed spectrum (Experiment, Lattice QCD, or other non-holographic models).

- This idea has been explored using the following:
  - Shifted bulk mass, Branz et. al, 2010.
  - Anomalous dimensions for glueballs, Braga and Boschi-Filho, 2012.
  - Generalized SWM, Afonin, 2013.
  - Anomalous dimensions for pseudoscalar and axial mesons, Martin et. al, 2019.
  - Deformed AdS, Capposouli et. al, 2019.
  - Non-linear Regge trajectories, Martin and Vega, 2021

DL proposal

Main goal

Following Hashimoto's works (2018, 2021) works, it is possible to train a neural network (NN) to solve Schrödinger inverse problem to find dilatons or deformation suitable to model hadronic mass spectrum.

- DL methods can define parameter spaces for ODEs solutions.
- DL are suitable to be used to background reconstruction.

Methodology

- Using experimental Regge trajectories data to train a NN (torch or tensorflow) to solve a Schrödinger-like potential to compute $\phi(z)$ and $h(z)$ as parametric function families.
- To set the parameter space.
- To compute other hadronic species as non-$q\,\bar{q}$ states (for example, tetraquarks).


Thank you
谢谢
Спасибо
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