中国科学院高能物理研究所

Institute of High Energy Physics, CAS

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CHINESE ACADEMY OF SCIENCES

Lattice QCD studies of exotic hadrons

Ying Chen

Institute of High Energy Physics,
Chinese Academy of Sciences, China

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Outline

I. Introduction
II. Glueballs and hybrids from lattice QCD
III. $T_{cc}^{+}(3875)$, X(3872) and $Z_c(3900)$ on the lattice
IV. Summary and perspectives

I. Introduction

1. Lattice QCD formalism

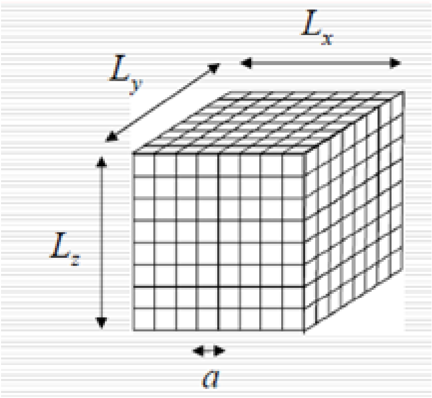
- **Path integral quantization on finite Euclidean spacetime lattices**

$$Z = \int DAD\psi D\bar{\psi}\, e^{iS[A,\psi,\bar{\psi}]} \rightarrow \int DU \det M[U]\; e^{-S_g[U]}$$

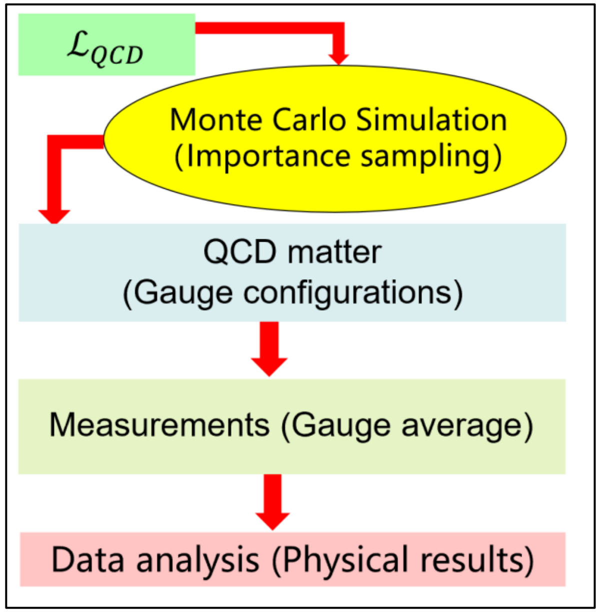
$$\langle\, \hat{\mathcal{O}}[U,\psi,\bar{\psi}]\rangle = \frac{1}{Z}\int DU\;\; \det M[U]\; e^{-S_g[U]}\;\; \mathcal{O}[U]$$

Green's functions

Field product



Spacetime discretization



- **Very similar to a statistical physics system**
- **Monte Carlo simulation——importance sampling according to** $\mathcal{P}[U] \propto \det M[U]\; e^{-S_g[U]}$

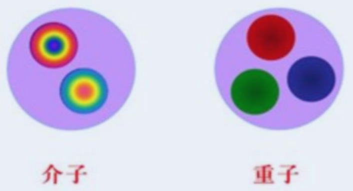
Gauge ensemble: $\{U_i(\text{spacetime}), i = 1, \dots, N\}$ ➡ $\langle\, \hat{\mathcal{O}}[U,\psi,\bar{\psi}]\rangle = \frac{1}{N}\sum_i \mathcal{O}[U_i] + O\left(\frac{1}{\sqrt{N}}\right)$

2. Exotic hadrons states

- ✓ Constituent quark model sorts hadrons into $q\bar{q}$ mesons and qqq baryons.
- ✓ Both quarks and gluons are fundamental degrees of freedom of QCD.
- ✓ Gluons can be also building blocks for hadrons:
  glueballs ($gg\cdots$), hybrids ($\bar{q}qg, qqqg, \cdots$)
- ✓ Multiquark states:
  Tetraquarks ($\bar{q}q\bar{q}q$) and Pentaquarks ($qqqq\bar{q}$)
  (hadron molecules or compact objects)
- ✓ Experimental candidates for multiquark states
  $X(3872)$, $Z_c(3900)$, $T_{cc}^{+}(3875)$, P_c states

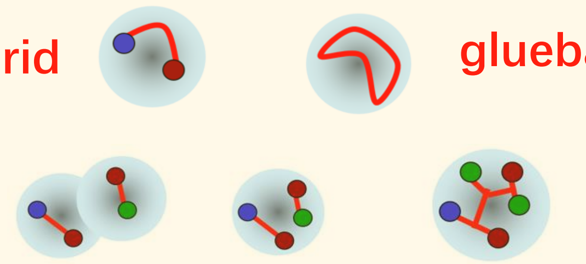
Quark Model

Conventional hadrons:



Exotic hadrons:

hybrid glueball

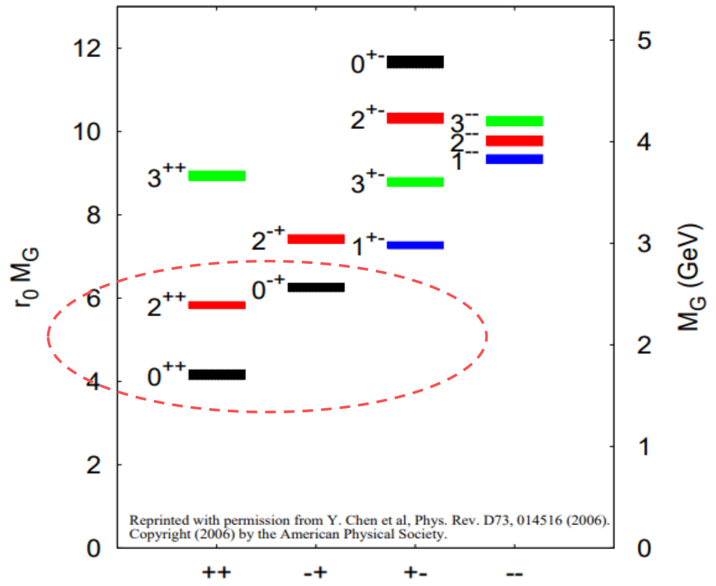


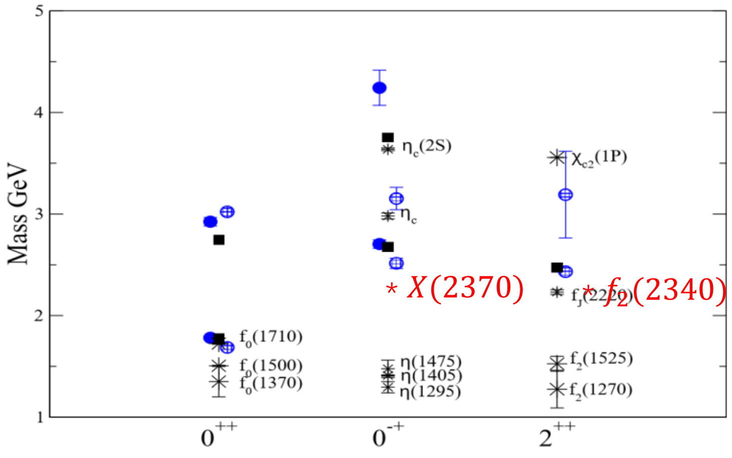
Molecule Tetraquark Pentaquark

- Quite a lot of phenomenological studies on multiquark states (see B.S. Zou, F.K. Guo and Q. Zhao's talks).
- I will give a brief overview of glueballs and hybrids in this talk.
- I will focus on the lattice studies on $X(3872)$, $Z_c(3900)$, $T_{cc}^{+}(3875)$.

II. Glueballs and hybrids from lattice QCD

1. Glueball masses





"50 Years of Quantum Chromodynamics",
F. Gross et al., Eur. Phys. J. C 83 (2023) 1125

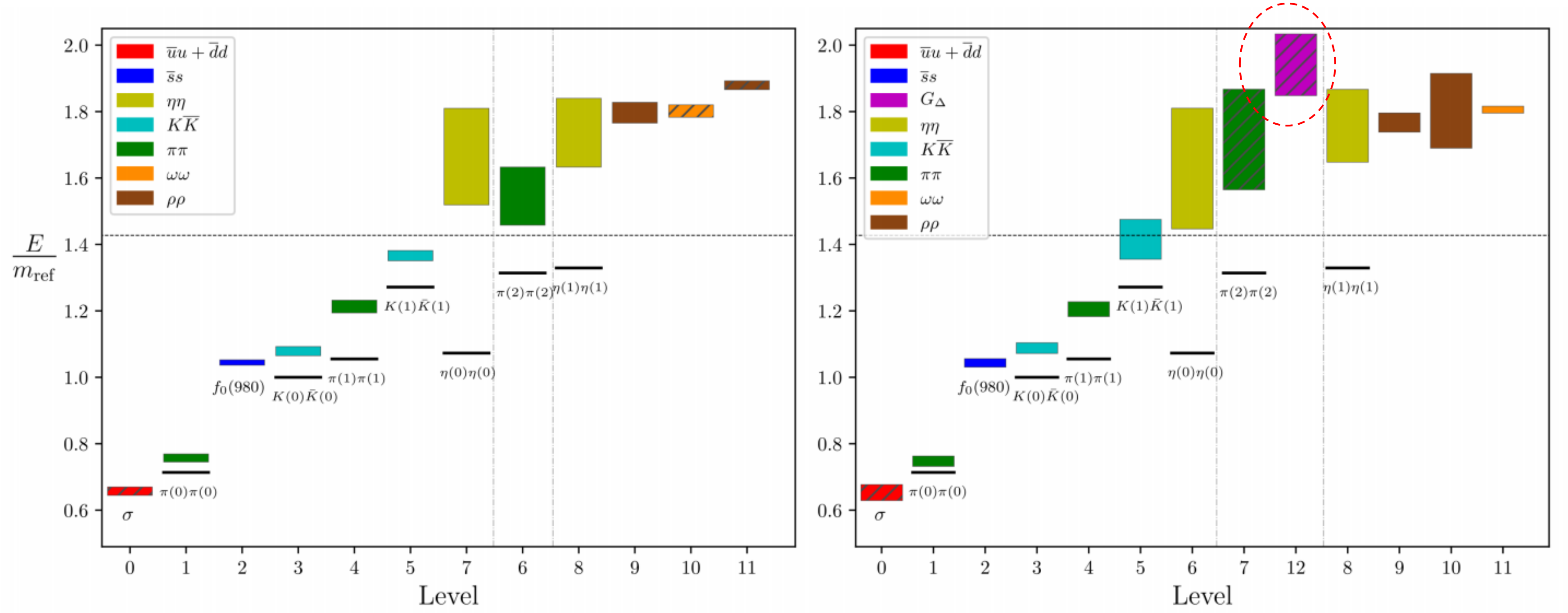
Glueball	Ref. [2475]	Ref. [2477]	Ref. [2478]	Ref. [2479]	Ref. [2480]	Ref. [2481]	Ref. [1104]
$\|0^{++}\rangle$	$1710 \pm 50 \pm 80$	1653 ± 26	1795 ± 60	1980	1780^{+140}_{-170}	1850 ± 130	1920
$\|2^{++}\rangle$	$2390 \pm 30 \pm 120$	2376 ± 32	2620 ± 50	2420	1860^{+140}_{-170}	2610 ± 180	2371
$\|0^{-+}\rangle$	$2560 \pm 40 \pm 120$	2561 ± 40	–	2220	2170 ± 110	2580 ± 180	

[2475] Y. Chen et al, Phys. Rev. D 73 (20060 014516, 2006
[2477] A. Athenodorou and M. Teper, JHEP 11 (2020) 172
[2478] E. Gregory et al., JHEP 10 (2012) 170
[2479] A.P. Szczepanniak and E.S. Swanson, Phys. Lett. B 577 (2003) 61
[2480] H.-X. Chen et al., Phys. Rev. D 104 (2021) 094050
[2481] M.Q. Huber et al., Eur. Phys. J. C 81 (2021) 1083
[1104] M. Rinaldi et al., Phys. Rev. D 104 (2021) 034016

Filled Squares: QQCD (Morningstar, PRD 60 (1999) 034509)
Open circles: full QCD, coarse lattice
Closed circles: full QCD, fine lattice
No meson or two-meson operators have been involved yet!
C.M. Richards et al., [UKQCD Collab.], Phys. Rev. D82, 034501 (2010).

Spectroscopy from lattice QCD: Scalar glueball

R. Brett et al. (HSC) AIP Conf. Proc. 2249 (2020) 030032 (arXiv: 1909.07306(hep-lat)



- **Quite a lot of operators: $\bar{q}q$, meson-meson, and glueball**
- **Black lines: two-meson thresholds**
- **Colored boxes: lattice energy levels (color corresponds to operator)**
- **Most states close to two-meson thresholds**
- **An additional state (around 1.9 GeV) observed when glueball operators are involved**

2. Glueballs in J/ψ radiative decays

- Glueballs are expected to be copiously produced in the gluon abundant J/ψ radiative decays.
- Theoretical prediction of their production rates are crucial for experiments to identify glueballs.
- There has been several lattice QCD studies on this topic in the quenched approximation.

- **Scalar glueball**

$\Gamma(J/\psi \to \gamma G_{0^+}) = 0.35(8)\ \text{keV}, \quad \Gamma/\Gamma_{tot} = 3.8(9) \times 10^{-3}$ — Gui, et al. (CLQCD), PRL110 (2013) 021601

$\Gamma(J/\psi \to \gamma G_{0^+}) = 0.45(4)\ \text{keV}, \quad \Gamma/\Gamma_{tot} = 4.8(5) \times 10^{-3}$ — Zou, et al., arXiv:2404.01564[hep-lat]

PDG: $\text{Br}(J/\psi \to \gamma f_0(1710) > 1.9 \times 10^{-3}$ **$f_0(1500)$ is produced 10 times less!**

- **Tensor glueball**

$\Gamma(J/\psi \to \gamma G_{2^+}) = 1.0(2)\ \text{keV}, \quad \Gamma/\Gamma_{tot} = 1.1(2) \times 10^{-2}$ — Yang, et al. (CLQCD), PRL111 (2013) 091601

BESIII observed $f_2(2340)$ in $J/\psi \to \gamma + (\eta\eta, \eta'\eta', K_S K_S, \phi\phi)$

Ablikim et al. (BESIII), PRD87(2013) 092009, PRD105(2022)072002, PRD98(2018)072003, PRD93(2016)112011

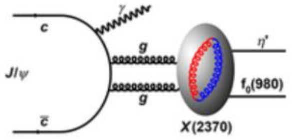
- **Pseudoscalar**

$m_{0^-} = 2395(14)\ \text{MeV}, \quad \text{Br}(J/\psi \to \gamma G_{0^-}) = 2.3(8) \times 10^{-4}$ — Gui, et al. (CLQCD), PRD100 (2019) 054511

✓ BESIII observed $X(2370)$ in $J/\psi \to \gamma + (\pi^+\pi^-\eta', K\bar{K}\eta')$

✓ The QNs of $X(2370)$ is determined to be $J^{PC} = 0^{-+}$

Ablikim et al. (BESIII), PRL106(2011)072002, EPJC80(2020)746 PRL130(2024)181901



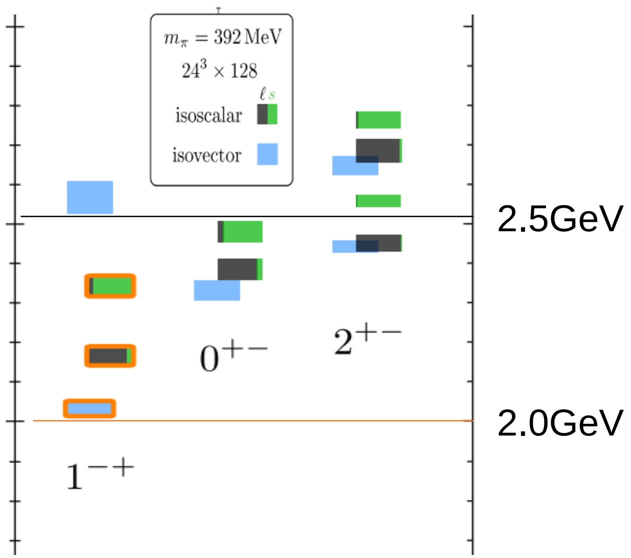
- **Connection with experiments**

"50 Years of Quantum Chromodynamics", Chap. 8.4: Glueballs, a fulfilled promise of QCD? (E. Klempt)
F. Gross et al., Eur. Phys. J. C 83 (2023) 1125

A.V. Sarantsev et al., PLB 816 (2021) 136227, E. Klempt et al., PLB 826 (2022)136906

3. Hybrids from lattice QCD

- Lattice QCD studies predict the light $(1^-)1^{-+}$ hybrid π_1 to have a mass in the range 1.7-2.1 GeV, and 1^{-+} charmonium-like hybrid η_{c1} to have a mass around 4.1-4.4 GeV.
- Flavor mixing angle of the two $(0^+)1^{-+}$ hybrid from $N_f = 2+1$ LQCD.
- π_1 decay from $N_f = 3$ LQCD.
- Branching fraction $J/\psi \to \gamma\eta_1$ for $N_f = 2$ (**F. Chen et al., PRD 107 (2023) 054511**)
- η_{c1} two-body decays (C. Shi et al., arXiv: 2306.12884[hep-lat] (PRD in press))

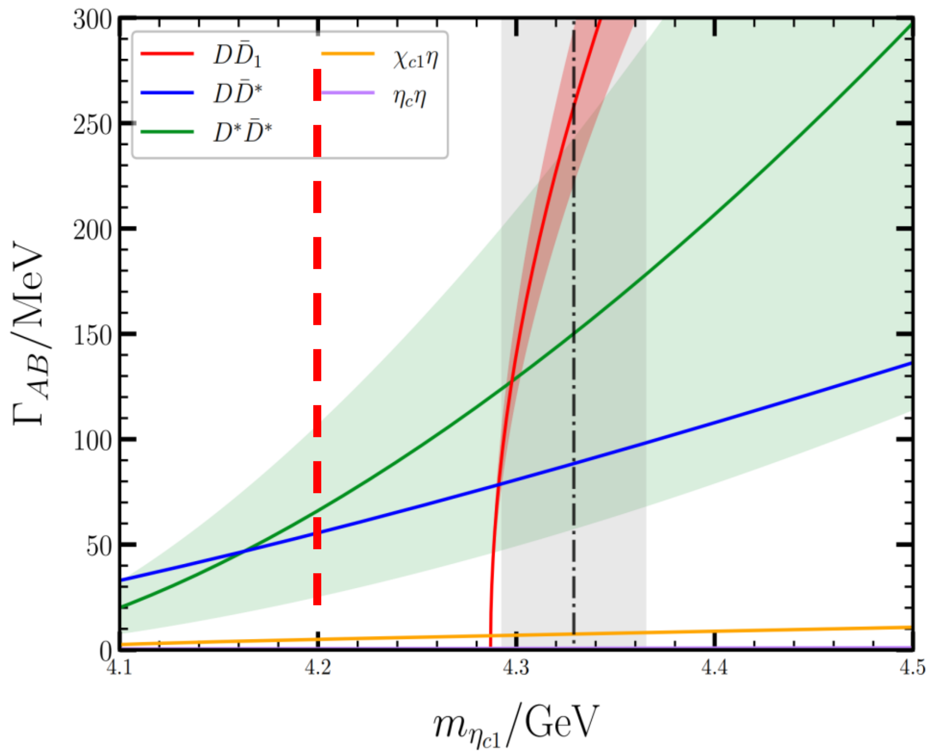


J. Dudek et al. (HSC), PRD 88(2013) 094505

	thr./MeV	$\lvert c_i^{\text{phys}}\rvert$/MeV	Γ_i/MeV
$\eta\pi$	688	$0 \to 43$	$0 \to 1$
$\rho\pi$	910	$0 \to 203$	$0 \to 20$
$\eta'\pi$	1098	$0 \to 173$	$0 \to 12$
$b_1\pi$	1375	$799 \to 1559$	$139 \to 529$
$K^*\bar{K}$	1386	$0 \to 87$	$0 \to 2$
$f_1(1285)\pi$	1425	$0 \to 363$	$0 \to 24$
$\rho\omega\{^1P_1\}$	1552	$\lesssim 19$	$\lesssim 0.03$
$\rho\omega\{^3P_1\}$	1552	$\lesssim 32$	$\lesssim 0.09$
$\rho\omega\{^5P_1\}$	1552	$\lesssim 19$	$\lesssim 0.03$
$f_1(1420)\pi$	1560	$0 \to 245$	$0 \to 2$
			$\Gamma = \sum_i \Gamma_i = 139 \to 590$

A.J. Woss (HSC), PRD 103 (2021) 054502

C. Shi et al., arXiv: 2306.12884[hep-lat]



The $m_{\eta_{c1}}$-dependence of partial decay widths

$$|D^*\bar{D}^*\rangle^{(I=0)}_{(C=+)} = \frac{1}{\sqrt{2}}\left(|D^{*+}D^{*-}\rangle + |D^{0*}\bar{D}^{0*}\rangle\right)^{(S=1)}_{(L=1)}$$

$L + S = \text{even}$

- For $m_{\eta_{c1}} = 4329(36)$ MeV, we have

$$\Gamma_{D_1\bar{D}} = 258(133)\ \mathrm{MeV}$$
$$\Gamma_{D^*\bar{D}^*} = 150(118)\ \mathrm{MeV}$$
$$\Gamma_{D^*\bar{D}^*} = 88(18)\ \mathrm{MeV}$$

$$\Gamma_{\chi_{c1}\eta} = \sin^2\theta \cdot 44(29)\ \mathrm{MeV}$$
$$\Gamma_{\eta_c\eta'} = \cos^2\theta \cdot 0.93(77)\ \mathrm{MeV}$$

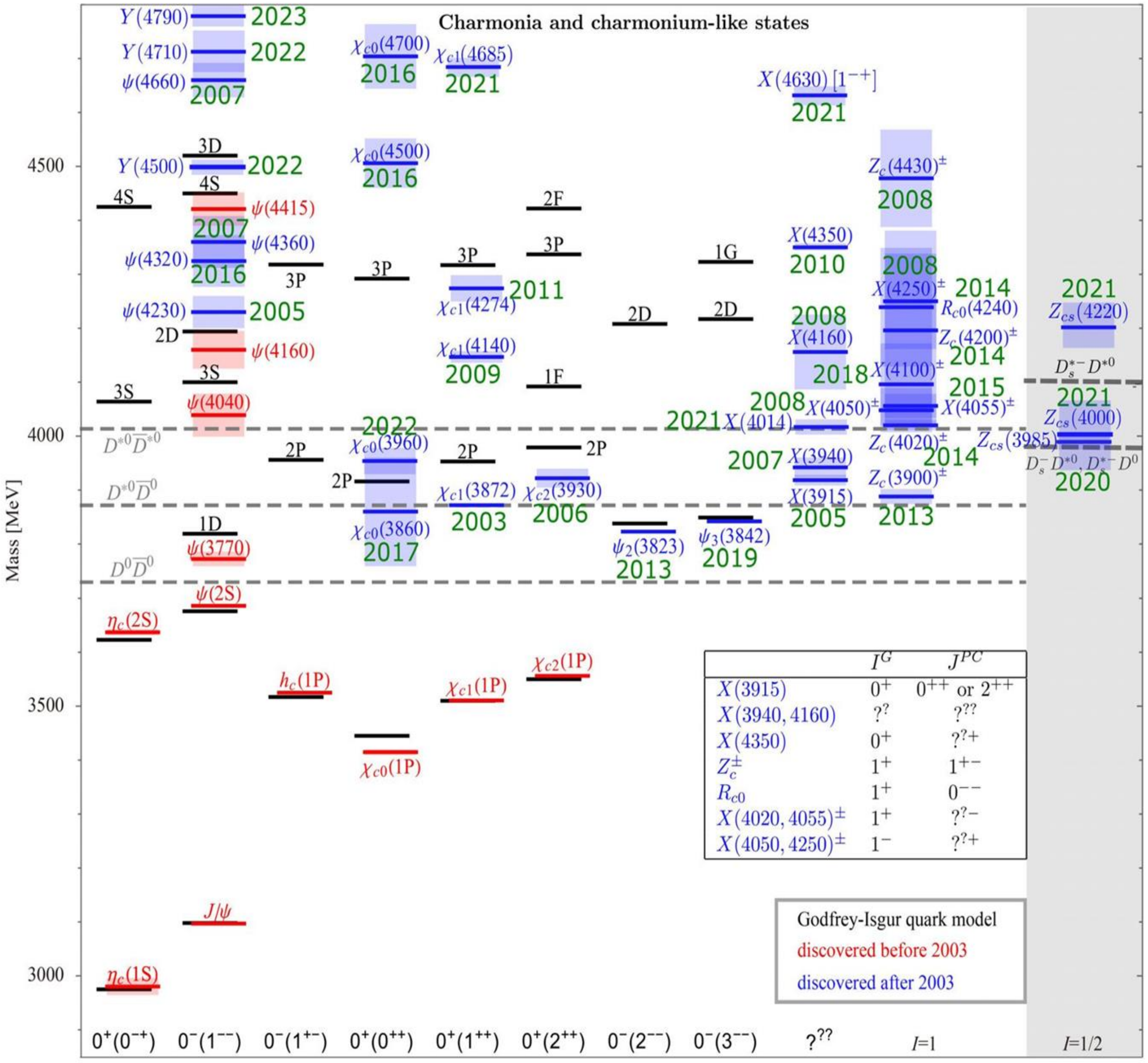
- Given the mass above, η_{c1} seems too wide to be identified easily in experiments.

- However, $\Gamma_{\eta_{c1}}$ is very sensitive to $m_{\eta_{c1}}$.
- If $m_{\eta_{c1}} \sim 4.2$ GeV, then $\Gamma_{\eta_{c1}} \sim 100$ MeV. The dominant decay channels are $D^*\bar{D}$ and $D^*\bar{D}^*$.
- Especially for $D^*\bar{D}^*$, the measurement of the polarization of D^* and $\bar{D}^*$ will help distinguish a 1^{-+} states from 1^{--} states.

- It is suggested that LHCb, BelleII and BESIII to search for η_{c1} in $D^*\bar{D}$ and $D^*\bar{D}^*$ systems !

III. $T_{cc}^{+}(3875)$, X(3872) and $Z_c(3900)$ on the lattice

- Ever since X(3872) found in 2003 by Belle, quite a lot of XYZ states observed.
- Unexpected by the naïve quark model and reside near open-charm or open-bottom thresholds.
- Several baryonic counterparts, such as P_c states observed by LHCb.
- Doubly charmed $T_{cc}^{+}(3875)$ observed by LHCb in 2021.
- These states spur intensive and extensive phenomenological studies. “Tetraquark” or “hadronic molecules”.
- Lattice QCD investigation through hadron-hadron scatterings.

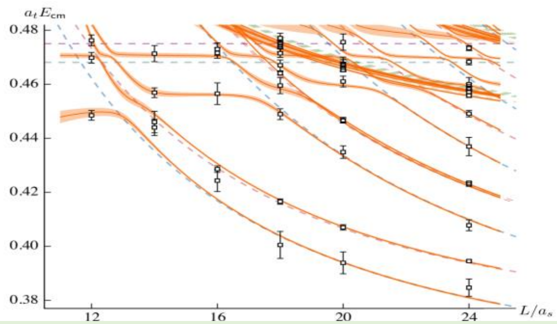


1. The methodology hadron-hadron scattering in lattice QCD

State-of-art Approach——Lüscher's formalism

(see R. Briceno et al., Rev. Mod. Phys. 90 (2018) 025001 for a review).

$$\det\left[F^{-1}\left(\vec{P},E,L\right)+\mathcal{M}(E)\right]=0$$



$E_n(L)$: Eigen-energies of lattice Hamiltonian.

- **Interpolation field operator set** for a given J^{PC}

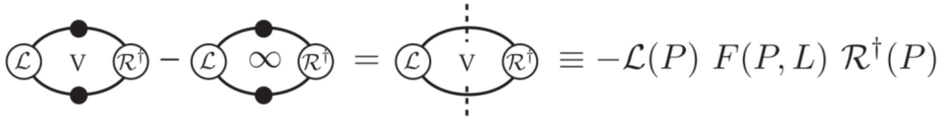
  $\mathcal{O}_i$: $\bar{q}_1\Gamma q_2$ $[\bar{q}_1\Gamma_1 q][\bar{q}\Gamma_2 q_2]$ $[q_1^T\Gamma_1 q][\bar{q}\Gamma_2\bar{q}_2^T]$, ...
- **Correlation function matrix** —— Observables

$$C_{ij}(t)\& =\left\langle\Omega\middle|\mathcal{O}_i(t)\mathcal{O}_j^\dagger(0)\middle|\Omega\right\rangle$$

$$=\sum_n\langle\Omega|\mathcal{O}_i|n\rangle\left\langle n\middle|\mathcal{O}_j^\dagger\middle|\Omega\right\rangle e^{-E_n t}$$

All the energy levels $E_n(L)$ are discretized.

$F\left(\vec{P},E,L\right)$: Mathematically known function matrix in the channel space (the explicit expression omitted

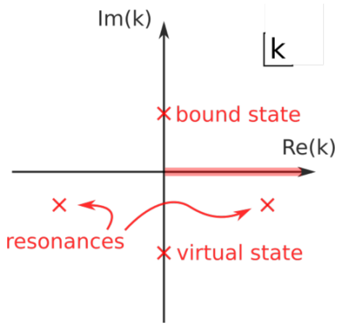


$\mathcal{M}(E)$: Scattering matrix.

- Unitarity requires

$$\mathcal{M}_{ab}^{-1}=(\mathcal{K}^{-1})_{ab}-i\delta_{ab}\frac{2q_a^*}{E_{cm}}$$

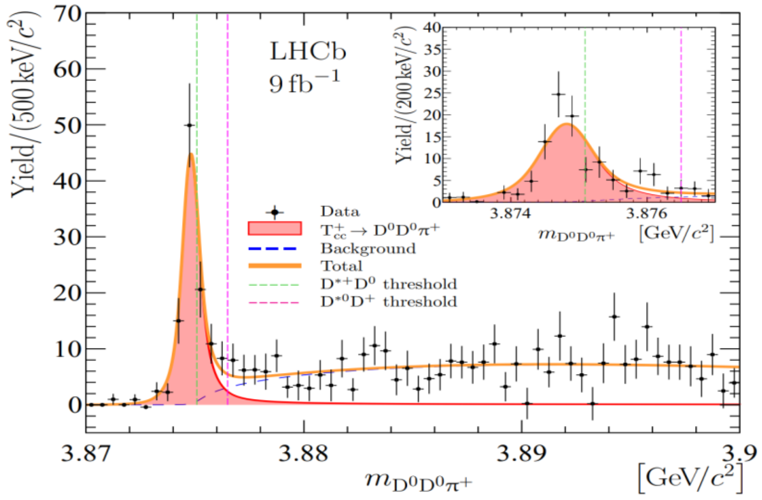
- $\mathcal{K}$ is a real function of s for real energies above kinematic threshold.
- The pole singularities of $\mathcal{M}(s)$ in the complex s-plane correspond to bound states, virtual states, resonances, etc..



$$k=\pm\frac{1}{2}\sqrt{s-4m^2}$$

2. Lattice studies of $T_{cc}^{+}(3875)$

LHCb discovered $T_{cc}^{+}(3875)$ in 2021 (LHCb, Nature Phys.18, 751 (2022), Nature Comm.13, 3551 (2022))



$$M_{T_{cc}} - (m_{D^0} + m_{D^{*+}}) = -273 \pm 61 \pm 5^{+11}_{-14}\ \mathrm{keV}$$

$$\Gamma_{BW} = 410 \pm 165 \pm 43^{+18}_{-38}\ \mathrm{keV}$$

$$\Gamma_{BW}^{U} = 48 \pm 2^{0}_{-14}\ keV$$

Isospin：Only observed in DD^{*+}, therefore $I = 0$

The minimum quark configuration：$cc\bar{u}\bar{d}$

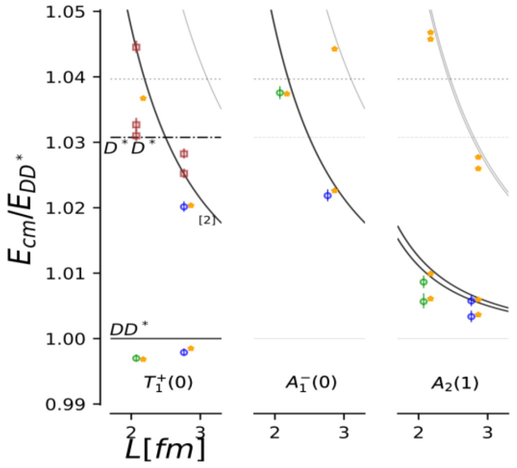
- Spured extensive and intensive phemonenological investigations
- Likely a DD^* hadronic molecule
- A series of lattice studies——make the things clearer!

Pole singularity: M. Padmanath and S. Prelovsek, Phys. Rev. Lett. 129 (2022) 032002
Dynamics underlying: S. Chen et al., Phys. Lett. B 833, 137391 (2022)
Interaction potential: Y. Lyu et al., Phys. Rev. Lett. 131 (2023)
Quark mass dependence: S. Collins et al., arXiv:2404.06399(hep-lat)

A. Pole singularity of $DD^*(I=0)$ scattering amplitude from lattice QCD

M. Padmanath and S. Prelovsek, Phys. Rev. Lett. 129 (2022) 032002

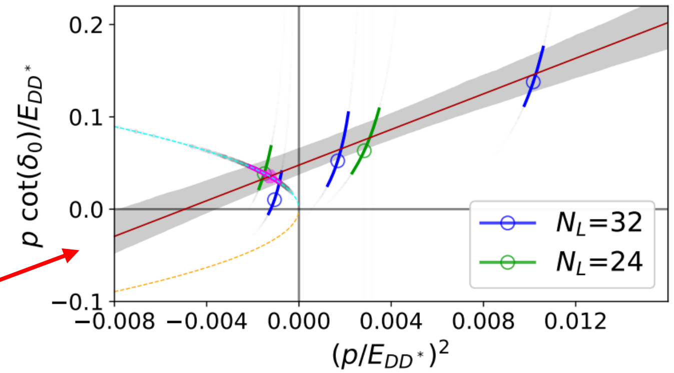
	m_D [MeV]	m_{D^*} [MeV]	M_{av} [MeV]	$a_{l=0}^{(J=1)}$ [fm]	$r_{l=0}^{(J=1)}$ [fm]	$\delta m_{T_{cc}}$ [MeV]	T_{cc}
lat. ($m_\pi \simeq 280$ MeV, $m_c^{(h)}$)	1927(1)	2049(2)	3103(3)	1.04(29)	$0.96(^{+0.18}_{-0.20})$	$-9.9^{+3.6}_{-7.2}$	virtual bound st.
lat. ($m_\pi \simeq 280$ MeV, $m_c^{(l)}$)	1762(1)	1898(2)	2820(3)	0.86(0.22)	$0.92(^{+0.17}_{-0.19})$	$-15.0(^{+4.6}_{-9.3})$	virtual bound st.
exp. [2, 38]	1864.85(5)	2010.26(5)	3068.6(1)	-7.15(51)	[-11.9(16.9),0]	$-0.36(4)$	bound st.



$$e^{2i\delta_l} = 1 + i\,2\rho t_l, \qquad \rho = \frac{2p}{\sqrt{s}}, \qquad \sqrt{s} = E_{cm} = \sqrt{m_D^2 + p^2} + \sqrt{m_{D^*}^2 + p^2}$$

S-wave scattering amplitude: $$t_0 = \frac{\sqrt{s}}{2}\frac{1}{p\cot\delta_0 - ip}$$

Effective range expansion (ERE): $$p\cot\delta_0 = \frac{1}{a_0} + \frac{1}{2}r_0 p^2$$



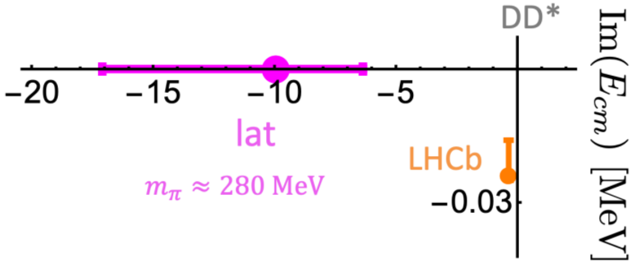
$$p = \pm i|p|$$
$$ip = \mp\sqrt{|p^2|},$$

Pole condition:
$$p\cot\delta_0 = ip$$

Lüscher's relation:

$$p\cot\delta_0(q^2) = \frac{2}{L\sqrt{\pi}}\mathcal{Z}_{00}(1,q^2) = \frac{1}{\pi L}\sum_{\vec{n}\in Z_3}\frac{1}{\vec{n}^2 - q^2}, \qquad q = \frac{Lp}{2\pi}$$

$$\delta m_{T_{cc}} = \mathrm{Re}(E_{cm}) - m_{D^0} - m_{D^{*+}} \text{ [MeV]}$$



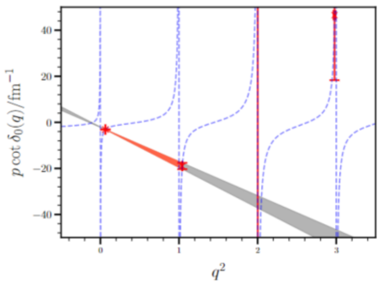
B. Investigation of the isospin-dependent interaction of DD^* scattering

(S. Chen et al., Phys. Lett. B 833, 137391 (2022))

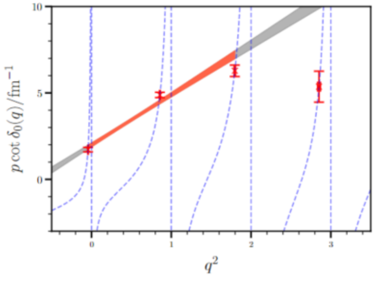
- DD^* energies and scattering momenta can be derived precisely
- Single-channel Lüscher's formula applied
- $I = 1\ DD^*$ is repulsive, $I = 0\ DD^*$ is repulsive (sign of a_0)
- Quark diagrams (after Wick's contraction) contributing to DD^* correlators

$$C^{(I)}_{DD^*}(t) = D + C_1 + (-)^{I+1}(C_2 + D')$$

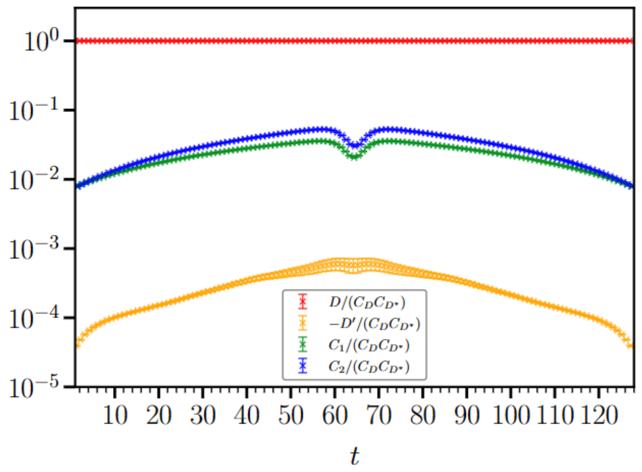
$$\boldsymbol{p \cot \delta_0(p) = \frac{1}{a_0} + \frac{1}{2} r_0 p^2 + \mathcal{O}(p^4)}$$



$I = 1$: repulsive



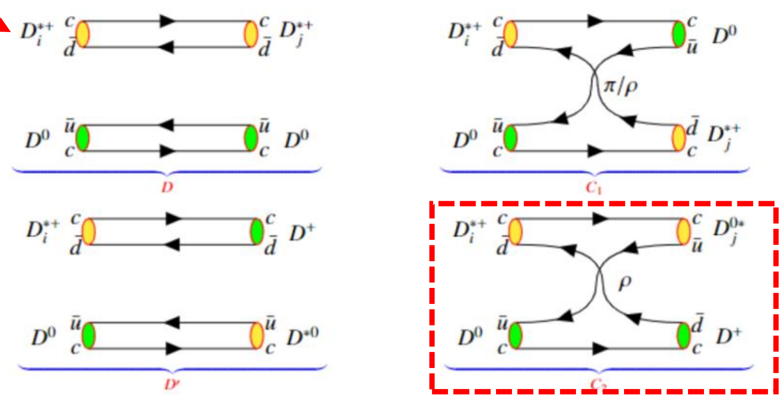
$I = 0$: attractive



$$\boldsymbol{\Delta E^{(I)}_{DD^*} \approx \epsilon_1 \delta E_1 + (-)^{I+1} \epsilon_2 \delta E_2}$$

$$\boldsymbol{(\epsilon_2 > \epsilon_1 > 0, \qquad \delta E_2 \geq \delta E_1)}$$

$$\begin{cases} \Delta E^{(I=0)}_{DD^*} < 0, \\ \Delta E^{(I=1)}_{DD^*} > 0, \end{cases}$$



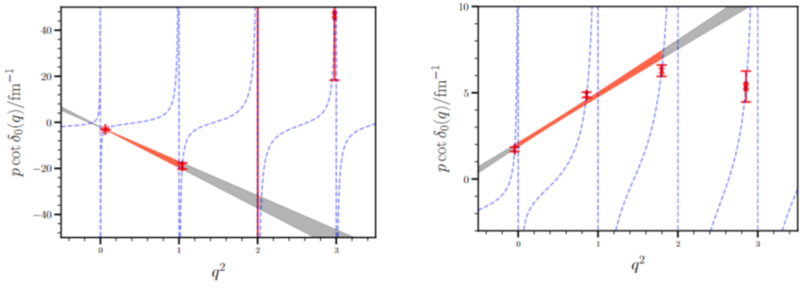
Schematic quark diagrams

- Initiatively interprets the underlying physics by analyzing the quark diagrams in lattice QCD calculations

B. Investigation of the isospin-dependent interaction of DD^* scattering

(S. Chen et al., Phys. Lett. B 833, 137391 (2022))

$$p \cot \delta_0(p) = \frac{1}{a_0} + \frac{1}{2} r_0 p^2 + \mathcal{O}(p^4)$$



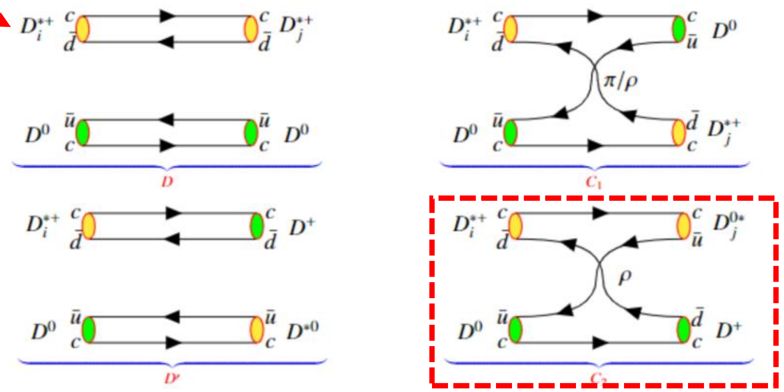
$I = 1$: repulsive $I = 0$: attractive

- DD^* energies and scattering momenta can be derived precisely
- Single-channel Lüscher's formula applied.
- $I = 1$ DD^* is repulsive, $I = 0$ DD^* is repulsive (sign of a_0)
- Quark diagrams (after Wick's contraction) contributing to DD^* correlators

$$C^{(I)}_{DD^*}(t) = D + C_1 + (-)^{I+1}(C_2 + D')$$

- ✓ D' term is negligible.
- ✓ C_2 term is responsible for the energy difference of $DD^*(I=1)$ and $DD^*(I=0)$.
- ✓ C_2 term can be understood as the exchange of charged vector ρ meson, which provides attractive (repulsive) interaction for $I = 0$ $(I = 1)$
- ✓ This is in qualitative agreement with phenomenological studies (Dong et al. CTP73 (2021) 125201, Feijoo et al, PRD104(2021)114015)

- Initiatively interprets the underlying physics by analyzing the quark diagrams in lattice QCD calculations



Schematic quark diagrams

C. Hadron-hadron interaction potential——HALQCD approach (Y. Lyu et al., Phys. Rev. Lett. 131 (2023))

Nambu-Bethe-Salpeter wave function

- (2+1)-flavor QCD on the 96^4 lattice with $m_\pi = 146.4$ MeV, L=8.1 fm
- Calculate the correlation functions

$$R(\vec{r},t) = e^{(m_{D^*}+m_D)t}\sum_{\vec{x}}\langle 0|D^*(\vec{x}+\vec{r},t)D(\vec{x},t)\bar{J}(0)|0\rangle = \sum_n A_n\psi_n(\vec{r})e^{-\Delta E_n t} + \cdots$$

- The function $R(\vec{r},t)$ satisfies the Shrödinger-type equation

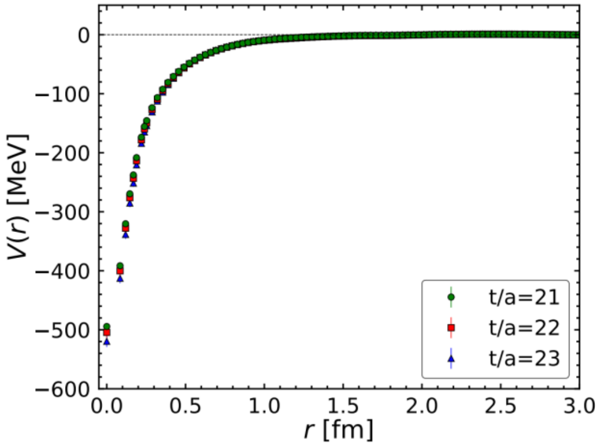
$$\left[\frac{1+3\delta^2}{8\mu}\partial_t^2 - \partial_t - H_0 + \cdots\right]R(\vec{r},t) = \int d\vec{r}'\,U(\vec{r},\vec{r}')R(\vec{r},t), \qquad H_0 = -\frac{\nabla^2}{2\mu}, \qquad \mu = \frac{m_{D^*}m_D}{m_{D^*}+m_D}, \qquad \delta = \frac{m_{D^*}-m_D}{m_{D^*}+m_D}$$

- Takes the leading term of derivative expansion of the non-local $U(\vec{r},\vec{r}')$

$$U(\vec{r},\vec{r}') \approx V(\vec{r})\delta(\vec{r}-\vec{r}'), \qquad V(r) = R^{-1}(\vec{r},t)\left[\frac{1+3\delta^2}{8\mu}\partial_t^2 - \partial_t - H_0 + \cdots\right]R(\vec{r},t)$$

- The DD^* potential in the $(I,J^P) = (0,1^+)$ channel is attractive.
- Short range: attractive diquark-antidiquark $(\bar{u}\bar{d} - cc)$
  Long range: two-pion exchange is favored:

$$V_{fit}^B(r;m_\pi) = \sum_{i=1,2} a_i e^{(-r/b_i)^2} + a_3\left(\frac{1}{r}e^{-m_\pi r}\right)^2 \cdots$$



- Different from phenomenological expectation that ρ-exchange dominates?

- Using the derived potential, the S-wave phase shifts δ_0 is obtained by solving the Schrödinger equation of DD^* system, which is put into the ERE

$$p\cot\delta_0(p) = \frac{1}{a_0} + \frac{1}{2}r_0 p^2 + \mathcal{O}(p^4)$$

- Extrapolate to the physical m_π,

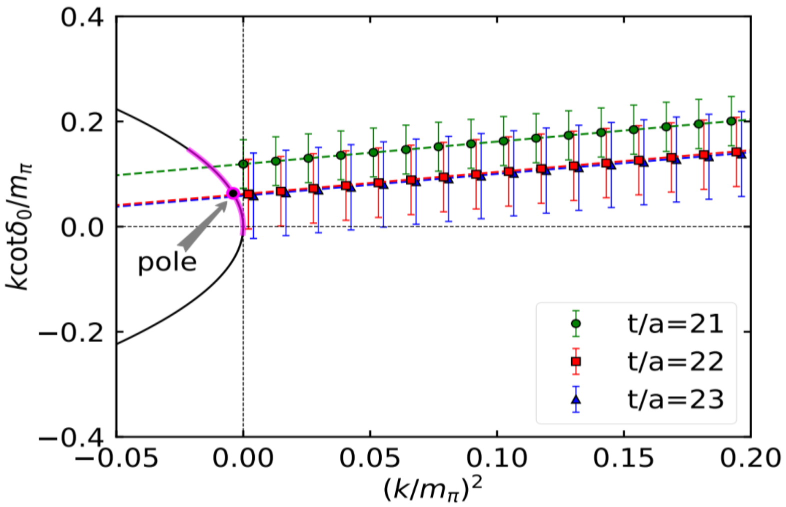
$$V^B_{fit}(r;m_\pi) \to V^B_{fit}\left(r;m_\pi^{\text{phys}}\right)$$

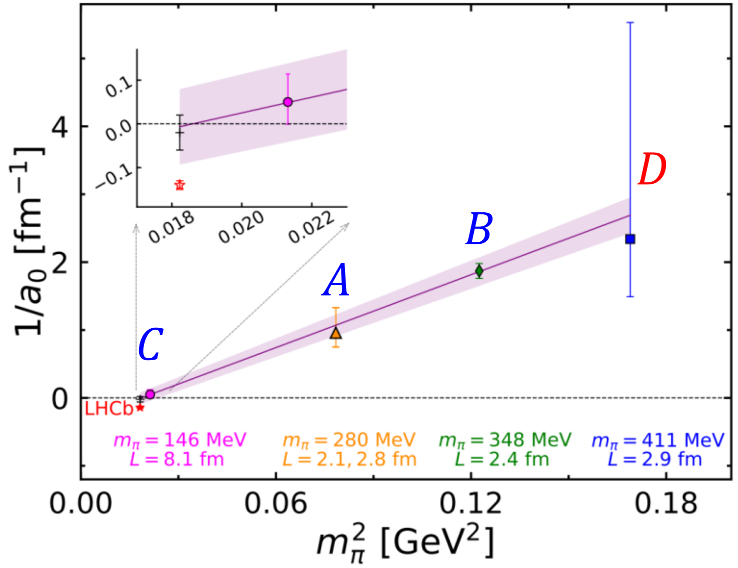
one gets

m_π [MeV]	146.4	135.0
$1/a_0$ [fm^{-1}]	$0.05(5)\,\binom{+4}{-1}$	$-0.02(4)$
r_{eff} [fm]	$1.14(6)\,\binom{+1}{-9}$	$1.14(8)$
κ_{pole} [MeV]	$-9(9)\,\binom{+1}{-8}$	$+3(8)$
E_{pole} [keV]	$-45(77)\,\binom{+02}{-99}$	$-10(37)$

consistent with the large negative scattering length a_0 of a bound state ($k = i\kappa_{\text{pole}}$).

- This result is consistent with the extrapolated a_0 using the existing lattice results.

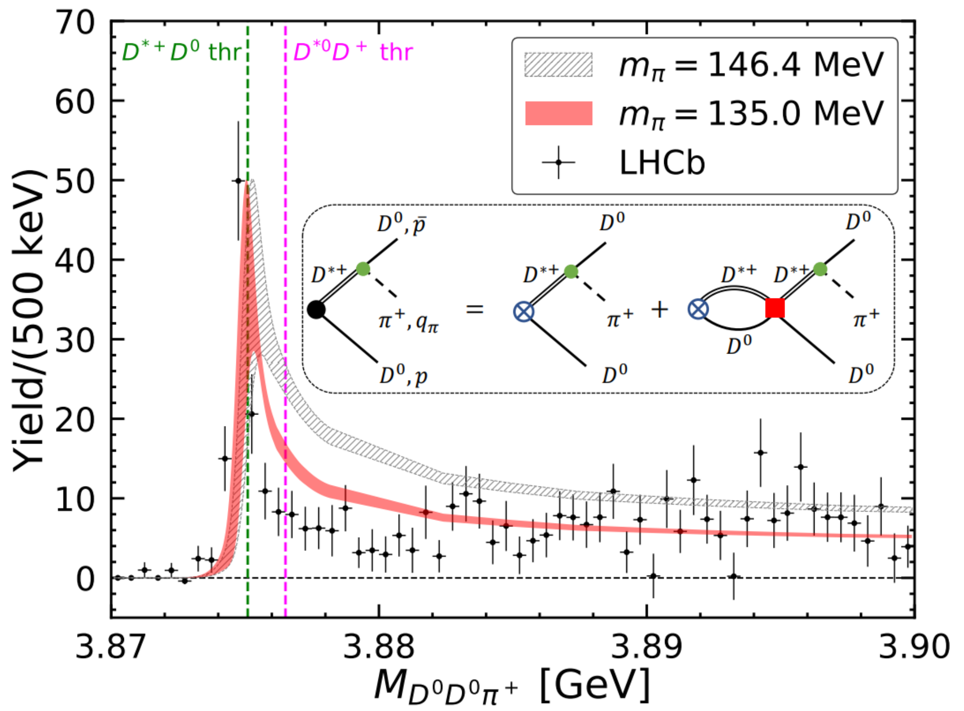




D: Y. Ikeda et al. (HALQCD) Phys. Lett. B 729 (2014) 84-90

- Fit to the $D^0D^0\pi^+$ mass spectrum of LHCb experimental data

- ✓ The gray band: the theoretical obtained by using $V^B_{fit}(r;m_\pi)$ at $m_\pi = 146.4$ MeV
- ✓ The red band: $D^0D^0\pi^+$ mass spectrum obtained by chiral extrapolated $V^B_{fit}(r;m_\pi)$ at $m_\pi = 135.0$ MeV
- ✓ Consistent with the trend of evolution from a near-threshold virtual state into a loosely bound state.



To summarize,

- ✓ The existing lattice results of $T^+_{cc}(3875)$ relevant studies are consistent with each other;
- ✓ These results support the existence of a DD^* bound state in the $I = 0$ channel.
- ✓ The interaction potential study (C) suggests that the two-pion exchange dominates the long range interaction, while study (B) supports the charged-ρ exchange that provides an attractive interaction for $I = 0$ DD^* system near the threshold, as expected by phenomenological studies.

D. One-pion-exchange left hand cut issue (M.-L. Du et al., PRL131(2023)131901)

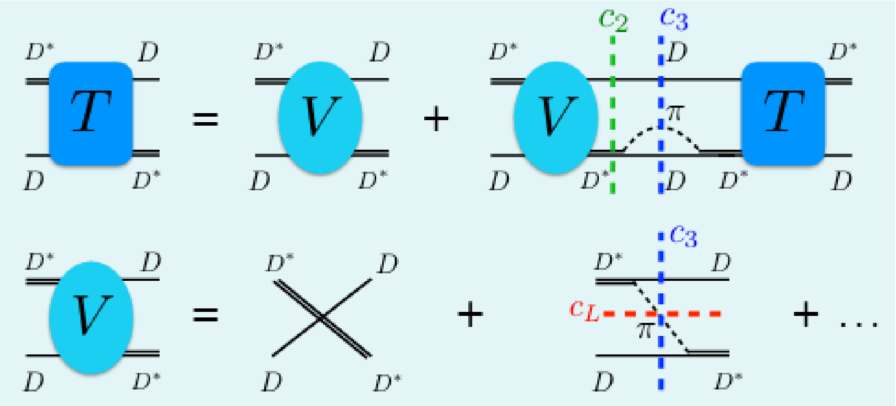
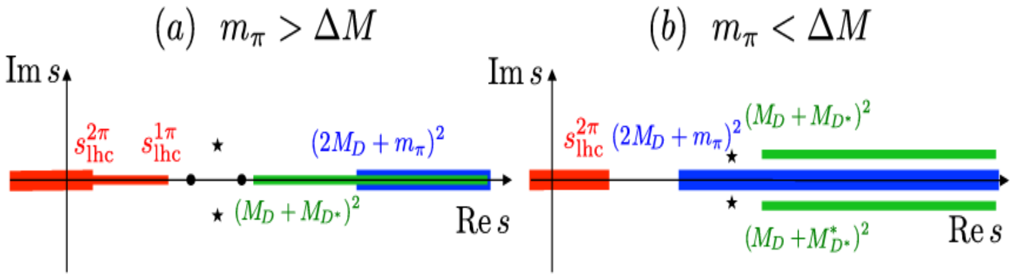


FIG. 1. The cut structure in the DD^* system: (i) the blue dotted vertical lines (c_3) indicate the three-body right-hand cuts, (ii) the green dotted vertical line (c_2) shows the two-body cut, and (iii) the red dotted horizontal line (c_L) is for the left-hand cut. T and V denote the amplitude and interaction potential, respectively.

$$G_\pi^{-1}(E,\boldsymbol{k}',\boldsymbol{k}) = \Delta M + \frac{p^2}{2\mu} - \frac{k^2+k'^2}{2M_D} - \omega_\pi(q^2)$$

One Pion Exchange (OPE)



New singularities emerge in the on-shell partial-wave amplitudes at imaginary values of $k^2 = k'^2 = p^2 < 0$:

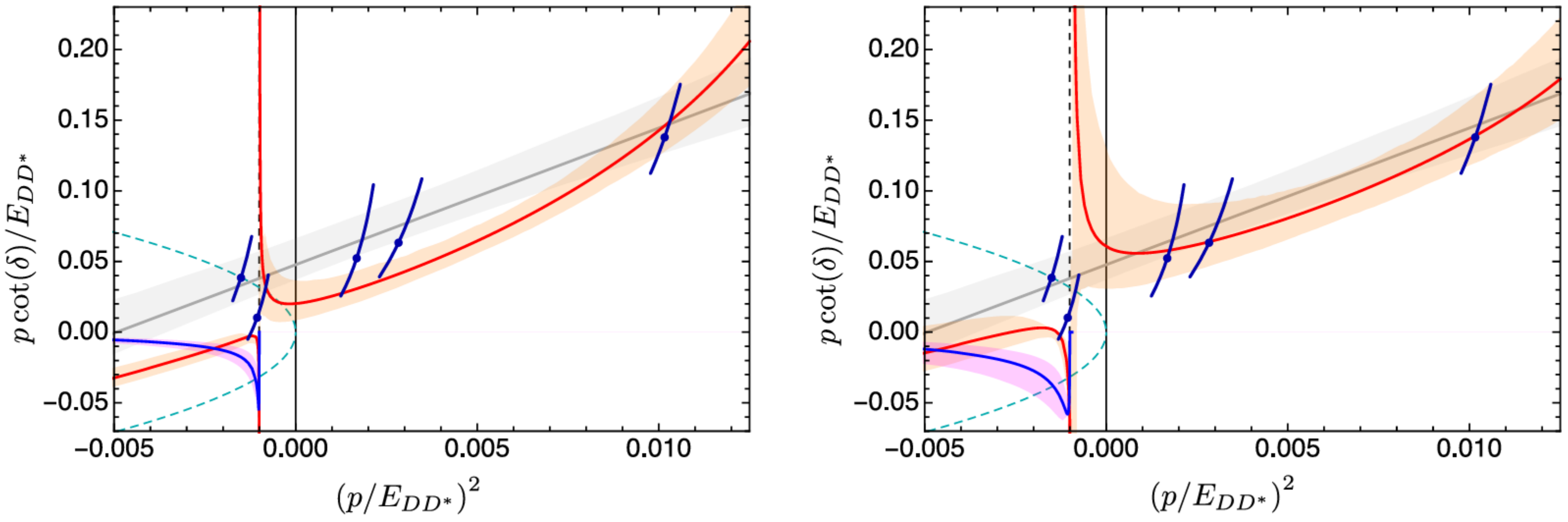
$$\left(p_{lhc}^{1\pi}\right)^2 \approx \frac{1}{4}\left[(\Delta M)^2 - m_\pi^2\right]$$

A lhc introduces nonanalyticity to $p\cot\delta$ and sets the upper bound on the convergence radius of ERE ($p\cot\delta$ acquires an imaginary part for $p^2 < \left(p_{lhc}^{1\pi}\right)^2$).

Stars: resonance poles.
Dots: virtual states poles

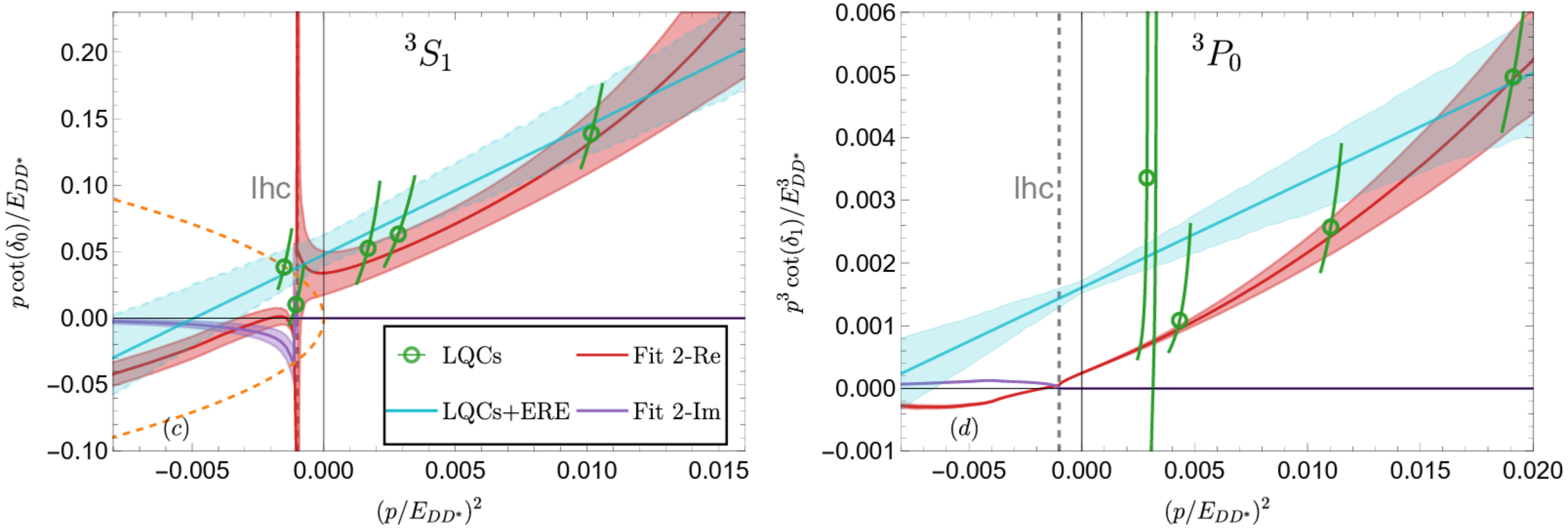
M.-L. Du et al., PRL131 (2023) 131901

Case studies on $T_{cc}^{+}(3872)$ relevant DD^* scattering. The data points are from lattice QCD calculation (M. Padmanath et al. PRL129(2022)032002)



L. Meng et al., PRD 109 (2024) L071506

Similar to the discussion above. The difference is that, the lattice finite volume energy levels are used to fix the parameters in the EFT involved. Then prediction of the EFT (red curves) are compared with ERE with out OPE.



2. Lattice QCD studies on X(3872)

S. Prelovsek, PRL111(2013)192001, H. Li, et al, arXiv: 2402.14541(hep-lat)

- **X(3872) found in 2003 by Belle, quite a lot of XYZ particles observed afterwards**

$$m_X = 3871.65 \pm 0.06 \text{ MeV}$$
$$\Gamma_x = 1.19 \pm 0.21 \text{ MeV}$$
$$I^G J^{PC} = 0^+ 1^{++}$$

- **X(3872) decays mainly to $D^0\bar{D}^{0*}$, a small fraction to $J/\psi\omega$, and also isospin violating $J/\psi\rho$**
- **Intensive and extensive phenomenological studies**
- **Interpreted as a $c\bar{c}$, a $D\bar{D}^*$ molecule or a tetraquark**
- **Main point of view: $D\bar{D}^* + c\bar{c}$**

- **Lattice QCD studies on $D\bar{D}^*$ and $c\bar{c}$ coupled channel effects.**
- **$J/\psi\omega$ channel is observed to be negligible**

- $m_\pi = 266$ MeV
  (S. Prelovsek, PRL111(2013)192001)
  A shallow bound state is observed

$$a_0 = -1.7 \pm 0.4\ fm,$$
$$r_0 = 0.5 \pm 0.1\ fm$$
$$E_B = -11 \pm 7 \text{ MeV}$$

  May correspond to $X(3872)$

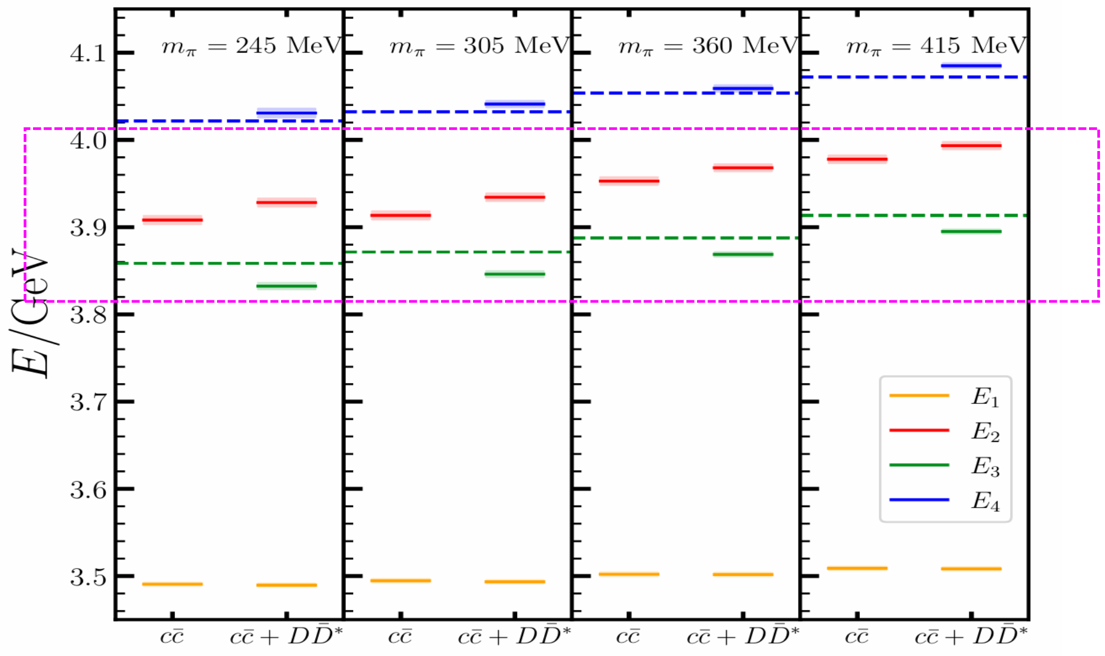
- $m_\pi = 250, 306, 360, 417$ MeV
  (H. Li, et al, arXiv: 2402.14541(hep-lat))
  Follow the strategy in the study above
  New interpretation of the lattice energy levels
  A likely bound state and a hint of a resonance around 4.0 GeV.

A. Existence of a bound state below $D\bar{D}^*$ threshold

Leuscher formula for S-wave scattering

$$p\cot\delta_0(p) = \frac{2}{\sqrt{\pi}\,L}\mathcal{Z}_{00}(1;q^2), \qquad q^2 \equiv \left(\frac{L}{2\pi}\right)^2 p^2$$

$$E_n(p_n) = \sqrt{m_D^2 + p_n^2} + \sqrt{m_{D^*}^2 + p_n^2}$$



Effective range expansion (ERE):

$$p\cot\delta_0(p) = \frac{1}{a_0} + \frac{1}{2}r_0p^2$$

Pole singularity of the scattering amplitude (in the infinite volume):

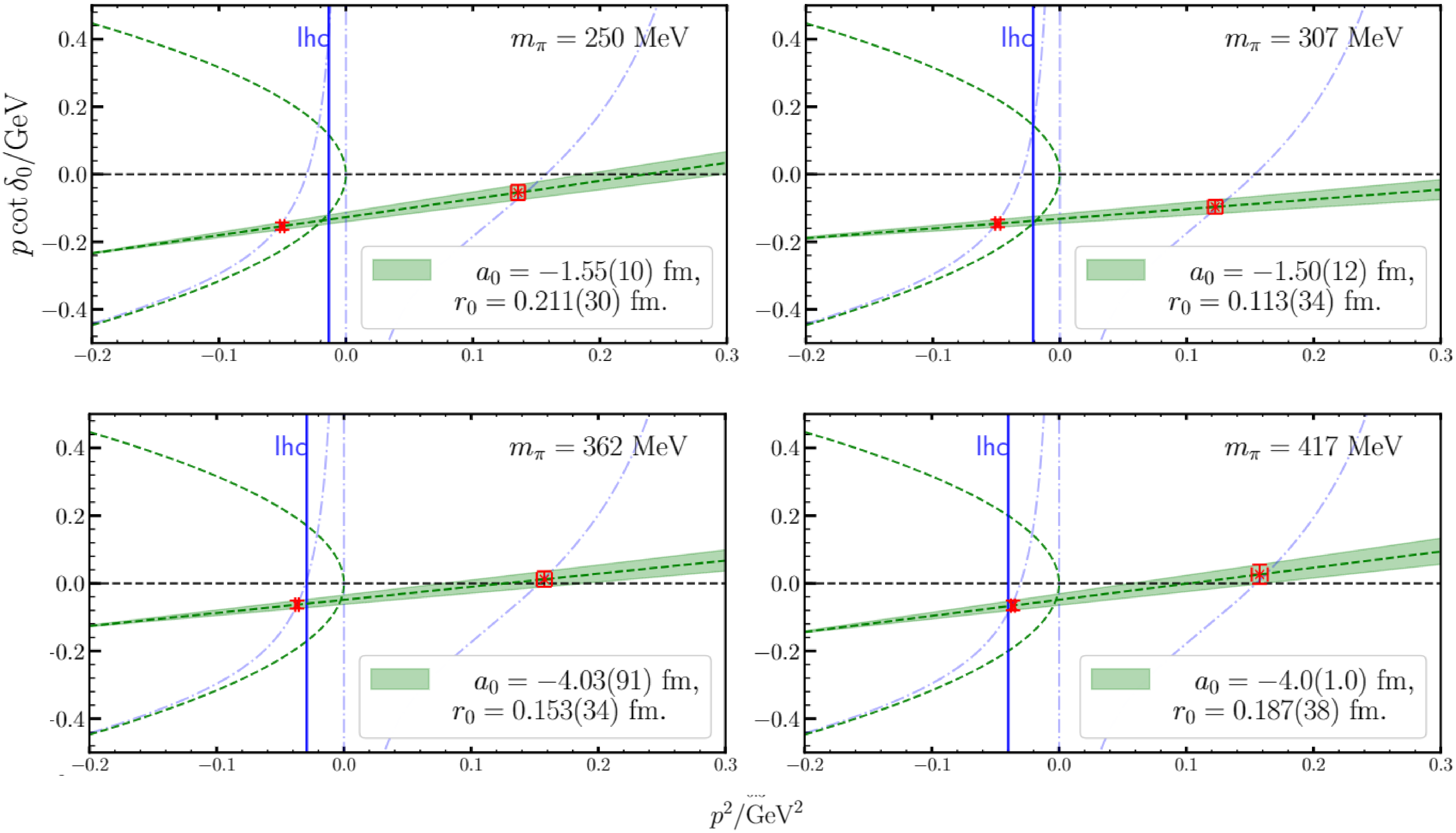
$$\mathcal{T} \propto \frac{1}{p\cot\delta_0(p) - ip}$$

- Solving ERE with E_2 and E_3, we can obtain the parameters (a_0, r_0)
- Using the derived (a_0, r_0) as the approximation in the $V \to \infty$ limit, then the pole equation gives the banding energy $E_B = E_{D\bar{D}^*}(p_B) - (m_D + m_{D^*})$, where p_B satisfies $p_B\cot\delta_0(p_B) - i\,p_B = 0$.

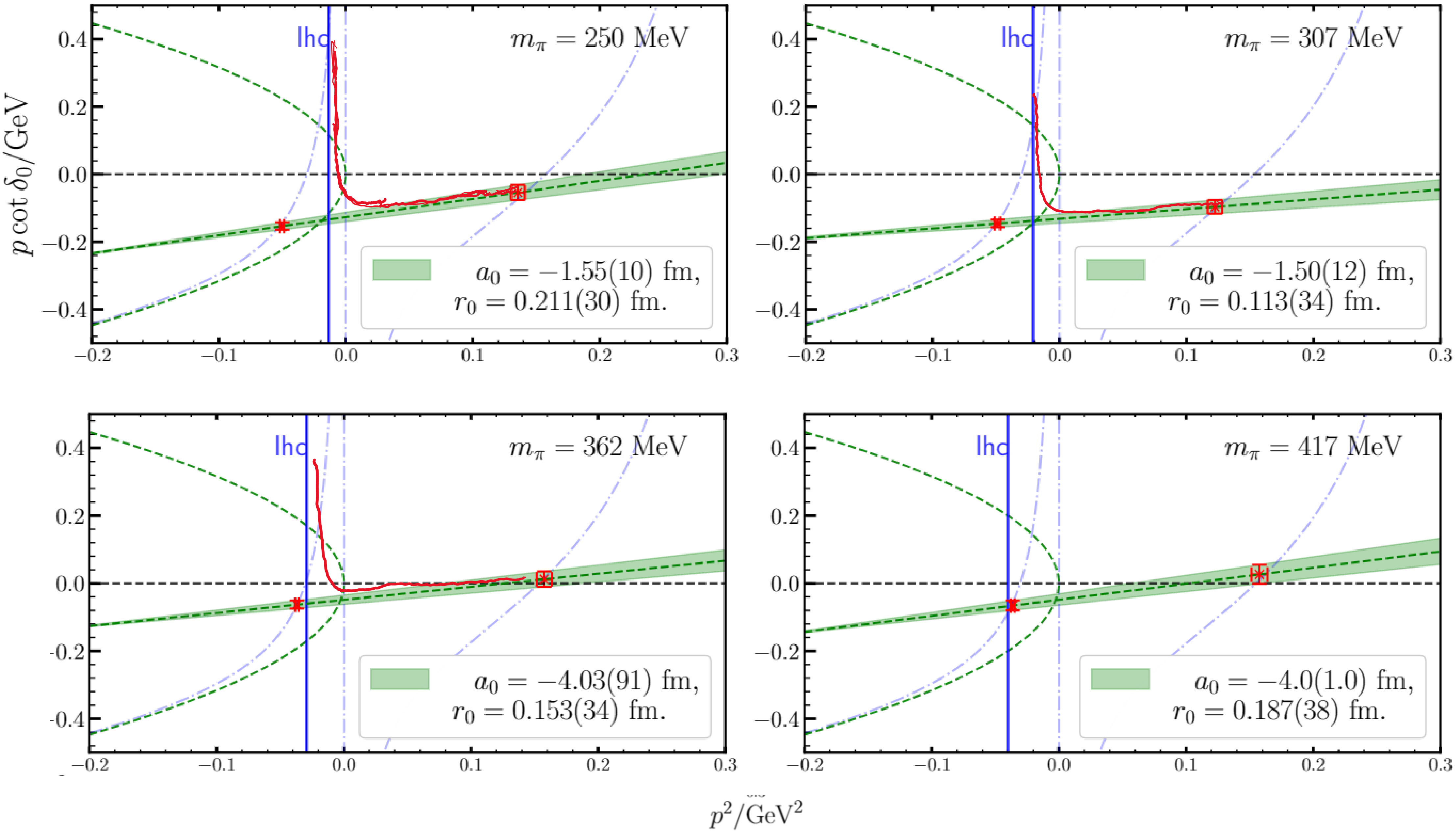
	m_π(MeV)	250(3)	307(2)	362(1)	417(1)
E_4	Δ_4(MeV) $= E_4 - E^{q=1}_{D\bar{D}^*}$	9.1(1.3)	8.9(1.2)	5.3(1.3)	12.8(1.3)
	p^2(GeV^2)	0.339(8)	0.335(6)	0.340(6)	0.342(4)
	$p\cot\delta_0(p)$(GeV)	-2.02(66)	-2.35(65)	-2.76(89)	-1.79(28)
	δ_0	$(163.9^{+4.0}_{-7.3})^\circ$	$(166.1^{+3.0}_{-5.1})^\circ$	$(168.1^{+2.9}_{-5.5})^\circ$	$(161.9^{+7.4}_{-3.2})^\circ$
E_3	Δ_3(MeV)$= E_3 - E^{q=0}_{D\bar{D}^*}$	70(3)	63(3)	80(3)	80(3)
	p^2(GeV^2)	0.135(5)	0.122(6)	0.158(6)	0.158(6)
	$p\cot\delta_0(p)$(GeV)	-0.054(19)	-0.097(19)	0.012(22)	0.026(24)
	δ_0	$(98.4^{+3.3}_{-3.6})^\circ$	$(105.4^{+3.0}_{-3.1})^\circ$	$(88.2^{+3.3}_{-3.3})^\circ$	$(86.2^{+4.0}_{-4.1})^\circ$
E_2	Δ_2(MeV)$= E_2 - E^{q=0}_{D\bar{D}^*}$	-26.1(9)	-25.4(11)	-19.0(7)	-18.6(8)
	p^2(GeV^2)	-0.050(2)	-0.049(2)	-0.037(1)	-0.036(1)
	$p\cot\delta_0(p)$(GeV)	-0.154(9)(*)	-0.146(10)(*)	-0.063(11)(*)	-0.066(13)
	$(p^{1\pi}_{\rm lhc})^2$(GeV^2)	-0.0135(4)	-0.0210(4)	-0.0292(3)	-0.0400(3)
	a_0 (fm)	-1.55(10)(*)	-1.50(12)(*)	-4.03(91)(*)	**−4.0(1.0)**
	r_0 (fm)	0.211(30)(*)	0.113(34)(*)	0.153(34)(*)	**0.187(38)**
	E_B (MeV)	$-9.7^{+2.1}_{-2.2}$ (*)	$-9.7^{+1.9}_{-2.0}$ (*)	$-1.3^{+0.6}_{-0.8}$ (*)	$\mathbf{-1.3^{+0.8}_{-1.0}}$

- E_2 : The lattice energy is lower than the $D\bar{D}^*$ threshold by 20 MeV or even more.
- a_0: Large negative, implies a bound state.
- r_0: Small positive, implies the compositeness $X \sim 1$ up to a $\mathcal{O}(p^2)$ correction
- (Y. Li et al., PRD105(2022)L071502).
- The bound state is predominantly a $D\bar{D}^*$ molecule.
- Maybe suffering from the Left Hand Cut (lhc) issue.

(M.-L. Du et al., PRL131(2023)131901, L. Meng et al., arXiv:2312.01930 [hep-lat])



lhc
$m_\pi = 250$ MeV
$a_0 = -1.55(10)$ fm,
$r_0 = 0.211(30)$ fm.
lhc
$m_\pi = 307$ MeV
$a_0 = -1.50(12)$ fm,
$r_0 = 0.113(34)$ fm.
lhc
$m_\pi = 362$ MeV
$a_0 = -4.03(91)$ fm,
$r_0 = 0.153(34)$ fm.
lhc
$m_\pi = 417$ MeV
$a_0 = -4.0(1.0)$ fm,
$r_0 = 0.187(38)$ fm.
$p \cot \delta_0/\text{GeV}$
p^2/GeV^2


lhc
$m_\pi = 250$ MeV
$a_0 = -1.55(10)$ fm,
$r_0 = 0.211(30)$ fm.
lhc
$m_\pi = 307$ MeV
$a_0 = -1.50(12)$ fm,
$r_0 = 0.113(34)$ fm.
lhc
$m_\pi = 362$ MeV
$a_0 = -4.03(91)$ fm,
$r_0 = 0.153(34)$ fm.
lhc
$m_\pi = 417$ MeV
$a_0 = -4.0(1.0)$ fm,
$r_0 = 0.187(38)$ fm.
$p \cot \delta_0/\mathrm{GeV}$
p^2/GeV^2

To summarize on the bound state:

- $\boldsymbol{m_\pi = 417}$ **MeV** : Free from the OPE lhc issue, a bound state exists in the $V \to \infty$ limit

$$E_B = -1.3^{+0.8}_{-1.0}\ MeV, \qquad X \approx 1 + \mathcal{O}(p^2)$$

  This pole may correspond to $X(3872)$, which is mainly a $D\bar{D}^*$ molecule.

- $\boldsymbol{m_\pi < 360}$ **MeV** : OPE lhc may have the effects on the existence and the pole position of a bound state.

- If the OPE lhc effects are similar to the case of $T_{cc}^{+}(3872)$ relevant scattering in that, ERE can give a ballpark description of the p^2 behavior of $p \cot \delta_0$, the singularity induced by OPE lhc permit the existence of a bound state, and result in a smaller binding energy.

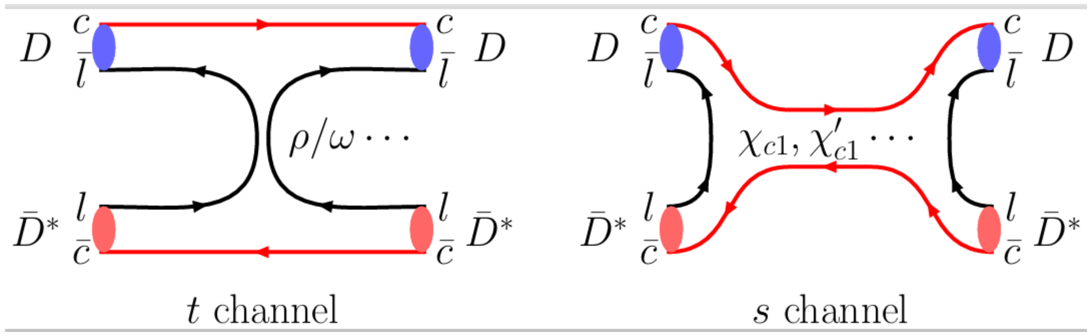
B. A possible resonance below 4.0 GeV

Where is $\chi_{c1}(2P)$?

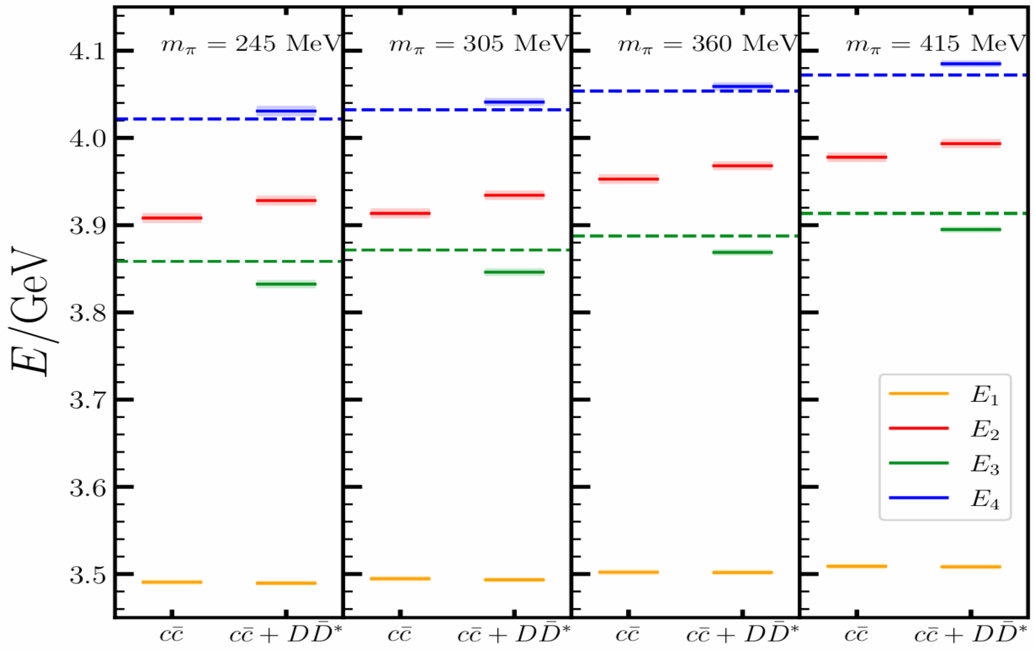
a) Non-relativistic quark model expects $\chi_{c1}(2P)$ with a mass around 3.95 GeV.
b) $X(3872)$ is likely a $D\bar{D}^*$ molecule.
c) There should be a state that has a large component of $\chi_{c1}(2\mathrm{P})$.
d) It might appear as a resonance.
e) The dynamics for the $D\bar{D}^*$ scattering in $0^{+}1^{++}$ channel

Multiplet	State	Expt.	Input (NR)	Theor. NR	Theor. GI
2P	$\chi_2(2^3\mathrm{P}_2)$			3972	3979
	$\chi_1(2^3\mathrm{P}_1)$			3925	3953
	$\chi_0(2^3\mathrm{P}_0)$			3852	3916
	$h_c(2^1\mathrm{P}_1)$			3934	3956

T. Barnes et al.,PRD72(2005)054026



Meson exchange Charmonium mediation



	m_π(MeV)	250(3)	307(2)	362(1)	417(1)
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	p^2(GeV^2)	0.339(8)	0.335(6)	0.340(6)	0.342(4)
	$p\cot\delta_0(p)$(GeV)	-2.02(66)	-2.35(65)	-2.76(89)	-1.79(28)
	δ_0	$(163.9^{+4.0}_{-7.3})^\circ$	$(166.1^{+3.0}_{-5.1})^\circ$	$(168.1^{+2.9}_{-5.5})^\circ$	$(161.9^{+7.4}_{-3.2})^\circ$
E_3	Δ_3(MeV)$= E_3 - E^{q=0}_{D\bar{D}^*}$	70(3)	63(3)	80(3)	80(3)
	p^2(GeV^2)	0.135(5)	0.122(6)	0.158(6)	0.158(6)
	$p\cot\delta_0(p)$(GeV)	-0.054(19)	-0.097(19)	0.012(22)	0.026(24)
	δ_0	$(98.4^{+3.3}_{-3.6})^\circ$	$(105.4^{+3.0}_{-3.1})^\circ$	$(88.2^{+3.3}_{-3.3})^\circ$	$(86.2^{+4.0}_{-4.1})^\circ$
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	p^2(GeV^2)	-0.050(2)	-0.049(2)	-0.037(1)	-0.036(1)
	$p\cot\delta_0(p)$(GeV)	-0.154(9)(*)	-0.146(10)(*)	-0.063(11)(*)	-0.066(13)
	$(p^{1\pi}_{\rm lhc})^2$(GeV^2)	-0.0135(4)	-0.0210(4)	-0.0292(3)	-0.0400(3)
	a_0 (fm)	-1.55(10)(*)	-1.50(12)(*)	-4.03(91)(*)	**−4.0(1.0)**
	r_0 (fm)	0.211(30)(*)	0.113(34)(*)	0.153(34)(*)	**0.187(38)**
	E_B (MeV)	$-9.7^{+2.1}_{-2.2}$ (*)	$-9.7^{+1.9}_{-2.0}$ (*)	$-1.3^{+0.6}_{-0.8}$ (*)	$\mathbf{-1.3^{+0.8}_{-1.0}}$

- E_3 : Gives a scattering phase around $\delta(E_3) \sim 90^\circ$;
- E_4: Gives a scattering phase close to $\delta(E_4) \sim 180^\circ$.
- Exactly as the expectation of the generalized Levinson's theorem.
- Hint at the existence of a resonance.

The possible existence of a resonance below 4.0 GeV

- **Breit-Wigner ansatz** for a resonance :

$$\mathcal{T} \approx \frac{1}{\cot\delta_0 - i} \sim \frac{1}{(m_R - E) - \frac{i\Gamma_R}{2}},$$

- Resonance parameters derived through

$$\delta_0(E) = \arctan\left(\frac{\Gamma_R}{2(m_R - E)}\right)$$

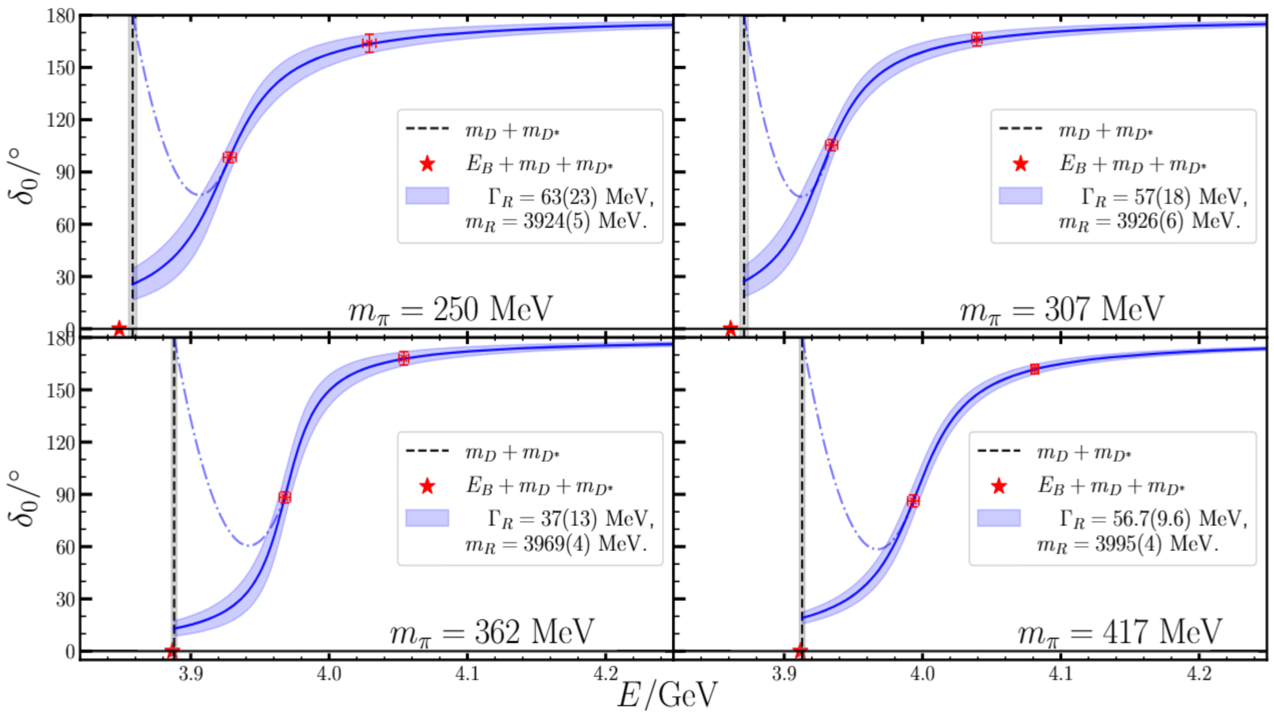
by using E_3 and E_4.

- Caution: The parameters (m_R, Γ_R) may change, since they are derived from only two energy levels.

- Only one experimental observation $X(3940)$:

$$m_X = 3942(9)\ \text{MeV}$$
$$\Gamma_X = 37^{+27}_{-17}\ \ \text{MeV}$$

(Belle, PRL98(2007)082001; PRL100(2008)202001)



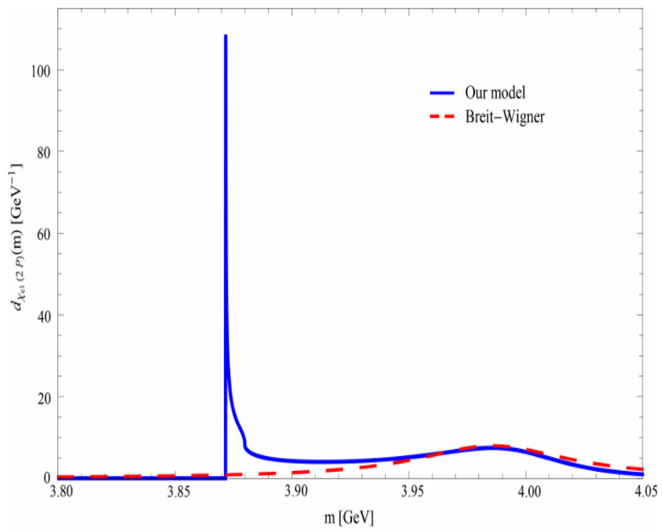
m_π(MeV)	250(3)	307(2)	362(1)	417(1)
m_R(MeV)	3924(5)	3926(6)	3969(4)	3995(4)
Γ_R(MeV)	63(23)	57(18)	37(13)	57(10)

Be caution that the resonance parameters may change !

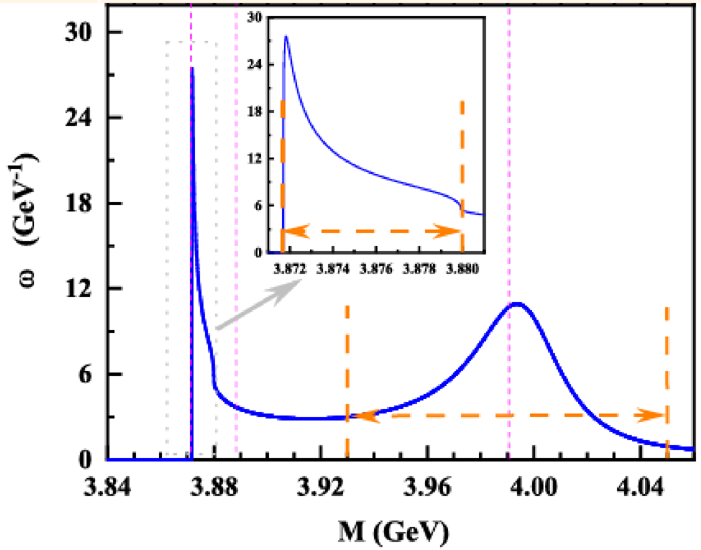
Related phenomenological studies

- Considering the mixing between $D\bar{D}^*$ and $\chi_{c1}(2P)$.

m_R (MeV)	3910-3925	3995	3990	3958
Γ_R (MeV)	$5-70$	72	~ 60	~ 17
Ref.	E. Cincioglu et al., EPJC76(2016)576	F. Giacosa et al., IJMPA34(2019)1950173	Q. Deng et al., 2312.10296	G.J. Wang et al., 2306.12406



E. Cincioglu et al., EPJC76(2016)576



Q. Deng et al., 2312.10296

Spectral function:

$$\omega(M) = \frac{1}{2\pi}\frac{\Gamma}{(M-m_R)^2+\Gamma^2/4}$$

3. Lattice QCD studies on $Z_c(3900)$

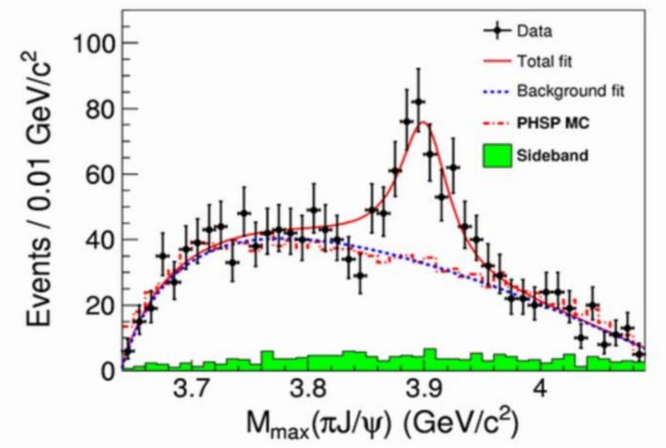
a. $D\bar{D}^*$ **scattering (**Y. Chen et al., PRD89(2014)094506, PRD92(2015)054507)
Single channel Lüscher' method, weak repulsive interaction.

b. Spectroscopy study (S. Prelovsek et al., PRD91(2015)014504)
No additional energy levels except for the interacting $D\bar{D}^*$ scattering states

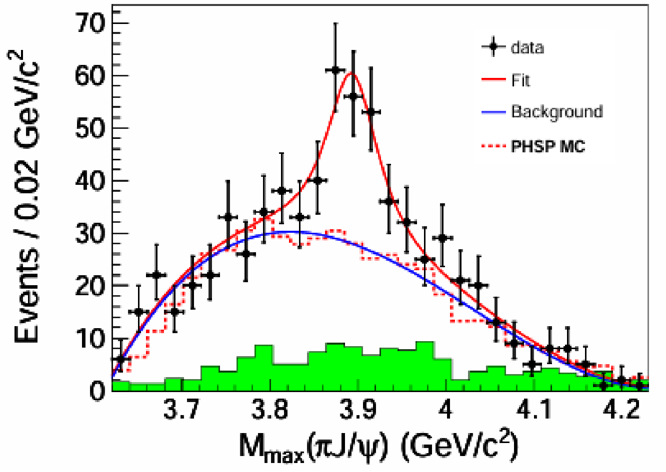
c. Potential matrix and scattering amplitudes
(Y. Ikeda et al (HAL Collab.), PRL117(2016) 242001)
HALQCD method, $D\bar{D}^* - J/\psi\pi - \eta_c\rho$ coupled channel effects
Interaction potentials——Off-diagonal potentials ($D\bar{D}^* - J/\psi\pi, D\bar{D}^* - \eta_c\rho$) are important.
$m_\pi = 410 - 710$ MeV, $Z_c(3900)$ may not be a usual resonance but a threshold cusp.

d. $D\bar{D}^* - J/\psi\pi$ coupled channel scattering (T. Chen et al., CPC43(2019)103103
Strong coupling effect between $D\bar{D}^*$ and $J/\psi\pi$ channels.
Lüscher' method and Ross-Shaw method is applied to analysis the finite volume energies.
Do not support the existence of a narrow near-threshold resonance

e. More scrutinized lattice QCD investigation is desired.



BESIII, PRL110(2013)252001



Belle, PRL110(2023)252002

Y. Ikeda et al (HAL Collab.), PRL117(2016) 242001

$$C^{\alpha\beta}(\vec{r},t) = \sum_{\vec{x}} \left\langle 0 \left| \phi_1^\alpha(\vec{x}+\vec{r},t)\phi_2^\beta(\vec{x},t) J^\beta(0) \right| 0 \right\rangle$$

$$= \sum_n \psi_n^\alpha(\vec{r}) A_n^\beta e^{-W_n t}$$

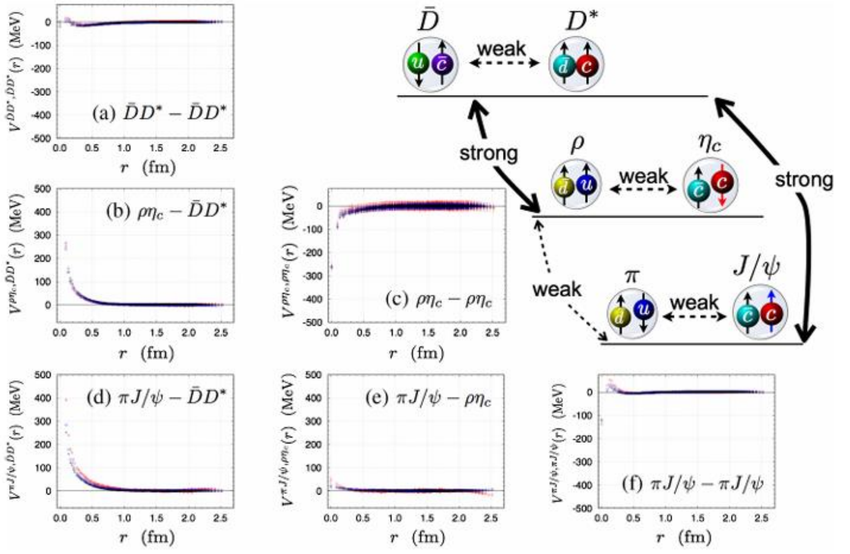
$$R^{\alpha\beta}(\vec{r},t) \equiv C^{\alpha\beta}(\vec{r},t) e^{(m_1^\alpha + m_2^\alpha)t}$$

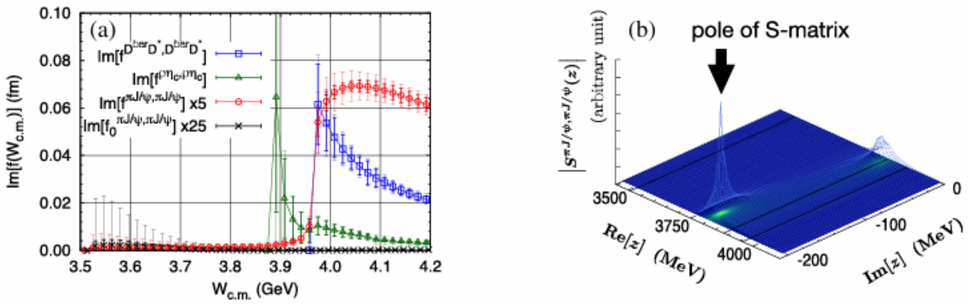
$$\left(-\frac{\partial}{\partial t} - H_0^\alpha\right) R^{\alpha\beta}(\vec{r},t) = \sum_\gamma \Delta^{\alpha\gamma} \int d\vec{r'}\, U\left(\vec{r},\vec{r'}\right) R^{\gamma\beta}\left(\vec{r'},t\right)$$

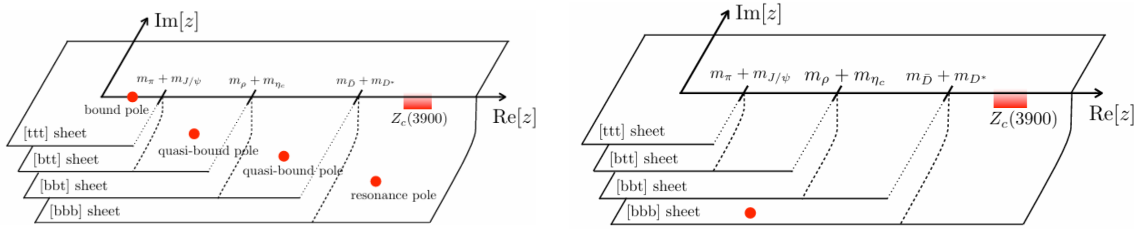
$$U^{\alpha\beta}\left(\vec{r},\vec{r'}\right) \approx V^{\alpha\beta}(\vec{r})\,\delta\left(\vec{r}-\vec{r'}\right) + \mathcal{O}(\nabla^2)$$

$$t^{\alpha\beta}(\vec{p}_\alpha,\vec{p}_\beta;W_{cm}) = V^{\alpha\beta}(\vec{p}_\alpha,\vec{p}_\beta) + \sum_\gamma \int d\vec{q}_\gamma \frac{\left(V^{\alpha\gamma}(\vec{p}_\alpha,\vec{q}_\gamma)\, t^{\gamma\beta}(\vec{q}_\gamma,\vec{p}_\beta;W_{cm})\right)}{W_{cm} - E_\gamma(\vec{q}_\gamma) + i\epsilon}$$

$$\mathrm{Im}\, f^{\alpha\alpha}(W_{cm}) = -\pi\mu^\alpha \mathrm{Im}\, t^{\alpha\alpha}(W_{cm})$$







V. Summary

- The present status of the lattice studies on glueballs is briefly overviewed. The lattice results of masses of glueballs and their productions rates in J/ψ radiative decays provide important theoretical inputs for experiments.
- Hybrid decays are now investigated in lattice QCD .
- The existing lattice QCD results relevant to $T_{cc}^{+}(3875)$ are consistent with each other and support the existence of a shallow $DD^{*}(I=0)$ bound state.
- However, the one-pion-exchange left hand cut issue should be considered in analyzing lattice data.
- Lattice QCD studies find evidence for a $0^{+}1^{++}$ bound state below $D\bar{D}^{*}$ threshold, which may correspond to $X(3872)$
- For $Z_c(3900)$, no consensus has been reach in the lattice QCD community.

Thank you for your Attention!