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# Nucleon charge and magnetisation distributions: flavour separation and zeroes

**Zhao-Qian Yao**

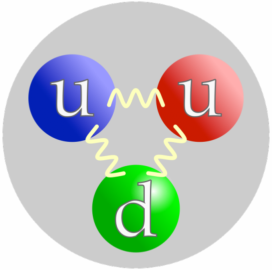
ECT*, Trento

15th May 2024, Nanjing

# Nucleon: Proton&Neutron



## Proton

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- Nature's most fundamental bound state;
- Proton($p$): $u + u + d$
- Neutron($n$): $d + d + u$

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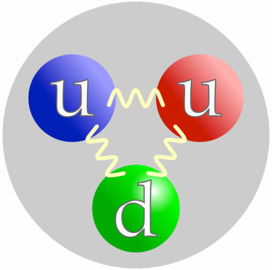


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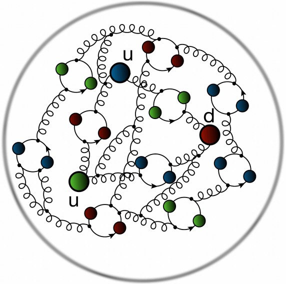


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- Consists of three valence-quark and infinitely many gluons and sea-quarks.
- The nucleon bound-state problem can be addressed by three-quark six-point Schwinger function.
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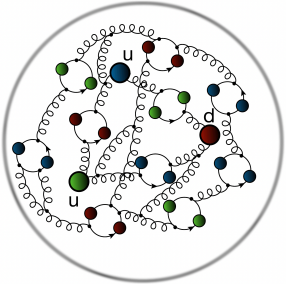


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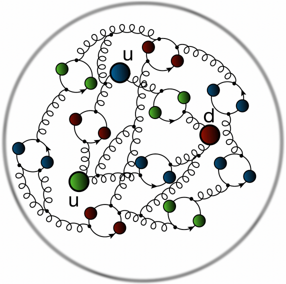


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# CSMs: Gap Equation - one quark

- **Dressed quark propagator**

$$S^{-1}(p) = i\gamma \cdot pA(p^2) + B(p^2)$$

- **Gap Equation**

$$S^{-1}(p) = Z_2 i\gamma \cdot p + Z_4 m^{\zeta_{19}} + Z_2^2 \int \frac{d^4 q}{(2\pi)^4} g^2 D_{\mu\nu}(p-q) \frac{\lambda^a}{2} \gamma_\mu S(q) \frac{\lambda^a}{2} \Gamma_\nu(q,p).$$

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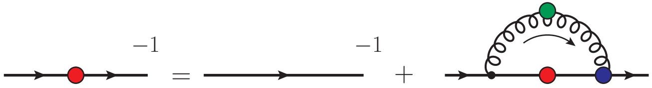

# CSMs: Bethe-Salpeter Equation - two quarks

- Mesons appear as poles in the 4-point Schwinger function;
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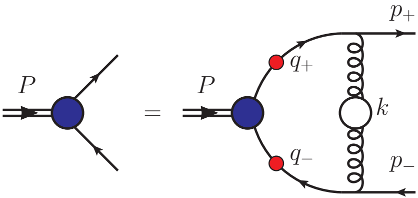

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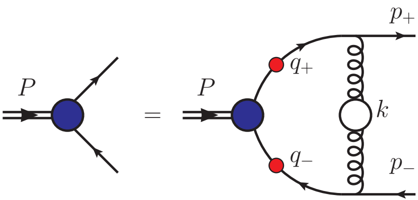


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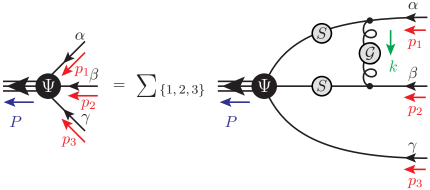
$$\Psi^{(3)}_{ABCD}(p,q,P) = \int_k \mathrm{K}_{AA'BB'}(k) S_{A'A''}(k_1) S_{B'B''}(\tilde{k}_2) \times \Psi_{A''B''CD}(p^{(3)}, q^{(3)}, P),$$



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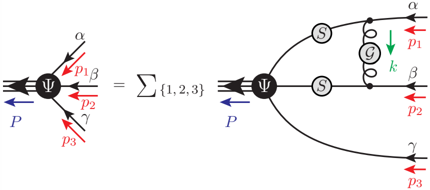

# CSMs: Faddeev Equation - three quarks

- **Faddeev Amplitude**

$$\Psi_{ABCD}(p,q,P) = \left( \sum_{\rho=0}^{1} \psi^{\rho}_{\alpha\beta\gamma\mathcal{I}}(p,q,P) \otimes \mathrm{F}^{\rho}_{abcd} \right) \otimes \frac{\epsilon_{rst}}{\sqrt{6}},$$

- the color term $\frac{\epsilon_{rst}}{\sqrt{6}}$ fixes the baryon to be a color singlet;
- the flavor terms $\mathrm{F}^{\rho}_{abcd}$ are the quark model $\mathrm{SU}(2)$ representations.

$$\mathrm{F}^0 : \text{mixed} - \text{antisymmetric}, \;\; \mathrm{F}^1 : \text{mixed} - \text{symmetric};$$

| | $\mathrm{F}^0$ | $\mathrm{F}^1$ |
|---|---|---|
| $p$ | $\frac{1}{\sqrt{2}}(udu - duu)$ | $\frac{1}{\sqrt{6}}(2uud - udu - duu)$ |
| $n$ | $\frac{1}{\sqrt{2}}(udd - dud)$ | $\frac{1}{\sqrt{6}}(udd + dud - 2ddu)$ |

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- **Spin-momentum Faddeev amplitude**
  It can be expressed in terms of the basis $\mathrm{X}_{i,\alpha\beta\gamma\mathcal{I}}(p,q,P)$ which satisfy the orthogonal relation

$$\frac{1}{4}\mathrm{Tr}[\overline{\mathrm{X}}_{i,\beta\alpha\delta\gamma}\mathrm{X}_{j,\alpha\beta\gamma\delta}] = \delta_{ij},$$

reading

$$\psi^{\rho}_{\alpha\beta\gamma\mathcal{I}}(p,q,P) := \sum_{i}^{64} f_i^{\rho}(p^2,q^2,z_0,z_1,z_2)\mathrm{X}_{i,\alpha\beta\gamma\mathcal{I}}(p,q,P),$$

$f_i^{\rho}$ (to be determined) depend on the five Lorentz-invariant variables.

$$p^2;\ q^2;\ z_0 = \widehat{p}_T\cdot\widehat{q}_T;\ z_1 = \widehat{p}\cdot\widehat{P};\ z_2 = \widehat{q}\cdot\widehat{P}.$$

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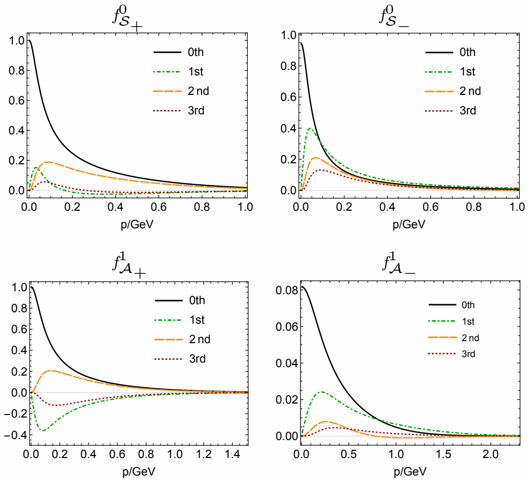
# CSMs: Faddeev Equation - three quarks

- The dominant contributions to $\psi^{\rho}_{\alpha\beta\gamma\mathcal{I}}(p,q,P)$ are given by

$$\mathcal{S}_{\pm} := \Lambda_{\pm}\gamma_5 C \otimes \Lambda_{+};$$

$$\mathcal{A}_{\pm} := \frac{1}{\sqrt{3}}\gamma_5\gamma_T^{\alpha}\Lambda_{\pm}\gamma_5 C \otimes \gamma_5\gamma_T^{\alpha}\Lambda_{+}.$$

$\Lambda_{+}(\hat{P}) = \frac{1}{2}(1+\gamma\cdot\hat{P})$ is the nucleon's positive-energy projector.




First four Chebyshev moments in the variable $z_1$ of the dressing functions evaluated at $q=0$. All the amplitudes normalise by $f^0_{\mathcal{S}_+}(p=0)$.

# Electromagnetic form factors

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**Electromagnetic currents:**

The nucleon electromagnetic current has the following form($N = p, n$)

$$J^\mu(Q) = i\Lambda_+(p_f)(F_1(Q^2)\gamma^\mu + \frac{F_2(Q^2)}{2m_N}\sigma^{\mu\nu}Q^\nu)\Lambda_+(p_i)$$

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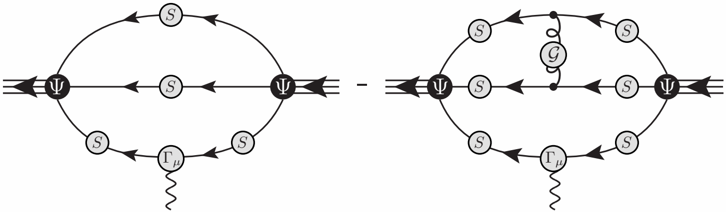
The charge and magnetisation distributions are ($\tau = Q^2/[4m_N^2]$):

$$G_E^N = F_1^N - \tau F_2^N, \quad G_M^N = F_1^N + F_2^N.$$

# Electromagnetic form factors

## Electromagnetic currents

$[J^{(3)}_{n'n}]^{\mu}_{\delta'\delta} =$



- $\Gamma_\mu$ is the dressed-photon+quark vertex.
- The complete current has three terms: $J_\mu(Q) = \sum_{a=1,2,3} J^a_\mu(Q)$.

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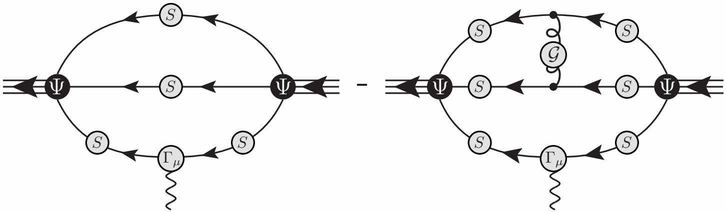


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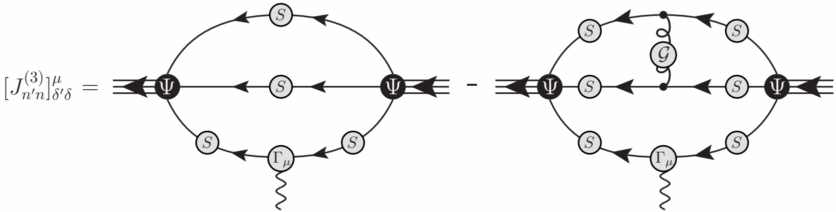


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| $\mu_p$ | $2.23$ | $2.793$ | |
| $\mu_n$ | $-1.33$ | $-1.913$ | |
| $\langle r_E^2\rangle^p$ | $0.788$ | $0.7070(7)$ | $0.717(14)$ |
| $\langle r_E^2\rangle^n$ | $-0.0621$ | $-0.1160(22)$ | |
| $\langle r_M^2\rangle^p$ | $0.672$ | $0.72(4)$ | $0.667(44)$ |
| $\langle r_M^2\rangle^n$ | $0.661$ | $0.75(2)$ | |

- The magnetic moments are $\sim 25\%$ small.
- It is worth highlighting the prediction

$$\langle r_E^2\rangle^p > \langle r_M^2\rangle^p$$

accords with SPM analyses of existing form factor data.

**Magnetic moments:**

$$\mu_N = G_M^N(Q^2=0).$$

**Radii:**

$$\langle r_{E,\,M}^2\rangle^N = -6\frac{d\ln G_{E,\,M}^N(Q^2)}{dQ^2}\bigg|_{Q^2=0},$$

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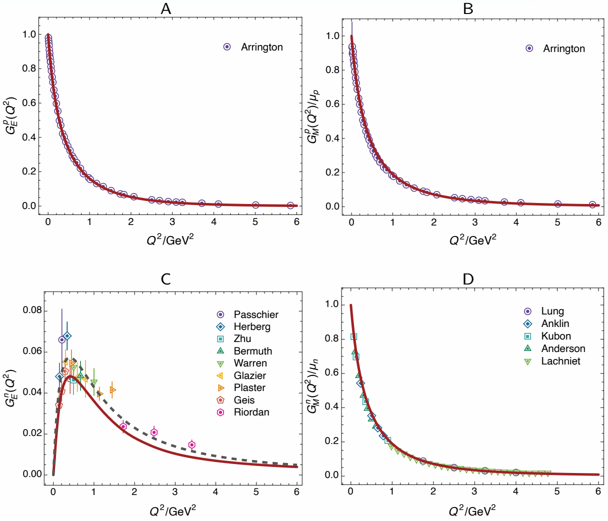
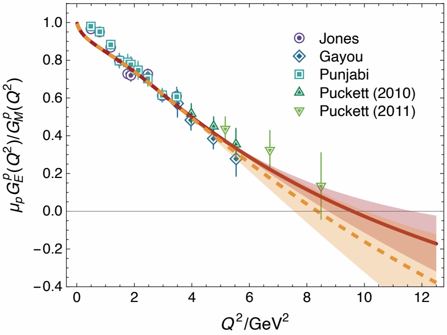
# Electromagnetic form factors $G^N_{E,M}(Q^2)$




- Faddeev equation predictions for each nucleon form factor.
- $G_E^n(Q^2)$ is difficult to explain because of its sensitivity to details of the neutron wave function.

# Electromagnetic form factors $\mu_p G_E^p(Q^2)/G_M^p(Q^2)$




- $Q^2 \lesssim 4\,\mathrm{GeV}^2$, directly calculated results agree well with experiment.
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  ($I$) independent SPM analyses of $G_{E,M}^p$;
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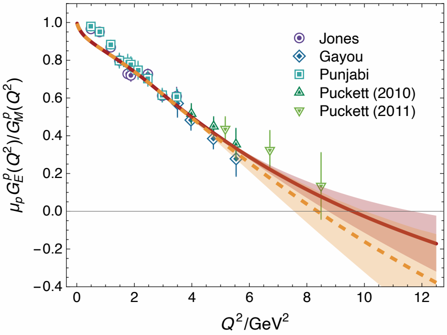


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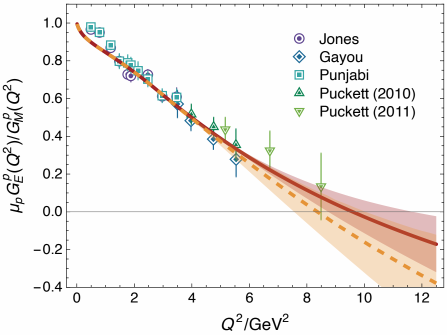


- **proton**

$$\text{SPM I:}\quad Q^2_{G_E^p-\text{zero}} = 8.37^{+1.68}_{-0.81}\,\text{GeV}^2\,,$$

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The averaged result: $Q^2_{G_E^p-\text{zero}} = 8.86^{+1.93}_{-0.86}\,\text{GeV}^2$.

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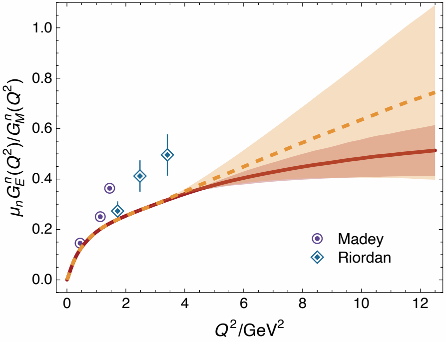


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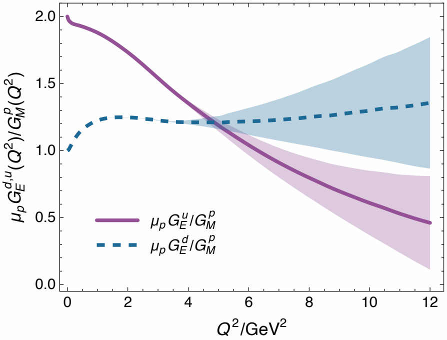
# Electromagnetic form factors Flavour separation




- The flavour separation of the charge and magnetisation form factors ($e_u = 2/3$, $e_d = -1/3$):

$$G_E^p = e_u G_E^{pu} + e_d G_E^{pd}\,, \qquad G_E^n = e_u G_E^{pd} + e_d G_E^{pu}\,.$$

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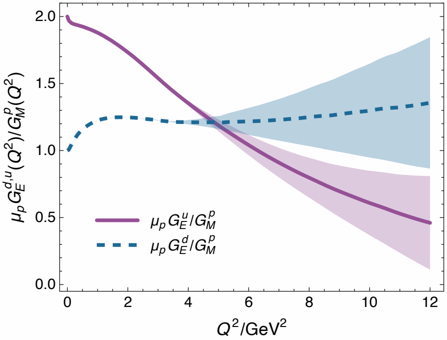


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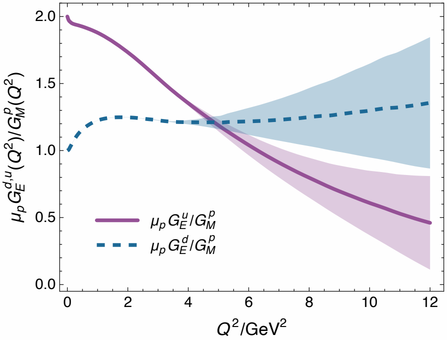


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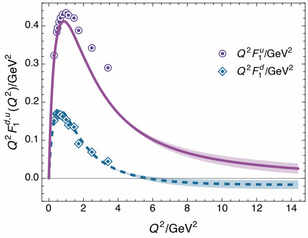


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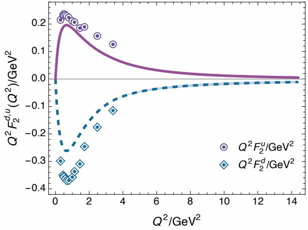


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with, $i = 1, 2$.

- A zero is projected in $F_1^d$ at

$$Q^2_{F_1^d-\text{zero}} = 5.73^{+1.46}_{-0.49}\,\text{GeV}^2.$$

This matches the result in the quark+diquark picture : $Q^2 = 7.0^{+1.1}_{-0.4}\,\text{GeV}^2$.

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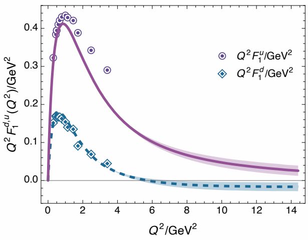


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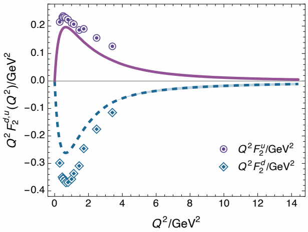


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# Outlook

**Based on Faddeev equation, we can study**

- baryon electroweak form factors, nucleon gravitational form factors;
- spectrum/structure of octet baryons;
- PDAs/PDFs of baryons;
- hybrid meson……

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# Acknowledgement

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Thank you!

# Appendix

- The key element is the quark+quark scattering kernel, for which the RL truncation is obtained by writing:

$$\mathcal{K}_{tu}^{rs}(k) = \mathcal{G}_{\mu\nu}(k)[i\gamma_\mu \frac{\lambda^a}{2}]_{ts}[i\gamma_\nu \frac{\lambda^a}{2}]_{ur}\,, \tag{1a}$$

$$\mathcal{G}_{\mu\nu}(k) = \tilde{\mathcal{G}}(y)\, T_{\mu\nu}(k)\,, \tag{1b}$$

$k^2\, T_{\mu\nu}(k) = k^2\delta_{\mu\nu} - k_\mu k_\nu$, $y = k^2$.$r, s, t, u$ represent colour, spinor, and flavour matrix indices.

# Appendix

$$\tilde{\mathcal{G}}(y) = \frac{8\pi^2}{\omega^4} D e^{-y/\omega^2} + \frac{8\pi^2 \gamma_m \mathcal{F}(y)}{\ln\left[\tau + (1 + y/\Lambda_{\rm QCD}^2)^2\right]}\,, \tag{2}$$

where $\gamma_m = 12/25$, $\Lambda_{\rm QCD} = 0.234\,{\rm GeV}$, $\tau = {\rm e}^2 - 1$, and $\mathcal{F}(y) = \{1 - \exp(-y/\Lambda_I^2)\}/y$, $\Lambda_I = 1\,{\rm GeV}$. We employ a mass-independent (chiral-limit) momentum-subtraction renormalisation scheme.

Contemporary studies employ $\omega = 0.8\,{\rm GeV}$. With $\omega D = 0.8\,{\rm GeV}^3$ and renormalisation point invariant quark current mass $\hat{m}_u = \hat{m}_d = 6.04\,{\rm MeV}$,

$$m_\pi = 0.14\,{\rm GeV};$$

$$m_N = 0.94\,{\rm GeV};$$

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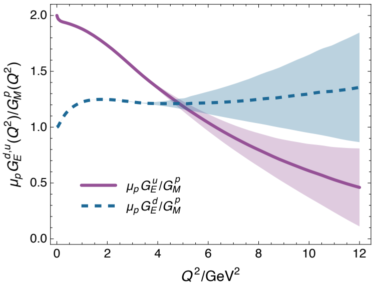
# Electromagnetic form factors $\mu_N G_E^N(Q^2)/G_M^N(Q^2)$

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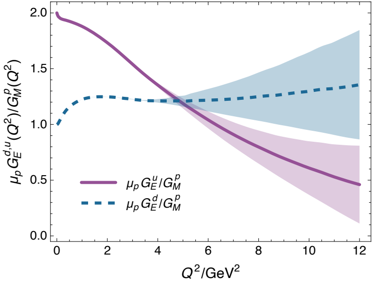


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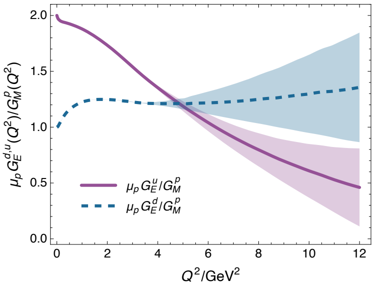


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# Flavor amplitudes

| state | $\mathrm{F}_{\mathcal{M}_{\mathcal{A}}}$ | $\mathrm{F}_{\mathcal{M}_{\mathcal{S}}}$ |
|---|---|---|
| $p$ | $\frac{1}{\sqrt{2}}(udu - duu)$ | $\frac{1}{\sqrt{6}}(2uud - udu - duu)$ |
| $n$ | $\frac{1}{\sqrt{2}}(udd - dud)$ | $\frac{1}{\sqrt{6}}(udd + dud - 2ddu)$ |

Table: Baryon octet flavor amplitudes; we define $\lambda_1\lambda_2\lambda_3 := \lambda_1 \otimes \lambda_2 \otimes \lambda_3$, and, $u^\dagger := (1\ 0)$, $d^\dagger := (0\ 1)$.