

Hadron physics with BESIII

Chang-Zheng Yuan

(yuancz@ihep.ac.cn)

(for the BESIII Collaboration)



Nanjing University

May 14 – 17, 2024

Selected topics in Hadron physics with BESIII

Chang-Zheng Yuan
(yuancz@ihep.ac.cn)
(for the BESIII Collaboration)



Nanjing University
May 14 – 17, 2024

BEPC(II) storage ring and BES(III) detector

Ground breaking: 1984

CM energy : 2 - 5 GeV

Major upgrade: 2004

Energy upgrade: 2024

World unique e^+e^-
accelerator in τ -charm
energy region

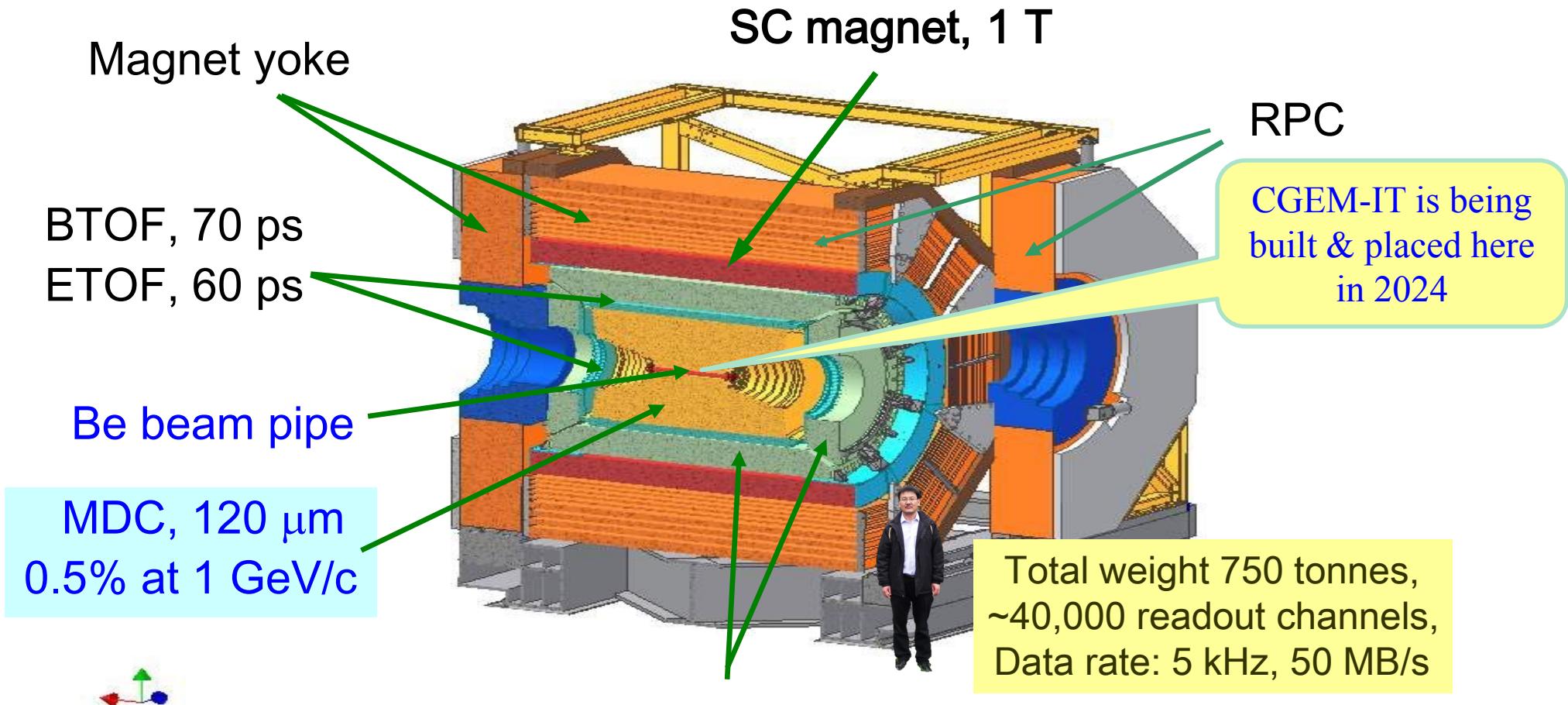
1989-2005 (BEPC): $L_{\text{peak}} = 1.0 \times 10^{31} / \text{cm}^2\text{s}$

2008-now (BEPCCII): $L_{\text{peak}} = 1.0 \times 10^{33} / \text{cm}^2\text{s}$ (Apr. 5, 2016)



IHEP, Beijing

BESIII detector



Has been in full operation since 2008,
all subdetectors are in very good status!

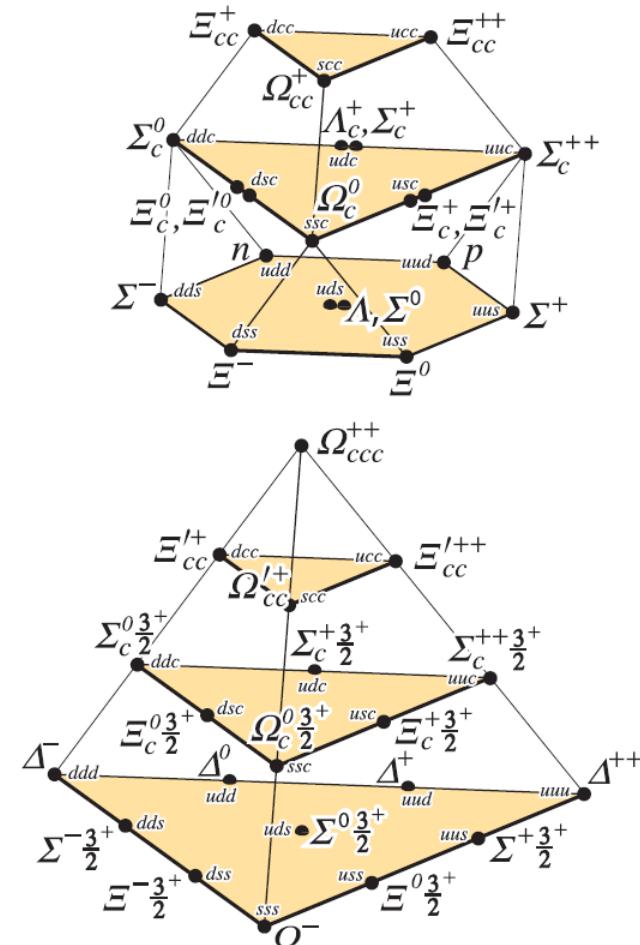
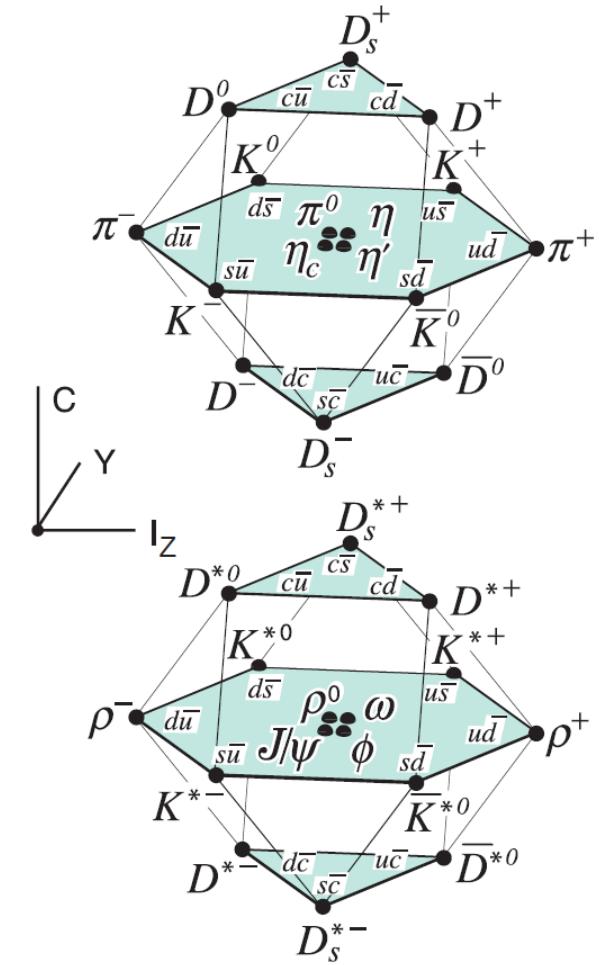
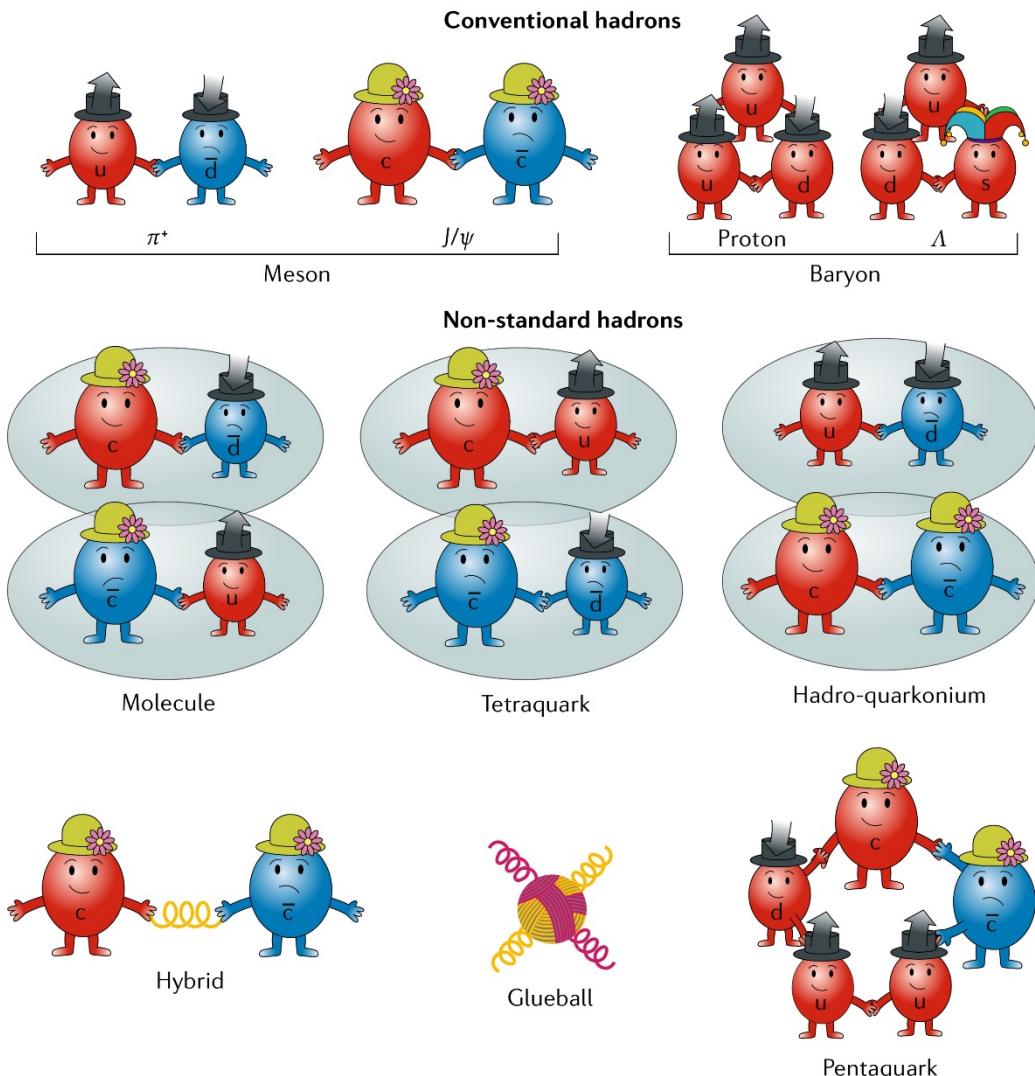
BESIII Collaboration



17 countries , 89 institutions, ~600 members

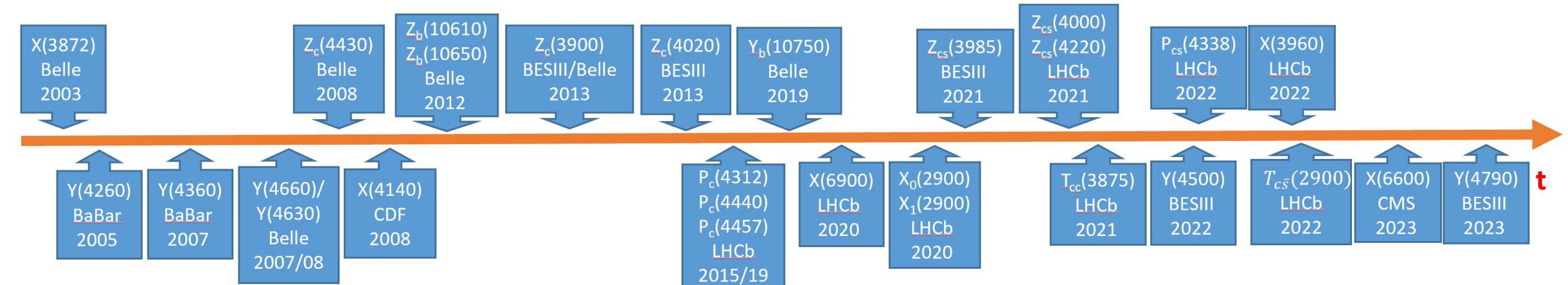


Hadrons: conventional & exotic



SU(4) multiplets of mesons & baryons

- Lots of states with heavy quarks (c, b) and exotic properties were observed since the discovery of the X(3872) in 2003!
- They are candidates of hadronic molecules, hybrids, and multiquark states.

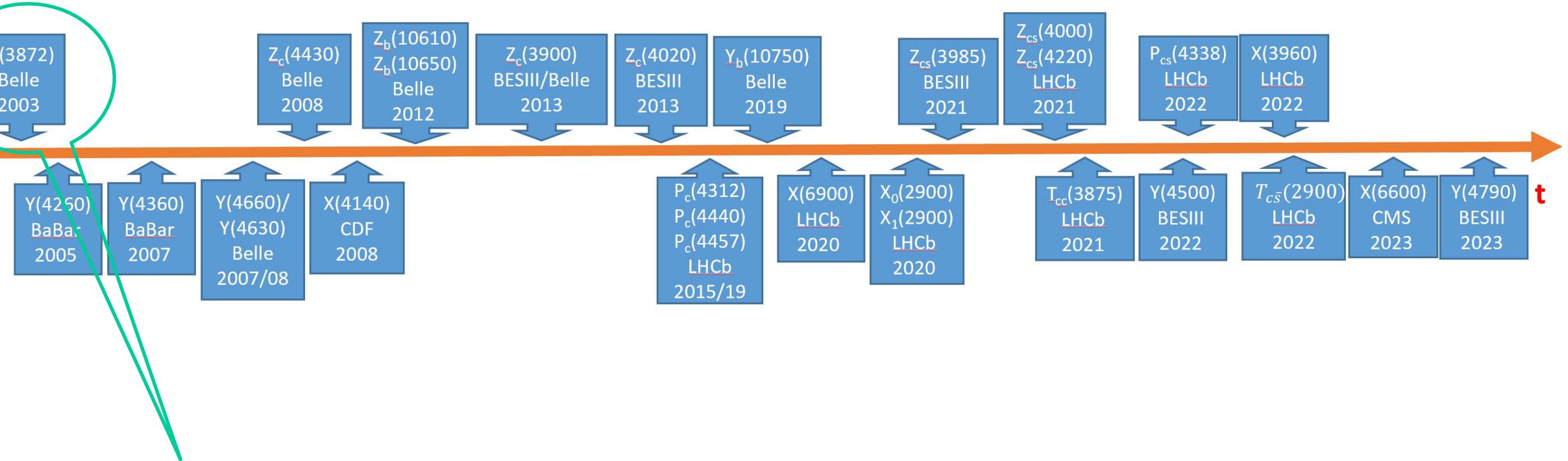


Z_Q : tetraquark with a $Q\bar{Q}$ pair
 P_Q : pentaquark with a $Q\bar{Q}$ pair
 Y : vectors, $J^{PC}=1^{--}$
 $T_{QQ'}$: tetraquark with QQ'
 X : other states

New spectrum emerges although more effort is needed to understand the nature of them.

This workshop:

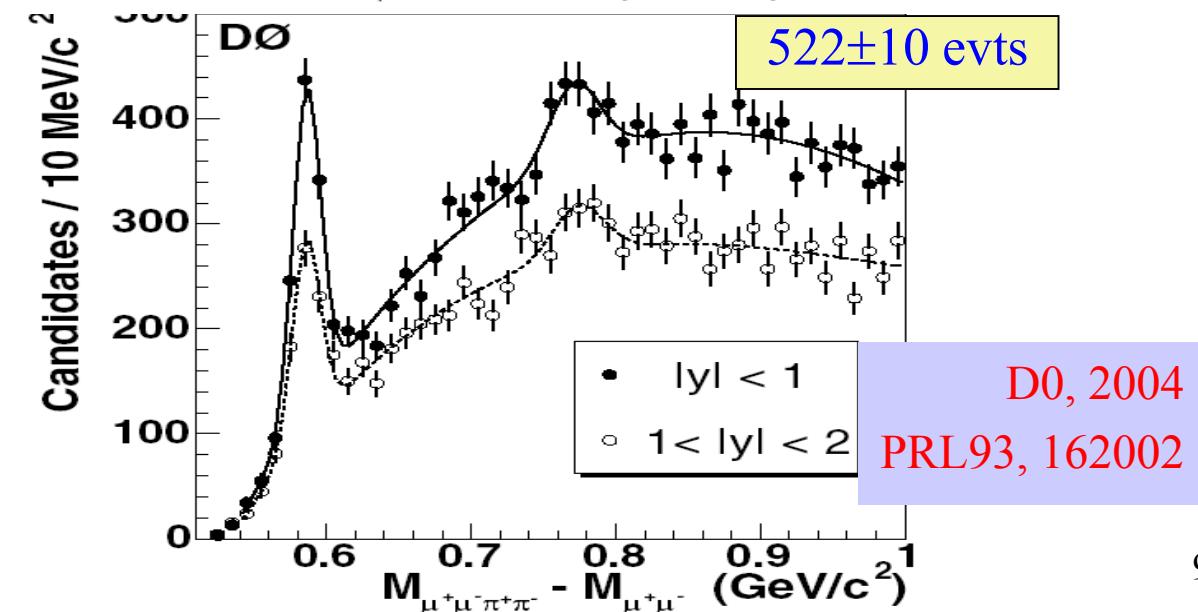
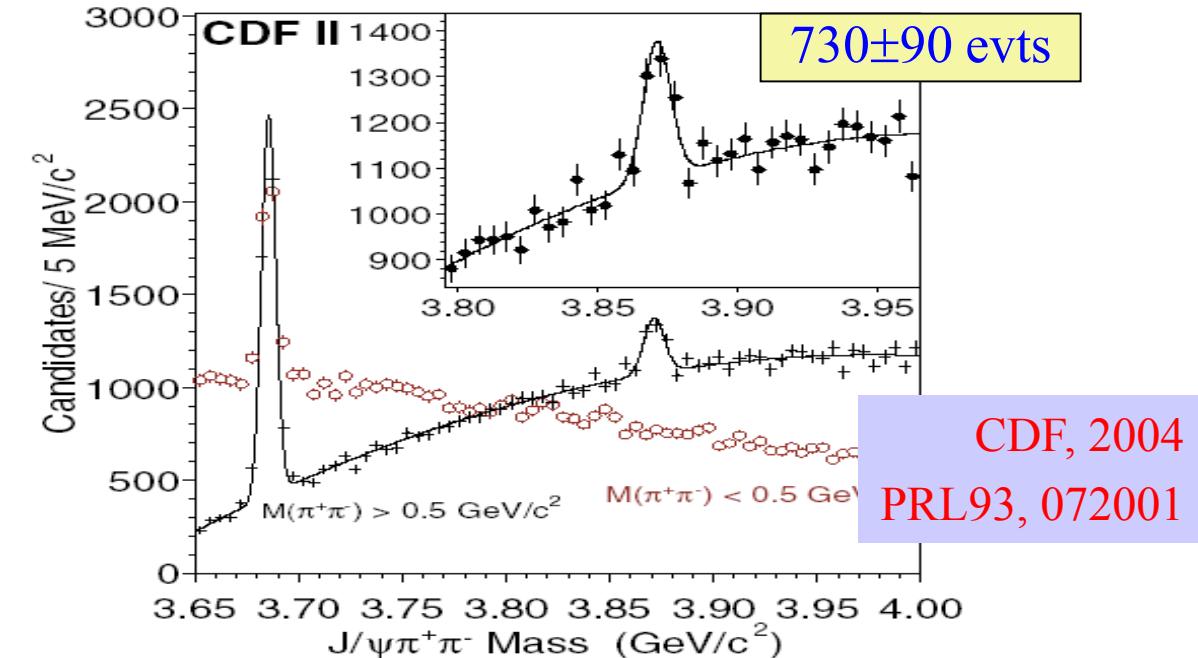
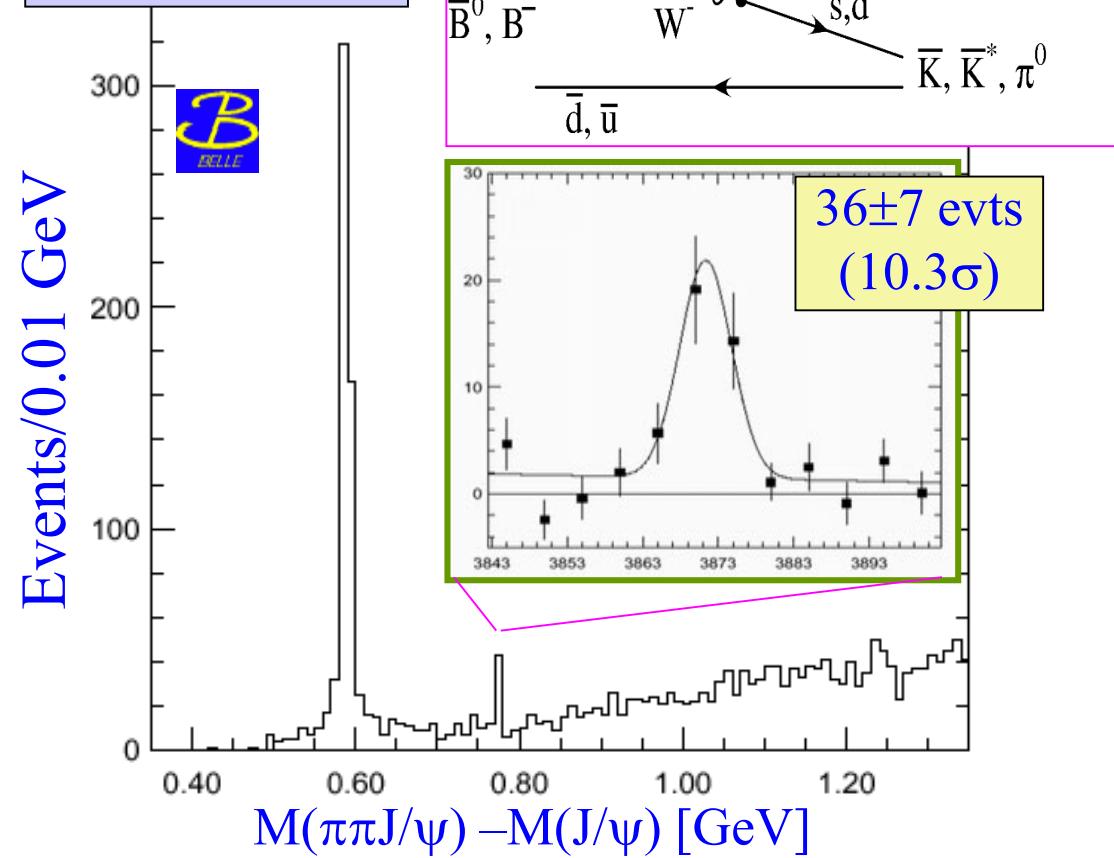
Bingsong Zou, Ying Chen, Feng-Kun Guo, Qiang Zhao, Jiajun Wu, ...



Lots of information on its quantum numbers, mass, width, production and decay properties,
and many new measurements are available

Discovery of the X(3872) [$\chi_{c1}(3872)$ in PDG2023]

Belle, 20030908,
PRL91, 262001



Mass of the X(3872)

VALUE(MeV)		EVTS	DOCUMENT ID	TECN	COMMENT
3871.65 ± 0.06	OUR AVERAGE				
3871.64 ± 0.06 ± 0.01		19.8k	1 AAIJ	2020S	LHCb $B^+ \rightarrow J/\psi \pi^+ \pi^- K^+$
3871.9 ± 0.7 ± 0.2		20	ABLIKIM	2014	BES3 $e^+ e^- \rightarrow J/\psi \pi^+ \pi^- \gamma$
3871.95 ± 0.48 ± 0.12		0.6k	AAIJ	2012H	LHCb $p p \rightarrow J/\psi \pi^+ \pi^- X$
3871.85 ± 0.27 ± 0.19		170	2 CHOI	2011	BELL $B \rightarrow K \pi^+ \pi^- J/\psi$
3873 ^{+1.8} _{-1.6} ± 1.3		27	3 DEL-AMO-SANCH..	2010B	BABR $B \rightarrow \omega J/\psi K$
3871.61 ± 0.16 ± 0.19		6k	4, 3 AALTONEN	2009AU	CDF2 $p \bar{p} \rightarrow J/\psi \pi^+ \pi^- X$
3871.4 ± 0.6 ± 0.1		93.4	AUBERT	2008Y	BABR $B^+ \rightarrow K^+ J/\psi \pi^+ \pi^-$
3868.7 ± 1.5 ± 0.4		9.4	AUBERT	2008Y	BABR $B^0 \rightarrow K_S^0 J/\psi \pi^+ \pi^-$
3871.8 ± 3.1 ± 3.0		522	5, 3 ABAZOV	2004F	D0 $p \bar{p} \rightarrow J/\psi \pi^+ \pi^- X$

$$M_{D0} + M_{D^*0} = 3871.69 \pm 0.11 \text{ MeV}$$

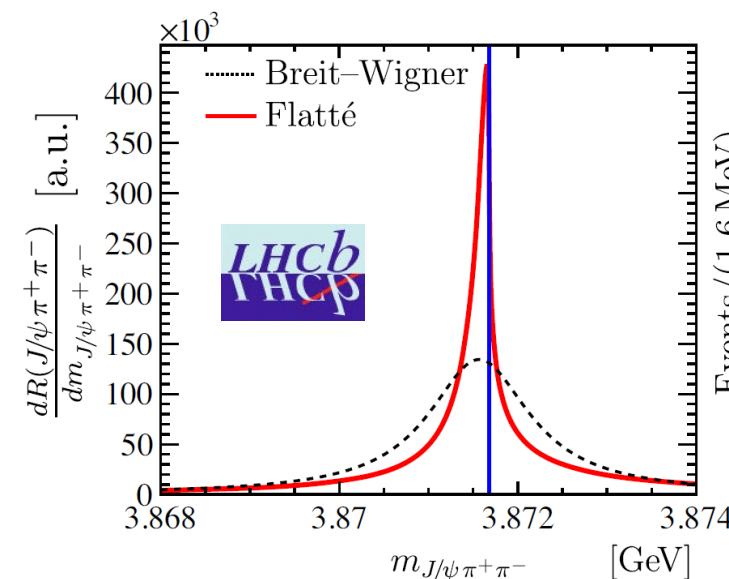
$$E_b = -0.04 \pm 0.12 \text{ MeV}$$

$$E_b(\text{deuteron}) = -2.2 \text{ MeV}$$

$$r_X = (8\mu |E_b|)^{-1/2} > 5 \text{ fm}$$

Width of the X(3872)

VALUE (MeV)	CL%	EVTS	DOCUMENT ID	TECN	COMMENT
1.19 ± 0.21	OUR AVERAGE	Error includes scale factor of 1.1.			
1.39 ± 0.24 ± 0.10		15.6k	¹ AAJ	2020AD LHCb	$p p \rightarrow J/\psi \pi^+ \pi^- X$
0.96 ^{+0.19} _{-0.18} ± 0.21	BW width!	4.2k	² AAJ	2020S LHCb	$B^+ \rightarrow J/\psi \pi^+ \pi^- K^+$

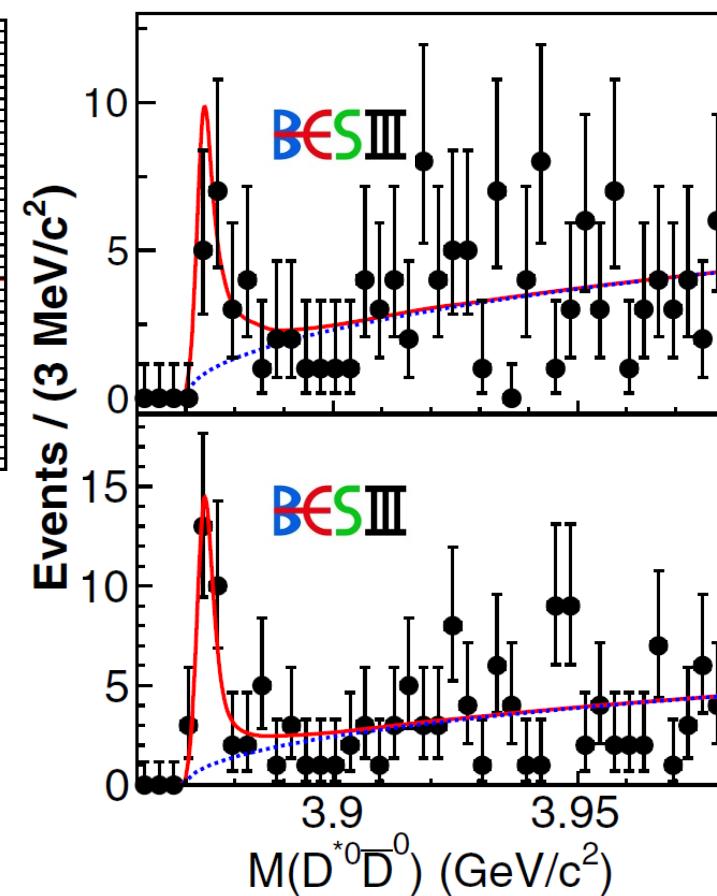
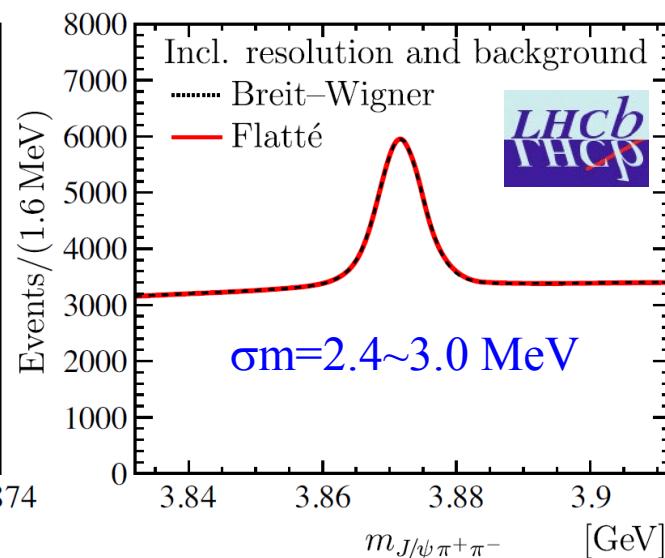


Flatté parametrization:

$$D(E) = E - E_f + \frac{i}{2} [g(k_1 + k_2) + \Gamma_\rho(E) + \Gamma_\omega(E) + \Gamma_0]$$

Depends strongly on g , coupling to $\bar{D}^0 D^{*0}$!

$$\text{FWHM} = 0.22^{+0.06+0.25}_{-0.08-0.17} \text{ MeV}$$



BESIII may supply crucial information on g & line shape.

Mass resolution $\sigma_m < 1 \text{ MeV}$!

PRL124, 242001 (2020)

A coupled channel analysis of the X(3872) line shape at BESIII

$$\frac{dB(D^0 \bar{D}^0 \pi^0)}{dE} = B \frac{1}{2\pi} \times \frac{g * k_{\text{eff}}(E)}{|D(E)|^2} \times \text{Br}(D^{*0} \rightarrow D^0 \pi^0)$$

$$\frac{dB(\pi^+ \pi^- J/\psi)}{dE} = B \frac{1}{2\pi} \times \frac{\Gamma_{\pi^+ \pi^- J/\psi}}{|D(E)|^2}$$

$$D(E) = E - E_X + \frac{1}{2} g * (\kappa_{\text{eff}}(E) + i k_{\text{eff}}(E) + \kappa_{\text{eff}}^c(E) + i k_{\text{eff}}^c(E)) + \frac{i}{2} \Gamma_0$$

$$k_{\text{eff}}(E) = \sqrt{\mu_p} \sqrt{\sqrt{(E - E_R)^2 + \Gamma^2/4} + E - E_R}$$

$$\begin{aligned} \kappa_{\text{eff}}(E) = & -\sqrt{\mu_p} \sqrt{\sqrt{(E - E_R)^2 + \Gamma^2/4} - E + E_R} \\ & + \sqrt{\mu_p} \sqrt{\sqrt{(E_X - E_R)^2 + \Gamma_X^2/4} - E_X + E_R} \end{aligned}$$

$$\Gamma_0 = \Gamma_{\pi^+ \pi^- J/\psi} + \Gamma_{\text{known}} + \Gamma_{\text{unknown}}$$

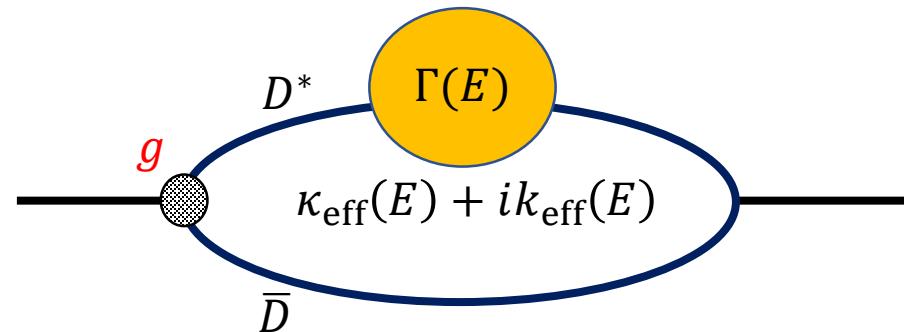
$$E_X = M_X - (m_{D^0} + m_{\bar{D}^0} + m_{\pi^0}) : \text{energy from } D^0 \bar{D}^0 \pi^0 \text{ thresh.}$$

superscript c: charged $D^{+} D^-$

*Due to the limited statistics, $\Gamma_{\text{unknown}} / \Gamma_{\pi^+ \pi^- J/\psi}$ is fixed

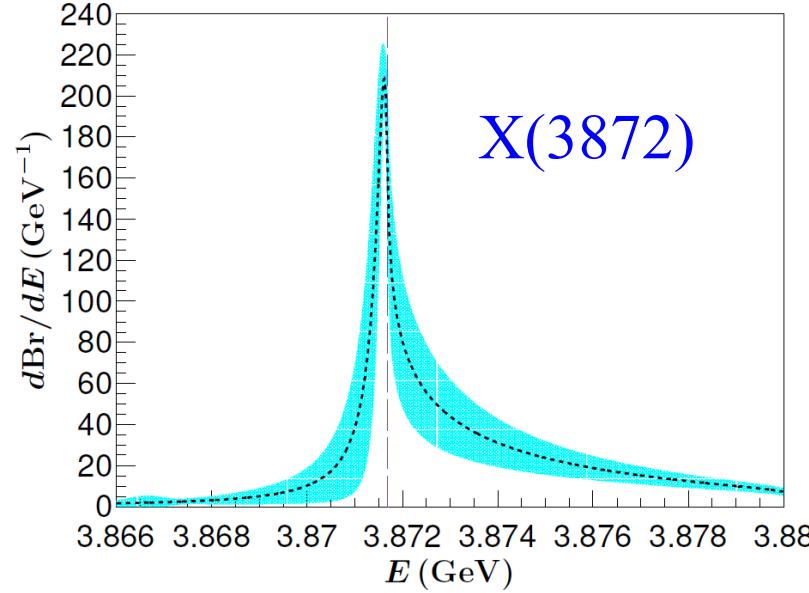
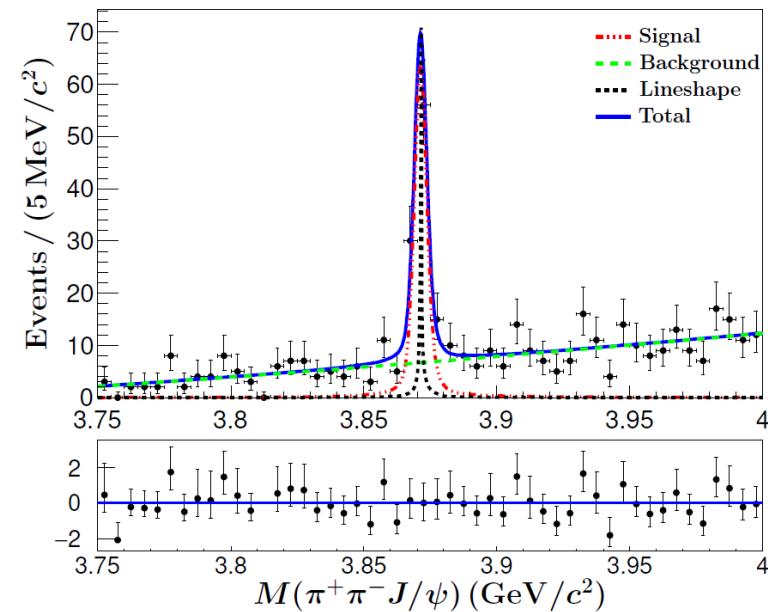
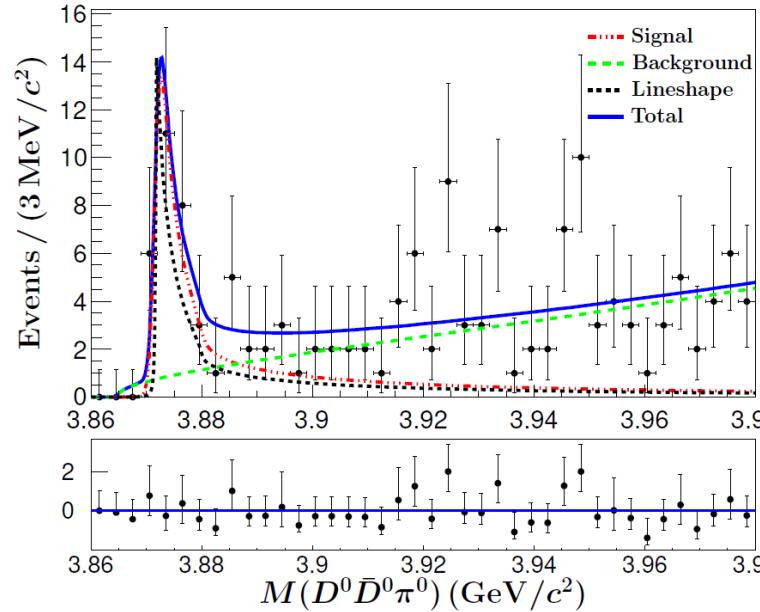
[Chunhua Li, Chang-Zheng Yuan, PRD 100, 094003 (2019)]

Hanhart, Kalashnikova, Nefediev, PRD 81, 094028 (2010)



Key features:

- Model independent
- Including the $D^* \bar{D}$ self energy terms
- Including the width of D^* , $\Gamma = \Gamma(E)$, $\Gamma_X = \Gamma(E_X)$
- Including the coupled channel effect
- Fit parameters: g , $\Gamma_{\pi^+ \pi^- J/\psi}$, M_X



Pole positions

Two sheets with respect to $D^{*0}\bar{D}^0$ branch cut

- Sheet I: $E - E_X - g\sqrt{-2\mu(E - E_R + i\Gamma/2)}$
- Sheet II: $E - E_X + g\sqrt{-2\mu(E - E_R + i\Gamma/2)}$

$$E_I = (7.04 \pm 0.15^{+0.07}_{-0.08}) + (-0.19 \pm 0.08^{+0.14}_{-0.19})i \text{ MeV}$$

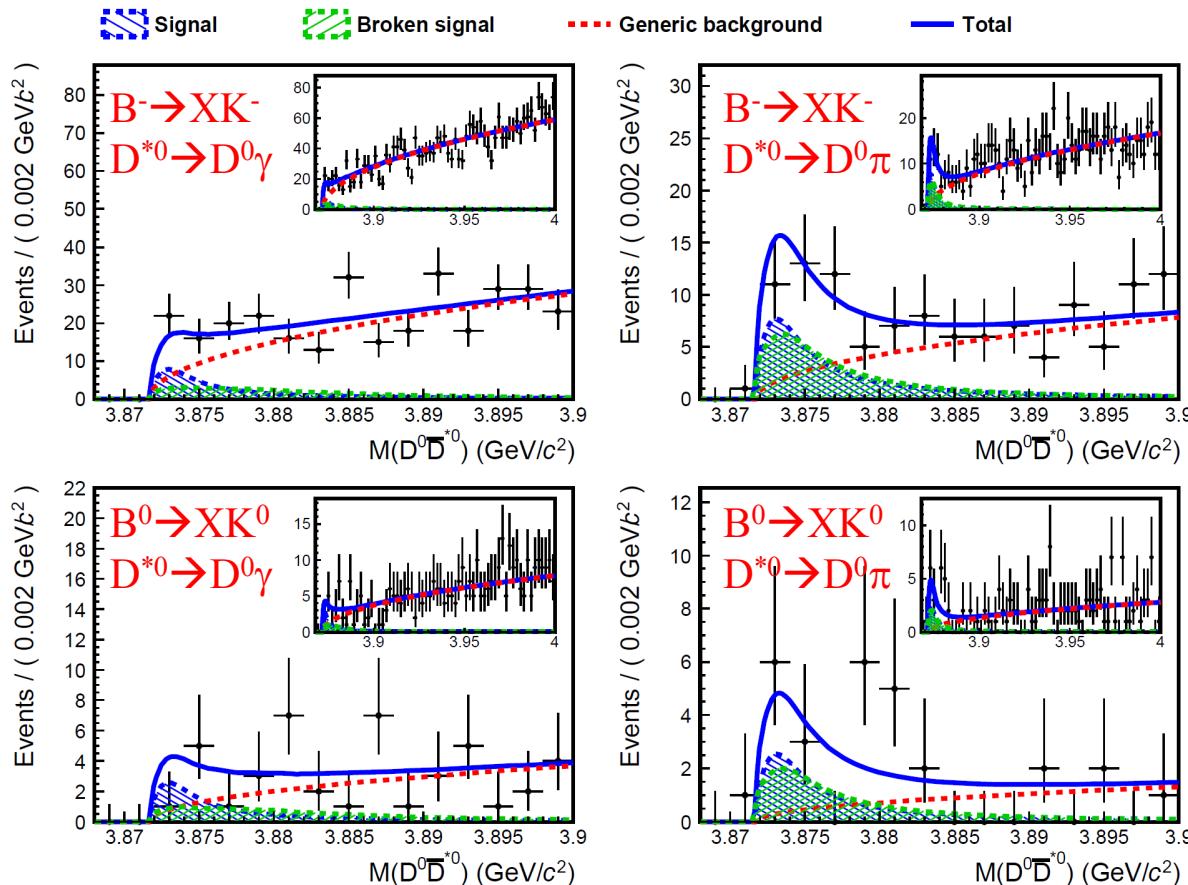
$$E_{II} = (0.26 \pm 5.74^{+5.14}_{-38.32}) + (-1.71 \pm 0.90^{+0.60}_{-1.96})i \text{ MeV}$$

Parameters	BESIII	LHCb
g	$0.16 \pm 0.10^{+1.12}_{-0.11}$	$0.108 \pm 0.003^{+0.005}_{-0.006}$
$Re[E_I]$ [MeV]	$7.04 \pm 0.15^{+0.07}_{-0.08}$	7.10
$Im[E_I]$ [MeV]	$-0.19 \pm 0.08^{+0.14}_{-0.19}$	-0.13
$\Gamma(\pi^+\pi^-J/\psi)/\Gamma(D^0\bar{D}^{*0})$	$0.05 \pm 0.01^{+0.01}_{-0.02}$	0.11 ± 0.03
FWHM (MeV)	$0.44^{+0.13}_{-0.35}{}^{+0.38}_{-0.25}$	$0.22^{+0.06}_{-0.08}{}^{+0.25}_{-0.17}$
Z	0.18	0.15

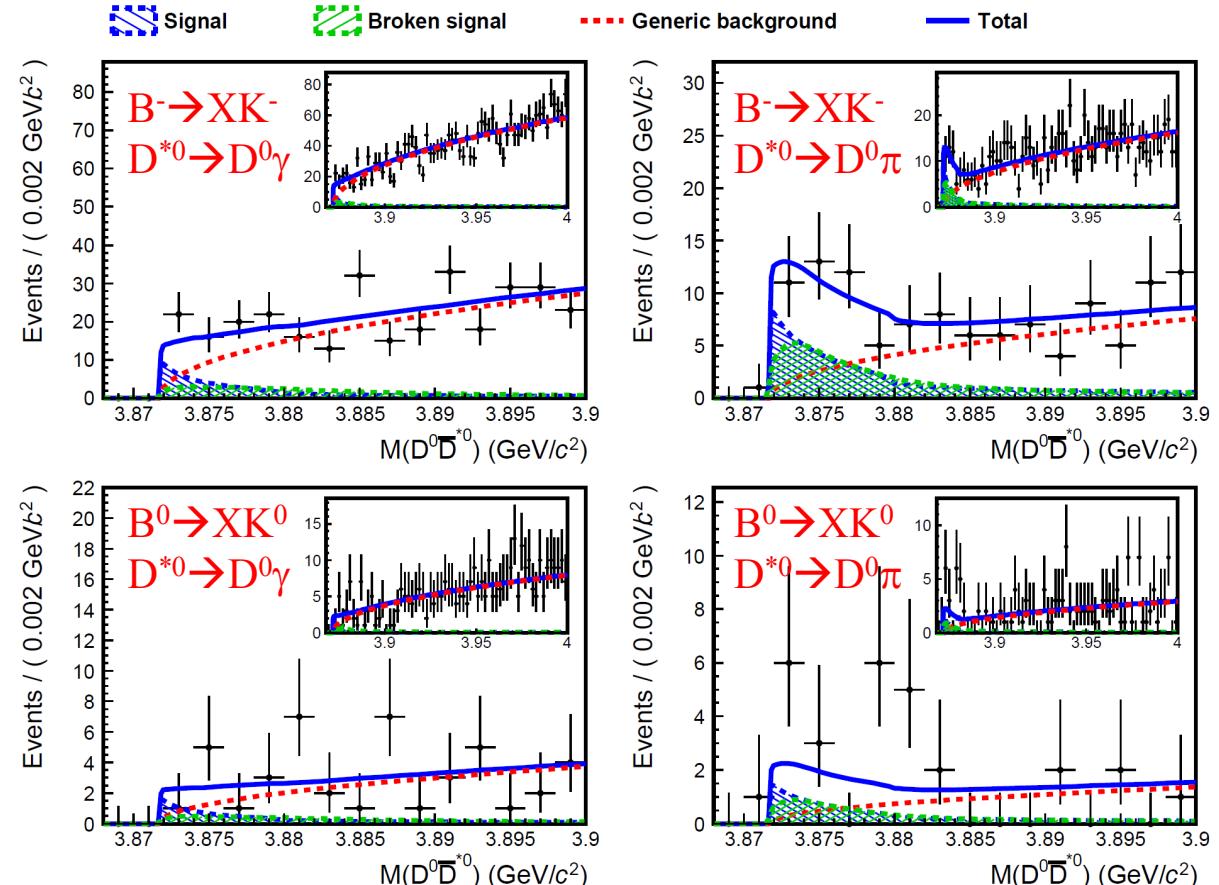
Weinberg's compositeness: $Z = 1$: pure elementary state; $Z = 0$: pure bound (composite) state.

X(3872) line shape @ Belle

BW parametrization

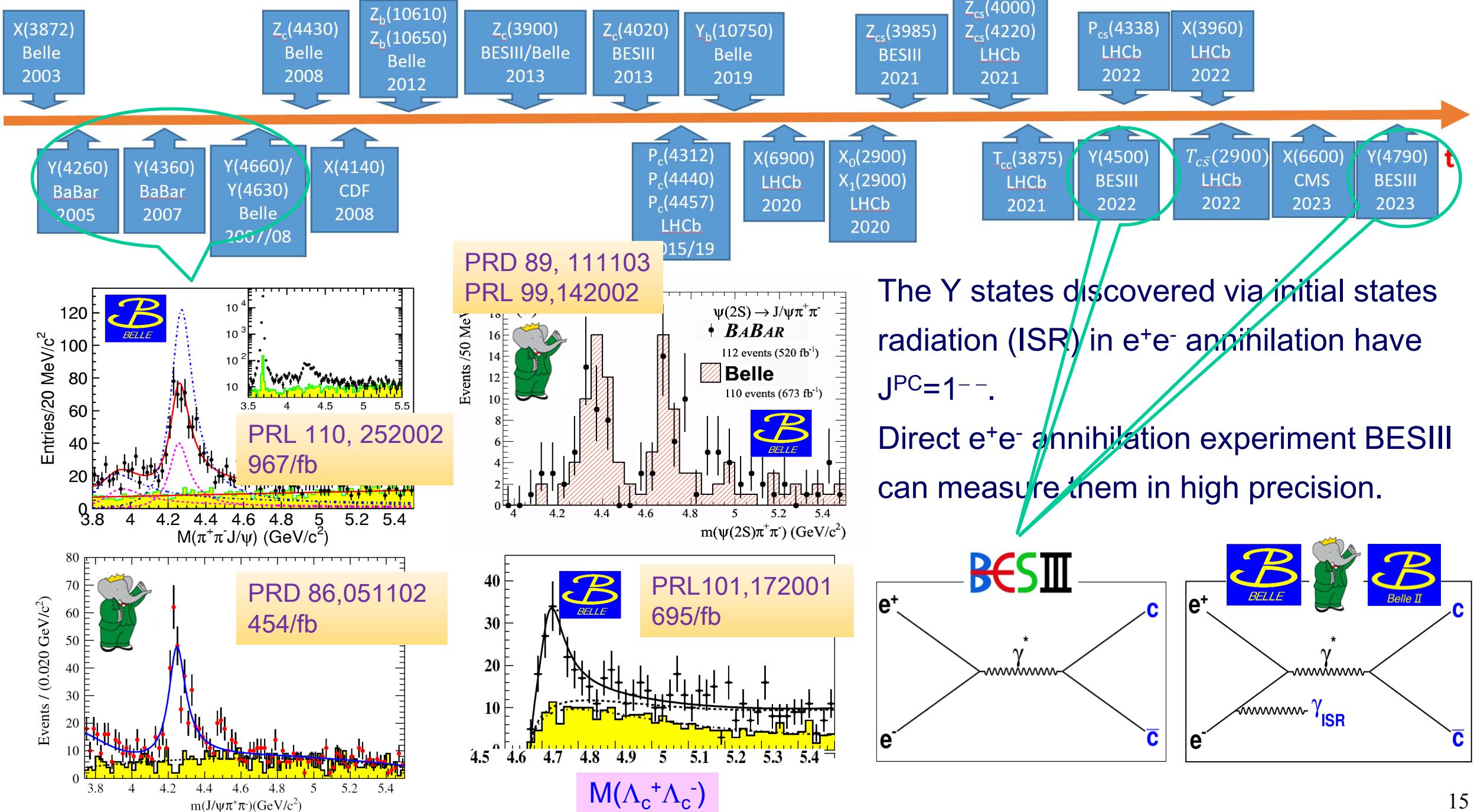


Flatté parametrization



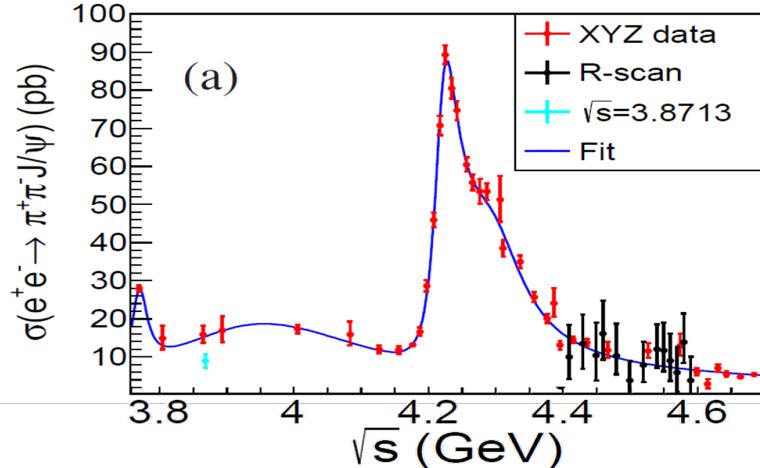
$m_{\text{BW}} = 3873.71^{+0.56}_{-0.50}(\text{stat}) \pm 0.13(\text{syst}) \text{ MeV}/c^2$,
 $\Gamma_{\text{BW}} = 5.2^{+2.2}_{-1.5}(\text{stat}) \pm 0.4(\text{syst}) \text{ MeV}$.

- Fit $D^0 \bar{D}^{*0}$ mode only, not a coupled-channel analysis
- BW is favored over Flatté parametrization
- coupled-channel analysis highly recommended

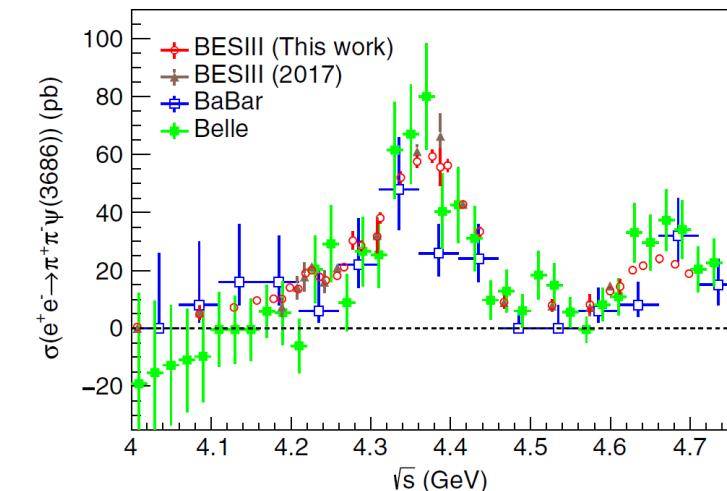


Y(4260) is now Y(4230) [$\psi(4230)$ in PDG2023]

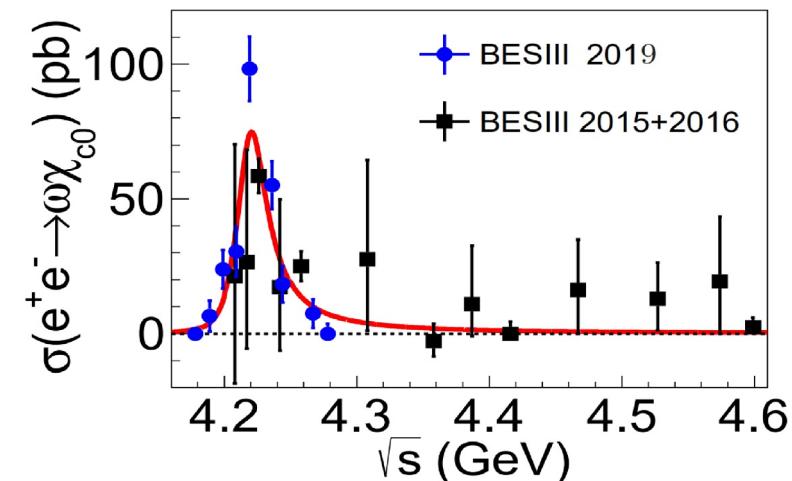
PRD106, 072001 (2022)



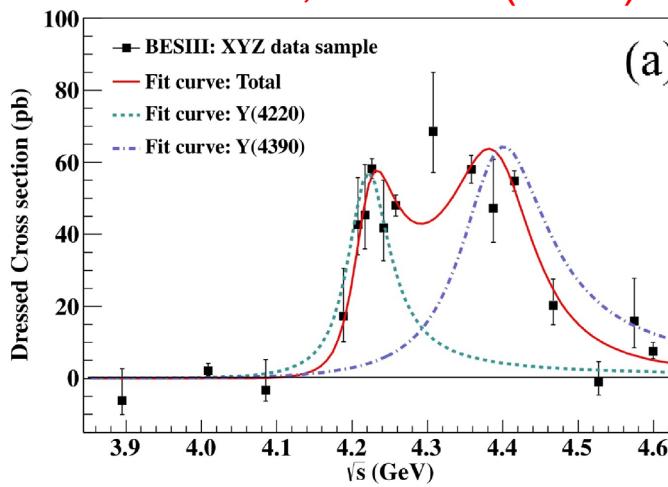
PRD104, 052102 (2021)



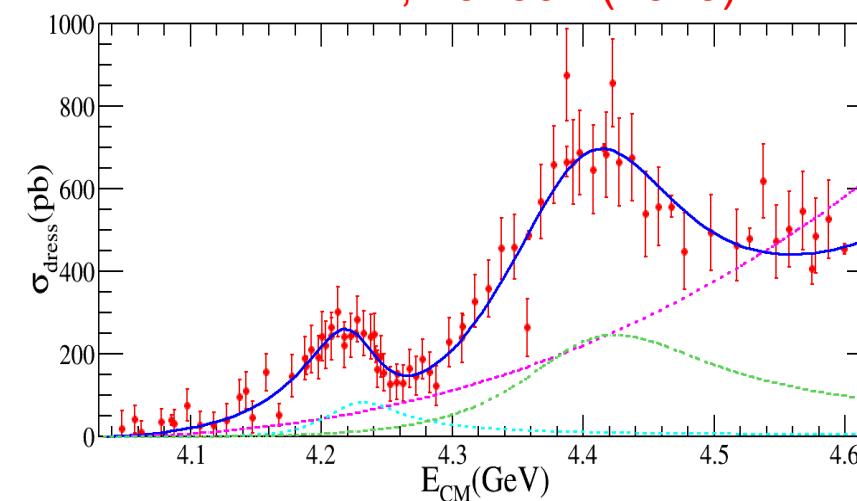
PRD99, 091103 (2019)



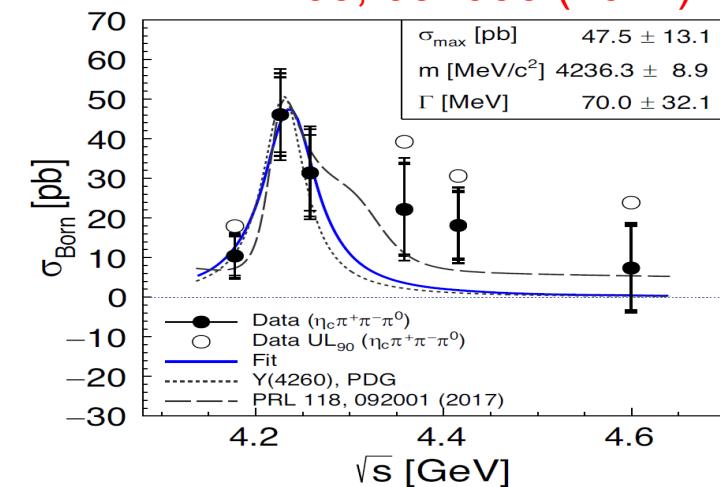
PRL118, 092002 (2017)



PRL122, 102002 (2019)



PRD 103, 032006 (2021)

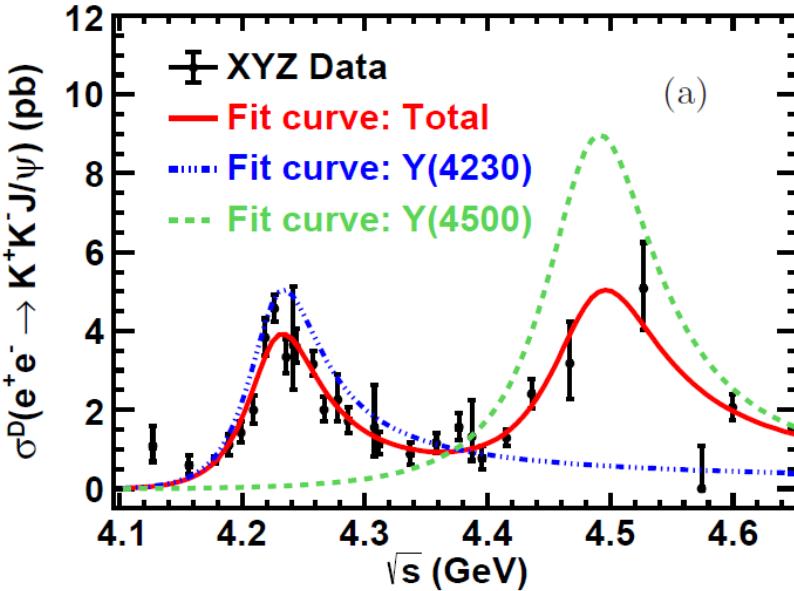


Y(4230) appears in $\omega\chi_{c0}$, $\pi^+\pi^-J/\psi$, $\pi^+\pi^-\psi'$, $\pi^+\pi^-h_c$, $D^0D^{*-}\pi^+$, $\eta_c\pi^+\pi^-\pi^0$, K^+K^-J/ψ , $D^{*0}D^{*-}\pi^+$,

Mass~4220 MeV, width~ 50 MeV!

15.6 fb^{-1} , Ecm=4.12-4.60 GeV

CPC46, 111002 (2022)

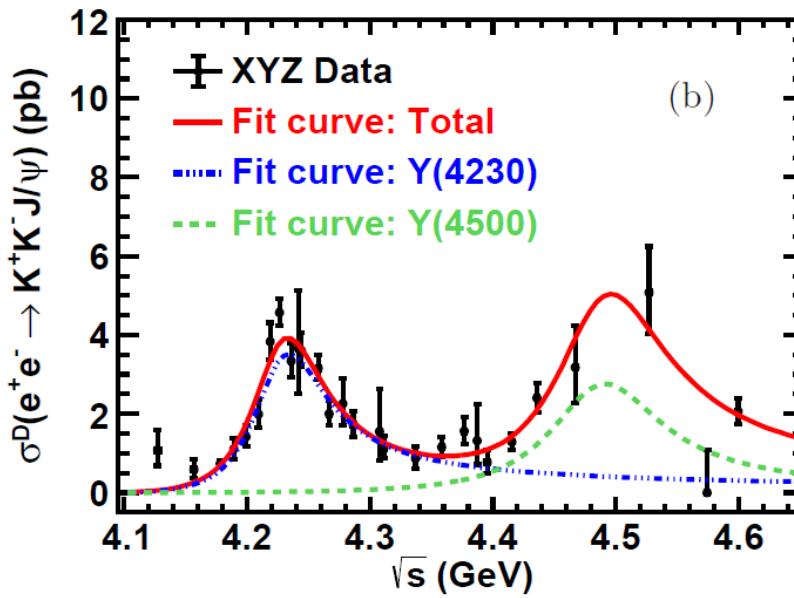


✓ First observation of $Y(4230) \rightarrow K^+K^-J/\psi$ (29σ)

$$0.02 < \frac{\mathcal{B}(Y(4230) \rightarrow K^+K^-J/\psi)}{\mathcal{B}(Y(4230) \rightarrow \pi^+\pi^-J/\psi)} < 0.26$$

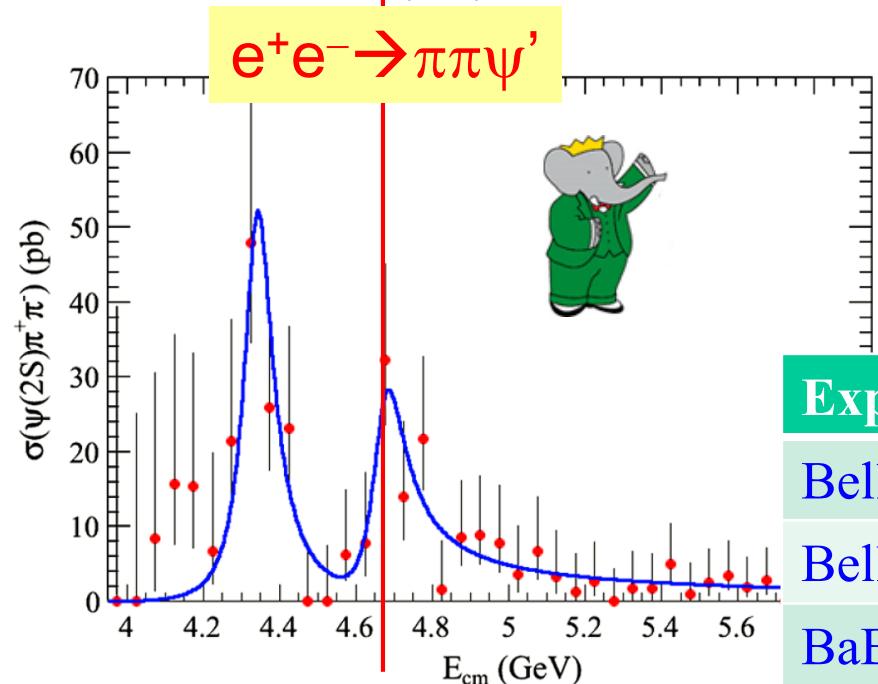
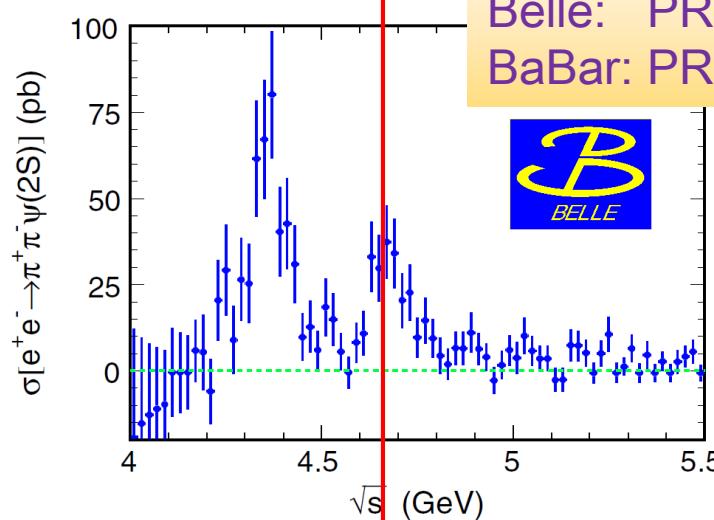
✓ Significance of the $Y(4500) > 8\sigma$

- A 5S-4D mixing state (J. Z. Wang et al., PRD 99, 114003 (2019))
- A heavy-antiheavy hadronic molecule
(X. K. Dong et al., Prog. Phys. 41, 65 (2021))
- A $(c\bar{s}\bar{c}\bar{s})$ state on LQCD (T. W. Chiu et al., PRD 73, 094510 (2006))

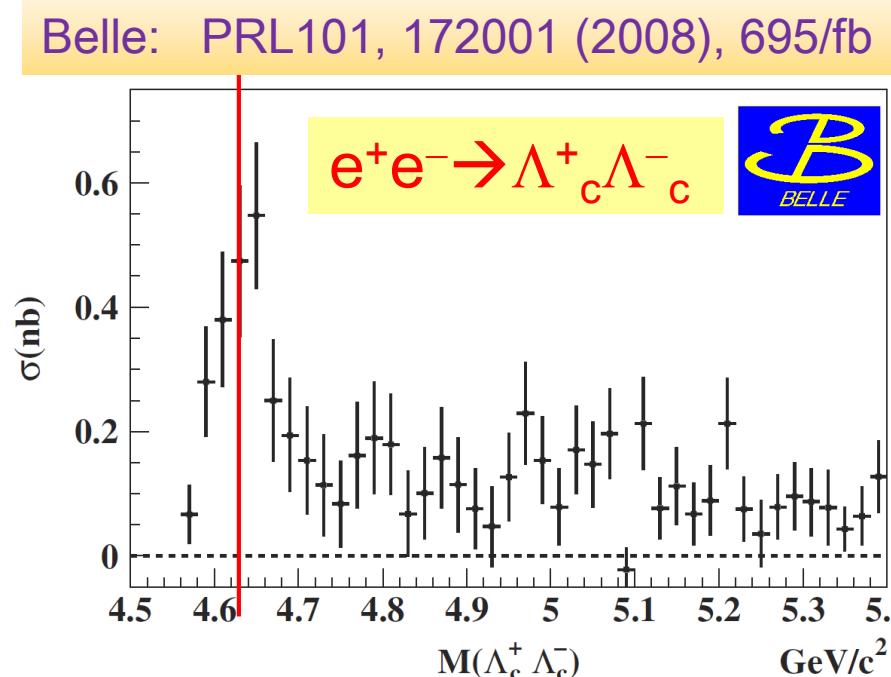


	Parameters	Solution I	Solution II
$Y(4230)$	$M(\text{MeV})$	$4225.3 \pm 2.3 \pm 21.5$	
	$\Gamma_{tot}(\text{MeV})$	$72.9 \pm 6.1 \pm 30.8$	
	$\Gamma_{ee}\mathcal{B}(\text{eV})$	$0.42 \pm 0.04 \pm 0.15$	$0.29 \pm 0.02 \pm 0.10$
$Y(4500)$	$M(\text{MeV})$	$4484.7 \pm 13.3 \pm 24.1$	
	$\Gamma_{tot}(\text{MeV})$	$111.1 \pm 30.1 \pm 15.2$	
	$\Gamma_{ee}\mathcal{B}(\text{eV})$	$1.35 \pm 0.14 \pm 0.06$	$0.41 \pm 0.08 \pm 0.13$
phase angle	$\varphi(\text{rad})$	$1.72 \pm 0.09 \pm 0.52$	$5.49 \pm 0.35 \pm 0.58$

$\Upsilon(4630) = \Upsilon(4660)$? Are there other decay modes?



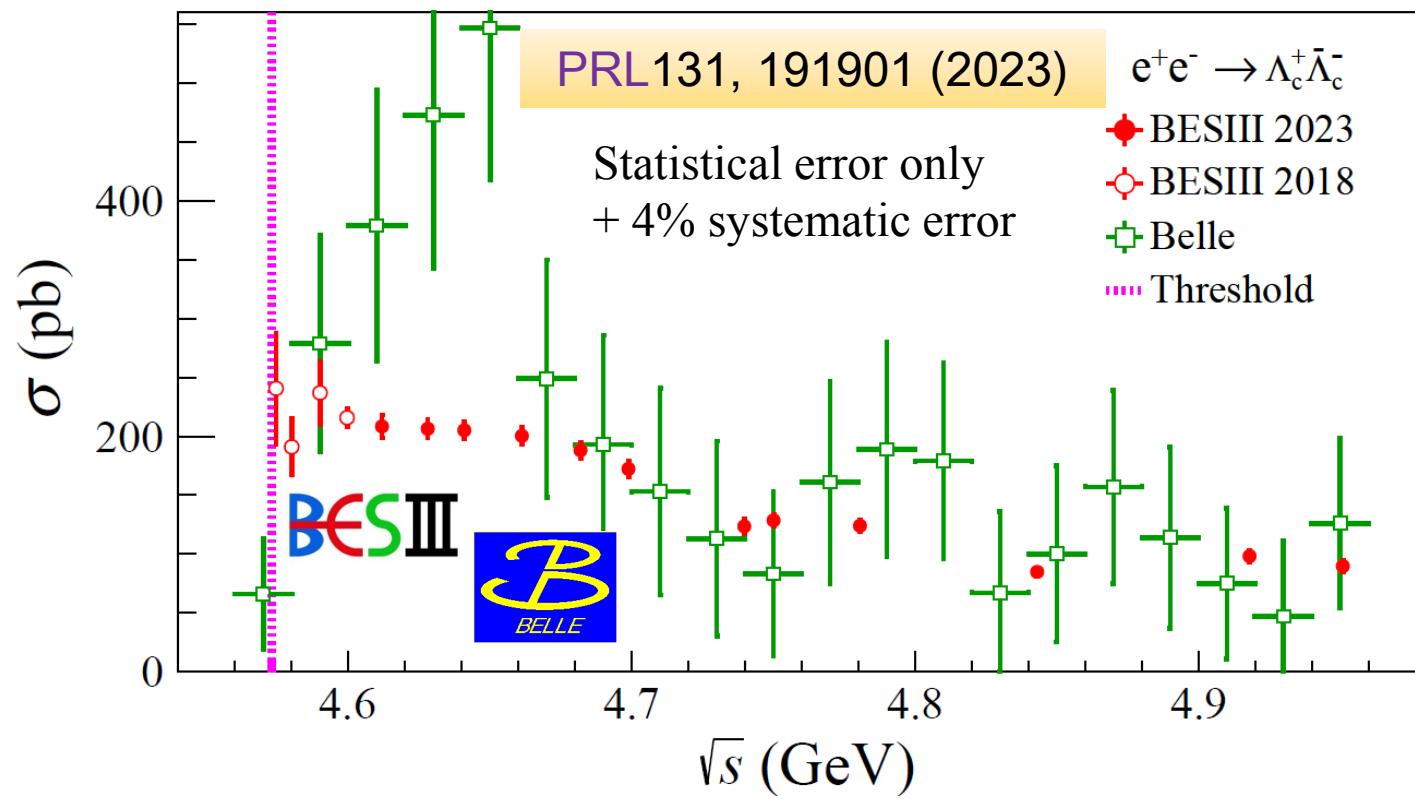
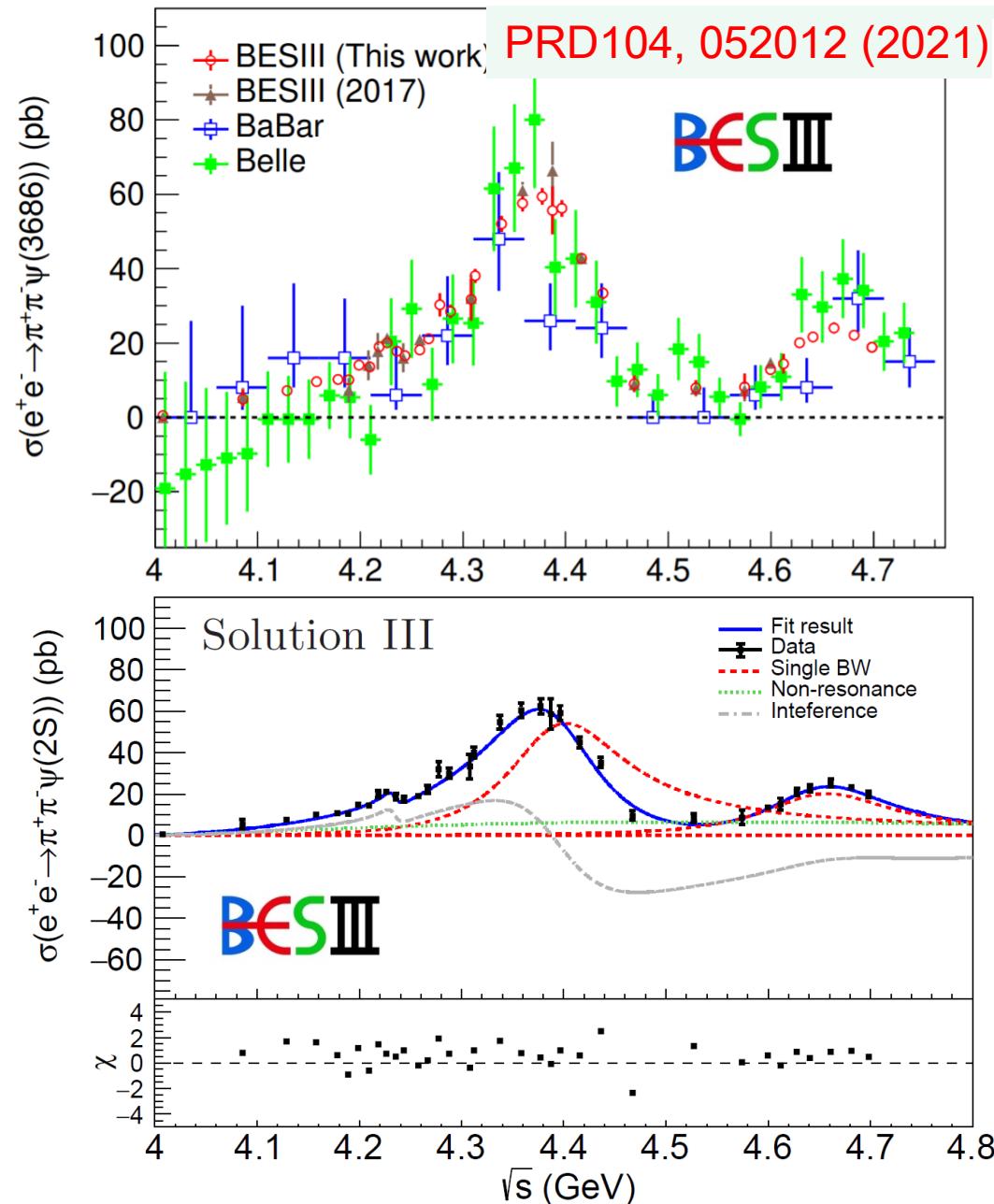
$\Upsilon(4660)$ discovered by Belle in 2007
 $\Upsilon(4630)$ discovered by Belle in 2008



Experiment	Mass (MeV)	Width (MeV)
Belle, $\Lambda_c^+\Lambda_c^-$	$4634^{+8}_{-7}{}^{+5}_{-8}$	$92^{+40}_{-24}{}^{+10}_{-21}$
Belle, $\pi\pi\psi'$	$4652 \pm 10 \pm 8$	$68 \pm 11 \pm 1$
BaBar, $\pi\pi\psi'$	$4669 \pm 21 \pm 3$	$104 \pm 48 \pm 10$

$e^+e^- \rightarrow \pi^+\pi^-\psi'$

Recent measurements

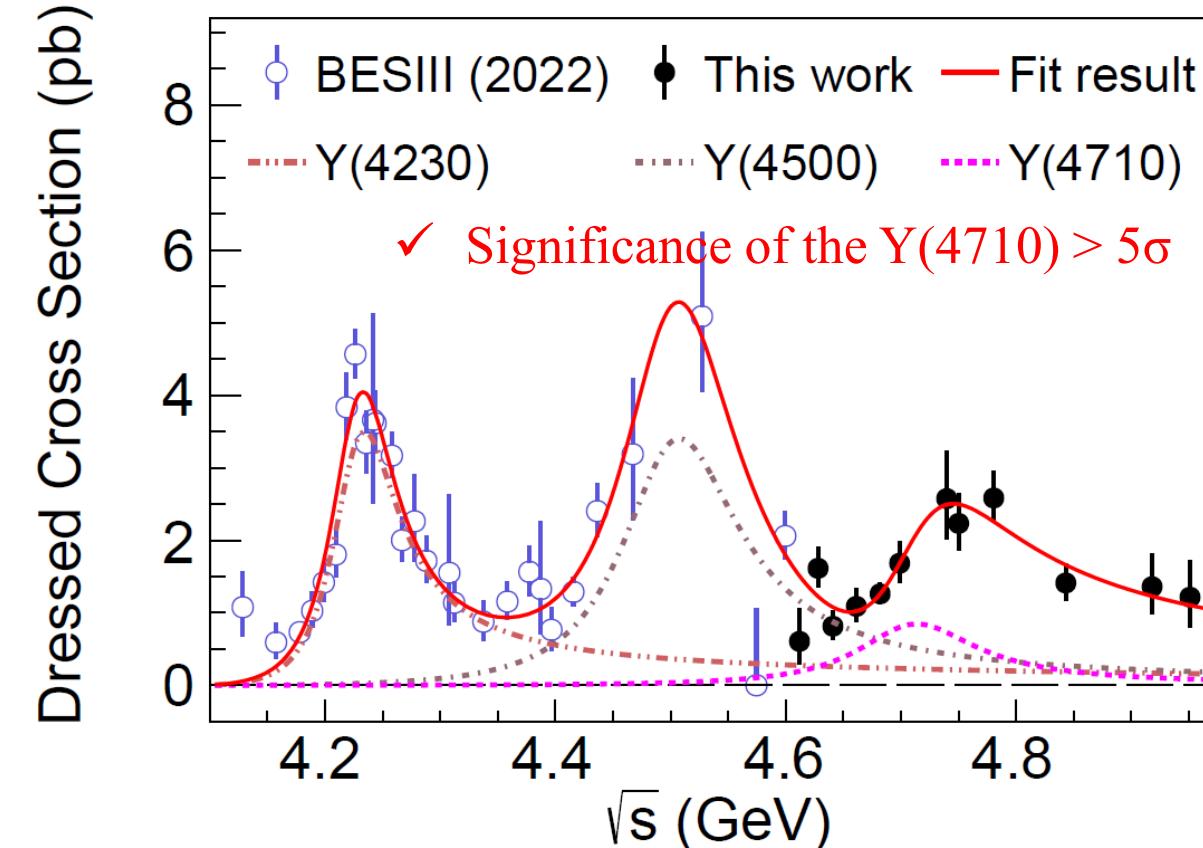
 $e^+e^- \rightarrow \Lambda^+_c\Lambda^-_c$ 

BESIII data confirmed Belle & BaBar measurements with much improved precision!

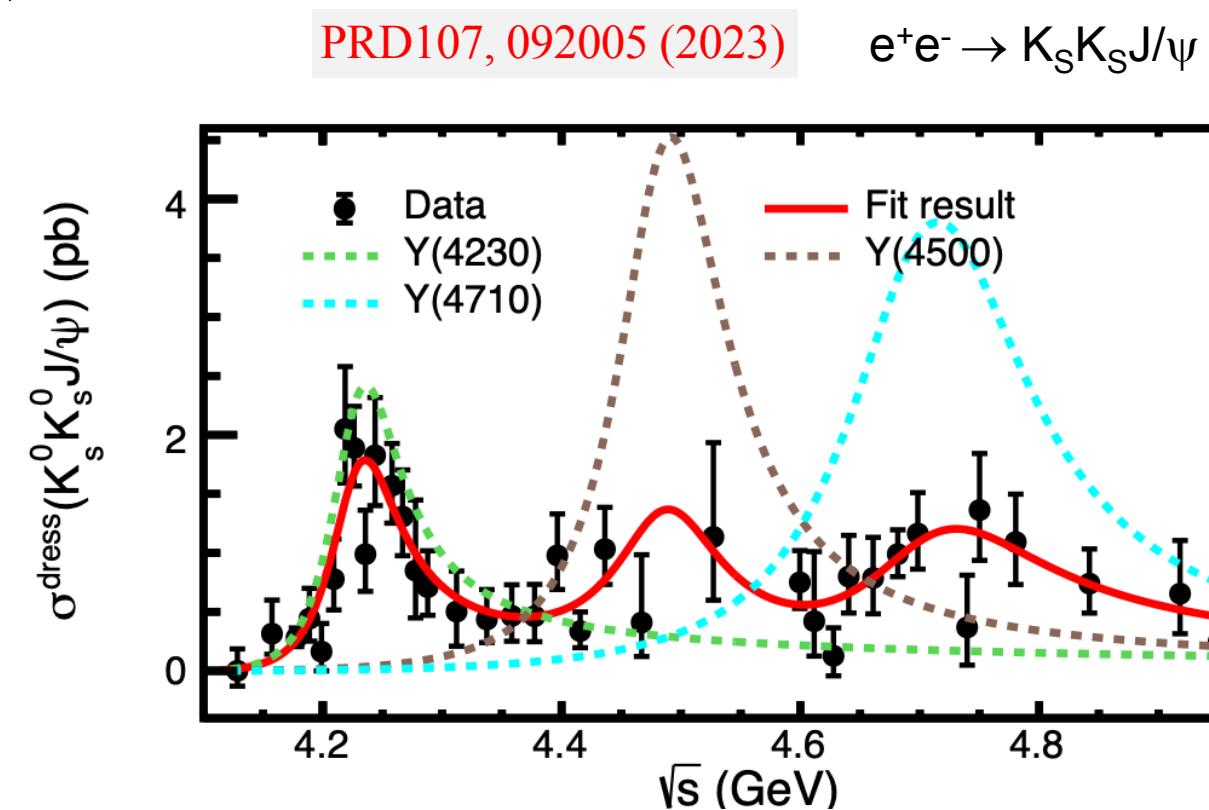
BESIII data did not confirm the Y(4630) in $e^+e^- \rightarrow \Lambda_c^+\Lambda_c^-$!

An even higher mass vector state $\Upsilon(4710)$ in KKJ/ψ

PRL131, 211902 (2023)

 $e^+e^- \rightarrow K^+K^-J/\psi$ 5.85 fb^{-1} , $E_{\text{cm}}=4.61-4.95 \text{ GeV}$ 

resonance	mass (MeV)	width (MeV)	note
Y(4230)	4226 ± 2	70 ± 4	Stat. only
Y(4500)	4499 ± 8	124 ± 20	Stat. only
Y(4710)	$4708^{+17}_{-15} \pm 21$	$126^{+27}_{-23} \pm 30$	$>5\sigma$

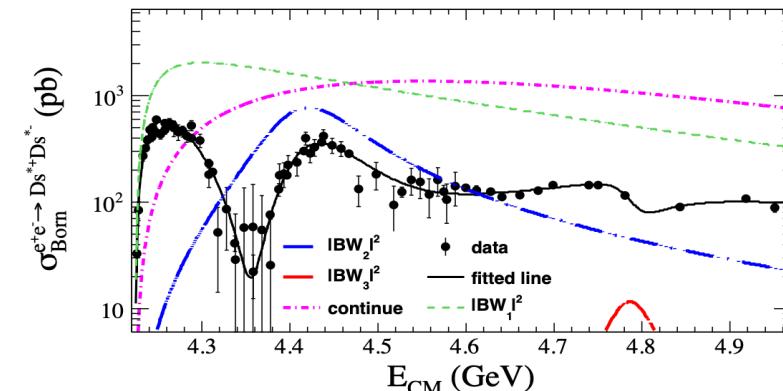
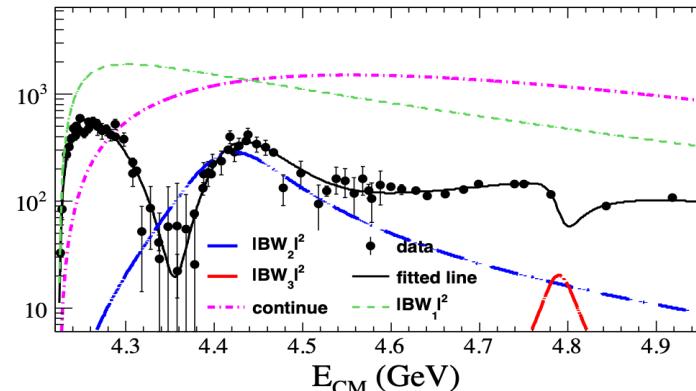
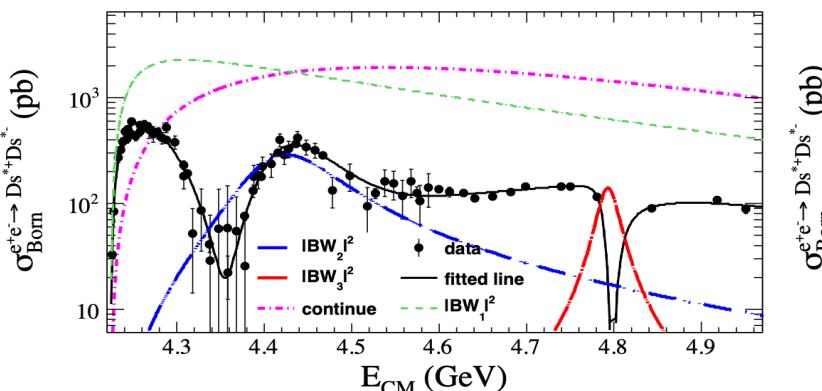
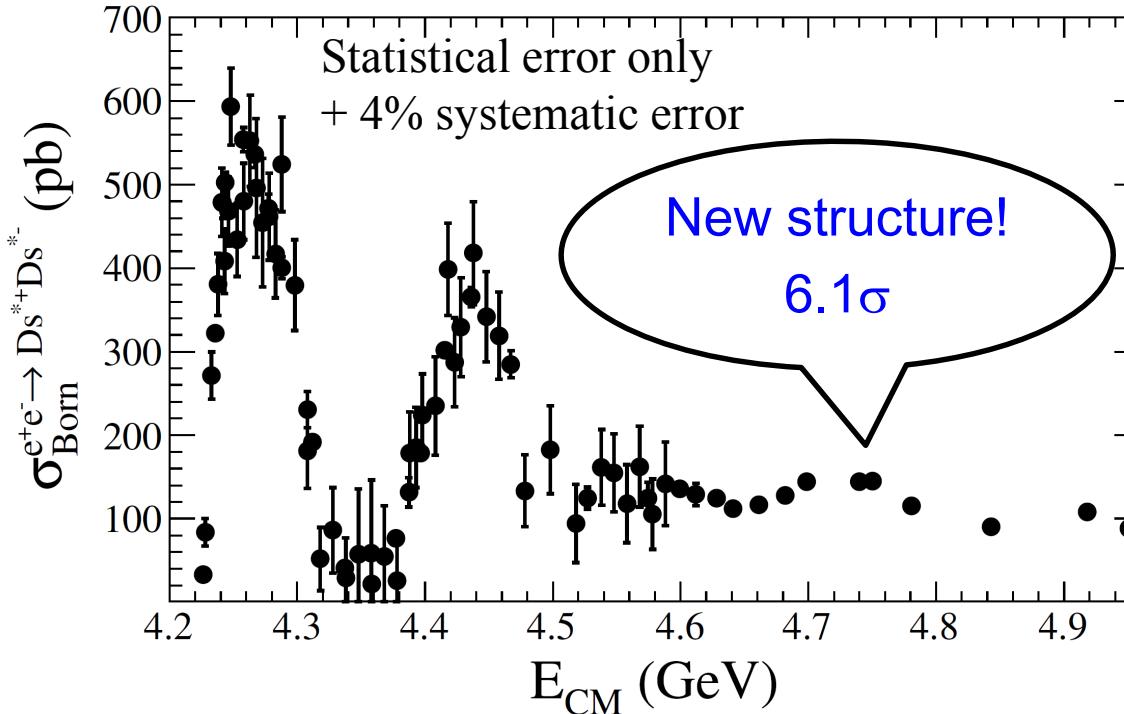


resonance	mass (MeV)	width (MeV)	note
Y(4230)	$4227 \pm 7 \pm 22$	$72 \pm 16 \pm 33$	
Y(4500)	Fixed	Fixed	1.4σ
Y(4710)	$4704 \pm 52 \pm 70$	$183 \pm 114 \pm 96$	4.0σ

5S vector charmonium states?

A new vector charmoniumlike state $\Upsilon(4790)$ in $e^+e^- \rightarrow D_s^{*+}D_s^{*-}$?

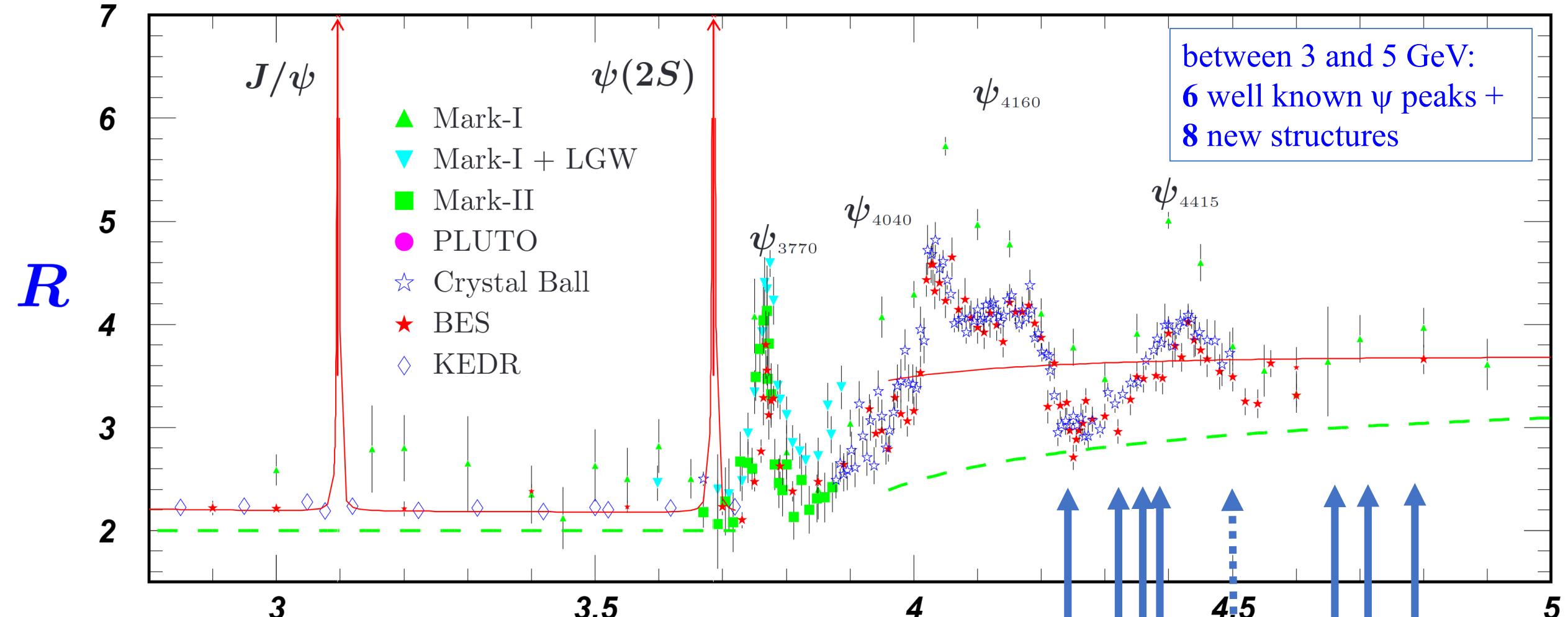
arXiv: 2305.10789, PRL 131, 151903 (2023)



- The peak position depends on the parametrization of the background amplitudes.
- Data at around 4.8 GeV are needed to understand the line shape.
- Could it be the $\Upsilon(4710)$ in KKJ/ψ ?

	Result 1	Result 2	Result 3
M_1 (MeV/c 2)	4186.5 ± 9.0	4193.8 ± 7.5	4195.3 ± 7.5
Γ_1 (MeV)	55 ± 17	61.2 ± 9.0	61.8 ± 9.0
M_2 (MeV/c 2)	4414.5 ± 3.2	4412.8 ± 3.2	4411.0 ± 3.2
Γ_2 (MeV)	122.6 ± 7.0	120.3 ± 7.0	120.0 ± 7.0
M_3 (MeV/c 2)	4793.3 ± 7.5	4789.8 ± 9.0	4786 ± 10
Γ_3 (MeV)	27.1 ± 7.0	41 ± 39	60 ± 35

How many vectors in charmonium energy region?



Besides vector charmonium ($c\bar{c}$) states, we also expect $c\bar{c}g$ hybrids, and $c\bar{c}q\bar{q}$ tetraquark states. Have they already been observed?

→ More theoretical/experimental efforts necessary!

$Y(4230)$, $Y(4320)$,
 $Y(4360)$, $Y(4390)$

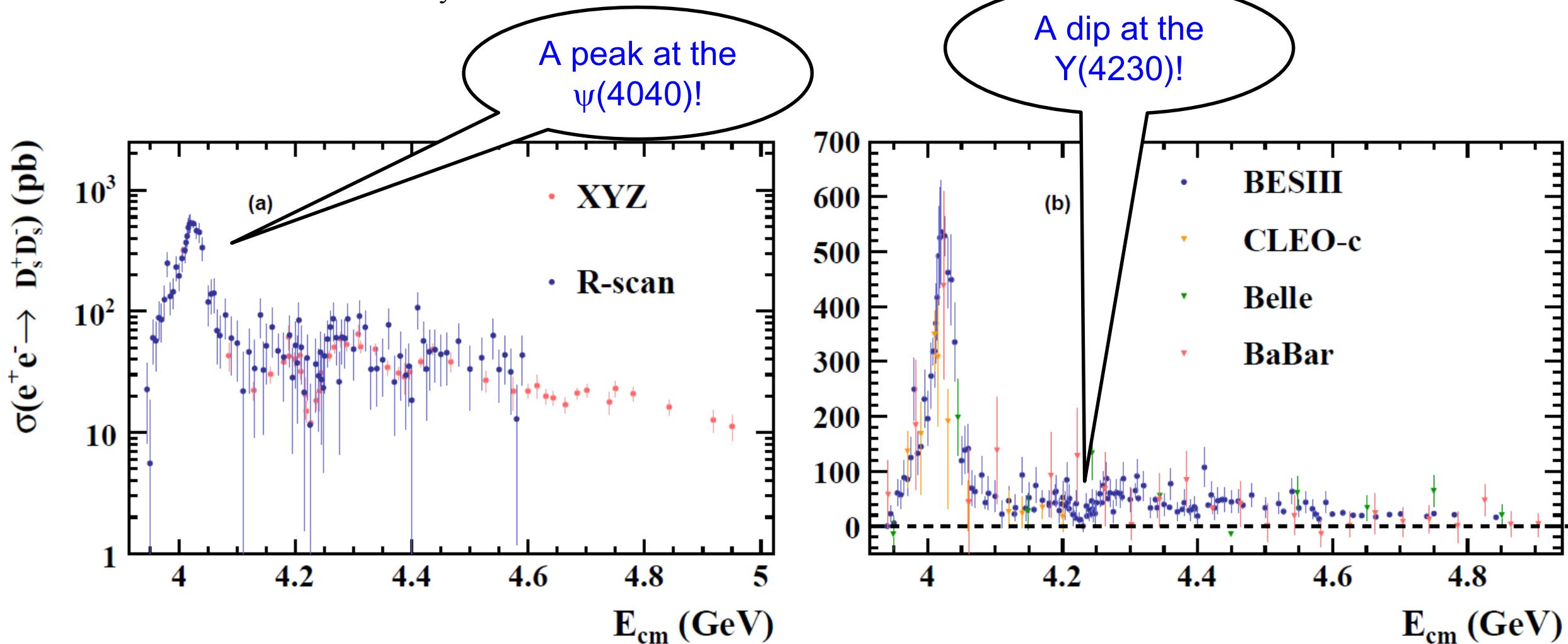
$Y(4500)$

$Y(4660)$, $Y(4710)$,
 $Y(4790)$

High precision measurement of $e^+e^- \rightarrow D_s^+ D_s^-$

arXiv: 2403.14998v1, submitted to PRL

Statistical error + 5.5% common systematic error

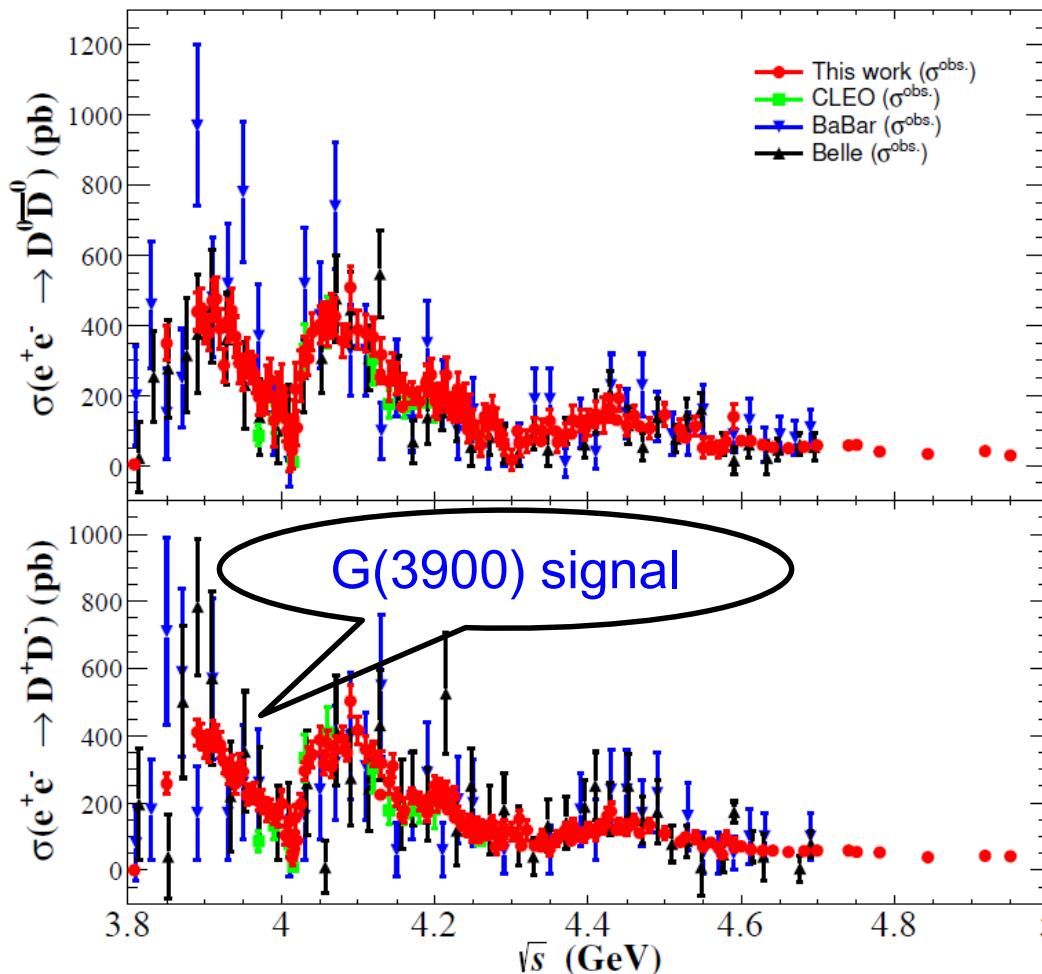


High precision measurement of $e^+e^- \rightarrow D^+D^-$ and $D^0\bar{D}^0$

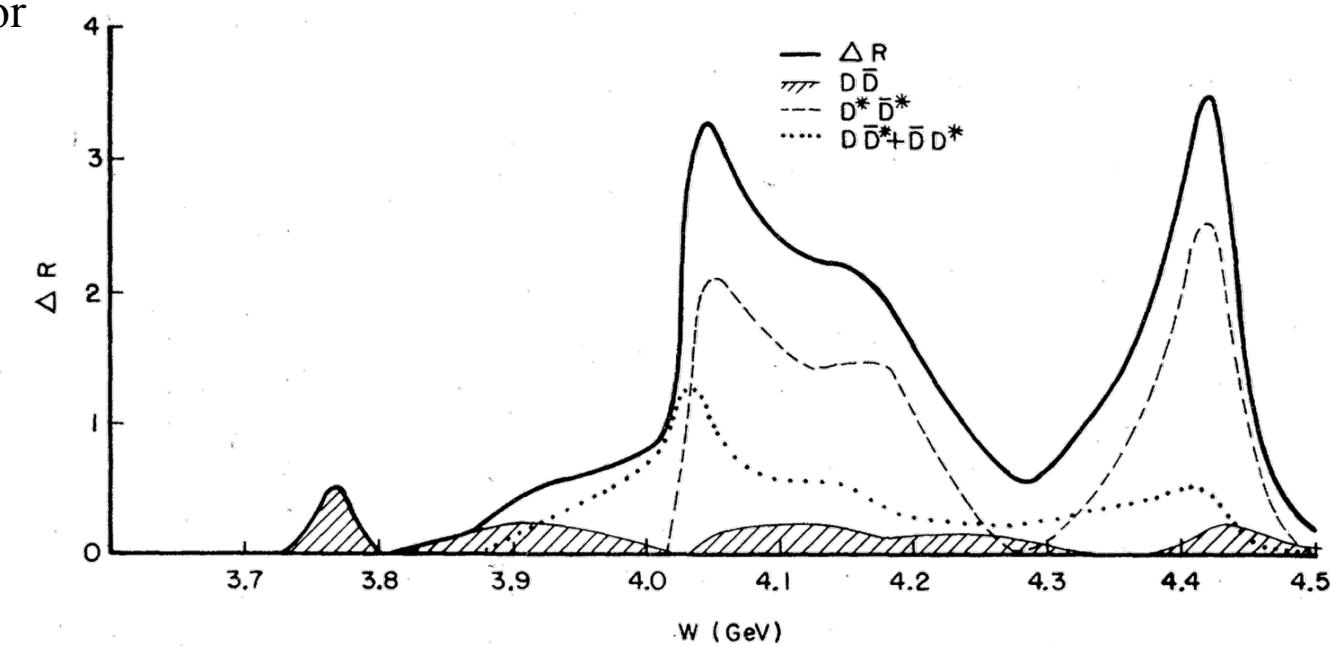
arXiv: 2402.03829v1, submitted to PRL

$D^0\bar{D}^0$: Statistical error + 7.0% common systematic error

D^+D^- : Statistical error + 6.5% common systematic error



Estia Eichten et al., Phys. Rev. D21 (1980) 203



Phenomenological studies:

- S. G. Salnikov & A. I. Milstein, arXiv:2404.06160
- N. Husken, et al., arXiv:2404.03896
- Z. Y. Lin, et al., arXiv:2403.01727
- S. X. Nakamura, et al., arXiv:2312.17658

Sophisticated models are necessary!

N. Husken, et al., arXiv:2404.03896

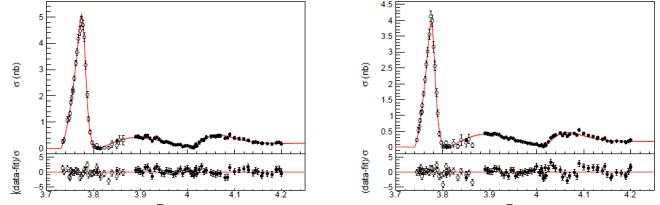


FIG. 2. Fit result for Model 1. Left: $e^+e^- \rightarrow D^0\bar{D}^0$. Right: $e^+e^- \rightarrow D^+\bar{D}^-$. Open data points are the Born cross section values based on observed cross sections, as reported in Ref. [18]; closed data points are from Ref. [1].

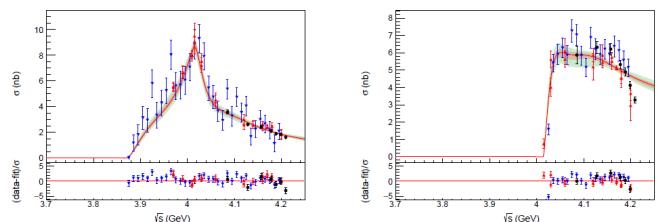
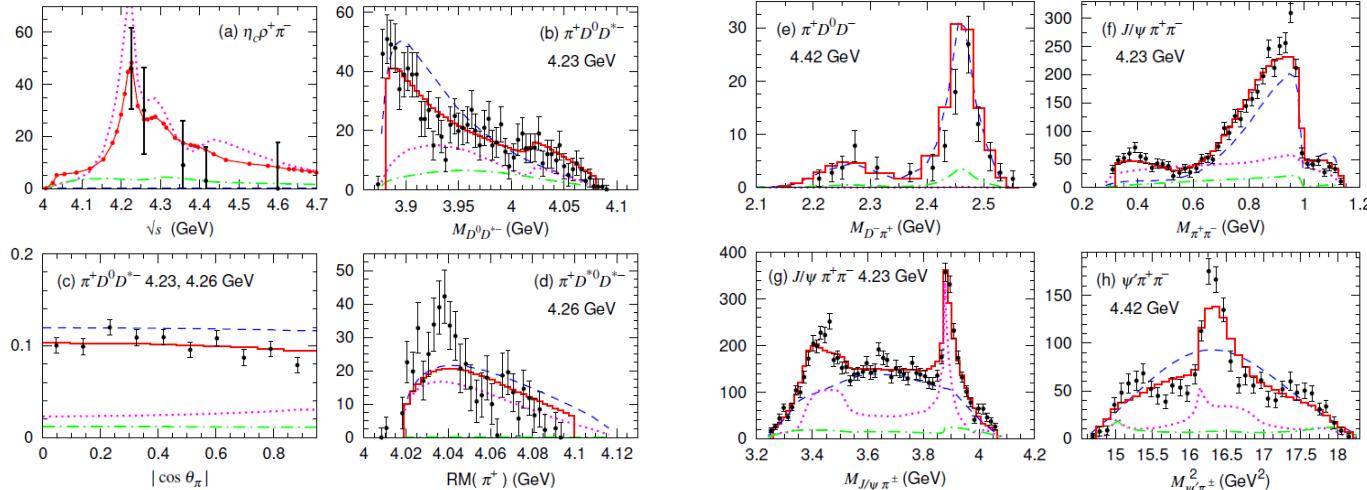


FIG. 3. Fit result for Model 1. Left: $e^+e^- \rightarrow D^*\bar{D}$. Right: $e^+e^- \rightarrow D^*\bar{D}^*$. The red region indicates the 68% confidence level, while green is the 90% confidence level. Black data points are from BESIII [21], red data is from CLEO-c [23, 24], blue data is from Belle [22].

S. X. Nakamura, et al., arXiv:2312.17658



S. G. Salnikov & A. I. Milstein, arXiv:2404.06160

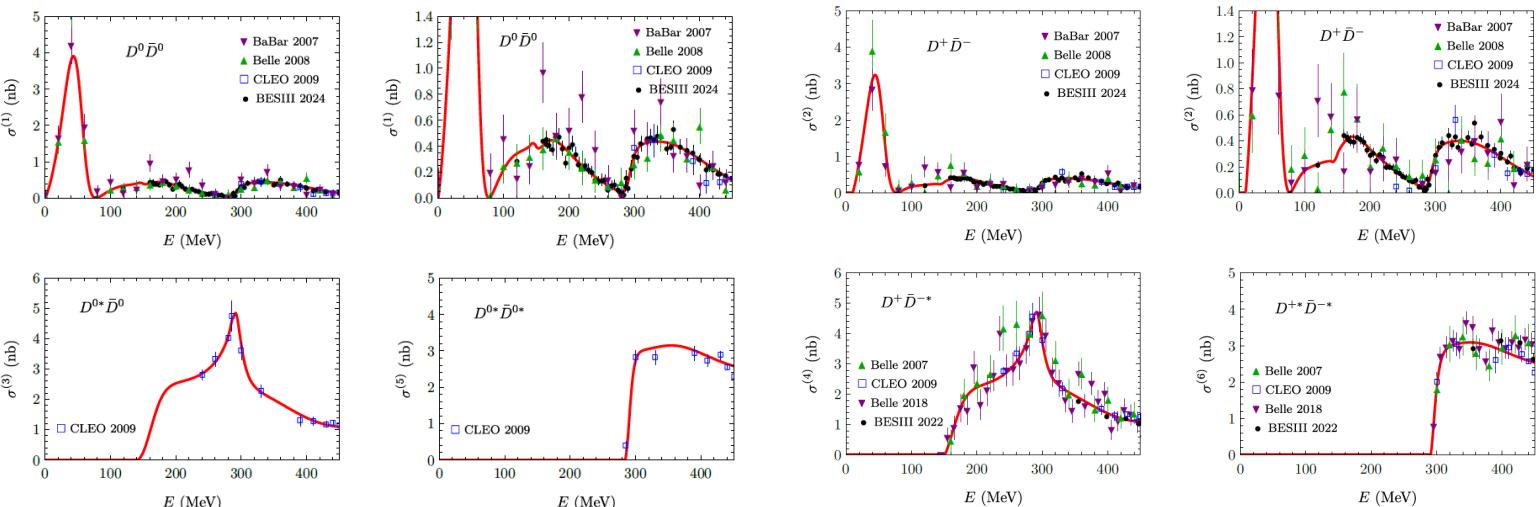


FIG. 1. Energy dependence of the cross sections for the production of neutral particles. Experimental data are taken from Refs. [32, 34–36, 39].

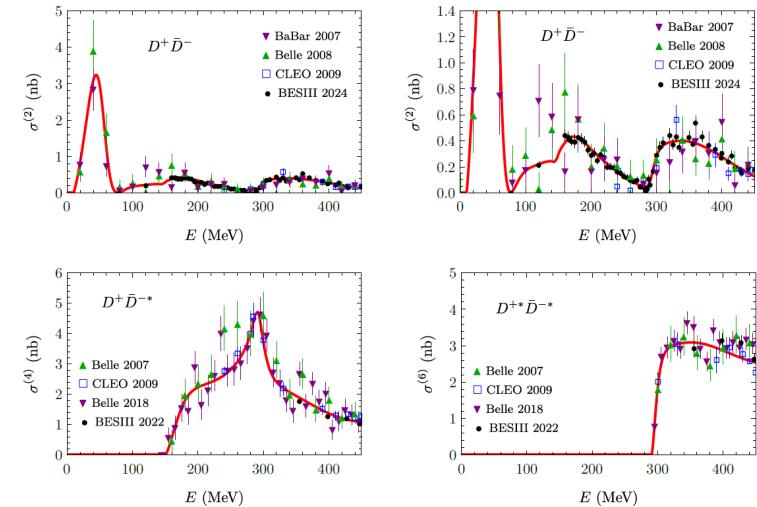
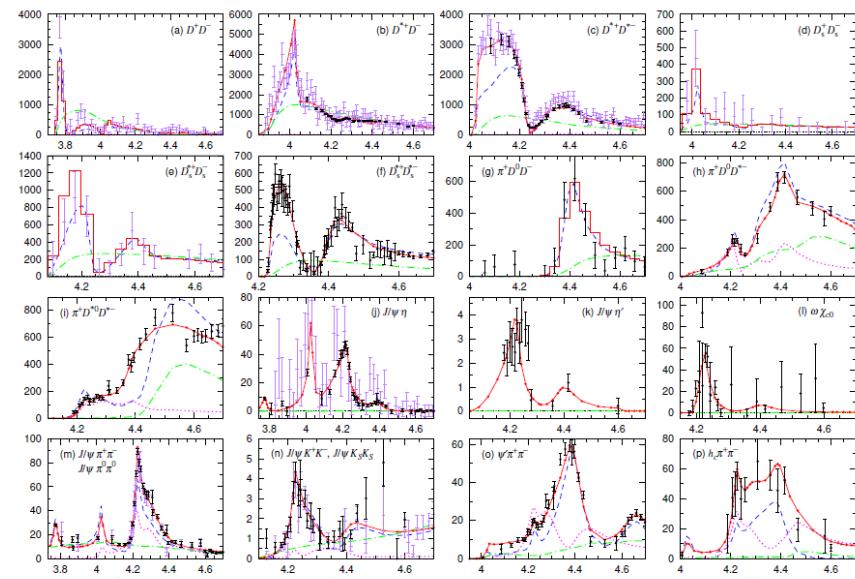
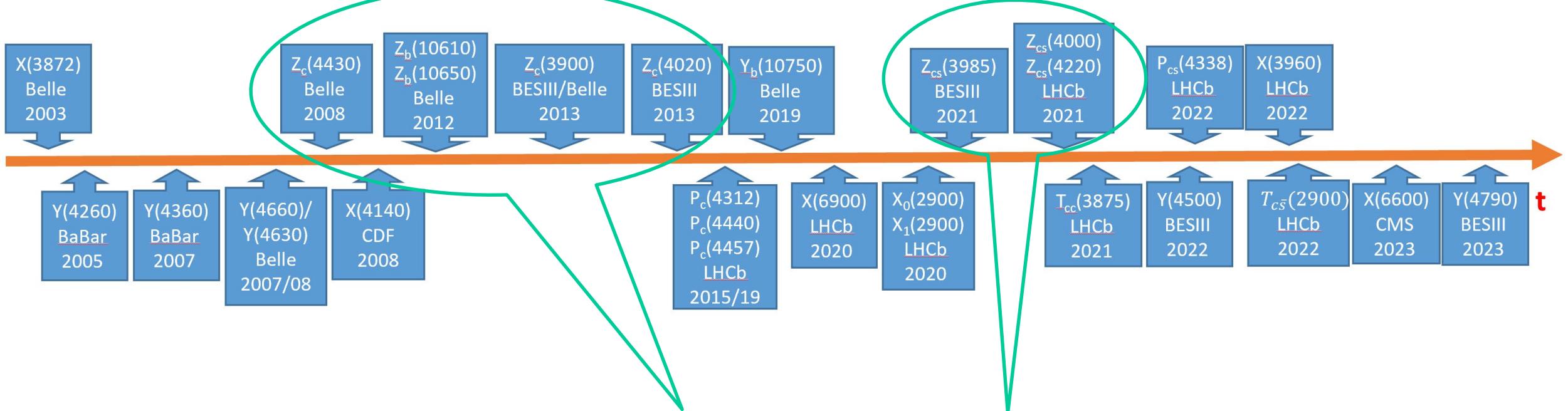
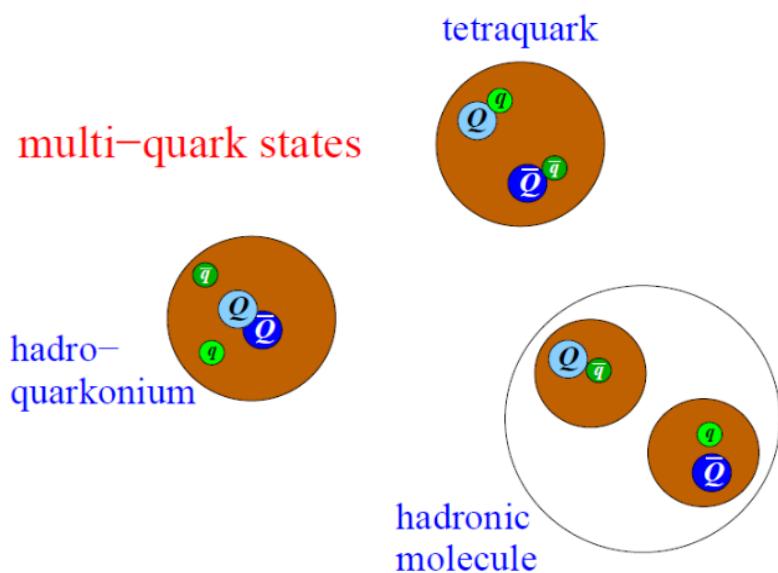


FIG. 2. Energy dependence of the cross sections for production of charged particles. Experimental data are taken from Refs. [32–39].

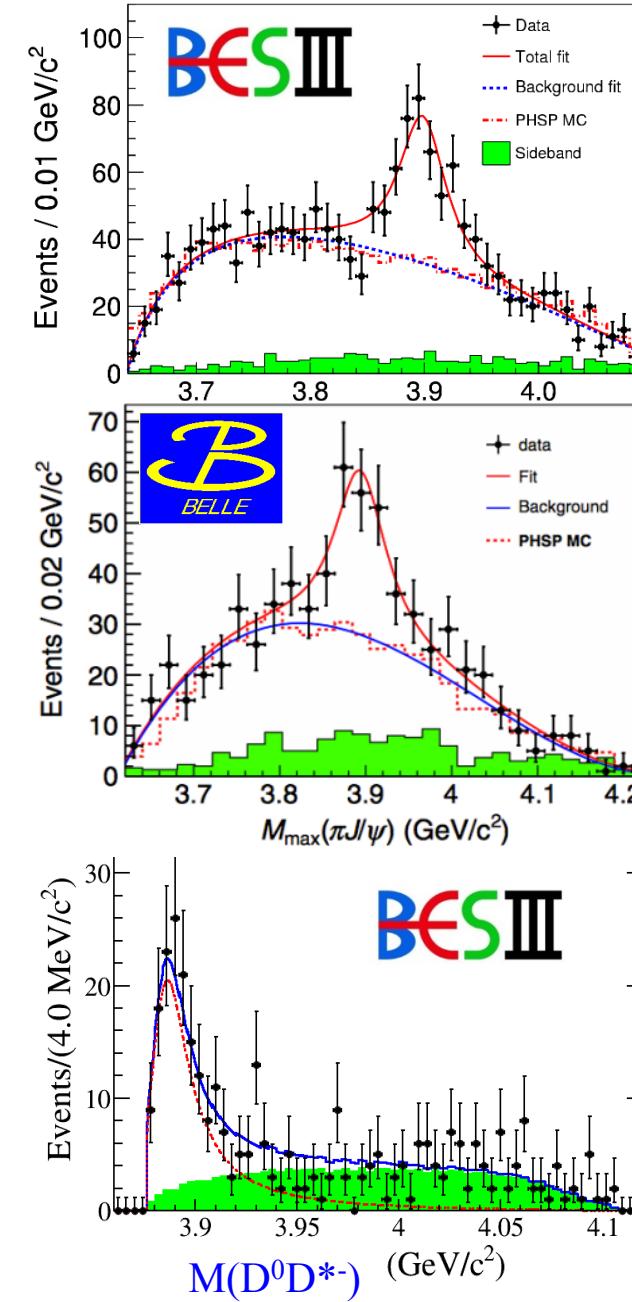




Charged quarkoniumlike states must have at least 4 quarks!

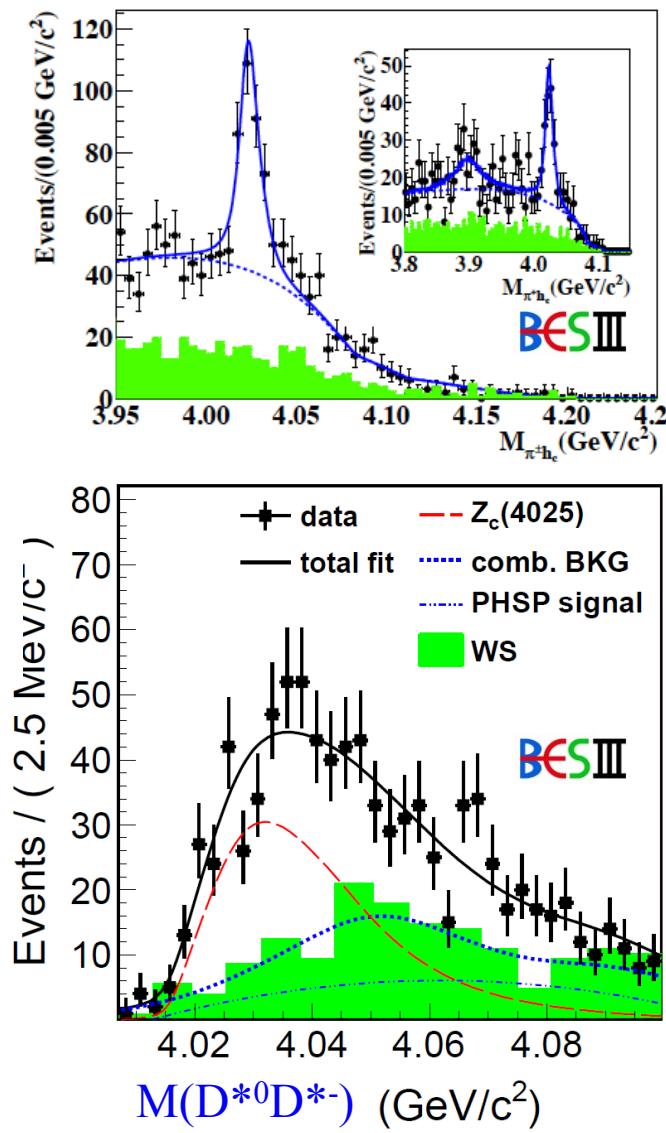


$Z_c(3900)$, 2013

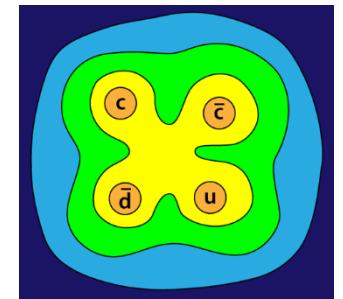
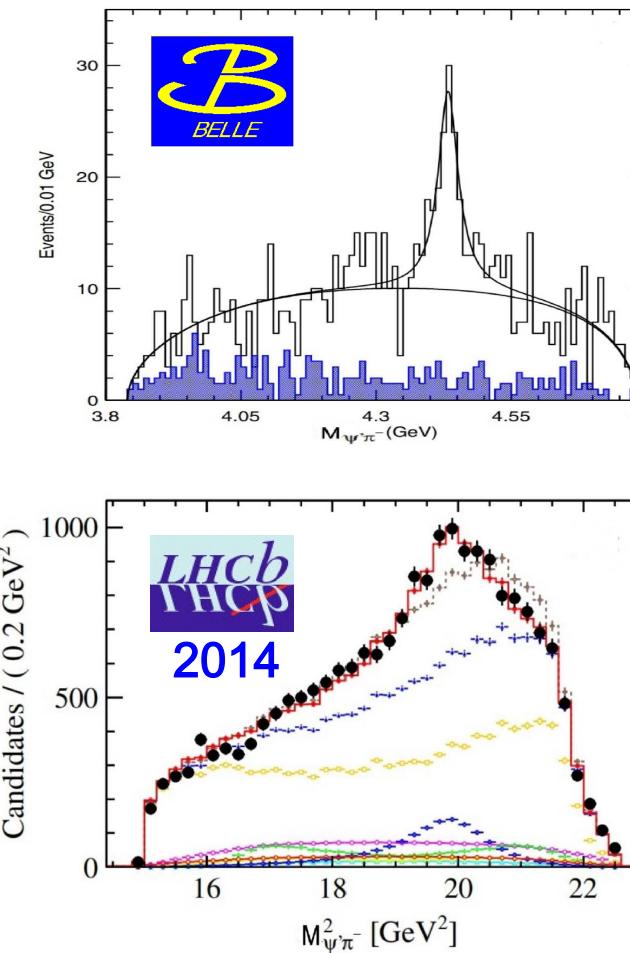


The Z_c states with u,d-quark

$Z_c(4020)$, 2013

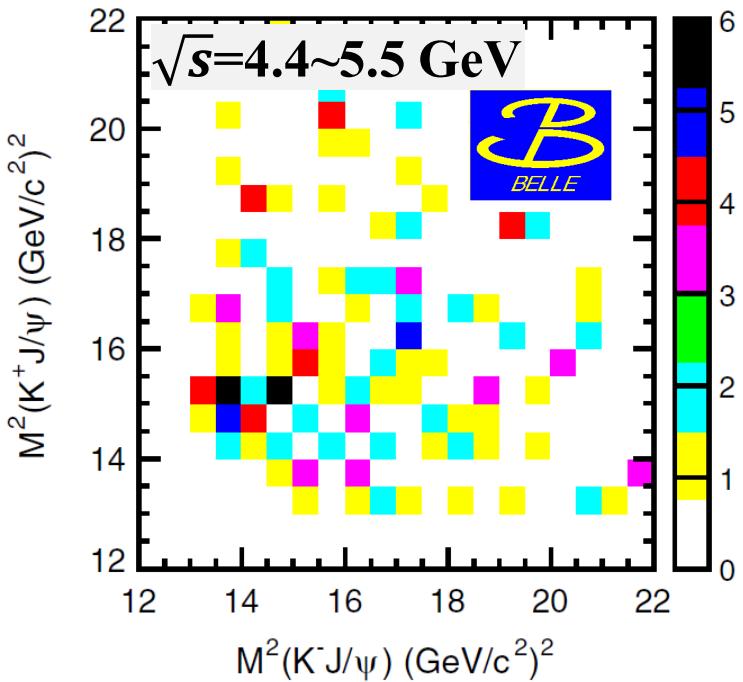


$Z_c(4430)$, 2008



All are observed in $\pi +$ charmonium (J/ψ , h_c , $\psi(2S)$) final states,
candidate $\bar{c} c \bar{d} u$
tetraquark states
→ Existence of states
with $d \rightarrow s$?
→ Search for states
decay into $K^\pm J/\psi$,
 $\bar{D}^* D_s + \bar{D} D_s^*$!

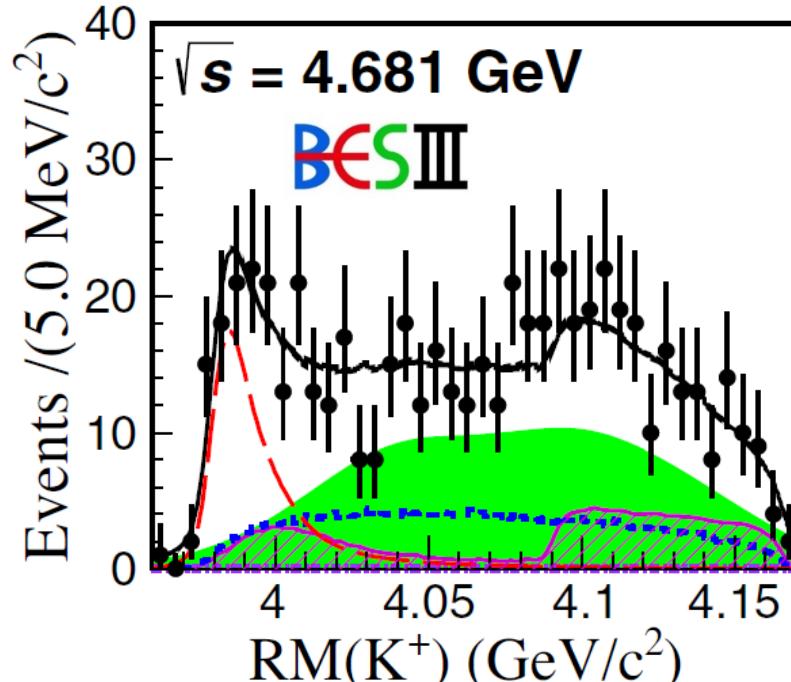
$e^+e^- \rightarrow K^+K^-J/\psi$



PRD 89, 072015 (2014)

No significant signal in
 $K^\pm J/\psi$ decay mode!
(statistics low!)

$e^+e^- \rightarrow K^+(D_s^- D^{*0} + D_s^* D^0)$

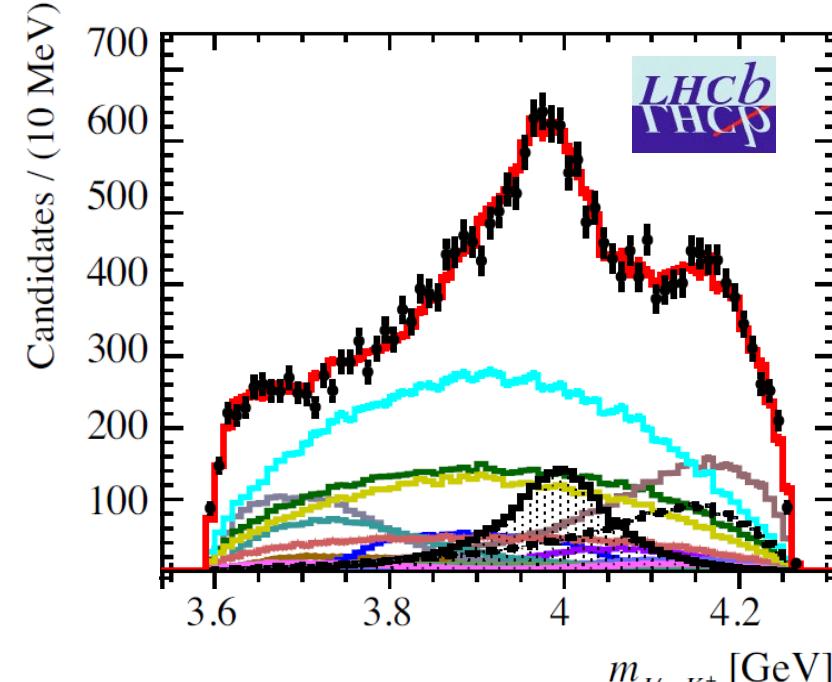


PRL 126, 102001 (2021)

$Z_{cs}(3985)$ in $\bar{D}^* D_s + \bar{D} D_s^*$ mode!

State	Signif.	JP	Mass (MeV)	Width (MeV)
$Z_{cs}(3985)$	5.3σ	??	$3982.5^{+1.8}_{-2.6} \pm 2.1$	$12.8^{+5.3}_{-4.4} \pm 3.0$
$Z_{cs}(4000)$	15σ	$1+$	$4003 \pm 6^{+4}_{-14}$	$131 \pm 15 \pm 26$
$Z_{cs}(4220)$	5.9σ	$1+$	$4216 \pm 24^{+43}_{-30}$	$233 \pm 52^{+97}_{-73}$

$B^+ \rightarrow J/\psi \phi K^+$



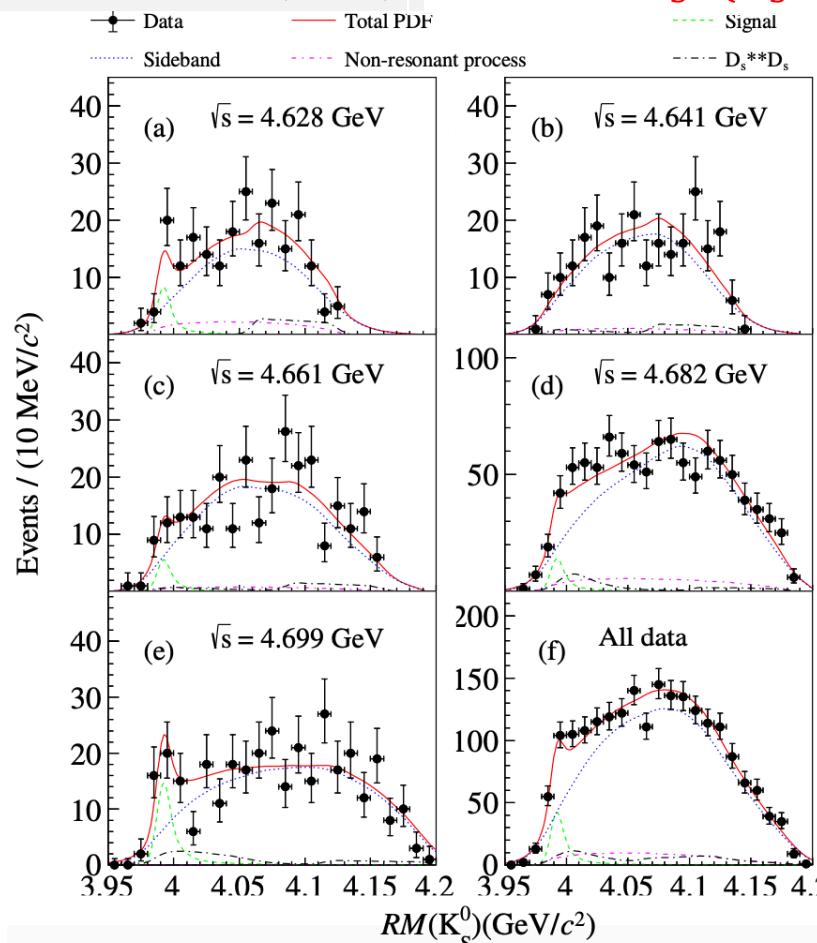
PRL 127, 082001 (2021)

$Z_{cs}(4000)$ and $Z_{cs}(4220)$
in $K^\pm J/\psi$ decay mode!

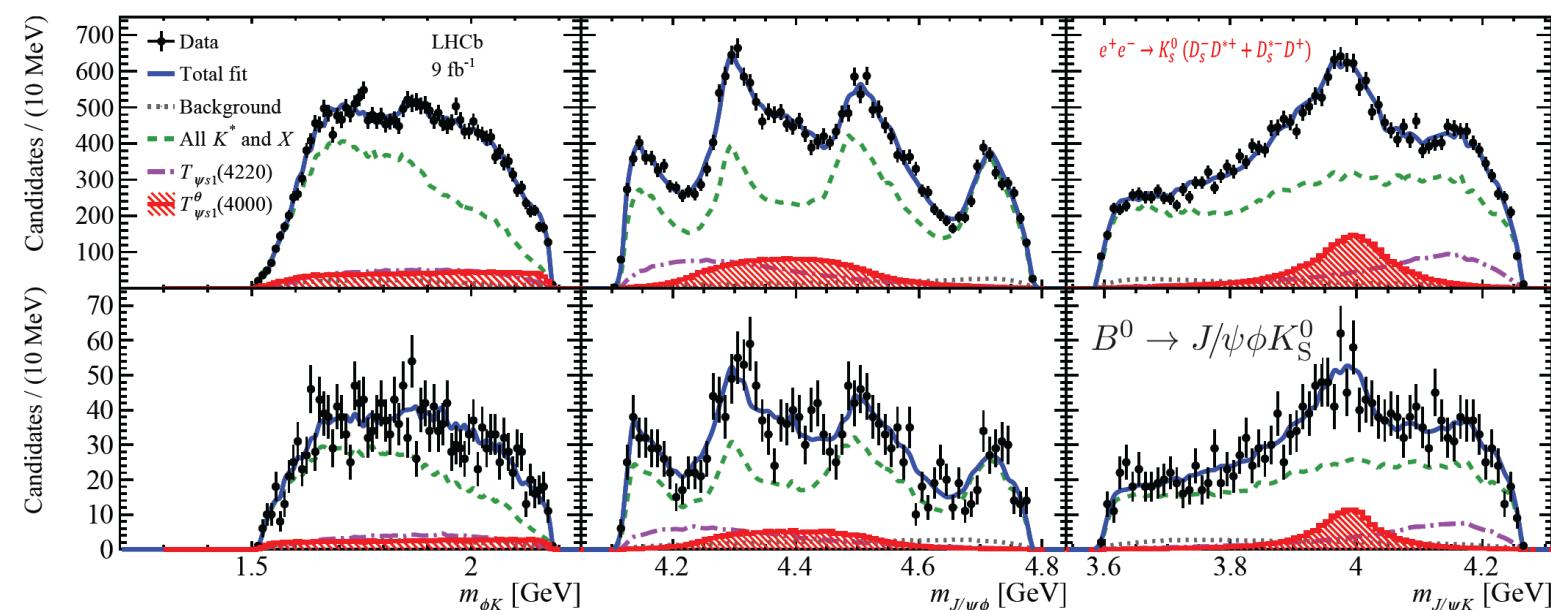
Widths very different!
Not the same state?
Same state but different
production dynamics?

Do their isospin partners exist? May BESIII see Z_{cs} in $e^+e^- \rightarrow K^+K^-J/\psi$?

PRL129, 112003 (2022)

 $e^+e^- \rightarrow K_S^0 (D_s^- D^{*+} + D_s^{*-} D^+)$  $B^0 \rightarrow J/\psi \phi K_S^0$

PRL131, 131901 (2023)

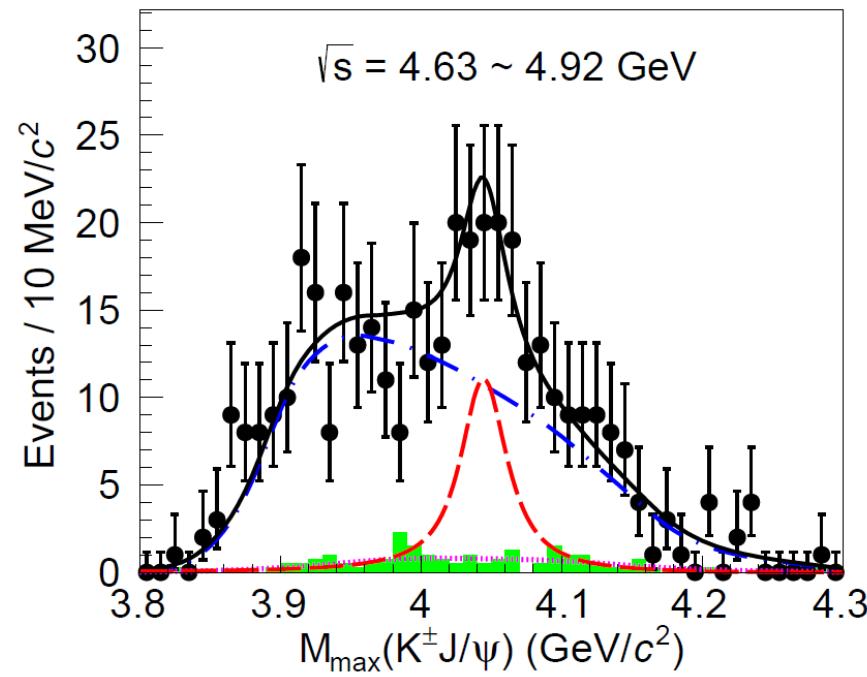
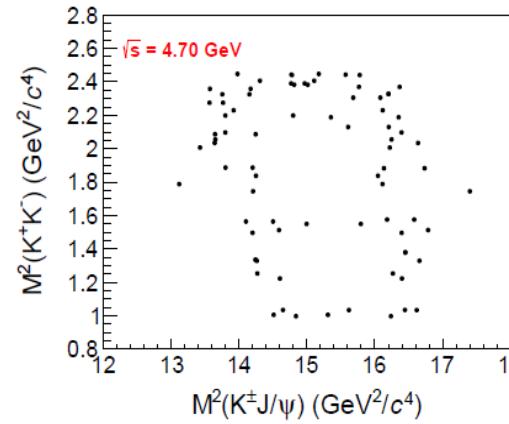
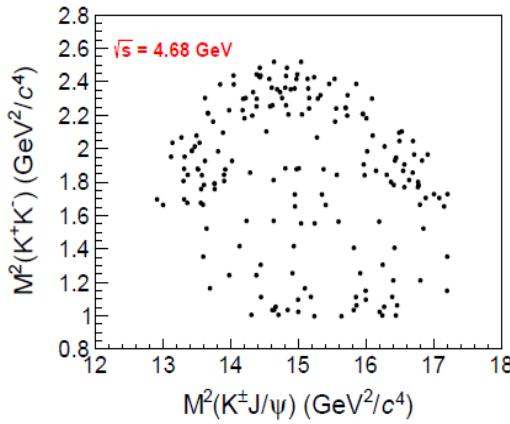
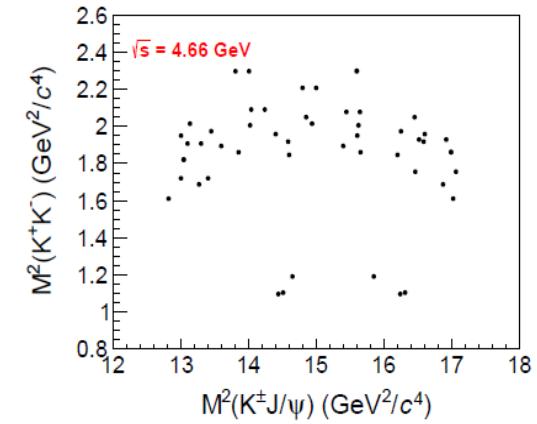
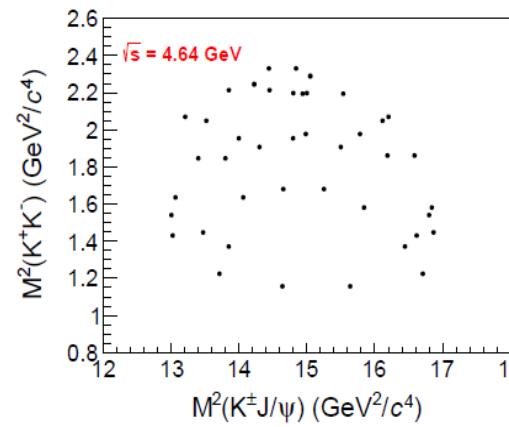
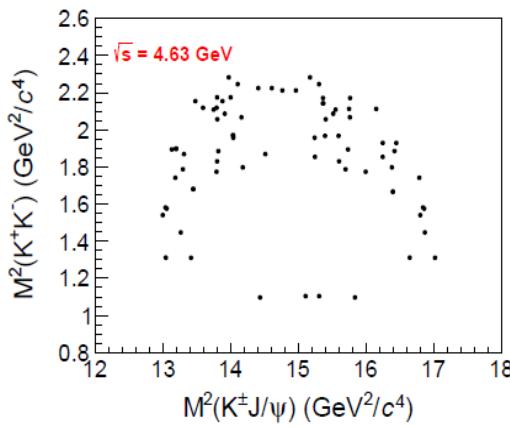
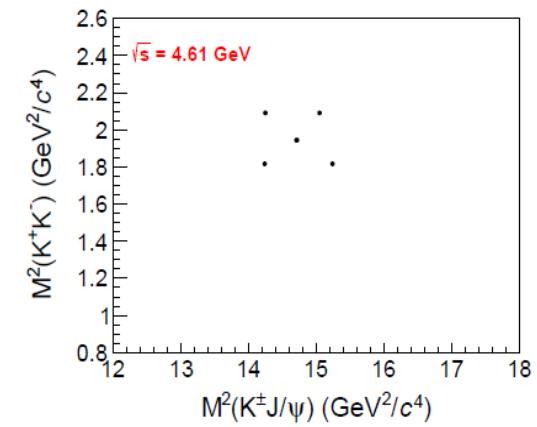
Significance $>4.0\sigma$ after including systematic uncertaintiesSignificance 5.4σ with isospin symmetry imposed

State	Mass (MeV/c ²)	Width (MeV)	Significance
$Z_{cs}(3985)^+$	$3985.2^{+2.1}_{-2.0} \pm 1.7$	$13.8^{+8.1}_{-5.2} \pm 4.9$	5.3σ
$Z_{cs}(3985)^0$	$3992.2 \pm 1.7 \pm 1.6$	$7.7^{+4.1}_{-3.8} \pm 4.3$	4.6σ

Mass (MeV)	Width (MeV)	Fit fraction (%)	ΔM (MeV)
$3991^{+12}_{-10} {}^{+9}_{-17}$	$105^{+29}_{-25} {}^{+17}_{-23}$	$7.9 \pm 2.5 {}^{+3.0}_{-2.8}$	$-12^{+11}_{-10} {}^{+6}_{-4}$

➤ Minimal quark content $c\bar{c}s\bar{d}$? Mass and width consistent with charged $Z_{cs} \rightarrow$ isospin partner

No significant structures in $K^\pm J/\psi$ decay mode!



$$M = 4044 \pm 6 \text{ MeV}$$

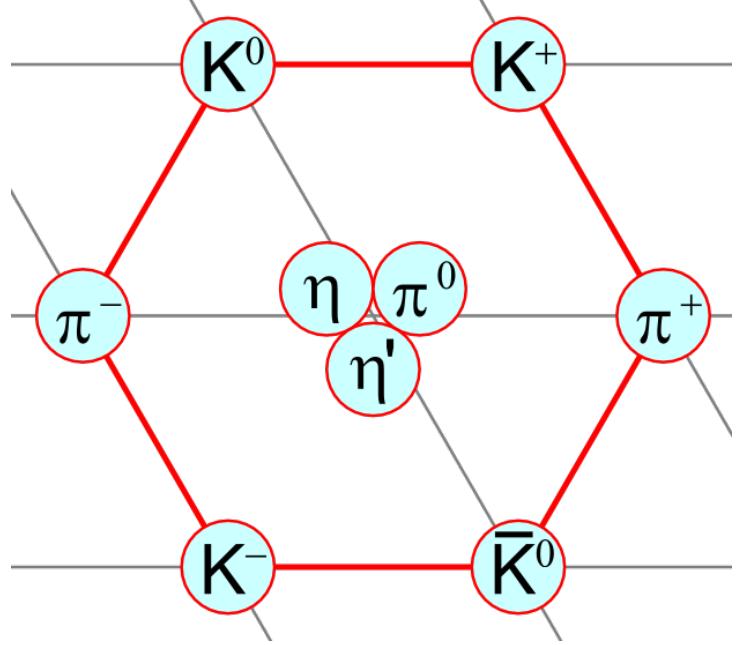
$$\Gamma = 36 \pm 16 \text{ MeV}$$

Significance: 2.3σ

$$\frac{B(Z_c(3900) \rightarrow J/\psi \pi^\pm)}{B(Z_c(3900) \rightarrow (D^* \bar{D})^\pm)} = 0.16 \pm 0.08$$

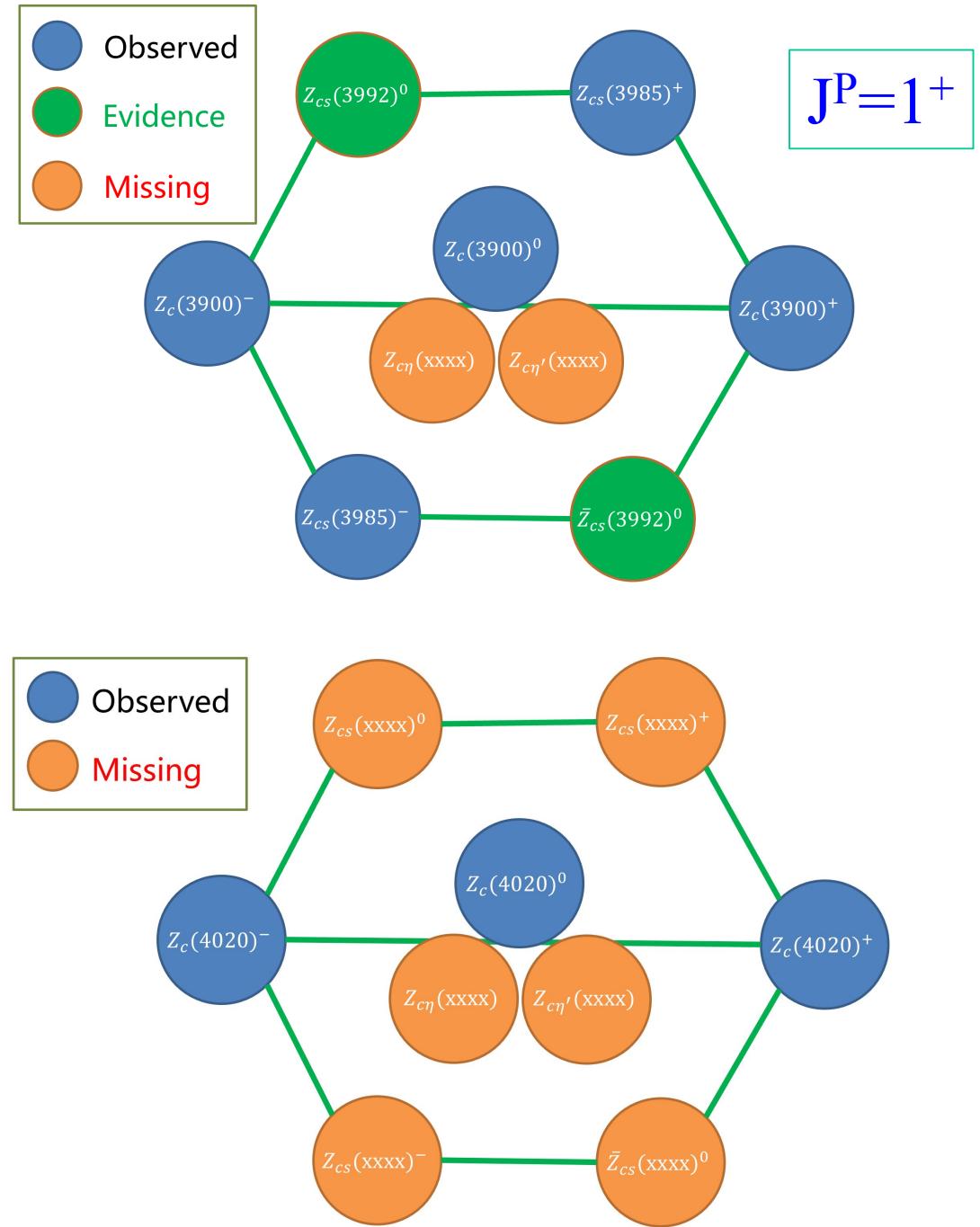
Calculated with data in PRL 112, 022001 (2014)

$$\frac{B(Z_{cs}(3985)^+ \rightarrow K^+ J/\psi)}{B(Z_{cs}(3985)^+ \rightarrow (\bar{D}^0 D_s^{*+} + \bar{D}^{*0} D_s^+))} < 0.03 \text{ @ 90% C.L.}$$

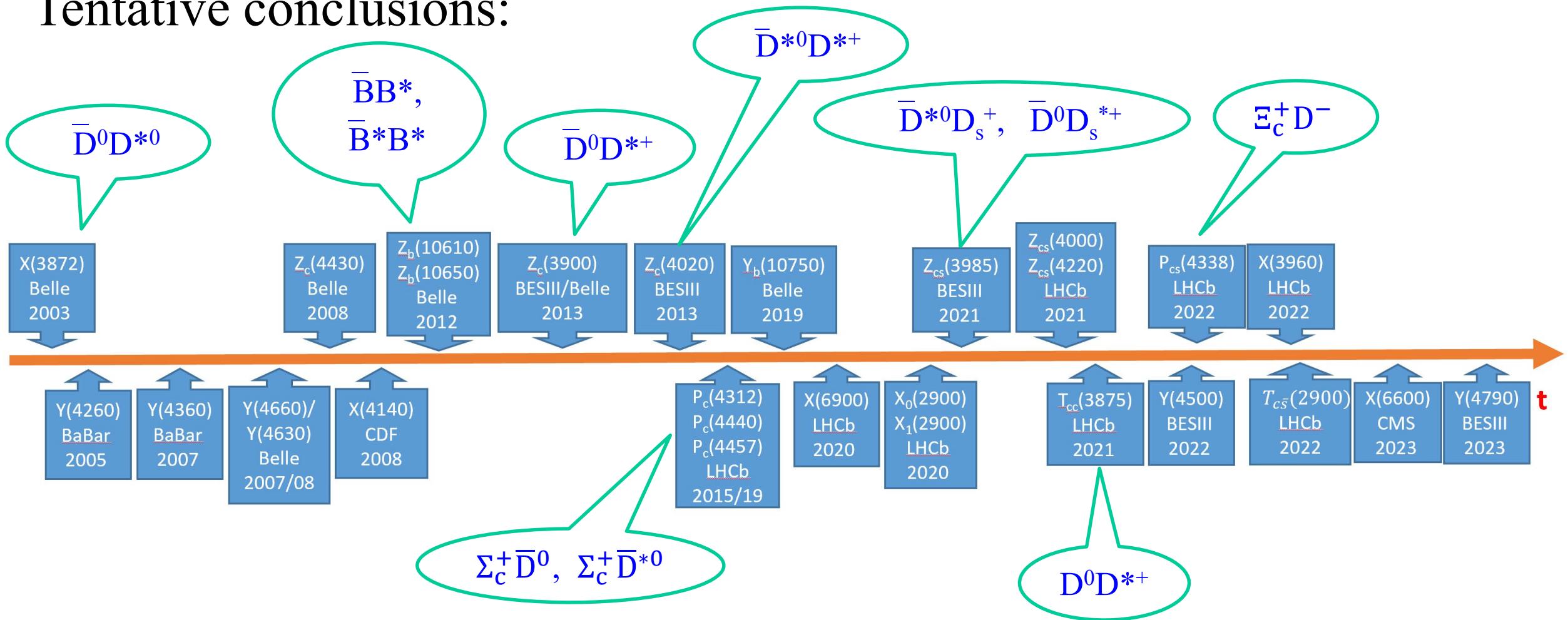


$+ J/\psi$

$+ h_c$

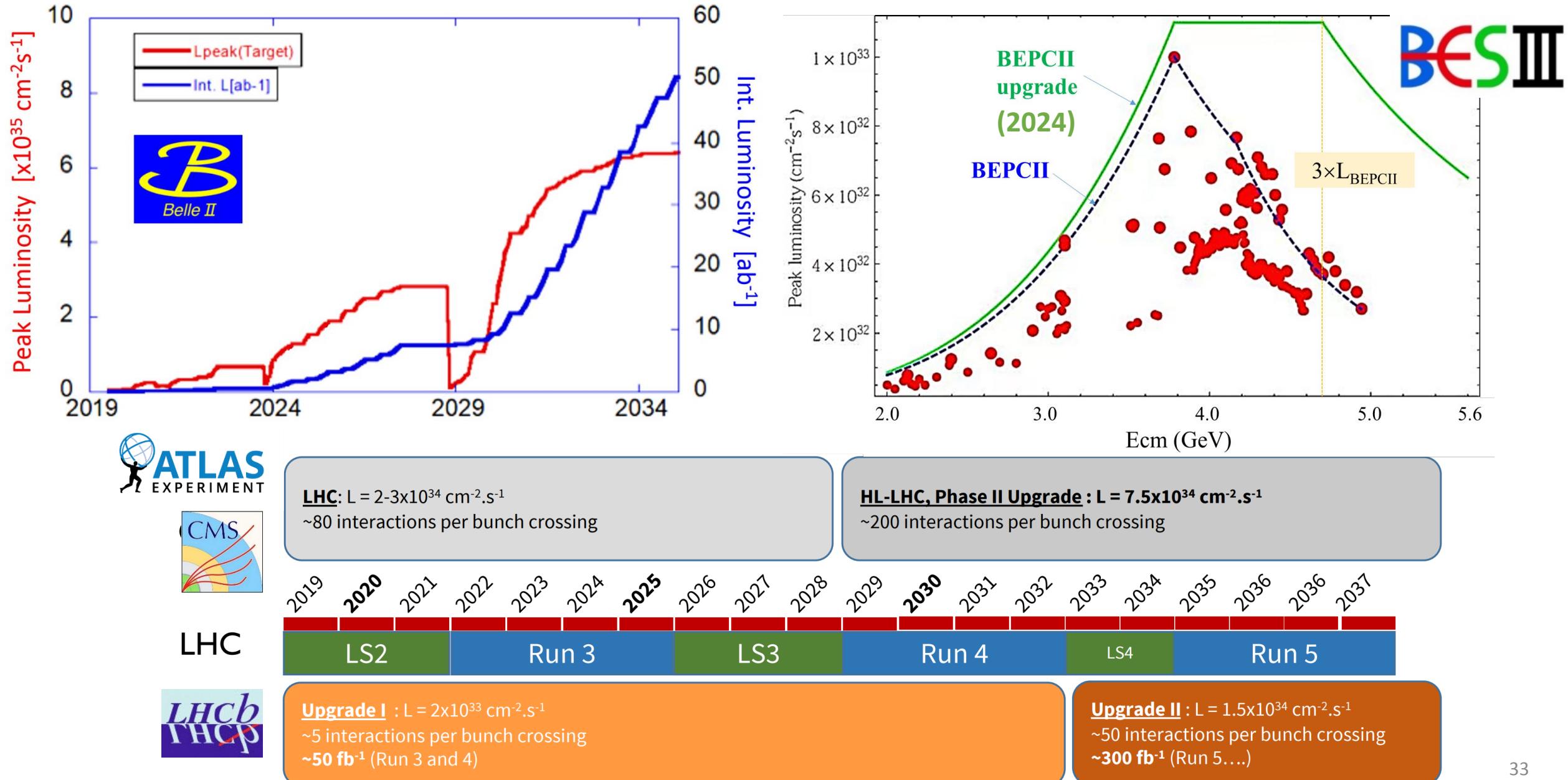


Tentative conclusions:



1. We did observe hadronic molecules close to the thresholds
2. There must be dynamics beyond molecule to explain many other states far from thresholds of narrow hadrons

More data are coming



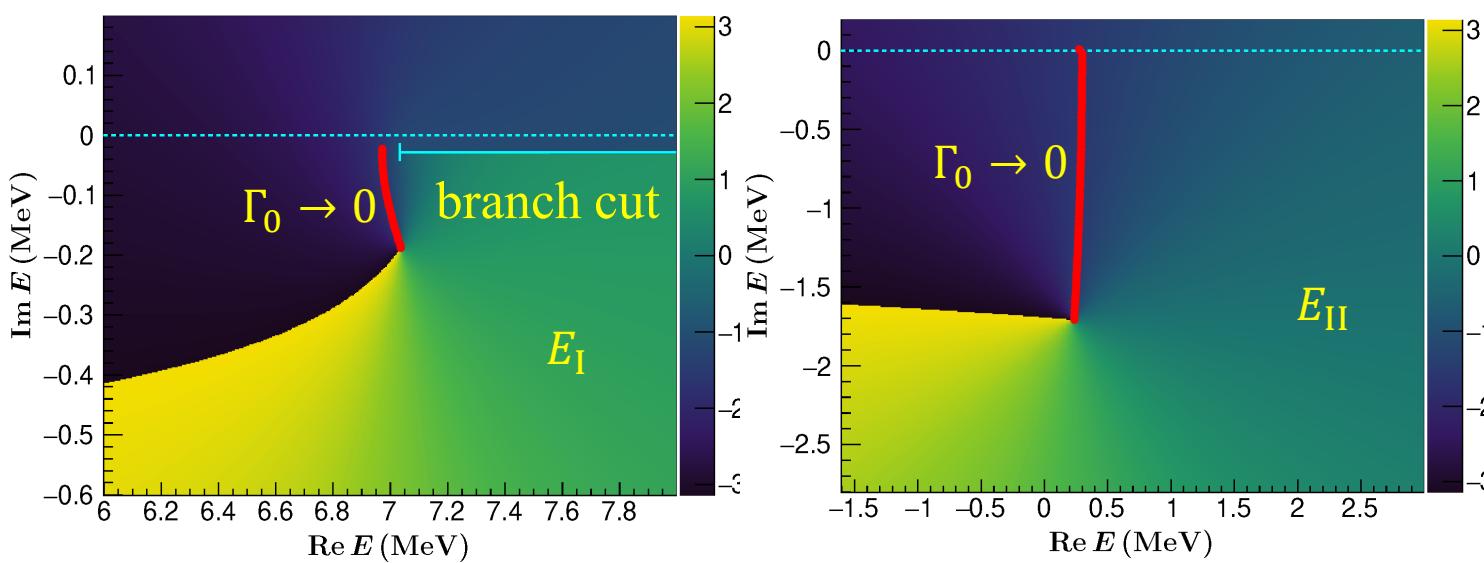
Summary

- Lots of progress in the experimental study of hadron spectroscopy.
- Spectroscopy of hadronic molecules to be further investigated.
- States formed by other dynamics may have been discovered.
- More results to come (Belle II, BESIII, LHCb, ...), and lots of opportunities and challenges ahead.
- Theoretical efforts needed to understand the hadron spectroscopy and the strong interaction.

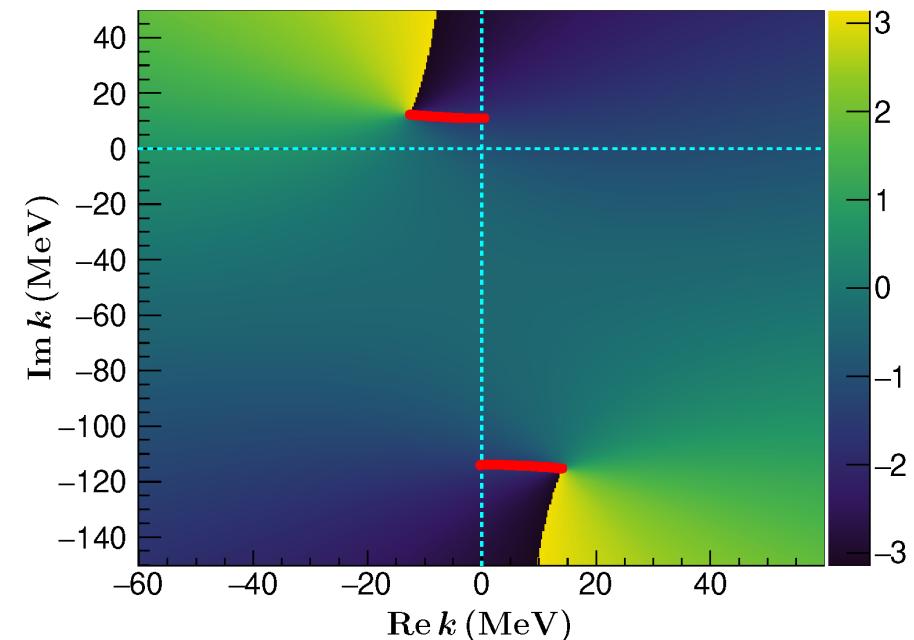
Thank you very much!

Backup slides

- Two sheets with respect to $D^{*0}\bar{D}^0$ branch cut
 - Sheet I: $E - E_X - g\sqrt{-2\mu(E - E_R + i\Gamma/2)}$
 - Sheet II: $E - E_X + g\sqrt{-2\mu(E - E_R + i\Gamma/2)}$
- $E_I = (7.04 \pm 0.15^{+0.07}_{-0.08}) + (-0.19 \pm 0.08^{+0.14}_{-0.19})i$ MeV
- $E_{II} = (0.26 \pm 5.74^{+5.14}_{-38.32}) + (-1.71 \pm 0.90^{+0.60}_{-1.96})i$ MeV



- Near threshold, scattering amplitude can be expanded as the power series of the momentum $k = \sqrt{2\mu(E - E_R)}$
- S-Wave $f^{-1}(E) \sim \frac{1}{a} + \frac{r_e}{2} k^2 - ik + \mathcal{O}(k^4)$
- In the limit of $\Gamma_0 \rightarrow 0$ and stable D^*
 - scattering length $a = (-16.5^{+7.0}_{-27.6}{}^{+5.6}_{-27.7})$ fm
 - effective range: $r_e = (-4.1^{+0.9}_{-3.3}{}^{+2.8}_{-4.4})$ fm



The effective range expansion

[S. Weinberg, Phys. Rev. 137, B672 (1965)]

$$a = -\frac{2(1-Z)}{(2-Z)} \frac{1}{\gamma} + \mathcal{O}(\beta^{-1})$$

$$r_e = -\frac{Z}{1-Z} \frac{1}{\gamma} + \mathcal{O}(\beta^{-1})$$

Z: field renormalization constant

- $Z = 0$: pure bound (composite) state
- $Z = 1$: pure elementary state

$\beta^{-1} \approx \frac{1}{m_\pi} \approx 1.4$ fm, for both deuteron and the $X(3872)$

$$\gamma = \sqrt{2\mu E_b}$$

Parameters	$X(3872)$	deuteron
Nearby threshold	$D^{*0}\bar{D}^0$	pn
a	$-16.5^{+7.0}_{-27.6} {}^{+5.6}_{-27.7}$ fm	-5.41 fm
r_e	$-4.1^{+0.9}_{-3.3} {}^{+2.8}_{-4.4}$ fm	1.75 fm
Range correction	negligible	important for r_e
Z	≈ 0.18	-

- Different sign, may suggest an elementary $c\bar{c}$ core
[A. Esposito PRD 105, L031503]
- Close to 0 but can not be solved model-independently due to the range correction

Effective Range Expansion → scattering length a and effective range r_e