Predicting and distinguishing gluon distributions in the proton and pion

Lei Chang(常雷)

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Nankai University

- Proton and pion distribution functions in counterpoint, arXiv: 2203.00753(PLB)
- Parton distributions of light quarks and antiquarks in the proton, arXiv:2201.07870(PLB)

Strong QCD from Hadron Structure Experiments-VI, 20240515, Nanjing

Profile Hadron

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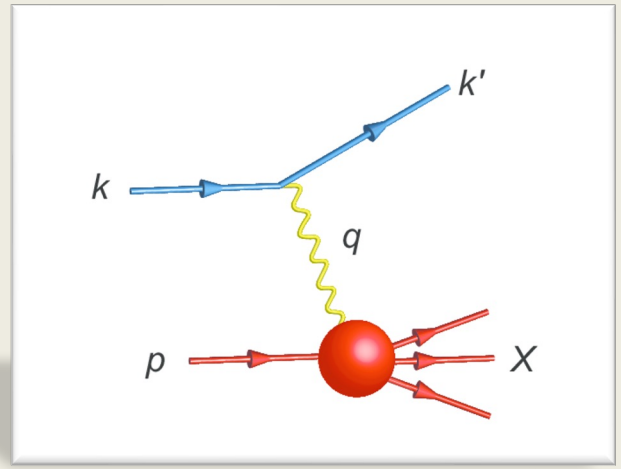
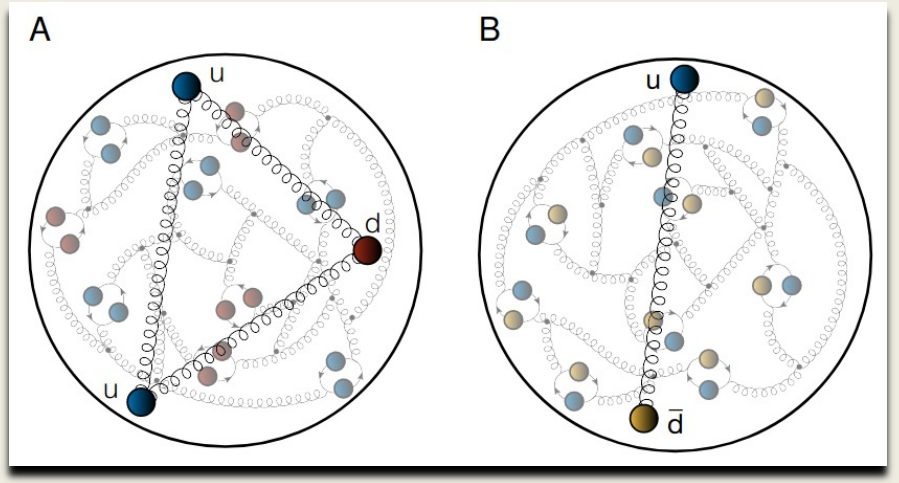
Secrets of QCD





- Phenomenological approaches…
- Global Fit…
- lQCD…direct simulation…
- Continuum Schwinger Methods…???

Emergent Hadron Mass-Paradigm

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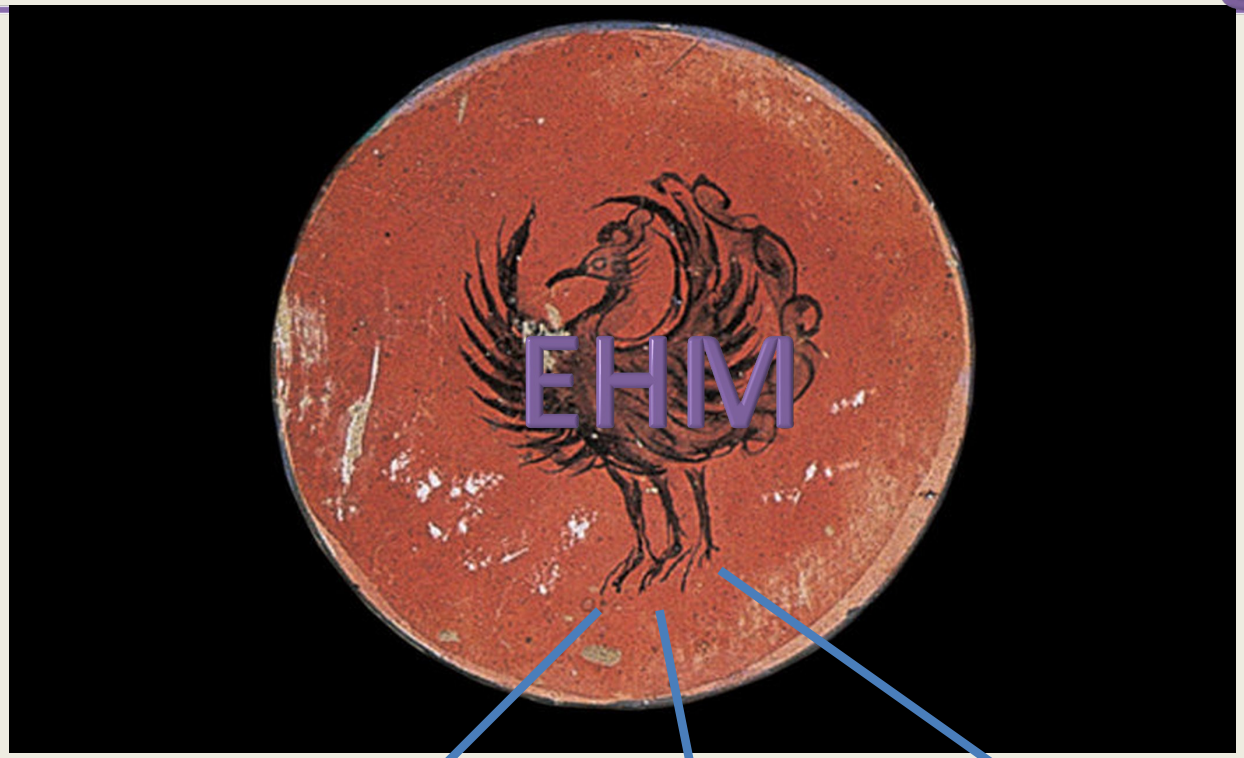
The Divine Bird
in Chinese Mythology



三足金乌
Three legged Golden Crow

a nonzero gluon mass-scale

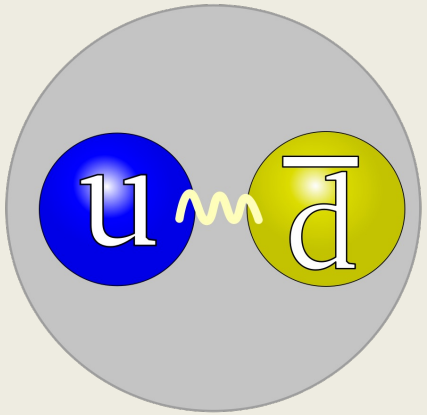
a process-independent effective charge

the dressed-quarks with constituent-like masses

Using CSM to profile hadron

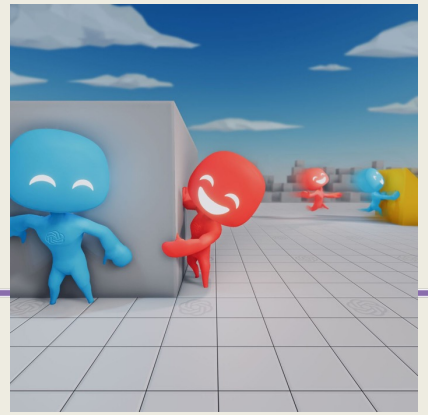
- **Imagine the Hadron at the Hadronic Scale**
- **Evolution**

- Theoretical calculations are perfomed at *some* low energy scale.



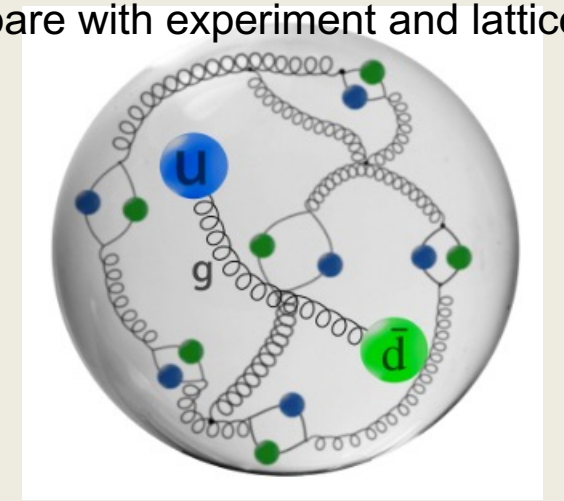
ζ_H

Hide gluon



- Then evolved via **DGLAP** equations to compare with experiment and lattice.

$\zeta > \zeta_H$



Seek gluon

Using CSM to profile hadron

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- **Imagine the Hadron at the Hadronic Scale**
- **Evolution**

Physics Letters B
Volume 93, Issue 3, 16 June 1980, Pages 313-317

Normalizing the renormalization group analysis of deep inelastic leptoproduction

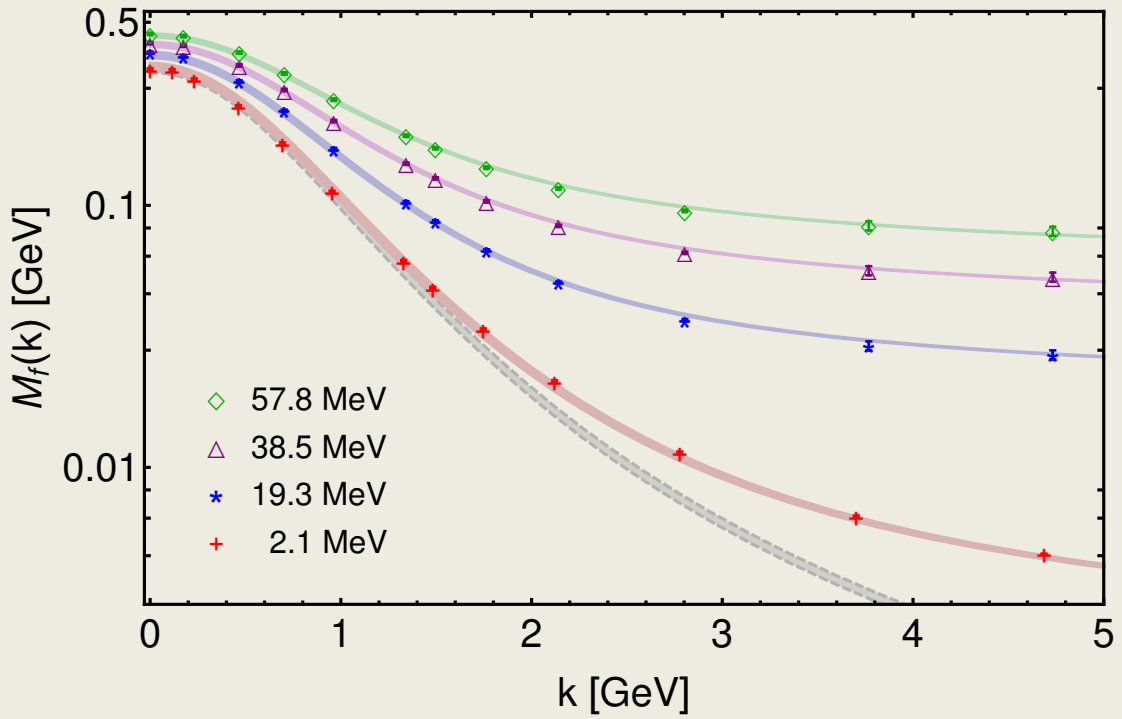
R.L. Jaffe [1], G.G. Ross [2]

“Constituent” quarks

Linking continuum and lattice quark mass functions via an effective charge

Lei Chang (Nankai U.), Yu-Bin Liu (Nankai U.), Khépani Raya (Nankai U. and Mexico U., ICN), J. Rodríguez-Quintero (Huelva U.), Yi-Bo Yang (Beijing, Inst. Theor. Phys. and HIAS, UCAS, Hangzhou and ICTP-AP, Beijing) (May 13, 2021)

Published in: *Phys.Rev.D* 104 (2021) 9, 094509 • e-Print: 2105.06596 [hep-lat]



- The strong interaction of a quark with its (gluon) surrounding gives rise to a “constituent” quark with effective mass M_0;
- M_0 is about $m_p/3$ and runs as a logarithm-corrected $1/k^2$ power-law in the ultraviolet region;
- This consistuent quark has the **finite size**(B. Povh and J. Hufner, PLB245(1990)653);

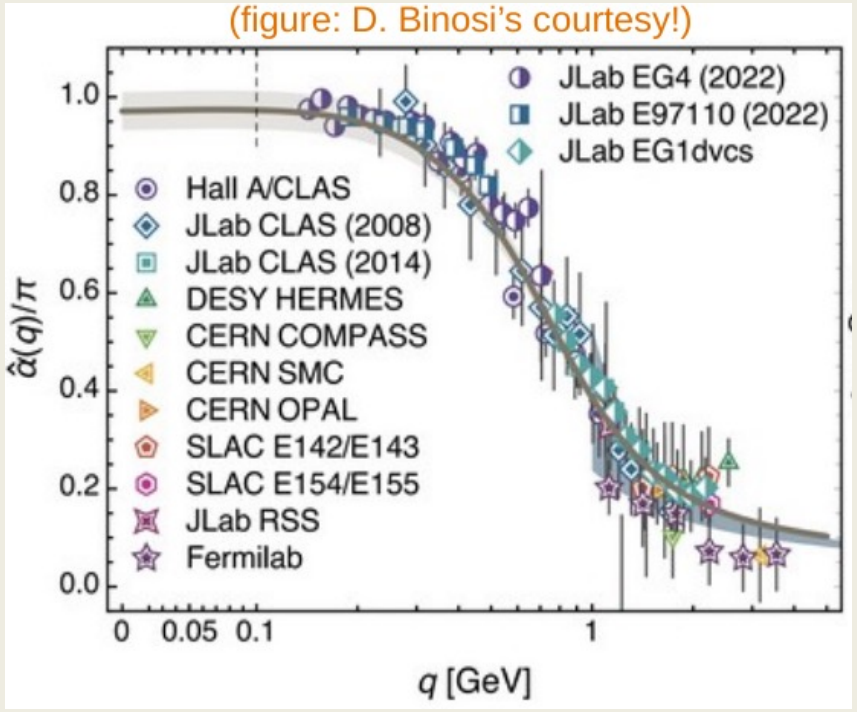
Dressed-Quark Anomalous Magnetic Moments

Lei Chang, Yu-Xin Liu, and Craig D. Roberts

Phys. Rev. Lett. **106**, 072001 (2011) - Published 16 February 2011

Natrual Scale:$m_p/3$

New perspective on hadronic scale !

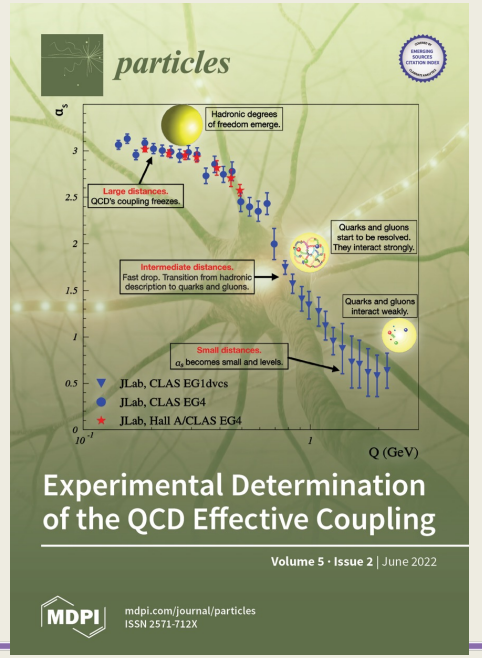


$$\hat{\alpha}(k^2) = \frac{\gamma_m \pi}{\ln\left[\frac{\mathcal{K}^2(k^2)}{\Lambda^2_{\rm QCD}}\right]}\,,\ \mathcal{K}^2(y) = \frac{a_0^2 + a_1 y + y^2}{b_0 + y}$$

Define a screening mass:

$$m_G := \mathcal{K}(k^2 = \Lambda^2_{QCD}) = 0.331\text{GeV}$$

The running coupling alters at m_G so that modes with $k^2<m^2$ are screened from interactions and theory enters a practically conformal domain.



A Hadronic Scale! Not parameter!

- Imagine the Hadron at the Hadronic Scale
- Evolution

Eur. Phys. J. Plus (2020) 135:211
https://doi.org/10.1140/epjp/s13360-020-00200-6

THE EUROPEAN PHYSICAL JOURNAL PLUS

Regular Article

Check for updates

Gluons and sea quarks in the proton at low scales

M. Diehl[a], P. Stienemeier[b]

Deutsches Elektronen-Synchrotron DESY, Notkestr. 85, 22607 Hamburg, Germany

Received: 9 May 2019 / Accepted: 10 October 2019 / Published online: 3 February 2020

Abstract We study the evolution of parton distributions down to low scales by considering several of their Mellin moments. For the initial conditions, we use a broad array of current parton density fits. Confirming earlier findings in the literature, we conclude that current determinations of parton distributions are incompatible with the idea that gluon or antiquark densities are generated by purely perturbative radiation as it is encoded in the DGLAP evolution equations.

Story of Pion: proposition view

Story of Pion: example

Maris, Roberts and Tandy, **Phys. Lett. B420**(1998) 267-273

- Pion's Bethe-Salpeter amplitude Solution of the Bethe-Salpeter equation

$$\Gamma_{\pi^j}(k;P) = \tau^{\pi^j}\gamma_5\left[iE_\pi(k;P) + \gamma\cdot P F_\pi(k;P) + \gamma\cdot k\, k\cdot P\, G_\pi(k;P) + \sigma_{\mu\nu}\, k_\mu P_\nu\, H_\pi(k;P)\right]$$

- Dressed-quark propagator

$$S(p) = \frac{1}{i\gamma\cdot p\, A(p^2) + B(p^2)}$$

- Axial-vector Ward-Takahashi identity entails(chiral limit)

$$f_\pi E(k;P|P^2=0) = B(k^2) + (k\cdot P)^2 \frac{d^2 B(k^2)}{d^2 k^2} + \ldots$$

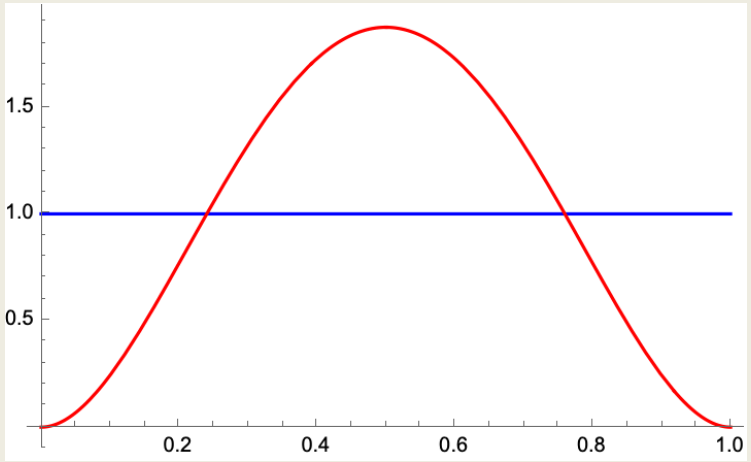
Boundaries of pion DF and moments at the hadronic scale

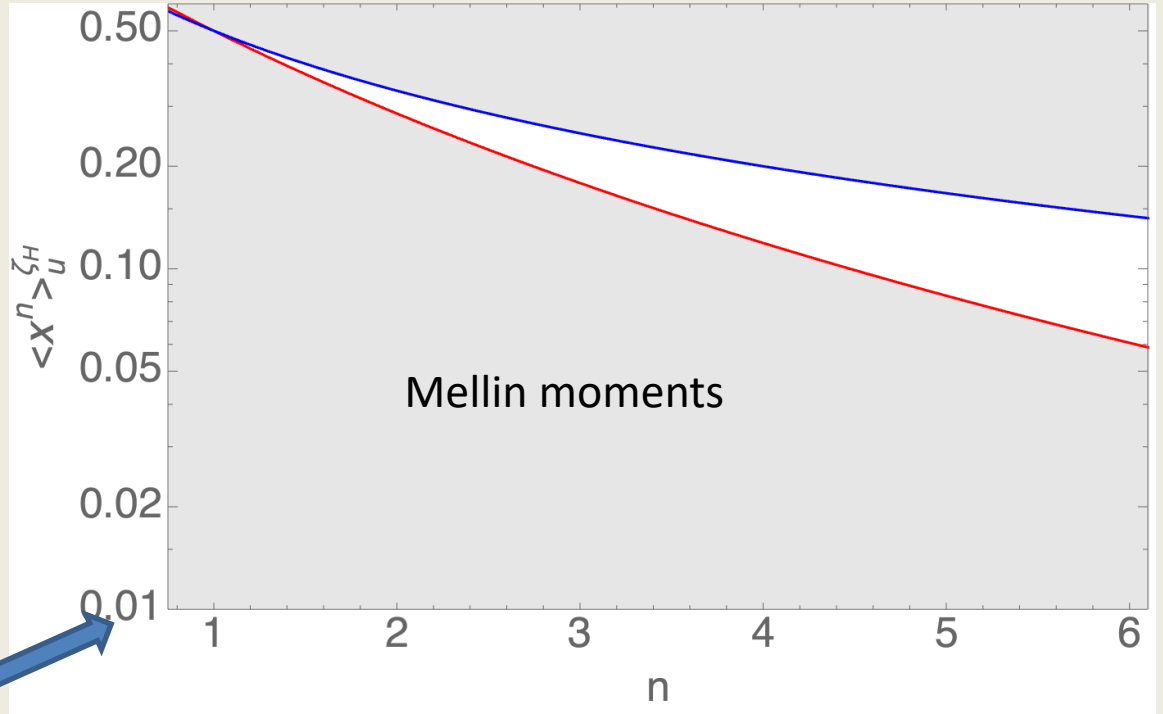
$$f_\pi E(k;P|P^2=0) = B(k^2) + (k\cdot P)^2 \frac{d^2B(k^2)}{d^2k^2} + \dots$$

Nakanishi representation

$$E(k;P) \sim \int_{-1}^{1} dz \rho(z) \frac{M^2}{k^2 + z\, k\cdot P + M^2}$$

$\rho(z) = 1 - z^2$ $\qquad$ $\rho(z) = \delta(1+z) + \delta(1-z)$





Boundaries of Mellin moments at the hadronic scale

Pion PDF: Lattice moments at hadronic scale

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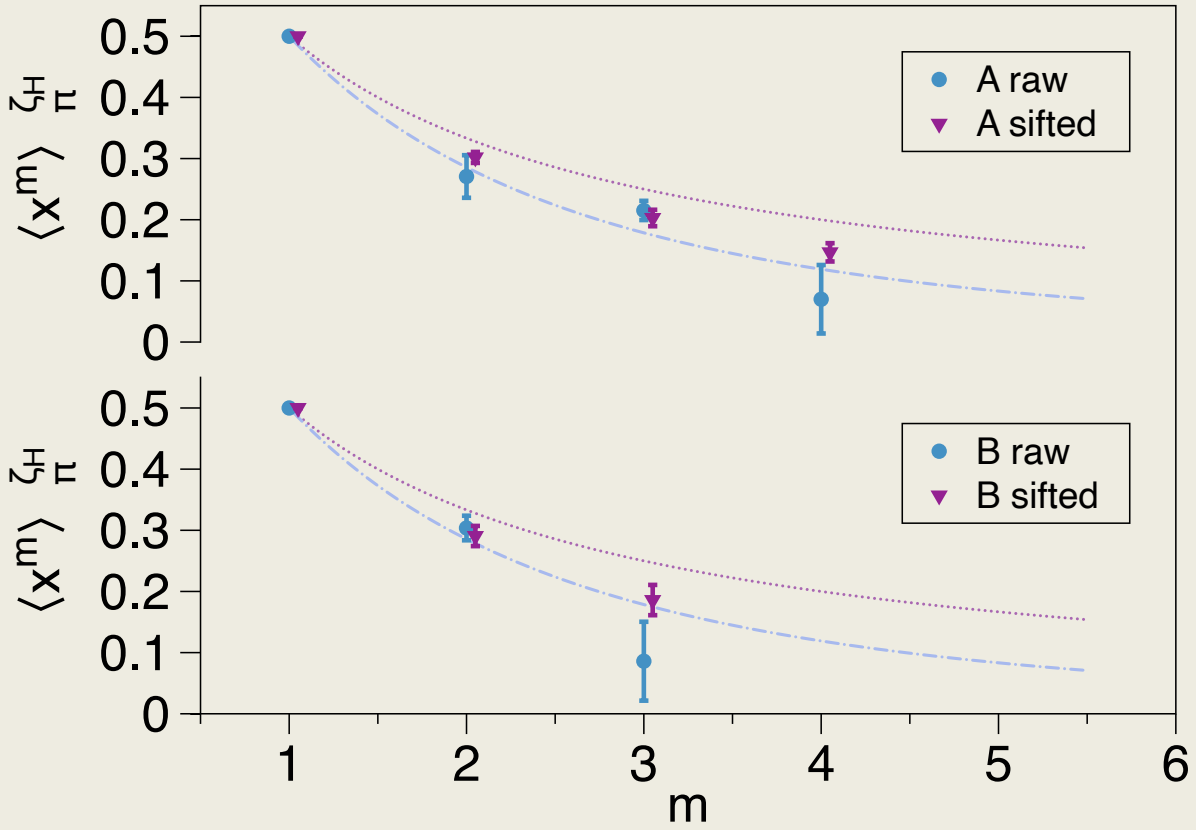
➢ **lattice QCD**:

PRD100(2019)114512,99(2019)074507,104(2021)054504

Sieving parton distribution function moments via the moment problem

Xiaobin Wang (Nankai U.), Minghui Ding (HZDR, Dresden), Lei Chang (Nankai U.) (Aug 28, 2023)

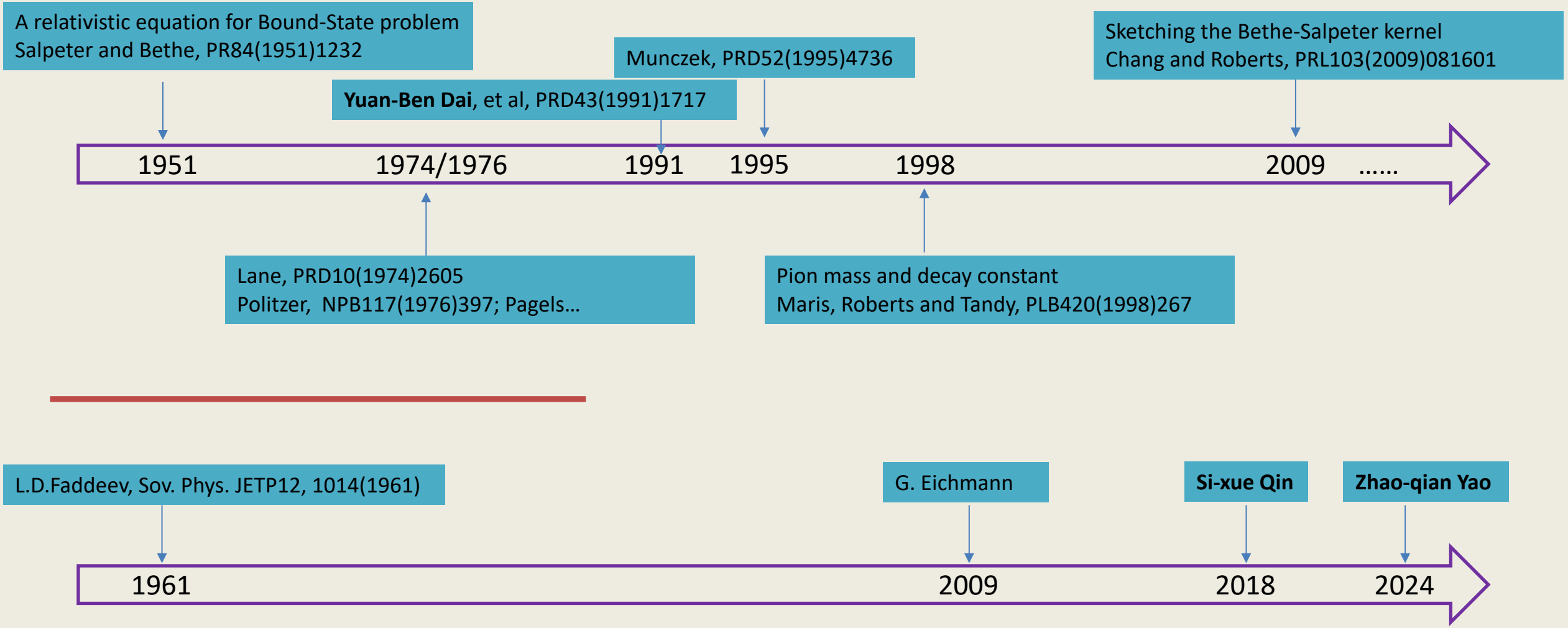
Published in: *Phys.Lett.B* 851 (2024) 138568 • e-Print: 2308.14871 [hep-ph]



- The Moment Problem: we propose a strategy to sieve the moments of the PDF by exploiting its properties such as continuity, unimodality, and symmetry;
- the mean of the moments, whose raw data are clearly not in the region between the two boundaries, enter the region after being sifted;
- Support “Hadronic scale”!

Using CSM to profile hadron

How to describe bound state (incomplete list example)



……Quark-Diquark Model of the Nucleon(many many papers)……

Models

✓ Propagators for quark and diquarks

$$S(\ell) = (-i\gamma\cdot\ell + M)\sigma_M(\ell^2),\ \sigma_M(s) = 1/[s+M^2],$$
$$\Delta^{0^+}(\ell) = \sigma_{M_0}(\ell^2),\ \Delta^{1^+}_{\sigma\rho}(\ell) = \delta_{\sigma\rho}\sigma_{M_1}(\ell^2),$$

✓ Pion BS and proton Faddeev amplitudes

$$E(k;P)\sim \int_{-1}^{1} dz\rho(z)\frac{M^2}{k^2 + z\,k\cdot P + M^2}$$

➢ $1/k^2$ behaviors

$$\psi^{0^+}(\ell;K) = \mathbb{I}\int_{-1}^{1} dz\,\omega(z)\hat{\sigma}_\Lambda(\ell_z^2)^2,$$
$$\psi^{1^+}_\rho(\ell;K) = \tfrac{1}{\sqrt{3}}\gamma_5\gamma^K_\rho\int_{-1}^{1} dz\,\omega(z)\hat{\sigma}_\Lambda(\ell_z^2)^2,$$

➢ $1/k^4$ behaviors

- Quark distribution in the diquarks

$$u_V^{0^+,1^+}(x;\zeta_\mathcal{H}) = q_{01}(x;\zeta_\mathcal{H}) = n_q x^2(1-x)^2 \mathrm{e}^{20x(1-x)-1},$$

➢ *My choice*

- Four parameters: M=0.4GeV, M_0=0.78GeV, M_1=0.92
  $r_{10} = a_{1^+}/a_{0^+}$

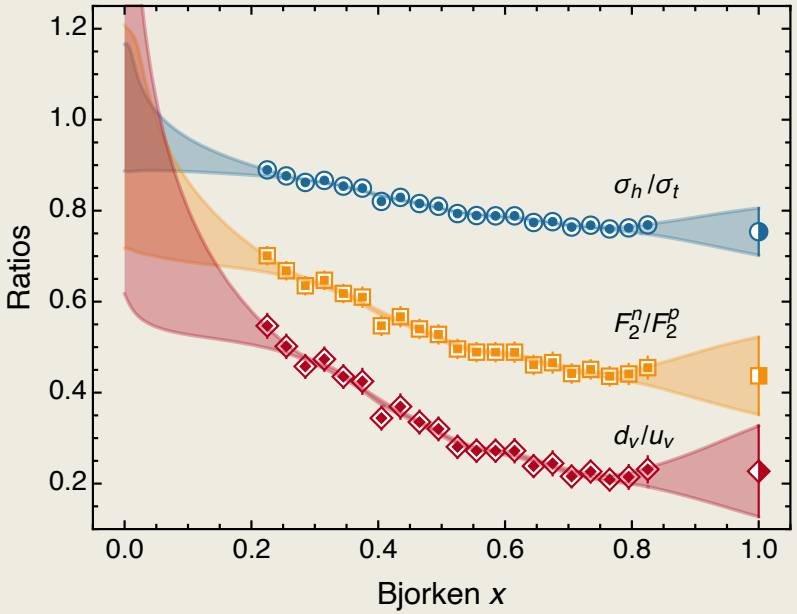
Large-x behavior

Constraint-I

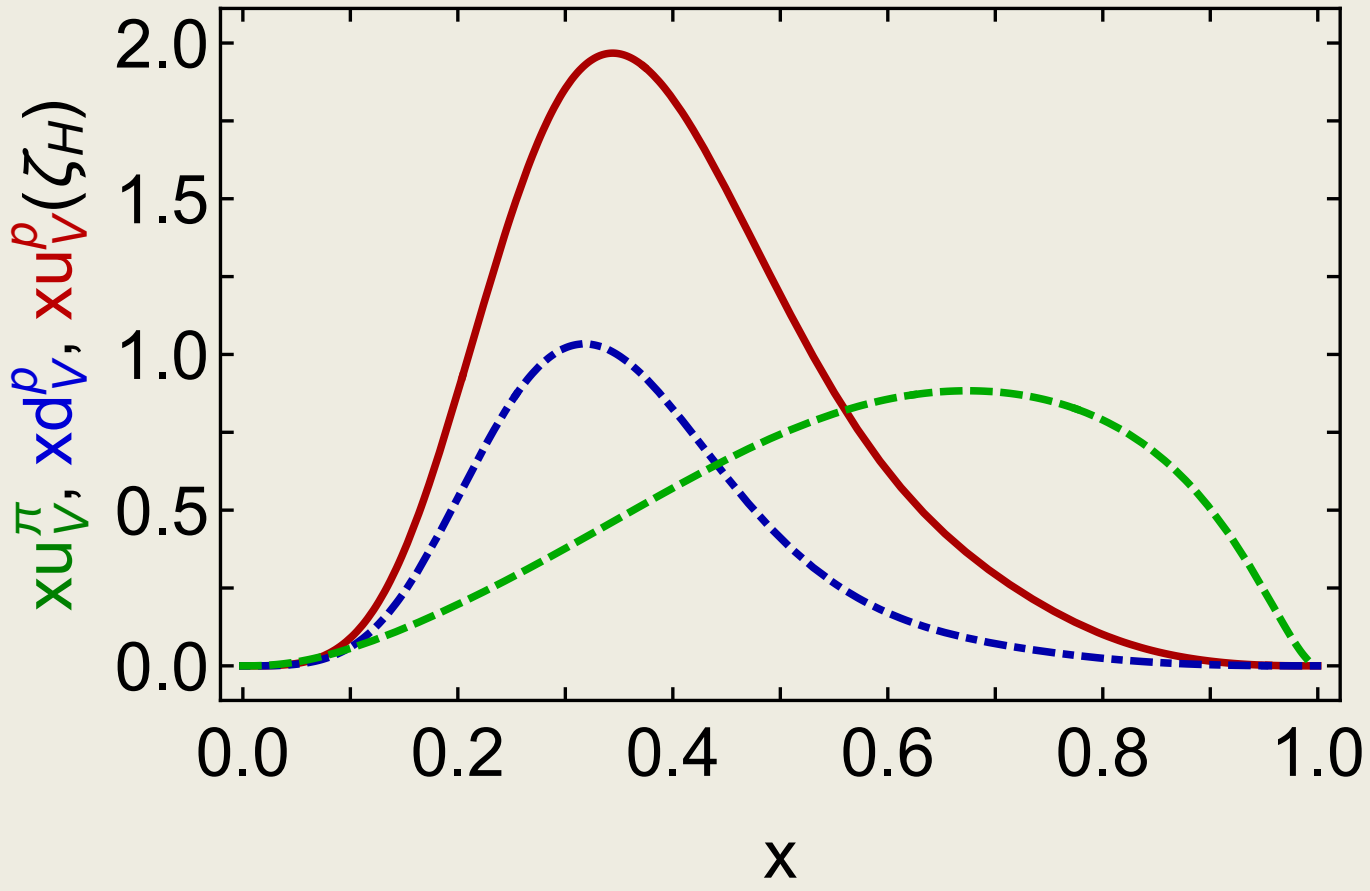
$$d^p(x;\zeta_{\mathcal{H}}),\, u^p(x;\zeta_{\mathcal{H}}) \overset{x\simeq 1}{\propto} (1-x)^3\,,$$

$$\bar{d}^\pi(x;\zeta_{\mathcal{H}}),\, u^\pi(x;\zeta_{\mathcal{H}}) \overset{x\simeq 1}{\propto} (1-x)^2\,;$$

Constraint-II



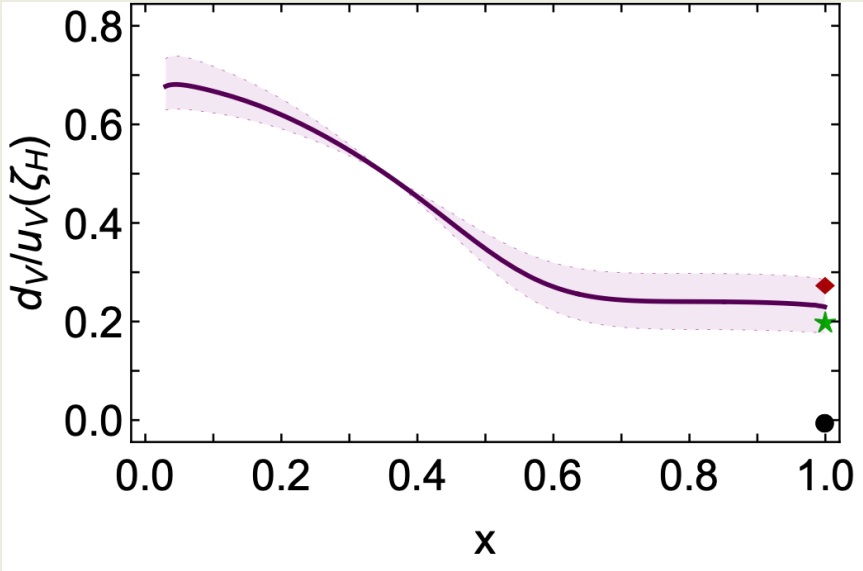
Distributions at the hadronic scale



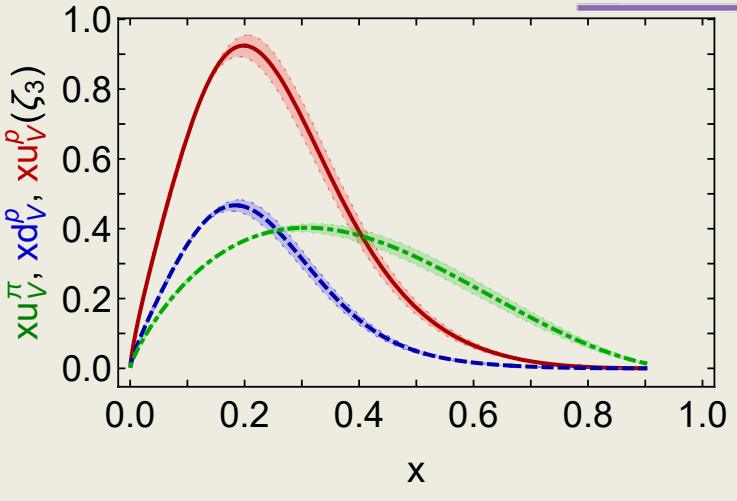
- Valence quarks carry all the momentum of hadron at the hadroni scale

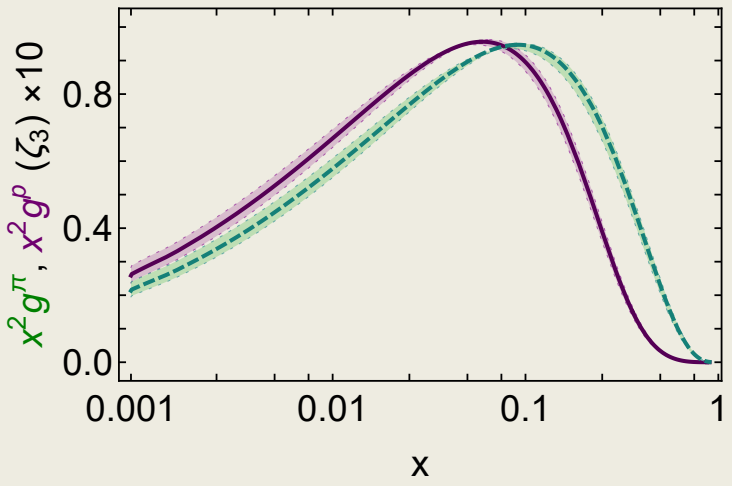
$$\langle x\rangle_{u_p}^{\zeta_{\mathcal{H}}} = 0.687\,,\ \langle x\rangle_{d_p}^{\zeta_{\mathcal{H}}} = 0.313\,,\ \langle x\rangle_{u_\pi}^{\zeta_{\mathcal{H}}} = 0.5\,,$$

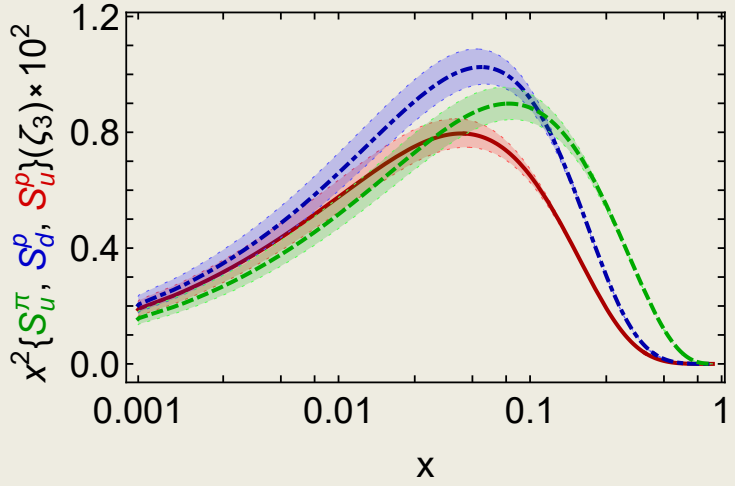
- Diquark correlations $u_V(x) \neq d_V(x)$ in the proton
- Pion is the Nature's most dilated PDF



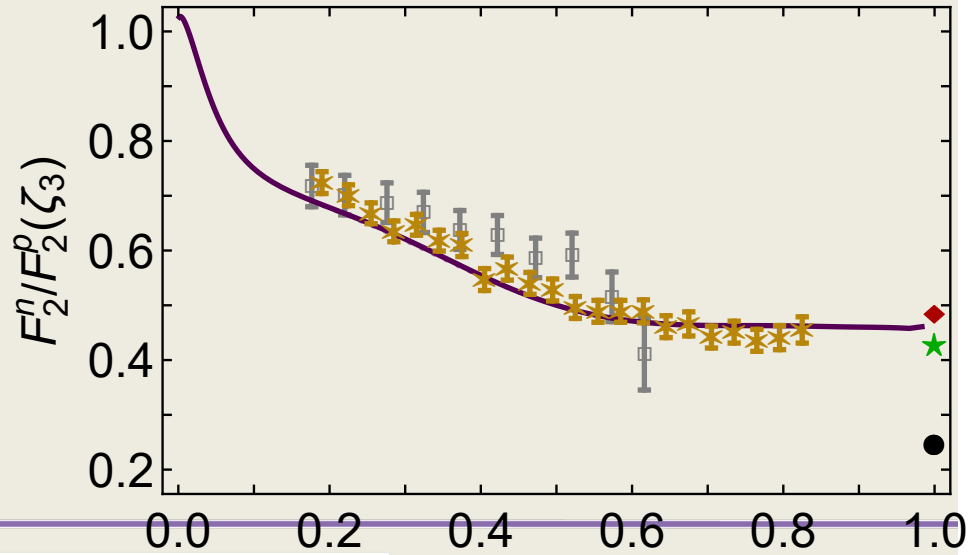
Evolution ($\zeta_3 = m_{J/\psi} = 3.097GeV$)







Pion possess significantly more support on the valence domain than those in the proton!



Using our results for the valence and sea DFs, it is straightforward to calculate the neutron-proton structure function ratio:

$$\frac{F_2^n(x;\zeta)}{F_2^p(x;\zeta)} = \frac{\mathcal{U}(x;\zeta) + 4\mathcal{D}(x;\zeta) + \Sigma(x;\zeta)}{4\mathcal{U}(x;\zeta) + \mathcal{D}(x;\zeta) + \Sigma(x;\zeta)}, \qquad (12)$$

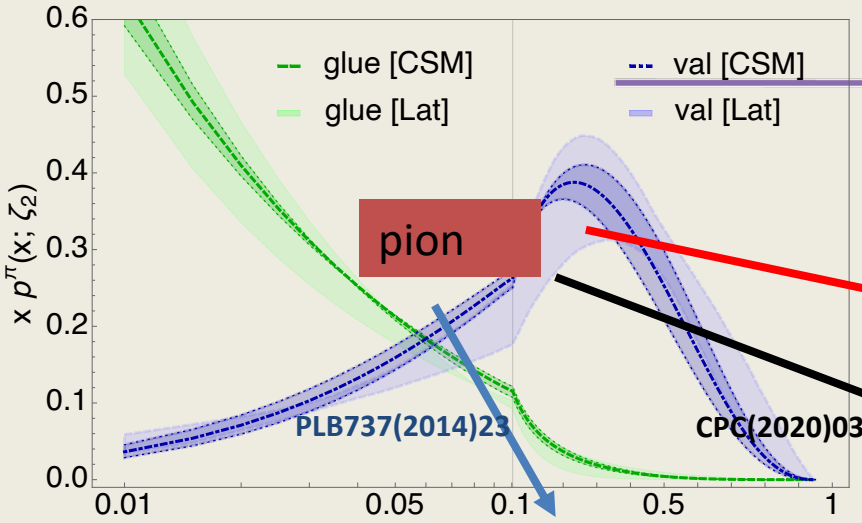
where, in terms of quark and antiquark DFs, $\mathcal{U}(x;\zeta) = u(x;\zeta) + \bar{u}(x;\zeta)$, $\mathcal{D}(x;\zeta) = d(x;\zeta) + \bar{d}(x;\zeta)$, $\Sigma(x;\zeta) = s(x;\zeta) + \bar{s}(x;\zeta) + c(x;\zeta) + \bar{c}(x;\zeta)$. The $\zeta = \zeta_3$ prediction is drawn in Fig. 4B: in comparison with modern data [69, MARATHON], it yields χ^2/degree-of-freedom = 1.3. Notably, both data and calculation indicate the presence of a significant axial-vector diquark component in the proton wave function [83, 84].

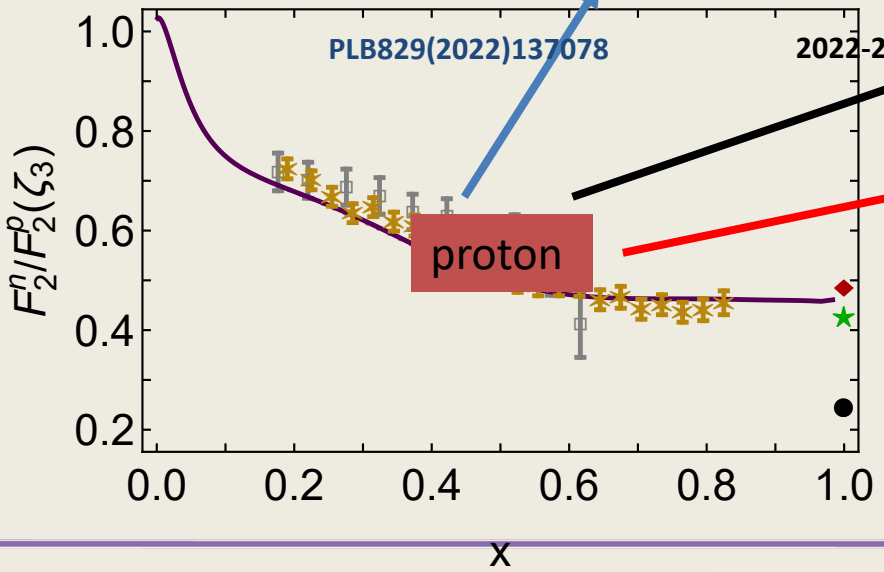
Moments ($\zeta_3 = m_{J/\psi} = 3.097GeV, \zeta_2 = 2GeV$)

Pion	u^π	$\bar{d}^\pi$	g^π	S_π^u	$S_\pi^{\bar{d}}$	S_π^s	S_π^c
$\langle x \rangle^{\zeta_2}$	24.0(1.1)	24.0(1.1)	41.0(1.2)	3.3(3)	3.3(3)	2.65(22)	1.33(5)
$\langle x^2 \rangle^{\zeta_2}$	9.5(7)	9.5(7)	3.7(1)	0.27(1)	0.27(1)	0.21(1)	0.092(2)
$\langle x^3 \rangle^{\zeta_2}$	4.7(4)	4.7(4)	0.92(6)	0.057(1)	0.057(1)	0.044(0)	0.018(1)
$\langle x \rangle^{\zeta_3}$	22.1(1.0)	22.1(1.0)	42.9(1.0)	3.7(3)	3.7(3)	3.0(2)	1.83(6)
$\langle x^2 \rangle^{\zeta_3}$	8.4(6)	8.4(6)	3.5(1)	0.27(1)	0.27(1)	0.22(1)	0.120(3)
$\langle x^3 \rangle^{\zeta_3}$	4.0(3)	4.0(3)	0.82(5)	0.056(0)	0.056(0)	0.044(0)	0.022(1)

Proton	u^p	d^p	g^p	S_p^u	S_p^d	S_p^s	S_p^c
$\langle x \rangle^{\zeta_2}$	32.9(1.4)	15.0(0.7)	40.9(1.1)	2.9(2)	3.7(3)	2.64(22)	1.32(5)
$\langle x^2 \rangle^{\zeta_2}$	8.7(6)	3.6(2)	2.4(1)	0.14(1)	0.21(1)	0.13(0)	0.059(2)
$\langle x^3 \rangle^{\zeta_2}$	2.9(3)	1.1(1)	0.39(2)	0.019(0)	0.030(1)	0.019(0)	0.008(0)
$\langle x \rangle^{\zeta_3}$	30.4(1.3)	13.8(0.6)	42.8(1.0)	3.3(3)	4.1(3)	3.0(2)	1.82(6)
$\langle x^2 \rangle^{\zeta_3}$	7.7(5)	3.2(2)	2.2(1)	0.15(1)	0.21(1)	0.14(0)	0.075(2)
$\langle x^3 \rangle^{\zeta_3}$	2.5(2)	0.9(1)	0.35(2)	0.019(0)	0.028(0)	0.019(0)	0.010(1)

- our approach entails that comparable momentum fractions in the proton and pion are identical
- This quantitative similarity also extends to the *charm* quark: we find $\langle x \rangle_{\zeta 2}$ = 1.32(5)%, $\langle x \rangle_{\zeta 3}$ = 1.82(6)%, compared with the values 1.7(4), 2.5(4)% in R.D. Ball, et al., Parton distributions from high-precision collider data, Eur. Phys. J. C 77 (10) (2017) 663.



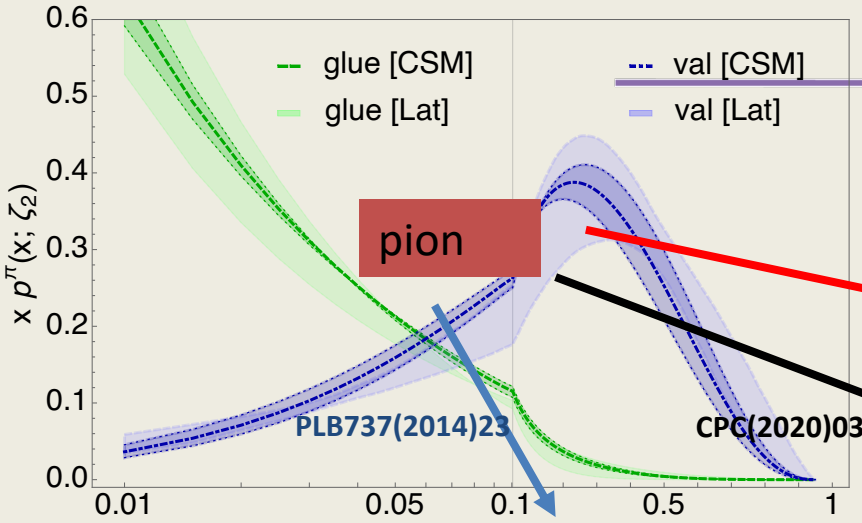


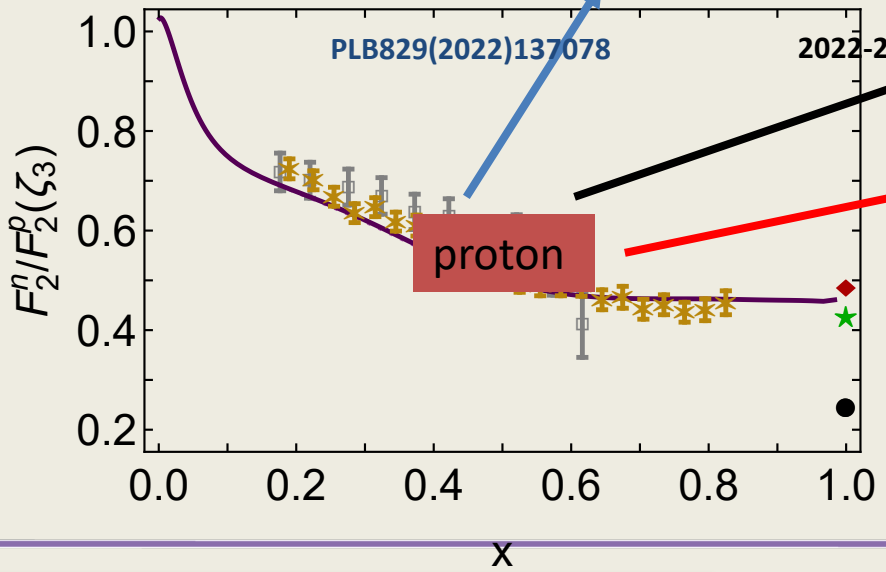
Toy model → Rainbow-Ladder truncation → Systematic

?

- Ending debate on the large-x behavior of pion and wait for the experiments
- Develop RL truncation of proton techniques and calculate the proton structure function directly

- Systematic error controls





Toy model → Rainbow-Ladder truncation → Systematic

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