

Lattice Computations of Parton Distributions

QUANTUM 3

NSF

U.S. DEPARTMENT OF ENERGY

Level 3
3,000
16 BONUS

p
0/3 n
1/3 Δ
0/2

Level 3
0
18 BONUS

All quarks have a flavor. Yum! Haha, not that kind of flavor.

p
0/3 n
0/3

Level 8
24,000
11 BONUS

CTEQ

RESEARCH CORPORATION for SCIENCE ADVANCEMENT

HUEY-WEN LIN

This work of HL is supported by the NSF under grant PHY 2209424 & 1653405, DOE under DE-SC0024053 and the Research Corporation for Science Advancement through the Cottrell Scholar Award

@LinQCD

Learn QCD on Your Phone!

§ Learn QCD on your phone

[Google Play Store](#)

[Apple Appstore](#)



Supported by the NSF under grant PHY 1653405



Outline

§ Lattice QCD in a Nutshell

§ Selected x -Dependent Parton Distributions

∞ Nucleon PDFs

∞ Pion/Kaon PDFs

∞ GPDs

§ Impact of Lattice-QCD PDFs on Global Fits

Biased selected results toward MSULat students and postdocs



What is Lattice QCD?

§ Lattice QCD is an ideal theoretical tool for investigating the strong-coupling regime of quantum field theories

§ Physical observables are calculated from the path integral

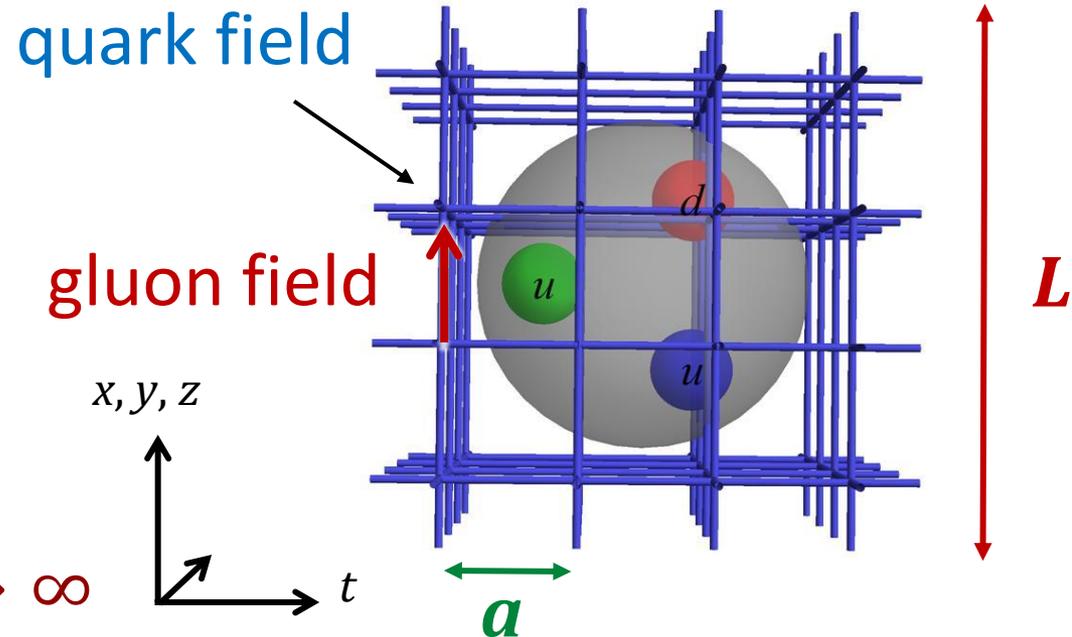
$$\langle 0|O(\bar{\psi}, \psi, A)|0\rangle = \frac{1}{Z} \int \mathcal{D}A \mathcal{D}\bar{\psi} \mathcal{D}\psi e^{iS(\bar{\psi}, \psi, A)} O(\bar{\psi}, \psi, A)$$

in **Euclidean** space

- ∞ Quark mass parameter (described by m_π)
- ∞ Impose a UV cutoff
discretize spacetime
- ∞ Impose an infrared cutoff
finite volume

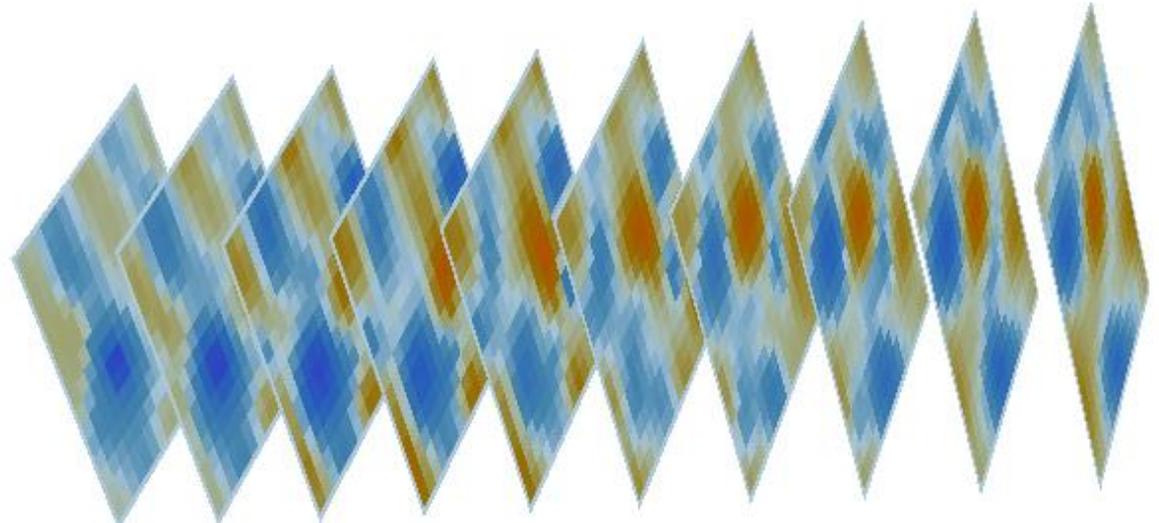
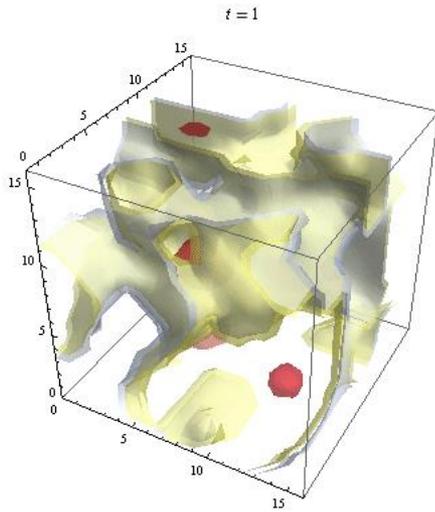
§ Recover physical limit

$$m_\pi \rightarrow m_\pi^{\text{phys}}, \quad a \rightarrow 0, \quad L \rightarrow \infty$$



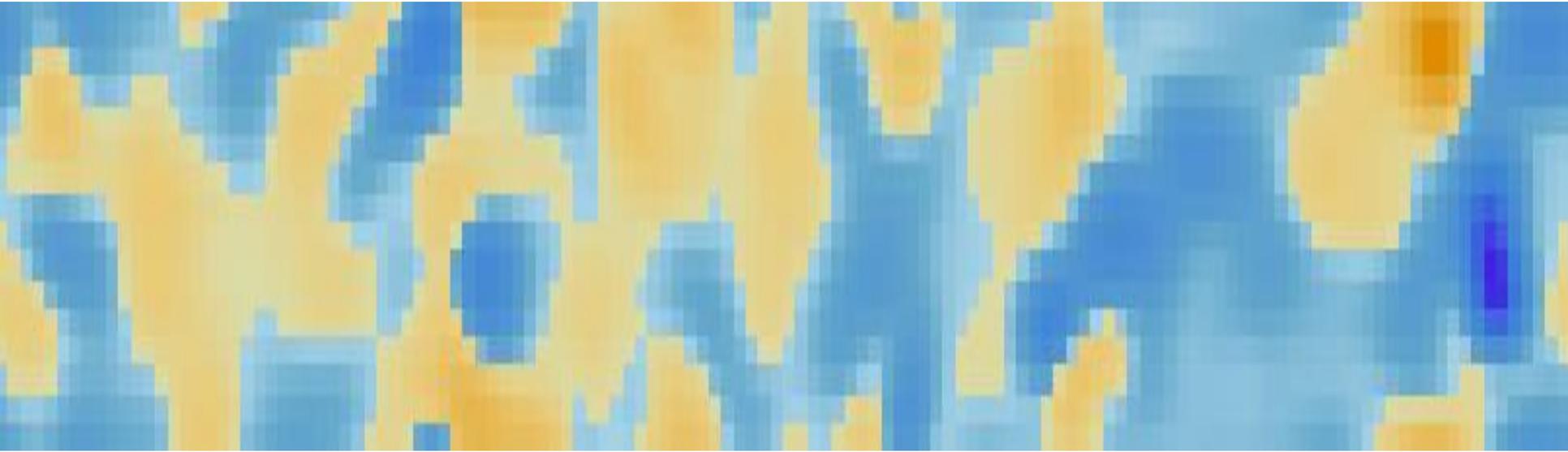
Anatomy of a Lattice Calculation

1. Start with QCD Vacuum (gauge configurations)



Anatomy of a Lattice Calculation

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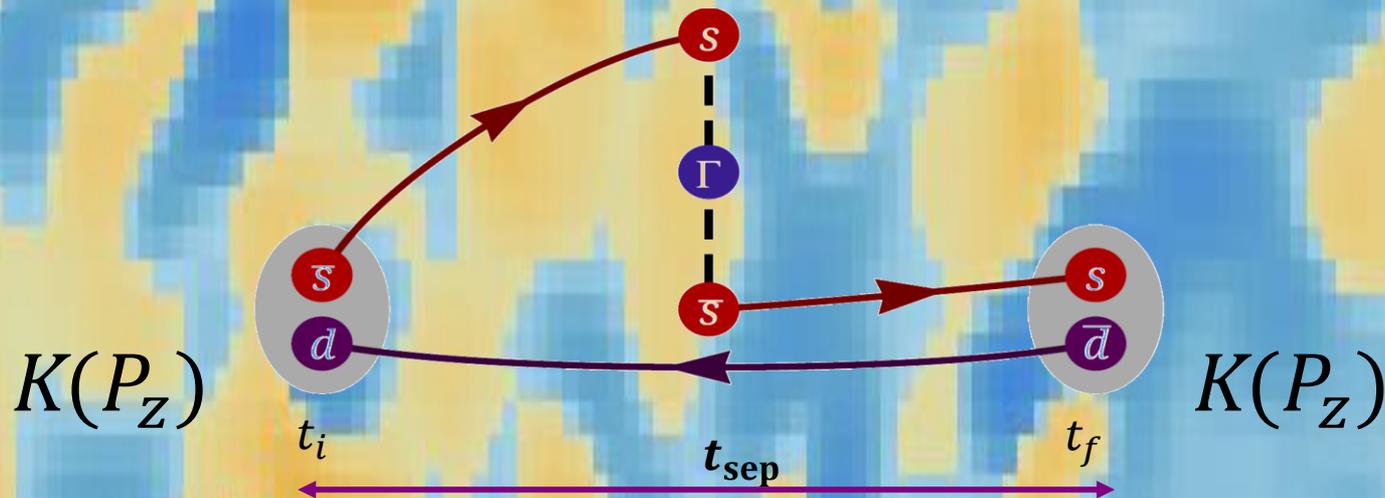


Thanks to MILC collaboration for sharing their 2+1+1 HISQ lattices

Anatomy of a Lattice Calculation

2. Correlators (hadronic observables)

- ∞ Invert Dirac operator matrix (rank 10^{12})
- ∞ Combine using color, spin and momentum into hadrons



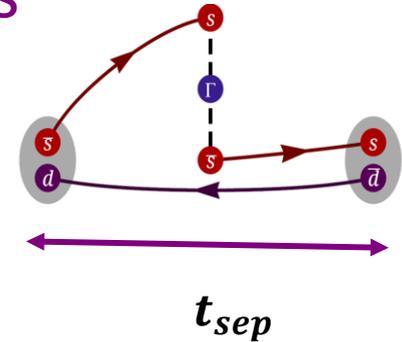
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Anatomy of a Lattice Calculation

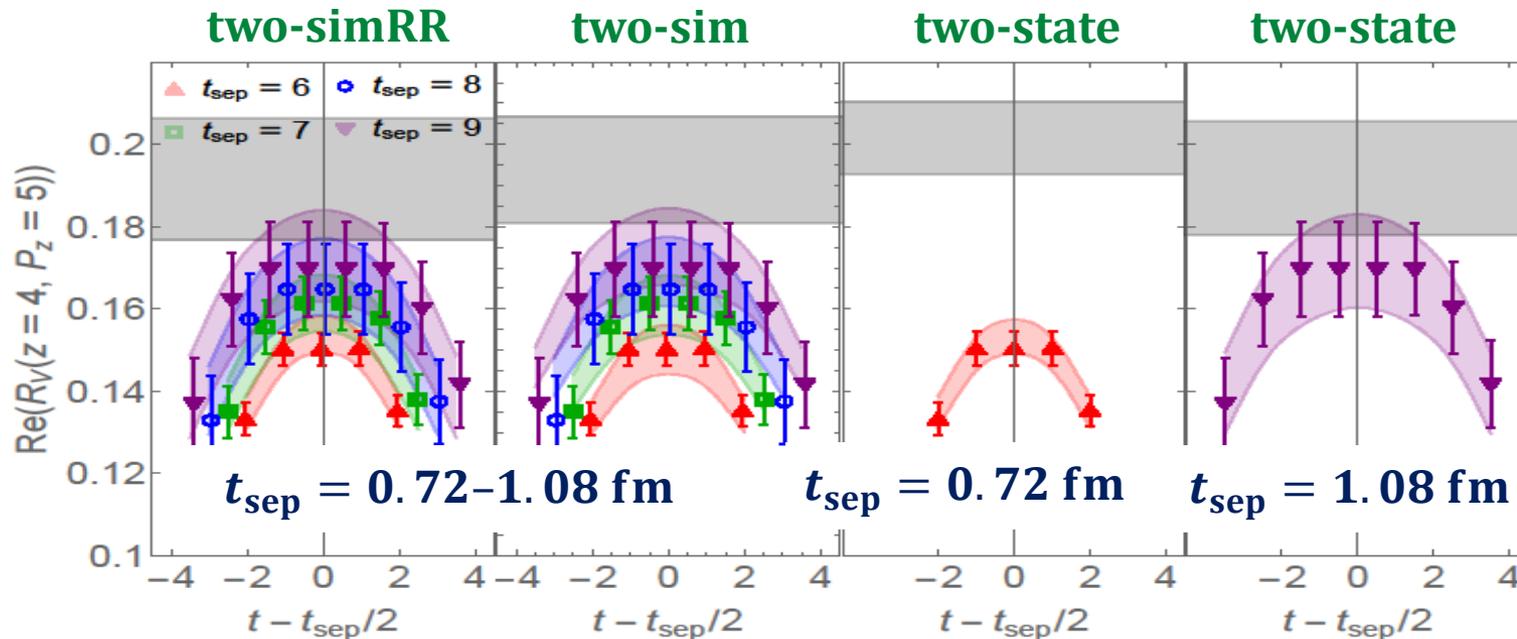
3. Extract reliable ground-state matrix elements

↻ Excited-state removal

↻ For example, kaon matrix element
at $M_\pi \approx 220$ MeV, $a \approx 0.12$ fm



HL et al. (MSULat), 2003.14128



stability in extracting matrix elements

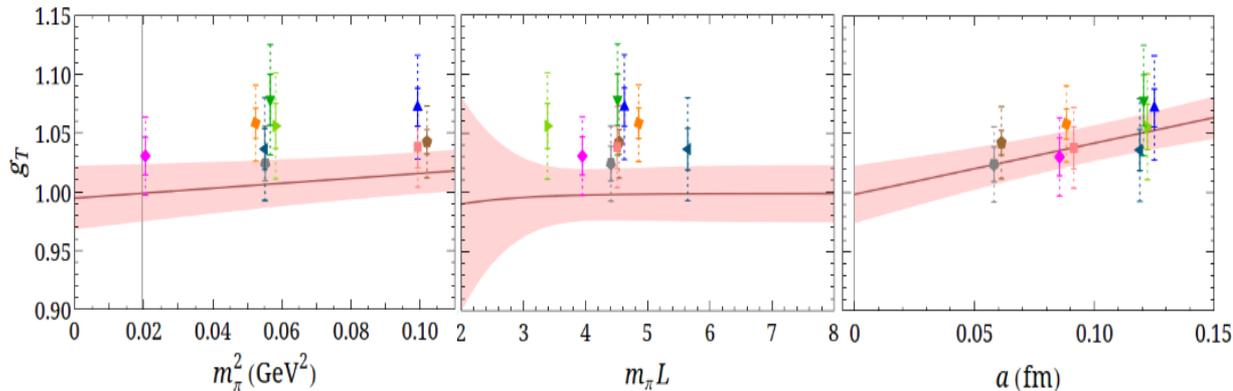
Anatomy of a Lattice Calculation

4. Systematic uncertainty (nonzero a , finite L , etc.)

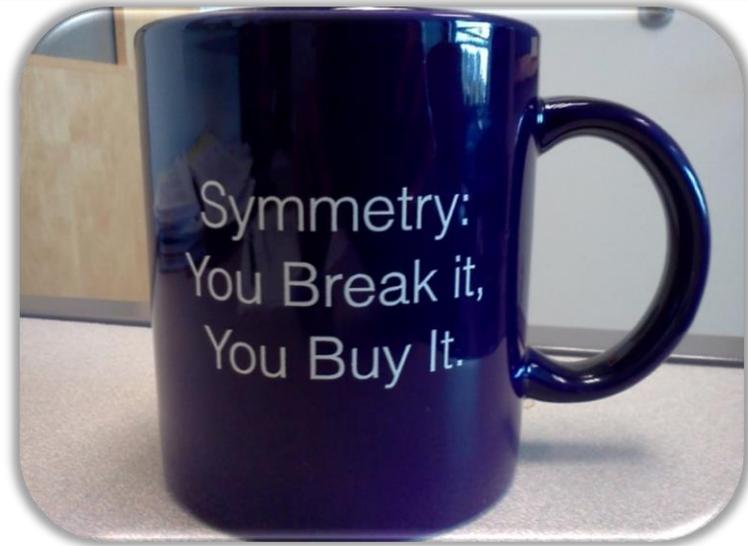
⌘ Nonperturbative renormalization, etc

⌘ Extrapolation to the continuum limit

$$(m_\pi \rightarrow m_\pi^{\text{phys}}, L \rightarrow \infty, a \rightarrow 0)$$



Lattice Structure Limitation



§ Lattice calculations rely on operator product expansion, only provide moments

$$\langle x^{n-1} \rangle_q = \int_{-1}^1 dx x^{n-1} q(x)$$

§ Limited to the lowest few moments

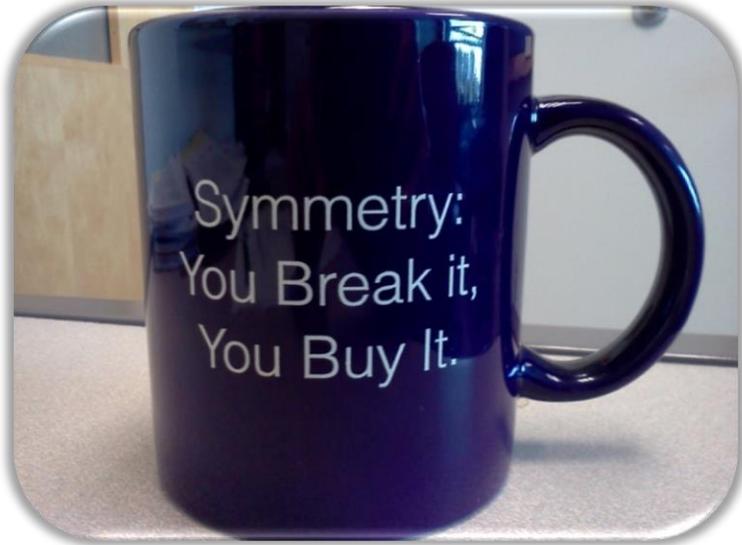
- ⌘ For higher moments, all ops mix with lower-dimension ops
- ⌘ Novel proposals to overcome this problem

W. Detmold and C. Lin, Phys. Rev. D73 (2006) 014501

Z. Davoudi and M. J. Savage, Phys. Rev. D86 (2012) 054505

A. Shindler, arXiv:2311.18704

Lattice Structure Limitation



§ Lattice calculations rely on operator product expansion, only provide moments

$$\langle x^{n-1} \rangle_q = \int_{-1}^1 dx x^{n-1} q(x)$$

§ Longstanding obstacle!

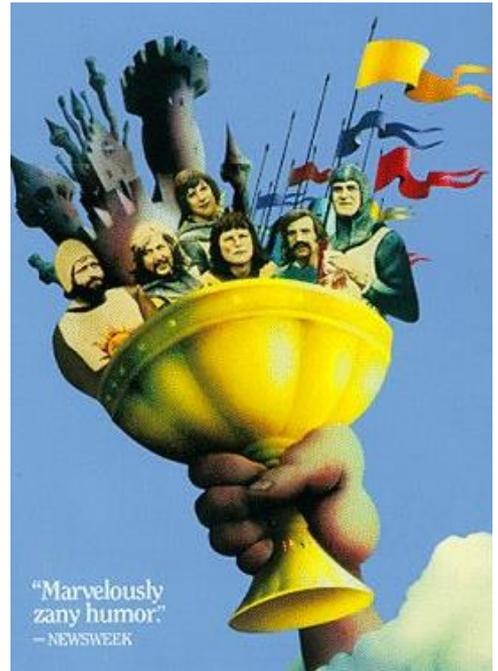
☞ Holy grail of structure calculations

§ Applies to many structure quantities:

☞ Parton Distribution Functions (PDFs)

☞ Generalized parton distributions (GPD)

☞ Transverse-momentum distributions (TMD)



A NEW HOPE

It is a period of war and economic uncertainty.

Turmoil has engulfed the galactic republics.

Basic truths at foundation of the human civilization are disputed by the dark forces of the evil empire.

A small group of QCD Knights from United Federation of Physicists has gathered in a remote location on the third planet of a star called Sol on the inner edge of the Orion-Cygnus arm of the galaxy.

The QCD Knights are the only ones who can tame the power of the Strong Force, responsible for holding atomic nuclei together, for giving mass and shape to matter in the Universe.

They carry secret plans to build the most powerful

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∞ **Nucleon PDFs**

∞ Pion/Kaon PDFs

∞ GPDs

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Direct x -Dependent Structure

§ Longstanding obstacle to lattice calculations!



↪ **Quasi-PDF**/large-momentum effective theory (LaMET)
(X. Ji, 2013; See 2004.03543 for review)

↪ **Pseudo-PDF** method: differs in FT (A. Radyushkin, 2017)

↪ Lattice cross-section method (**LCS**) (Y Ma and J. Qiu, 2014, 2017)

↪ Hadronic tensor currents (Liu et al., hep-ph/9806491, ... 1603.07352)

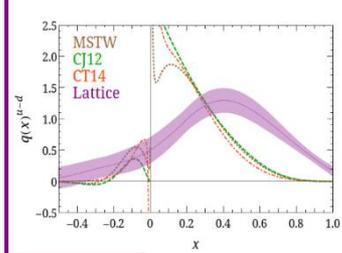
↪ Euclidean correlation functions (RQCD, 1709.04325)

↪ ...

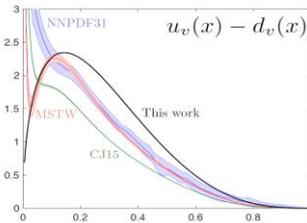
Lattice Parton Calculations

§ Rapid developments!

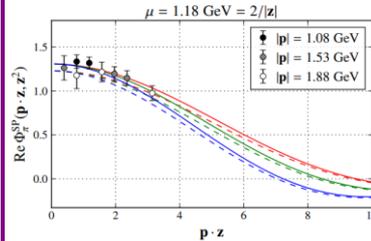
First unpol. PDF
lattice calculation



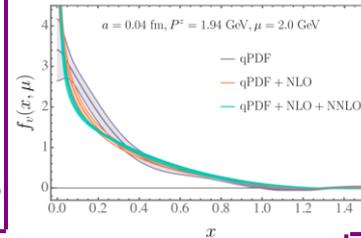
First lattice
pseudo-PDFs



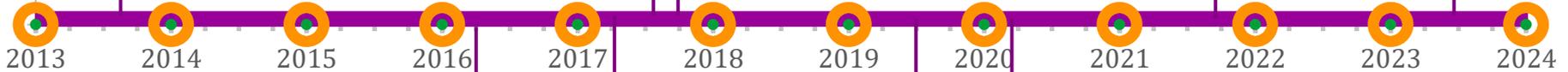
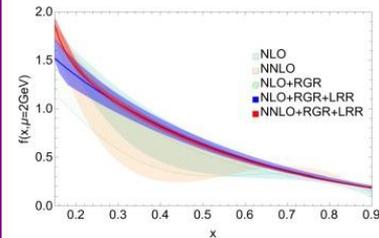
Euclidean
correlation
functions



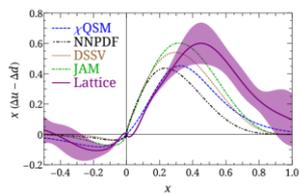
1st NNLO
PDF



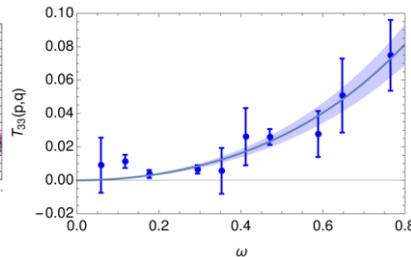
1stPDF
w/
LRR+RGR



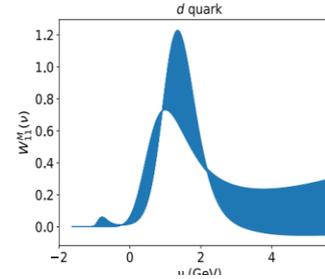
Pol. PDFs and
mass corrections



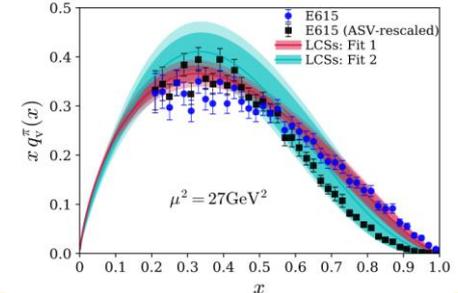
Compton
amplitude



Hadronic tensor



LCS



Direct x -Dependent Structure

§ Longstanding obstacle to lattice calculations!

Quantities that can be calculated on the lattice today

=

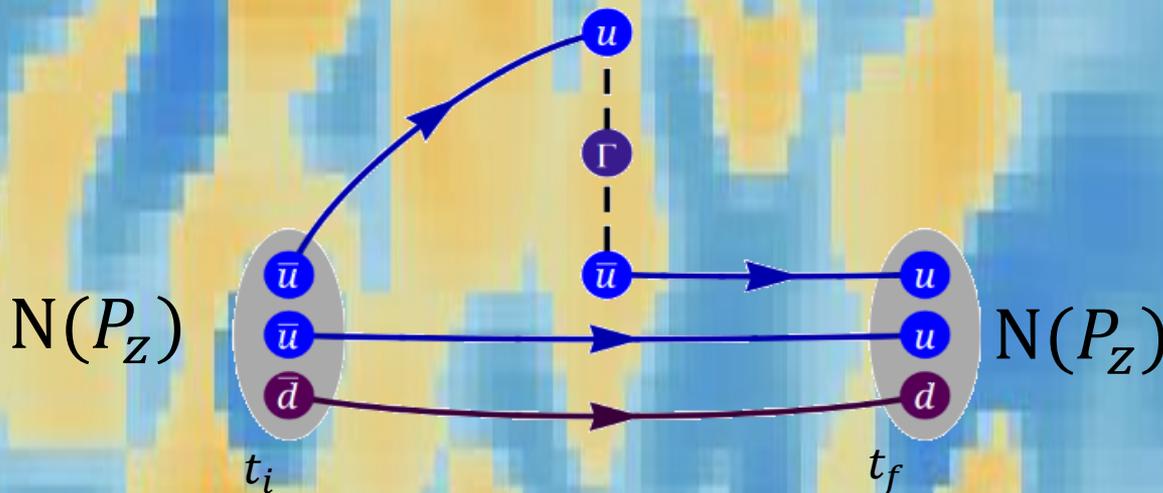
Σ

Wanted PDFs, GPDs, etc.

\times

pQCD-calculate d kernel

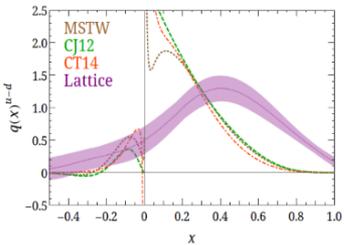
Quasi-PDF & Pseudo-PDF method



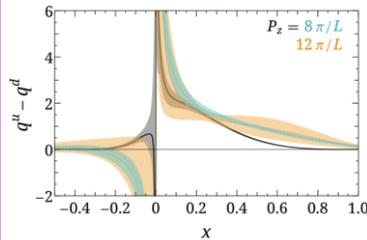
Lattice Parton Calculations

§ Physics quantity milestones

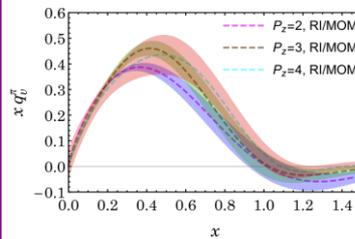
First unpol. lattice PDF



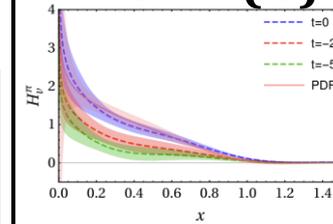
First PDFs at M_π^{phys}



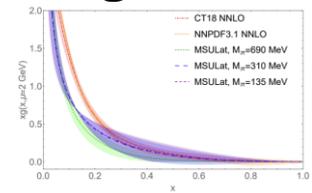
Pion v-PDF



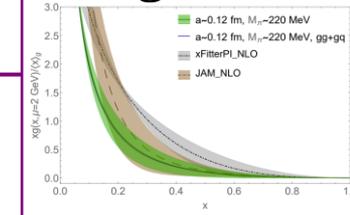
1st GPD (π)



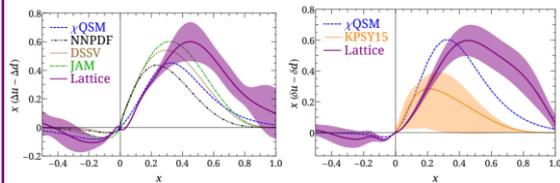
N g -PDF



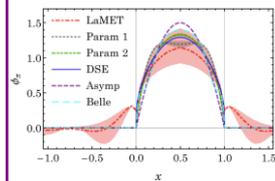
π g -PDF



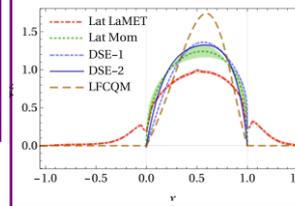
Pol. PDFs and mass corrections



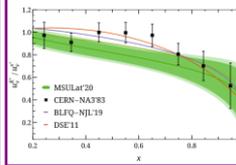
Pion DA



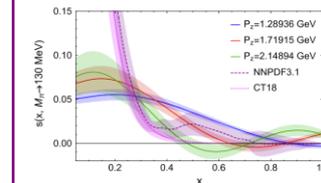
Kaon DA



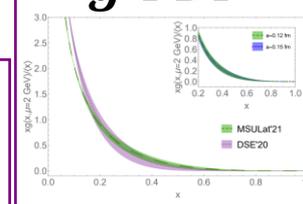
K PDF



s, c PDF



Kaon g -PDF



Lattice Example Results

§ Summary of physical pion mass PDFs results

unpolarized



$$u(x) - d(x)$$

longitudinally polarized



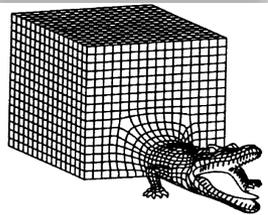
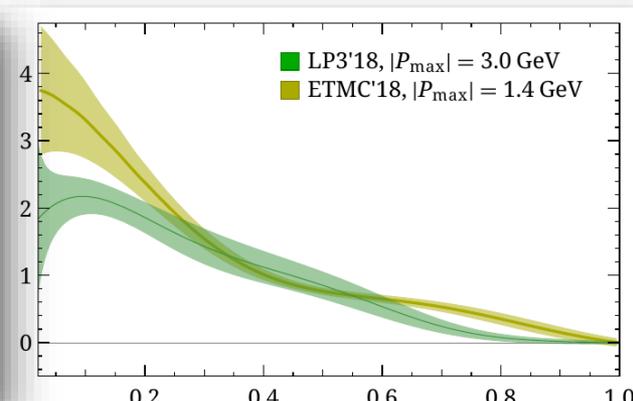
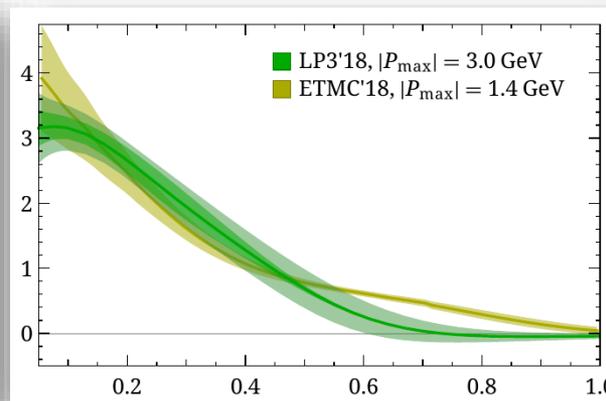
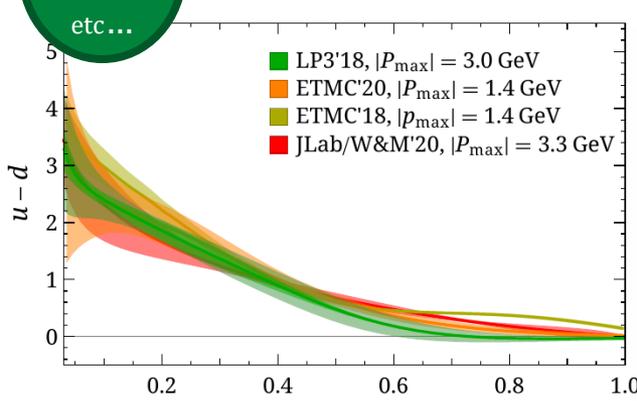
$$\Delta u(x) - \Delta d(x)$$

transversely polarized



$$\delta u(x) - \delta d(x)$$

Wanted
PDFs,
GPDs,
etc...



Finite volume,
Discretization,

...



2006.08636 (PDFLattice2019)

Lattice Example Results

§ Summary of physical pion mass PDFs results

unpolarized



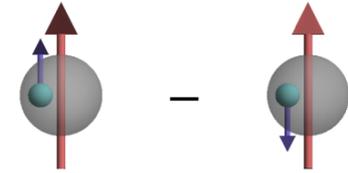
$$u(x) - d(x)$$

longitudinally polarized



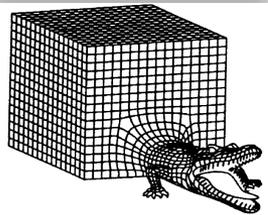
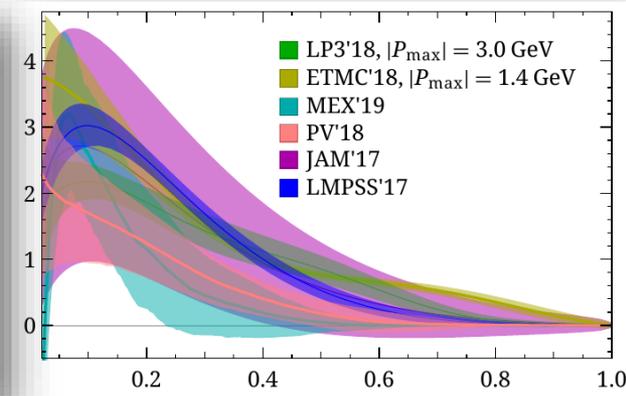
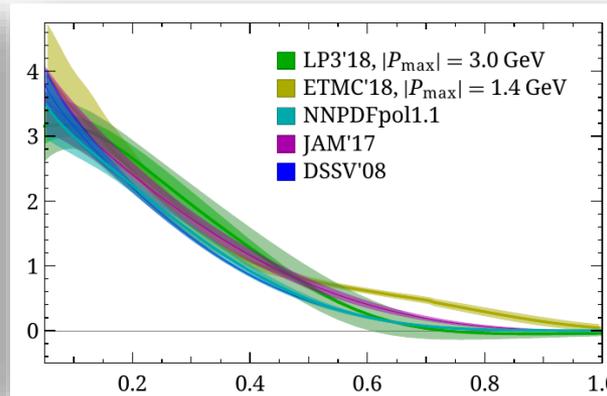
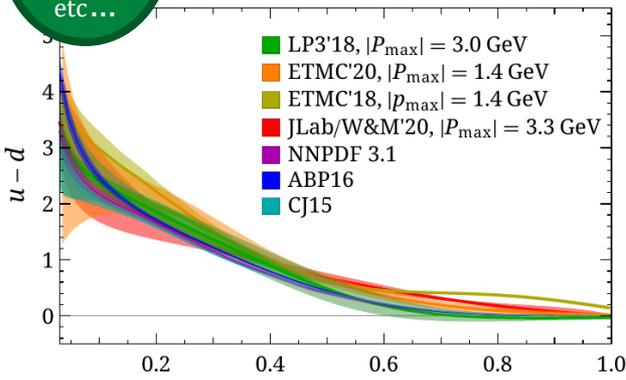
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Finite volume,
Discretization,

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2006.08636 (PDFLattice2019)

Isvector PDFs Update

§ Nucleon isovector PDF calculated directly at physical pion mass

∞ NNLO matching & treat leading-renormalon effects



∞ Leading-renormalon resummation (LRR)

R. Zhang, et. al.

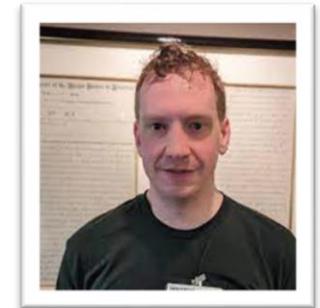
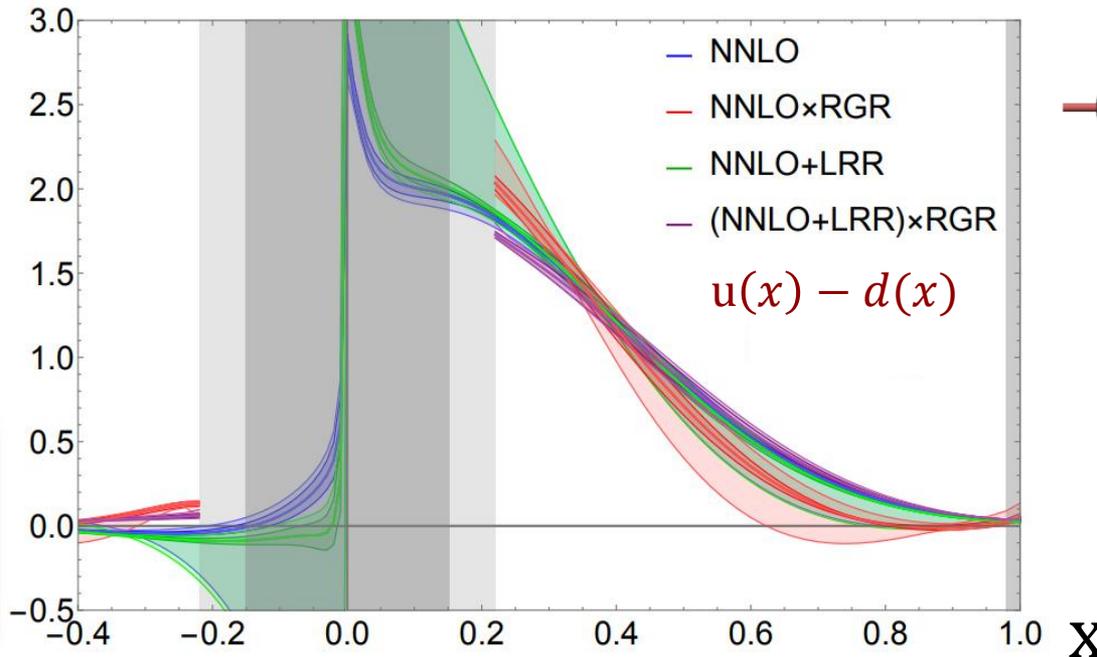
∞ Renormalization-group resummation (RGR)

PLB 844, 138081 (2023)

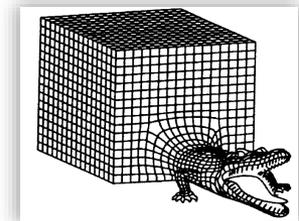
∞ $N_f=2+1+1$ clover/HISQ, $a\sim 0.09$ fm

J. Holligan, HL (MSULat), 2312.10829 [hep-lat]

Wanted
PDFs,
GPDs,
etc...



P: Jack Holligan



Isvector PDFs Update

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R. Zhang, et. al.

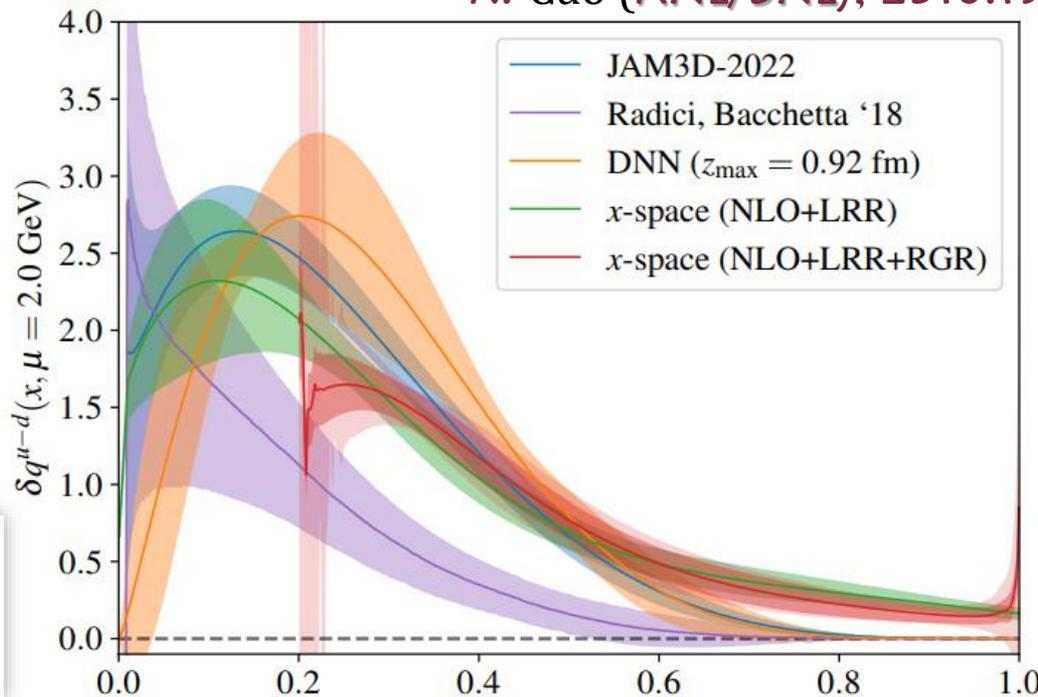
∞ Renormalization-group resummation (RGR)

PLB 844, 138081 (2023)

∞ $N_f=2+1$ clover/HISQ, $a\sim 0.076$ fm

X. Gao (ANL/BNL), 2310.19047 [hep-lat]

Wanted
PDFs,
GPDs,
etc...

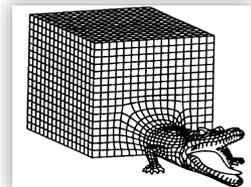


Transversity



$$\delta u(x) - \delta d(x)$$

X



Continuum PDF

§ Nucleon PDFs using quasi-PDFs in the continuum limit

∞ Lattice details: clover/2+1+1 HISQ (MSULat)

$a \approx \{0.06, 0.09, 0.12\}$ fm,

$M_\pi \in \{135, 220, 310\}$ -MeV pion,

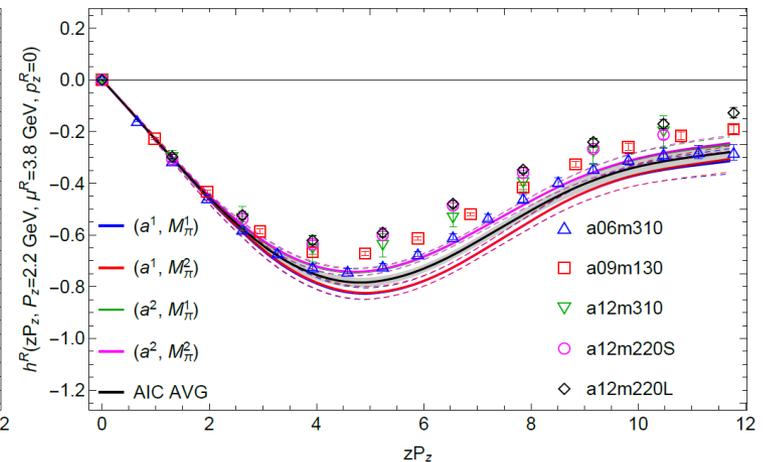
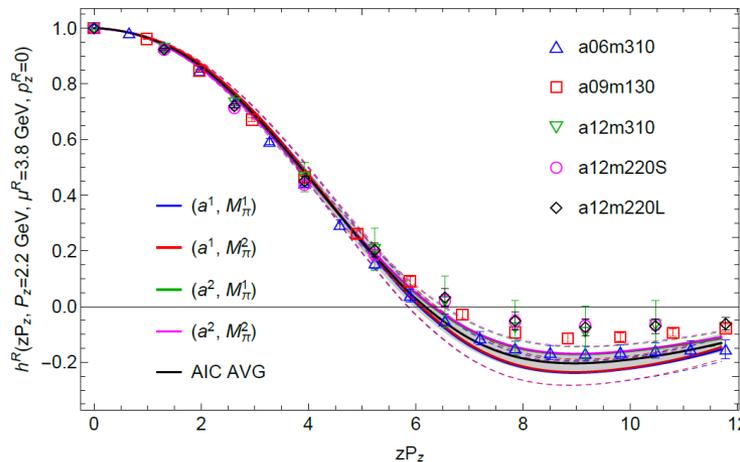
$M_\pi L \in \{3.3, 5.5\}$.

$P_z \approx 2$ GeV 2011.14971, HL et al (MSULat)



∞ Naïve extrapolation to physical-continuum limit

Quantities that can be calculated on the lattice



Continuum PDF

§ Nucleon PDFs using quasi-PDFs in the continuum limit

⌘ Lattice details: clover/2+1+1 HISQ (MSULat)

$a \approx \{0.06, 0.09, 0.12\}$ fm,

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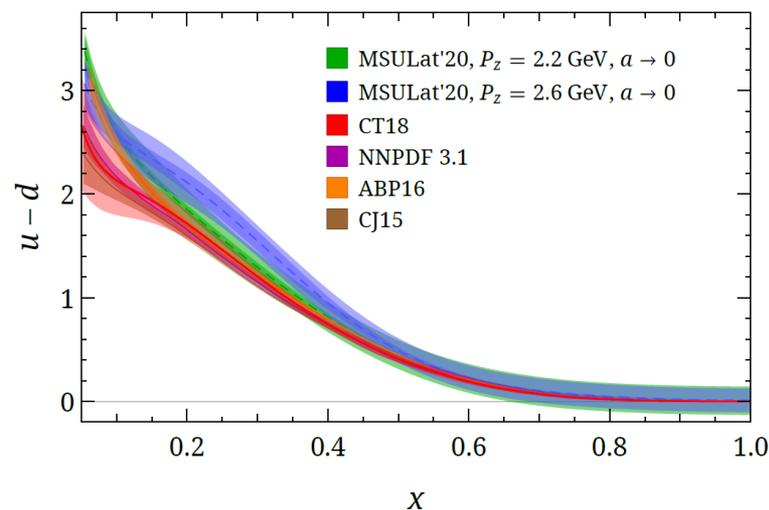
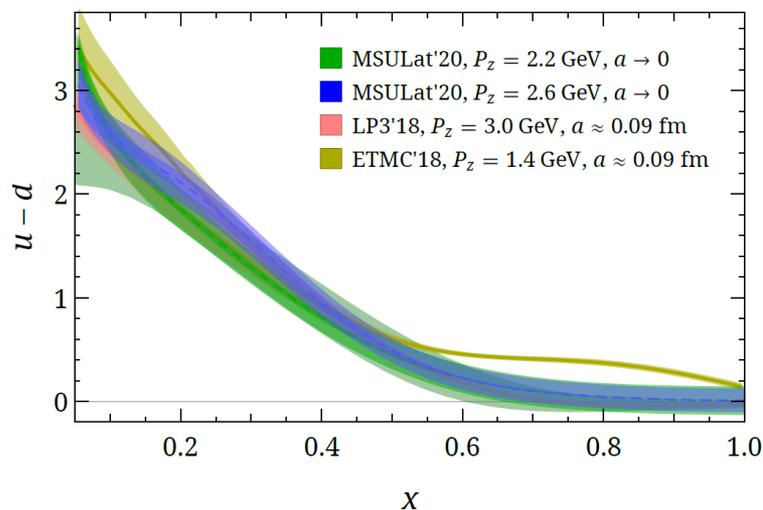
$P_z \approx 2$ GeV

2011.14971, HL et al (MSULat)



⌘ Naïve extrapolation to physical-continuum limit

$u(x) - d(x)$



Continuum PDF

§ Nucleon PDFs using quasi-PDFs in the continuum limit

∞ Lattice details: clover/2+1 clover (LPC)

$a \approx \{0.49, 0.64, 0.85, 0.98\}$ fm,

$M_\pi \in [222, 354]$ -MeV pion,

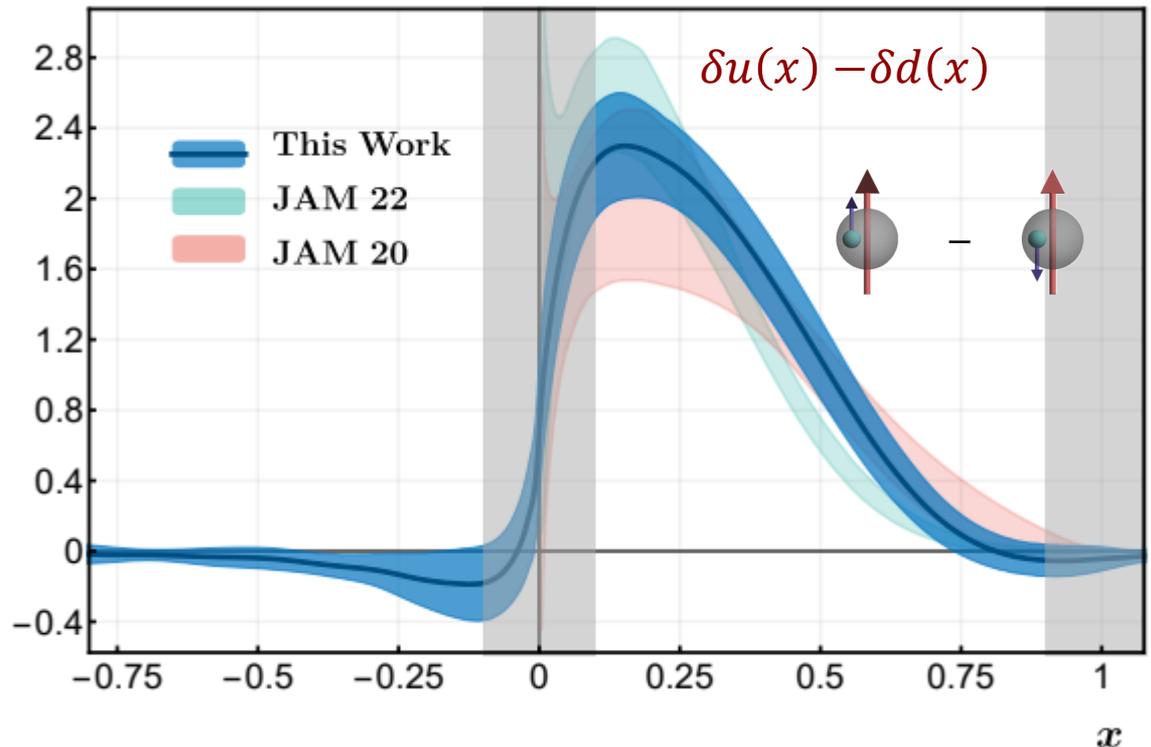
$M_\pi L \in [3.9, 8.1]$.

$P_z \approx \in [1.8, 2.8]$.



F. Yao et al (LPC), 2208.08008

Wanted
PDFs, GPDs,
etc...



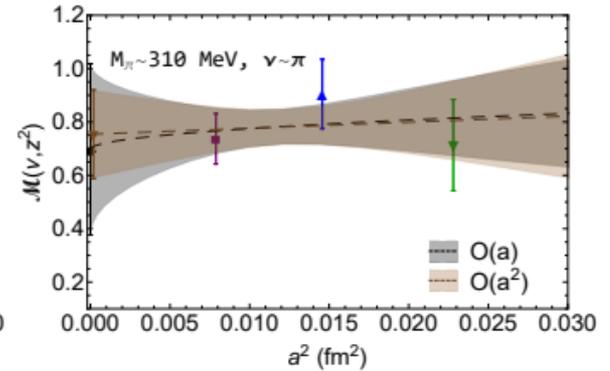
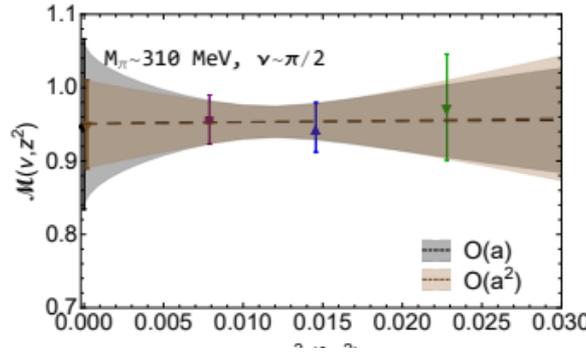
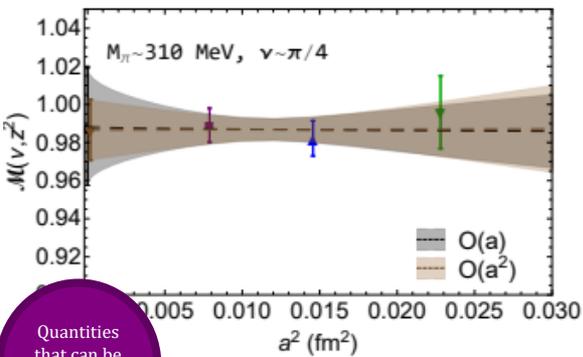
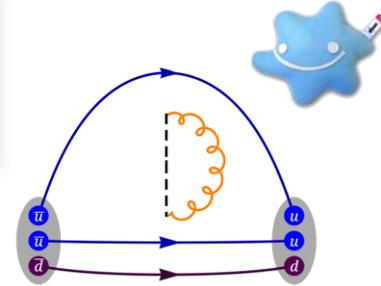
Gluon PDF in Nucleon

§ Continuum Gluon PDF w/ pseudo-PDF

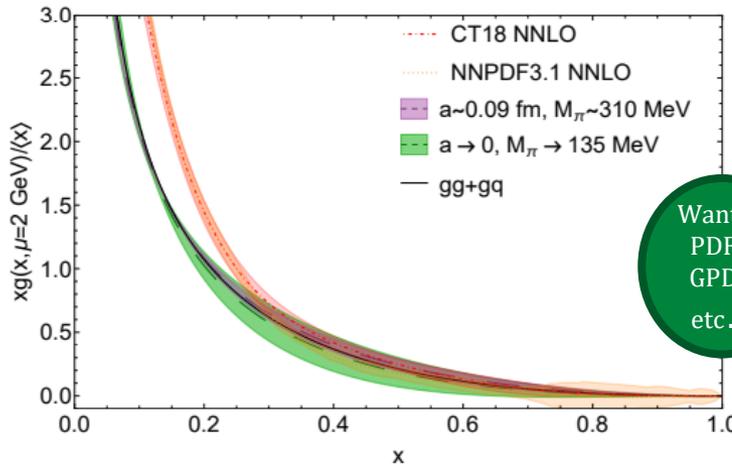
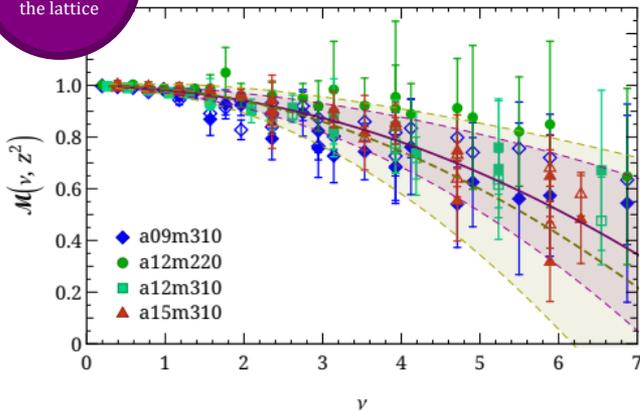
∞ 2+1+1 HISQ {0.09, 0.12, 0.15} fm,

[220,310,700]-MeV pion, 10^5 - 10^6 statistics

Z. Fan et al (MSULat), 2210.09985



Quantities that can be calculated on the lattice



Wanted PDFs, GPDs, etc...



G: Zhouyou Fan

Outline

§ Lattice QCD in a Nutshell

§ Selected x -Dependent Parton Distributions

↪ Nucleon PDFs

↪ **Pion/Kaon PDFs**

↪ GPDs

§ Impact of Lattice-QCD PDFs on Global Fits

Biased selected results toward MSULat students and postdocs



Direct x -Dependent Structure

§ Longstanding obstacle to lattice calculations!

Quantities that can be calculated on the lattice today

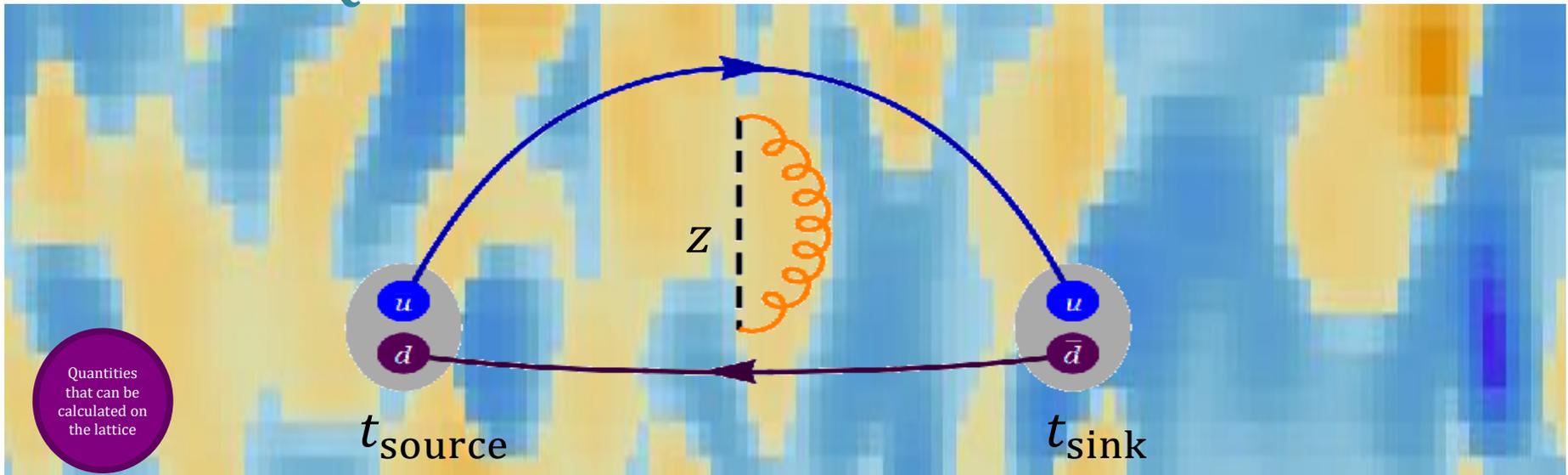
= Σ

Wanted PDFs, GPDs, etc.

\times

pQCD-calculate d kernel

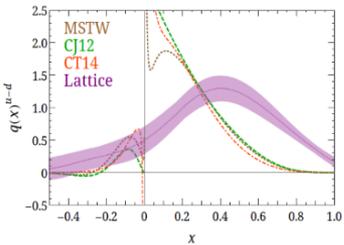
Quasi-PDF & Pseudo-PDF method



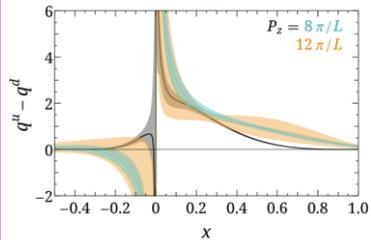
Lattice Parton Calculations

§ Physics quantity milestones

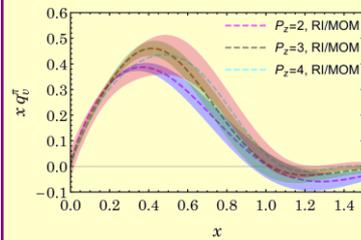
First unpol. lattice PDF



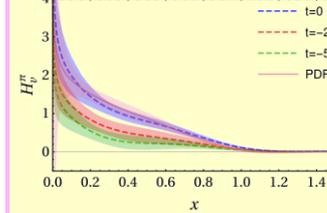
First PDFs at M_π^{phys}



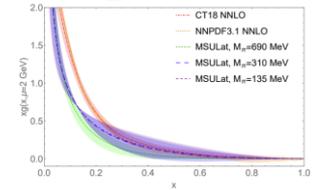
Pion v-PDF



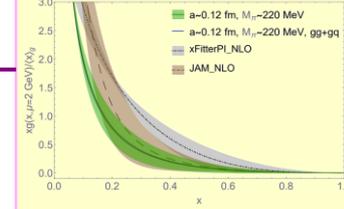
1st GPD (π)



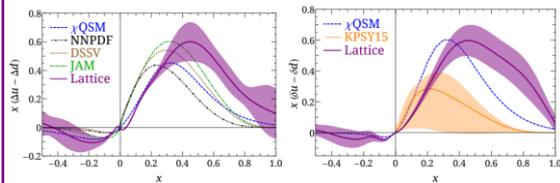
N g-PDF



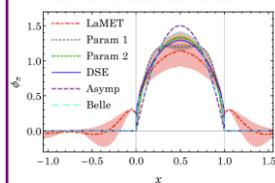
π g-PDF



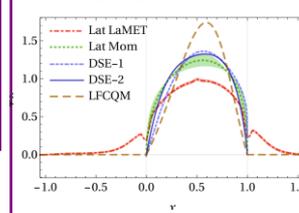
Pol. PDFs and mass corrections



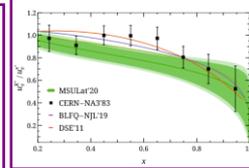
Pion DA



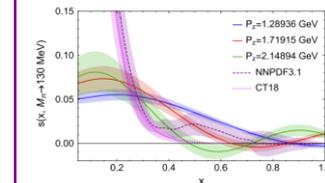
Kaon DA



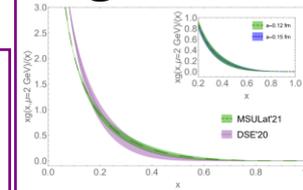
K PDF



s, c PDF

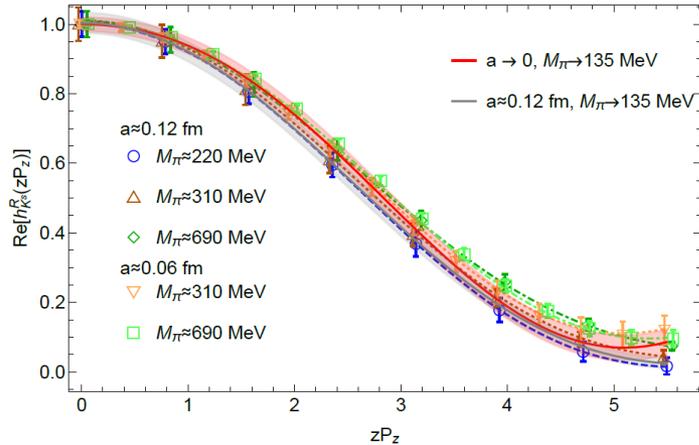
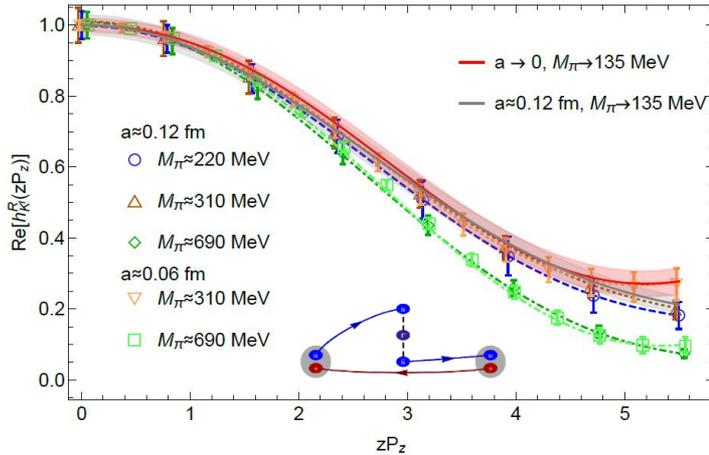


Kaon g-PDF



Meson Valence-quark PDFs

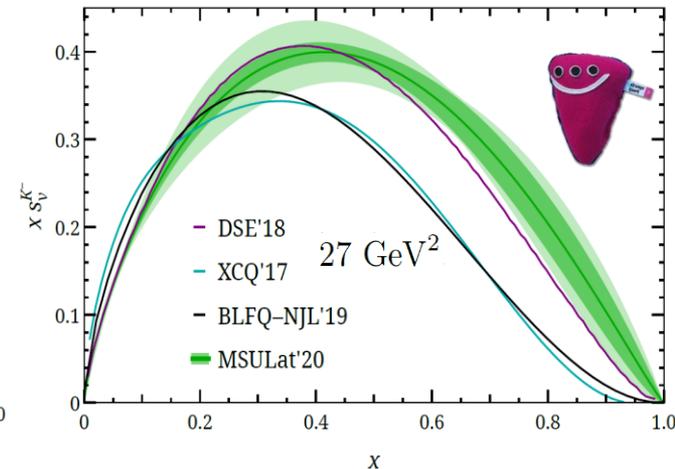
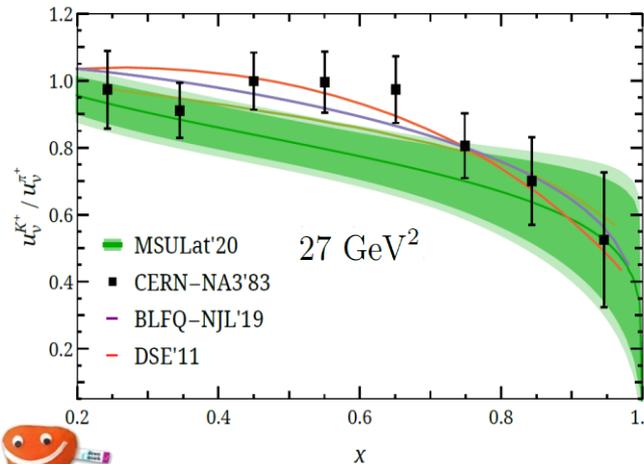
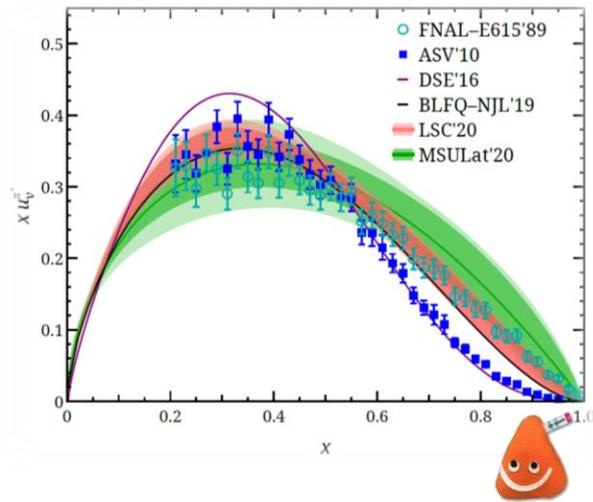
§ Pion/kaon PDFs using quasi-PDF in the continuum limit



Quantities that can be calculated on the lattice

Wanted PDFs, GPDs, etc...

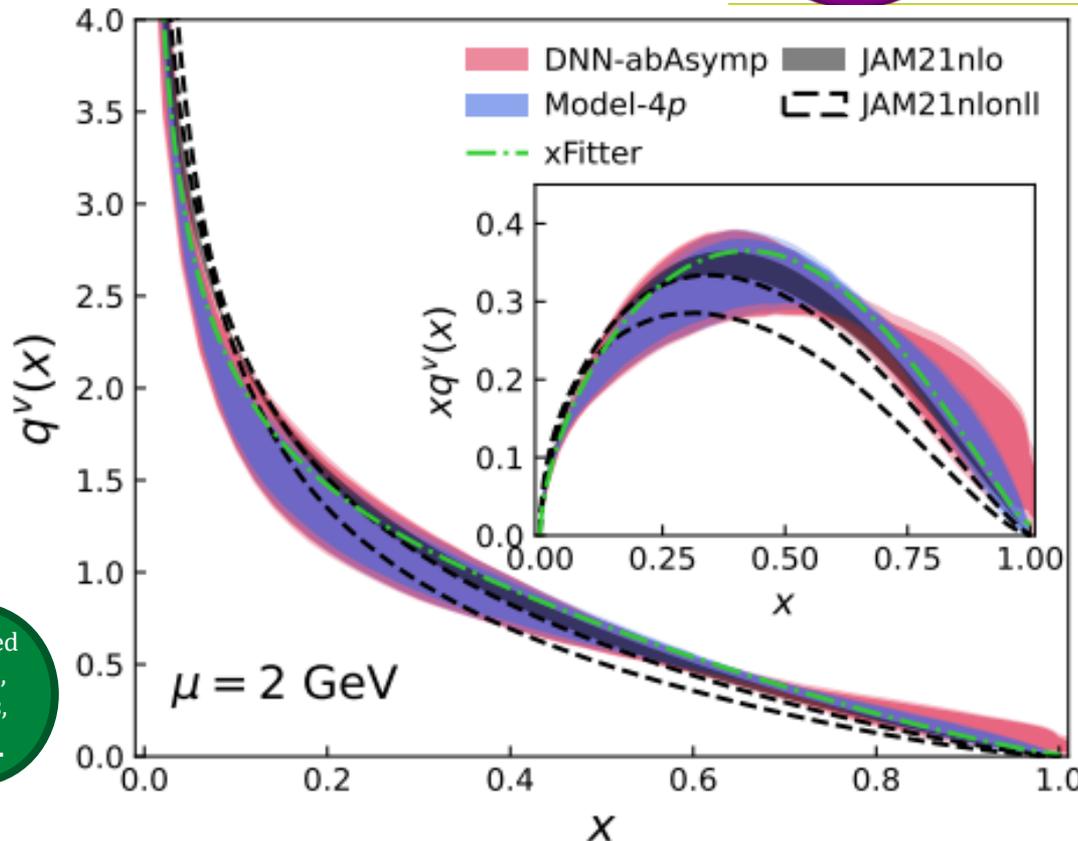
MSULat, 2003.14128



Valence-quark PDFs Update

§ Pion PDFs calculated directly at physical pion mass

↻ with NNLO matching



↻ $N_f=2+1$ clover/HISQ
 $a \sim 0.076$ fm

ANL/BNL, Phys. Rev. D 106, 114510 (2022)

Wanted PDFs, GPDs, etc...

Valence-quark PDFs Update

§ Pion PDFs calculated directly at physical pion mass

∞ NNLO matching & treat leading-renormalon effects

∞ Leading-renormalon resummation (LRR)

∞ Renormalization-group resummation (RGR)

∞ $N_f=2+1+1$ clover/HISQ, $a\sim 0.09$ fm

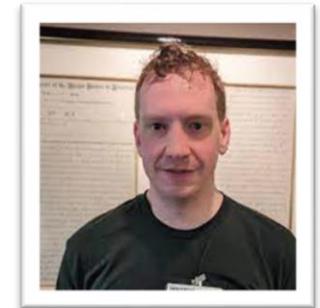
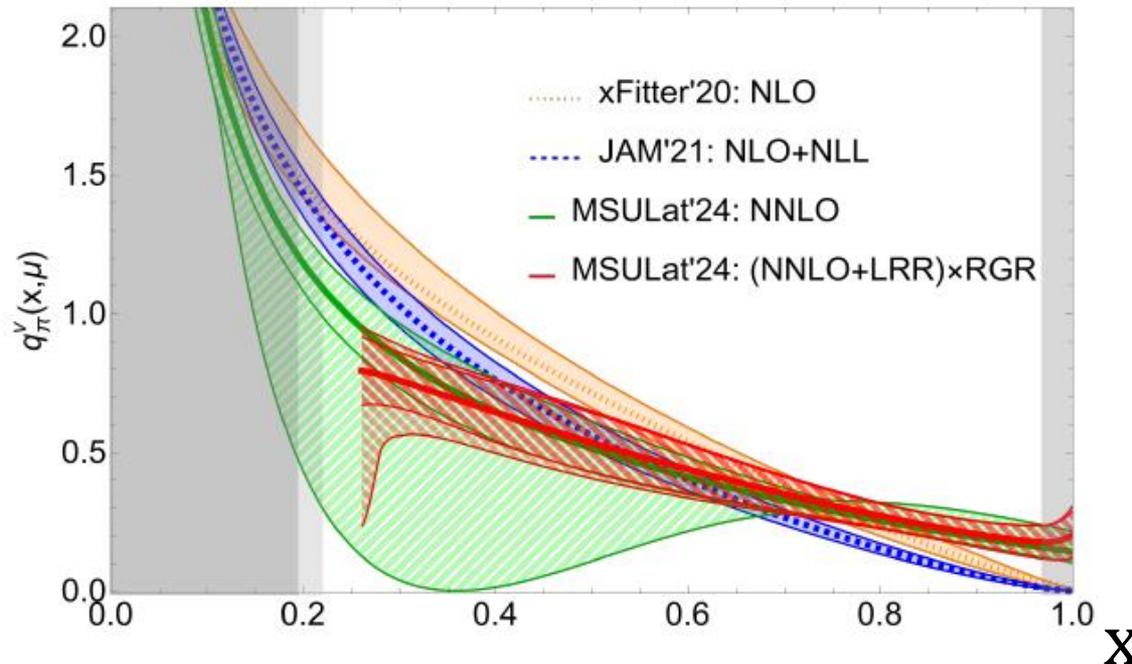
R. Zhang, et. al.

PLB 844, 138081 (2023)



J. Holligan, HL (MSULat), [10.1088/1361-6471/ad3162](https://doi.org/10.1088/1361-6471/ad3162)

Wanted
PDFs,
GPDs,
etc...

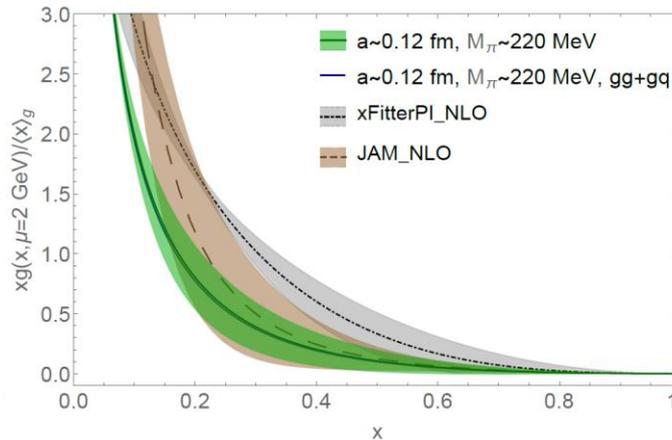
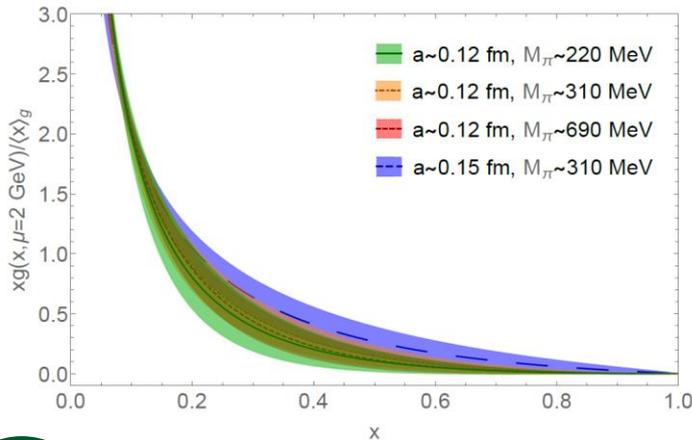


P: Jack Holligan

Meson Gluon PDFs



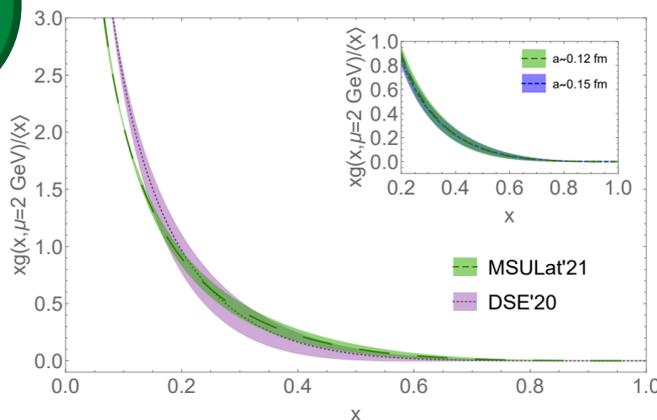
§ First pion and kaon gluon PDFs $g(x)/\langle x \rangle$ using pseudo-PDF



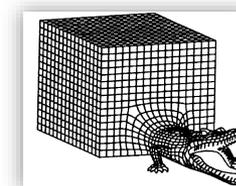
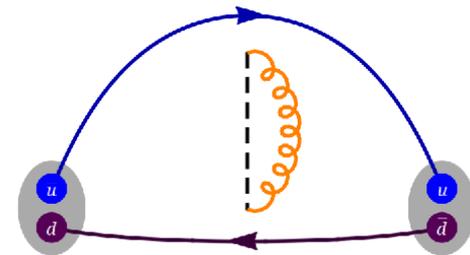
G: Zhouyou Fan

Wanted
PDFs,
GPDs,
etc...

2104.06372, Fan et al. (MSULat); 2112.03124, Salas-Chavira et al. (MSULat)



G: Alejandro Salas-Chavira



finite-volume,
discretization,
heavy quark
mass, ...

§ What does lattice QCD say about $g(x)$?

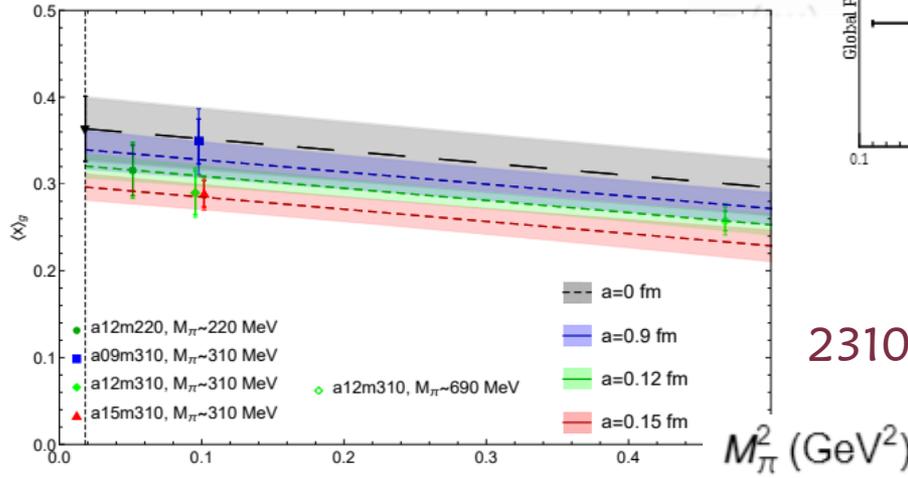
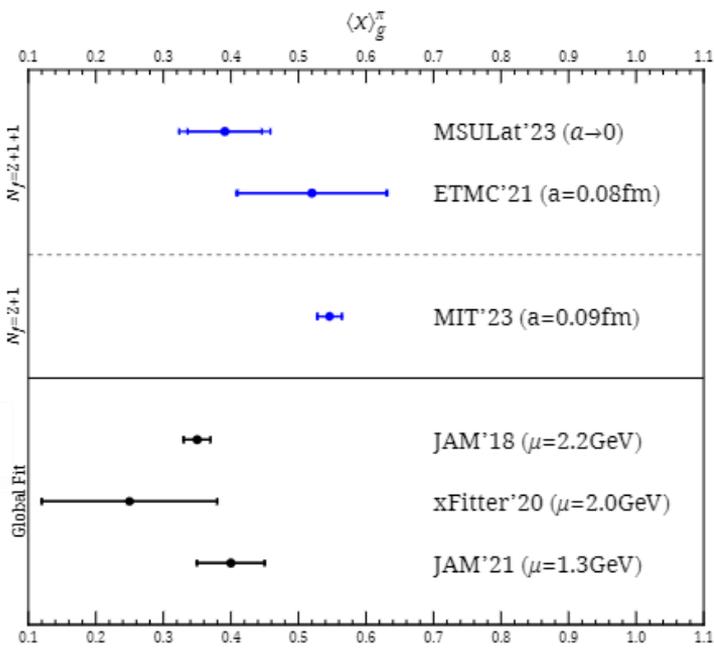
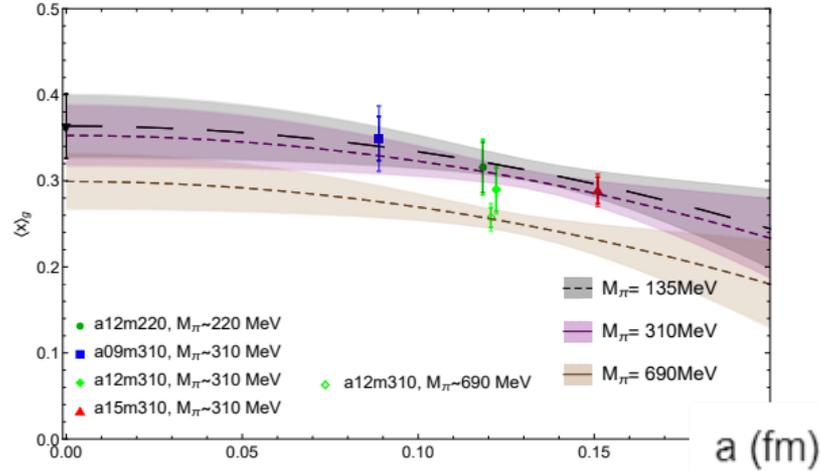


Pion Gluon PDF Update



§ Study discretization systematic in $\langle x \rangle_{\{\pi, g\}}$

∞ Lattice details: clover/HISO. HISO. $a \sim \{0.15, 0.12, 0.09\}$ fm



2310.12034, Good et al. (MSULat)



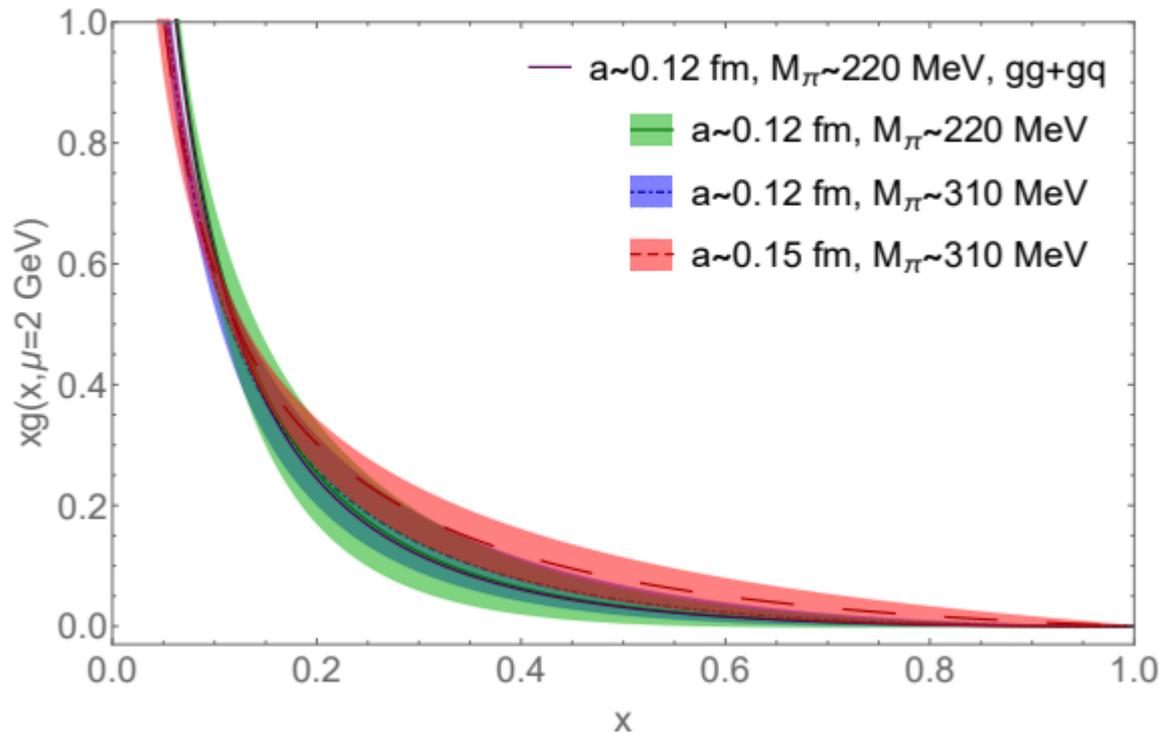
G: Bill Good



Pion Gluon PDF Update

§ Back to Pion gluon PDF $g(x)$

↻ Update previous calculated $g(x)/\langle x \rangle$ in 2021



2310.12034, Good et al. (MSULat)



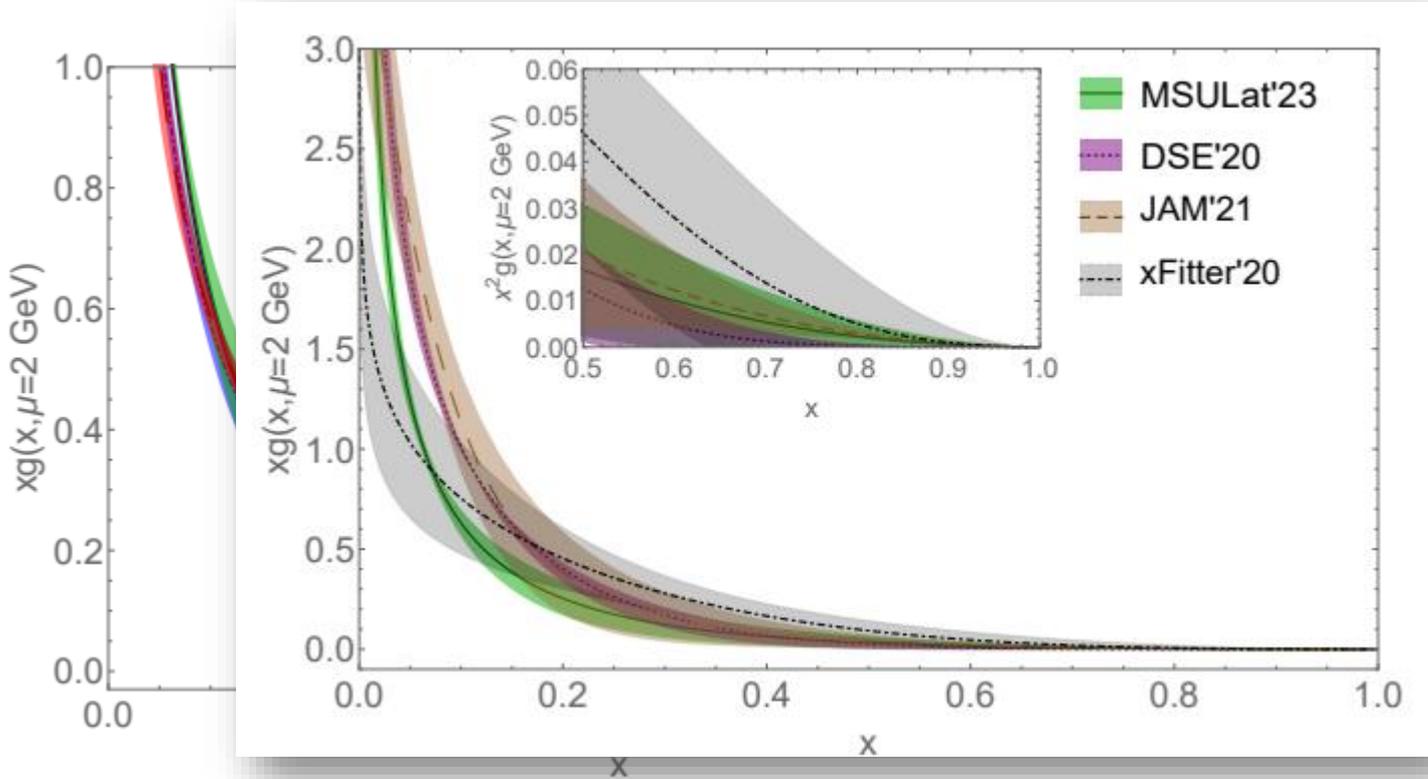
G: Bill Good



Pion Gluon PDF Update

§ Back to Pion gluon PDF $g(x)$

↻ Update previous calculated $g(x)/\langle x \rangle$ in 2021

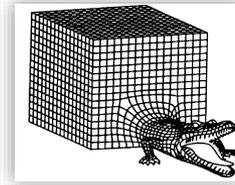


2310.12034, Good et al. (MSULat)

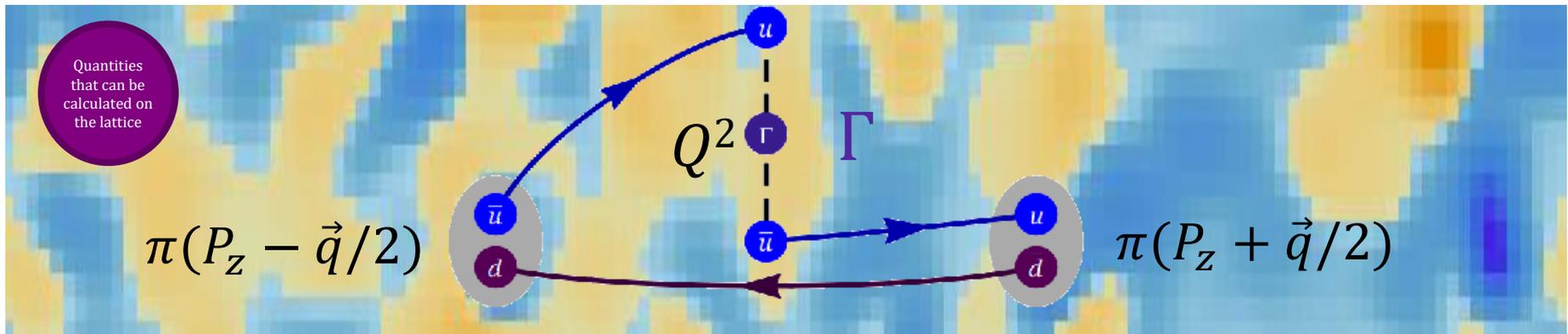
G: Bill Good

Generalized Parton Distributions

Single-ensemble result



finite-volume,
discretization,
heavy quark mass,
...



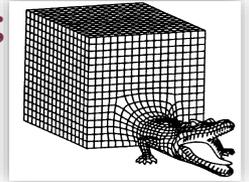
First Lattice GPDs

§ First glimpse into pion GPD using **Quasi-PDF/LaMET**

∞ Lattice details: clover/HISQ, **0.12fm**, **310-MeV** pion mass

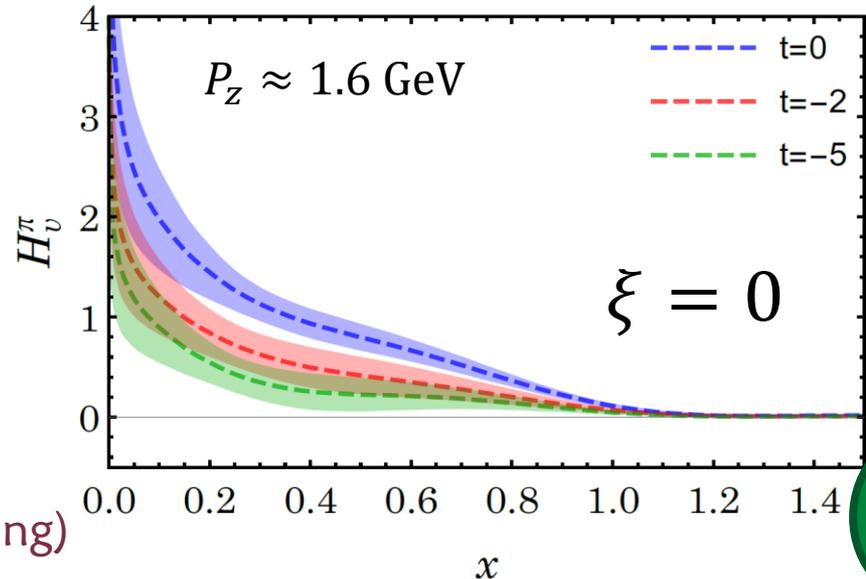
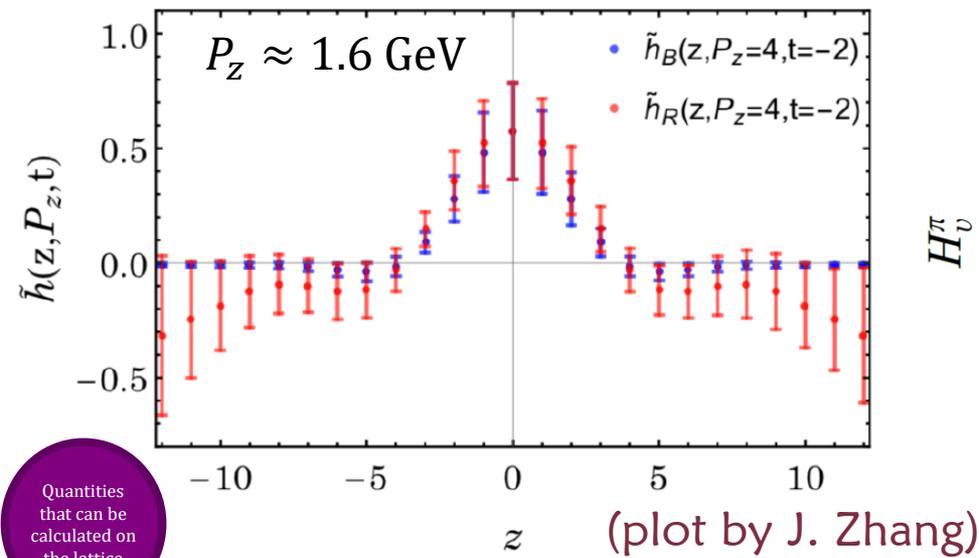
$$P_z \approx 1.3, 1.6 \text{ GeV}$$

MILC, Phys. Rev. D, 82 (2010), 074501;
Phys. Rev. D, 87 (2013), 0545056



J. Chen, HL, J. Zhang, 1904.1237;

$$H_q^\pi(x, \xi, t, \mu) = \int \frac{d\eta^-}{4\pi} e^{-ix\eta^- P^+} \left\langle \pi(P + \Delta/2) \left| \bar{q} \left(\frac{\eta^-}{2} \right) \gamma^+ \Gamma \left(\frac{\eta^-}{2}, -\frac{\eta^-}{2} \right) q \left(-\frac{\eta^-}{2} \right) \right| \pi(P - \Delta/2) \right\rangle$$



Quantities that can be calculated on the lattice

Wanted PDFs, GPDs, etc...

Valence-Quark Pion GPD

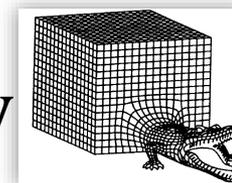
§ Pion GPD (H^π) using quasi-PDFs at physical pion mass

☞ Lattice details: clover/2+1+1 HISQ

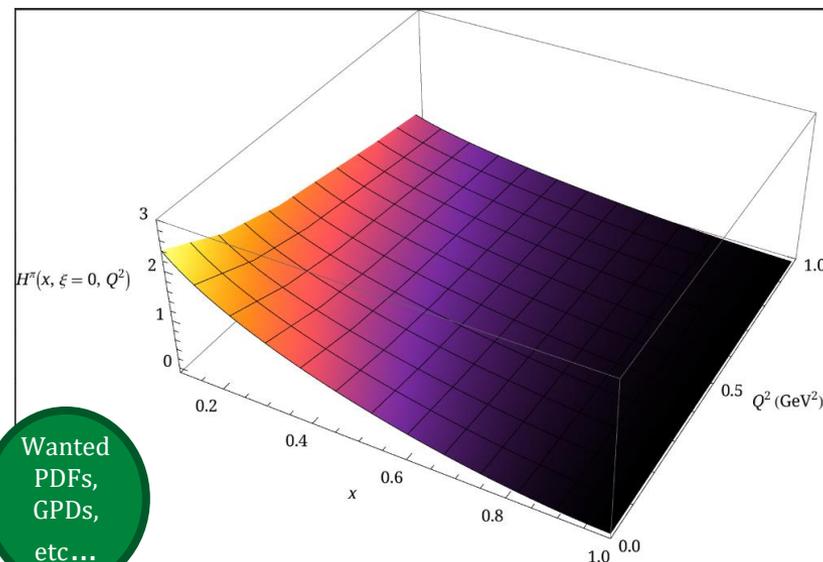
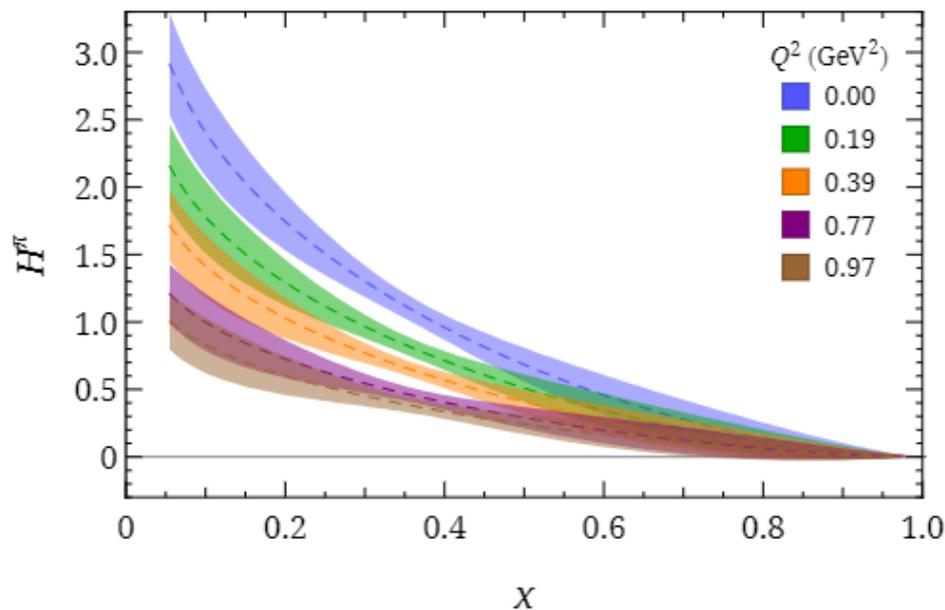
0.09 fm, 135-MeV pion mass, $P_z \approx 1.7$ GeV

☞ $\xi = 0$ valence-quark Pion GPD results

HL (MSULat), Phys. Lett. B 846 (2023) 138181



finite-volume,
discretization,



Wanted
PDFs,
GPDs,
etc...

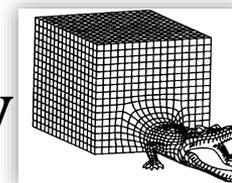
Valence-Quark Pion GPD

§ Pion GPD (H^π) using quasi-PDFs at physical pion mass

∞ Lattice details: clover/2+1+1 HISQ

0.09 fm, 135-MeV pion mass, $P_z \approx 1.7$ GeV

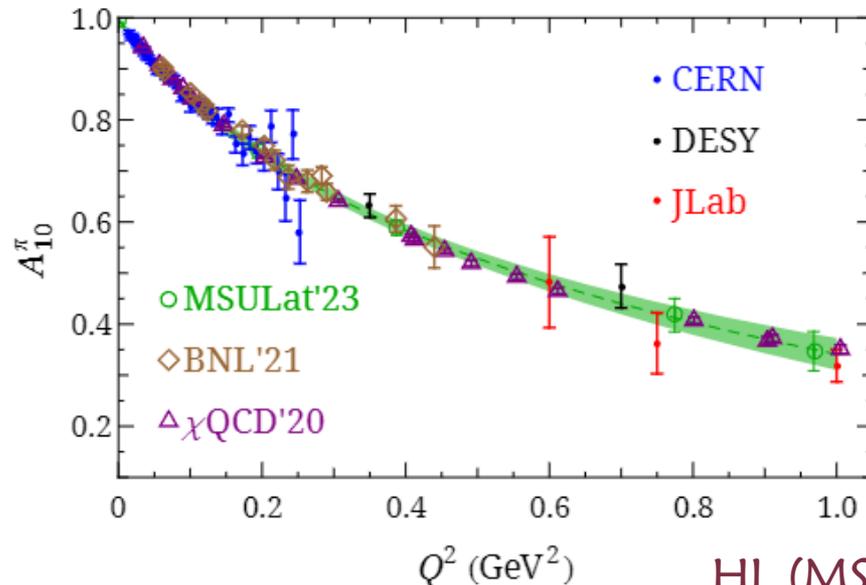
∞ $\xi = 0$ valence-quark Pion GPD results



finite-volume,
discretization,



$$\int_{-1}^{+1} dx x^{n-1} \text{[3D surface plot]} = A_{ni}^\pi(t)$$



HL (MSULat), Phys. Lett. B 846 (2023) 138181

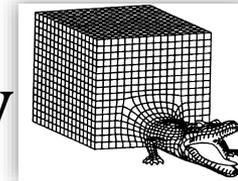
Pion Tomography

§ Nucleon GPD using quasi-PDFs at physical pion mass

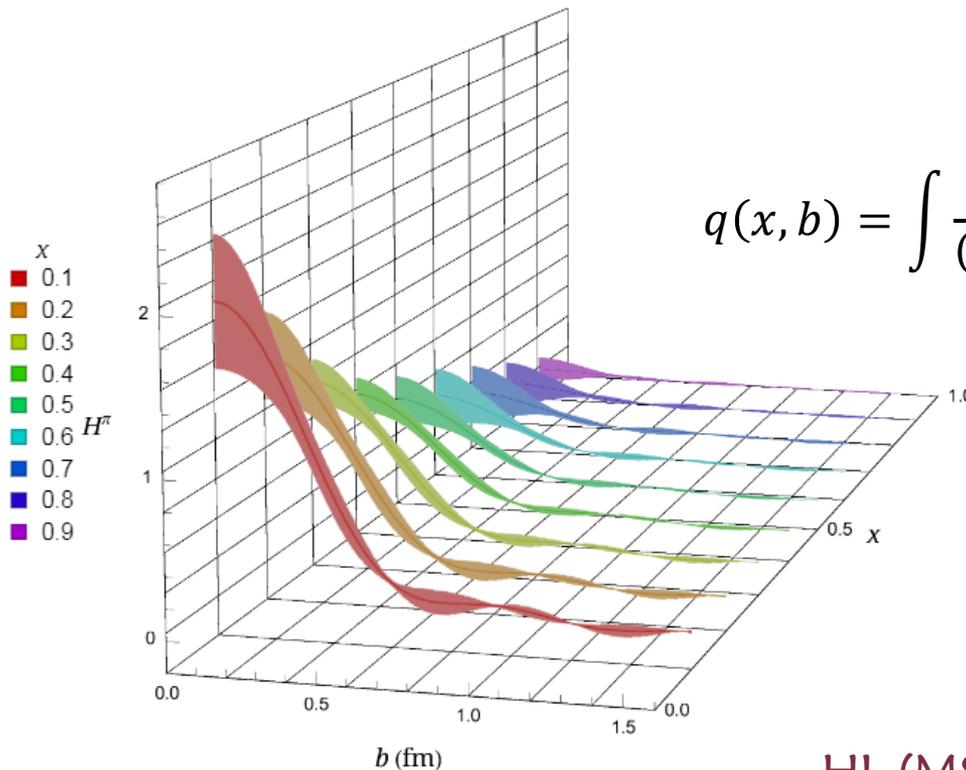
∞ Lattice details: clover/2+1+1 HISQ

0.09 fm, 135-MeV pion mass, $P_z \approx 1.7$ GeV

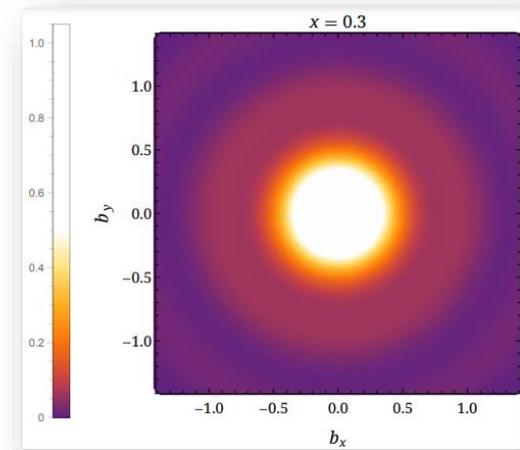
∞ $\xi = 0$ valence-quark Pion GPD results



finite-volume,
discretization,



$$q(x, b) = \int \frac{d\vec{q}}{(2\pi)^2} H(x, \xi = 0, t = -\vec{q}^2) e^{i\vec{q} \cdot \vec{b}}$$

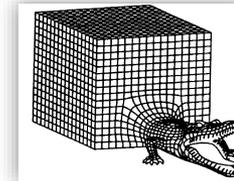


HL (MSULat), Phys. Lett. B 846 (2023) 138181

Nucleon Tomography

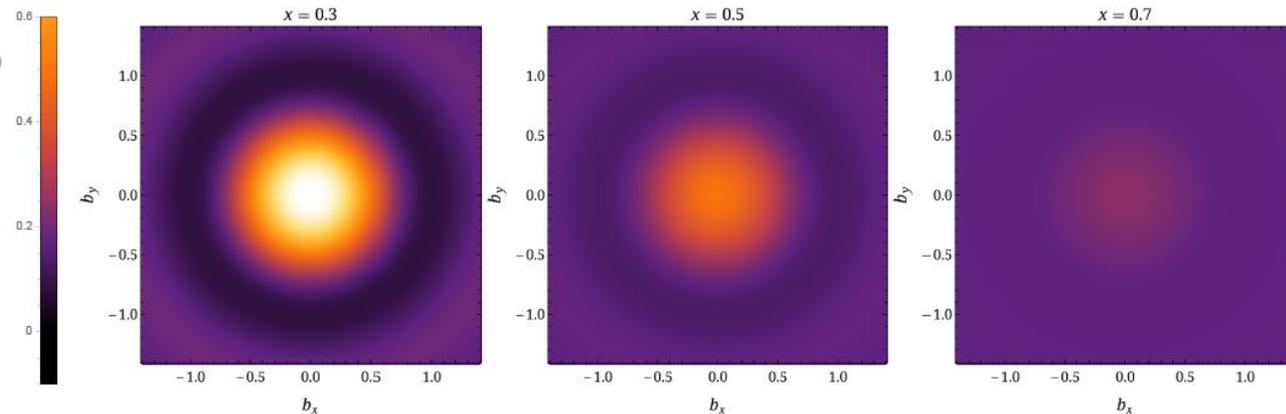
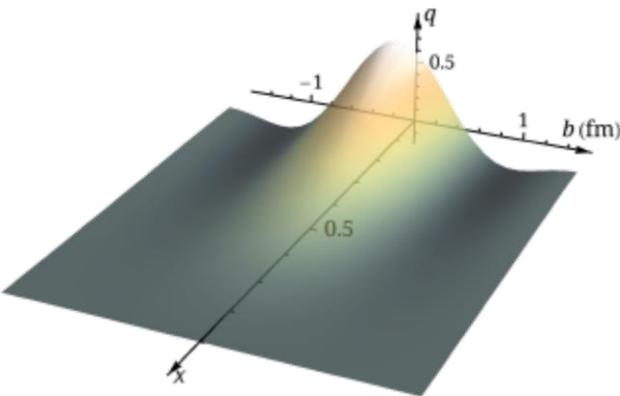
§ Nucleon GPD using quasi-PDFs at physical pion mass

- ∞ Lattice details: clover/2+1+1 HISQ
0.09 fm, 135-MeV pion mass, $P_z \approx 2$ GeV
- ∞ $\xi = 0$ isovector nucleon GPD results



finite-volume,
discretization,

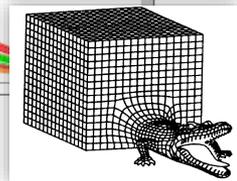
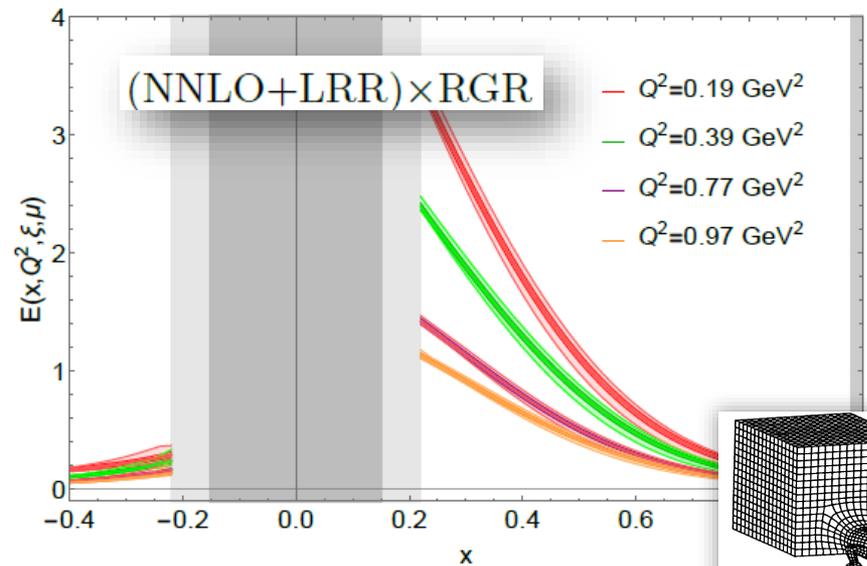
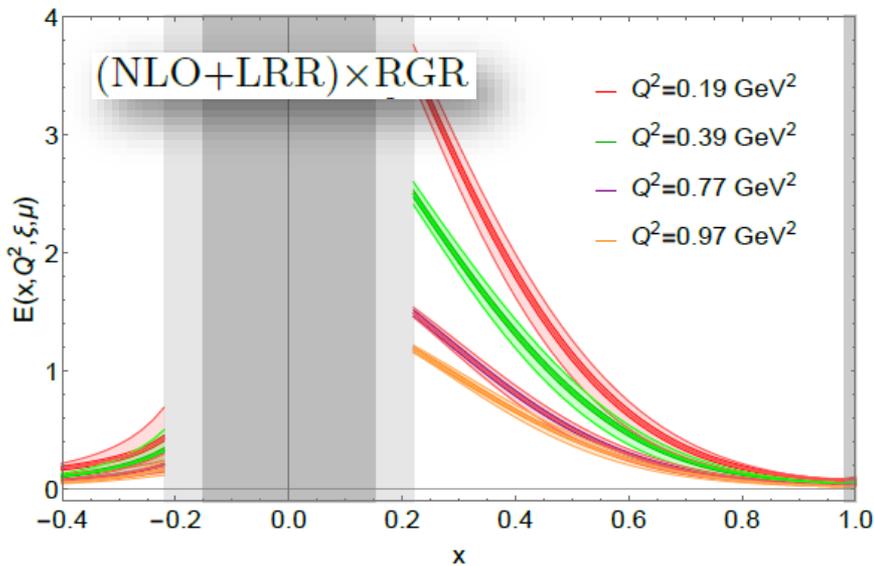
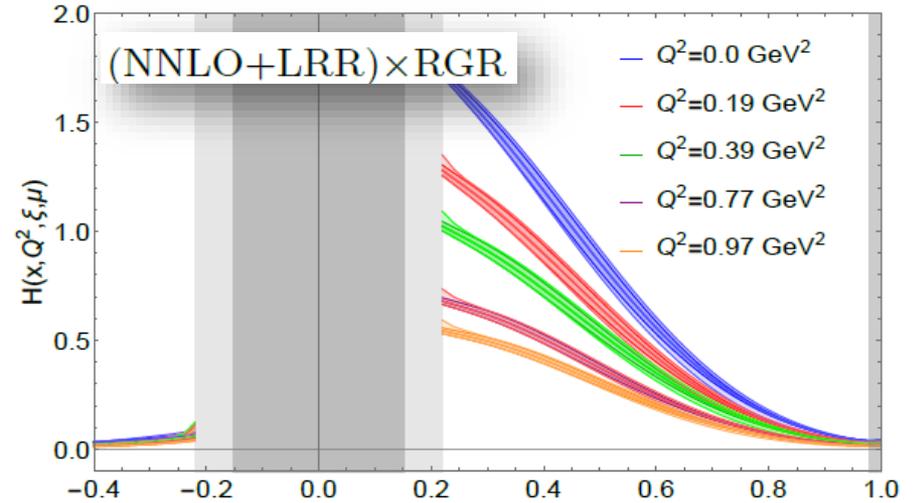
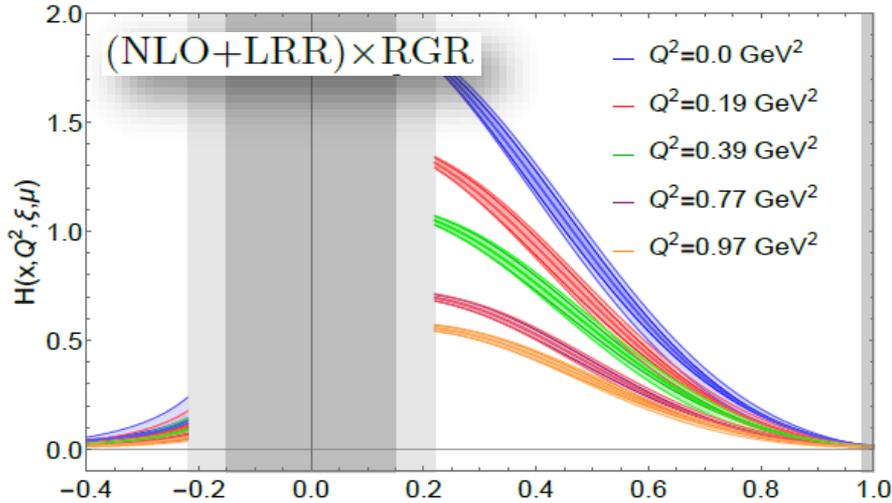
$$q(x, b) = \int \frac{d\vec{q}}{(2\pi)^2} H(x, \xi = 0, t = -\vec{q}^2) e^{i\vec{q} \cdot \vec{b}}$$



HL, Phys.Rev.Lett. 127 (2021) 18, 182001

Also see work done by ANL/BNL/ETMC, [2209.05373](#), [2310.13114](#)

$\xi=0$ GPDs



J. Holligan, HL (MSULat), 2312.10829 [hep-lat]

Outline

§ Lattice QCD in a Nutshell

§ Selected x -Dependent Parton Distributions

↪ Nucleon PDFs

↪ Pion/Kaon PDFs

↪ GPDs

CTEQ

§ Impact of Lattice-QCD PDFs on Global Fits

Biased selected results toward MSULat students and postdocs



First Lattice Strange PDF

§ Results by MSULat/quasi-PDF method

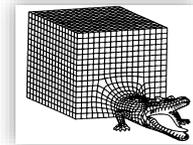
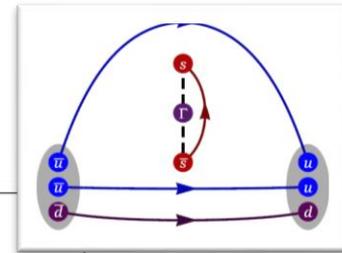
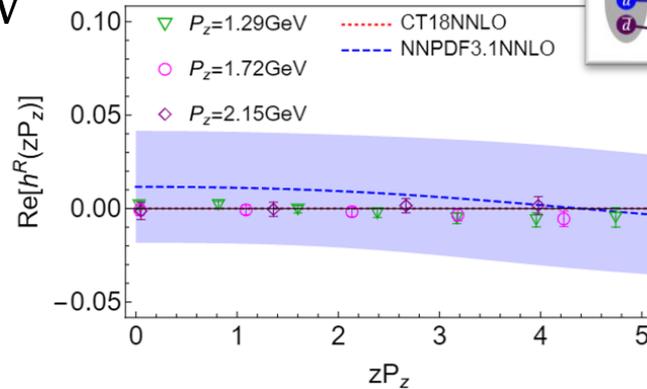
- ☞ Clover on 2+1+1 HISQ, 0.12-fm 310-MeV QCD vacuum
- ☞ Extrapolated to $M_\pi \approx 140$ MeV

R. Zhang et al (MSULat),
2005.01124

$$\text{Re}[h(z)] \propto \int dx (s(x) - \bar{s}(x)) \cos(xzP_z)$$



G: Rui Zhang



Quantities that can be calculated on the lattice

Lattice Strangeness Asymmetry Impact

§ Results by MSULat/quasi-PDF method

- ☞ Clover on 2+1+1 HISQ, 0.12-fm 310-MeV QCD vacuum
- ☞ Extrapolated to $M_\pi \approx 140$ MeV

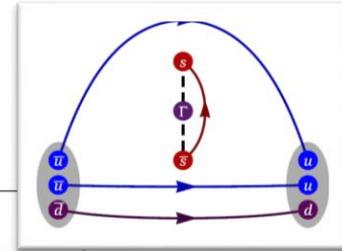
R. Zhang et al (MSULat),
2005.01124

$$\text{Re}[h(z)] \propto \int dx (s(x) - \bar{s}(x)) \cos(xzP_z)$$

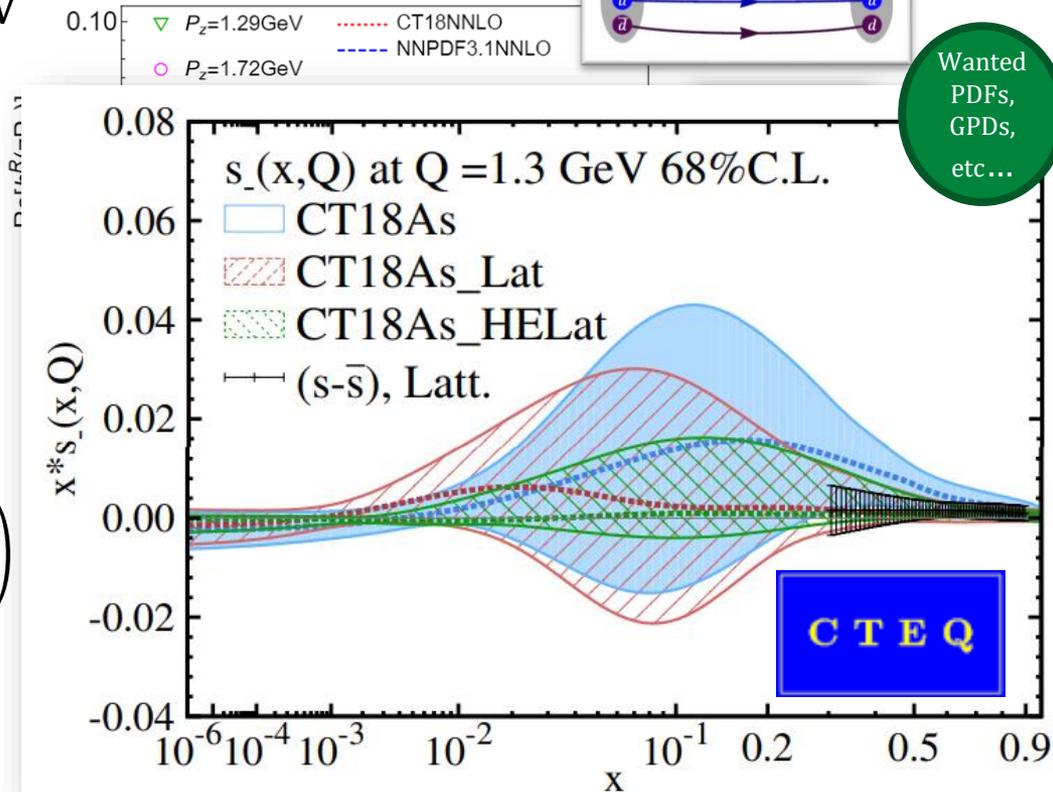
§ From quasi-PDF to PDF

$$\tilde{f}_q(x, P_z) = \int_{-1}^1 \frac{dy}{|y|} f_q(y) C_{q/q}(x, y, P_z, \mu) + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}^2}{x^2 P_z^2}, \frac{\Lambda_{\text{QCD}}^2}{(1-x)^2 P_z^2}\right)$$

T. Hou, HL, M. Yan, C. Yuan,
2211.11064



Wanted
PDFs,
GPDs,
etc...



§ The strangeness asymmetry $s(x, Q) - \bar{s}(x, Q)$ at $x > 0.2$ is difficult to measure, but can be predicted in lattice QCD

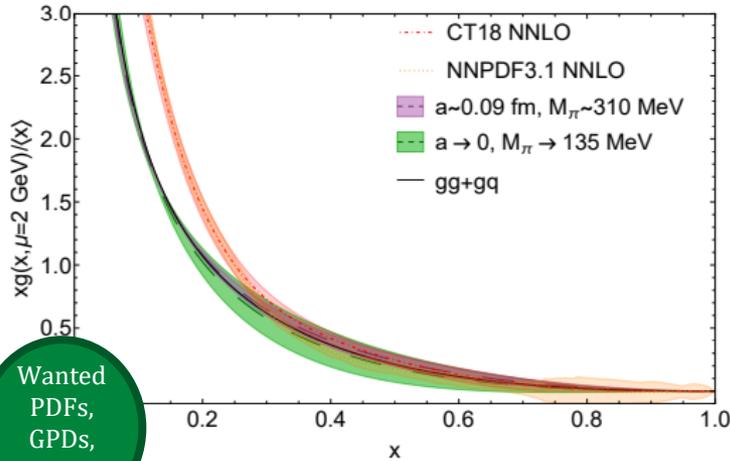
Lattice Gluon PDF Impact

§ First continuum Gluon PDF w/ pseudo-PDF

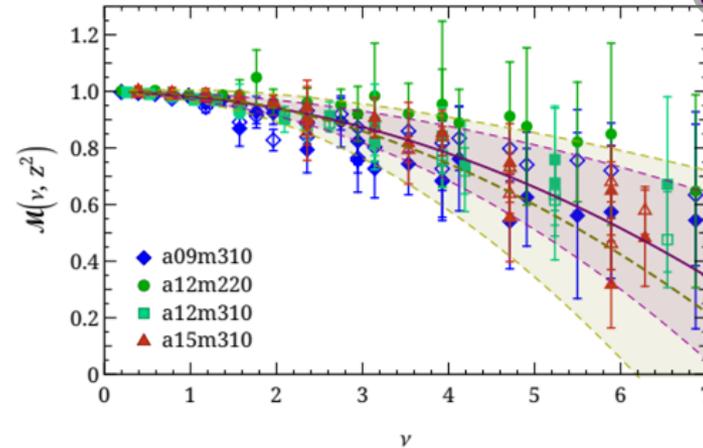
∞ 2+1+1 HISQ {0.09, 0.12, 0.15} fm,

[220,310,700]-MeV pion, 10^5 - 10^6 statistics

2210.09985, W. Good et al (MSULat)



Wanted PDFs, GPDs, etc...

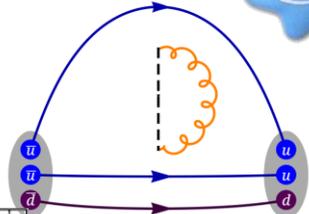
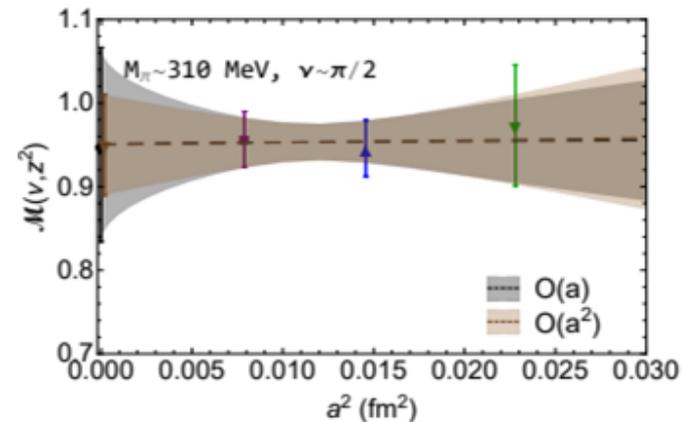


Quantities that can be calculated on the lattice

§ Can use $a \approx 0.09$ fm results as

∞ Best estimate of gluon PDF

∞ Aim for future precision



Lattice Gluon PDF Impact

§ Preliminary study with CTEQ-TEA analysis

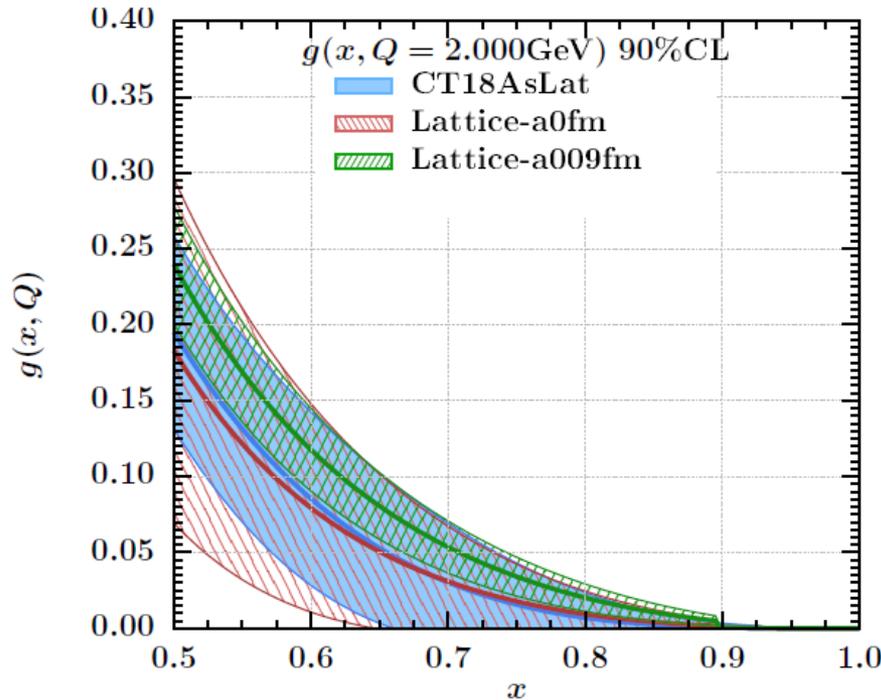


☞ Take lattice inputs in the region where no strong experimental data constraints, $x \in [0.4, 0.7]$

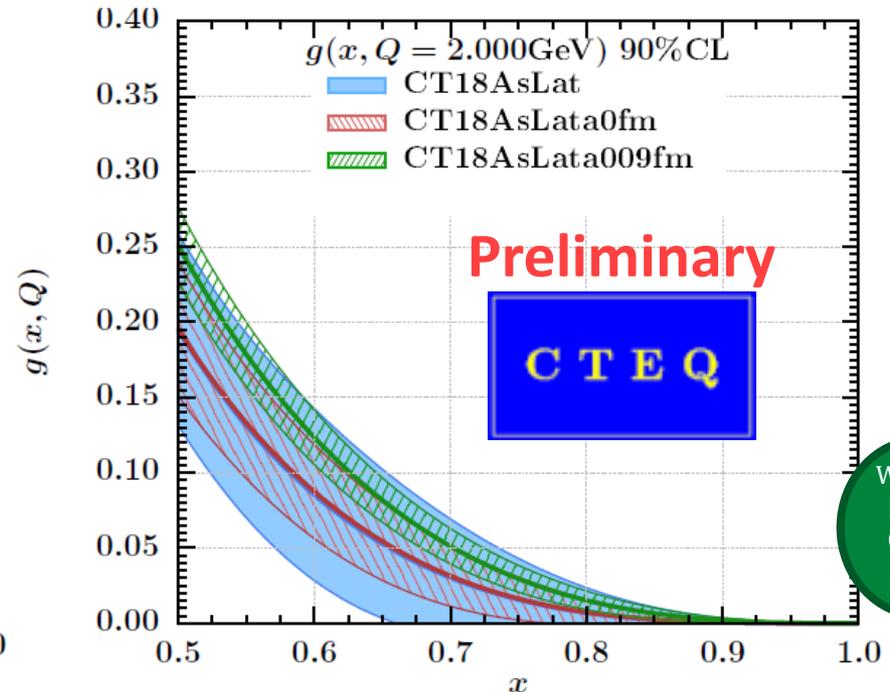
☞ Using e-pump for re-weighting

Plots by Alim Ablat (Xinjiang U.)

Before



After



Wanted
PDFs,
GPDs,
etc...

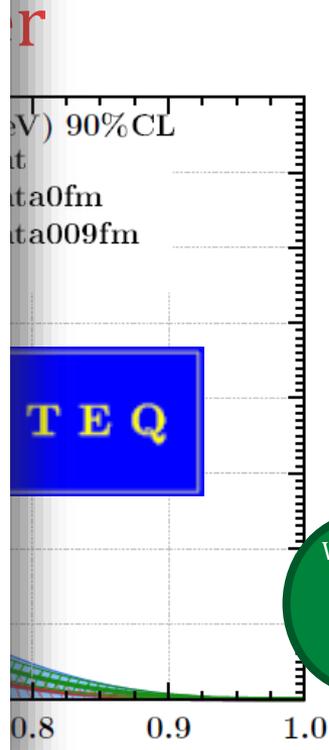
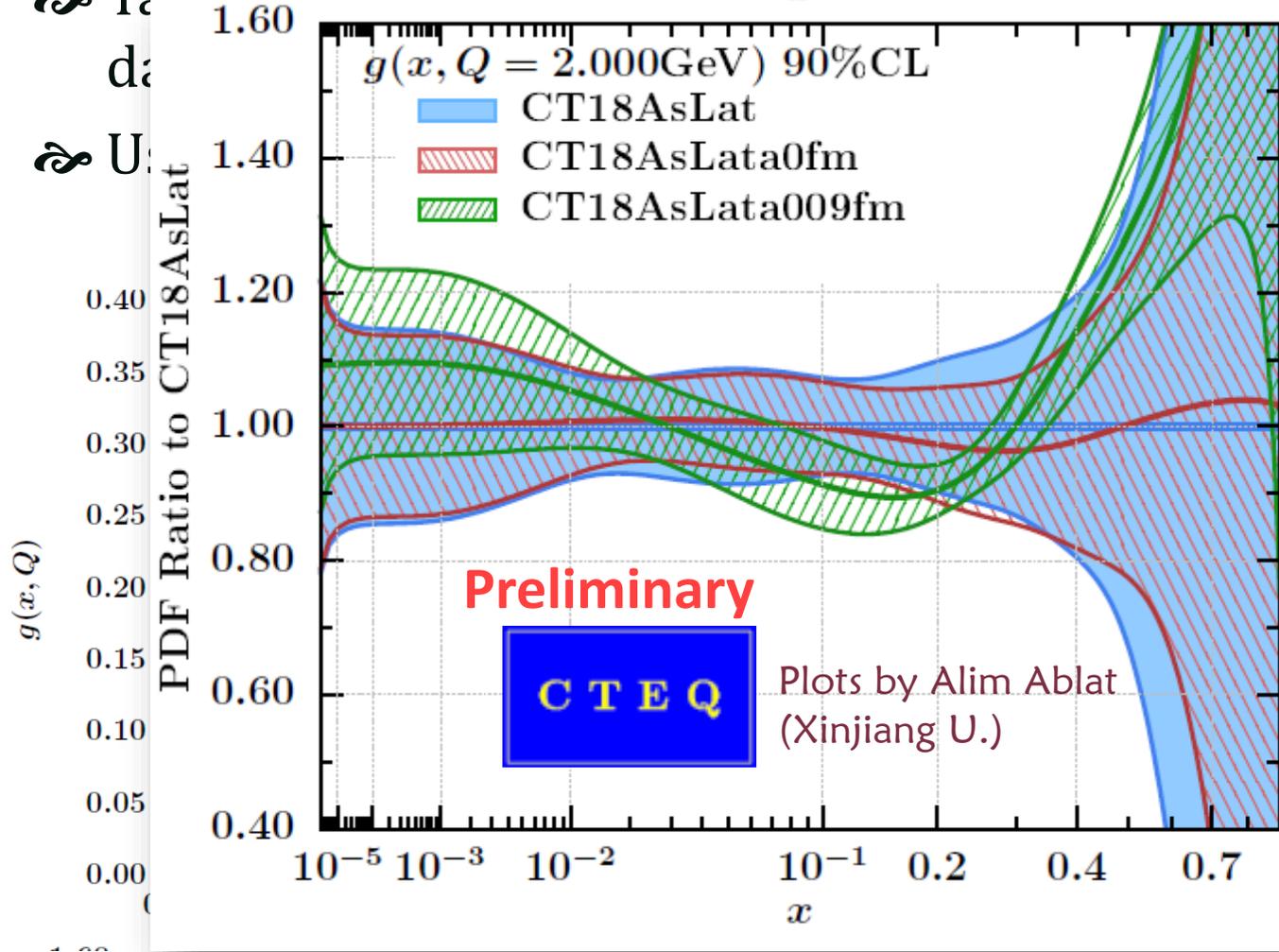
Lattice Gluon PDF Impact

§ Preliminary study with CTEQ-TEA analysis



Take lattice inputs in the region where no strong experimental data

Use



Wanted PDFs, GPDs, etc...

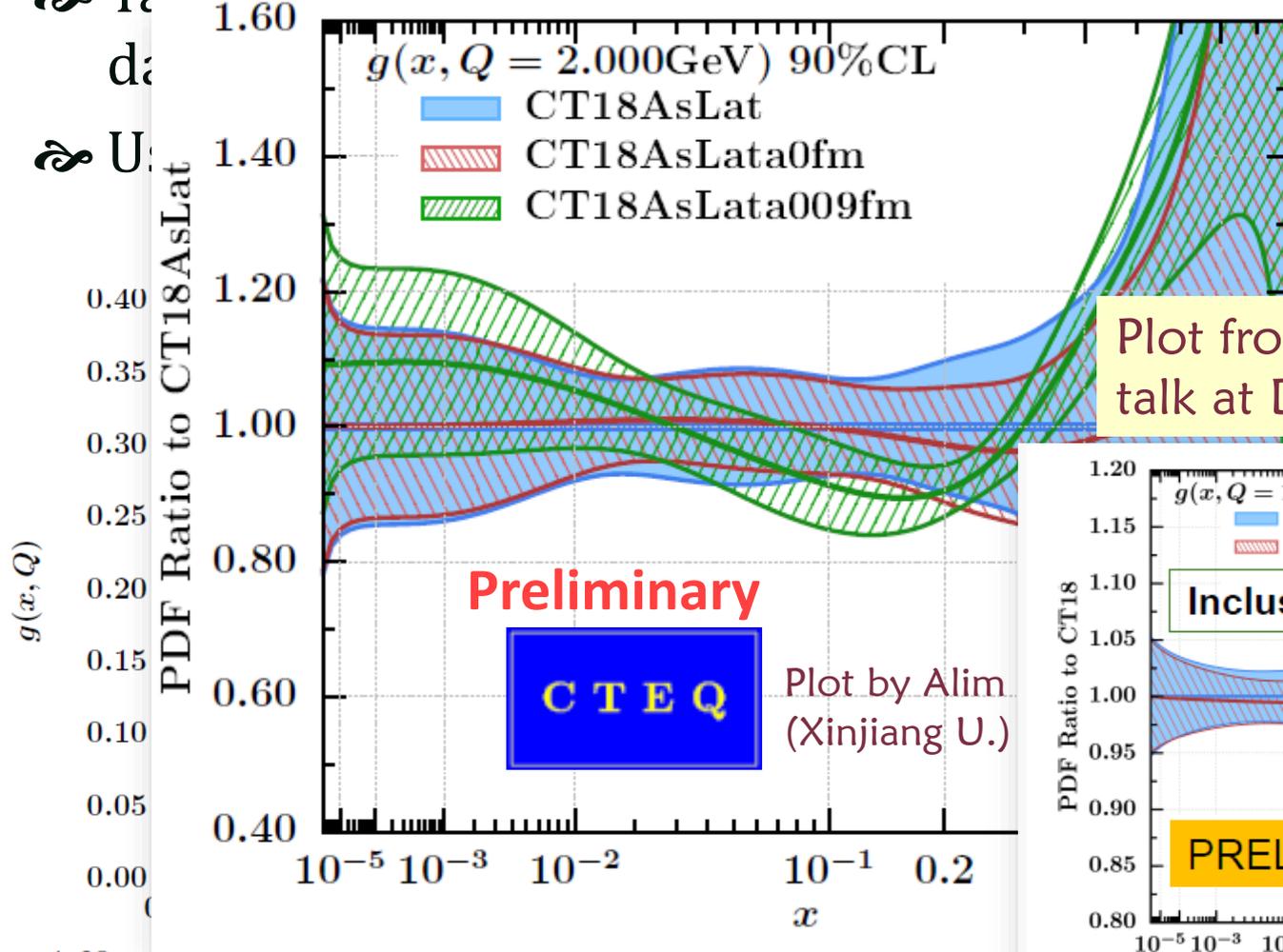
Lattice Gluon PDF Impact

§ Preliminary study with CTEQ-TEA analysis

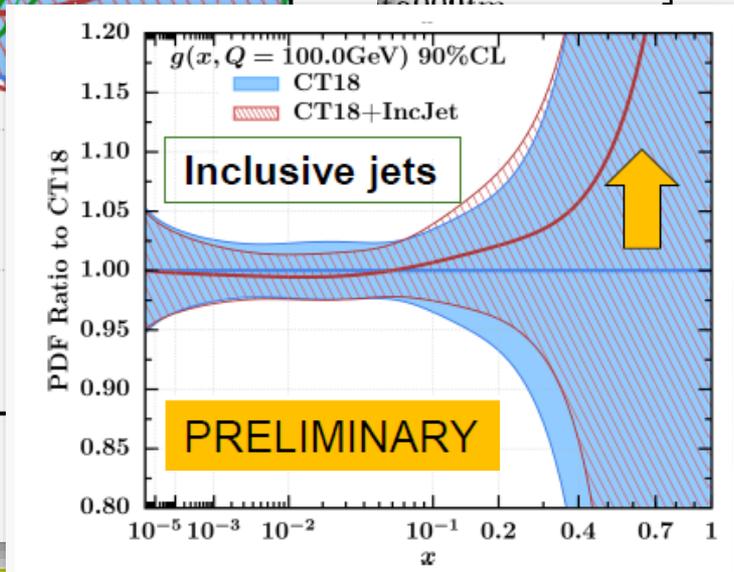


Take lattice inputs in the region where no strong experimental data

Use



Plot from P. Nadolsky's talk at DIS2024



nted
Fs,
Ds,
...

First Lattice Charm PDF

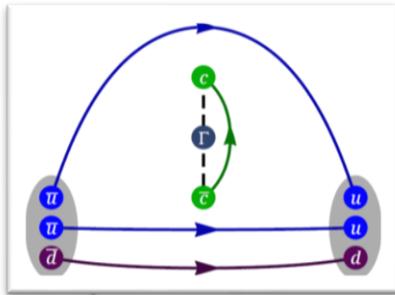


§ Large uncertainties in global PDFs

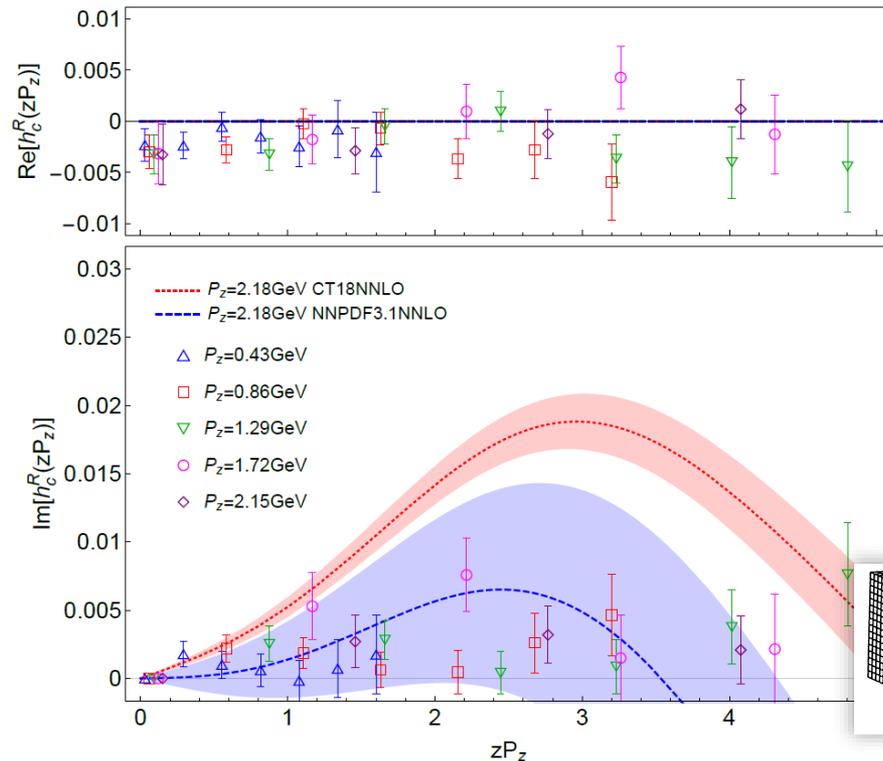
§ Results by MSULat/quasi-PDF method

☞ Clover on 2+1+1 HISQ 0.12-fm 310-MeV QCD vacuum

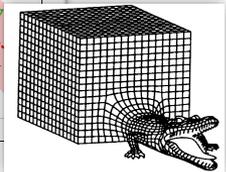
2005.01124, R. Zhang et al (MSULat)



- suggest a symmetric $c - \bar{c}$ distribution
- much smaller than strange PDF



Quantities that can be calculated on the lattice



First Lattice Charm PDF

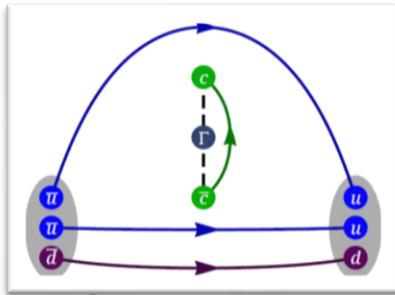


§ Large uncertainties in global PDFs

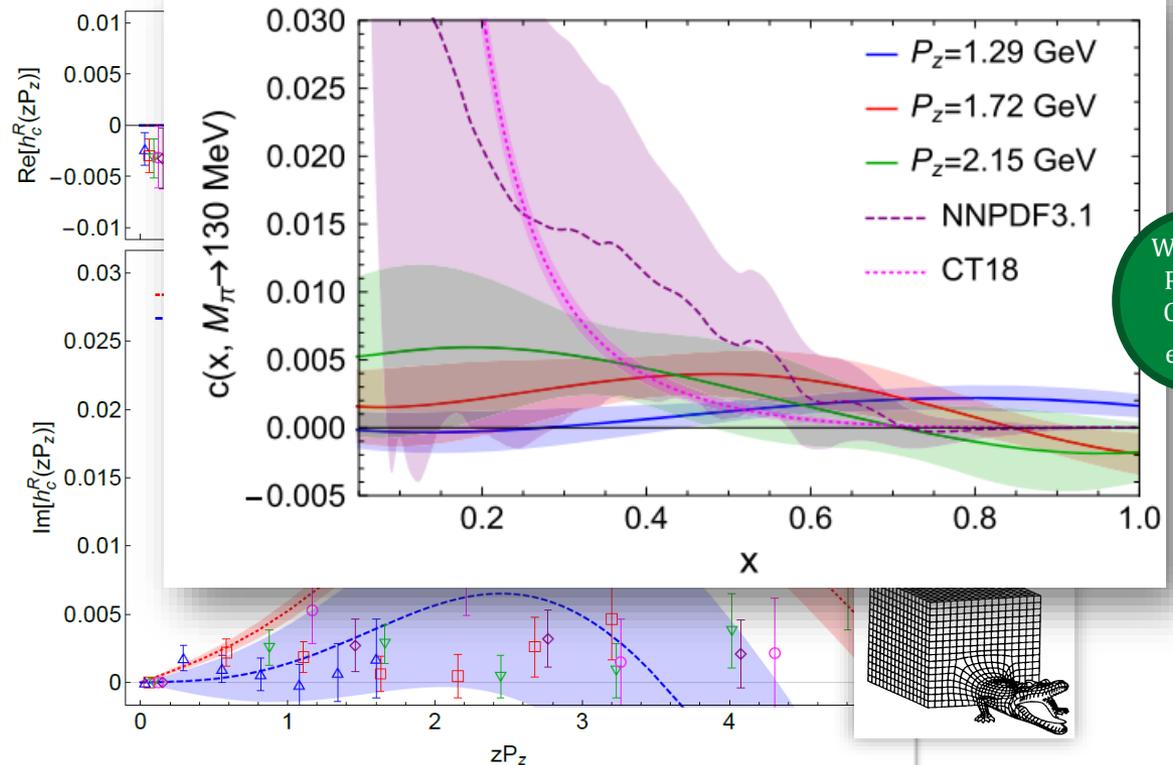
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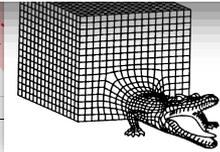
2005.01124, R. Zhang et al (MSULat)



- suggest a symmetric $c - \bar{c}$ distribution
- much smaller than strange PDF



Wanted PDFs, GPDs, etc...



Lattice Progress & Challenges

§ Beyond the standard twist-2 collinear PDFs

- ↻ Generalized parton distributions (GPDs) for the pion and unpolarized/polarized nucleon
- ↻ Transverse-momentum- dependent distributions (TMDs)
 - ↻ Collins-Soper kernel, soft function and wavefunctions
- ↻ Twist-3 PDFs and GPDs

For more details and references, refer to 2202.07193

§ Challenges ahead for precision PDFs

- ↻ Large momentum is essential
 - ↻ With sufficient statistics nucleons may reach 5 GeV
- ↻ Methods for signal-to-noise improvement
 - ↻ Gluonic observables, new ideas for large momentum
- ↻ Access small-x physics; some methods have inverse problem in PDF extraction, more computational resources, etc.

Summary and Outlook

§ Exciting era using LQCD to study x -dependent PDFs

§ Overcoming longstanding limitations

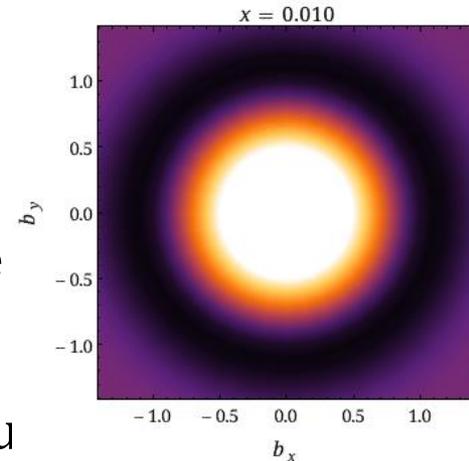
⇒ Bjorken- x dependence of parton distributions
now widely studied

⇒ More study of systematics planned for the near future

§ Lattice strange and gluon PDFs can have impacts

⇒ Treat lattice matrix elements as expt inputs in the futu

§ Precision and progress are limited by resources

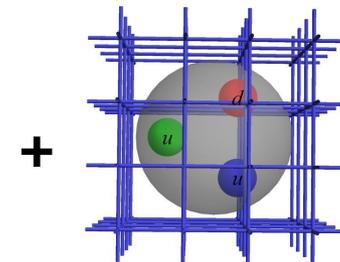


Theory
Input



Global Analysis
of PDFs/GPDs

Exp't
Input



EXCLAIM

Thanks to MILC collaboration for sharing their 2+1+1 HISQ lattices & USQCD/NSF/DOE for computational resources
This work is partially sponsored by grants NSF PHY 1653405 & 1653405, DOE DE-SC0024053 & RCSA Cottrell Scholar

Award

Huey-Wen Lin — Strong QCD from Hadron Structure Experiments @ Nanjing University

Students Wanted

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