Nucleon Structure From Lattice QCD Simulations at the Physical Point

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RESEARCH • TECHNOLOGY • INNOVATION

Strong QCD from Hadron Structure Experiments - VI
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Nucleon Structure with Physical Point Ensembles

Outline

- Matrix elements on the lattice
- Lattice methods
- $N_f=2+1+1$ ensembles at physical point (twisted mass + clover)
- Statistics and uncertainties
- Selected results

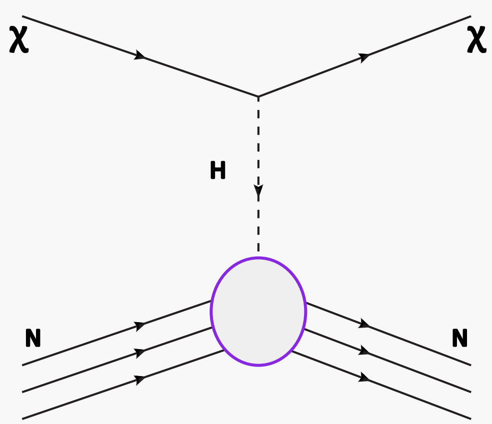
Extended Twisted Mass Collaboration

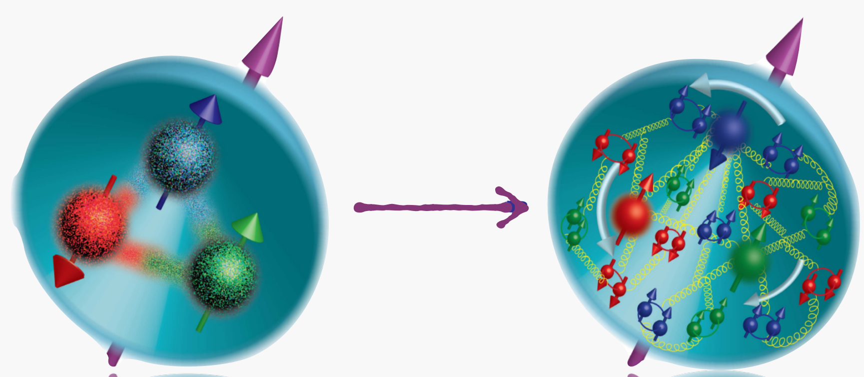
ETM Collaboration

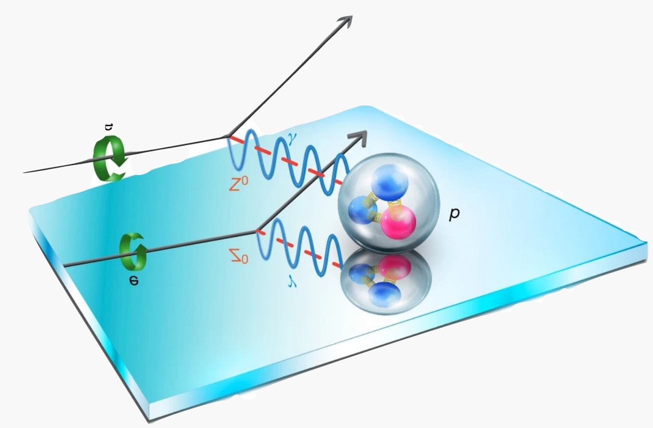
Cyprus (Univ. of Cyprus, Cyprus Inst.), *Germany* (Berlin/Zeuthen, Bonn, Wuppertal), *Italy* (Rome I, II, III, Parma), *Poland* (Poznan), *Switzerland* (Bern), *US* (Temple, PA)

Nucleon Matrix Elements

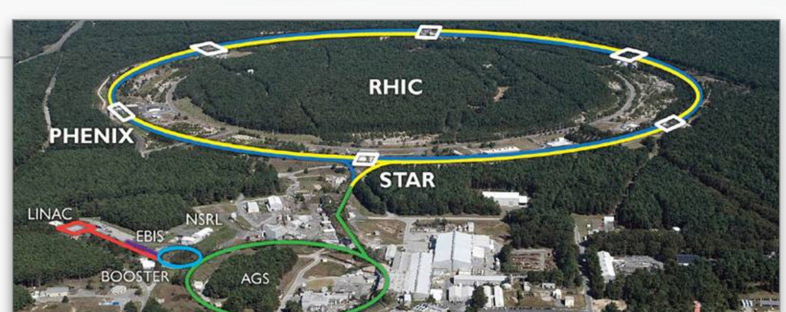
- Scalar and tensor charges ➜ novel interactions/dark matter searches
- Axial matrix elements ➜ origin of nucleon spin
- σ-terms ➜ mass decomposition of nucleon
- Electromagnetic form factors ➜ radii and moments well known experimentally
- Axial form factors ➜ PCAC and pion pole dominance relations
- Strange form factors ➜ connect to weak charges and constraints on new physics
- Momentum fraction, moments of PDFs and GPDs, …

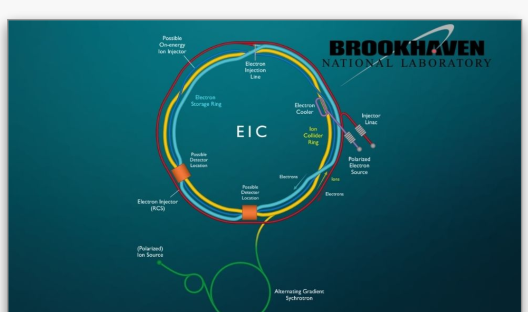






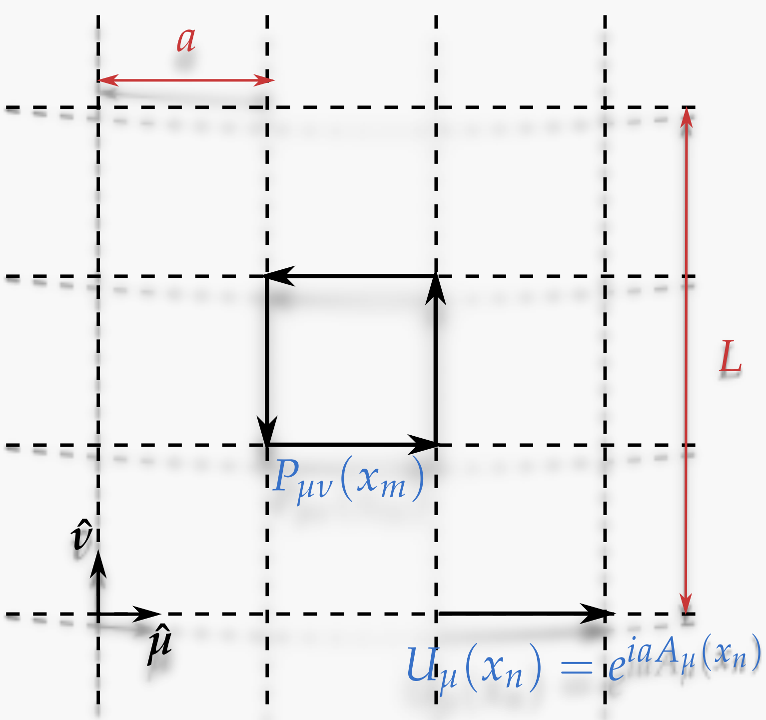
JLab · Mainz · PSI · RHIC · Planned Electron Ion Collider





Lattice QCD — ab initio simulation of QCD

- Freedom in choice of:
  - Bare quark masses (cost increases as $m_{PS} \to m_\pi$)
  - Lattice spacing (cost increases as $a \to 0$)
  - Volume (cost increases as $L^3 \to \infty$)
- Choice of discretisation scheme; e.g. Clover, Twisted Mass, Staggered, Overlap, Domain Wall
  - Each with their own trade-offs and advantages



Eventually, all schemes must agree:

- At the continuum limit: $a \to 0$
- At infinite volume limit $L \to \infty$
- At physical quark mass

Lattice QCD

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}[U] \mathcal{O}(D_f^{-1}[U], U) \left(\prod_{f=u,d,s,c} \mathrm{Det}(D_f[U]) \right) e^{-S_{QCD}[U]}$$

Simulation

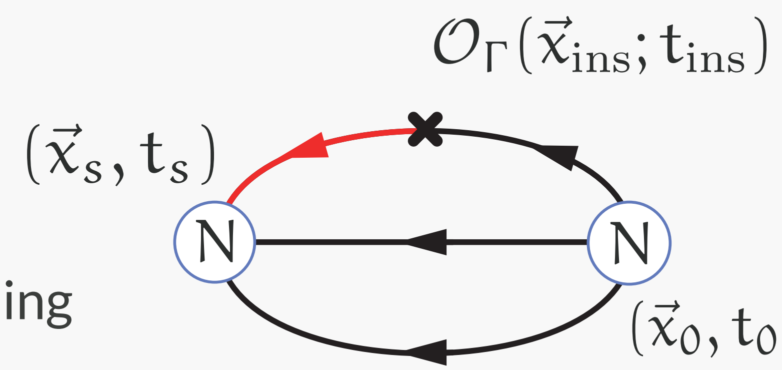
- Markov chain Monte Carlo to generate ensembles of gluon-field configurations $\{U\}$

$$P[U] = \frac{1}{Z} \left(\prod_{f=u,d,s,c} \mathrm{Det}(D_f[U]) \right) e^{-S_{QCD}[U]}$$

Analysis

- Construction of hadron correlation functions on background field configurations:

$$\langle N(p', s') | \mathcal{O} | N(p, s) \rangle$$

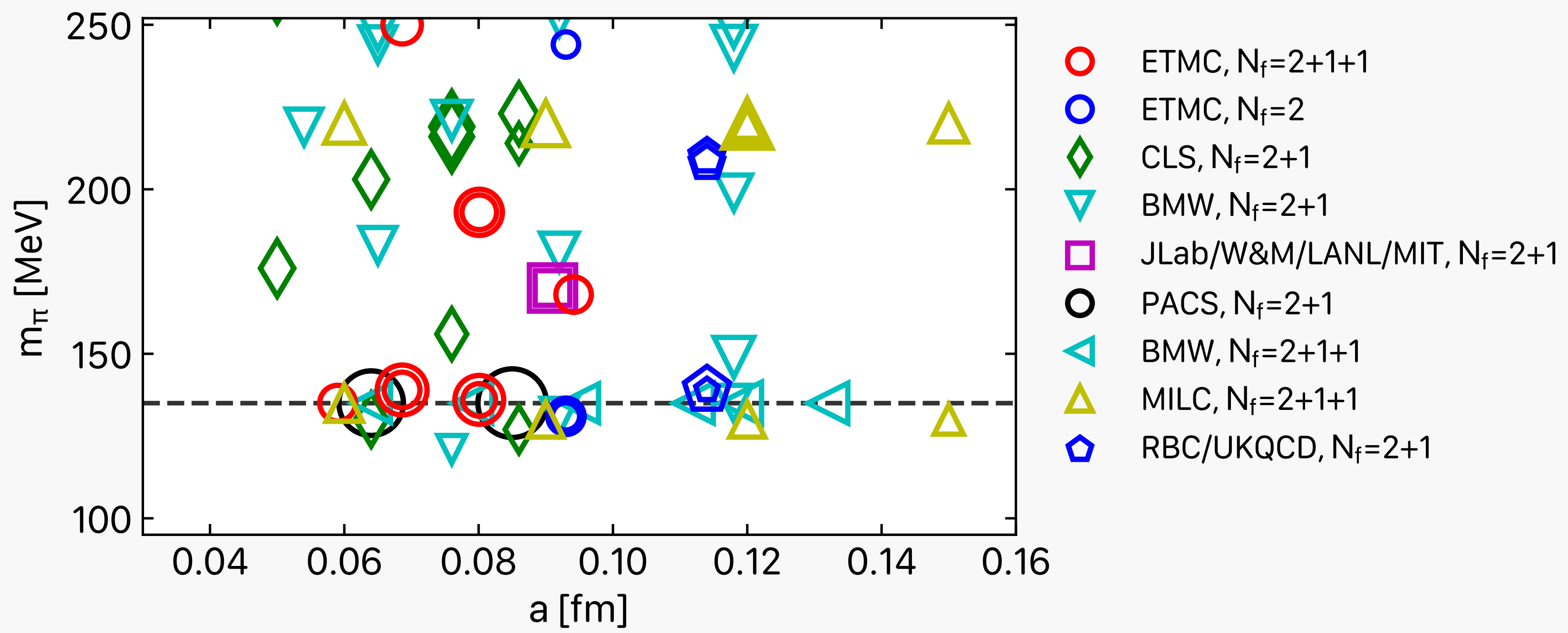


Data analysis – post-processing

- Statistical analysis, resampling
- Statistical and stochastic errors
- Continuum and infinite volume extrapolation

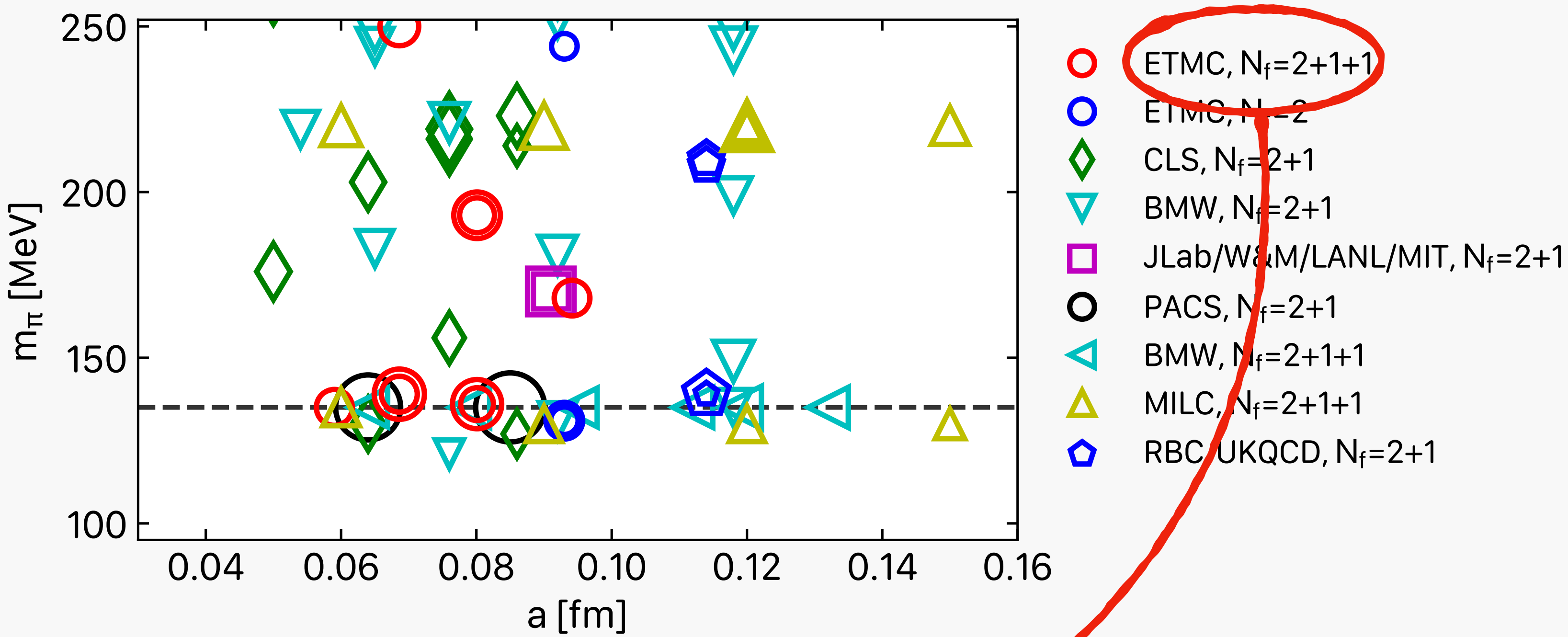
Ensembles

Landscape of ensembles used for nucleon structure



Ensembles

Landscape of ensembles used for nucleon structure



ETMC: three $N_f=2+1+1$ ensembles at physical pion mass

Ens. ID (abbrv.)	Vol.	a [fm]
cB211.072.64 (cB64)	64×128	0.080
cC211.060.80 (cC80)	80×160	0.068
cD211.054.96 (cD96)	96×192	0.057

- Three lattice spacings at physical point
- Ongoing generation of finer ensembles and larger volumes
- ***This talk:*** 3 ensembles with:

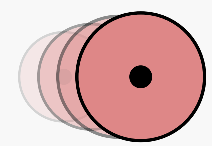
  $a = 0.057 - 0.08$ fm

Local matrix elements

$$G_\Gamma(P;\vec{q};t_s,t_{ins}) = \sum_{\vec{x}_s\,\vec{x}_{ins}} e^{-i\vec{q}.\vec{x}_{ins}} P^{\alpha\beta} \langle \bar{\chi}_N^\beta(\vec{x}_s;t_s) | \mathcal{O}^\Gamma(\vec{x}_{ins};t_{ins}) | \chi_N^\alpha(\vec{0};0)\rangle$$

Unpolarised

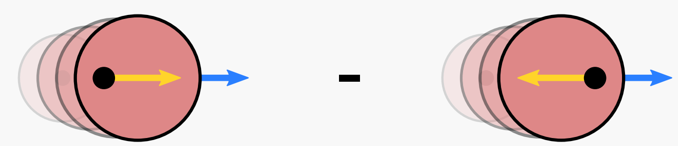
$$\mathcal{O}_V^{\mu\mu_1\mu_2\ldots\mu_n} = \bar{\psi}\gamma^{\{\mu} iD^{\mu_1} iD^{\mu_2} \ldots iD^{\mu_n\}}\psi$$



$$\langle 1\rangle_{u-d} = g_V,\quad \langle x\rangle_{u-d},\quad \ldots$$

Helicity

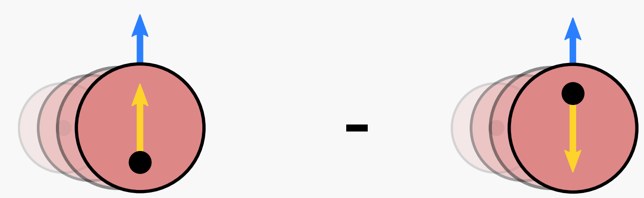
$$\mathcal{O}_A^{\mu\mu_1\mu_2\ldots\mu_n} = \bar{\psi}\gamma_5\gamma^{\{\mu} iD^{\mu_1} iD^{\mu_2} \ldots iD^{\mu_n\}}\psi$$



$$\langle 1\rangle_{\Delta u-\Delta d} = g_A,\quad \langle x\rangle_{\Delta u-\Delta d},\quad \ldots$$

Transverse

$$\mathcal{O}_T^{\nu\mu\mu_1\mu_2\ldots\mu_n} = \bar{\psi}\sigma^{\nu\{\mu} iD^{\mu_1} iD^{\mu_2} \ldots iD^{\mu_n\}}\psi$$



$$\langle 1\rangle_{\delta u-\delta d} = g_T,\quad \langle x\rangle_{\delta u-\delta d},\quad \ldots$$

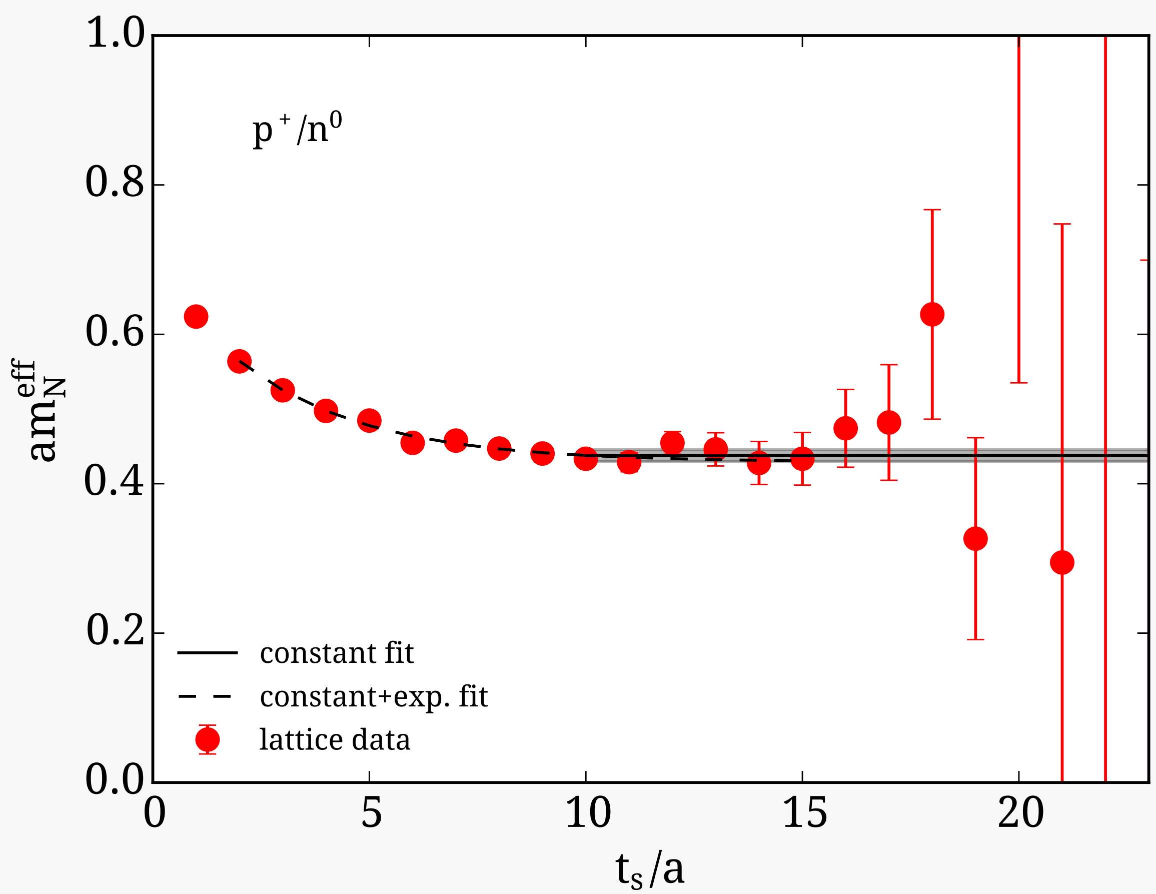
Renormalization of lattice operators for proper continuum limit: $\mathcal{O} = Z_\mathcal{O}\mathcal{O}^{\text{lat}}$

- In general, Z determined non-perturbatively in e.g. RI'-MOM scheme, converted to $\overline{MS}$ at 2 GeV [ETM, Phys. Rev. D104 (2021) 7, 074515 2104.13408]
- Z_A also available via a hadronic scheme [ETM, Phys. Rev. D107 (2023) 7, 074506 2206.15084]
- $\mathcal{O}_V$ also available via a lattice conserved current

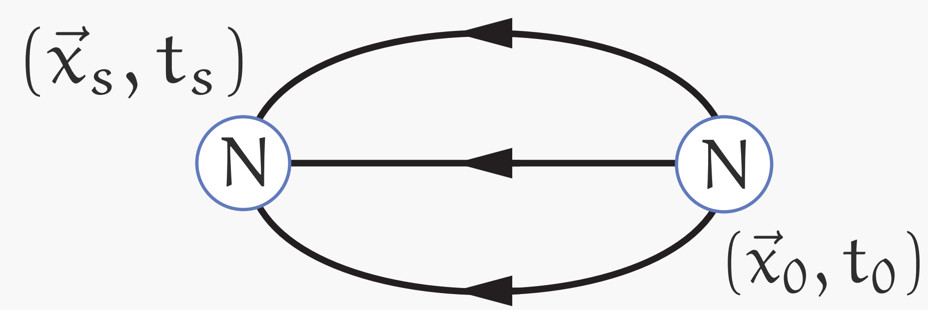
Nucleon structure on the lattice

Two-point correlation functions

- **Statistical error:** $N^{-1/2}$ with Monte Carlo samples
- Correlation functions exponentially decay with time-separation
- Contamination from higher energy states



$$\sum_{\vec{x}_s} \Gamma^{\alpha\beta} \langle \bar{\chi}_N^{\beta}(x_s) | \chi_N^{\alpha}(0) \rangle = c_0 e^{-E_0 t_s} + c_1 e^{-E_1 t_s} + \ldots$$



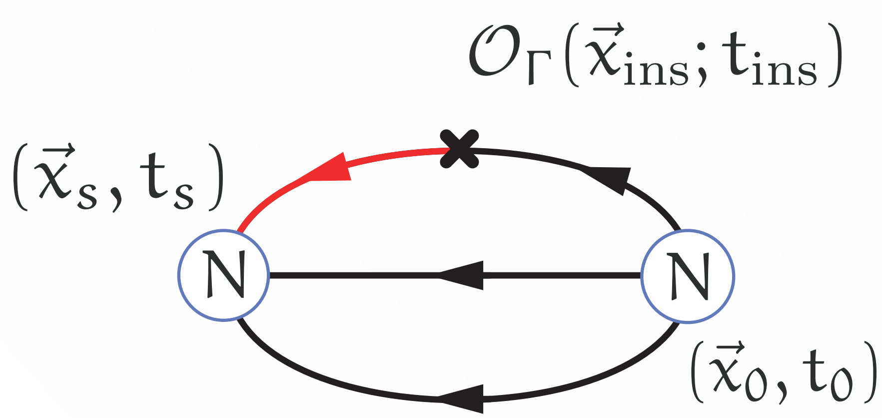
Matrix elements on the Lattice

General three-point function:

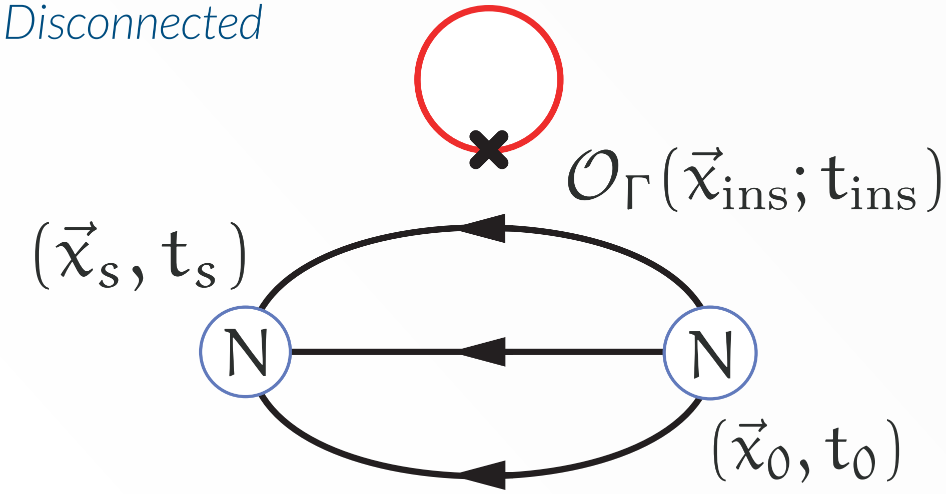
$$G_\Gamma(P;\vec{q};t_s,t_{ins}) = \sum_{\vec{x}_s\,\vec{x}_{ins}} e^{-i\vec{q}.\vec{x}_{ins}} P^{\alpha\beta} \langle \bar{\chi}^\beta_N(\vec{x}_s;t_s)|\mathcal{O}^\Gamma(\vec{x}_{ins};t_{ins})|\chi^\alpha_N(\vec{0};0)\rangle$$

At quark level gives rise to both so-called connected and disconnected contributions

Connected



Disconnected



Matrix elements on the Lattice

General three-point function:

$$G_\Gamma(P;\vec{q};t_s,t_{ins}) = \sum_{\vec{x}_s\,\vec{x}_{ins}} e^{-i\vec{q}.\vec{x}_{ins}} P^{\alpha\beta} \langle \bar{\chi}^\beta_N(\vec{x}_s;t_s)|\mathcal{O}^\Gamma(\vec{x}_{ins};t_{ins})|\chi^\alpha_N(\vec{0};0)\rangle$$

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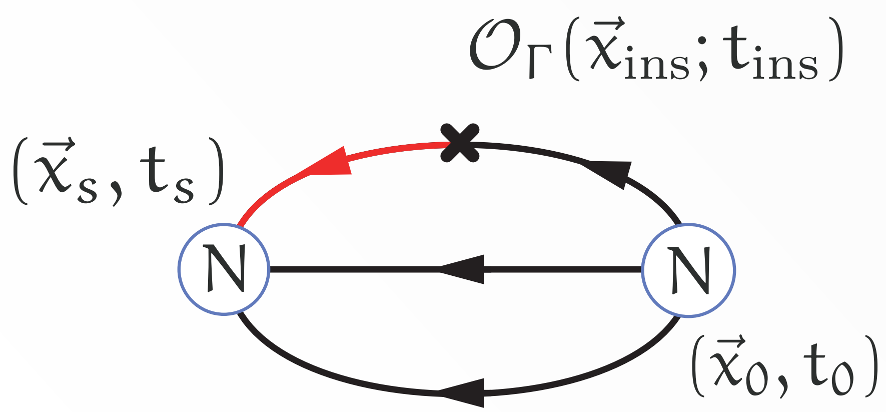
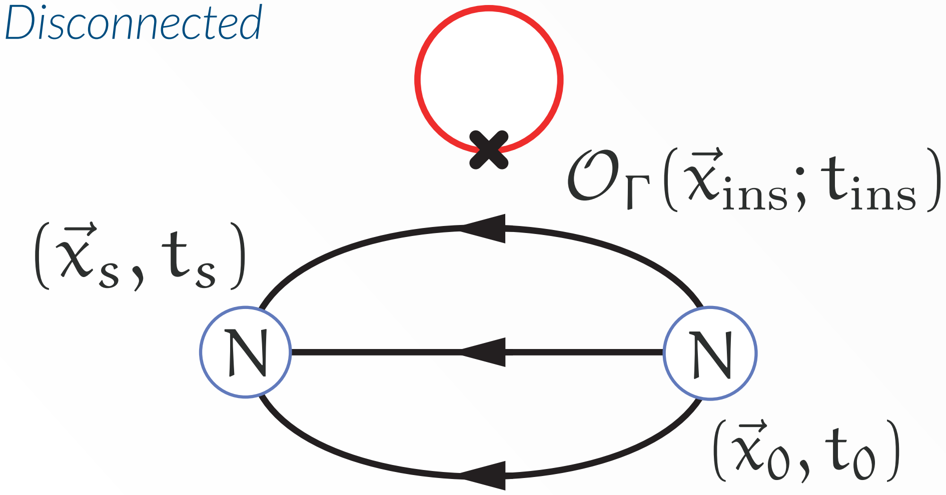
Connected

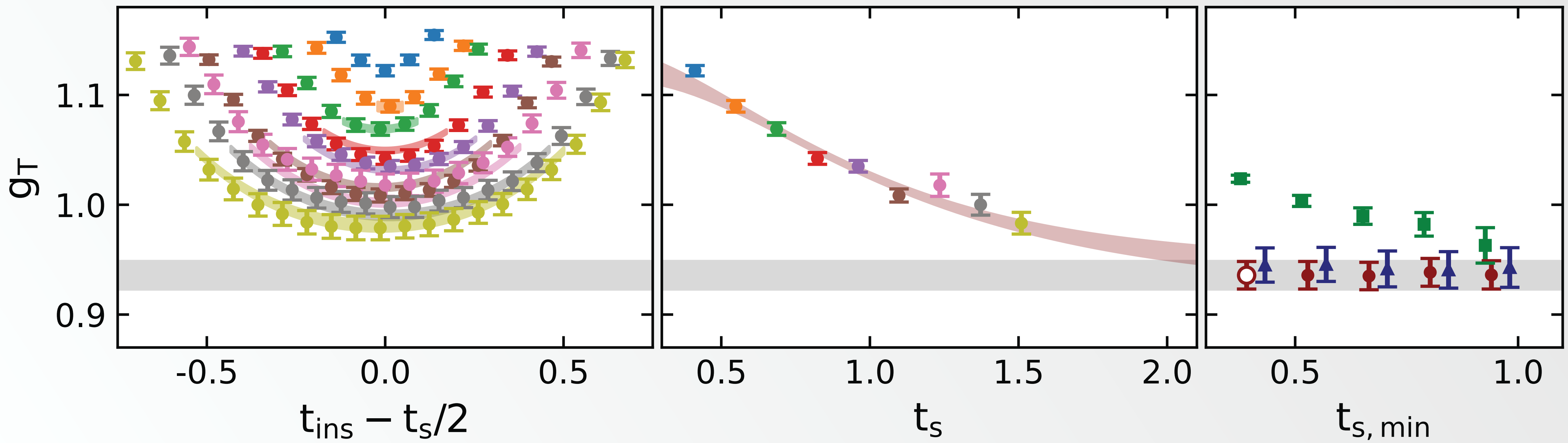


Disconnected



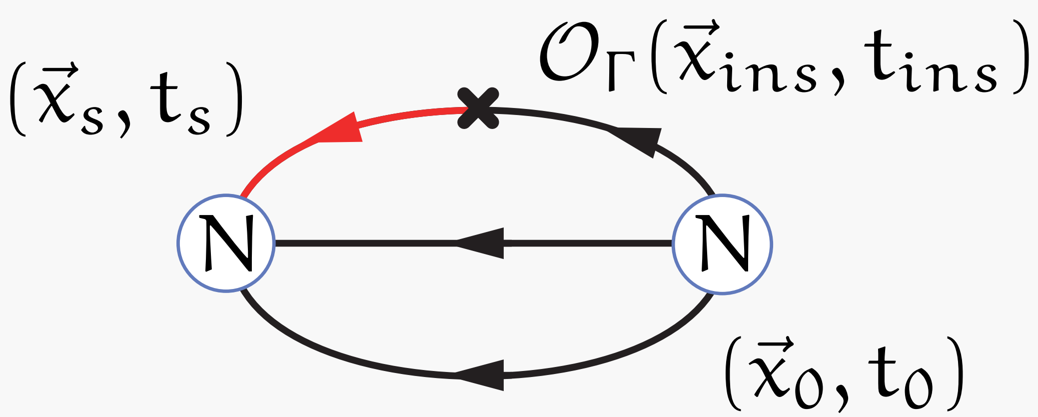
Stochastic evaluation of loop – stochastic error in addition to statistical

Treatment of excited states

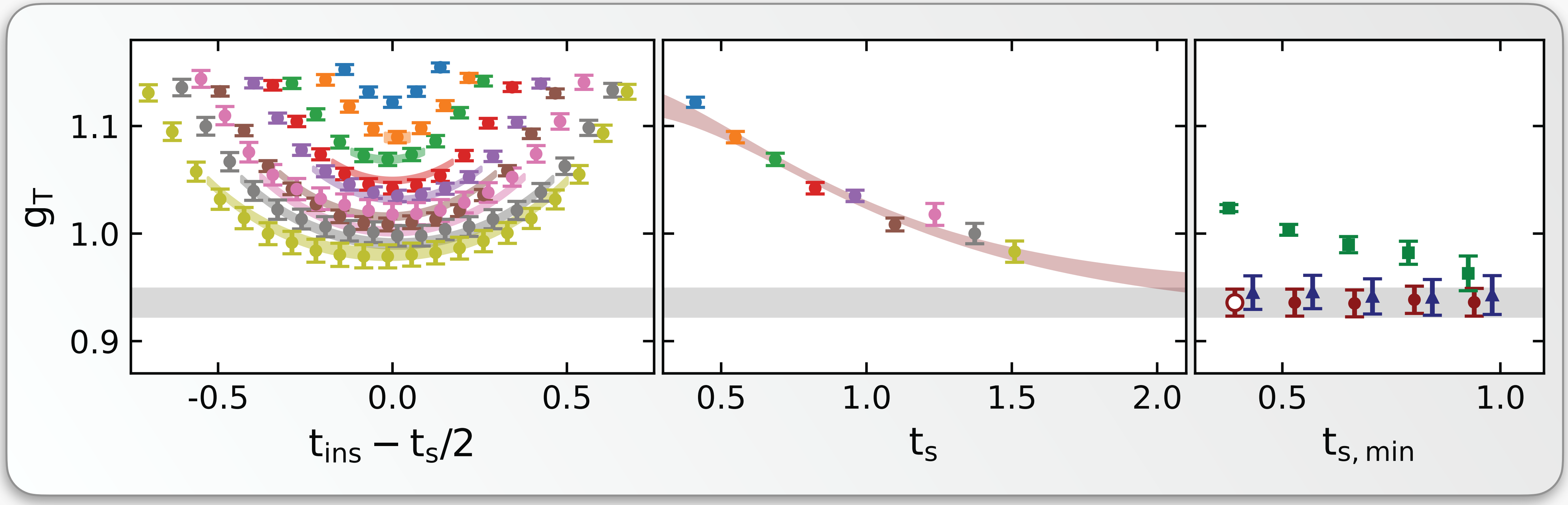


Example from intermediate a

- Isovector tensor charge (only connected)
- Increasing statistics with separation t_s
- Summation, two- and three-state fits



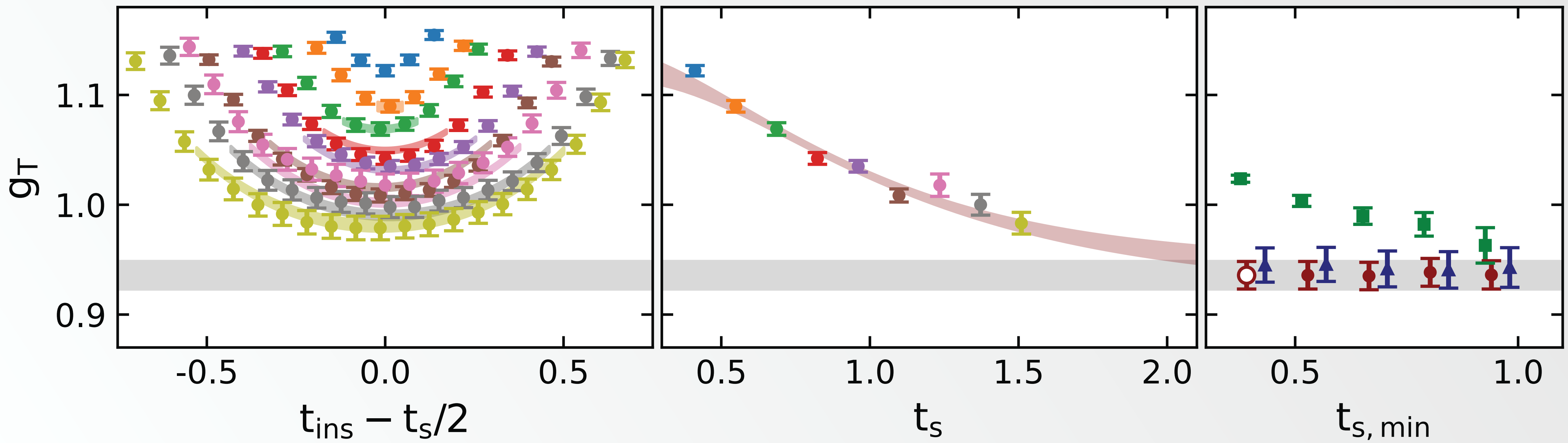
Treatment of excited states



Summation method

$$S_\Gamma(\vec{q};t_s) = \sum_{t_{ins}=\tau}^{t_s-\tau} R_\Gamma(\vec{q};t_s;t_{ins}) \rightarrow \mathcal{M}t_s + C$$

Treatment of excited states



Two-state fit

$$G_\Gamma(\vec{q}; t_s, t_{ins}) = \sum_{i=0}^{1}\sum_{j=0}^{1} c_{ij} e^{-E_i(0)(t_s - t_{ins})} e^{-E_j(\vec{q})t_{ins}}$$

$$\mathcal{M} = \frac{c_{00}}{\sqrt{a_0(\vec{0})a_0(\vec{q})}}$$

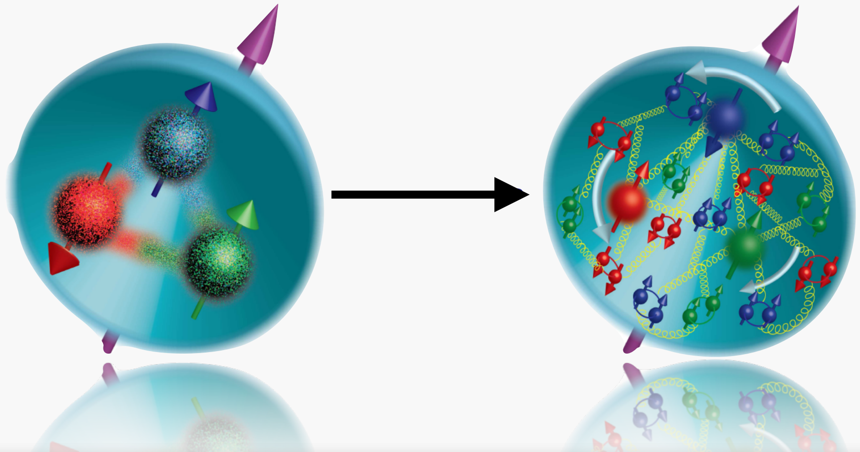
$$G(\vec{q}; t_s) = a_0(\vec{q})e^{-\varepsilon_0(\vec{q})t_s} + a_1(\vec{q})e^{-\varepsilon_1(\vec{q})t_s}$$

Nucleon axial charge

Matrix element of the axial current

Isovector case well known from β-decay: $\langle \mathrm{p} | \bar{\mathrm{u}} \gamma_5 \gamma_\mathrm{k} \mathrm{d} | \mathrm{n} \rangle$

Flavor–separated contributions to axial charge relate to quark intrinsic spin contributions to nucleon spin

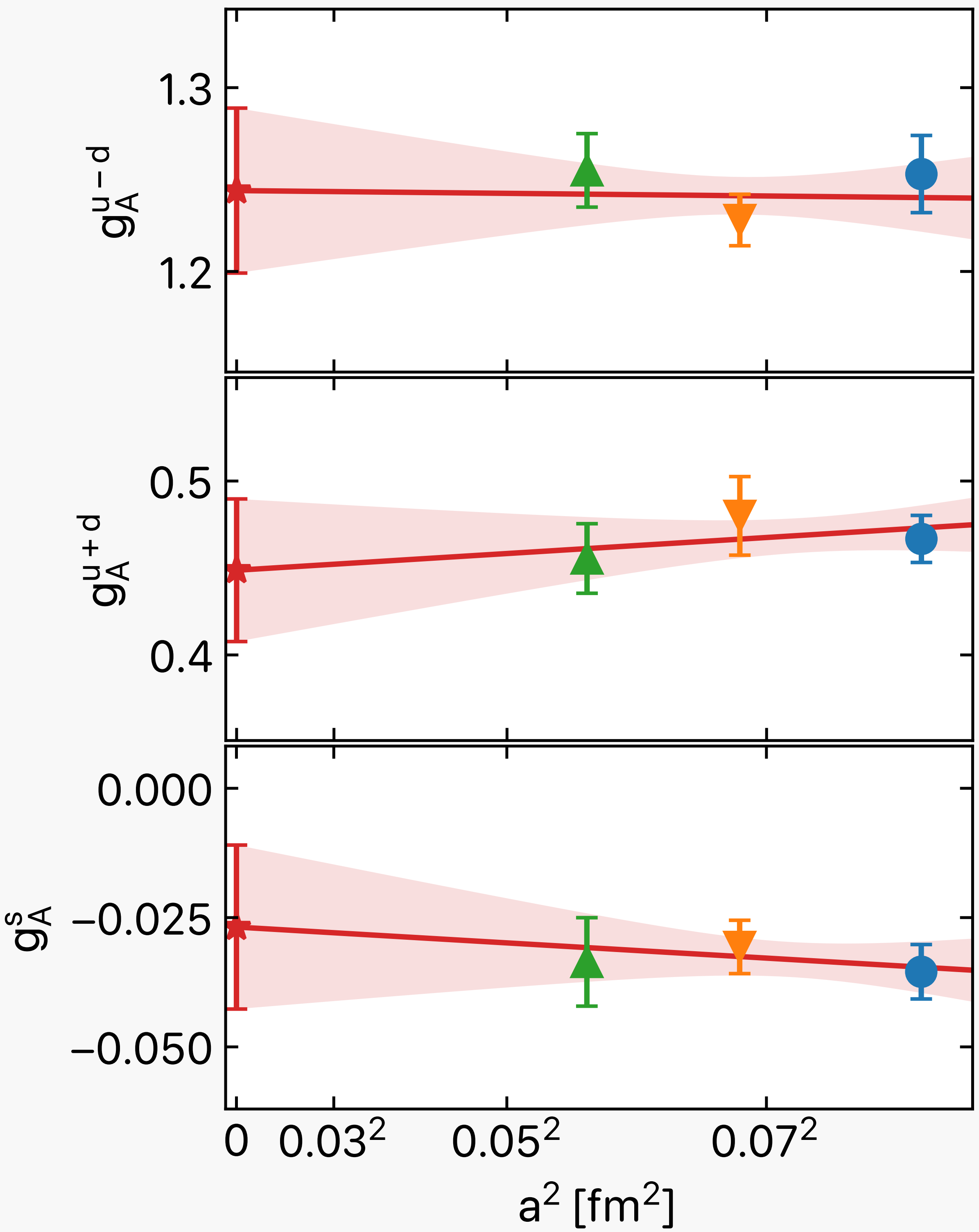


$$\frac{1}{2}\Delta\Sigma = \frac{1}{2}\sum_{q=u,d,s,\ldots} g_A^q$$

Quark intrinsic spin contributions to nucleon spin

- Need linear combination of isovector (u-d) and isoscalar (u+d) contributions for individual up- and down-quarks
- Strange quark contribution is sea-quark contribution only (disconnected diagrams)

Nucleon axial charge



Recent result by ETM collaboration; three values of a at physical m_π

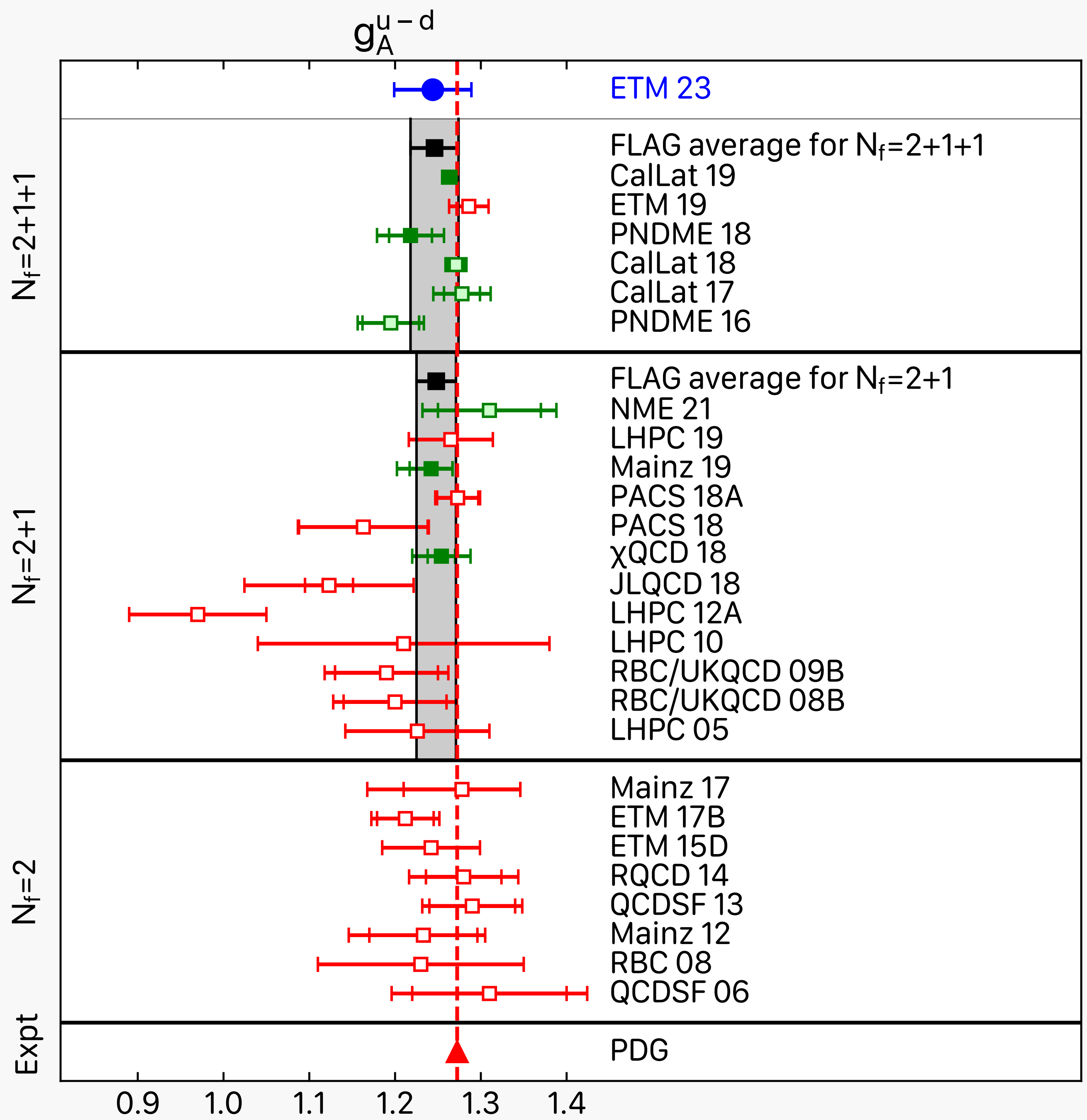
ETM collab., Phys. Rev. D 109 (2024) 3, 034503 [arXiv:2309.05774]

- Errors for each ensemble include *statistical* and *systematic* due to excited state contamination
- Model averaged to evaluate systematic errors due to excited state contamination (see e.g. `arXiv:2208.14983`)

Preliminary!

Complete analysis for flavour separated charges ongoing

Nucleon axial charge



Latest FLAG21 values

- ETM23 consistent with FLAG average
- Agreement for g_A means confidence for more challenging quantities
- E.g.
  - Scalar ME, σ-terms

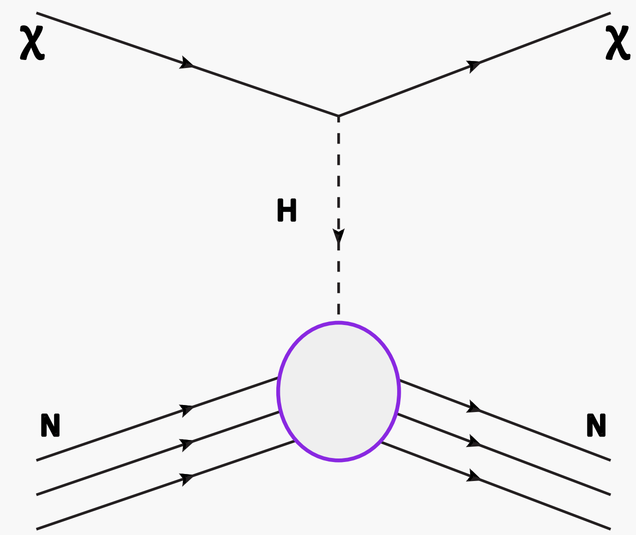
$$\sigma_{\pi N} = m_{ud}\langle N|\bar{u}u + \bar{d}d|N\rangle$$

  - Tensor ME

$$g_T = \langle 1\rangle_{\delta u - \delta d} \leftarrow \langle N|\bar{u}\sigma_{\mu\nu}u + \bar{d}\sigma_{\mu\nu}d|N\rangle$$

Scalar charge – σ-terms

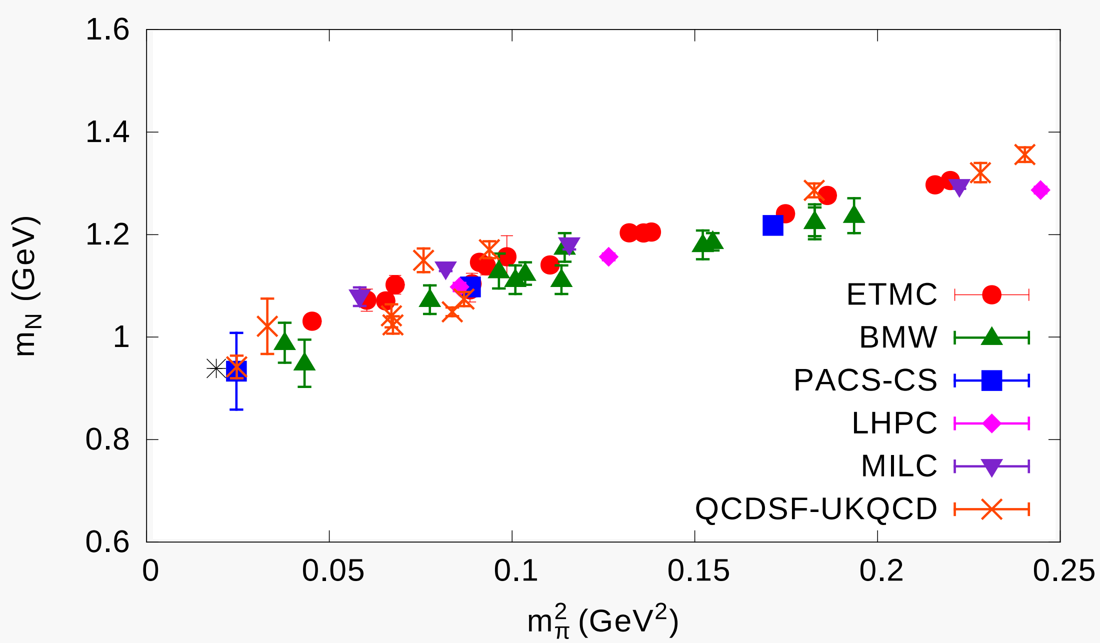
- Pion nucleon σ-term: $\sigma_{\pi N} = m_{ud}\langle N|\bar{u}u + \bar{d}d|N\rangle$
- Strange σ-term: $\sigma_s = m_s\langle N|\bar{s}s|N\rangle$
- Enter super-symmetric candidate particle scattering cross sections with nucleon (e.g. neutralino through Higgs)



1. Direct calculation of matrix elements

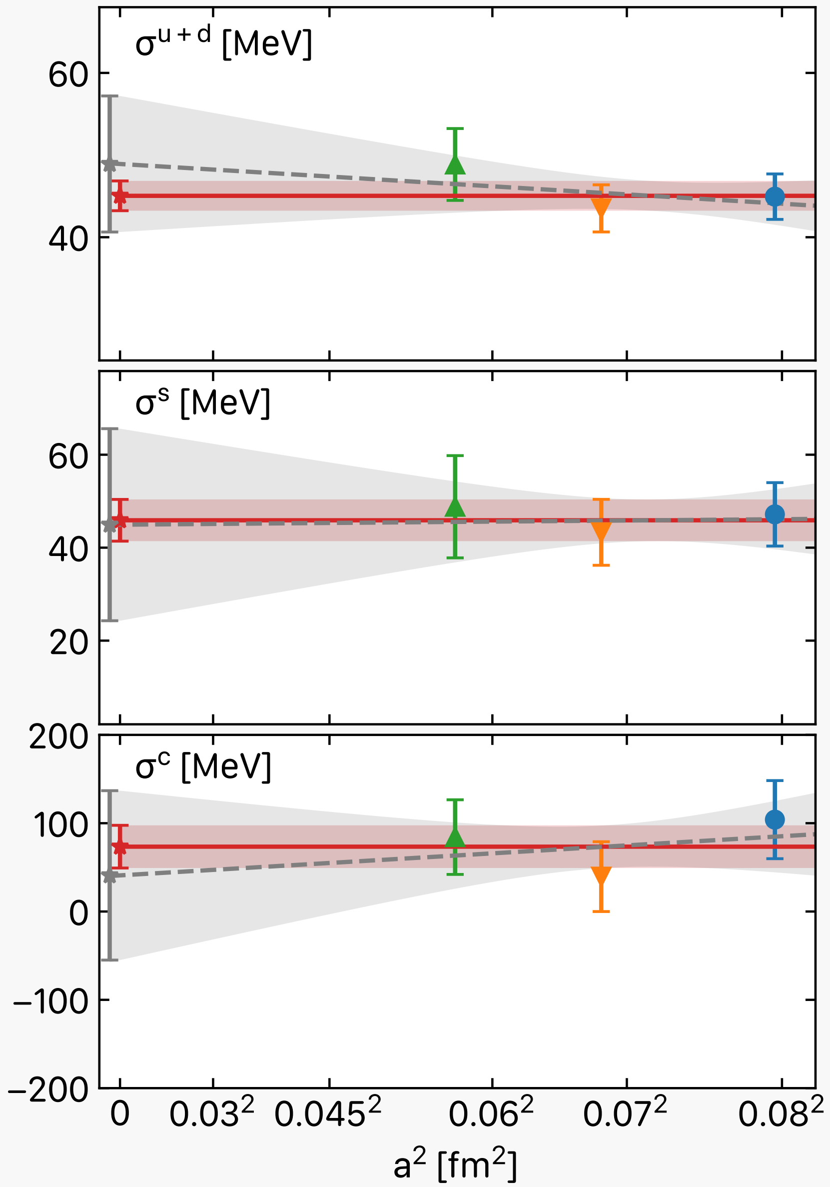
Involves disconnected contributions

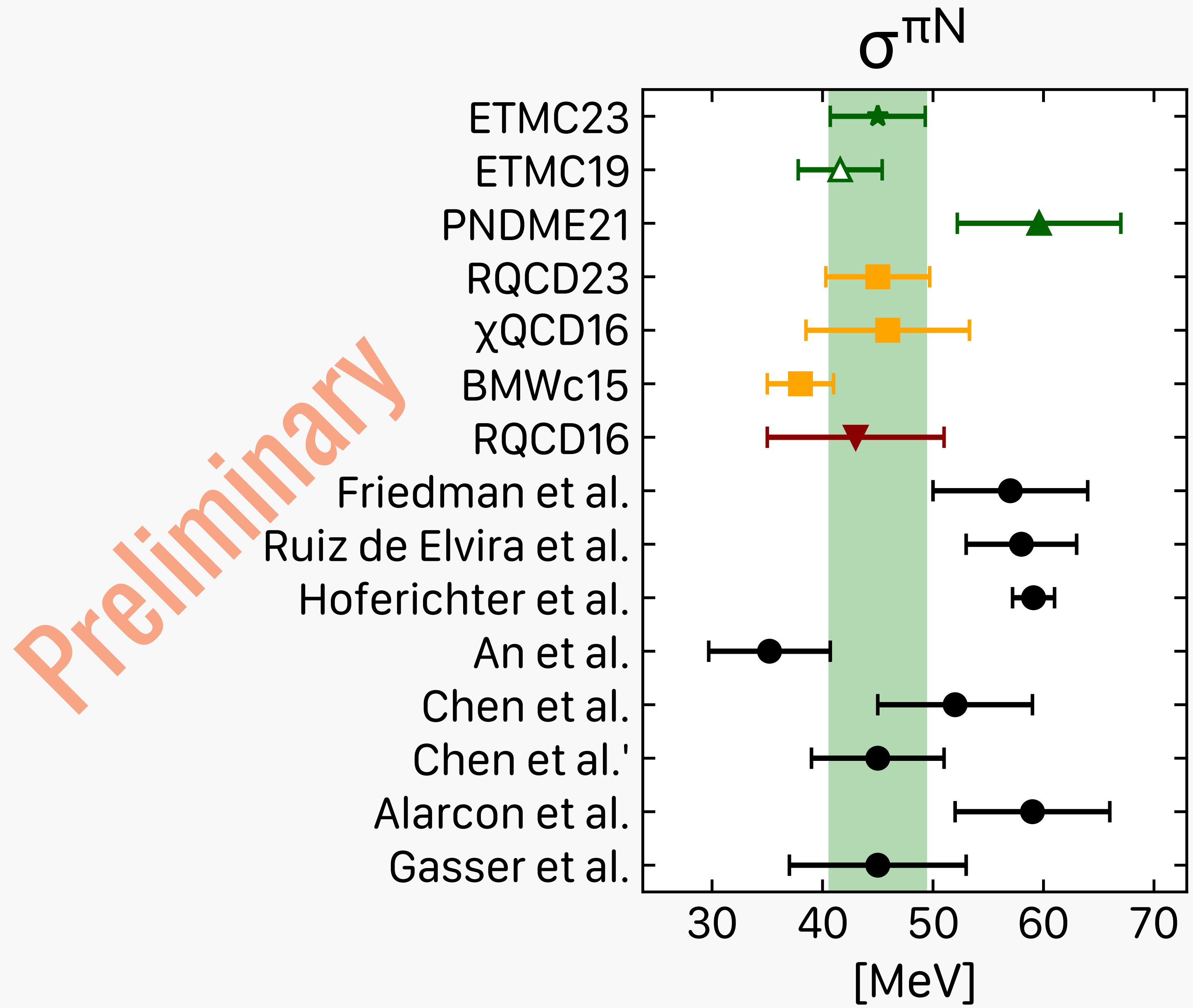
2. Through Feynman - Hellmann theorem: $\sigma_{\pi N} = m_{ud}\frac{\partial m_N}{\partial m_{ud}}$ $\sigma_s = m_s\frac{\partial m_N}{\partial m_s}$



- Reliance on effective theories for dependence on m_π
- Weak dependence on m_s

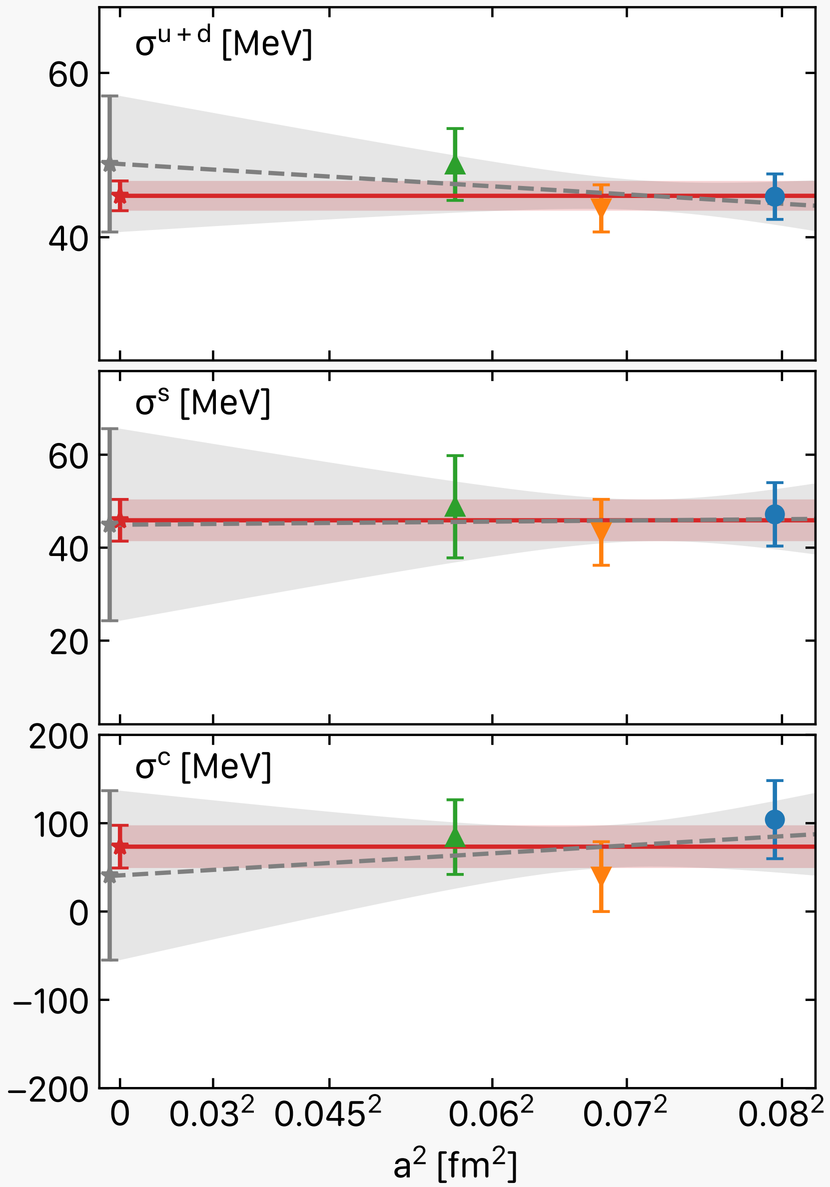
Scalar charge – σ-terms



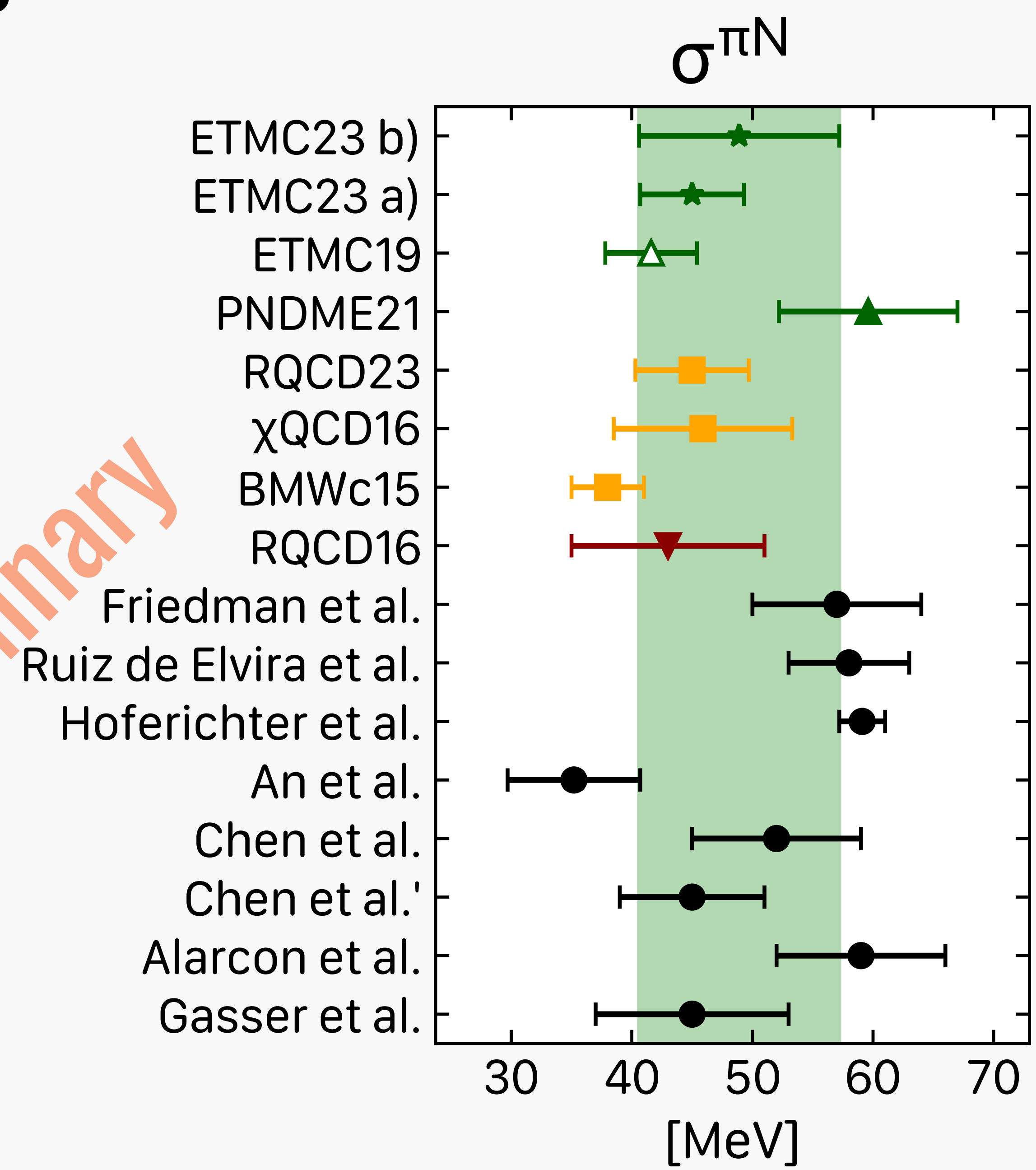


- One result using FH method (BMW)
- Los Alamos group [R. Gupta et al. PRL 127 (2021) 24]:
  - → $59.6(7.4)\ MeV$, whit explicit πN as prior

Scalar charge – σ-terms

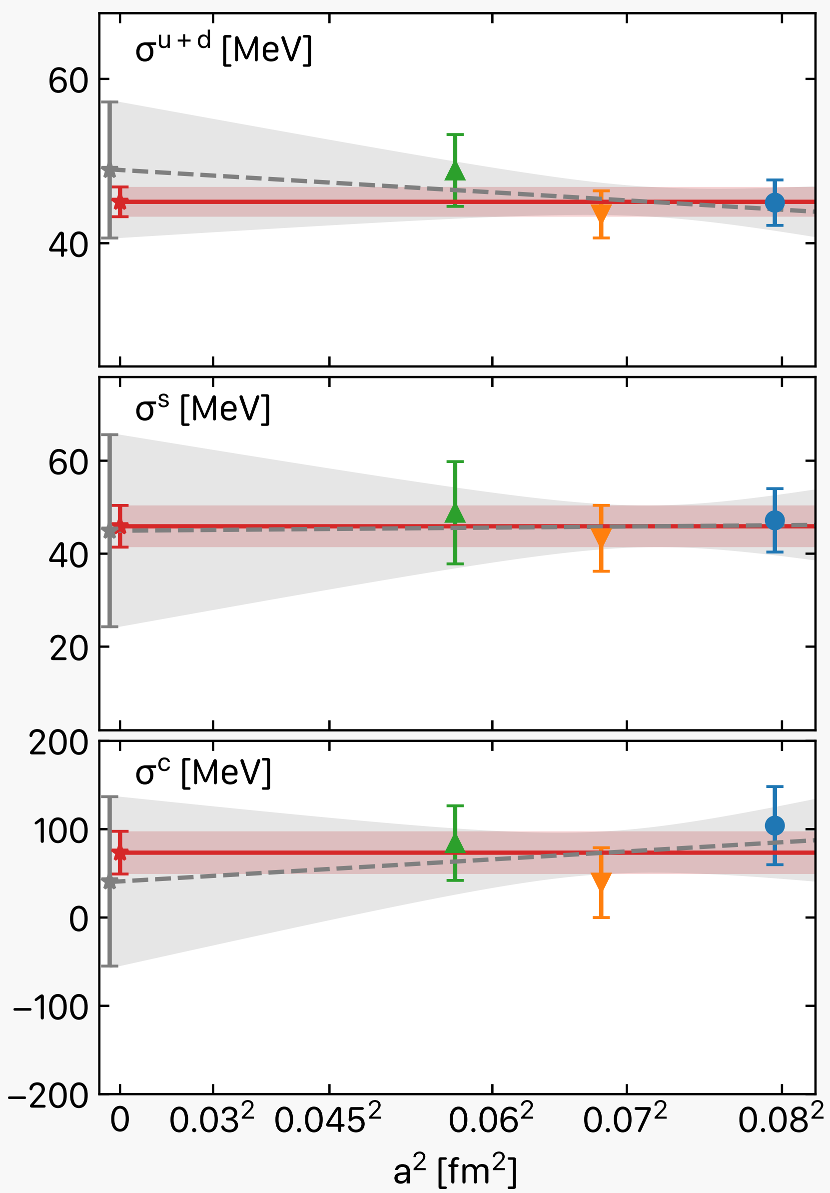


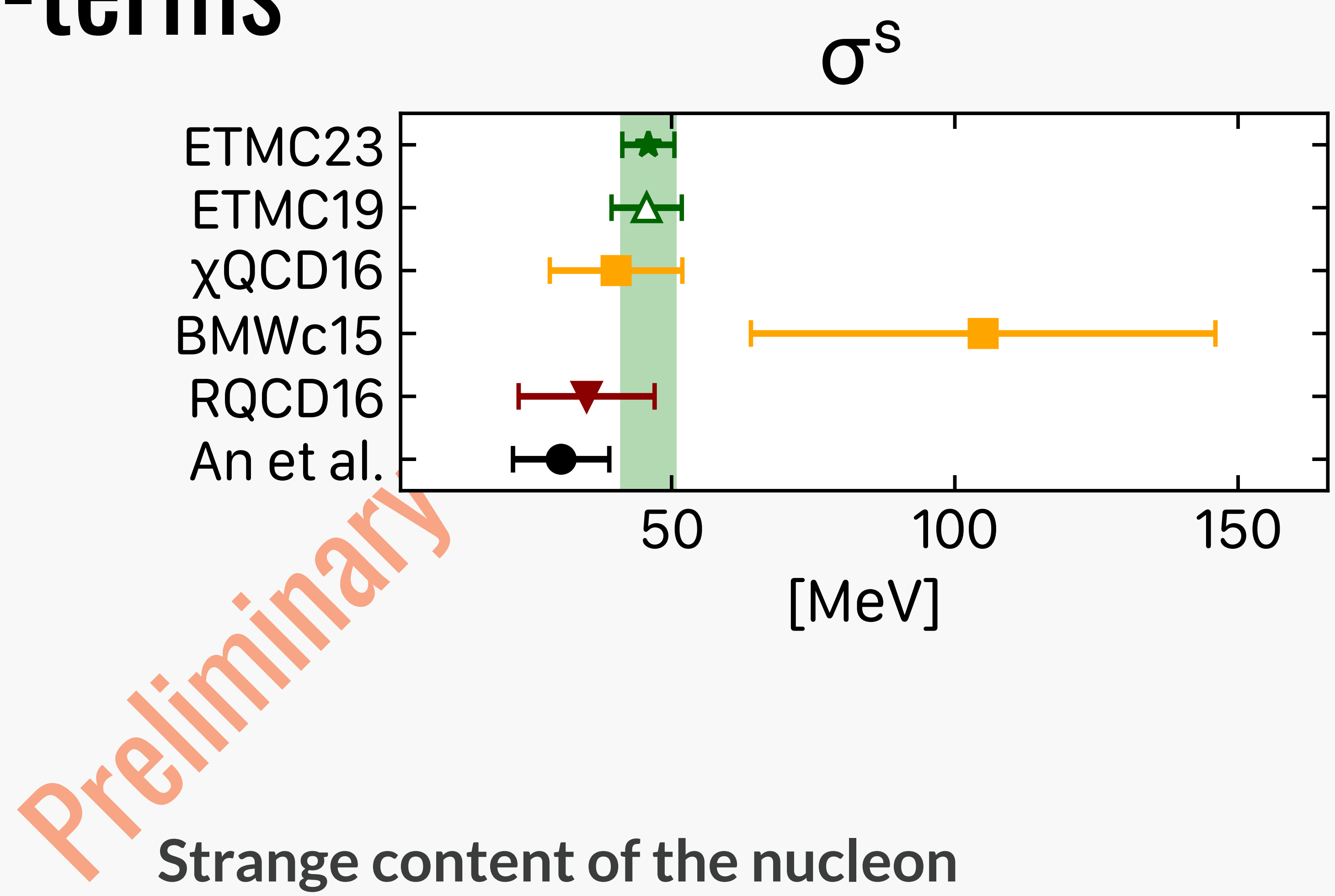
Preliminary



- One result using FH method (BMW)
- Los Alamos group: $59.6(7.4)\ MeV$, when explicitly including πN energy as prior
- Our result with linear continuum extrapolation (Grey band in left plot)

Scalar charge – σ-terms





Strange content of the nucleon

- Weaker dependence on lattice spacing
- Overall general agreement between lattice formulations

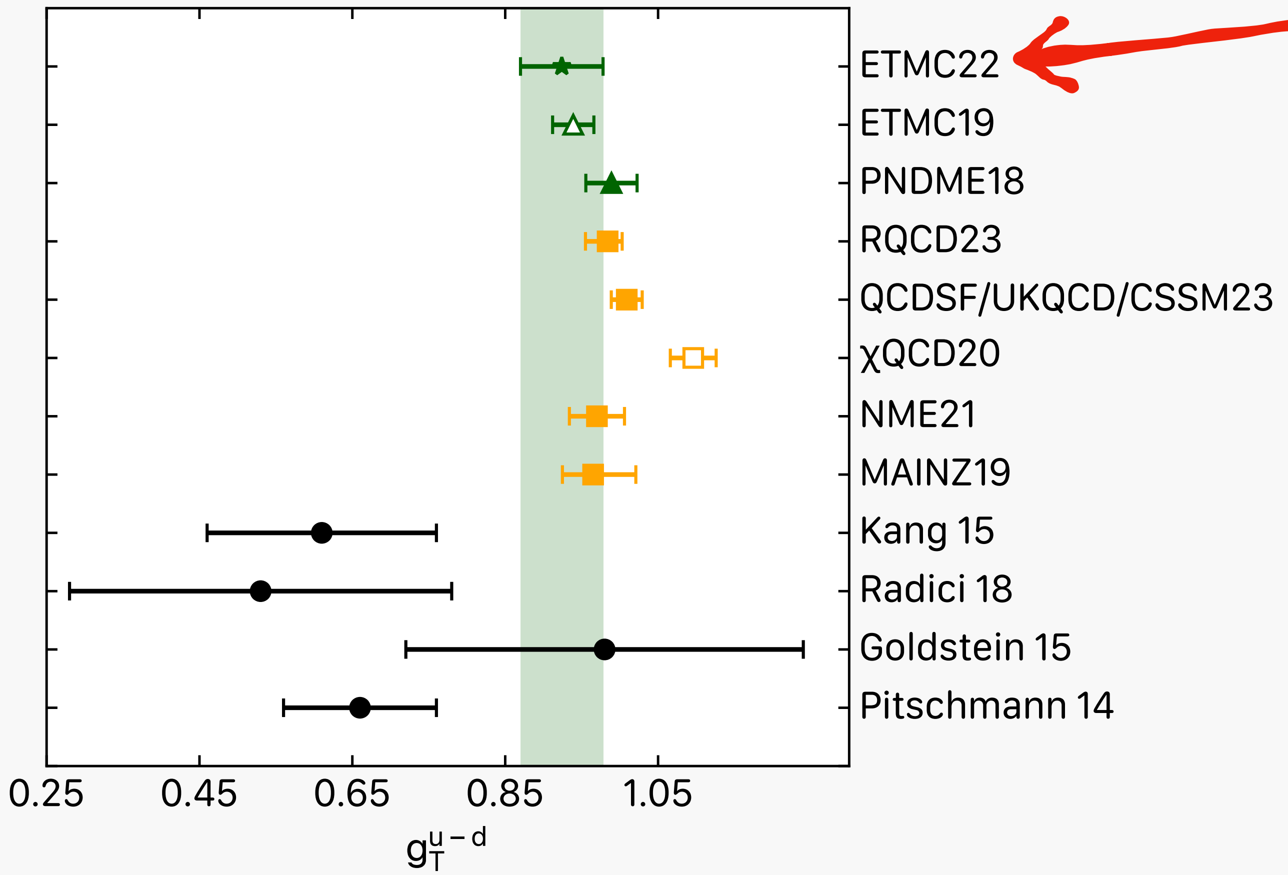
Tensor charge

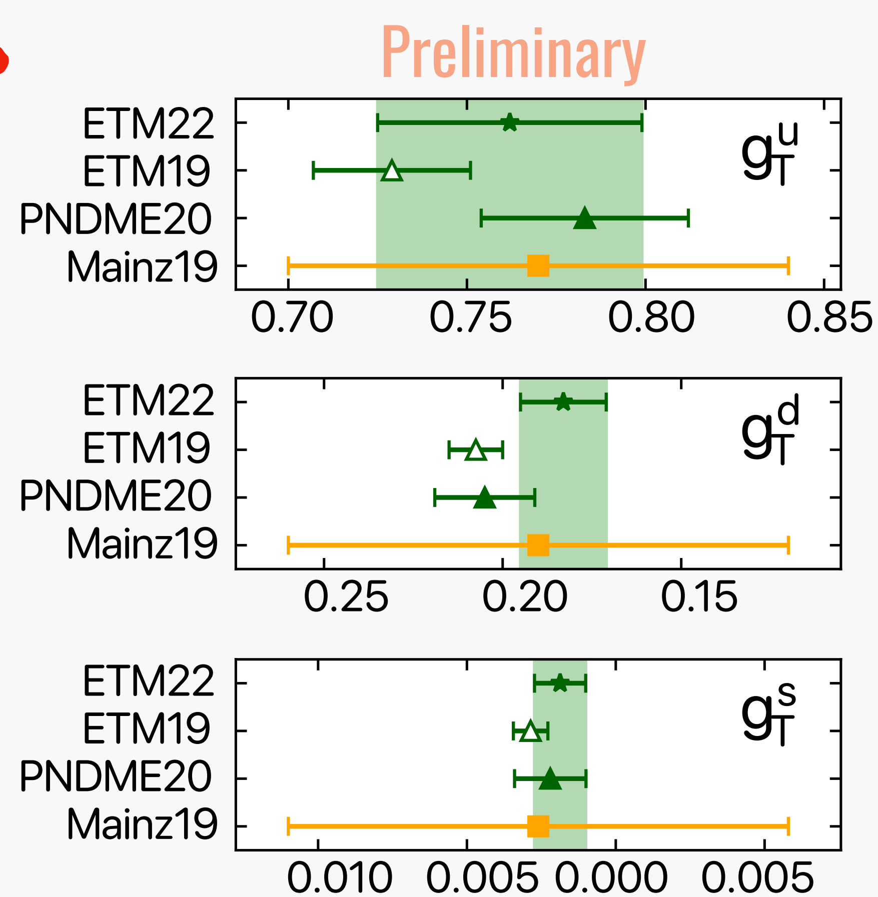
- Tensor matrix element

$$g_T = \langle 1 \rangle_{\delta q} \leftarrow \langle N | \bar{q} \sigma_{\mu\nu} q | N \rangle \quad \delta q \in \{\delta u + \delta d,\ \delta u - \delta d,\ \delta s, \ldots\}$$

- Novel CP-violating interactions, non-zero nEDM
- Moment of transversity PDF; Can be used to constrain experimental analyses, e.g. JAM [Phys. Rev. D 106 (2022) 3, 034014 arXiv:2205.00999]

"First moments of the nucleon transverse quark spin densities using lattice QCD", Phys. Rev. D107 (2023) 5, 054504 [arXiv:2202.09871]





Nucleon Electromagnetic Form Factors

Matrix element:

$$\langle N(p',s')|j^\mu|N(p,s)\rangle = \sqrt{\frac{M_N^2}{E_N(\mathbf{p}')E_N(\mathbf{p})}}\,\bar{u}(p',s')\mathcal{O}^\mu u(p,s)$$

$$\mathcal{O}^\mu = \gamma_\mu F_1(q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{2M_N}F_2(q^2), \quad q = p' - p$$

Dirac and Pauli (F_1 and F_2) **/ Sachs Electric and Magnetic** (G_E and G_M) **form-factors:**

$$G_E(q^2) = F_1(q^2) + \frac{q^2}{(2M_N)^2}F_2(q^2) \qquad G_M(q^2) = F_1(q^2) + F_2(q^2)$$

Isovector & Isoscalar currents:

$$j_\mu^v = \bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d,$$

$$j_\mu^s = \bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d$$

Assuming mass degenerate up and down quarks

$$F^p - F^n = F^u - F^d$$

$$F^p + F^n = \frac{1}{3}(F^u + F^d)$$

Treatment of excited states

Summation method

$$S_\Gamma(\vec{q};t_s) = \sum_{t_{ins}=\tau}^{t_s-\tau} R_\Gamma(\vec{q};t_s;t_{ins}) \to \mathcal{M}t_s + C$$

Two-state fit

$$G_\Gamma(\vec{q};t_s,t_{ins}) = \sum_{i=0}^{1}\sum_{j=0}^{1} c_{ij} e^{-E_i(0)(t_s-t_{ins})} e^{-E_j(\vec{q})t_{ins}}$$

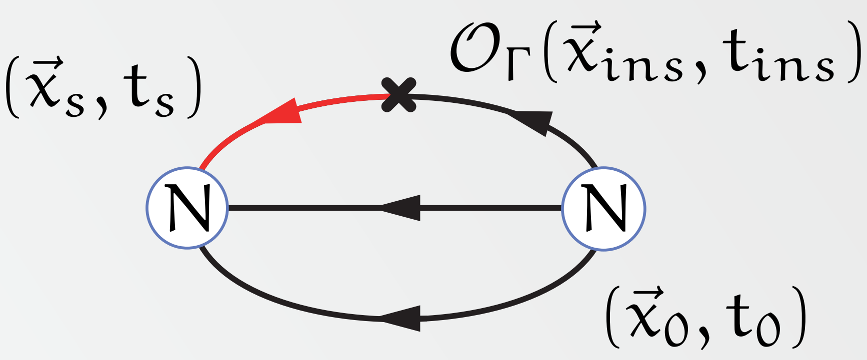
$$\mathcal{M} = \frac{c_{00}}{\sqrt{a_0(\vec{0})a_0(\vec{q})}}$$

$$G(\vec{q};t_s) = a_0(\vec{q})e^{-\varepsilon_0(\vec{q})t_s} + a_1(\vec{q})e^{-\varepsilon_1(\vec{q})t_s}$$

Ground state:

$$E_0(0) = \varepsilon_0(0)$$

$$E_0(\vec{q}) = \varepsilon_0(\vec{q}) = [\varepsilon_0^2(\vec{0}) + (\frac{2\pi}{L}\vec{q})^2]^{\frac{1}{2}}$$

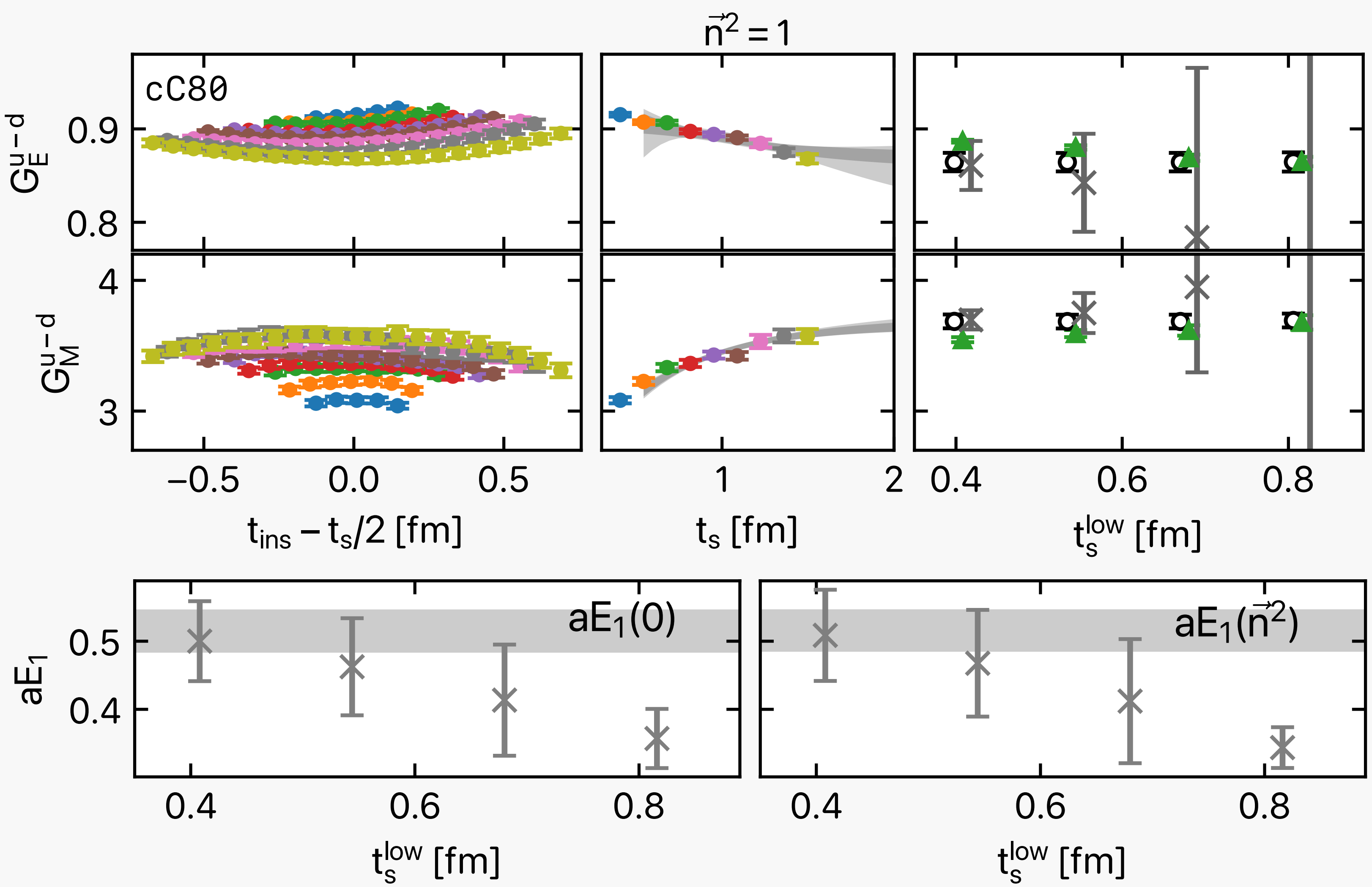


Excited states:

In general, $E_1(\vec{q}) \neq \varepsilon_1(\vec{q})$ and $E_1(0) \neq \varepsilon_1(0)$ *

*E.g. arXiv:1905.06470

Treatment of excited states



Summation

Two-state,

$E_1(0) = \varepsilon_1(0)$

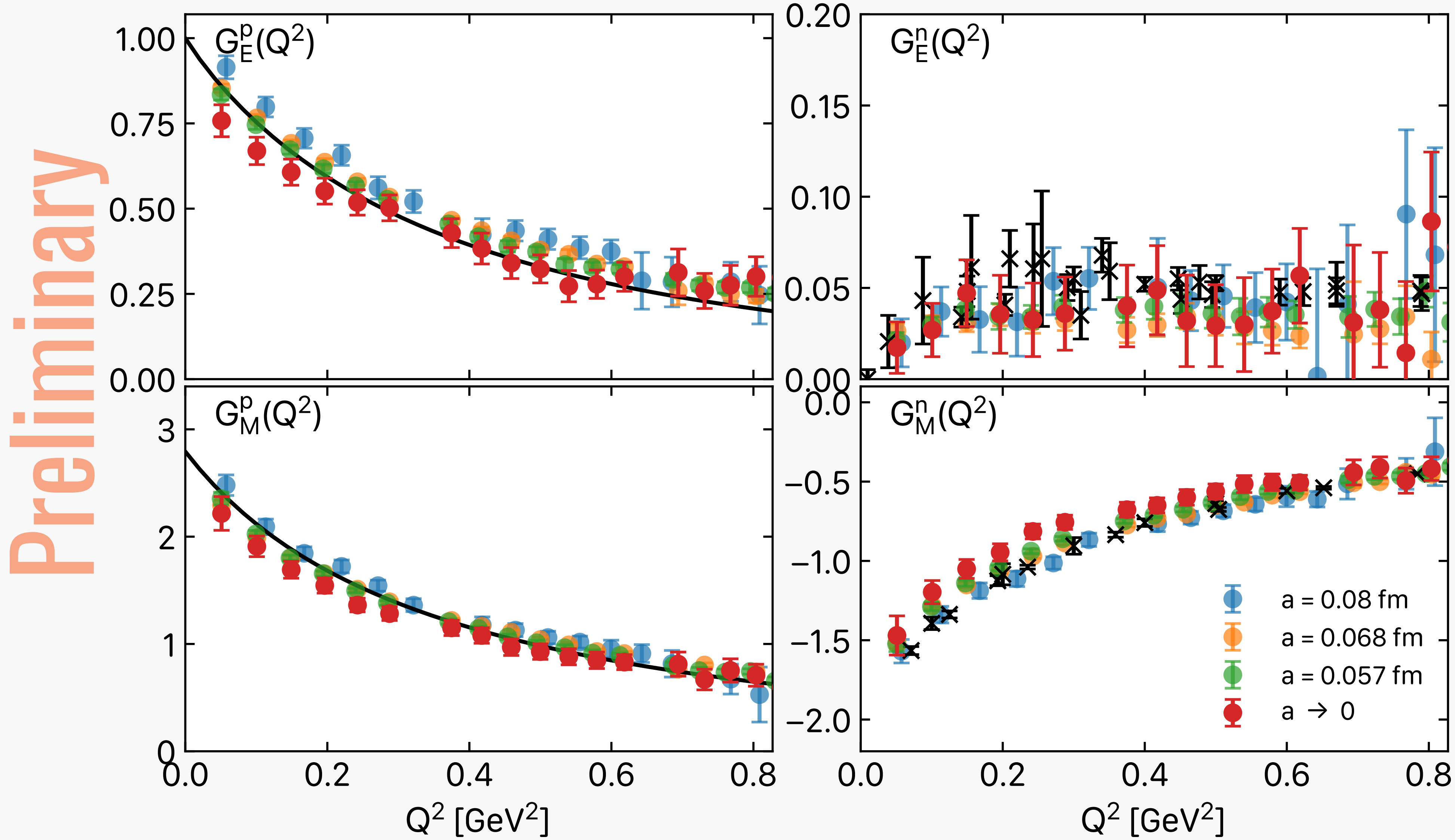
$E_1(\vec{q}) = \varepsilon_1(\vec{q})$

Two-state,

$E_1(\vec{q}) \neq \varepsilon_1(\vec{q})$

$E_1(0) \neq \varepsilon_1(0)$

Proton & neutron form factors



- Includes disconnected for $u+d$ combination
- Black curve for proton → from *z*-expansion fits to world data (experiment)
- Black crosses for neutron → world data (experiment)
- Ongoing analysis for proton and neutron radii, magnetic moment

Axial Form Factors

Matrix element:

$$\langle N(p', s')|A^3_\mu|N(p, s)\rangle = \sqrt{\frac{M_N^2}{E_N(\mathbf{p}')E_N(\mathbf{p})}}\bar{u}(p', s')\mathcal{O}^\mu u(p, s)$$

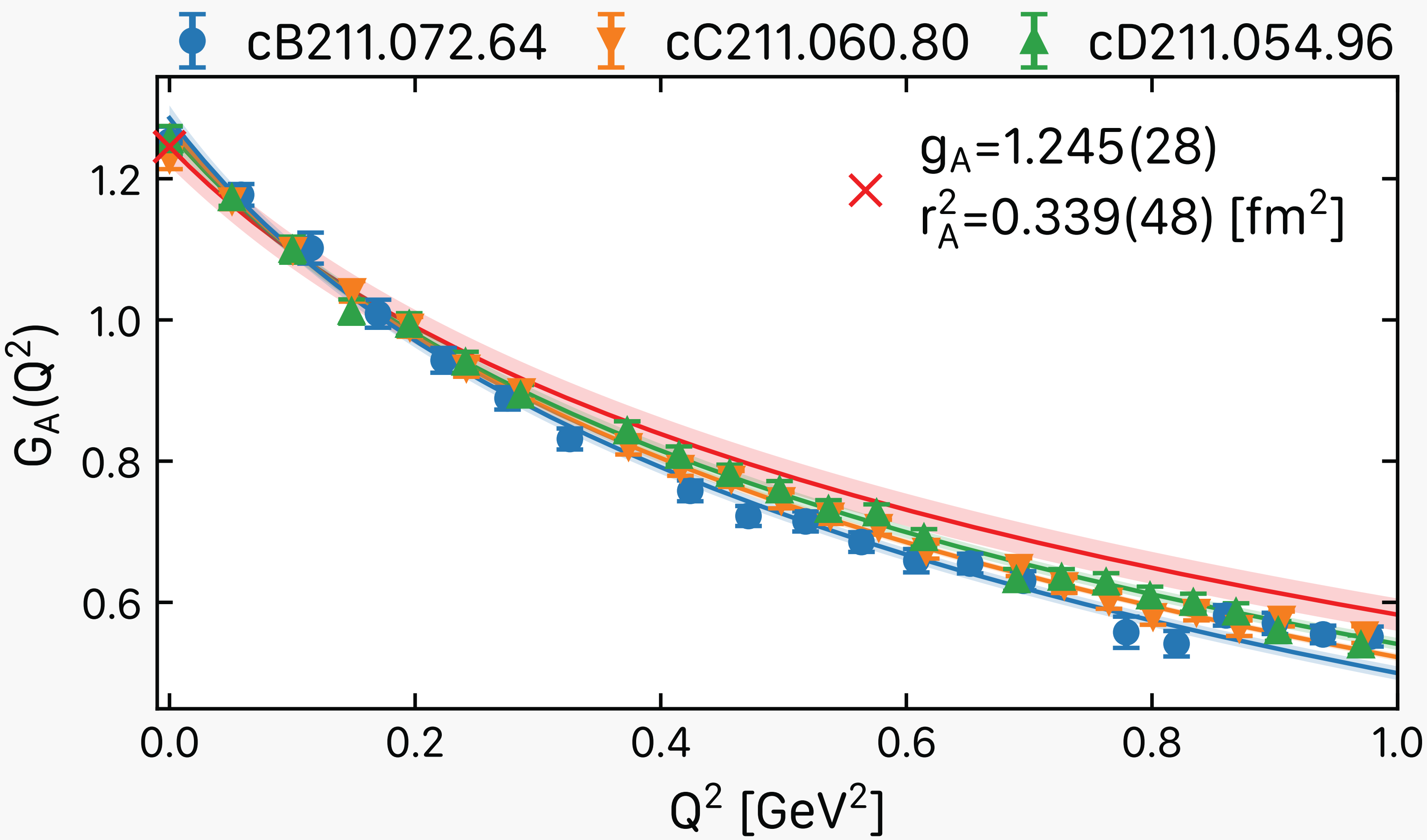
$$\mathcal{O}^\mu = \gamma_\mu\gamma_5 G_A(q^2) + \frac{q^\mu\gamma_5}{2M_N}G_P(q^2), \quad q = p' - p$$

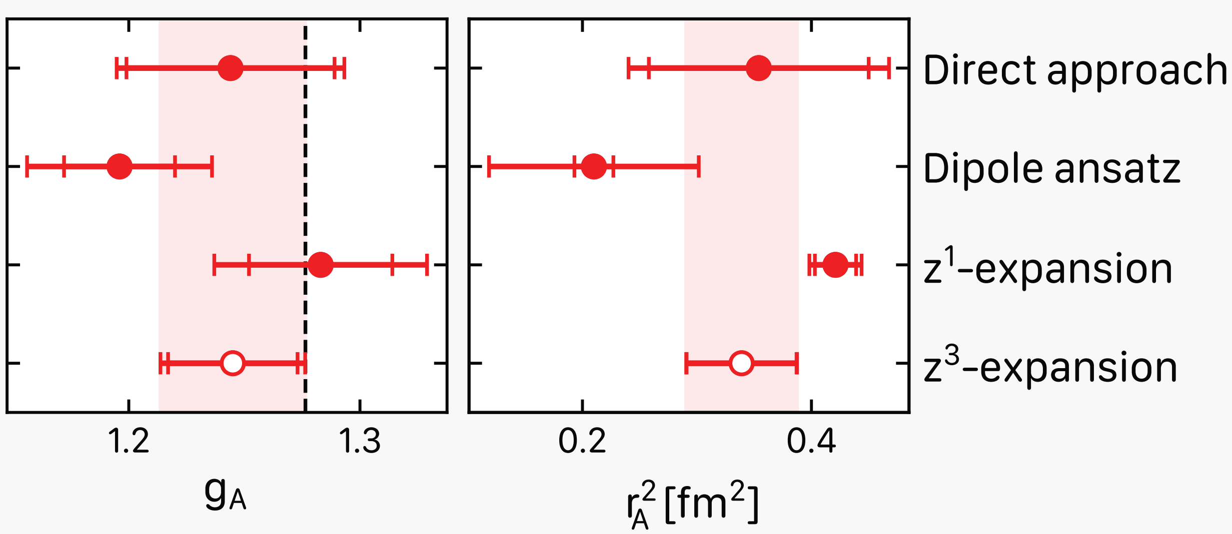
Axial (G_A**)** and **Induced Pseudoscalar (**G_P**)** form factors

- Known to less accuracy experimentally compared to EM
  - Via elastic scattering: $\nu_\mu + n \to \mu^- + p$
  - Via charged pion electroproduction
- Required in neutrino oscillation experiments. Traditionally modelled with a dipole form:
  $G_A(Q^2) = \frac{g_A}{(1 + Q^2/M_A^2)^2}$ to extract the "axial mass" M_A
- Note, the lattice calculation of the isovector case:

$$\langle p(p')|\bar{u}\gamma_5\gamma_\mu d|n(p)\rangle \xrightarrow{p\leftrightarrow n} \langle N(p')|\bar{u}\gamma_5\gamma_\mu u - \bar{d}\gamma_5\gamma_\mu d|N(p)\rangle$$

Axial Form Factor





- Nice agreement of results independent of fit
- Compare to experiment (dashed line)

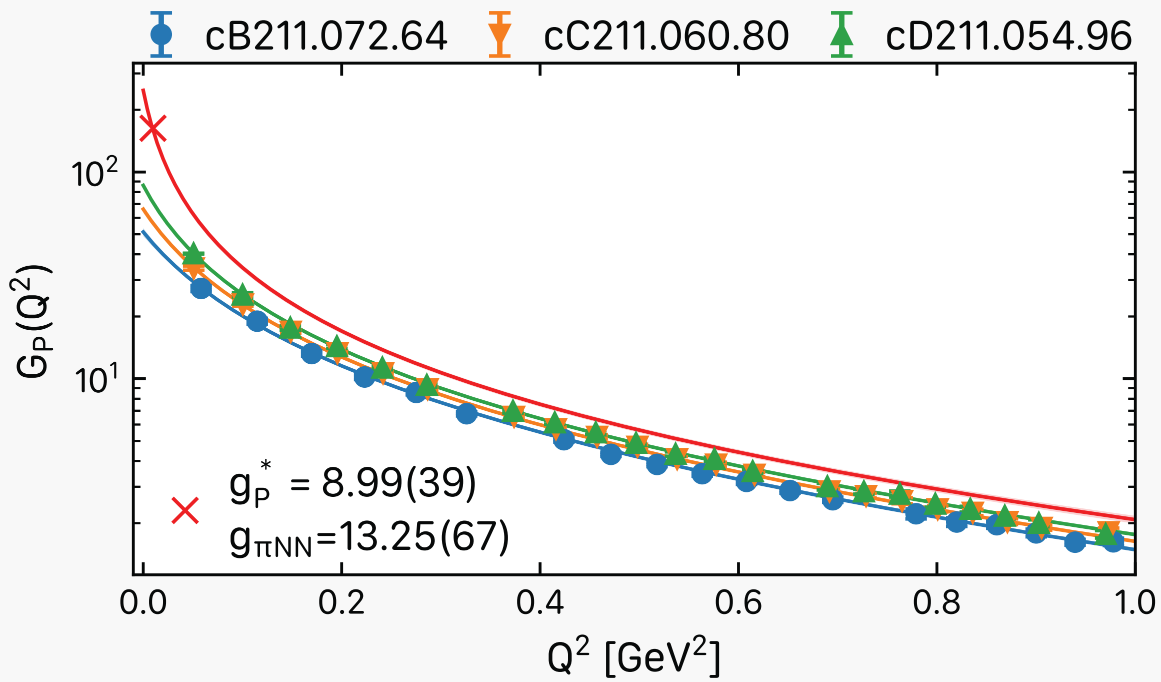
ETM collab., Phys. Rev. D 109 (2024) 3, 034503 [arXiv:2309.05774]

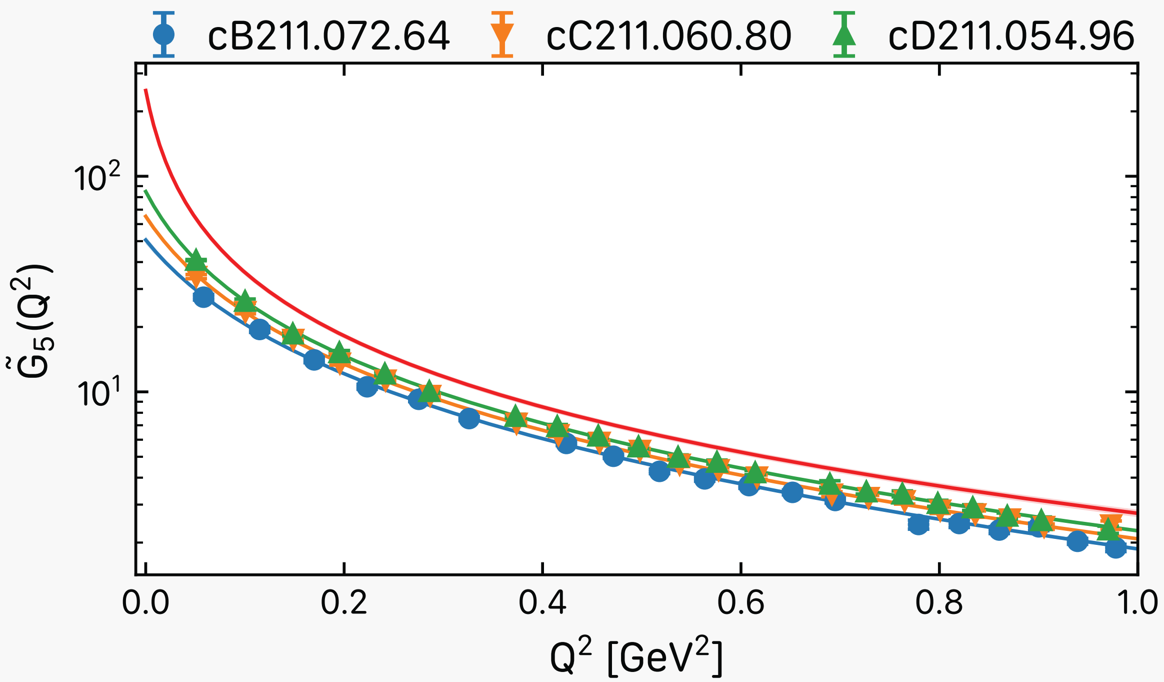
Axial & Pseudoscalar form factors

Matrix element:

$$\langle N(p',s')|\bar{u}\gamma_5 u - \bar{d}\gamma_5 d|N(p,s)\rangle = \bar{u}(p',s')\gamma_5 G_5(Q^2) u(p,s)$$

Pseudoscalar form factor (G_5)





$$g_{\pi NN} = G_{\pi NN}(-m_\pi^2), \quad g_P^* = \frac{m_\mu}{2m_N} G_P(0.88 m_\mu^2) \qquad \tilde{G}_5(Q^2) = \frac{4m_N}{m_\pi^2} m_q G_5(Q^2)$$

$$\frac{F_\pi m_\pi^2}{m_\pi^2 + Q^2} G_{\pi NN}(Q^2) = m_q G_5(Q^2)$$

Axial & Pseudoscalar form factors

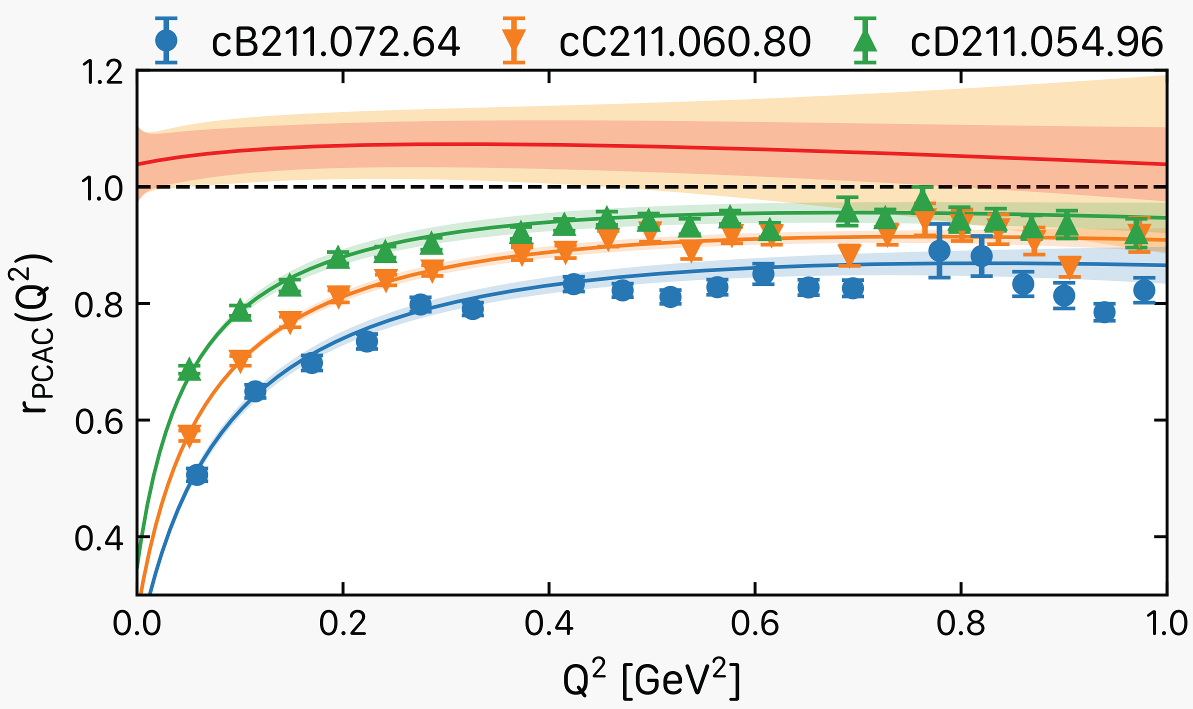
Check relation between axial and pseudoscalar form factors

– From PCAC relation

$$\partial^{\mu} A_{\mu} = 2 m_q P$$

– Between nucleon states:

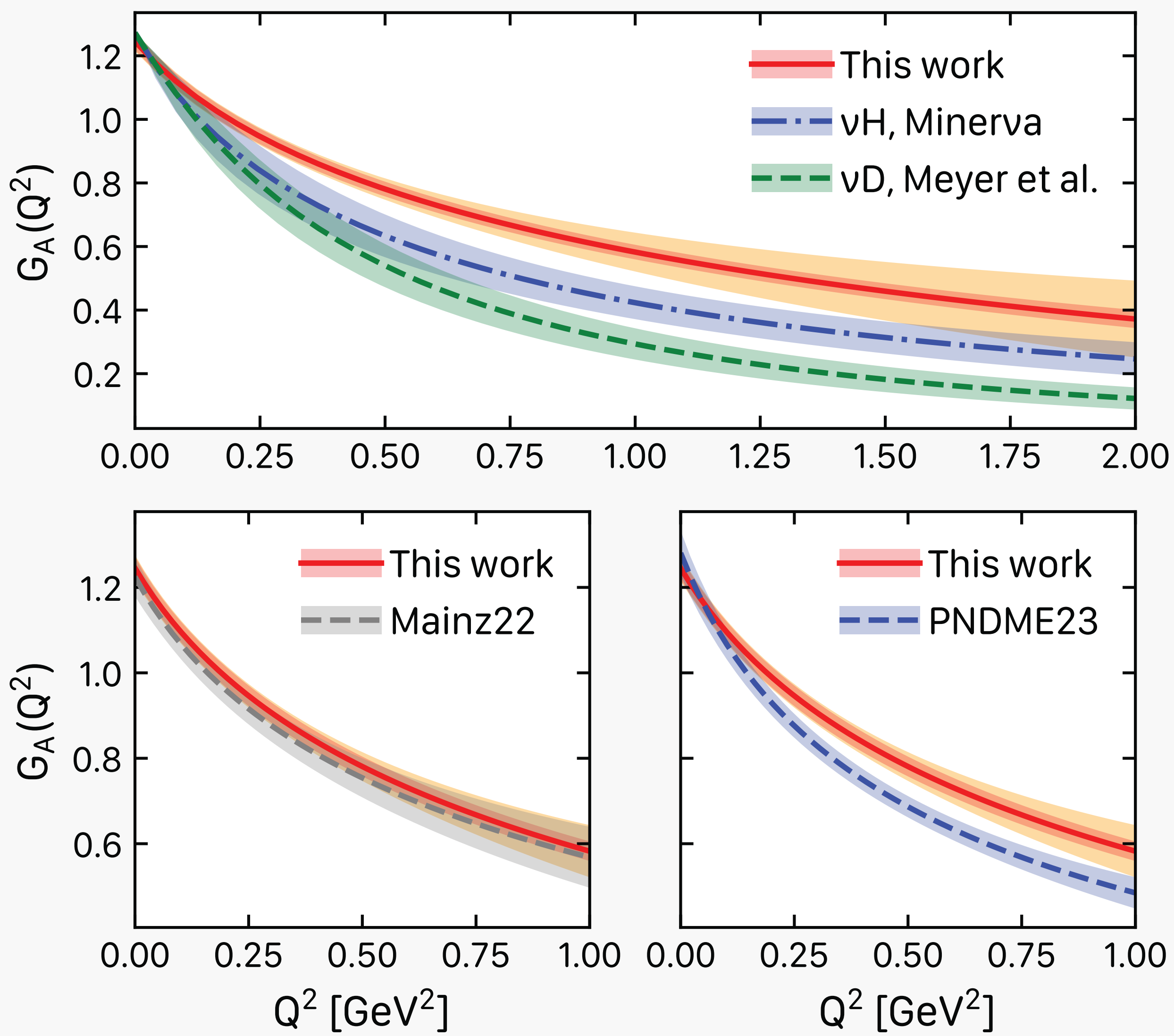
$$G_A(Q^2) - \frac{Q^2}{4m_N^2} G_P(Q^2) = \frac{m_q}{m_N} G_5(Q^2)$$

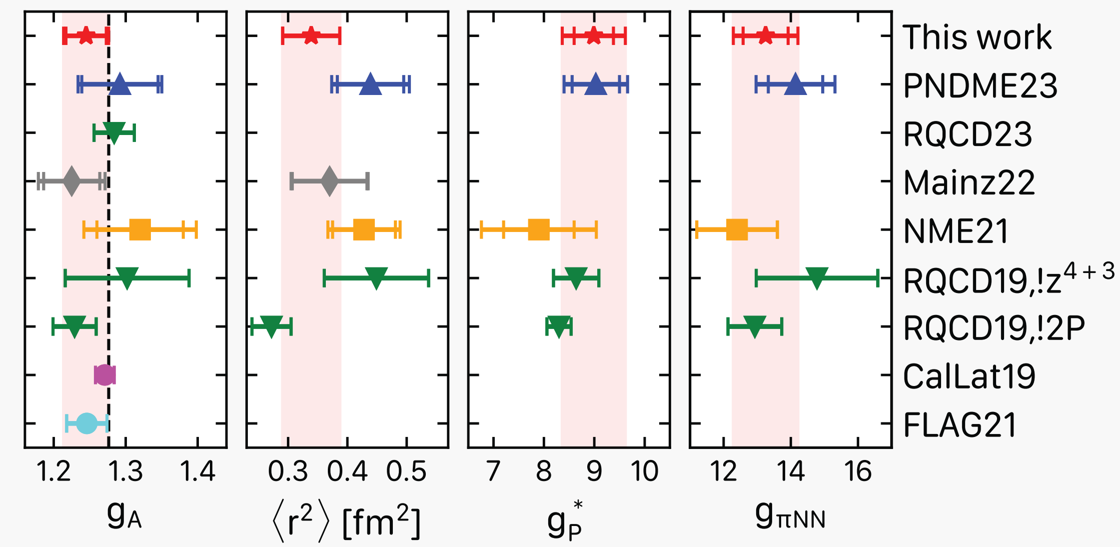


– Relation from PCAC restored at continuum limit ($a \to 0$)

Axial & Pseudoscalar form factors

Comparison with experiment and other lattice results





- Good agreement between lattice results (*non-trivial*: each has varying systematics)
- Note the MINERνA result (2023):
  - $r_A = 0.73(17)$ fm or
  - $(r_A)^2 = 0.53(25)$ fm^2

Summary & Outlook

- Lattice QCD with physical point ensembles at multiple lattice spacings
  - Here, three lattice spacings ➜ Continuum limit directly at physical point
- Reproduction of well-known nucleon structure quantities, e.g. axial charge
- Requires thorough study of systematic uncertainties
  - Main systematic is excited state effects
- Reproduction of quantities known well experimentally at physical point
  - Axial charge and proton electromagnetic form factors
- Impact on quantities less well-known
  - σ-terms, tensor charge, axial form factors, strange and charm contents

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Με τη συγχρηματοδότηση της Ευρωπαϊκής Ένωσης

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