Reaction Theory for
Resonance Electro- and Photoproduction

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Strong QCD 2024
Strong QCD from Hadron Structure Experiments - VI

JÜLICH FORSCHUNGSZENTRUM

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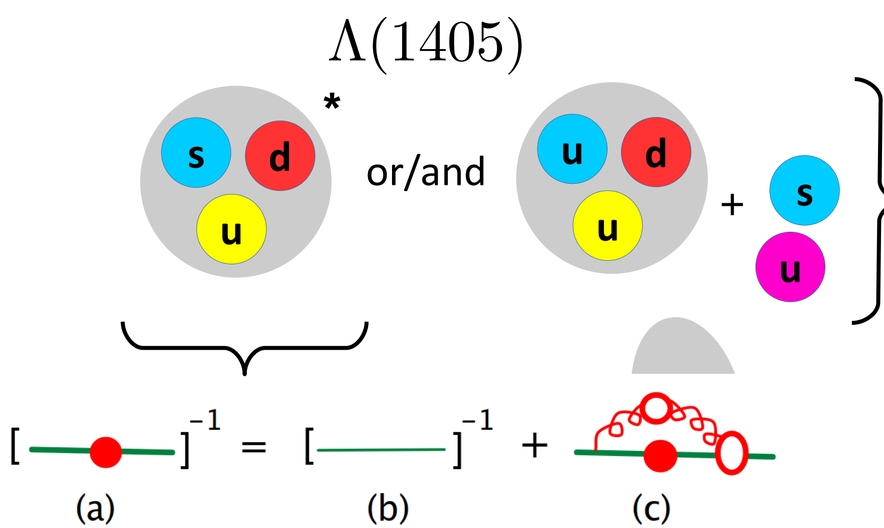
With slides/material from J. Hergenrather & M. Mai

Degrees of freedom: Quarks or hadrons

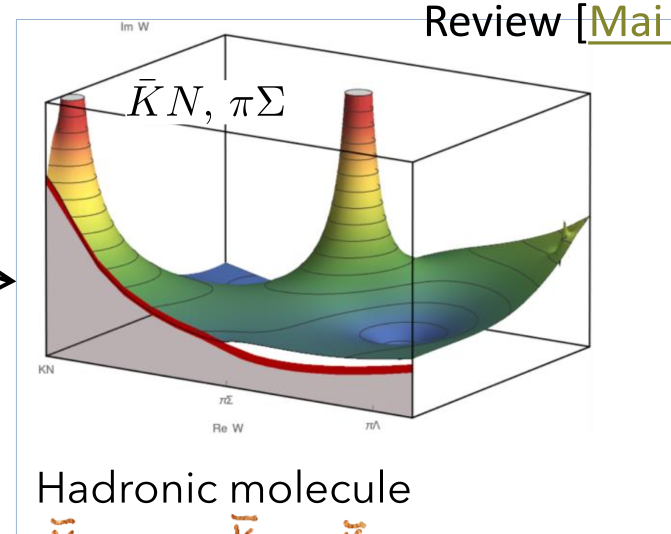
- Resonance review [Mai 2022]

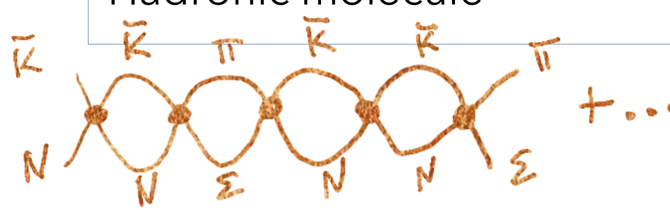
QCD at low energies

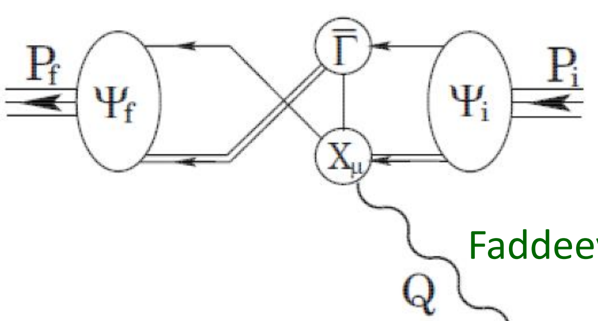
Non-perturbative dynamics	→ rich spectrum of excited states
How many states are there?	→ missing resonance problem (does it exist?)
What are they?	→ 2-quark/3-quark, hadron molecules, …



Review [Mai 2021]



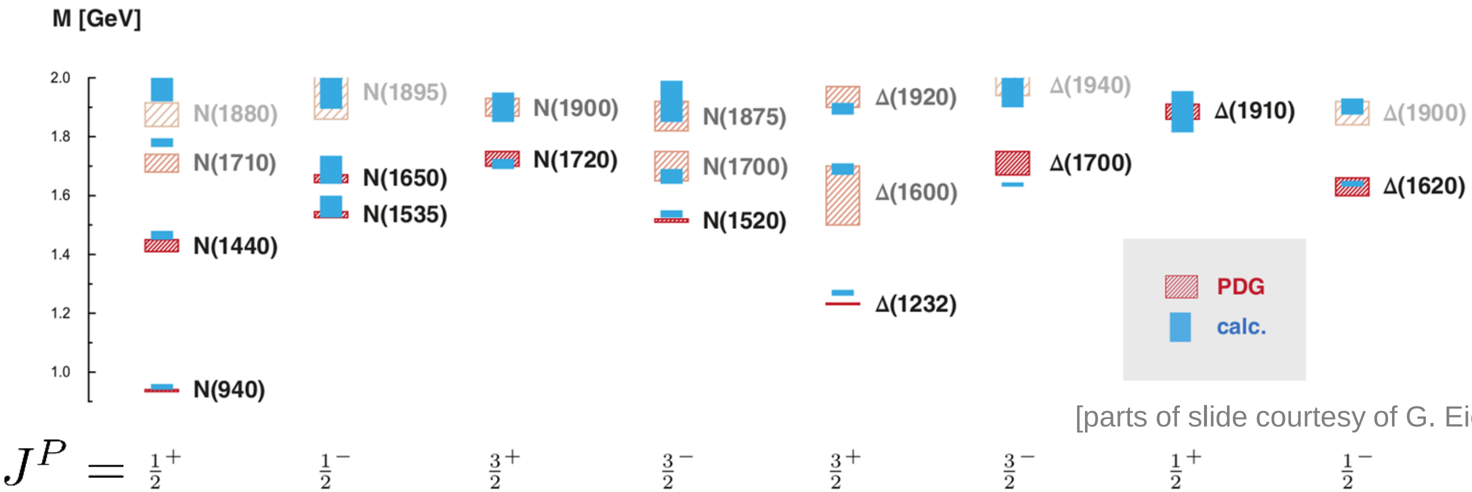




Faddeev Eq. / DSE (Binosi, Cloet, Chang, Eichman, Roberts,…);

Light baryons from diquark dynamics

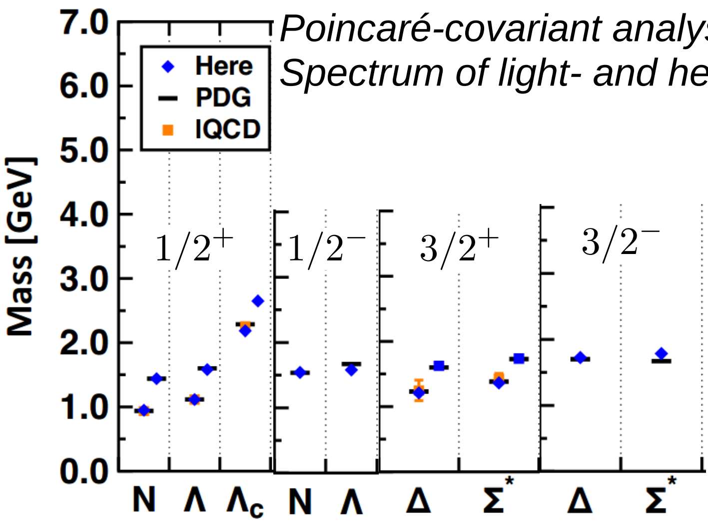
Quark-diquark with reduced pseudoscalar + vector diquarks: [Eichmann (2016)]

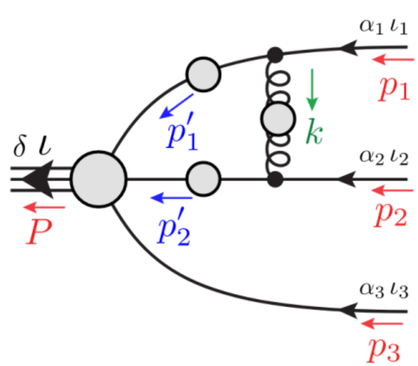


Not much of a missing resonance problem left…

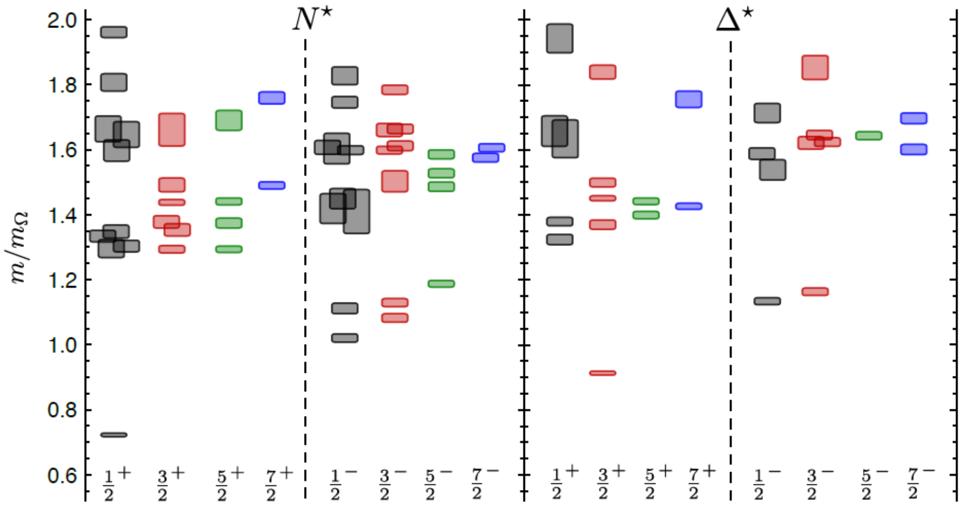
[parts of slide courtesy of G. Eichmann, Few Body 2018]

Poincaré-covariant analysis of heavy-quark baryons, [Qin 2018]
Spectrum of light- and heavy-baryons, [Qin 2019]



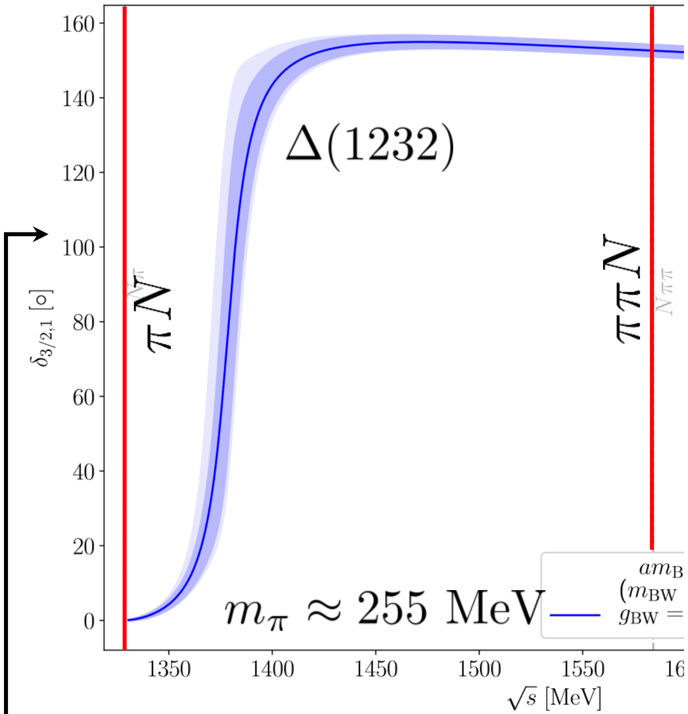


Lattice QCD for excited baryons



$m_\pi = 396$ MeV [Edwards *et al.*, Phys.Rev. D84 (2011)]

- Pioneering spectroscopic calculations
- Information on existence, width & properties of resonances requires
  - Meson-baryon interpolating operators
  - Detailed finite-volume analysis



[G. Silvi et. al., arXiv: 2101.00689]

See also: Bulava et al., [2208.03867] → Morningstar talk on Monday

How about $\pi\pi N$? 3B Dynamics?

Phenomenology of the baryon spectrum

Review by [Thiel, Afzal, Wunderlich 2022]

Dynamical coupled-channel approaches

[MD, M. Mai, J. Haidenbauer, T. Sato, upcoming review]

- ANL-Osaka (former: EBAC) [Kamano et al.]
- Dubna-Mainz-Taipei model [Tiator]
- Jülich-Bonn(-Washington) [Rönchen]
- …
- Characteristics:
  - Direct fit to data (pion & photon-induced)
  - Simultaneous fit to data of different final states
  - Integral scattering equation as needed for proper treatment of three-body channels ($\pi\pi N$) & inclusion of lefthand cut

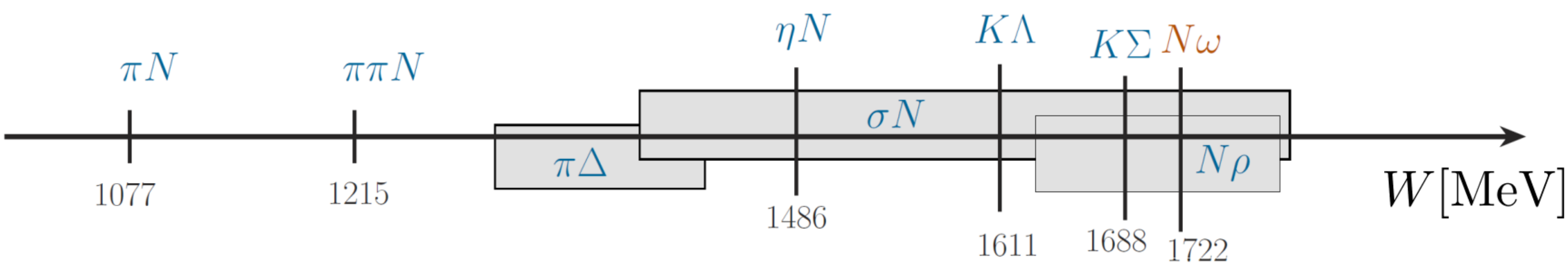
Note: Only a sub-class of analysis efforts; see, e.g., Bonn-Gatchina group K-matrix approach

JBW DCC approach (Jülich-Bonn-Washington)

The scattering equation in partial-wave basis

$$\langle L'S'p'|T^{IJ}_{\mu\nu}|LSp\rangle = \langle L'S'p'|V^{IJ}_{\mu\nu}|LSp\rangle + \sum_{\gamma,L''S''}\int_0^\infty dq \quad q^2 \quad \langle L'S'p'|V^{IJ}_{\mu\gamma}|L''S''q\rangle \frac{1}{W- E_\gamma(q)+i\epsilon}\langle L''S''q|T^{IJ}_{\gamma\nu}|LSp\rangle$$

- channels ν, μ, γ:



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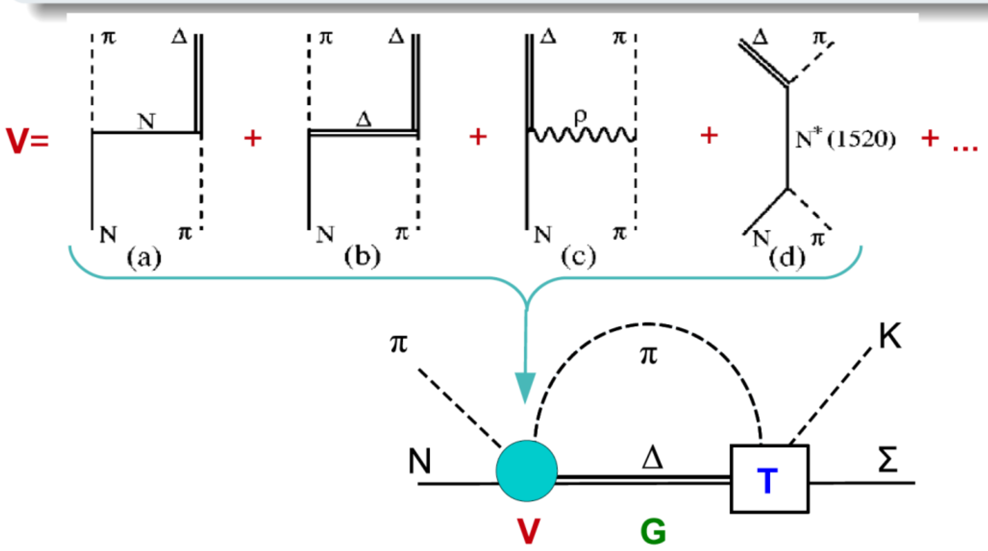


- potentials V constructed from effective $\mathcal{L}$
- s-channel diagrams: T^P
  genuine resonance states
- t- and u-channel: T^{NP}
  dynamical generation of poles
  partial waves strongly correlated
- contact terms

Transitions in s, t, and u-channels

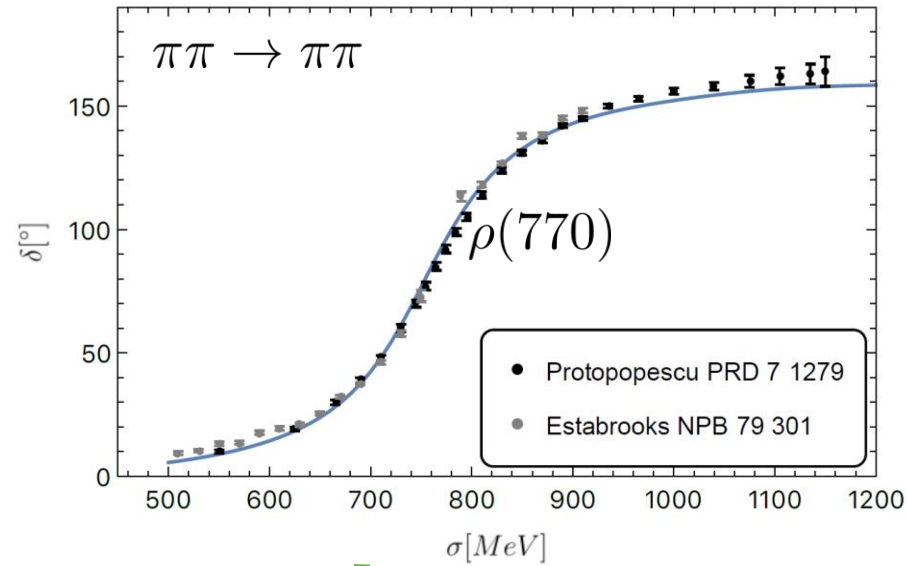
- 21 s-channel exchanges (resonance)
- Contact terms
- t and u-channel exchanges:

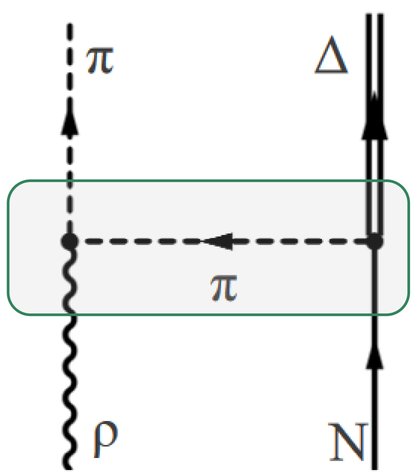
[Yu-Fei Wang 2022]

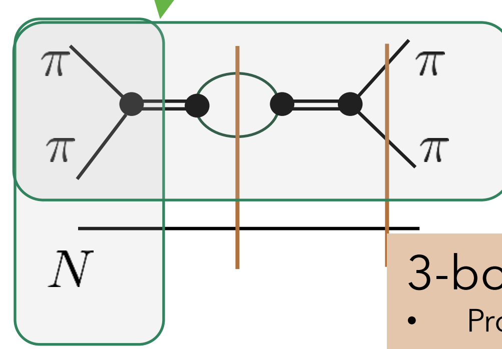
μ	πN	ηN	$K\Lambda$	$K\Sigma$	ωN	$\pi\Delta$	σN	ρN
πN	$(\pi\pi)_\sigma$, $(\pi\pi)_\rho$, N, Δ	a_0, N	K^*, Σ, Σ^*	K^*, Λ, Σ, Σ^*	ρ, N	ρ, N, Δ	π, N	π, ω, a_1, N, Δ, C
ηN		N, f_0	K^*, Λ	K^*, Σ, Σ^*	ω, N			
$K\Lambda$			ω, f_0, ϕ, Ξ, Ξ^*	ρ, a_0, Ξ, Ξ^*	K, K^*, Λ			
$K\Sigma$				ρ, ω, ϕ, f_0, a_0, Ξ, Ξ^*	K, K^*, Σ, Σ^*			
ωN					σ, N			
$\pi\Delta$						ρ, N, Δ	π	π, N
σN							σ, N	
ρN								ρ, N, Δ, C

Three-body channels $\sigma N, \pi\Delta, \rho N$

- Resonant sub-channels
- Fit 2→2 amplitude to 2→ 2 scattering data
- Include as sub-channel in 3-body amplitude:
- 3-body unitarity: Requires, e.g.



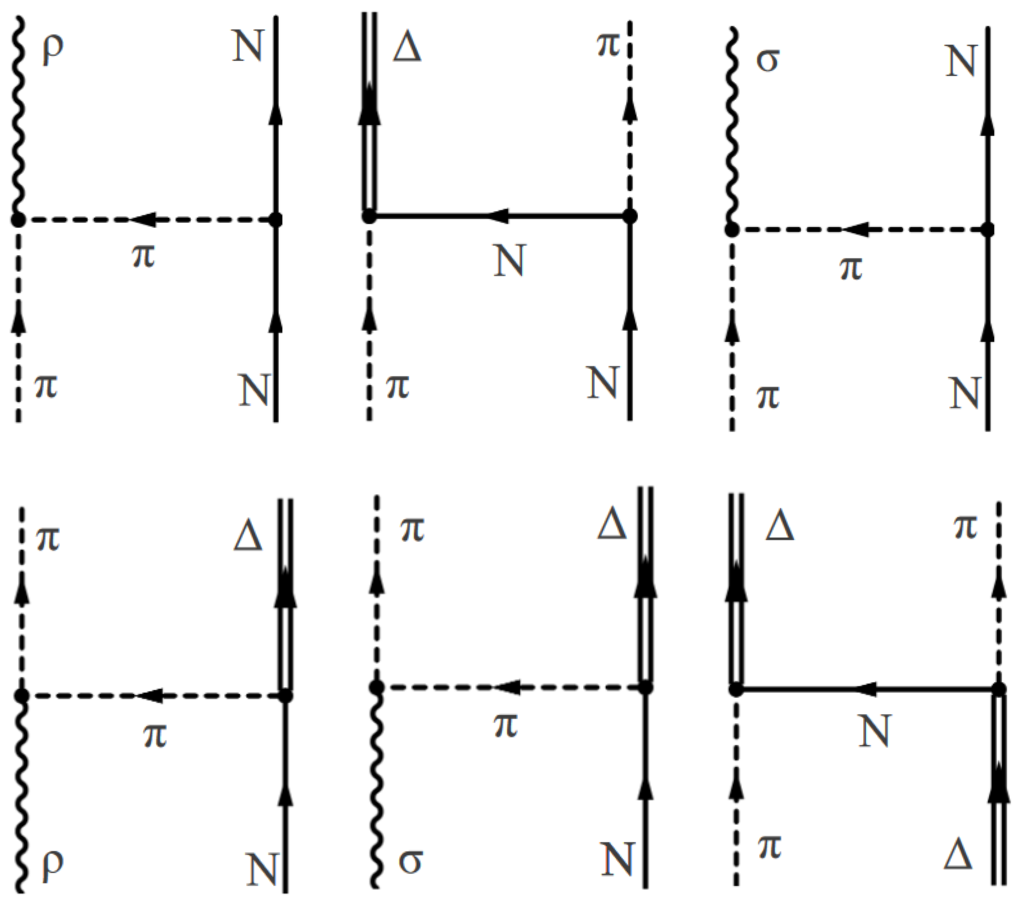




3-body cuts in
- Propagation
- exchange

2→ 3 and 3→3 body unitarity

- Unitarity requires certain transition amplitudes



2→3

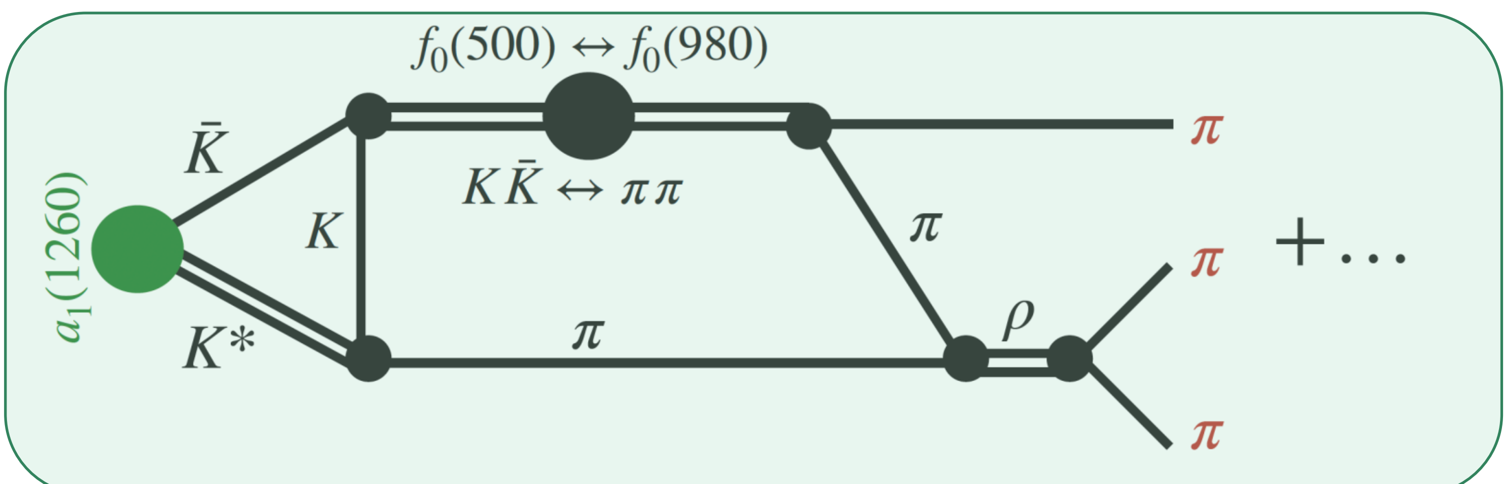
Unitarity requires formulation of scattering via integral equation
→ ANL-Osaka & JB(W)

3→3

Unitary amplitudes for meson analysis

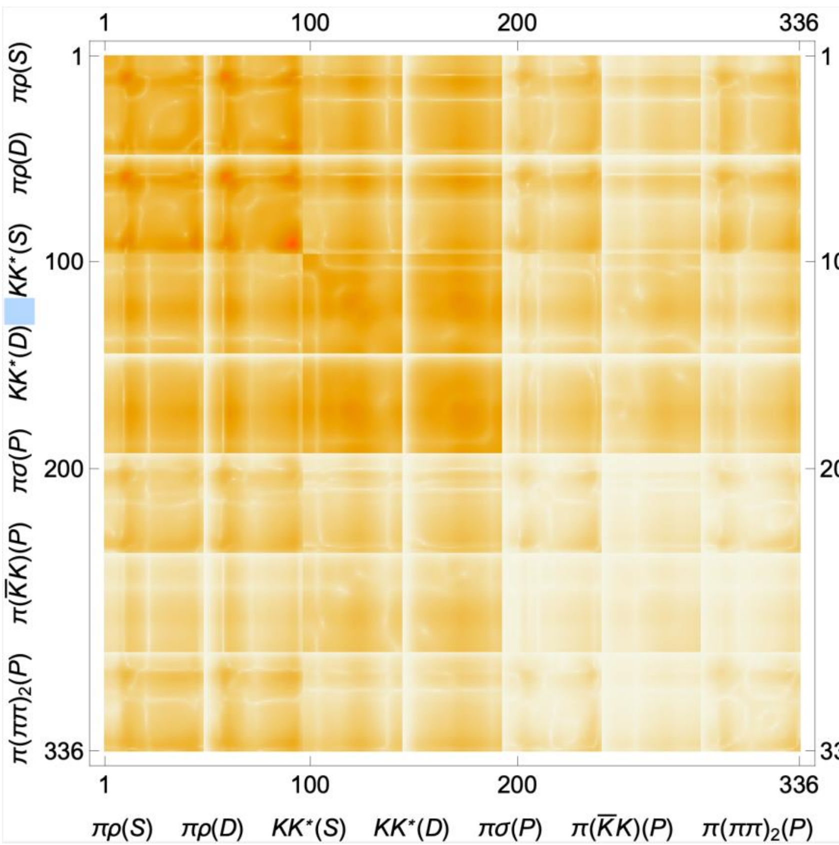
[Y. Feng, F. Gil, R. Molina, M. Mai, V. Shastry, A. Szczepaniak, et al.]

- Coupled-channel, coupled-partial wave amplitudes
- Unitarity manifest
- In-flight transitions of isobars: $\pi\pi \leftrightarrow K\bar{K}$
- All isospins: $I = 0,\ (1/2),\ 1, (3/2),\ 2$
- All subsystems up to P-wave, including $f_0(500),\ \rho,\ f_0(980), K^*,\ (\kappa)$
- Example:



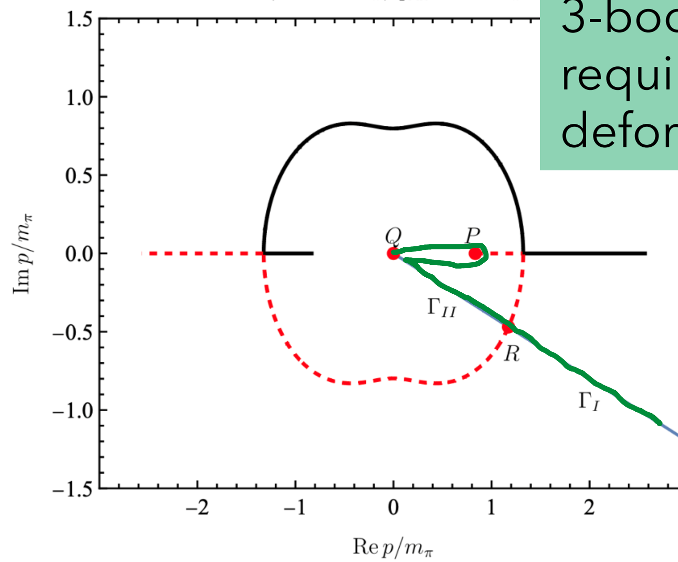
Prelim results

- 7-channel model T-matrix (at fixed 3body energy)

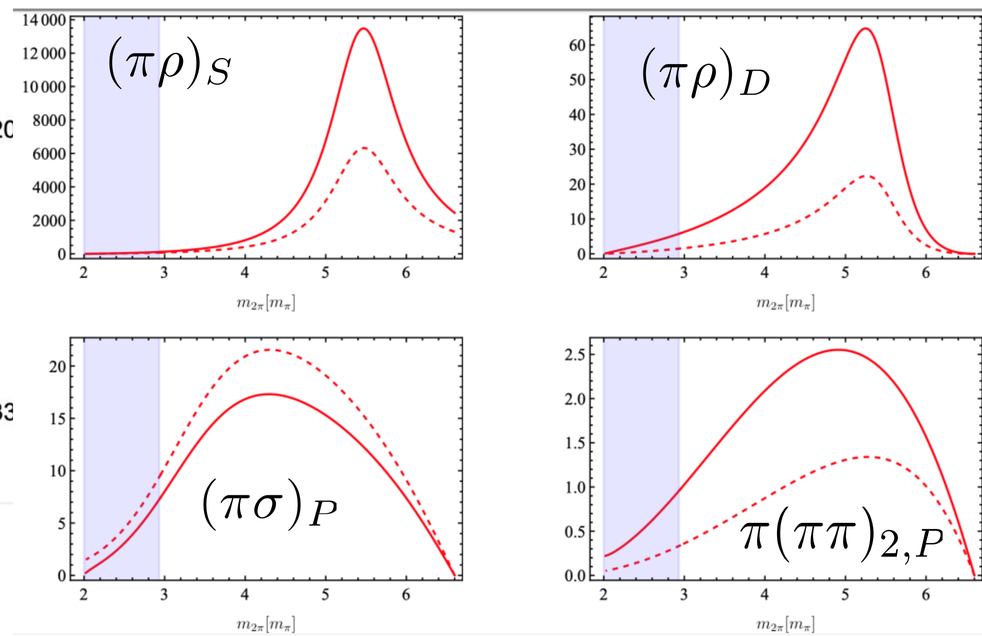


→ Spectator momentum ⊗channels

- 4-channel model: Production amplitudes

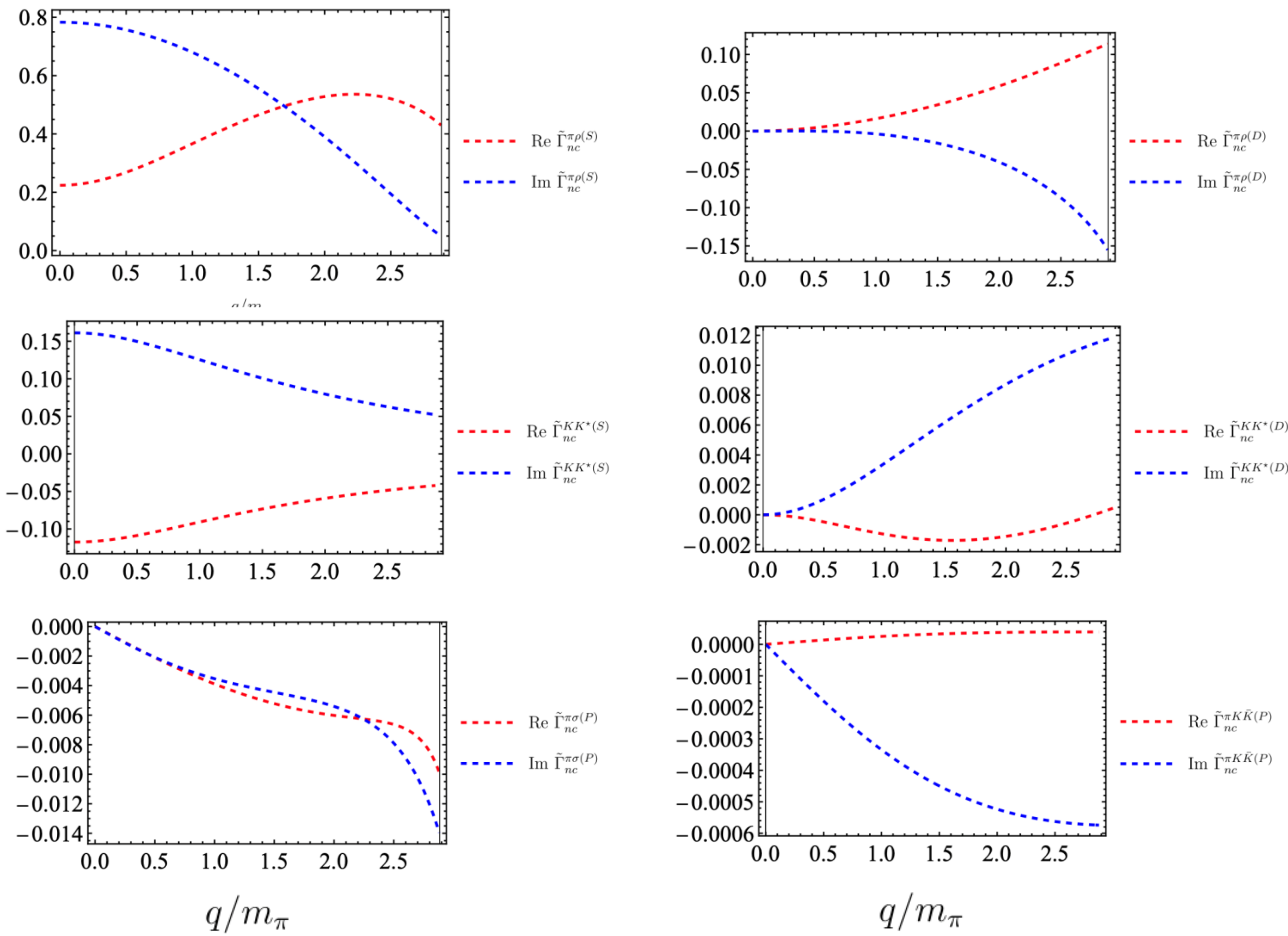


3-body cuts require contour deformation



Production amplitude 7-channel model

Only the (non-trivial) rescattering piece $3m_\pi < W < 2m_K + m_\pi$



$+(p(\pi\pi)_2)_S$

JB: Data base

- $\pi N \to X$: $>$ **7,000** data points ($\pi N \to \pi N$: GW-SAID WI08 (ED solution))
- $\gamma N \to X$:

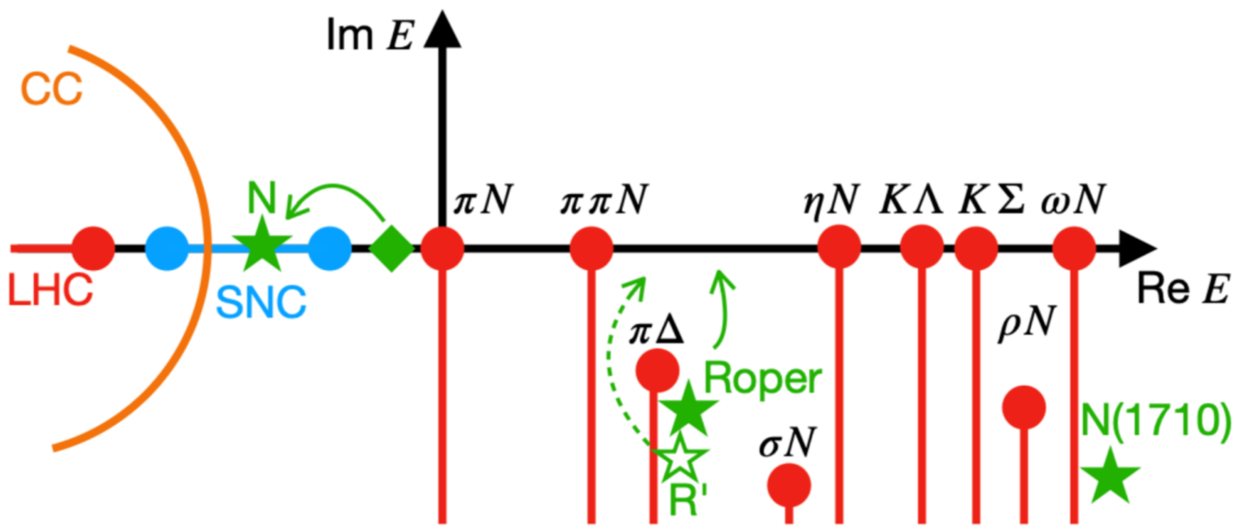
New: $\pi N \to \omega N$ [Yu-Fei Wang 2022]
Upcoming data from JParc

Reaction	Observables (# data points)	p./channel
$\gamma p \to \pi^0 p$	$d\sigma/d\Omega$ (18721), Σ (2927), P (768), T (1404), $\Delta\sigma_{31}$ (140), G (393), H (225), E (467), F (397), $C_{x'_L}$ (74), $C_{z'_L}$ (26)	25,542
$\gamma p \to \pi^+ n$	$d\sigma/d\Omega$ (5961), Σ (1456), P (265), T (718), $\Delta\sigma_{31}$ (231), G (86), H (128), E (903)	9,748
$\gamma p \to \eta p$	$d\sigma/d\Omega$ (9112), Σ (403), P (7), T (144), F (144), E (129)	9,939
$\gamma p \to K^+\Lambda$	$d\sigma/d\Omega$ (2478), P (1612), Σ (459), T (383), $C_{x'}$ (121), $C_{z'}$ (123), $O_{x'}$ (66), $O_{z'}$ (66), O_x (314), O_z (314),	5,936
$\gamma p \to K^+\Sigma^0$	$d\sigma/d\Omega$ (4271), P (422), Σ (280), T (127), $C_{x',z'}$ (188), $O_{x,z}$ (254)	5,542
$\gamma p \to K^0\Sigma^+$	$d\sigma/d\Omega$ (242), P (78)	320
	in total	**57,027**

New interface [https://jbw.phys.gwu.edu/]

Partial-Wave Analytic structure

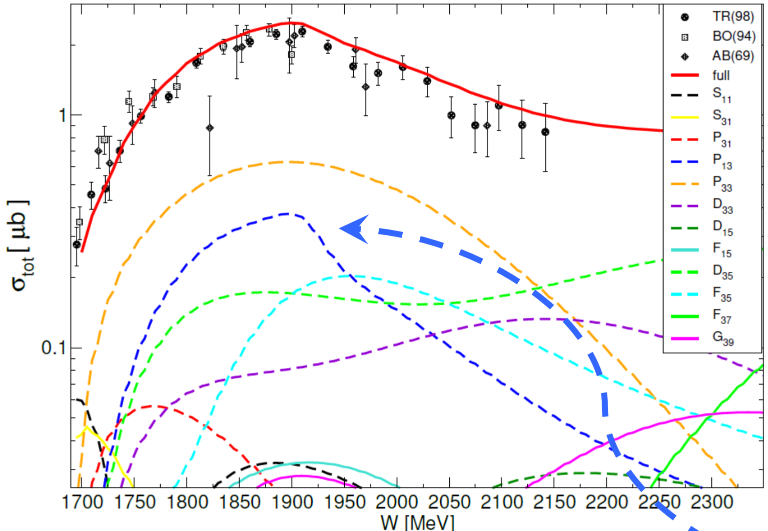
- Branch points indicate thresholds
- Partial-wave amplitudes have more cuts than plane-wave amplitude
- Example: The structure of the P11 amplitude

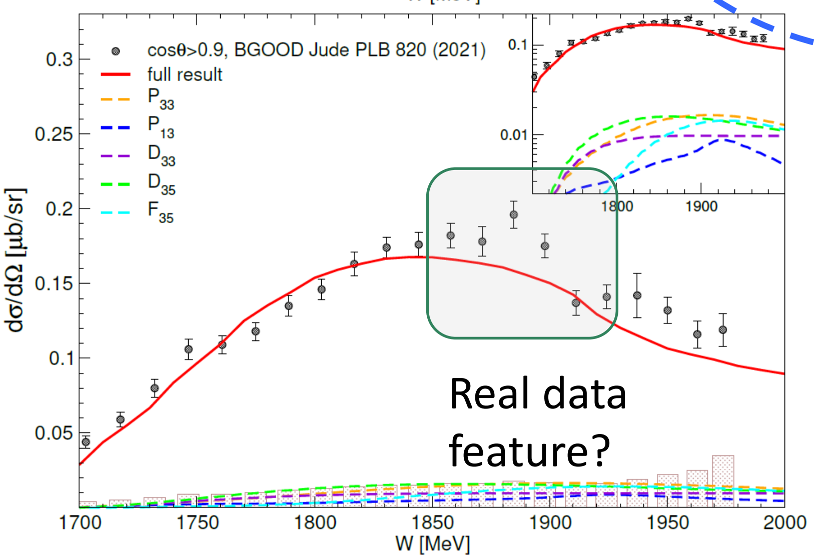


Resonances in $K\Sigma$ photoproduction

- [D. Roenchen et al. (EPJA 2022)]
- [Webpage all results]

$\gamma p \to K^+\Sigma^0$





Similarly: $K^0\Sigma^+$

- dominant partial waves: $I = 3/2$

Exception: P_{13} partial wave ($I = 1/2$):

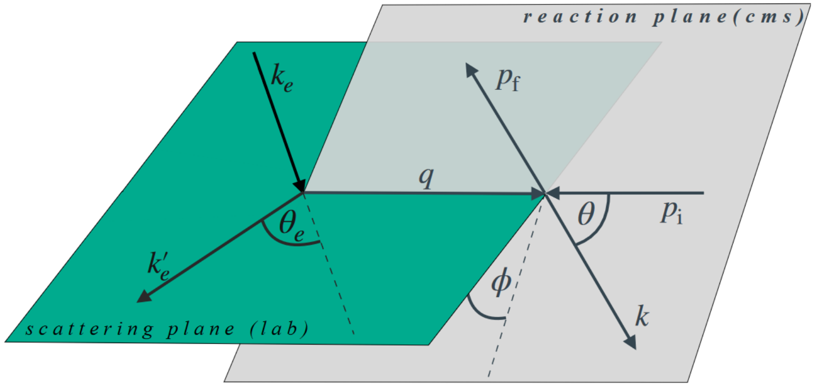
$N(1720)$ $3/2^+$ $****$	Re E_0 [MeV]	-2Im E_0 [MeV]	$\frac{\Gamma^{1/2}_{\pi N}\Gamma^{1/2}_{K\Sigma}}{\Gamma_{tot}}$ [%]	$\theta_{\pi N\to K\Sigma}$ [deg]
2022	1726	185	5.9	82
2017	1689(4)	191(3)	0.6(0.4)	26(58)
PDG 2021	1675 ± 15	250^{+150}_{-100}	—	—

$N(1900)$ $3/2^+$ $****$	Re E_0 [MeV]	-2Im E_0 [MeV]	$\frac{\Gamma^{1/2}_{\pi N}\Gamma^{1/2}_{K\Sigma}}{\Gamma_{tot}}$ [%]	$\theta_{\pi N\to K\Sigma}$ [deg]
2022	1905	93	1.3	-40
2017	1923(2)	217(23)	10(7)	$-34(74)$
PDG 2021	1920 ± 20	150 ± 50	4 ± 2	110 ± 30

- drop in cross section ("cusp-like structure") due to $N(1900)3/2^+$

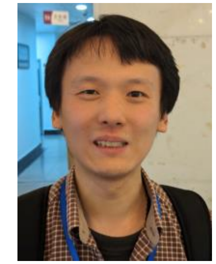
$N(1535)$ ½⁻ $****$	Re E_0 [MeV]	-2Im E_0 [MeV]
2022	1504(0)	74 (1)
2017	1495(2)	112(1)
PDG 2021	1510 ± 10	130 ± 20

New, wide dynamically generated states in J^P=3/2⁻




J. Hergenrather


M. Mai


Yu-Fei Wang

Coupled-Channel Electroproduction

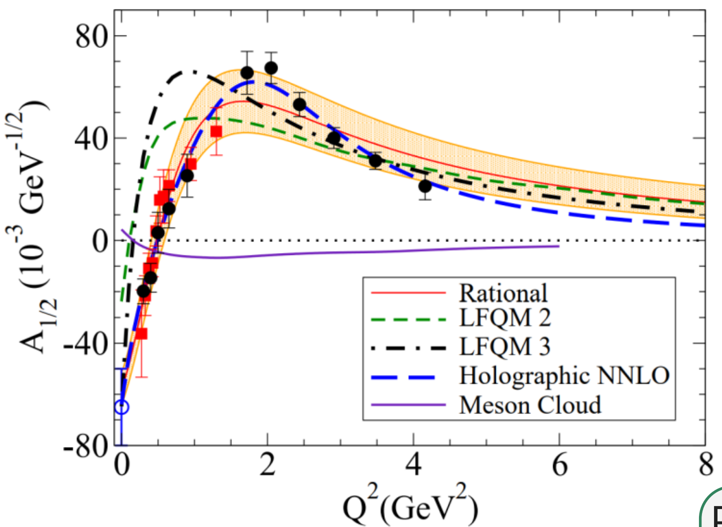
First coupled-channel electroproduction analysis with different final states

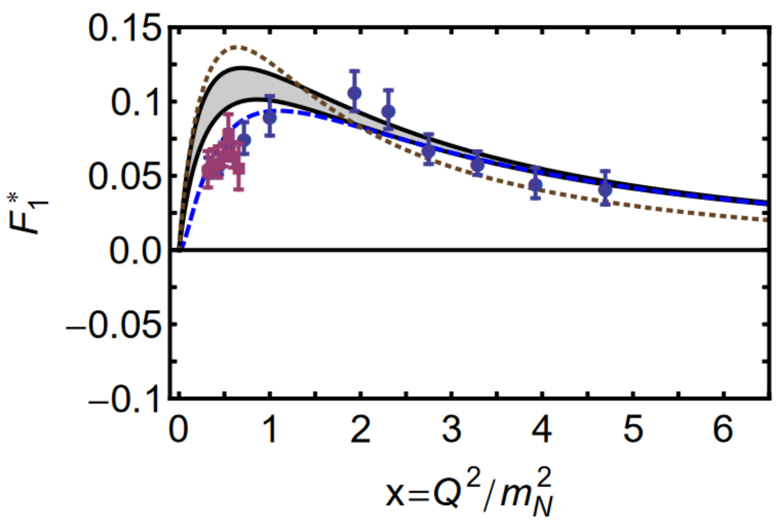
[M. Mai, Yu-Fei Wang et al. (2020 -)]

Theory:

- Siegert's theorem manifestly fulfilled (consequence of gauge invariance)
- Watson theorem fulfilled
- Coupled-channel unitarity fulfilled
- General expansion of electroproduction kernel in Laurent series
- Resonances, background, channels have independent Q^2 dependence for flexible fit

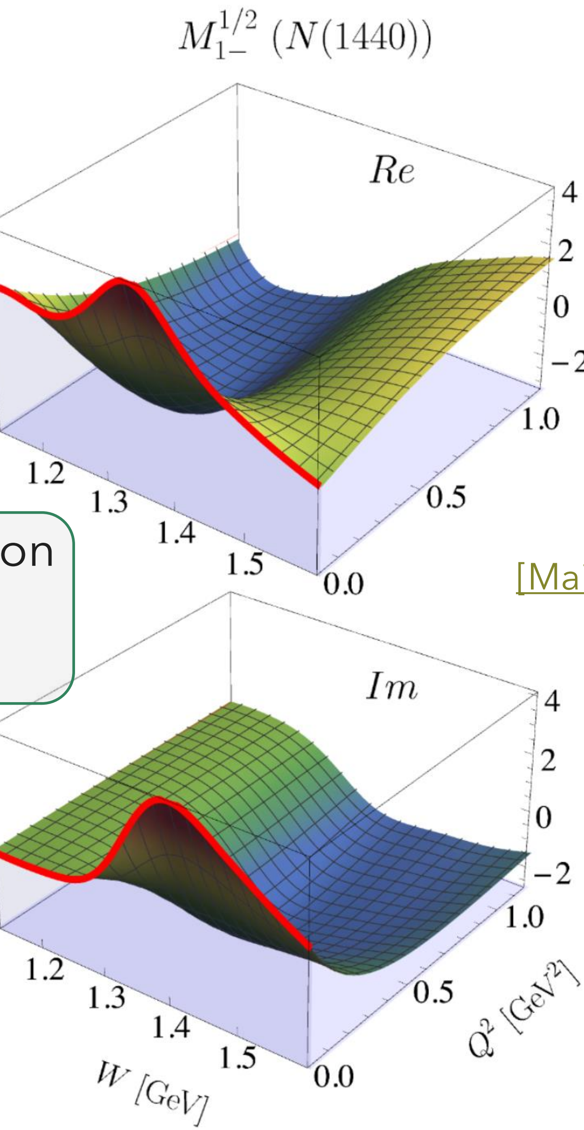
Electroproduction reveals resonance structure





Proton-Roper Transition [Burkert] [Segovia]

Photoproduction included as boundary

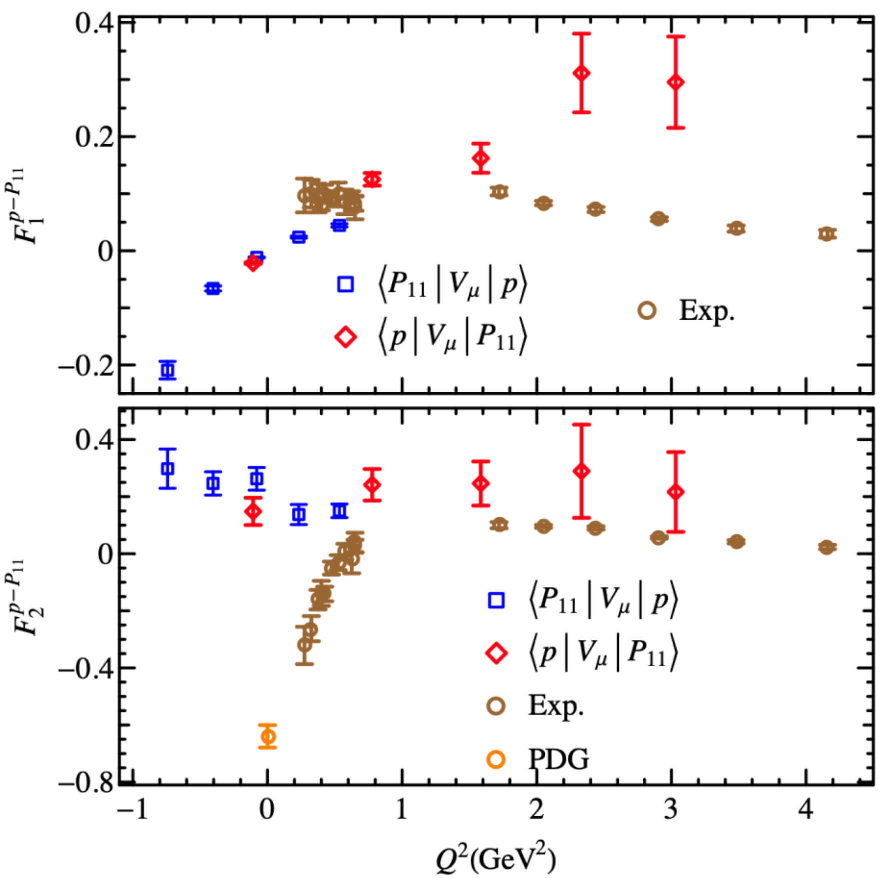


[Mai]

Lattice explorations of TFFs

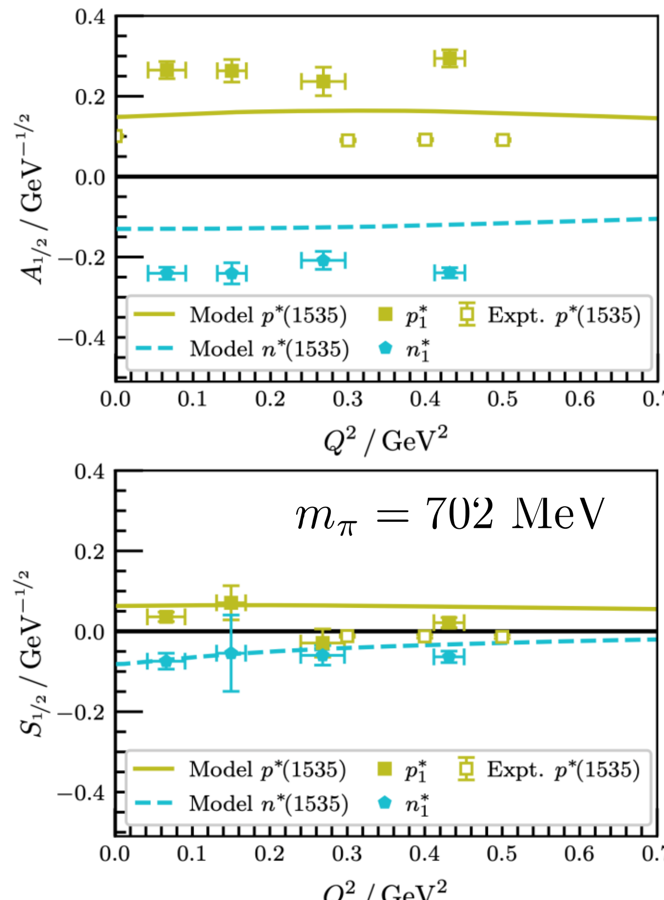
- Proton-Roper

  [H.-W. Lin et al. (2008)]



- N(1535)1/2⁻

  [F.-M. Stokes et al. (2024)]



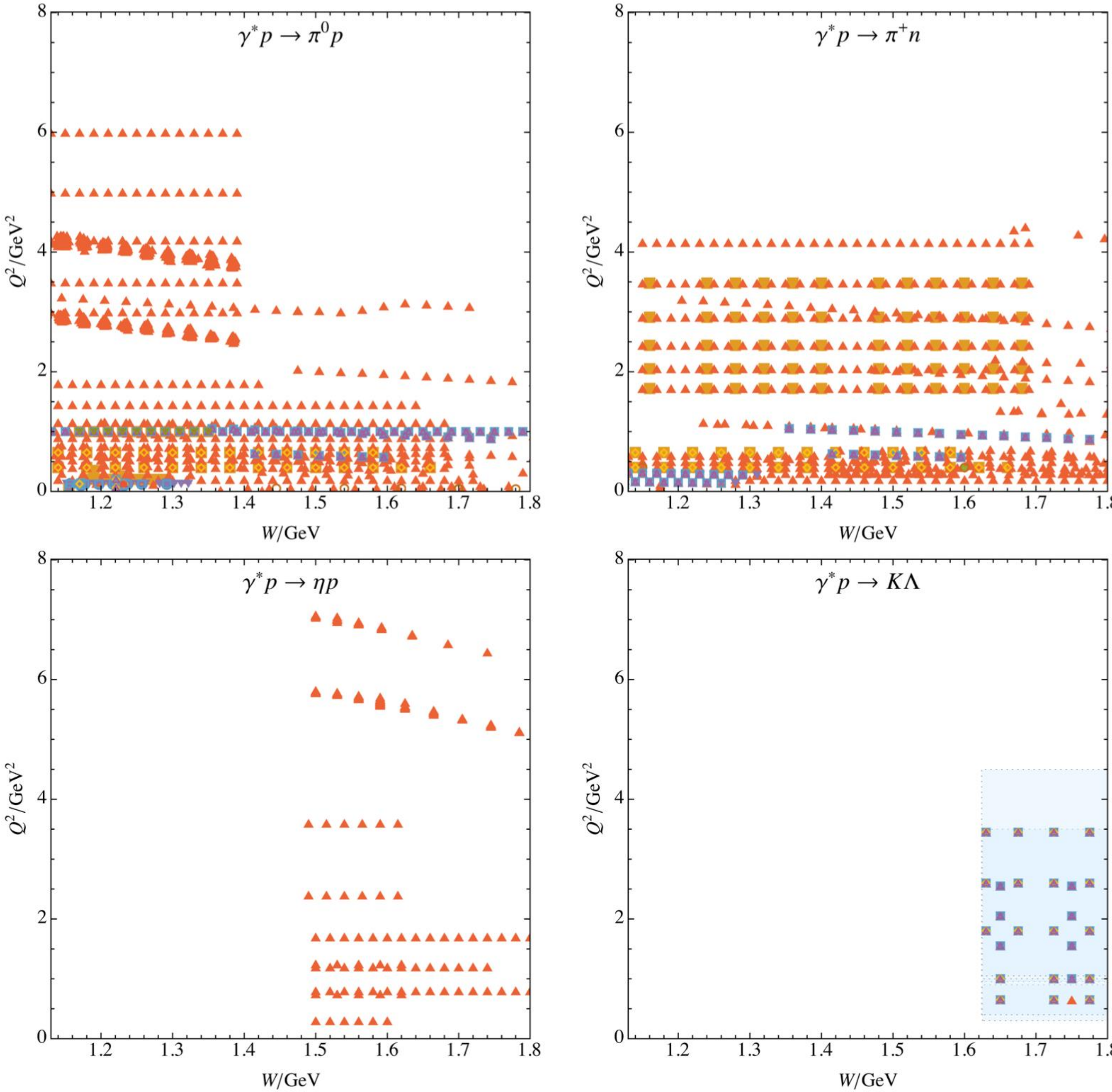
Electroproduction Analysis efforts

- **MAID**: electroproduction of pions, eta mesons, and kaons in separate approaches [Tiator 2007]
- **JM (JLab)** approach: single-pion analysis, double pion analysis [Mokeev, PRC 2023]
- **ANL-Osaka:** Single-pion electroproduction, using multi-channel model. [Kamano, Lee, Nakamura, Sato, 2016]
- **JBW**: simultaneous analysis of multiple electroproduction final states, using multi-channel model
- **Bonn-Gatchina**: Upcoming calculations

JBW Electroproduction data base

Type	$N_{\text{data}}^{\pi^0 p}$	$N_{\text{data}}^{\pi^+ n}$	$N_{\text{data}}^{\eta p}$	$N_{\text{data}}^{K\Lambda}$
ρ_{LT}	45	–	–	–
$\rho_{LT'}$	2768	5068	–	–
σ_L	–	2	–	–
$d\sigma/d\Omega$	48135	44266	3665	2055
$\sigma_T + \epsilon\sigma_L$	384	182	–	204
σ_T	30	2	–	–
σ_{LT}	373	138	–	204
$\sigma_{LT'}$	214	208	–	156
σ_{TT}	327	123	–	204
K_{D1}	1527	–	–	–
P_Y	–	2	–	–
Total	53804	49989	3665	2823

- Data base grown over decades with recent input mostly by CLAS, MAMI.
- Far from complete: Kinematic gaps & consistency issues. Need to combine information from different (W, Q^2) regions
- Need to combine information from simultaneous analysis of different final states ($\pi N/\eta N/K Y/\pi\pi N, \ldots$) to extract resonance helicity couplings



$\gamma^* p \to \pi^0 p$
$\gamma^* p \to \pi^+ n$
$\gamma^* p \to \eta p$
$\gamma^* p \to K\Lambda$
Q^2/GeV^2
W/GeV
0
2
4
6
8
1.2
1.3
1.4
1.5
1.6
1.7
1.8

Fit details: Weighted vs. unweighted χ^2

- Meson production data bases are heterogeneous:
  - A few polarization measurements with large error bars (small weight in χ^2)
  - Many cross section data with smaller error bars (large weight in χ^2)
  - … but those **few** polarization possess **great** power to discriminate solutions
- Introduce **weighted** vs. **unweighted** χ^2:

$$\chi^2_{\rm wt} = \sum_{j\in\{\pi^0 p,\pi^+ n,\eta p\}} \frac{N_{\rm all}}{3N_j} \sum_{i=1}^{N_j} \left(\frac{\mathcal{O}^{\rm exp}_{ji} - \mathcal{O}_{ji}}{\Delta^{\rm stat}_{ji} + \Delta^{\rm syst}_{ji}} \right)^2 .$$

$$\chi^2_{\rm reg} = \sum_{i=1}^{N_{\rm all}} \left(\frac{\mathcal{O}^{\rm exp}_{i} - \mathcal{O}_{i}}{\Delta^{\rm stat}_{i} + \Delta^{\rm syst}_{i}} \right)^2$$

- Quote results for both cases

Fit Strategies (πN)

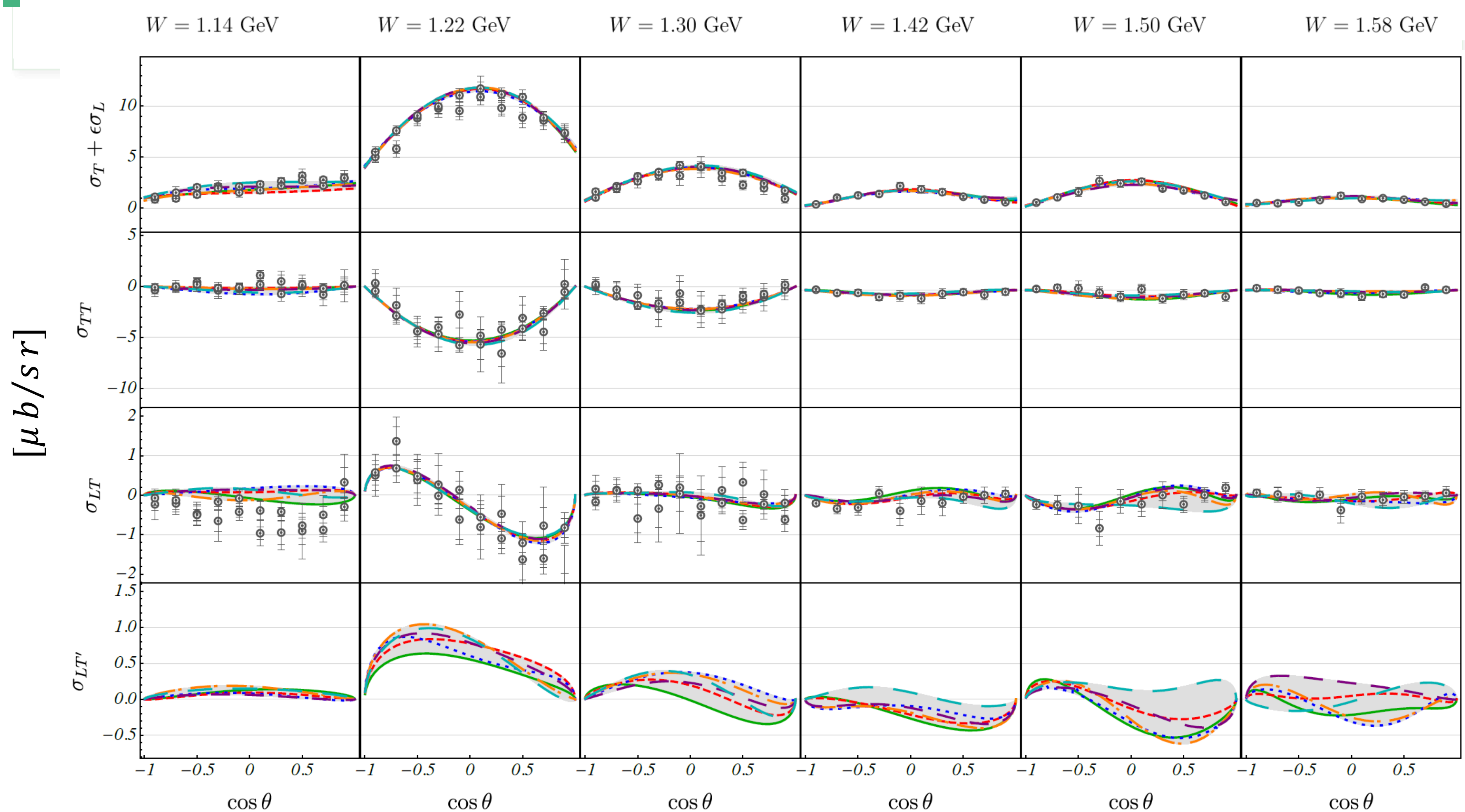
[M. Mai et al, 2021]

- Different fit strategies for $N \approx 85{,}000$ data in $\gamma^* N \to \pi N, \eta N$:
  - Sequential S → S+P → S+P+D waves;
  - Subsets of data until full data set reached
  - Simultaneous fit all parameters (209) set to zero without any (!) guidance
  - Extend data range from $0 < Q^2 < 4$ Gev² to $0 < Q^2 < 6$ Gev² to check for stability

Fit	σ_L		$d\sigma/d\Omega$		$\sigma_T + \epsilon\sigma_L$		σ_T		σ_{LT}		$\sigma_{LT'}$		σ_{TT}		K_{D1}		P_Y		ρ_{LT}		$\rho_{LT'}$		$\chi^2_{\rm dof}$
	$\pi^0 p$	$\pi^+ n$	$\pi^0 p$	$\pi^+ n$	$\pi^0 p$	$\pi^+ n$	$\pi^0 p$	$\pi^+ n$	$\pi^0 p$	$\pi^+ n$	$\pi^0 p$	$\pi^+ n$	$\pi^0 p$	$\pi^+ n$	$\pi^0 p$	$\pi^+ n$	$\pi^0 p$	$\pi^+ n$	$\pi^0 p$	$\pi^+ n$	$\pi^0 p$	$\pi^+ n$	
$\mathfrak{F}_1$	–	9	65355	53229	870	418	87	88	1212	133	862	762	4400	251	4493	–	234	–	525	–	3300	10294	1.77
$\mathfrak{F}_2$	–	4	69472	55889	1081	619	65	78	1780	150	1225	822	4274	237	4518	–	325	–	590	–	3545	10629	1.69
$\mathfrak{F}_3$	–	8	66981	54979	568	388	84	95	1863	181	1201	437	3934	339	4296	–	686	–	687	–	3556	9377	1.81
$\mathfrak{F}_4$	–	22	63113	52616	562	378	153	107	1270	146	1198	1015	4385	218	5929	–	699	–	604	–	3548	11028	1.78
$\mathfrak{F}_5$	–	20	65724	53340	536	528	125	81	1507	219	1075	756	4134	230	5236	–	692	–	554	–	3580	11254	1.81
$\mathfrak{F}_6$	–	18	71982	58434	1075	501	29	68	1353	135	1600	1810	3935	291	5364	–	421	–	587	–	3932	11475	1.78

χ^2

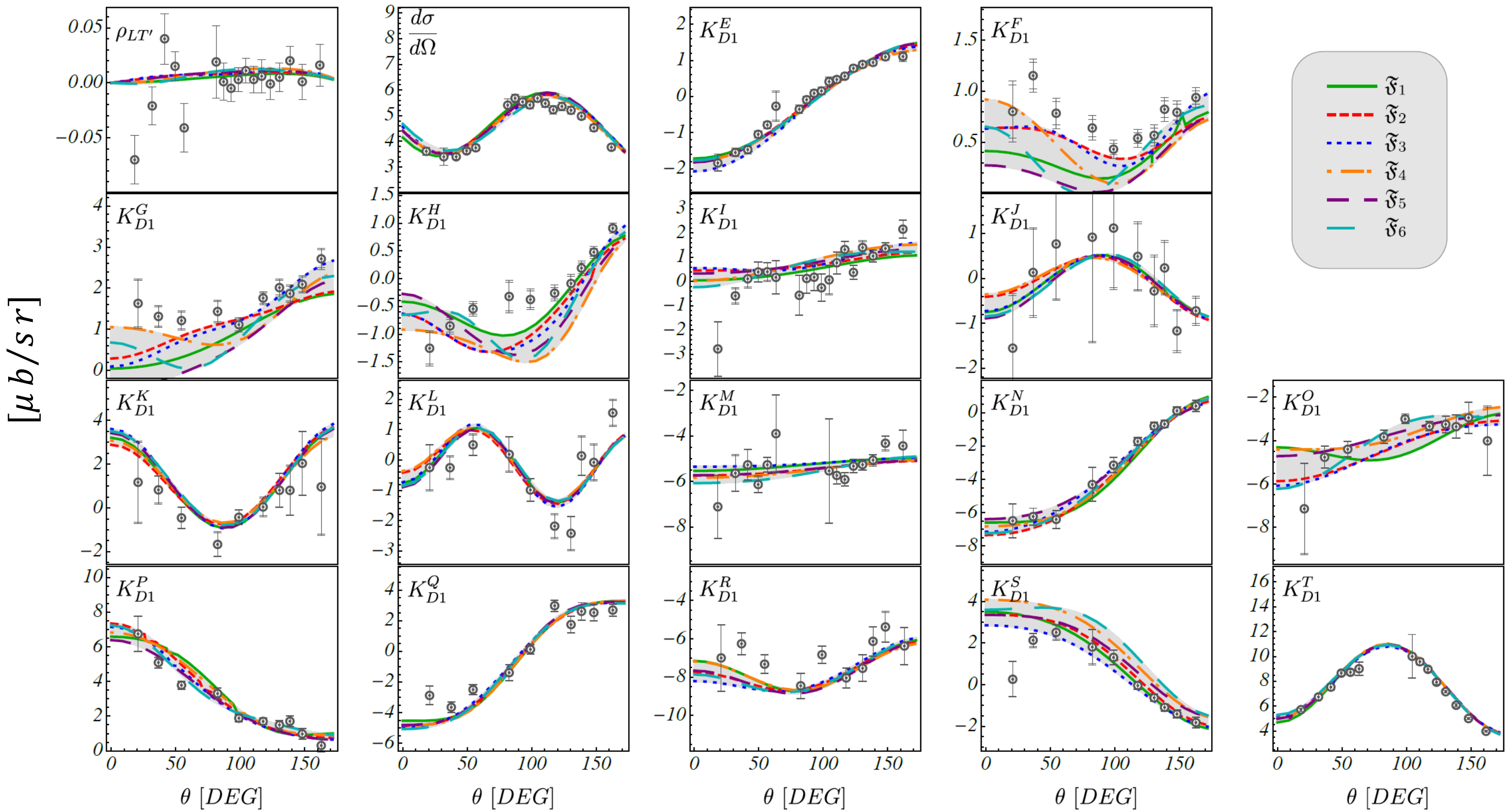
Structure functions $\pi^0 p$ (not fitted)



$Q^2 = 0.9$ Gev^2

Data: CLAS, Phys. Rev. C (2003) 0301012 [nucl-ex], Phys. Rev. Lett. (2002) 0110007 [hep-ex]

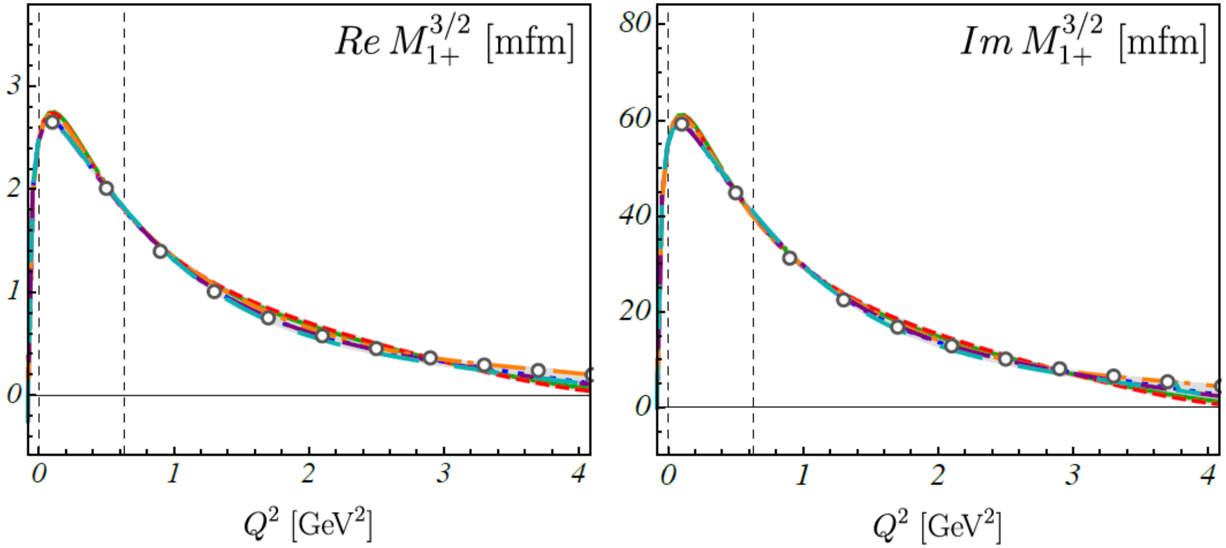
Description of Polarization Observables (πN)

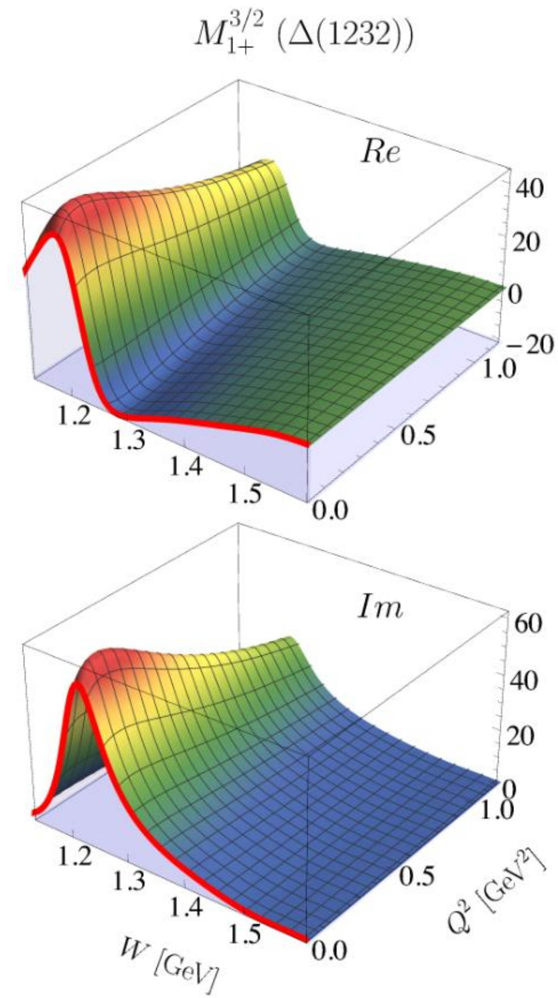


$\pi^0 p$, Q^2=1 GeV^2, W=1.23 GeV, ϕ=15^0

J. J. Kelly, Phys. Rev. Lett. 95 (2005).

Large Multipoles



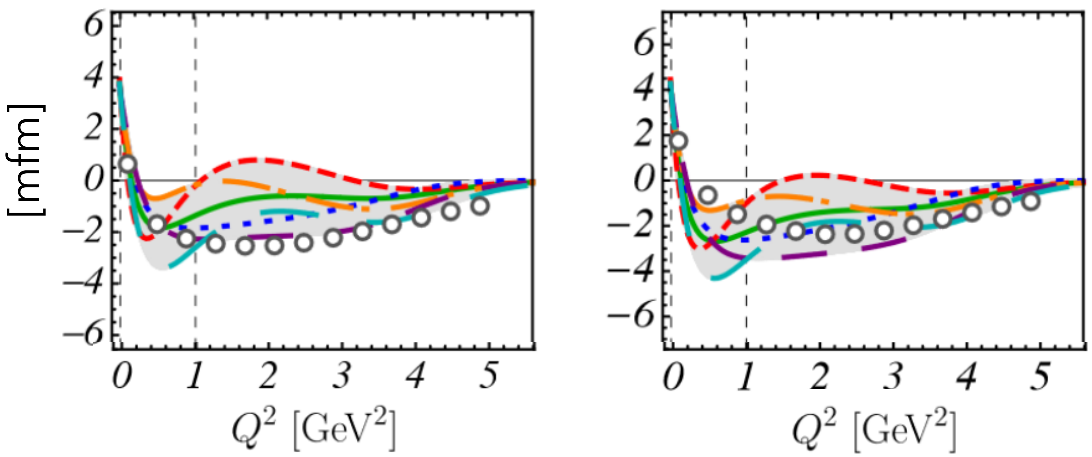


Fit strategies 1-6 together with MAID (open dots) for the magnetic multipole of the $\Delta(1232)$ Drechsel et al., EPJA (2007) 0710.0306 [nucl-th]

Prominent multipoles are well determined

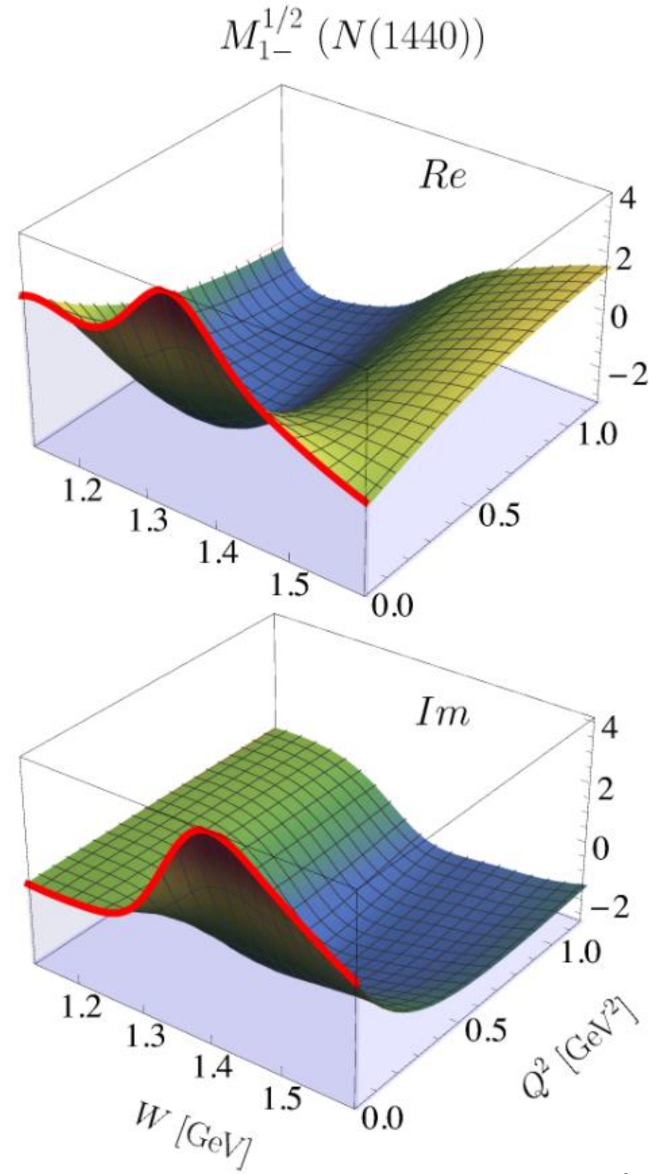
(Strategy 1 only)

Roper Multipole



(W=1.38 GeV fixed)

- Zero-transition (agrees with MAID)
- Extensive exploration of parameter space reveals ambiguities in PWA and reflects systematic uncertainties



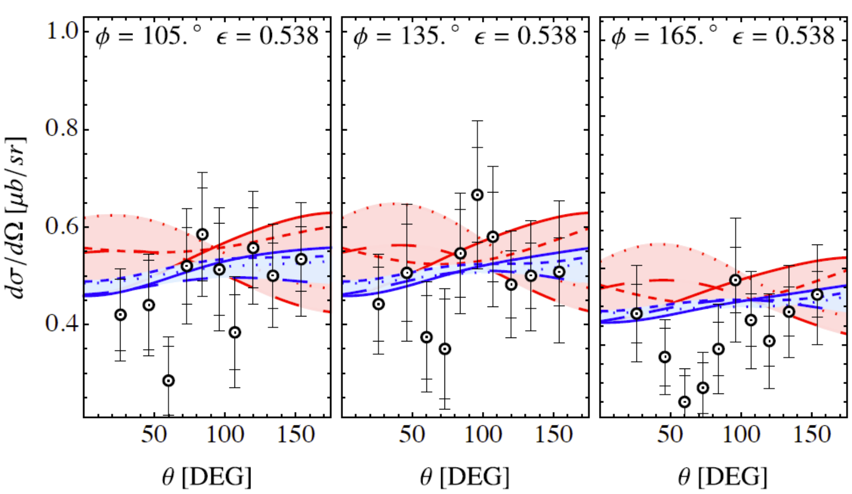
(Strategy 1 only)

η Electroproduction

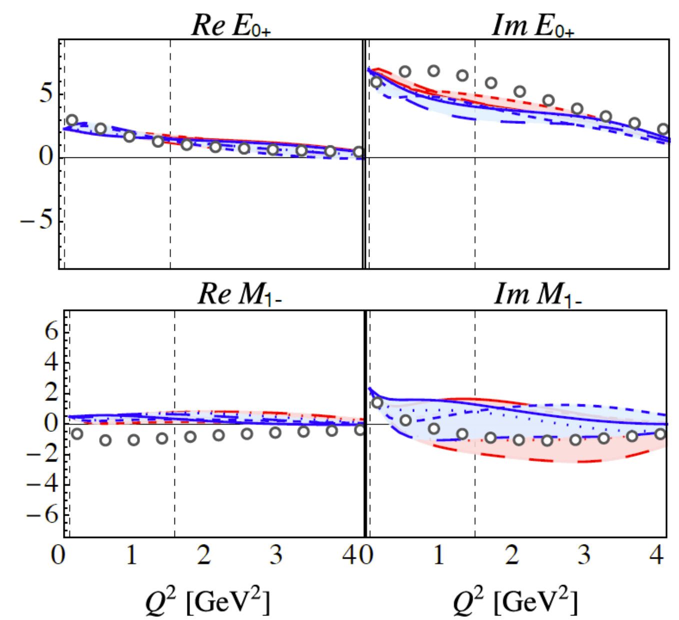
[M. Mai et al., PRC (2022)]

- $N^{\eta p}_{data} = 1,874$ (only $d\sigma/d\Omega$) (84,842 in total)
- kinematic range: $0 < Q^2 < 4$ GeV^2, $1.13 < W < 1.6$ GeV
- 8 different fit strategies: 4 with standard χ^2, 4 with weighted χ^2 to account for the smaller $N^{\eta p}_{data}$
  $\rightarrow$ better data description with weighted fit strategies:

Selected fit results: $\gamma^* p \rightarrow \eta p$ at $W = 1.5$ GeV, $Q^2 = 1.2$ GeV^2. Data: Denizli et al. (CLAS) PRC 76 (2007)



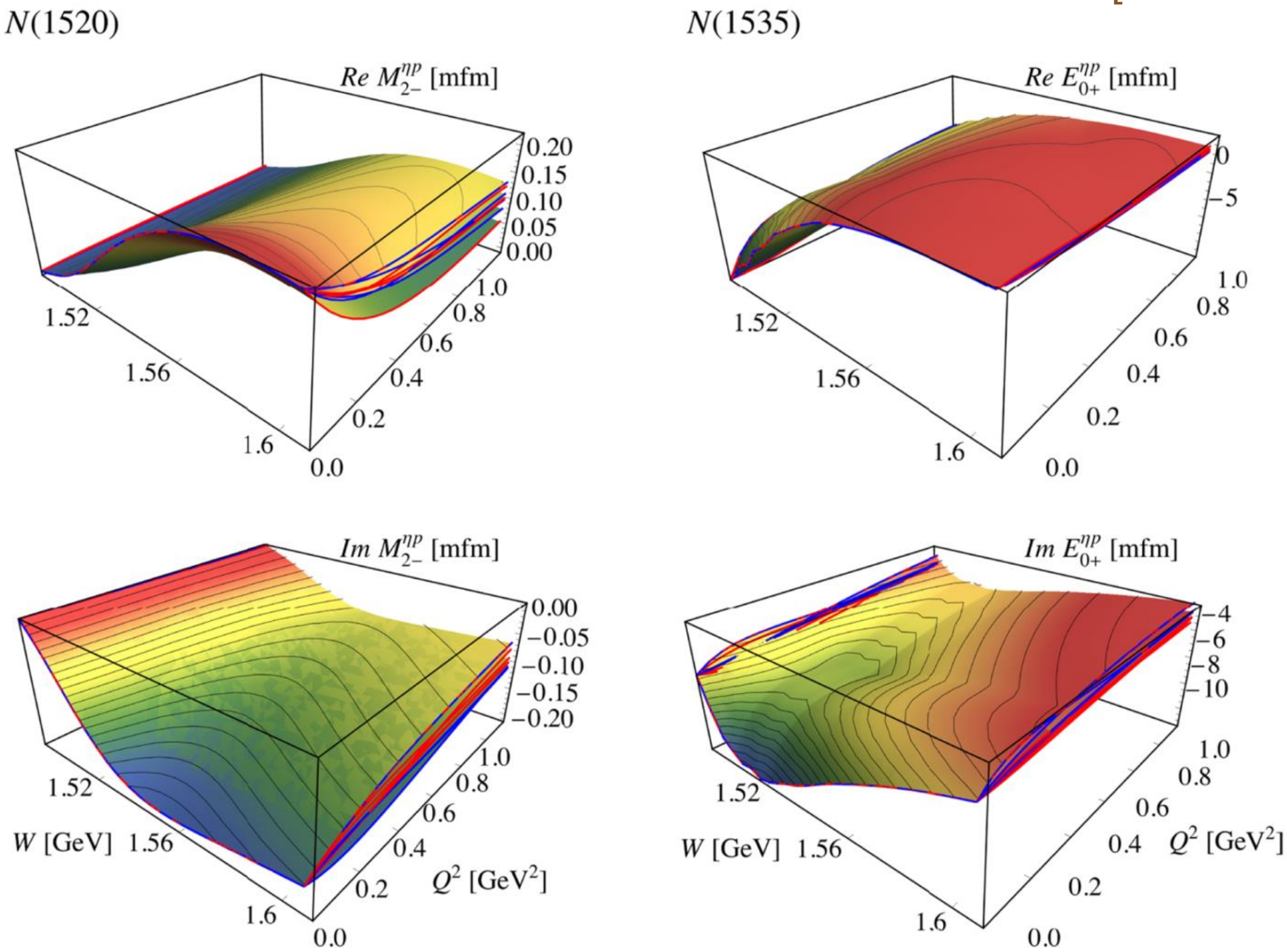
Selected multipoles at $W = 1535$ MeV



Dots: eta-MAID, arXiv:0110034

η Multipoles: Resonances disappear at high Q^2

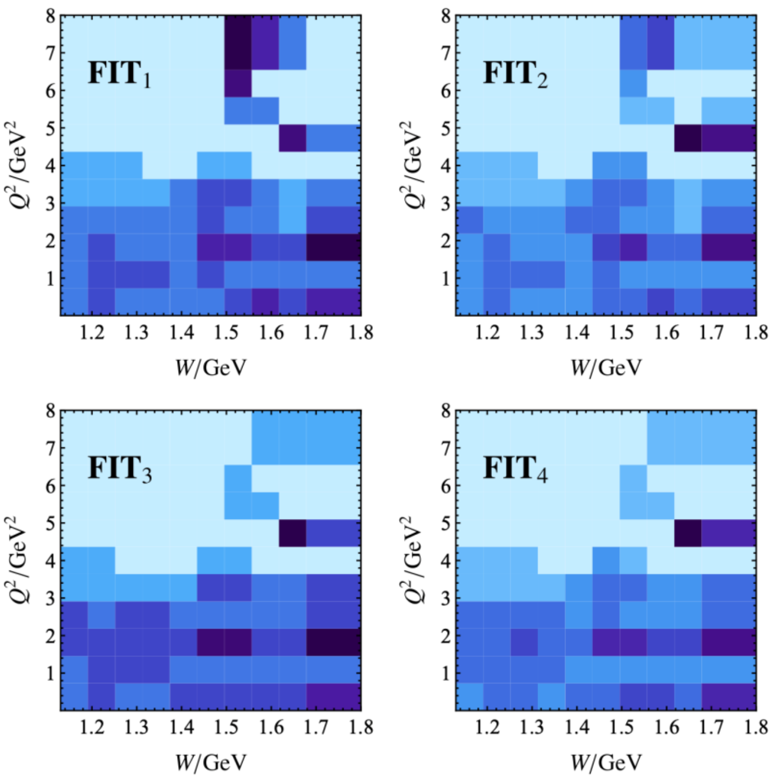
[M. Mai et al., PRC (2022)]

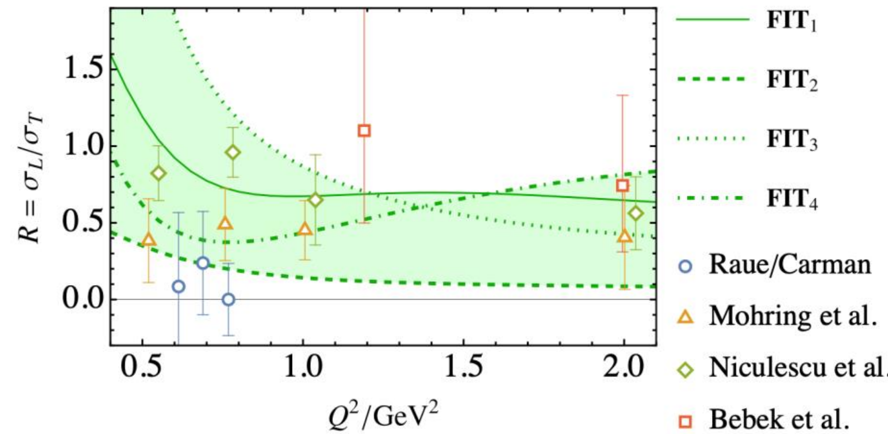


Kaon electroproduction

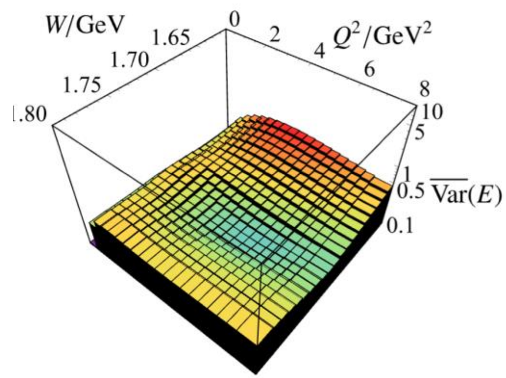
[M. Mai et al., EPJA 2023]

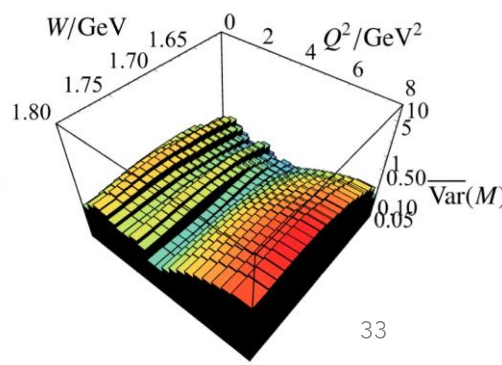
	$\chi^2_{\rm dof}$	$\chi^2_{\rm pp}(\pi^0 p)$	$\chi^2_{\rm pp}(\pi^+ n)$	$\chi^2_{\rm pp}(\eta p)$	$\chi^2_{\rm pp}(K^+\Lambda)$
FIT$_1$	1.42	1.40	1.47	1.49	0.70
FIT$_2$	1.35	1.38	1.35	1.40	0.58
	$\chi^2_{\rm wt,dof}$	$\chi^2_{\rm pp}(\pi^0 p)$	$\chi^2_{\rm pp}(\pi^+ n)$	$\chi^2_{\rm pp}(\eta p)$	$\chi^2_{\rm pp}(K^+\Lambda)$
FIT$_3$	1.12	1.44	1.61	1.08	0.33
FIT$_4$	1.06	1.42	1.44	1.09	0.32





$$\overline{\mathrm{Var}}(W,Q^2)(X) := \sum_{\ell\pm} \frac{\mathrm{Var}\{|\mathrm{X}_{\ell\pm,\mathrm{i}}|\}}{\mathrm{Mean}\{|\mathrm{X}_{\ell\pm,\mathrm{i}}|\}+\varepsilon}$$

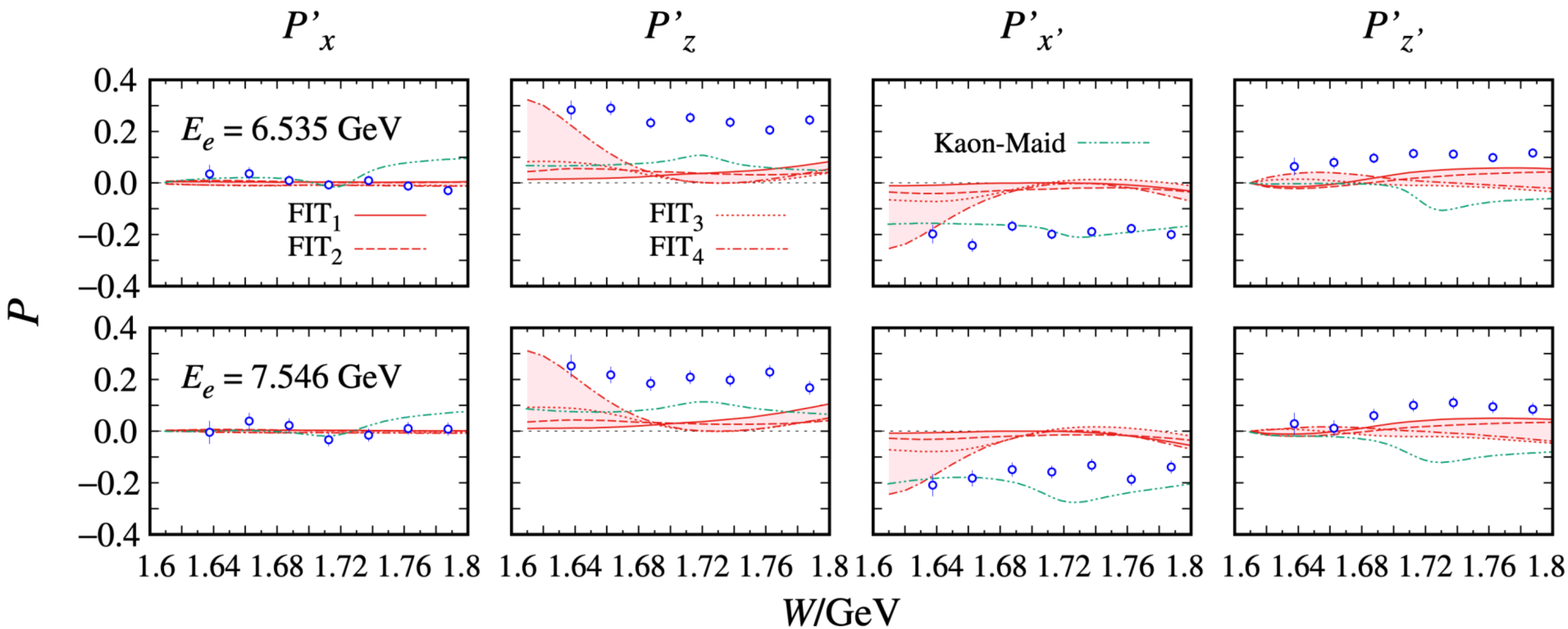




K Λ Challenges: Polarization measurements

- During analysis, first polarization data became available by CLAS:
- Predicted, not fitted

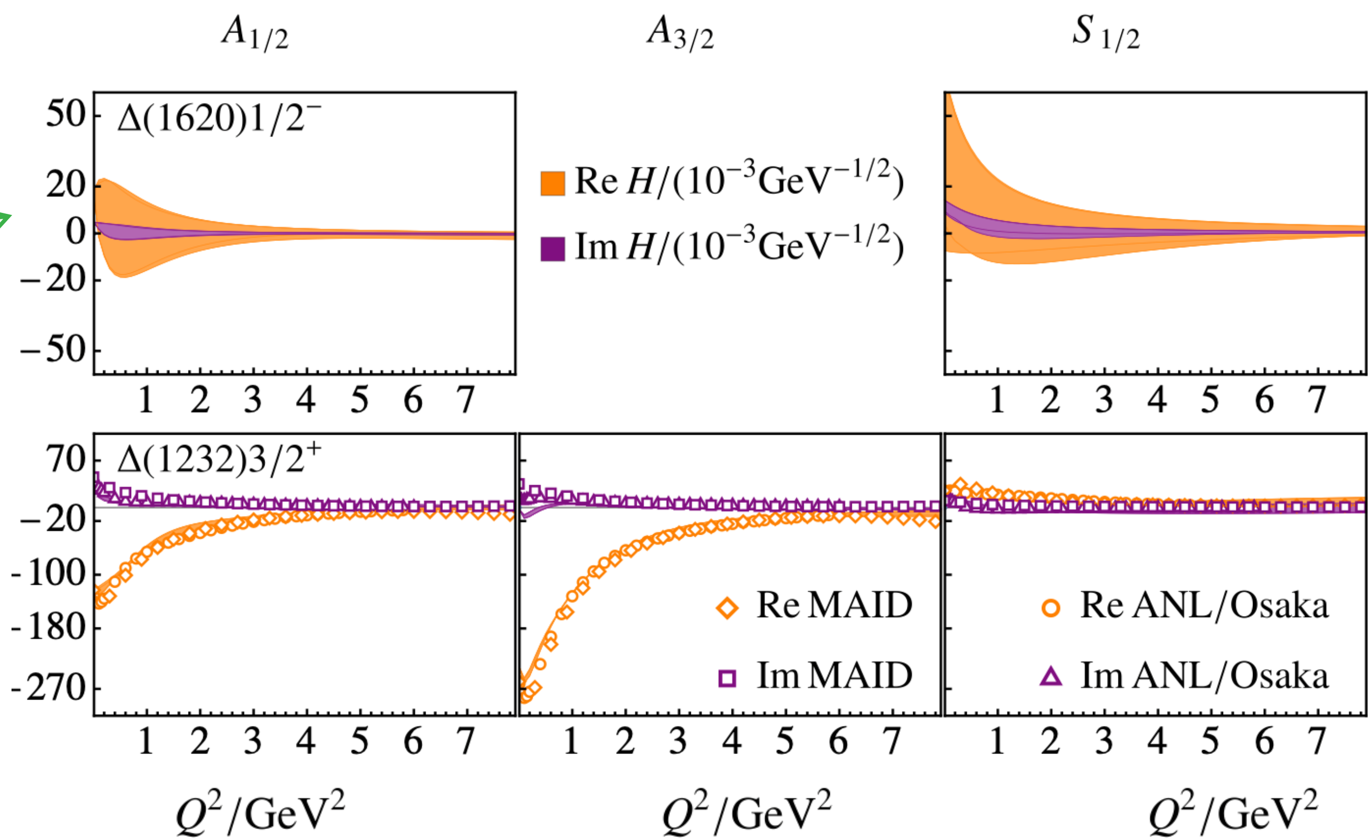
Beam-recoil polarization transfer [Carman et al. 2022]



Helicity Couplings

[Yu-Fei Wang et al., 2024]

- (Selected results)

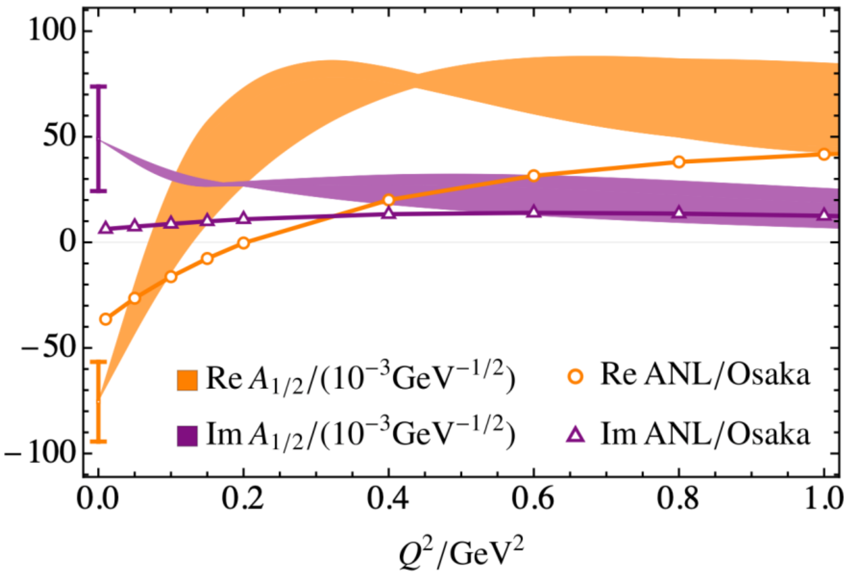


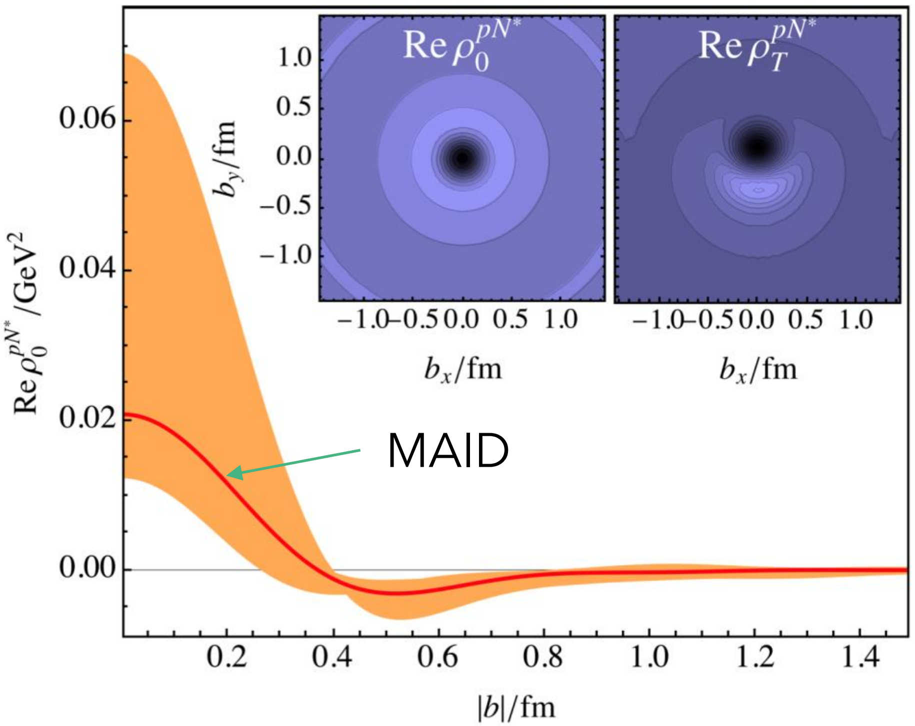
Compares qualitatively with [Mokeev et al., 2022]

Results for the Roper resonance

Charge density structure

[approx./ following Tiator et al., (2009)]





Summary

- Juelich-Bonn-Washington/JBW model: Phenomenology of excited baryons through coupled-channels, two- and three-body dynamics.
- Renewed effort to explore additional reaction channels in the last years:
  - $\gamma p \to K\Sigma$
  - $\pi N \to \omega N$
  - $\gamma^* p \to \pi N, \eta N, K\Lambda$ (Electroproduction)
- First global electroproduction analysis of different final states
- Extensive exploration of parameter space with good χ^2 (better than MAID) leads to *significant* variance of some multipoles.
- Many Transition Form factors at the pole exctracted for the first time.
- Many hyperon polarization data changed (α_- decay parameter of Λ changed)

[D.G. Ireland et al., PRL, 1904.07616]

- How to find a minimal resonance spectrum? Model selection.

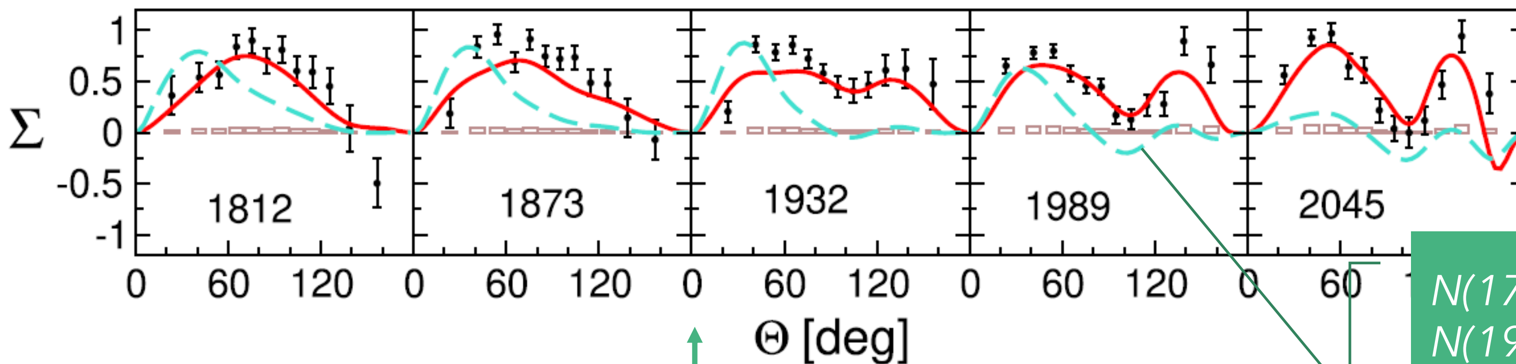
[J. Landay et al., PRD, 1810.00075]

- Data aspects: How to get solid statistical statements out of a heterogeneous data base dominated by systematic errors? [New experiments: Klong, Epecur,..]

(spare slides)

2022 Update in other reactions

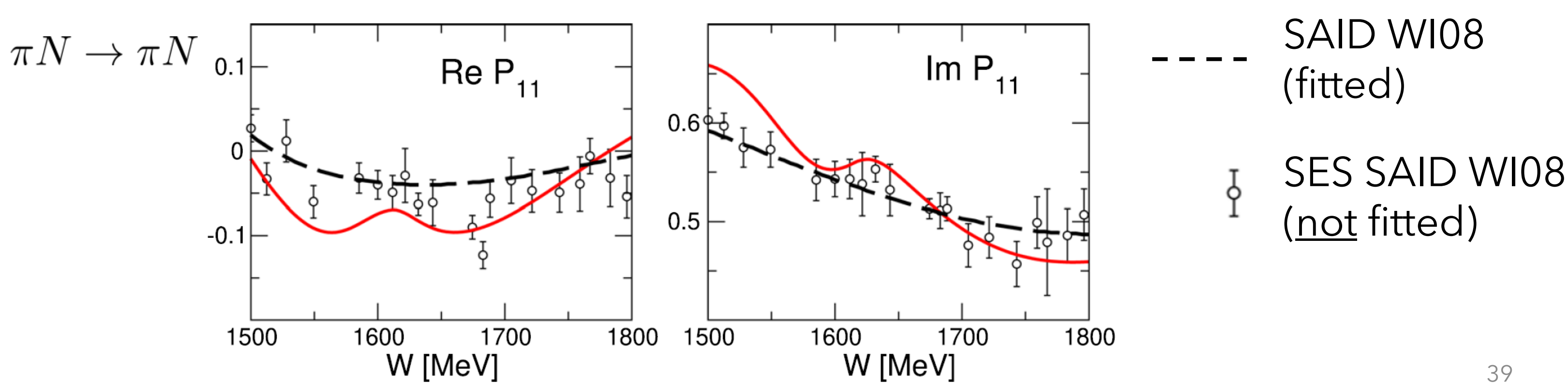
- Beam asymmetry in η photoproduction (different W)



$\eta' p$ threshold of $W = 1896$ MeV.

N(1720)3/2$^+$ off
N(1900)3/2$^+$ off

- N(1710)1/2+ returns with large ηN and KΛ branching ratios



Parametrization - Details

- Dependence on virtuality: Channel and resonance-wise:

$$\tilde{F}_\mu(Q^2) = \tilde{F}_D(Q^2)\, e^{-\beta^0_\mu Q^2/m^2}\, P^N(Q^2/m^2, \vec{\beta}_\mu)$$
$$\tilde{F}_i(Q^2) = \tilde{F}_D(Q^2)\, e^{-\delta^0_i Q^2/m^2}\, P^N(Q^2/m^2, \vec{\delta}_i)$$

- Factorization:

$$\alpha^{NP}_{\mu\gamma^*}(p, W, Q^2) = \tilde{F}_\mu(Q^2)\alpha^{NP}_{\mu\gamma}(p, W)$$
$$\gamma^c_{\gamma^*;i}(W, Q^2) = \tilde{F}_i(Q^2)\gamma^c_{\gamma;i}(W)$$