



Nucleon electroweak elastic & transition form factors

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&
Peng Huanwu Center for Fundamental Theory

May 15th, 2024

Strong QCD from Hadron Structure Experiments - VI

Non-Perturbative QCD:

- **Hadrons, as bound states, are dominated by non-perturbative QCD dynamics – Two emergent phenomena**
 - **Confinement:** Colored particles have never been seen isolated
 - ✓ Explain how quarks and gluons bind together
 - **Dynamical Chiral Symmetry Breaking (DCSB):** Hadrons do not follow the chiral symmetry pattern
 - ✓ Explain the most important mass generating mechanism for visible matter in the Universe
- Neither of these phenomena is apparent in QCD's Lagrangian, HOWEVER, They play a dominant role in determining the characteristics of real-world QCD!

Non-Perturbative QCD:

➤ From a quantum field theoretical point of view, these emergent phenomena could be associated with dramatic, dynamically driven changes in the analytic structure of QCD's Schwinger functions (propagators and vertices). The Schwinger functions are solutions of the quantum equations of motion (**Dyson-Schwinger equations**).

➤ **Dressed-quark propagator:**



- Mass generated from the interaction of quarks with the gluon.
- Light quarks acquire a **HUGE** constituent mass.
- Responsible of the 98% of the mass of the proton and the large splitting between parity partners.

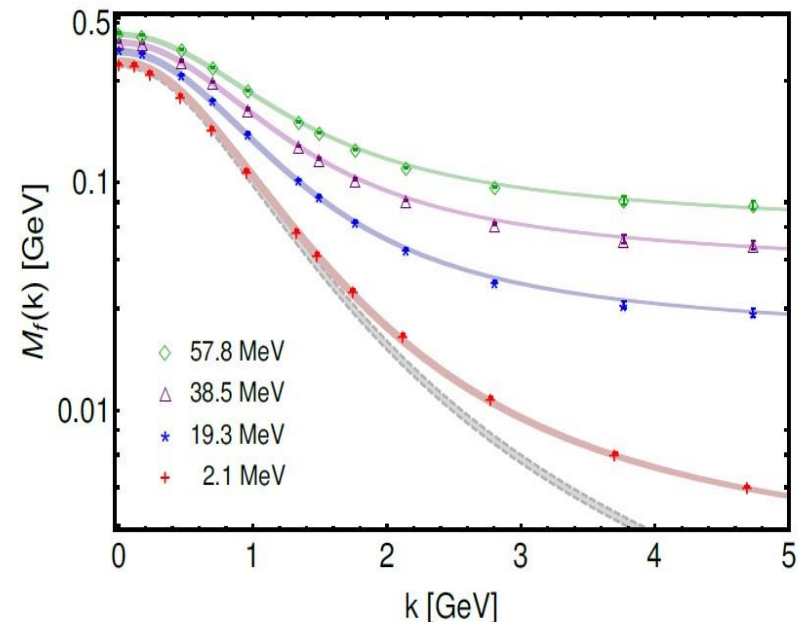
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- Lei Chang, Yu-Bin Liu, Khépani Raya, J. Rodríguez-Quintero, and Yi-Bo Yang, Phys. Rev. D 104, no.9, 094509 (2021)

Continuum Schwinger function methods (CSMs)

Quark propagator:

$$\text{---}\text{---}\text{---}^{-1} = \text{---}\text{---}\text{---}^{-1} + \text{---}\text{---}\text{---}$$

Ghost propagator:

$$\text{---}\text{---}\text{---}^{-1} = \text{---}\text{---}\text{---}^{-1} + \text{---}\text{---}\text{---}$$

Ghost-gluon vertex:

$$\text{---}\text{---}\text{---} = \text{---}\text{---}\text{---} + \text{---}\text{---}\text{---}$$

Quark-gluon vertex:

$$\text{---}\text{---}\text{---} = \text{---}\text{---}\text{---} + \text{---}\text{---}\text{---} + \text{---}\text{---}\text{---} + \text{---}\text{---}\text{---} + \text{---}\text{---}\text{---} + \text{---}\text{---}\text{---} + \text{---}\text{---}\text{---} + \text{---}\text{---}\text{---}$$

Gluon propagator:

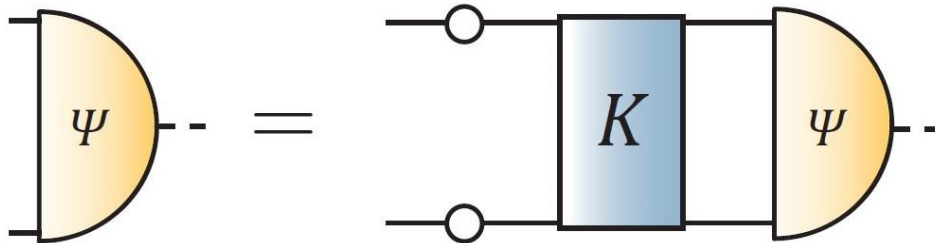
$$\text{---}\text{---}\text{---}^{-1} = \text{---}\text{---}\text{---}^{-1} + \text{---}\text{---}\text{---} + \text{---}\text{---}\text{---} + \text{---}\text{---}\text{---} + \text{---}\text{---}\text{---} + \text{---}\text{---}\text{---} + \text{---}\text{---}\text{---} + \text{---}\text{---}\text{---}$$

Hadrons: Bound-states in QFT

➤ **Mesons:** a 2-body bound state problem in QFT

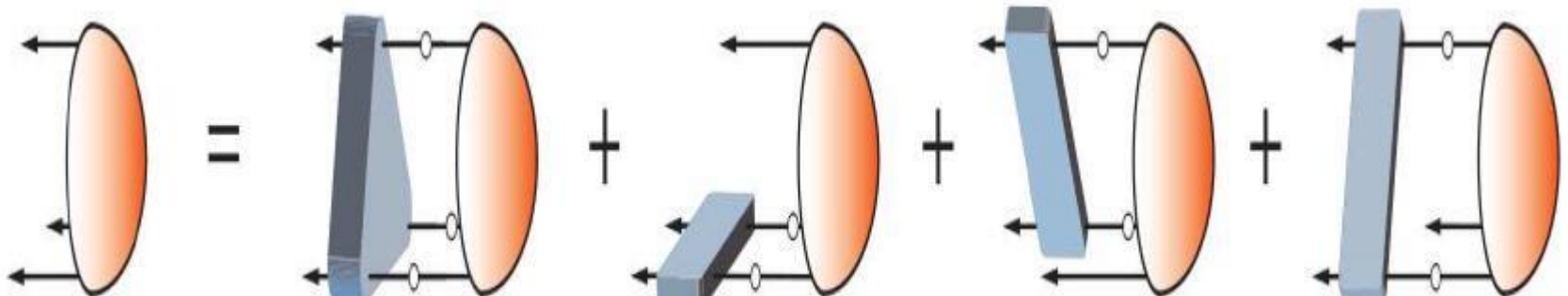
➤ Bethe-Salpeter Equation

➤ **K - fully amputated, two-particle irreducible, quark-antiquark scattering kernel**



➤ **Baryons:** a 3-body bound state problem in QFT

➤ Faddeev equation: sums all possible quantum field theoretical exchanges and interactions that can take place between the three dressed-quarks that define its valence quark content.




2-body correlations

- Mesons: quark-antiquark correlations -- **color-singlet**
- Diquarks: quark-quark correlations within a **color-singlet** baryon.
- Diquark correlations:
 - In our approach: non-pointlike color-antitriplet and fully interacting.
 - Diquark correlations are soft, they possess an electromagnetic size.
 - Owing to properties of charge-conjugation, a diquark with spin-parity J^P may be viewed as a partner to the analogous $J^{\{-P\}}$ meson.

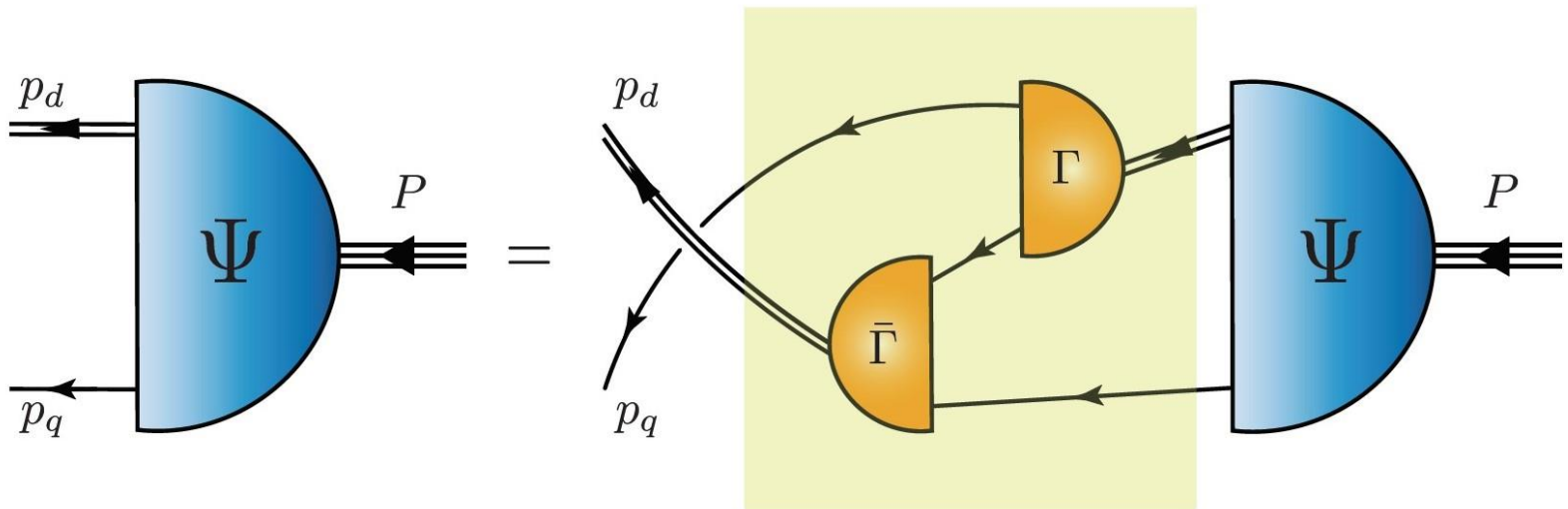
$$\Gamma_{q\bar{q}}(p; P) = - \int \frac{d^4 q}{(2\pi)^4} g^2 D_{\mu\nu}(p - q) \frac{\lambda^a}{2} \gamma_\mu S(q + P) \Gamma_{q\bar{q}}(q; P) S(q) \frac{\lambda^a}{2} \gamma_\nu$$
$$\Gamma_{qq}(p; P) C^\dagger = - \frac{1}{2} \int \frac{d^4 q}{(2\pi)^4} g^2 D_{\mu\nu}(p - q) \frac{\lambda^a}{2} \gamma_\mu S(q + P) \Gamma_{qq}(q; P) C^\dagger S(q) \frac{\lambda^a}{2} \gamma_\nu$$

quark-diquark Faddeev equation

- Quantum numbers:
 - $(I=0, J^P=0^{++})$: isoscalar-scalar diquark
 - $(I=1, J^P=1^{++})$: isovector-pseudovector diquark
 - $(I=0, J^P=0^{-})$: isoscalar-pseudoscalar diquark
 - $(I=0, J^P=1^{-})$: isoscalar-vector diquark
 - $(I=1, J^P=1^{-})$: isovector-vector diquark

- Three-body bound states  Quark-diquark two-body bound states

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quark-diquark Faddeev equation



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Review

Diquark correlations in hadron physics: Origin, impact and evidence

M.Yu. Barabanov¹, M.A. Bedolla², W.K. Brooks³, G.D. Cates⁴, C. Chen⁵, Y. Chen^{6,7}, E. Cisbani⁸, M. Ding⁹, G. Eichmann^{10,11}, R. Ent¹², J. Ferretti¹³
✉, R.W. Gothe¹⁴, T. Horn^{15,12}, S. Liuti⁴, C. Mezrag¹⁶, A. Pilloni⁹, A.J.R. Puckett¹⁷, C.D. Roberts^{18,19} ✉ ... B.B. Wojtsekhowski¹² ✉

- The electromagnetic nucleon-to- $\Delta(1600)$ transition form factors

PHYSICAL REVIEW D **100**, 034001 (2019)

Transition form factors: $\gamma^* + p \rightarrow \Delta(1232), \Delta(1600)$

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- In 2023, those (parameter-free) predictions were confirmed in an analysis of data obtained using the CLAS detector at JLab

PHYSICAL REVIEW C **108**, 025204 (2023)

First results on nucleon resonance electroexcitation amplitudes from $ep \rightarrow e'\pi^+\pi^-p'$ cross sections at $W = 1.4\text{--}1.7$ GeV and $Q^2 = 2.0\text{--}5.0$ GeV²

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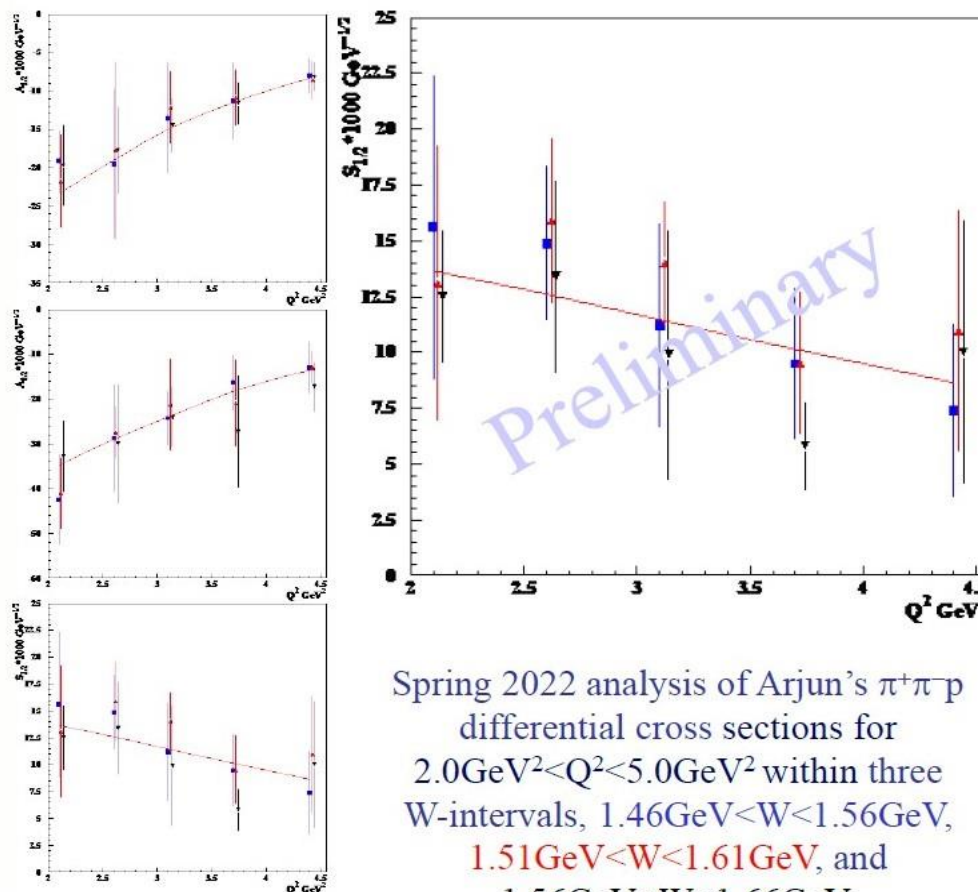
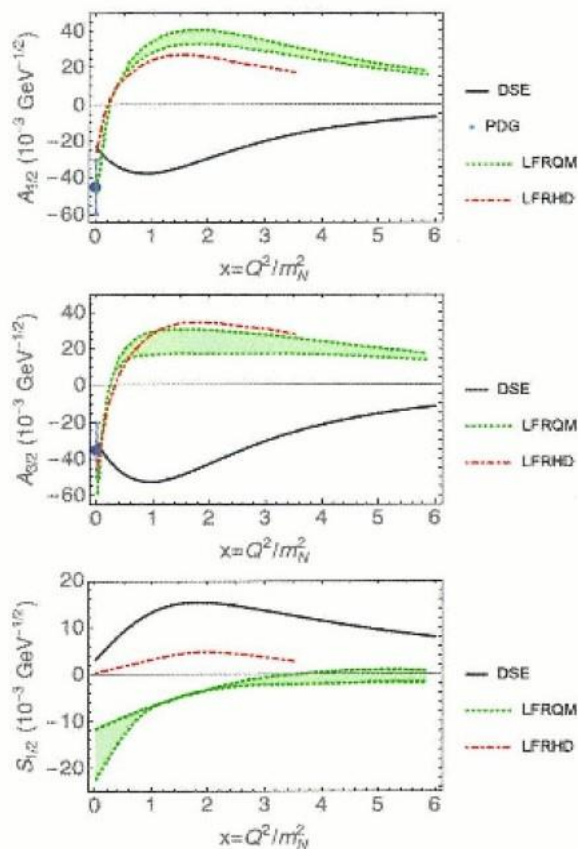
⁴Skobeltsyn Institute of Nuclear Physics and Physics Department, Lomonosov Moscow State University, 119234 Moscow, Russia

⁵University of Connecticut, Storrs, Connecticut 06269, USA

$\Delta(1600)3/2^+$ Form Factors in CSM Approach

Viktor Mokeev

CSM predictions of the $\Delta(1600)3/2^+$ electrocouplings



Spring 2022 analysis of Arjun's $\pi^+\pi^-\rho$ differential cross sections for $2.0\text{GeV}^2 < Q^2 < 5.0\text{GeV}^2$ within three W -intervals, $1.46\text{GeV} < W < 1.56\text{GeV}$, $1.51\text{GeV} < W < 1.61\text{GeV}$, and $1.56\text{GeV} < W < 1.66\text{GeV}$.

Ya Lu et al., PRD 100, 034001 (2019)

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Nucleon axial and pseudoscalar form factors from the covariant Faddeev equation

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(Dated: November 2, 2018)

We compute the axial and pseudoscalar form factors of the nucleon in the Dyson-Schwinger approach. To this end, we solve a covariant three-body Faddeev equation for the nucleon wave function and determine the matrix elements of the axialvector and pseudoscalar isotriplet currents. Our only input is a well-established and phenomenologically successful ansatz for the nonperturbative quark-gluon interaction. As a consequence of the axial Ward-Takahashi identity that is respected at the quark level, the Goldberger-Treiman relation is reproduced for all current-quark masses. We discuss the timelike pole structure of the quark-antiquark vertices that enters the nucleon matrix elements and determines the momentum dependence of the form factors. Our result for the axial charge underestimates the experimental value by 20–25% which might be a signal of missing pion-cloud contributions. The axial and pseudoscalar form factors agree with phenomenological and lattice data in the momentum range above $Q^2 \sim 1 \dots 2 \text{ GeV}^2$.

PACS numbers: 11.80.Jy 12.38.Lg, 11.40.Ha 14.20.Dh

I. INTRODUCTION

The nucleon's axial and pseudoscalar form factors are of fundamental significance for the properties of the nucleon that are probed in weak interaction processes. Their momentum dependence can be experimentally tested by (anti)neutrino scattering off nucleons or nuclei, charged pion electroproduction and muon capture processes; see [1–3] for reviews. Both form factors are experimentally hard to extract and therefore considerably less well known than their electromagnetic counterparts. Precisely measured is only the low-momentum limit g_A of the axial form factor which is determined from neutron β -decay. Planned experiments at major facilities are expected to change this situation in the near future.

The theoretical calculation of the nucleon's axial and pseudoscalar form factors requires genuinely non-perturbative methods. Chiral perturbation theory has been successful in this respect [1, 4, 5] although it is generally limited to the region of low momentum transfer. Recent studies in lattice gauge theory are getting closer to the physical pion mass region [6–8] but finite-volume effects become increasingly important. Another non-perturbative approach is the one via functional meth-

The study of axial and pseudoscalar form factors in the functional approach has so far been limited to an approximation where the nucleon is treated as a bound object of a quark and a diquark that interact via quark exchange [12, 13]. The entire gluonic substructure appears here only implicitly within the dressing of quark and diquark propagators as well as diquark vertex functions. There are several conceptual issues that complicate the treatment of form factors in the quark-diquark model. First, the requirement of current conservation induces the appearance of intricate 'seagull' diagrams [14]. Such terms have been taken into account for electromagnetic form factors, but their implementation in the case of axial form factors has not yet been possible for technical reasons [13]. Second, to comply with chiral Ward identities, a current-conserving quark-diquark model requires vector diquarks in addition to the usual scalar and axialvector diquark degrees of freedom [15]. Such an elaborate treatment of the quark-diquark model has not yet been performed.

The situation is somewhat different when the nucleon is treated as a genuine three-body problem. The resulting Faddeev equation in rainbow-ladder truncation has been solved only recently for the nucleon and Δ masses [16–17] and the corresponding nucleon electro-

The study of axial and pseudoscalar form factors in the functional approach has so far been limited to an approximation where the nucleon is treated as a bound object of a quark and a diquark that interact via quark exchange [12, 13]. The entire gluonic substructure appears here only implicitly within the dressing of quark and diquark propagators as well as diquark vertex functions. There are several conceptual issues that complicate the treatment of form factors in the quark-diquark model. First, the requirement of current conservation induces the appearance of intricate 'seagull' diagrams [14]. Such terms have been taken into account for electromagnetic form factors, but their implementation in the case of axial form factors has not yet been possible for technical reasons [13]. Second, to comply with chiral Ward identities, a current-conserving quark-diquark model requires vector diquarks in addition to the usual scalar and axialvector diquark degrees of freedom [15]. Such an elaborate treatment of the quark-diquark model has not yet been performed.

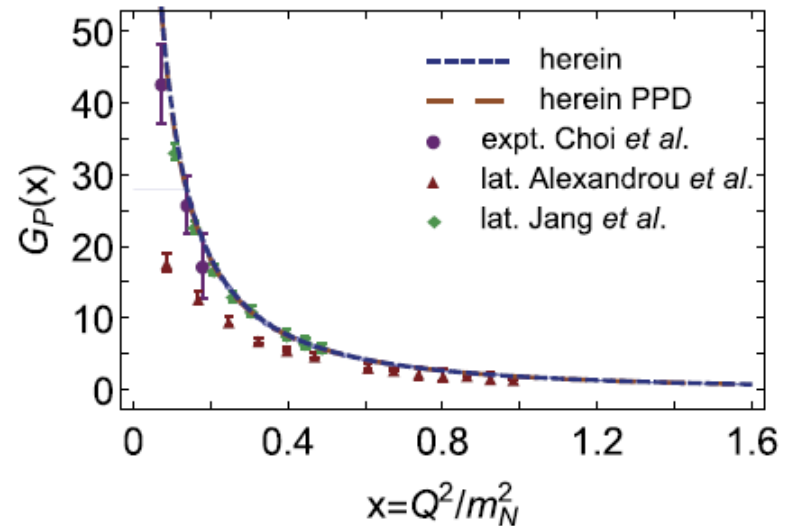
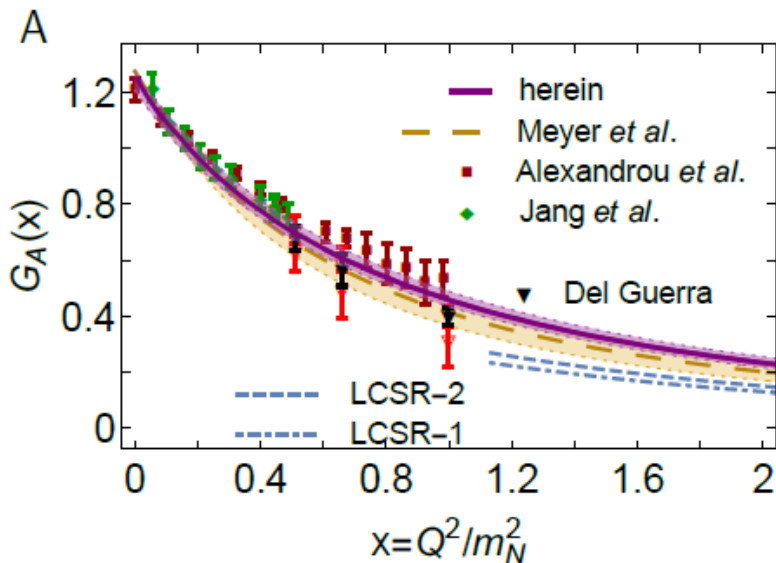
Goldberger-Treiman relation and $g_{\pi NN}$ from the three quark BS / Faddeev approach in the NJL model

Noriyoshi Ishii (Erlangen - Nuremberg U.) (Apr 28, 2000)

Published in: *Nucl.Phys.A* 689 (2001) 793-845 • e-Print: nucl-th/0004063 [nucl-th]

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PHYSICAL REVIEW D **105**, 094022 (2022)

Nucleon axial-vector and pseudoscalar form factors and PCAC relations

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Axial Form Factors

- Nucleon Axial Form Factor at large momentum transfers
- $\Delta(1232)$ Axial Form Factors
- $N(940) \rightarrow \Delta(1232)$ Axial and Pseudoscalar Transition Form Factors



Nucleon axial form factor at large momentum transfers

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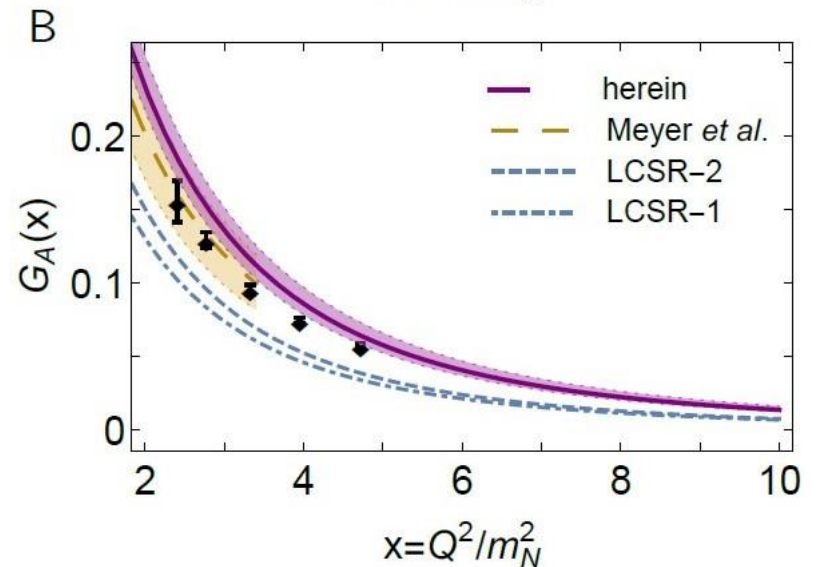
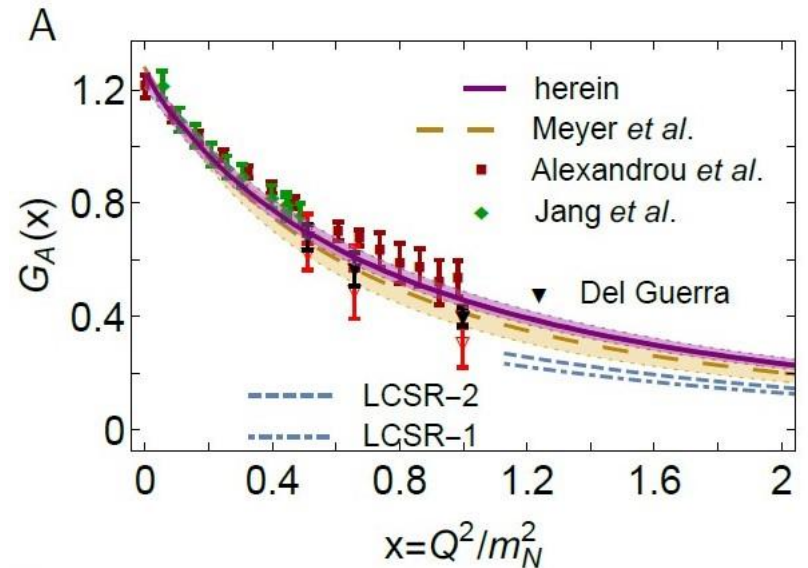
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Large Q^2 Nucleon Axial Form Factor

- Parameter-free CSM predictions to $Q^2 = 10 \cdot m_N^2$
- CSM prediction agrees with available data: small & large Q^2





Δ -Baryon axialvector and pseudoscalar form factors, and associated PCAC relations

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Δ -Baryon axialvector and pseudoscalar form factors, and associated PCAC relations

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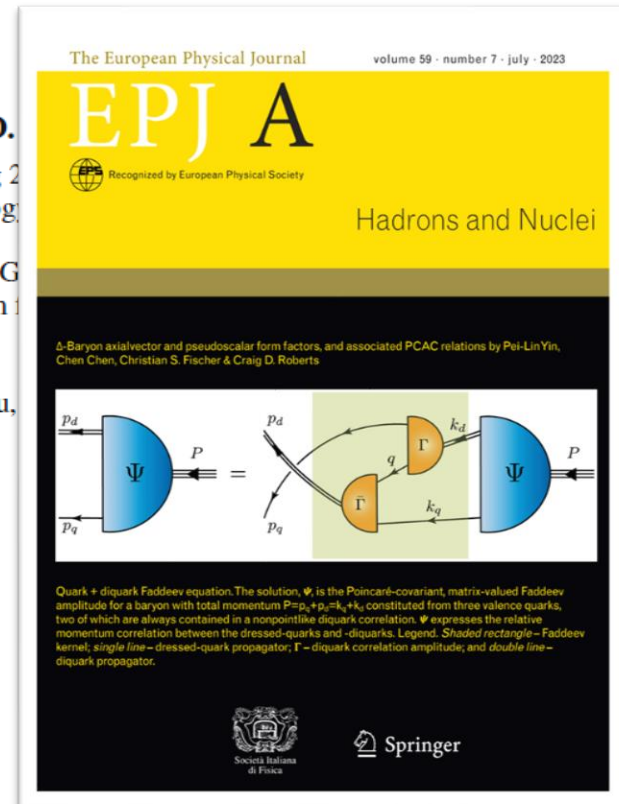
⁴ Institut für Theoretische Physik, Justus-Liebig-Universität Gießen, 35392 Gießen, G

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⁶ School of Physics, Nanjing University, Nanjing 210093, Jiangsu, China

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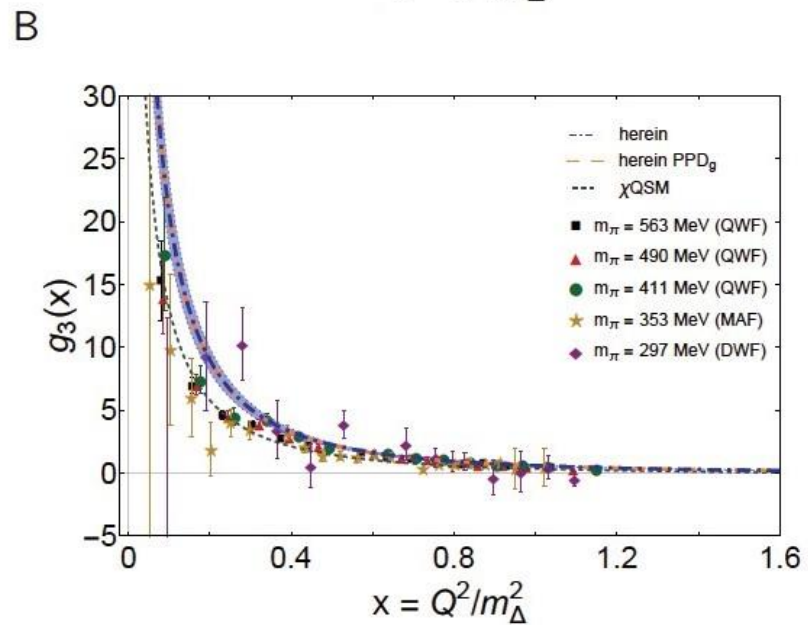
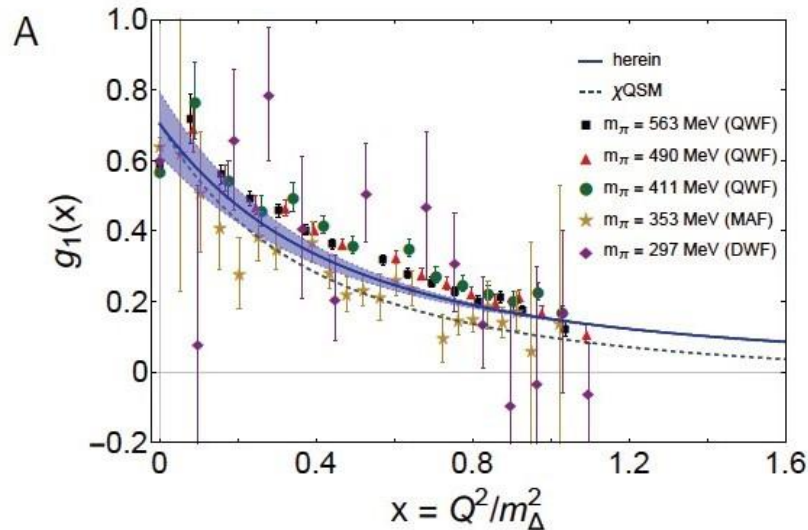
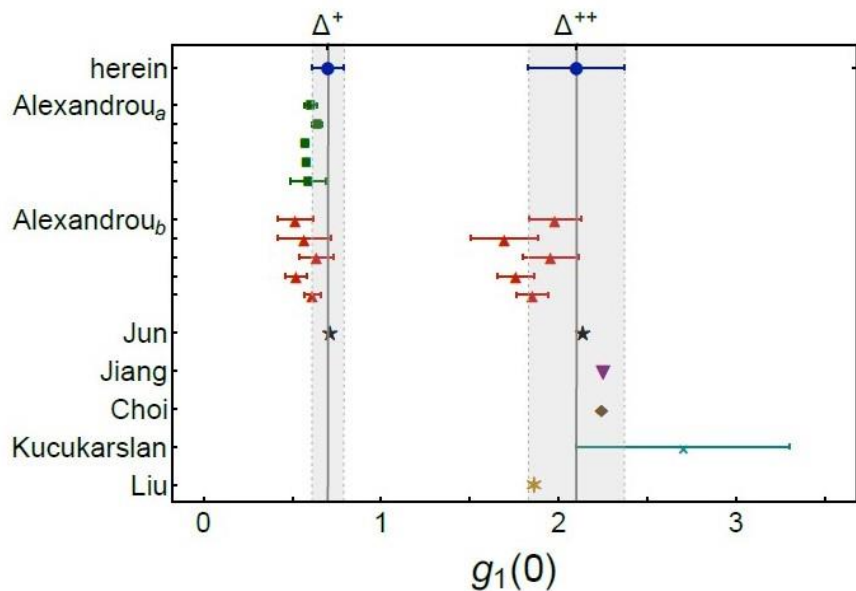
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Gießen,

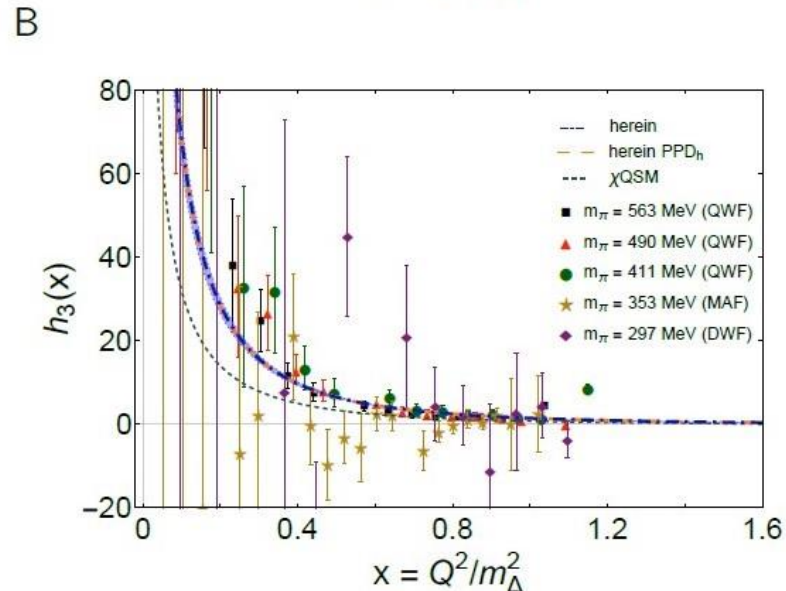
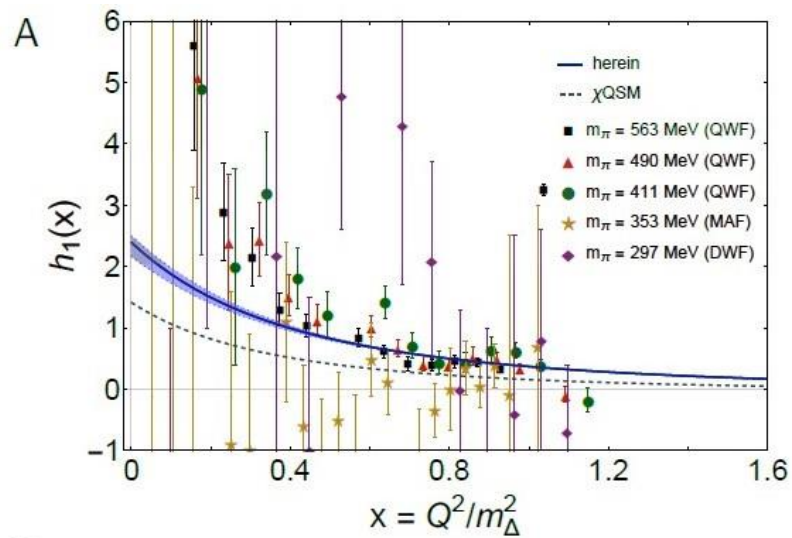
$\Delta(1232)$ Axial Form Factors

➤ Axial charge: $g_1(0) = 0.71(9)$



$\Delta(1232)$ Axial Form Factors

- $h_1(x)$: regular
- $h_1(0) = 2.25(17)$



$N(940) \rightarrow \Delta(1232)$ Axial and Pseudoscalar Form Factors

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Nucleon to Δ axial and pseudoscalar transition form factors

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(Dated: 2023 December 21)

A symmetry-preserving continuum approach to the calculation of baryon properties in relativistic quantum field theory is used to predict all form factors associated with nucleon-to- Δ axial and pseudoscalar transition currents, thereby unifying them with many additional properties of these and other baryons. The new parameter-free predictions can serve as credible benchmarks for use in analysing existing and anticipated data from worldwide efforts focused on elucidation of ν properties.

2023

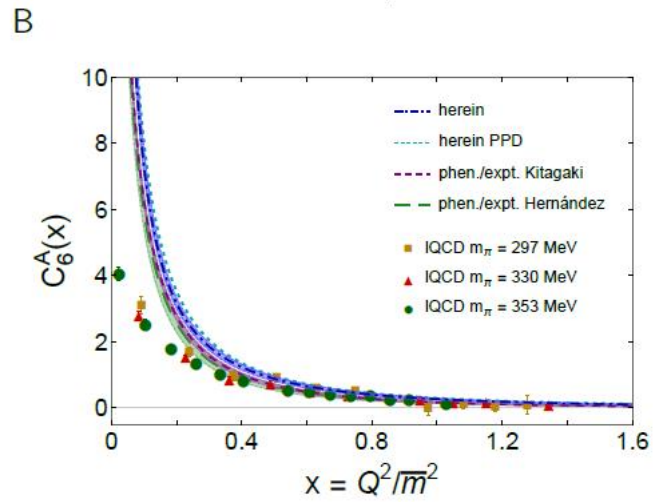
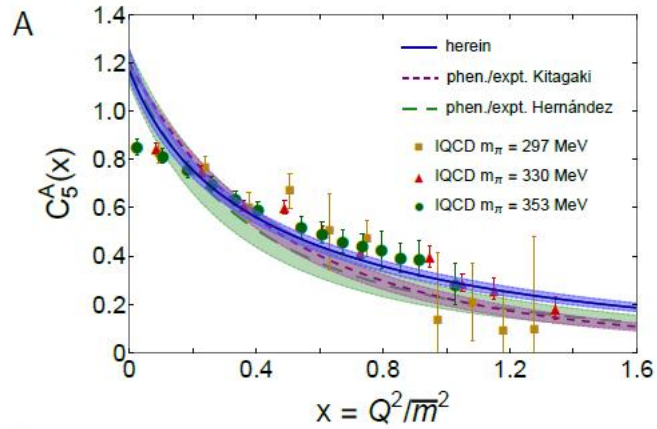
$$J_{\mu}^{\Delta N}(K, Q) = \bar{u}_{\alpha}^{\Delta}(P_f) \Gamma_{(5)(\mu), \alpha}(K, Q) u^N(P_i),$$

$$\Gamma_{\mu, \alpha}^{\text{EM}}(K, Q) = b \left[\frac{i\omega}{2\lambda_+} (G_M^* - G_E^*) \gamma_5 \varepsilon_{\alpha\mu\gamma\delta} K_{\gamma} \hat{Q}_{\delta} - G_E^* T_{\alpha\gamma}^Q T_{\gamma\mu}^K - \frac{i\tau}{\omega} G_C^* \hat{Q}_{\alpha} K_{\mu} \right],$$

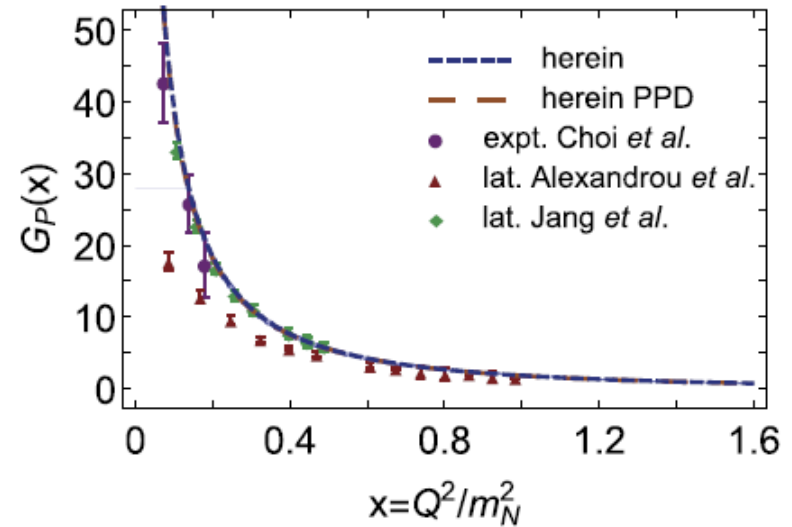
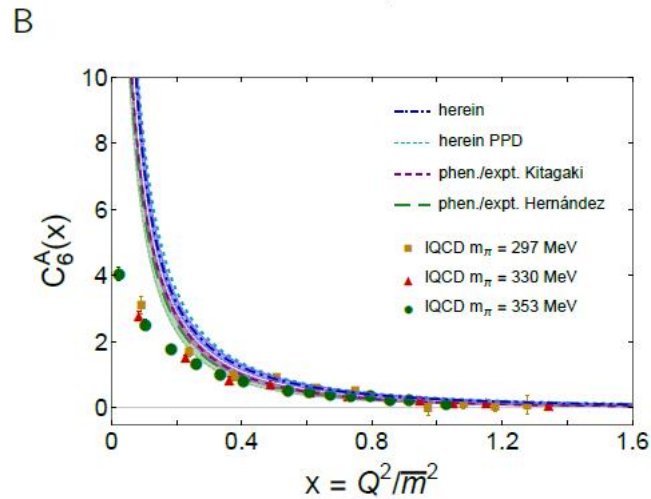
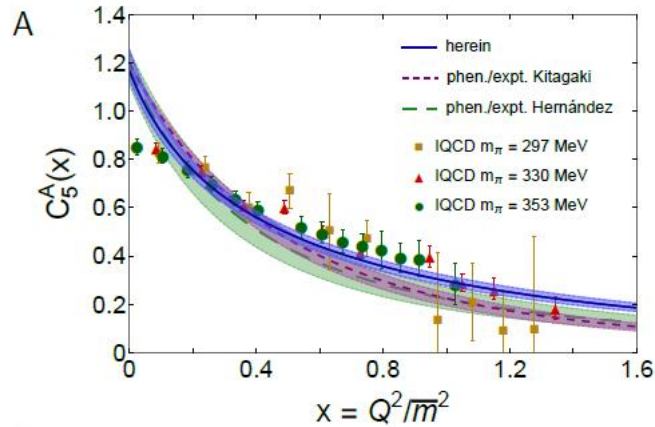
$$\Gamma_{5\mu, \alpha}^{\text{AX}}(Q) = \sqrt{\frac{2}{3}} \left[i(\gamma_{\mu} Q_{\lambda} - \delta_{\mu\lambda} Q) \frac{C_3^A}{m_N} - (\delta_{\mu\lambda} (P_f \cdot Q) - P_{f\mu} Q_{\lambda}) \frac{C_4^A}{m_N^2} + \delta_{\mu\lambda} C_5^A - Q_{\mu} Q_{\lambda} \frac{C_6^A}{m_N^2} \right],$$

$$\Gamma_{5, \alpha}^{\text{PS}}(Q) = \sqrt{\frac{2}{3}} \left[i \frac{Q_{\lambda}}{4m_N} \frac{m_{\pi}^2}{Q^2 + m_{\pi}^2} \frac{f_{\pi}}{m_q} G_{\pi N \Delta}(Q^2) \right].$$

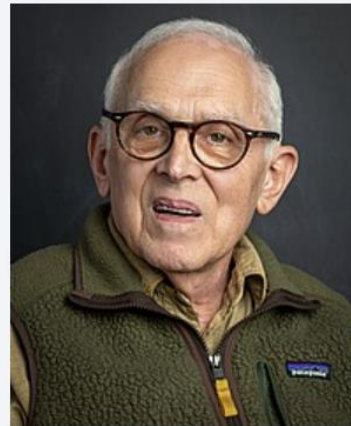
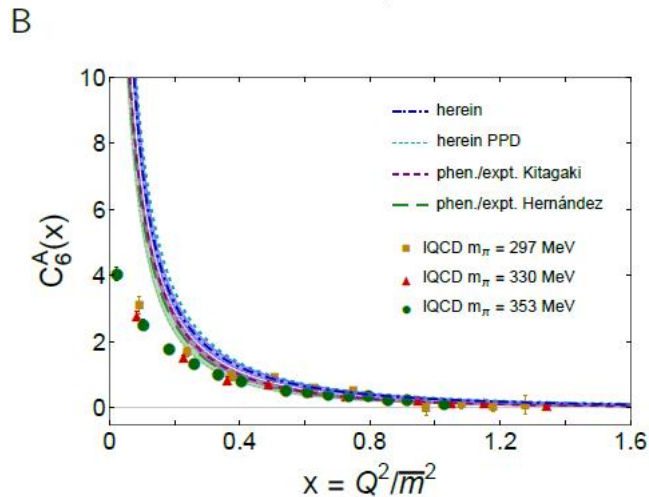
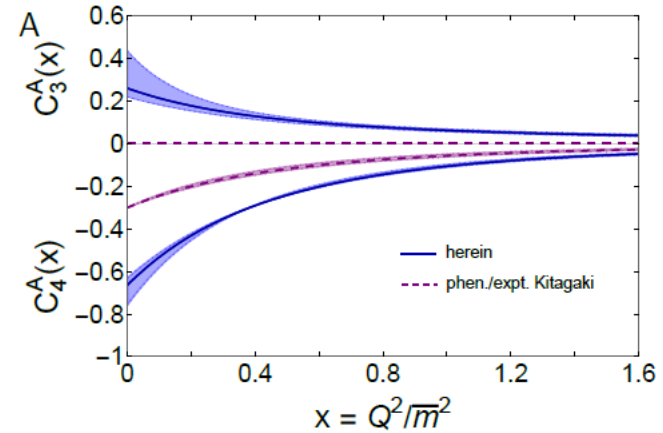
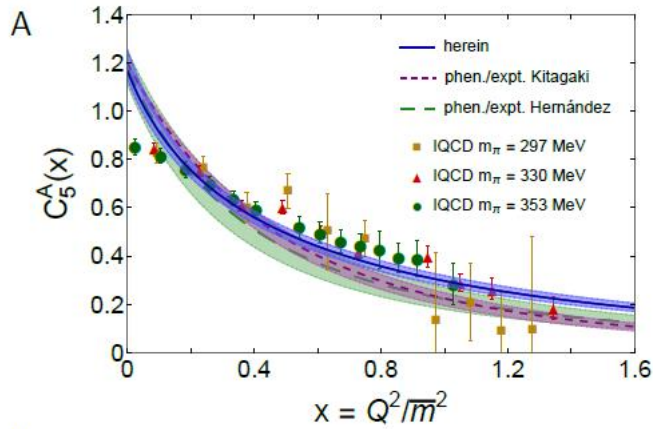
$N(940) \rightarrow \Delta(1232)$ Axial and Pseudoscalar Form Factors



$N(940) \rightarrow \Delta(1232)$ Axial and Pseudoscalar Form Factors



$N(940) \rightarrow \Delta(1232)$ Axial and Pseudoscalar Form Factors



Stephen L. Adler

PROFESSOR EMERITUS

School of Natural Sciences
Particle Physics

Photoproduction, electroproduction and weak single pion production in the (3,3) resonance region

#1

Stephen L. Adler (Princeton, Inst. Advanced Study) (1968)

Published in: *Annals Phys.* 50 (1968) 189-311

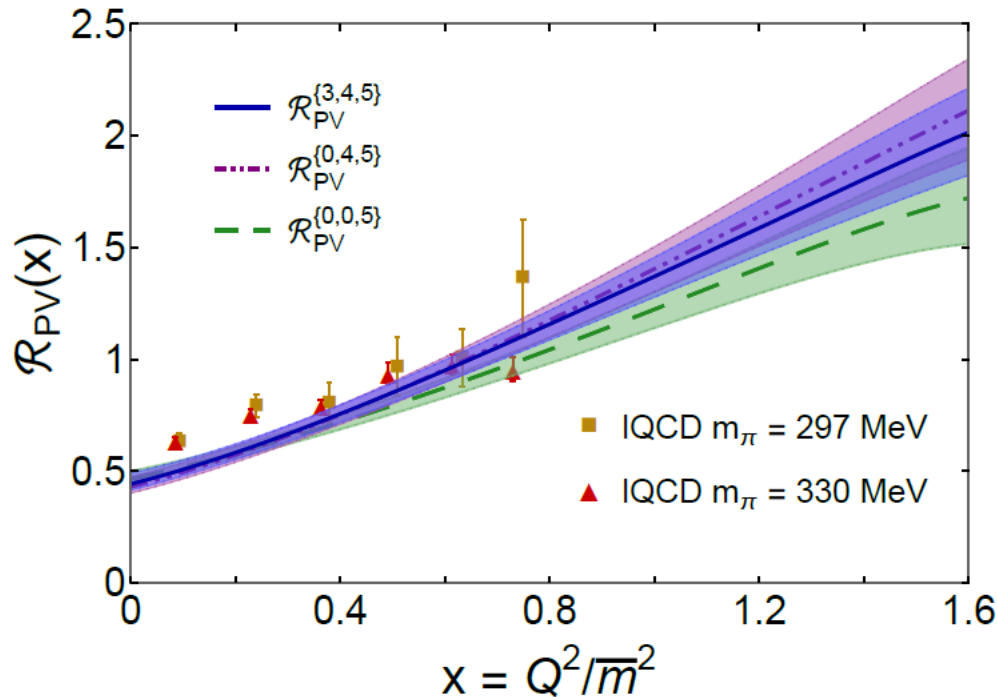
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$N(940) \rightarrow \Delta(1232)$ Axial and Pseudoscalar Form Factors

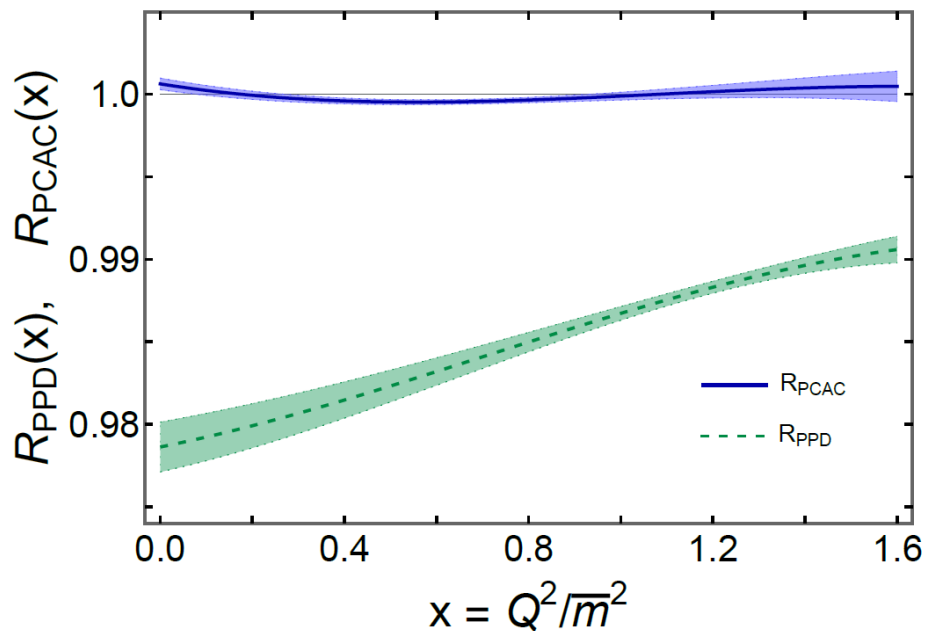
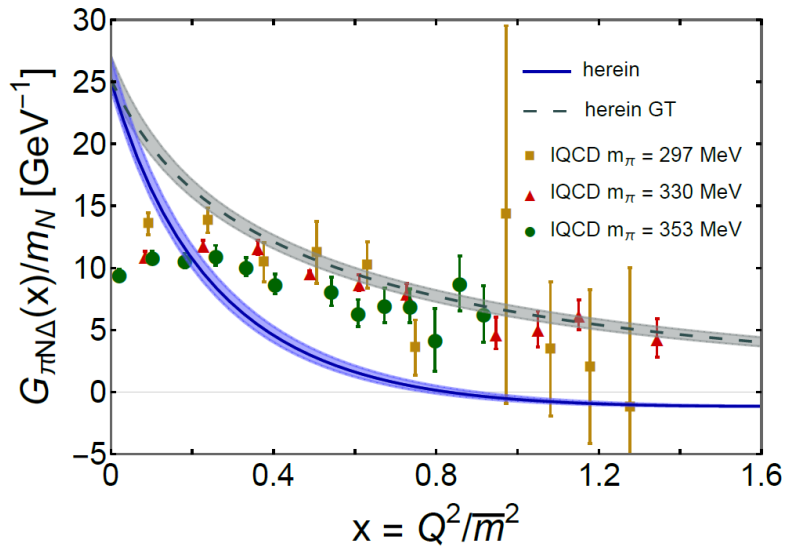
➤ The parity violation asymmetry

$$\mathcal{R}_{PV}^{\{3,4,5\}}(Q^2) := \frac{C_5^A}{C_3^V} \left[1 + \frac{m_\Delta^2 - Q^2 - m_N^2}{2m_N^2} \frac{C_4^A}{C_5^A} - \frac{m_N^2 + Q^2 + 2m_N m_\Delta - 3m_\Delta^2}{4m_N m_\Delta} \frac{C_3^A}{C_5^A} \right]$$



$N(940) \rightarrow \Delta(1232)$ Axial and Pseudoscalar Form Factors

➤ Pseudoscalar form factor



Summary & Perspective

- Nucleon Axial Form Factor at large momentum transfers
- $\Delta(1232)$ Axial Form Factors
- $N(940) \rightarrow \Delta(1232)$ Axial and Pseudoscalar Transition Form Factors
- Longer plan:

	Spectrum		EM FFs		AX FFs		GFFs	
	q-dq	qqq	q-dq	qqq	q-dq	qqq	q-dq	qqq
$N(940)1/2^+$	✓	✓	✓	✓	✓	✓		
$\Delta(1232)3/2^+$	✓	✓	✓	✓	✓			
$N(1440)1/2^+$	✓		✓					
$N(1535)1/2^-$	✓							
$N(1520)3/2^-$	✓							
Hyperons ($P = +$)	✓	✓		✓				
Hyperons ($P = -$)								

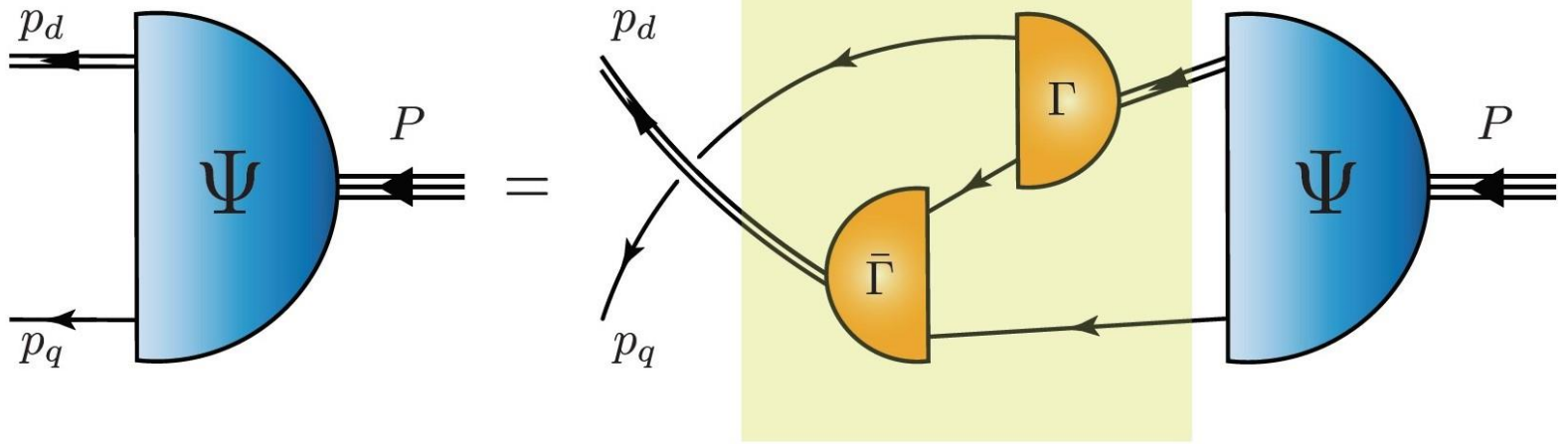
Summary & Perspective

- Nucleon Axial Form Factor at large momentum transfers
- $\Delta(1232)$ Axial Form Factors
- $N(940) \rightarrow \Delta(1232)$ Axial and Pseudoscalar Transition Form Factors
- Longer plan:

	Spectrum		EM FFs		AX FFs		GFFs	
	q-dq	qqq	q-dq	qqq	q-dq	qqq	q-dq	qqq
$N(940)1/2^+$	✓	✓	✓	✓	✓	✓		
$\Delta(1232)3/2^+$	✓	✓	✓	✓	✓			
$N(1440)1/2^+$	✓		✓					
$N(1535)1/2^-$	✓							
$N(1520)3/2^-$	✓							
Hyperons ($P = +$)	✓	✓		✓				
Hyperons ($P = -$)								

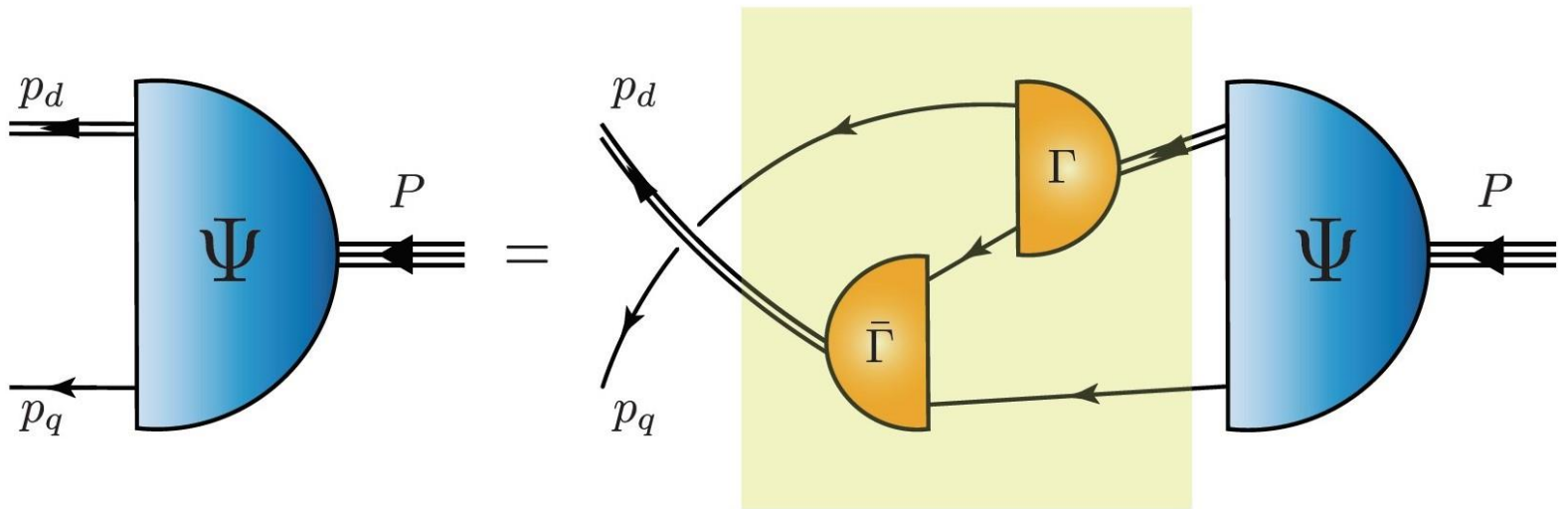
Thank you!

How to solve?



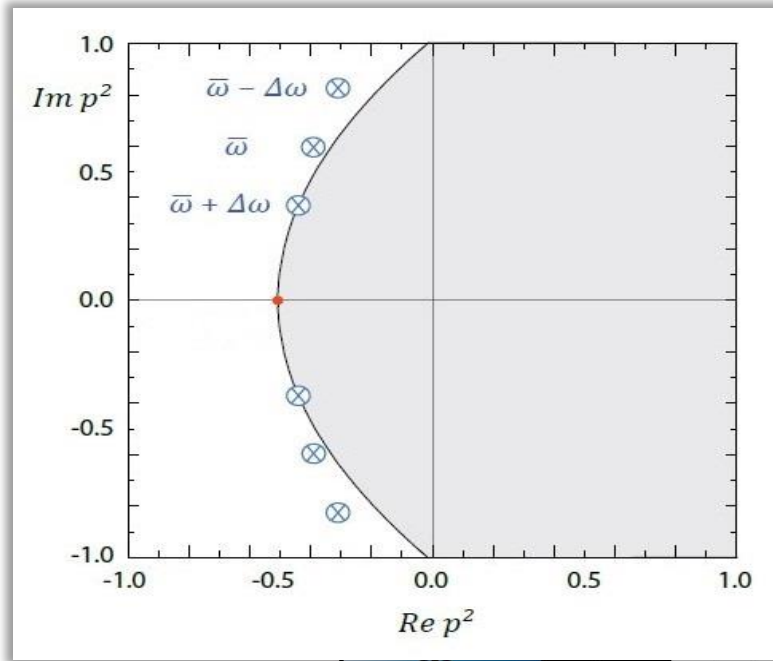
How to solve?

- ◆ The dressed-quark propagator
- ◆ Diquark amplitudes
- ◆ Diquark propagators
- ◆ **Faddeev amplitudes**

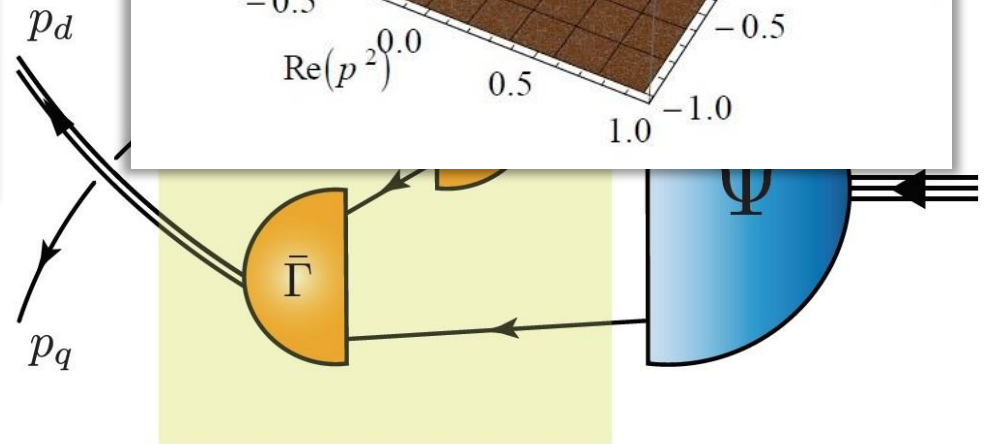
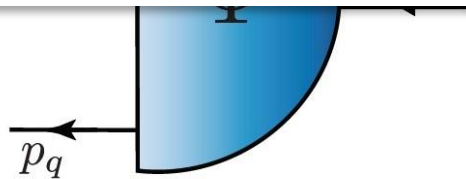
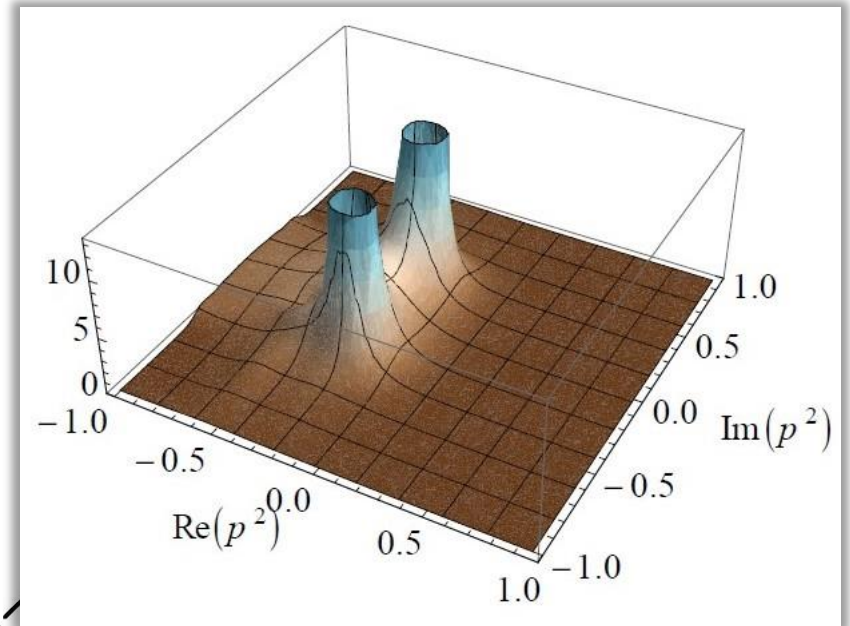


How to solve?

◆ The dressed-quark propagator

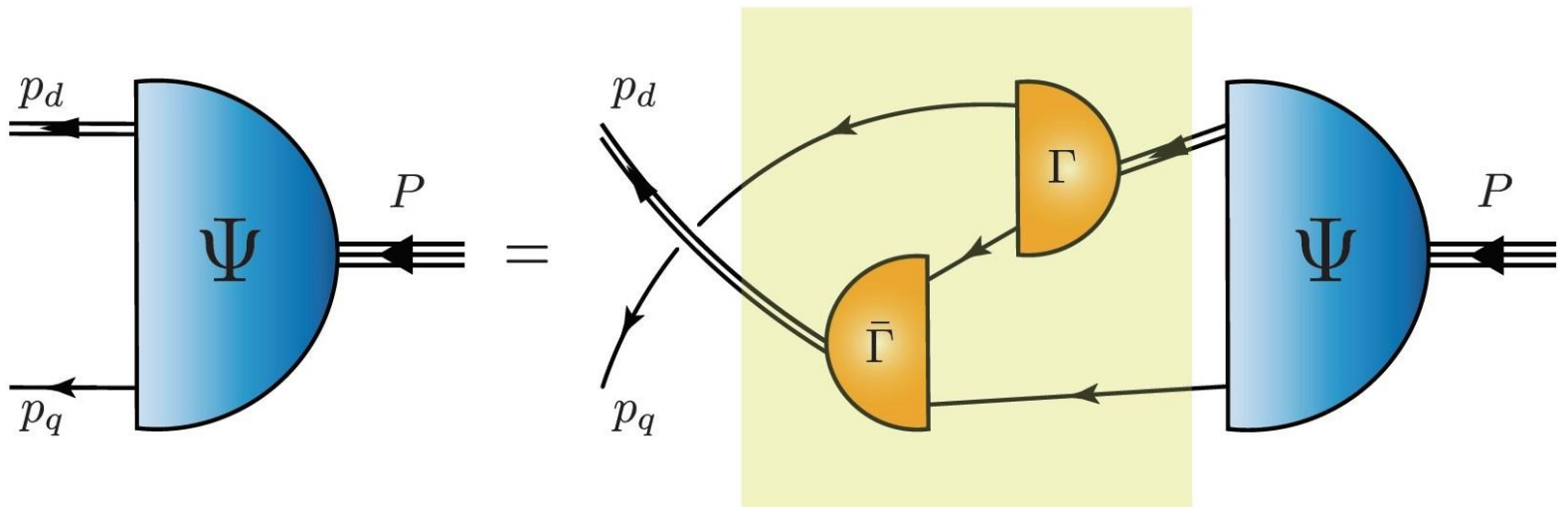


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QCD-kindred model

- ◆ The dressed-quark propagator
- ◆ Diquark amplitudes
- ◆ Diquark propagators
- ◆ **Faddeev amplitudes**



QCD-kindred model

➤ **Diquark masses (in GeV):**

$$m_{[ud]_{0+}} = 0.80 \text{ GeV}$$

$$m_{\{uu\}_{1+}} = m_{\{ud\}_{1+}} = m_{\{dd\}_{1+}} = 0.89 \text{ GeV}$$

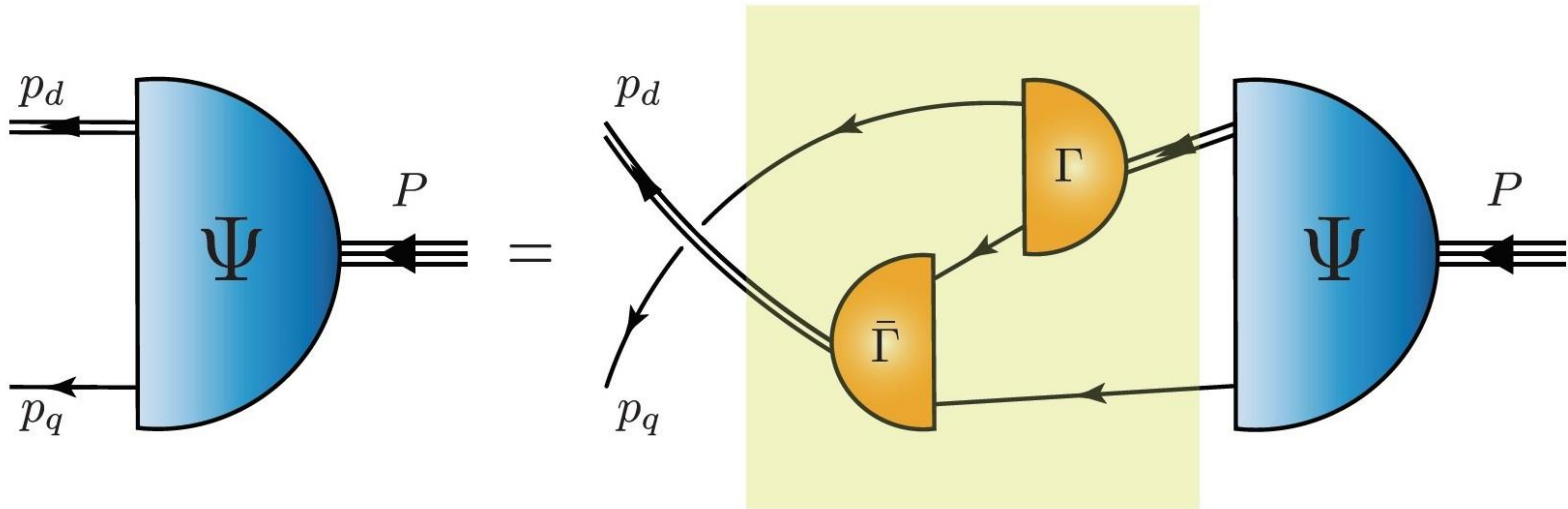
$$m_N = 1.18 \text{ GeV}$$

$$m_R = 1.72 \text{ GeV}$$

$$m_\Delta = 1.35 \text{ GeV}$$



- **These two values provide for a good description of numerous dynamical properties of the nucleon, Δ -baryon and low-lying excitations, e.g., N(1440), N(1535), Δ (1600).**



Proton Spin Structure

- Flavour separation of proton axial charge
- **d**-quark receives large contribution from probe+quark in presence of axialvector diquark

$$\frac{g_A^d}{g_A^u} = 0^+ \& 1^+ -0.32(2)$$

$$\frac{g_A^d}{g_A^u} = 0^+ \text{ only } -0.054(13)$$

- Experiment: **0.27(4)**
- Hadron scale: $g_A^u + g_A^d (+g_A^s = 0) = 0.65(2) \Rightarrow$ quarks carry **65%** of the proton spin
- Poincaré-covariant proton wave function: remaining **35%** lodged with quark+diquark orbital angular momentum



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Perspective on polarised parton distribution functions and proton spin

P. Cheng (程鹏)^{a,b}, Y. Yu (俞杨)^{a,b}, H.-Y. Xing (邢惠瑜)^{a,b}, C. Chen (陈晨)^{c,d,*},
Z.-F. Cui (崔著钊)^{a,b,*}, C.D. Roberts^{a,b,*}



Large Q^2 Nucleon Axial Form Factor

➤ Light-front transverse density profiles

➤ Scalar diquark only:

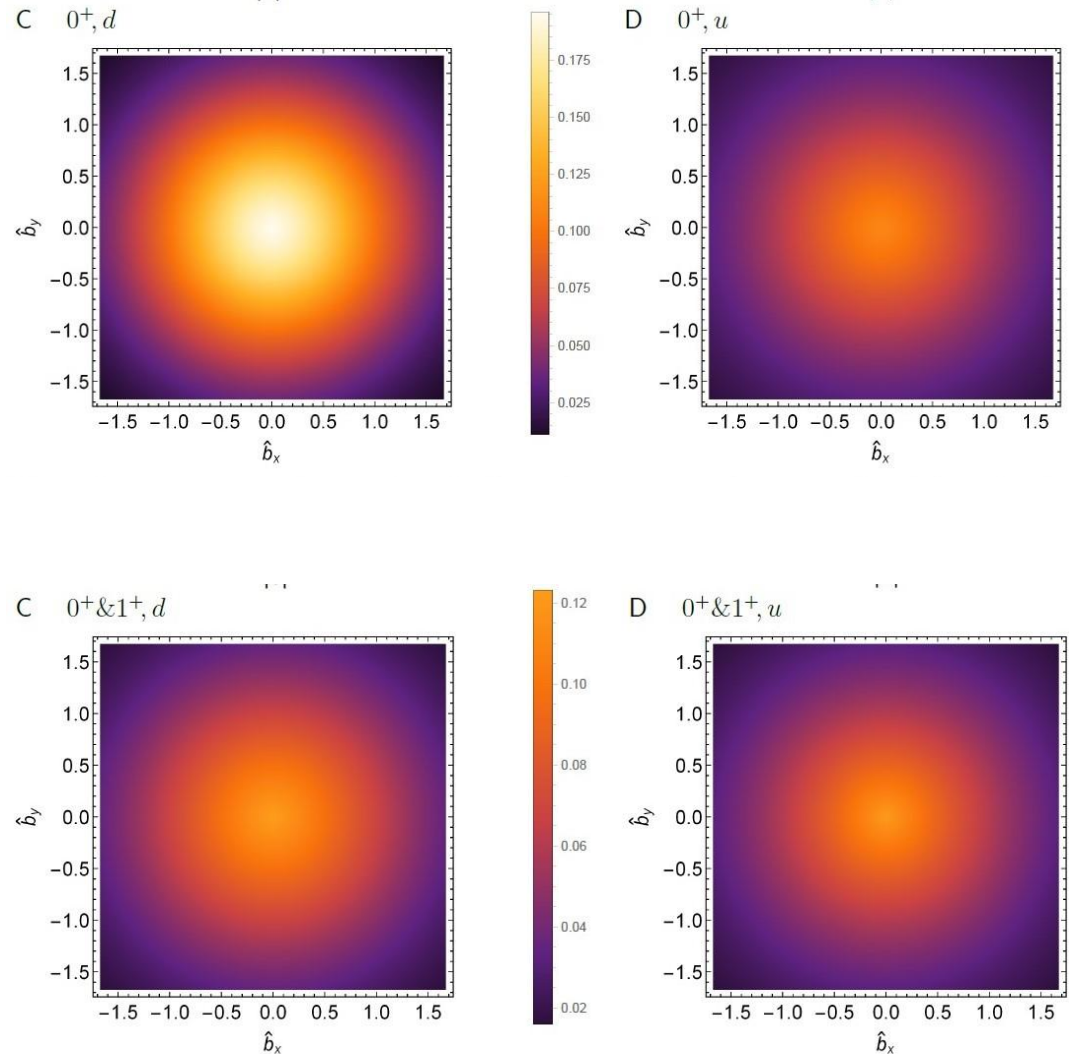
- magnitude of the d quark contribution to GA is just 10% of that from the u quark
- d quark is also much more localized

$$r_{A_d}^\perp \approx 0.5 r_{A_u}^\perp$$

➤ Scalar + axial-vector diquarks:

- d and u quark transverse profiles are quite similar

$$r_{A_d}^\perp \approx 0.9 r_{A_u}^\perp$$



The axial current – G_A & G_P

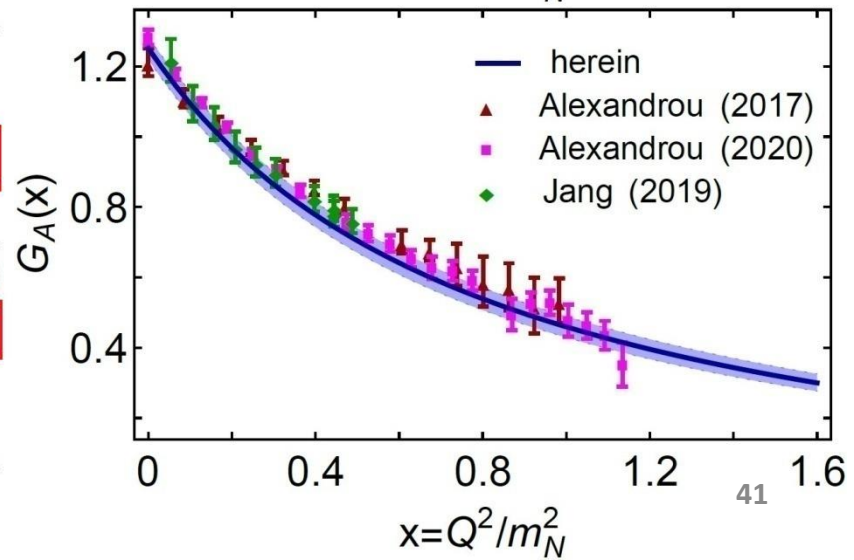
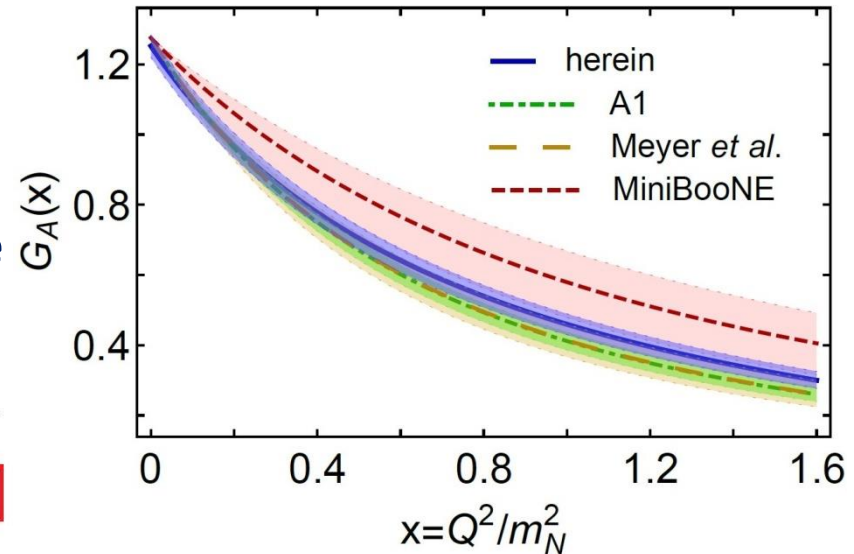
$$J_{5\mu}^j(K, Q) = \bar{u}(P_f) \frac{\tau^j}{2} \gamma_5 \left[\gamma_\mu G_A(Q^2) + i \frac{Q_\mu}{2m_N} G_P(Q^2) \right] u(P_i)$$

➤ Two form factors:

- G_A – axial form factor
- G_P – induced pseudoscalar form factor

➤ G_A can reliably be represented by dipole characterised by mass-scale m_A

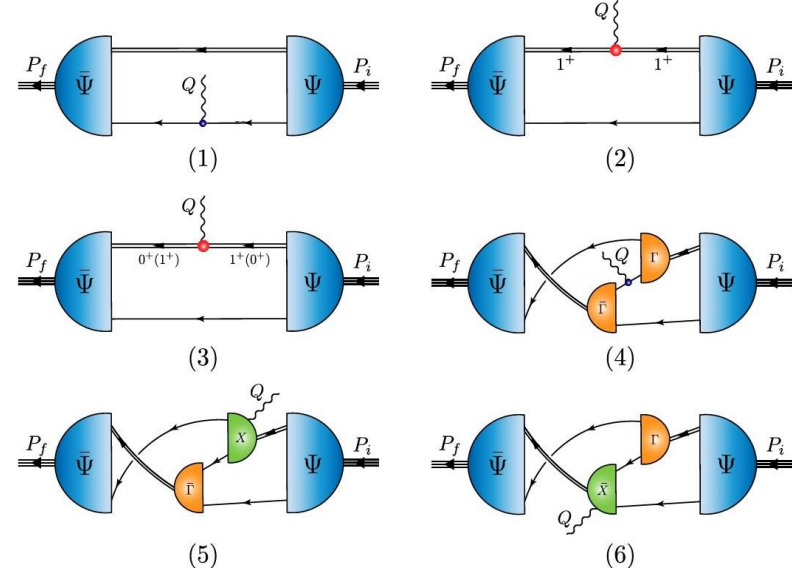
	g_A	$m_N \langle r_A^2 \rangle^{1/2}$	m_A/m_N
Herein	1.25(03)	3.25(04)	1.23(03)
Faddeev ₃ [31]	0.99(02)	2.63(06)	1.32(03)
Exp [4]	1.2756(13)	–	–
Exp [13]	–	3.02(11)	1.15(04)
Exp [14]	–	3.23(72)	1.15(08)
Exp [17]	–	2.41(31)	1.44(18)
IQCD [57]	1.21(3)(2)	2.45(08)(03)	1.41(04)(02)
IQCD [58]	1.30(6)	3.57(30)	0.97(16)
IQCD _d [59]	1.23(3)	2.48(15)	1.39(09)
IQCD _z [59]	1.30(9)	3.19(30)	1.09(11)



Fractions of $G_A(0)$, $G_P(0)$ and $G_5(0)$

TABLE I. Referring to Fig. 3, separation of $G_A(0)$, $G_P(0)$ and $G_5(0)$ into contributions from various diagrams, listed as a fraction of the total $Q^2 = 0$ value. Diagram (1): $\langle J \rangle_q^S$ – weak-boson strikes dressed-quark with scalar diquark spectator; and $\langle J \rangle_q^A$ – weak-boson strikes dressed-quark with axial-vector diquark spectator. Diagram (2): $\langle J \rangle_{qq}^{AA}$ – weak-boson interacts strikes axial-vector diquark with dressed-quark spectator. Diagram (3): $\langle J \rangle_{dq}^{SA+AS}$ – weak-boson mediates transition between scalar and axial-vector diquarks, with dressed-quark spectator. Diagram (4): $\langle J \rangle_{ex}$ – weak-boson strikes dressed-quark “in-flight” between one diquark correlation and another. Diagrams (5) and (6): $\langle J \rangle_{sg}$ – weak-boson couples inside the diquark correlation amplitude. The listed uncertainty in these results reflects the impact of $\pm 5\%$ variations in the diquark masses in Eq. (16), *e.g.* $0.71_{1\mp} \Rightarrow 0.71 \mp 0.01$.

	$\langle J \rangle_q^S$	$\langle J \rangle_q^A$	$\langle J \rangle_{qq}^{AA}$	$\langle J \rangle_{dq}^{SA+AS}$	$\langle J \rangle_{ex}$	$\langle J \rangle_{sg}$
$G_A(0)$	$0.71_{4\mp}$	$0.064_{2\pm}$	$0.025_{5\pm}$	0.130_{\mp}	$0.072_{32\pm}$	0
$G_P(0)$	$0.74_{4\mp}$	$0.070_{5\pm}$	$0.025_{5\pm}$	0.130_{\mp}	$0.22_{4\pm}$	$-0.19_{1\mp}$
$G_5(0)$	$0.74_{4\mp}$	$0.069_{5\pm}$	$0.025_{5\pm}$	0.130_{\mp}	$0.22_{4\pm}$	$-0.19_{1\mp}$



➤ Projections:

$$G_A = -\frac{1}{4(1+\tau)} \text{tr}_D [J_{5\mu} \gamma_5 \gamma_\mu^T],$$

$$G_P = \frac{1}{\tau} \left(G_A - \frac{Q_\mu}{4im_N \tau} \text{tr}_D [J_{5\mu} \gamma_5] \right),$$

$$G_5 = \frac{1}{2\tau} \text{tr}_D [J_5 \gamma_5],$$

➤ $G_P(0) \sim G_5(0)$

$$G_P \sim \frac{Q_\mu}{\tau^2} \text{tr}_D [J_{5\mu} \gamma_5] \sim \frac{1}{\tau} \text{tr}_D [J_5 \gamma_5] \sim G_5,$$

when $Q^2 \sim 0 \text{ GeV}^2$.

QCD-kindred model

➤ **The dressed-quark propagator**

$$S(p) = -i\gamma \cdot p \sigma_V(p^2) + \sigma_S(p^2)$$

➤ **algebraic form:**

$$\begin{aligned} \bar{\sigma}_S(x) = & 2\bar{m}\mathcal{F}(2(x + \bar{m}^2)) \\ & + \mathcal{F}(b_1x)\mathcal{F}(b_3x)[b_0 + b_2\mathcal{F}(\epsilon x)], \end{aligned} \quad (\text{A3a})$$

$$\bar{\sigma}_V(x) = \frac{1}{x + \bar{m}^2} [1 - \mathcal{F}(2(x + \bar{m}^2))], \quad (\text{A3b})$$

with $x = p^2/\lambda^2$, $\bar{m} = m/\lambda$,

$$\mathcal{F}(x) = \frac{1 - e^{-x}}{x}, \quad (\text{A4})$$

$\bar{\sigma}_S(x) = \lambda\sigma_S(p^2)$ and $\bar{\sigma}_V(x) = \lambda^2\sigma_V(p^2)$. The mass scale, $\lambda = 0.566$ GeV, and parameter values,

$$\frac{\bar{m} \quad b_0 \quad b_1 \quad b_2 \quad b_3}{0.00897 \quad 0.131 \quad 2.90 \quad 0.603 \quad 0.185}, \quad (\text{A5})$$

associated with Eq. (A3) were fixed in a least-squares fit to light-meson observables [79,80]. [$\epsilon = 10^{-4}$ in Eq. (A3a) acts only to decouple the large- and intermediate- p^2 domains.]

QCD-kindred model

➤ The dressed-quark propagator

$$S(p) = -i\gamma \cdot p \sigma_V(p^2) + \sigma_S(p^2)$$

- Based on solutions to the gap equation that were obtained with a dressed gluon-quark vertex.
- Mass function has a real-world value at $p^2 = 0$, NOT the highly inflated value typical of **RL** truncation.
- Propagators are entire functions, consistent with sufficient condition for confinement and completely unlike known results from **RL** truncation.
- Parameters in quark propagators were fitted to a diverse array of meson observables. **ZERO** parameters changed in study of baryons.
- Compare with that computed using the DCSB-improved gap equation kernel (DB).
The parametrization is a sound representation of numerical results, although simple and introduced long beforehand.

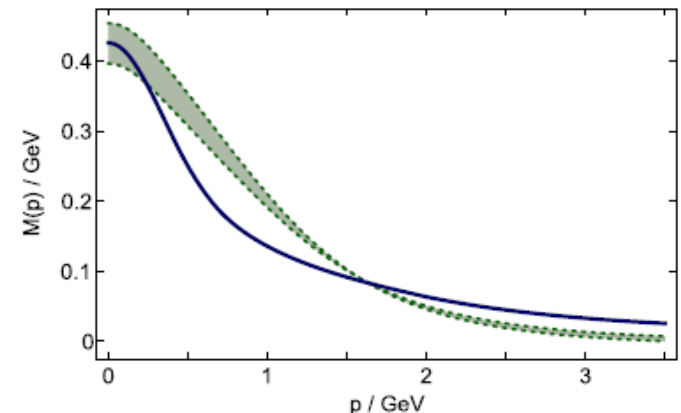


FIG. 6. Solid curve (blue)—quark mass function generated by the parametrization of the dressed-quark propagator specified by Eqs. (A3) and (A4) (A5); and band (green)—exemplary range of numerical results obtained by solving the gap equation with the modern DCSB-improved kernels described and⁴⁴used in Refs. [16,81–83].

QCD-kindred model

- **Diquark amplitudes:** five types of correlation are possible in a $J=1/2$ bound state: isoscalar scalar ($I=0, J^P=0^+$), isovector pseudovector, isoscalar pseudoscalar, isoscalar vector, and isovector vector.

- The **LEADING** structures in the correlation amplitudes for each case are, respectively (Dirac-flavor-color),

$$\Gamma^{0+}(k; K) = g_{0+} \gamma_5 C \tau^2 \vec{H} \mathcal{F}(k^2 / \omega_{0+}^2),$$

$$\vec{\Gamma}_{\mu}^{1+}(k; K) = i g_{1+} \gamma_{\mu} C \vec{\tau} \vec{H} \mathcal{F}(k^2 / \omega_{1+}^2),$$

$$\Gamma^{0-}(k; K) = i g_{0-} C \tau^2 \vec{H} \mathcal{F}(k^2 / \omega_{0-}^2),$$

$$\Gamma_{\mu}^{1-}(k; K) = g_{1-} \gamma_{\mu} \gamma_5 C \tau^2 \vec{H} \mathcal{F}(k^2 / \omega_{1-}^2),$$

$$\vec{\Gamma}_{\mu}^{1-}(k; K) = i g_{1-} [\gamma_{\mu}, \gamma \cdot K] \gamma_5 C \vec{\tau} \vec{H} \mathcal{F}(k^2 / \omega_{1-}^2),$$

- **Simple form. Just one parameter: diquark masses.**
- **Match expectations based on solutions of meson and diquark Bethe-Salpeter amplitudes.**

➤ The diquark propagators

$$\Delta^{0\pm}(K) = \frac{1}{m_{0\pm}^2} \mathcal{F}(k^2/\omega_{0\pm}^2),$$

$$\Delta_{\mu\nu}^{1\pm}(K) = \left[\delta_{\mu\nu} + \frac{K_\mu K_\nu}{m_{1\pm}^2} \right] \frac{1}{m_{1\pm}^2} \mathcal{F}(k^2/\omega_{1\pm}^2).$$

➤ The *\mathcal{F} -functions*: Simplest possible form that is consistent with infrared and ultraviolet constraints of confinement (IR) and $1/q^2$ evolution (UV) of meson propagators.

➤ Diquarks are **confined**.

- free-particle-like at spacelike momenta
- pole-free on the timelike axis
- This is **NOT** true of **RL** studies. It enables us to reach arbitrarily high values of momentum transfer.

QCD-kindred model

➤ The Faddeev amplitudes:

$$\begin{aligned}
 \psi^\pm(p_i, \alpha_i, \sigma_i) = & [\Gamma^{0+}(k; K)]_{\sigma_1 \alpha_1 \sigma_2 \alpha_2} \Delta^{0+}(K) [\varphi_{0^+}^\pm(\ell; P) u(P)]_{\sigma_3}^{\alpha_3} \\
 & + [\Gamma_\mu^{1+j}] \Delta_{\mu\nu}^{1+} [\varphi_{1^+ \nu}^{j\pm}(\ell; P) u(P)] \\
 & + [\Gamma^{0-}] \Delta^{0-} [\varphi_{0^-}^\pm(\ell; P) u(P)] \\
 & + [\Gamma_\mu^{1-}] \Delta_{\mu\nu}^{1-} [\varphi_{1^- \nu}^\pm(\ell; P) u(P)], \quad (9)
 \end{aligned}$$

➤ Quark-diquark vertices:

$$\varphi_{0^+}^\pm(\ell; P) = \sum_{i=1}^2 \beta_i^\pm(\ell^2, \ell \cdot P) S^i(\ell; P) \mathcal{G}^\pm,$$

where $\mathcal{G}^{+(-)} = \mathbf{I}_D(\gamma_5)$ and

$$\varphi_{1^+ \nu}^{j\pm}(\ell; P) = \sum_{i=1}^6 \alpha_i^{j\pm}(\ell^2, \ell \cdot P) \gamma_5 \mathcal{A}_\nu^i(\ell; P) \mathcal{G}^\pm,$$

$$S^1 = \mathbf{I}_D, \quad S^2 = i\gamma \cdot \hat{\ell} - \hat{\ell} \cdot \hat{P} \mathbf{I}_D$$

$$\mathcal{A}_\nu^1 = \gamma \cdot \ell^\perp \hat{P}_\nu, \quad \mathcal{A}_\nu^2 = -i\hat{P}_\nu \mathbf{I}_D, \quad \mathcal{A}_\nu^3 = \gamma \cdot \hat{\ell}^\perp \hat{\ell}_\nu^\perp$$

$$\varphi_{0^-}^\pm(\ell; P) = \sum_{i=1}^2 \rho_i^\pm(\ell^2, \ell \cdot P) S^i(\ell; P) \mathcal{G}^\mp,$$

$$\mathcal{A}_\nu^4 = i\hat{\ell}_\nu^\perp \mathbf{I}_D, \quad \mathcal{A}_\nu^5 = \gamma_\nu^\perp - \mathcal{A}_\nu^3, \quad \mathcal{A}_\nu^6 = i\gamma_\nu^\perp \gamma \cdot \hat{\ell}^\perp - \mathcal{A}_\nu^4,$$

$$\varphi_{1^- \nu}^\pm(\ell; P) = \sum_{i=1}^6 \nu_i^\pm(\ell^2, \ell \cdot P) \gamma_5 \mathcal{A}_\nu^i(\ell; P) \mathcal{G}^\mp,$$

QCD-kindred model

- Both the Faddeev amplitude and wave function are Poincare covariant, i.e. they are qualitatively identical in all reference frames.
- Each of the scalar functions that appears is frame independent, but the frame chosen determines just how the elements should be combined.
- In consequence, the manner by which the dressed quarks' spin, S , and orbital angular momentum, L , add to form the total momentum J , is **frame dependent**: L , S are not independently Poincare invariant.
- The set of baryon **rest-frame** quark-diquark angular momentum identifications:

$${}^2S: S^1, \mathcal{A}_v^2, (\mathcal{A}_v^3 + \mathcal{A}_v^5),$$

$${}^2P: S^2, \mathcal{A}_v^1, (\mathcal{A}_v^4 + \mathcal{A}_v^6),$$

$${}^4P: (2\mathcal{A}_v^4 - \mathcal{A}_v^6)/3,$$

$${}^4D: (2\mathcal{A}_v^3 - \mathcal{A}_v^5)/3,$$

- The scalar functions associated with these combinations of Dirac matrices in a Faddeev wave function possess the identified angular momentum correlation between the quark and diquark.