NANJING UNIVERSITY

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Emergence of a gluon mass

Dynamical mass generation in QCD

$$\mathcal{L}_{\rm QCD} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \bar{\psi}^i_f\left(i\gamma^\mu D_\mu - m_f\right)_{ij}\psi^j_f + \frac{1}{2\xi}(\partial^\mu A^a_\mu)^2 - \bar{c}^a\partial^\mu D^{ab}_\mu c^b$$

- At the level of the Lagrangian:
  - **Gluons** are massless;
  - **Quarks** have **current masses**, but **far smaller than the hadrons they constitute**
- Perturbation theory cannot generate mass at any finite order
- Vast majority of the observable mass is **generated by the nonperturbative QCD dynamics**.
- To study **dynamical mass generation**, we look at the behavior of the nonperturbative QCD Schwinger functions (propagators and vertices):

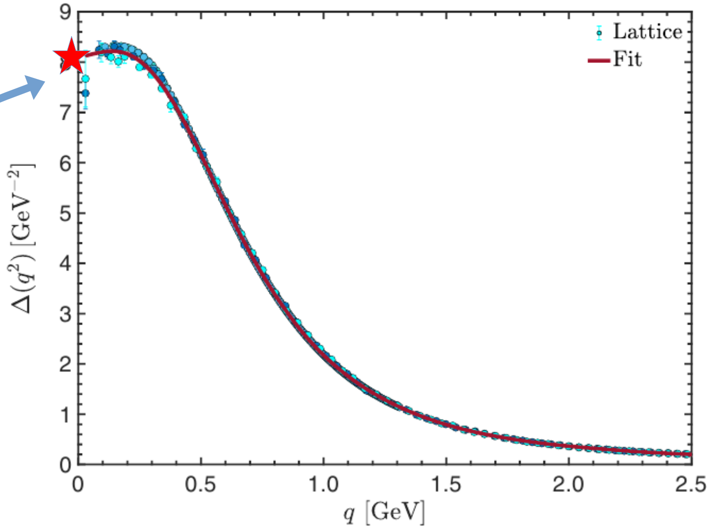
M. N. F. and J. Papavassiliou, Particles **6**, no.1, 312-363 (2023).
M. Ding, C. D. Roberts and S. M. Schmidt, Particles 6, 57-120 (2023).
J. Papavassiliou, Chin. Phys. C 46, no.11, 112001 (2022).
C. D. Roberts, Symmetry 12, no.9, 1468 (2020).

Mass generation leaves **distinctive signals in the infrared** momentum region of several Schwinger functions.

Gluon mass generation

Gluon self-interactions can generate a dynamical mass
J. M. Cornwall, Phys. Rev. D26, 1453 (1982)

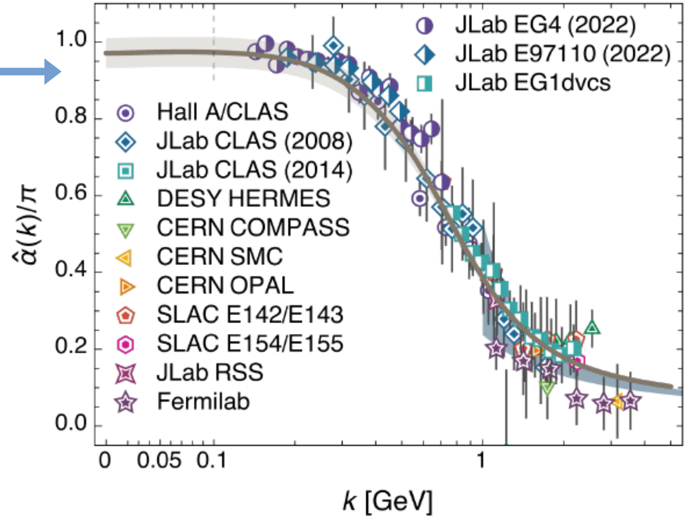
Lattice QCD: The gluon propagator saturates at the origin:
I. L. Bogolubsky, et al, Phys. Lett. B **676**, 69-73 (2009).
A. Cucchieri and T. Mendes, Phys. Rev. D **81**, 016005 (2010).
A. C. Aguilar, C. O. Ambrósio, F. De Soto, M. N. F., B. M. Oliveira, J. Papavassiliou and J. Rodríguez-Quintero, Phys. Rev. D **104**, no.5, 054028 (2021).
A. Ayala, et al, Phys. Rev. D **86**, 074512 (2012).
D. Binosi, C. D. Roberts and J. Rodriguez-Quintero, Phys. Rev. D **95**, no.11, 114009 (2017).
A. C. Aguilar, et al, Eur. Phys. J. C **80**, no.2, 154 (2020).



- Unequivocal signal of gluon mass generation.

- Many important implications for QCD. Crucially, allows the construction of an IR finite and process independent effective charge. (cf. Roberts', Pepe's and Lei Chang's talks).

M. N. F. and J. Papavassiliou, Particles **6**, no.1, 312-363 (2023).
M. Ding, C. D. Roberts and S. M. Schmidt, Particles **6**, 57-120 (2023).
C. D. Roberts, Symmetry **12**, no.9, 1468 (2020).



- All symmetries must be explicitly preserved.

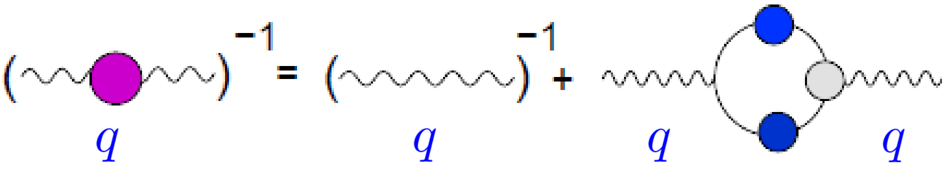
How can the gluon acquire a mass gap?

Schwinger mechanism

A gauge boson may acquire mass, dynamically and without violating gauge symmetry if its vacuum polarization function develops a pole at zero momentum transfer.

J. S. Schwinger, Phys. Rev. 125, 397 (1962); Phys. Rev. 128, 2425 (1962)

Dyson-Schwinger equation for gauge boson propagator



$$\Delta^{-1}(q^2) = q^2[1 + \Pi(q^2)]$$

If, for some reason

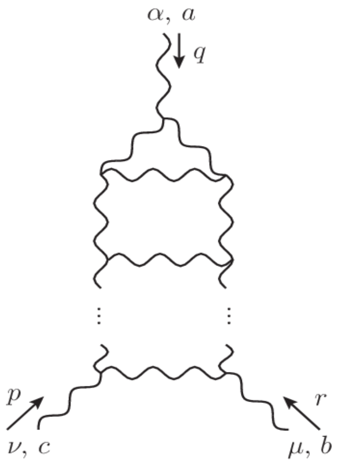
$$\lim_{q\to 0} \Pi(q^2) = \frac{c}{q^2}\,, \qquad c > 0$$

$$\Delta^{-1}(0) = c > 0$$

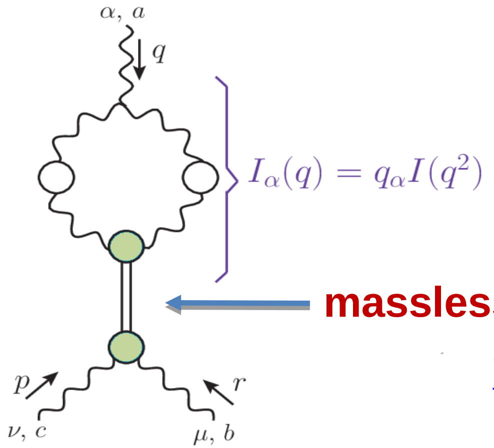
But how can the vacuum polarization acquire such a pole?

Massless bound state formalism

If the interaction is sufficiently strong $\longrightarrow$ formation of **massless bound states**



$q \to 0$



Longitudinally coupled

massless colored excitation

$$\frac{i\delta^{ab}}{q^2}$$

Vertices of the theory acquire **longitudinally coupled poles at zero gluon momentum, e.g.:**

$$\mathbb{\Gamma}_{\alpha\mu\nu}(q,r,k) = \underbrace{\Gamma_{\alpha\mu\nu}(q,r,k)}_{\text{pole-free}} + \frac{q_\alpha}{q^2} g_{\mu\nu} 2(q\cdot r)\boxed{\mathbb{C}(r^2)} + \dots$$

Residue functions

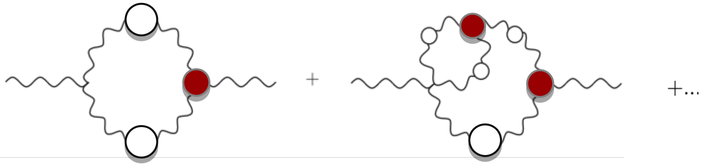
E. Eichten and F. Feinberg, Phys. Rev. D **10**, 3254-3279 (1974).
J. Smit, Phys. Rev. D **10**, 2473 (1974).

A. C. Aguilar, D. Ibanez, V. Mathieu, and J. Papavassiliou, Phys. Rev. D **85**, 014018 (2012).
G. Eichmann, J. M. Pawlowski and J. M. Silva, Phys. Rev. D **104**, no.11, 114016 (2021).

Massless bound state formalism

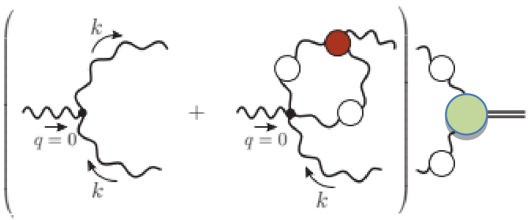
Massless poles in the three-gluon vertex lead to pole in the gluon vacuum polarization:

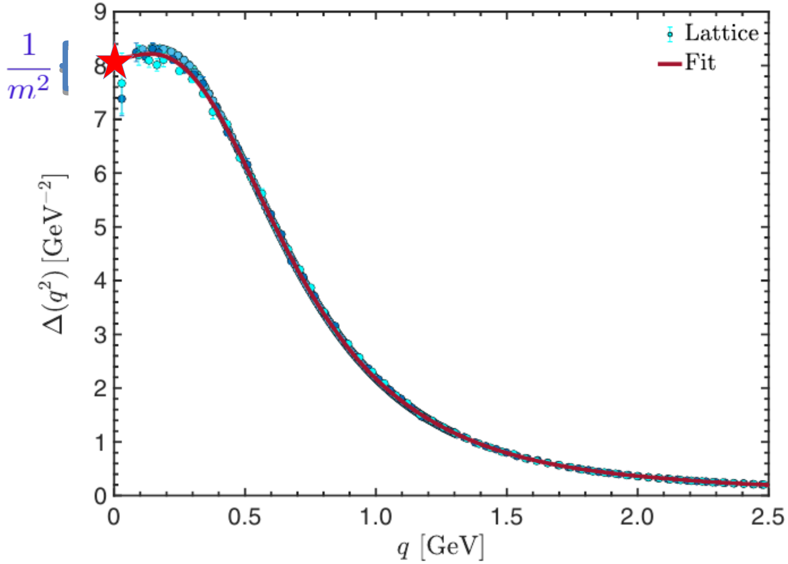
$$\Delta^{-1}(q^2) = q^2 +$$



$q \to 0$

$$m^2 =$$





A. C. Aguilar, D. Ibanez, V. Mathieu and J. Papavassiliou, Phys. Rev. D 85, 014018 (2012).
D. Binosi, D. Ibanez and J. Papavassiliou, Phys. Rev. D 86, 085033 (2012).

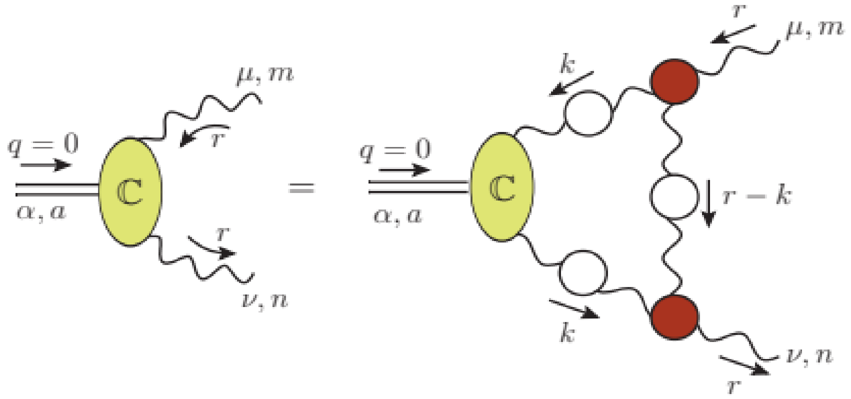
Bethe-Salpeter equation

The formation of massless bound state is **dynamical and governed by a Bethe-Salpeter equation.**

Recalling:

$$\mathrm{I\Gamma}_{\alpha\mu\nu}(q,r,k) = \Gamma_{\alpha\mu\nu}(q,r,k) + \frac{q_\alpha}{q^2} g_{\mu\nu} 2(q\cdot r)\mathbb{C}(r^2) + \dots$$

The function $\mathbb{C}(r^2)$ satisfies the equation



BS amplitude

A. C. Aguilar, D. Ibanez, V. Mathieu and J. Papavassiliou, Phys. Rev. D 85, 014018 (2012).
A. C. Aguilar, M. N. F. and J. Papavassiliou, Phys. Rev. D 105, no.1, 014030 (2022).

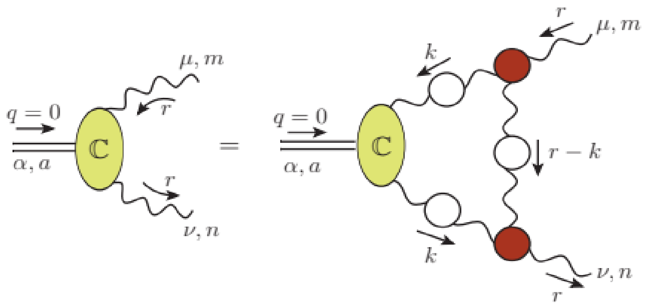
Bethe-Salpeter equation

The Bethe-Salpeter equation admits **nontrivial solutions compatible with lattice ingredients** for the:

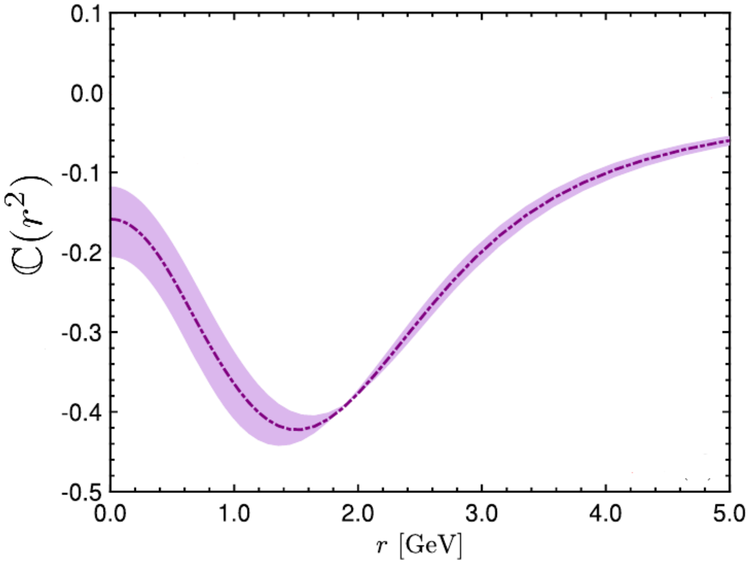
- Propagator;
- Vertex;
- and, value of the coupling

$$\alpha_s \approx 0.3 \ \ @ \ \ \mu = 4.3 \text{ GeV}$$

A. C. Aguilar, D. Ibanez, V. Mathieu and J. Papavassiliou, Phys. Rev. D 85, 014018 (2012).
D. Binosi and J. Papavassiliou, Phys. Rev. D 97, no.5, 054029 (2018).
A. C. Aguilar, D. Binosi, C. T. Figueiredo and J. Papavassiliou, Eur. Phys. J. C 78, no.3, 181 (2018).



BS amplitude



A. C. Aguilar, M. N. F. and J. Papavassiliou, Phys. Rev. D 105, no.1, 014030 (2022).

Schwinger mechanism poles in lattice results?

Now, the **lattice can also compute the three-gluon vertex. Can we see longitudinal poles in it?**

Unfortunately, no!

The Schwinger mechanism **poles are longitudinally coupled**

$$\mathbb{\Gamma}_{\alpha\mu\nu}(q,r,k) = \underbrace{\Gamma_{\alpha\mu\nu}(q,r,k)}_{\text{pole-free}} + \underbrace{\frac{q_\alpha}{q^2} g_{\mu\nu} 2(q\cdot r)\mathbb{C}(r^2)}_{\text{massless pole}} + \dots$$

But **lattice simulations only access transverse tensor structures.**

→ **Lattice extracts the pole-free part of the vertex.**

A. Athenodorou, D. Binosi, P. Boucaud, F. De Soto, J. Papavassiliou, J. Rodriguez-Quintero and S. Zafeiropoulos, Phys. Lett. B 761, 444-449 (2016).
A. C. Aguilar, F. De Soto, M. N. F., J. Papavassiliou and J. Rodríguez-Quintero, Phys. Lett. B 818, 136352 (2021).

A smoking gun signal?

Question:

Is there a smoking-gun signal of the massless bound state poles, which can be tested with lattice inputs?

Answer:

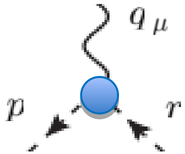
Yes, the **displacement of the Ward identities** satisfied by the vertices.

- The key observation is that the **Schwinger mechanism preserves the gauge symmetry**.
- **Gauge symmetry** relates 2- and 3- point functions through the **Ward identities, which are sensitive to longitudinal poles**.

A. C. Aguilar, M. N. F. and J. Papavassliou, Phys. Rev. D 105, no.1, 014030 (2022).
A. C. Aguilar, F. De Soto, M. N. F., J. Papavassiliou, F. Pinto-Gómez, C. D. Roberts and J. Rodríguez-Quintero, Phys. Lett. B **841**, 137906 (2023).

A toy example: scalar QED

Schwinger mechanism **off**



Ward-Takahashi identity

$$\underbrace{q^\mu \Gamma_\mu(q,r,p)}_{\text{pole-free}} = D^{-1}(p^2) - D^{-1}(r^2)$$

$$\Big\downarrow \begin{matrix} q \to 0 \\ p \to -r \end{matrix} \quad \text{Taylor expansion}$$

Textbook Ward identity

$$\Gamma_\mu(0,r,-r) = \frac{\partial D^{-1}(r^2)}{\partial r^\mu}$$

Schwinger mechanism **on**

$$\mathrm{I\!\Gamma}_\mu(q,r,p) = \underbrace{\Gamma_\mu(q,r,p)}_{\text{pole-free}} + \frac{q_\mu}{q^2} C(q,r,p)$$

The Ward-Takahashi identity does not change

$$\begin{aligned} q^\mu \mathrm{I\!\Gamma}_\mu(q,r,p) &= q^\mu \Gamma_\mu(q,r,p) + C(q,r,p) \\ &= D^{-1}(p^2) - D^{-1}(r^2) \end{aligned}$$

$$q \to 0 \ \Big\downarrow \quad \text{Taylor expansion}$$

Displaced Ward identity

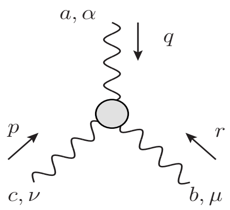
$$\underbrace{\Gamma_\mu(0,r,-r)}_{\text{pole-free}} = \frac{\partial D^{-1}(r^2)}{\partial r^\mu} - 2r_\mu \underbrace{\left[\frac{\partial C(q,r,p)}{\partial p^2}\right]_{q=0}}_{\mathbb{C}(r^2)}$$

Displacement = BS amplitude

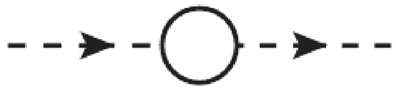
Ward identity displacement in QCD

The **same idea applies to QCD**, just more complicated due to **non-Abelian Slavnov-Taylor identities**:

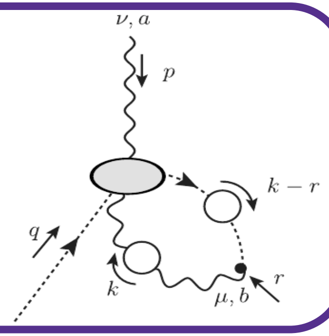
$$q^{\alpha}\mathrm{I}\!\Gamma_{\alpha\mu\nu}(q,r,p) = F(q^2)[\Delta^{-1}(p^2)P^{\sigma}_{\nu}(p)H_{\sigma\mu}(p,q,r) - \Delta^{-1}(r^2)P^{\sigma}_{\mu}(r)H_{\sigma\nu}(r,q,p)]$$



$:= \mathrm{I}\!\Gamma^{abc}_{\alpha\mu\nu}(q,r,p)$



$H_{\mu\nu}(r,q,p) :=$



Then, assume the three-gluon vertex has a massless bound state pole:

$$\mathrm{I}\!\Gamma_{\alpha\mu\nu}(q,r,k) = \Gamma_{\alpha\mu\nu}(q,r,k) + \frac{q_{\alpha}}{q^2}g_{\mu\nu}2(q\cdot r)\mathbb{C}(r^2) + \dots$$

And expand around $q = 0$

Ward identity displacement in QCD

$$q^\alpha \mathrm{I\Gamma}_{\alpha\mu\nu}(q,r,p) = F(q^2)[\Delta^{-1}(p^2)P_\nu^\sigma(p)H_{\sigma\mu}(p,q,r) - \Delta^{-1}(r^2)P_\mu^\sigma(r)H_{\sigma\nu}(r,q,p)]$$

$q \to 0$ Isolate classical tensor structure

Ward identity

$$L_{\mathrm{sg}}(r^2) = F(0)\left[\frac{\mathcal{W}(r^2)}{r^2}\Delta^{-1}(r^2) + \frac{\partial\Delta^{-1}(r^2)}{\partial r^2}\right] + \boxed{\mathbb{C}(r^2)}$$

Displacement = BS amplitude

- **Ingredients can be computed with lattice simulations.**
- **Combine ingredients and determine if there is a nontrivial displacement.**

A. C. Aguilar, M. N. F. and J. Papavassliou, Phys. Rev. D 105, no.1, 014030 (2022).
A. C. Aguilar, F. De Soto, M. N. F., J. Papavassiliou, F. Pinto-Gómez, C. D. Roberts and J. Rodríguez-Quintero, Phys. Lett. B **841**, 137906 (2023).

Ward identity displacement in QCD

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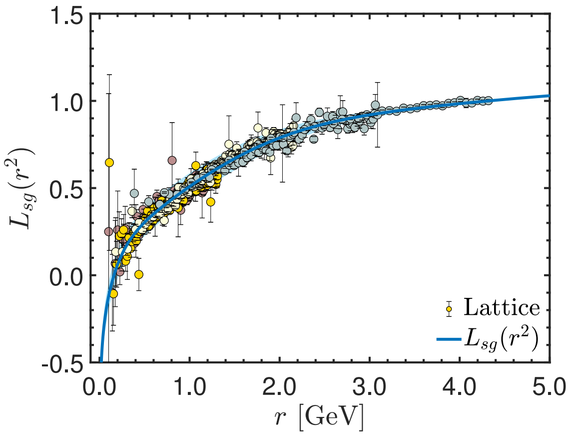

Soft-gluon form factor of the three-gluon vertex

$$P_{\mu}^{\mu'}(r)P_{\nu}^{\nu'}(r)\mathrm{I\Gamma}_{\alpha\mu'\nu'}(0,r,-r) = 2L_{\rm sg}(r^2)r_{\alpha}P_{\mu\nu}(r)$$

$$P_{\mu\nu}(q) := g_{\mu\nu} - q_{\mu}q_{\nu}/q^2$$

A. C. Aguilar, C. O. Ambrosio, F. De Soto, M.N. F., B. M. Oliveira, J. Papavassiliou and J. Rodriguez-Quintero, Phys. Rev. D 104 no.5, 054028, (2021).

Ward identity displacement in QCD

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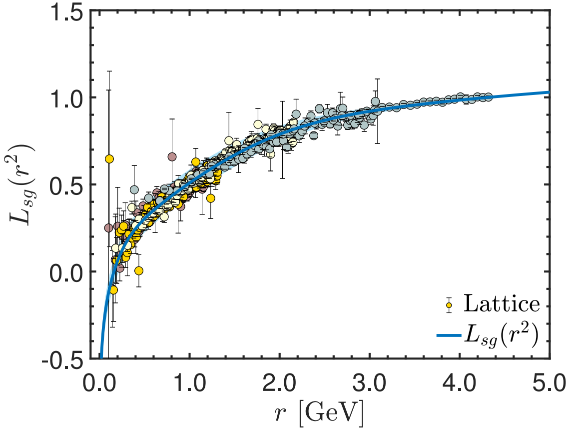

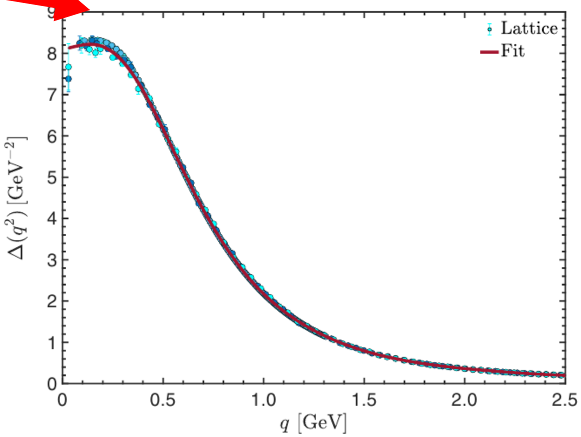

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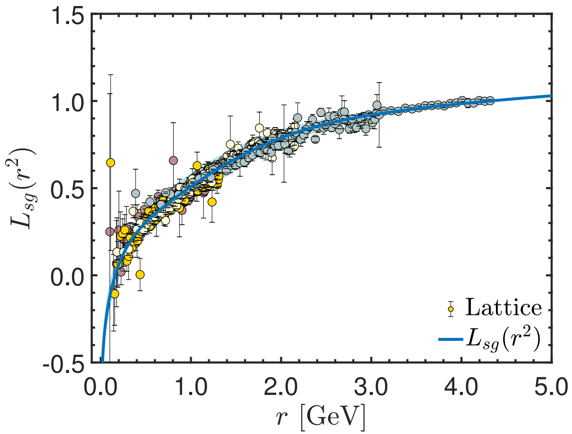

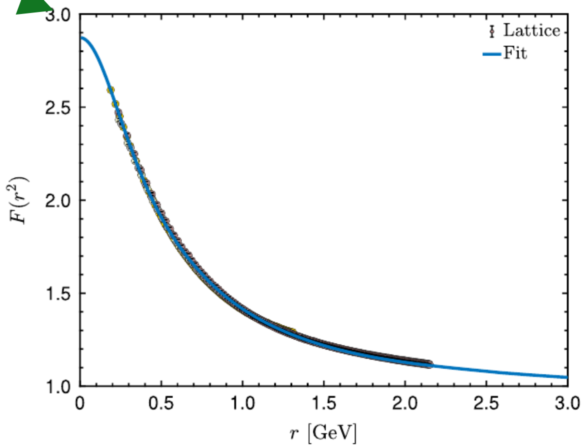

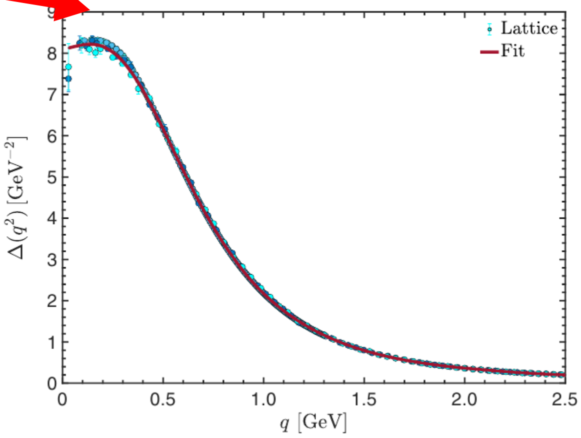

Ward identity displacement in QCD

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★ **Only one ingredient not yet determined directly by lattice simulations.**

Ward identity displacement in QCD

$$q^{\alpha}\mathrm{I\Gamma}_{\alpha\mu\nu}(q,r,p) = F(q^2)[\Delta^{-1}(p^2)P_{\nu}^{\sigma}(p)H_{\sigma\mu}(p,q,r) - \Delta^{-1}(r^2)P_{\mu}^{\sigma}(r)H_{\sigma\nu}(r,q,p)]$$

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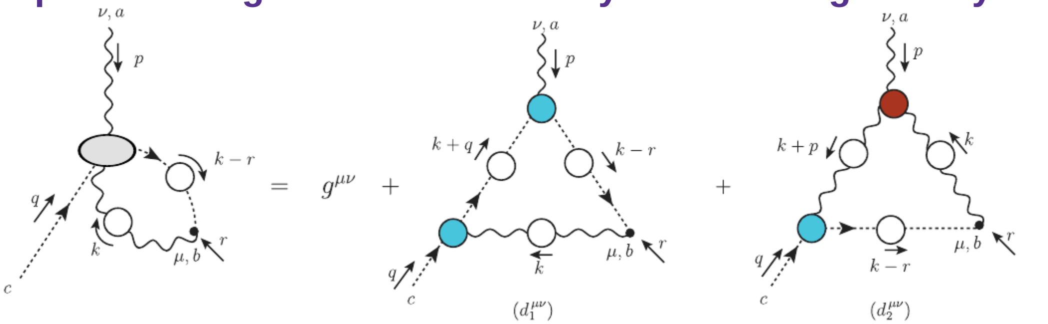
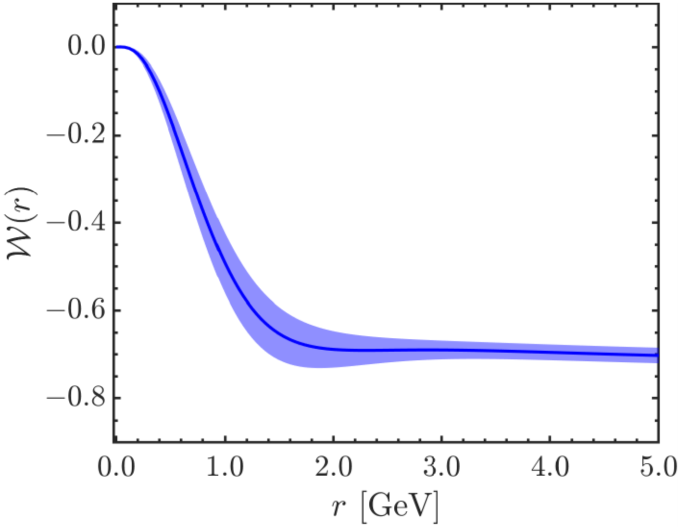
Partial derivative of the ghost-gluon kernel
Computed through a lattice driven Dyson-Schwinger analysis





A. C. Aguilar, F. De Soto, M. N. F., J. Papavassiliou, F. Pinto-Gómez, C. D. Roberts and J. Rodríguez-Quintero, Phys. Lett. B **841**, 137906 (2023).

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$$L_{\rm sg}(r^2) = F(0)\left[\frac{\mathcal{W}(r^2)}{r^2}\Delta^{-1}(r^2) + \frac{\partial \Delta^{-1}(r^2)}{\partial r^2}\right] + \mathbb{C}(r^2)$$

Displacement = BS amplitude

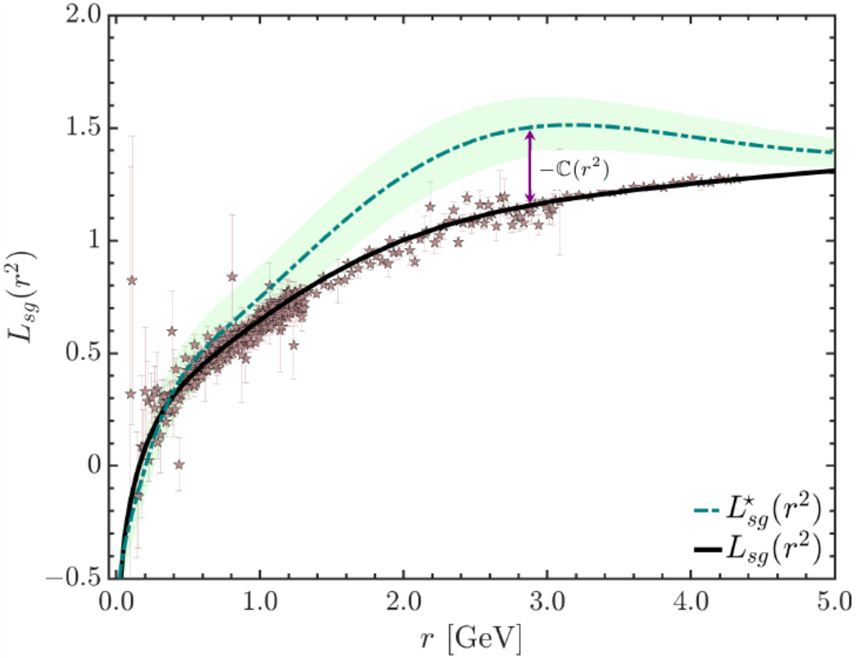
★ **Combine ingredients and determine if there is a nontrivial displacement.**

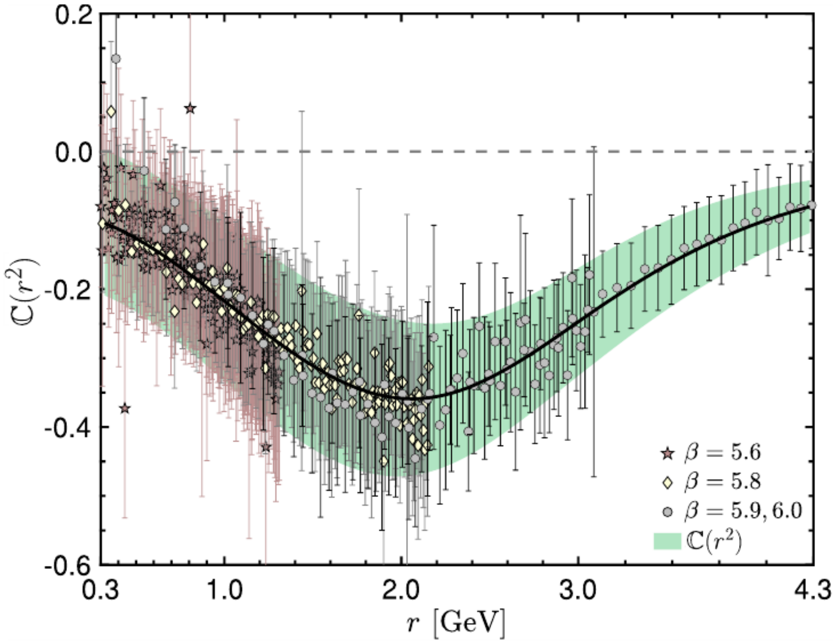
Results for $\mathbb{C}(r^2)$

With $\mathcal{W}(r^2)$ at hand, we can compute $L^{\star}_{sg}(r^2)$ and determine $\mathbb{C}(r^2)$ as a **WI displacement**

$$L^{\star}_{sg}(r^2) = F(0)\left[\frac{\mathcal{W}(r^2)}{r^2}\Delta^{-1}(r^2) + \frac{d\Delta^{-1}(r^2)}{dr^2}\right]$$

$$\mathbb{C}(r^2) = L_{sg}(r^2) - L^{\star}_{sg}(r^2)$$





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Results for $\mathbb{C}(r^2)$

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$$L^{\star}_{sg}(r^2) = F(0)\left[\frac{\mathcal{W}(r^2)}{r^2}\Delta^{-1}(r^2) + \frac{d\Delta^{-1}(r^2)}{dr^2}\right]$$

$$\mathbb{C}(r^2) = L_{sg}(r^2) - L^{\star}_{sg}(r^2)$$

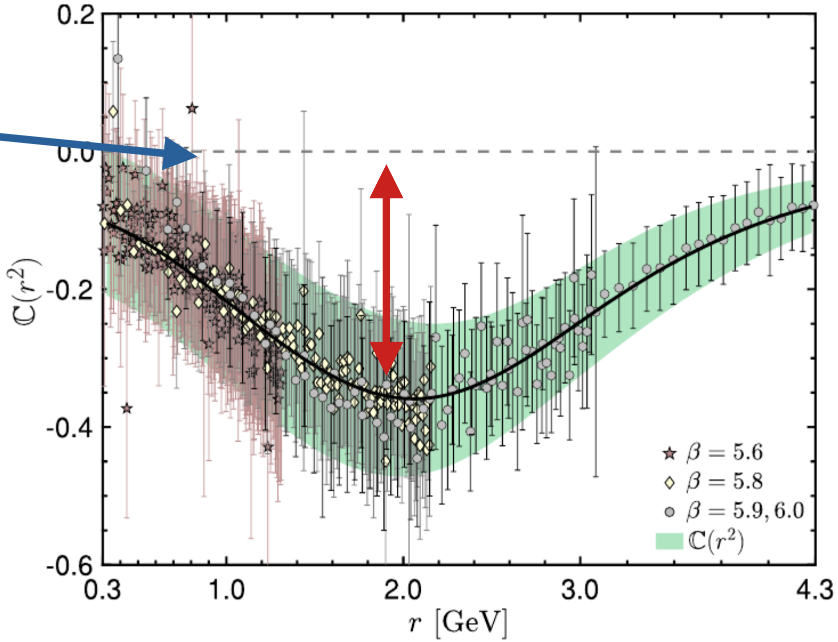
- $\mathbb{C}(r^2)$ obtained is clearly nonzero.
- Define the **null hypothesis**,

$$\mathbb{C}(r^2) = \mathbb{C}_0 := 0$$

***p*-value of null hypothesis is tiny:**

$$P_{\mathbb{C}_0} = \int_{\chi^2=2\,630}^{\infty} \chi^2_{\rm PDF}(515, x)dx = 7.3 \times 10^{-280}$$

- Even if the errors were doubled, the null hypothesis would still be discarded at the 5σ level.



A. C. Aguilar, F. De Soto, M. N. F., J. Papavassiliou, F. Pinto-Gómez, C. D. Roberts and J. Rodríguez-Quintero, Phys. Lett. B **841**, 137906 (2023).

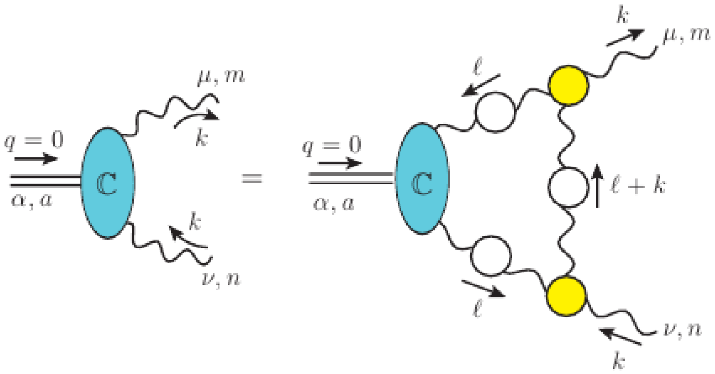
Results for $\mathbb{C}(r^2)$

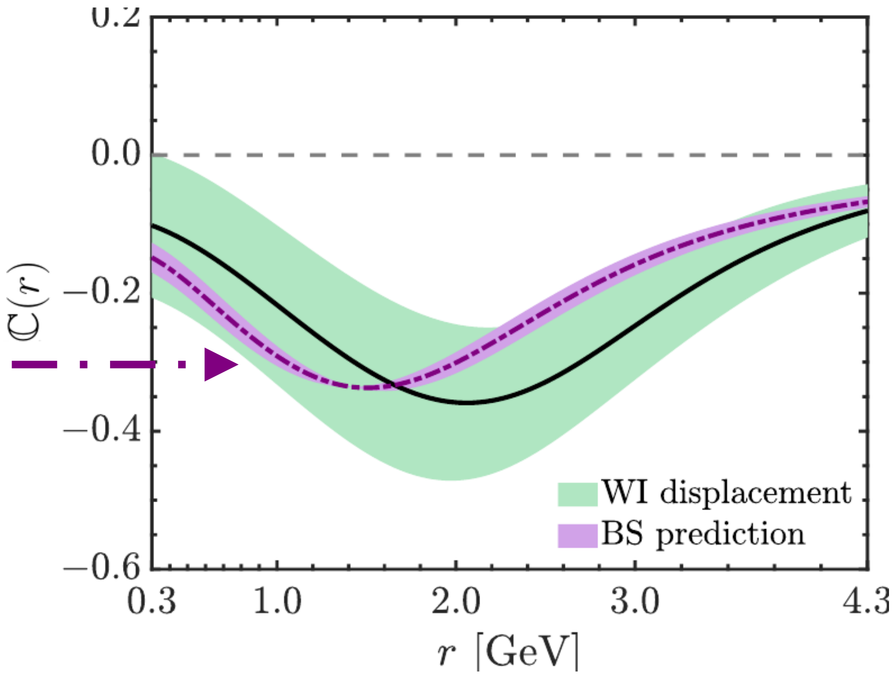
With $\mathcal{W}(r^2)$ at hand, we can compute $L^{\star}_{sg}(r^2)$ and determine $\mathbb{C}(r^2)$ as a **WI displacement**

$$L^{\star}_{sg}(r^2) = F(0)\left[\frac{\mathcal{W}(r^2)}{r^2}\Delta^{-1}(r^2) + \frac{d\Delta^{-1}(r^2)}{dr^2}\right]$$

$$\mathbb{C}(r^2) = L_{sg}(r^2) - L^{\star}_{sg}(r^2)$$

- Moreover, we find good agreement with the BSE prediction.



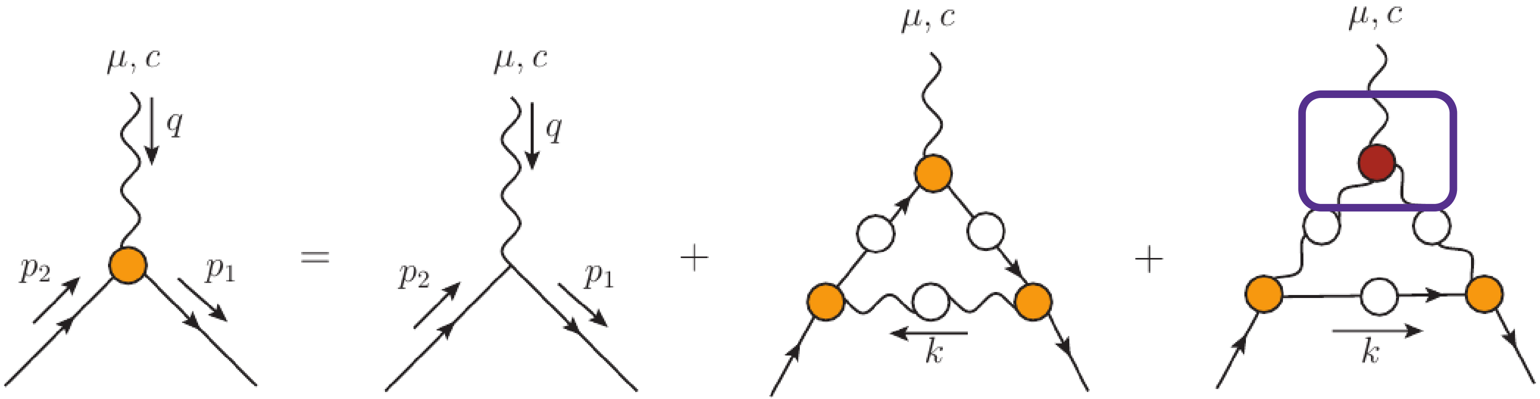


A. C. Aguilar, M. N. F. and J. Papavassiliou, Phys. Rev. D 105, no.1, 014030 (2022).
M. N. F. and J. Papavassiliou, Particles **6**, no.1, 312-363 (2023).

Poles in other vertices: including dynamical quarks

The Dyson-Schwinger equations couple vertices of different species and number of external legs.

- If a **longitudinally coupled pole** is generated **in the three-gluon vertex**, it tends to **spread out to other vertices as well**.
- In particular, the **quark-gluon vertex picks up a longitudinally coupled pole**:



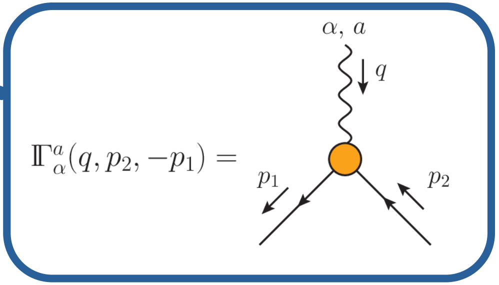
This allows additional tests of the Schwinger mechanism, and studying the role of dynamical quarks.

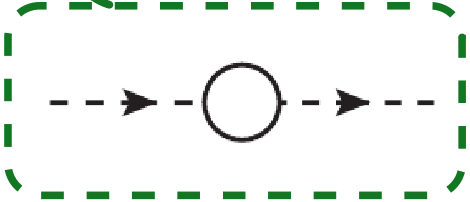
A. C. Aguilar, M. N. F., D. Ibañez and J. Papavassiliou, Eur. Phys. J. C 83, no.10, 967 (2023).

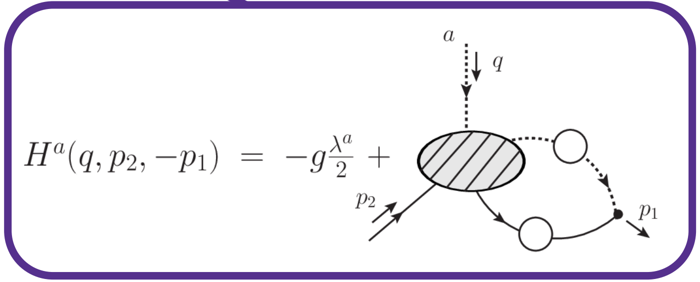
Ward identity displacement of the quark-gluon vertex

The same idea of Ward identity displacement applies to the quark-gluon vertex. We start with the STI

$$q^{\alpha}\mathrm{I\Gamma}_{\alpha}(q,p_2,-p_1) = F(q^2)[S^{-1}(p_1)H(q,p_2,-p_1) - \overline{H}(-q,p_1,-p_2)\,S^{-1}(p_2)]$$







Again, assume that the vertex has a massless bound state pole:

$$\mathrm{I\Gamma}_{\alpha}(q,p_2,-p_1) = \Gamma_{\alpha}(q,p_2,-p_1) + \frac{q_\alpha}{q^2}\not{q}\boxed{Q_3(p_2^2) +} \ldots$$

BS amplitude

And expand around *q* = *0*

Ward identity displacement of the quark-gluon vertex

$$q^{\alpha}\mathrm{I\Gamma}_{\alpha}(q,p_2,-p_1) = F(q^2)[S^{-1}(p_1)H(q,p_2,-p_1) - \overline{H}(-q,p_1,-p_2)\,S^{-1}(p_2)]$$

$q \to 0$

Isolate classical tensor structure

Ward identity

A. C. Aguilar, D. Binosi, D. Ibañez and J. Papavassiliou, Phys. Rev. D **90**, no.6, 065027 (2014).

$$\lambda_1(p^2) = \underbrace{F(0)A(p^2)\left\{\left[1+4p^2K_4(p^2)\right] - 2K_1(p^2)\mathcal{M}(p^2)\right\}}_{\lambda_1^{\star}(p^2)} - \boxed{Q_3(p^2)}$$

Displacement = BS amplitude

- Ingredients can be computed using lattice results.

O. Oliveira, P. J. Silva, J. I. Skullerud and A. Sternbeck, Phys. Rev. D **99**, no.9, 094506 (2019).
A. Kizilersü, O. Oliveira, P. J. Silva, J. I. Skullerud and A. Sternbeck, Phys. Rev. D **103**, no.11, 114515 (2021).

- Combine ingredients and determine if there is a nontrivial displacement.

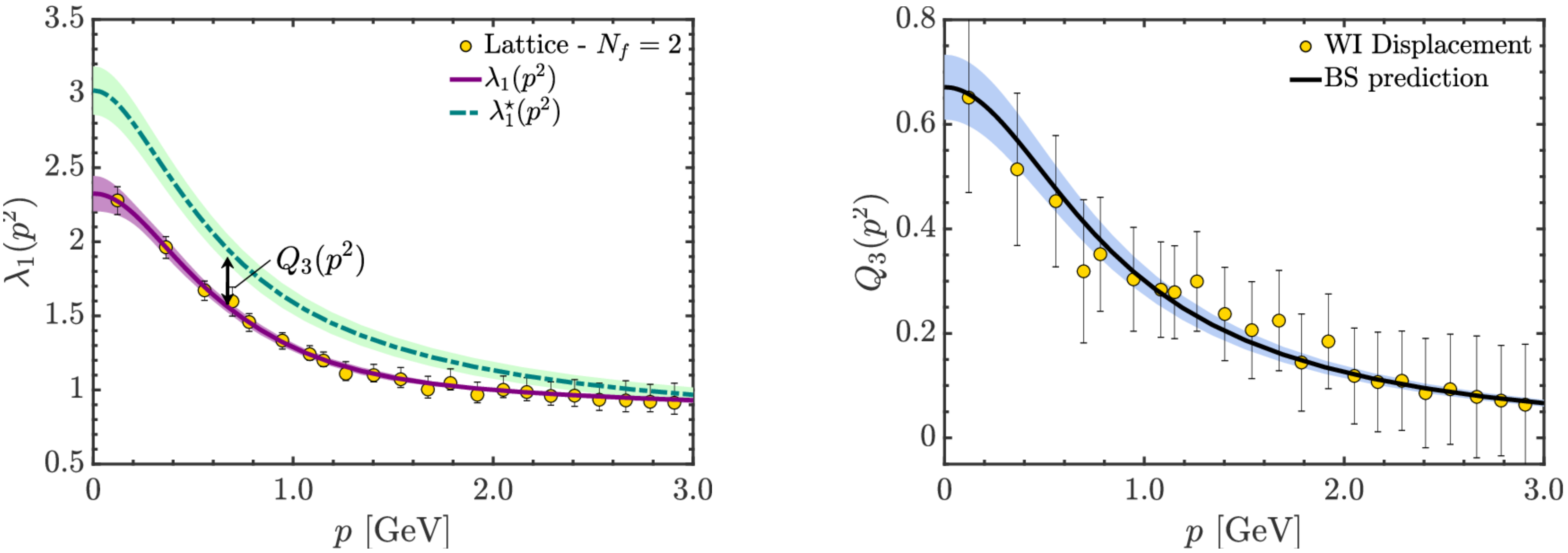
A. C. Aguilar, M. N. F., D. Ibañez and J. Papavassiliou, Eur. Phys. J. C 83, no.10, 967 (2023).

Results for $Q_3(p^2)$

We are in position to compute $\lambda_1^\star(p^2)$ and then obtain $Q_3(p^2)$ from the **WI displacement**

$$\lambda_1^\star(p^2) = F(0)A(p^2)\left\{\left[1+4p^2K_4(p^2)\right] - 2K_1(p^2)\mathcal{M}(p^2)\right\}$$

$$Q_3(p^2) = \lambda_1^\star(p^2) - \lambda_1(p^2)$$



A. C. Aguilar, M. N. F., D. Ibañez and J. Papavassiliou, Eur. Phys. J. C 83, no.10, 967 (2023).

Conclusions

- **Dynamical mass generation in QCD is key** for hadron phenomenology.

- **Gluon self-interactions** generate a **gluon mass through the Schwinger mechanism,** via the formation of **massless bound state poles in the three-gluon vertex.**

- Leads to **displacements of the Ward identities, whose amplitudes coincide with BS amplitudes of the massless bound states.**

- The **occurrence of such displacements can be tested in QCD**, by combining **lattice and Dyson-Schwinger results** for the propagators and vertices.

- **We obtain clear displacements which agree with the Bethe-Salpeter predictions.**

Backup slides

Massless bound state formalism

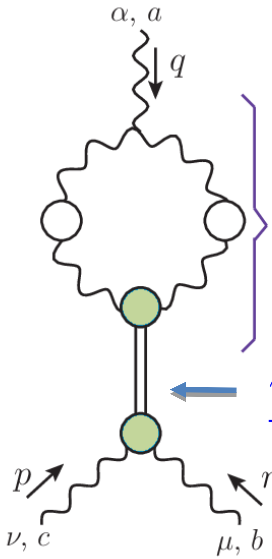
Important: These bound states are not *glueballs*!

Glueballs:

- Color singlets.
- Massive.
- Appear in the spectrum.

Schwinger mechanism poles:

- Colored states.
- Massless.
- Do not appear in the spectrum (would-be Goldstone boson, eaten to generate the gluon mass)



$\frac{i\delta^{ab}}{q^2}$ **massless colored excitation**

V. Mathieu, N. Kochelev and V. Vento, Int. J. Mod. Phys. E 18, 1-49 (2009).

J. Smit, Phys. Rev. D **10**, 2473 (1974).
E. Eichten and F. Feinberg, Phys. Rev. D 10, 3254-3279 (1974).
A. C. Aguilar, D. Ibanez, V. Mathieu, and J. Papavassiliou, Phys. Rev. D 85, 014018 (2012).

Schwinger mechanism poles do not show in lattice results

A typical vertex form factor on the lattice is given by:

$$\mathcal{A}(q,r,p) = \frac{\Gamma_0^{\alpha'\mu'\nu'}(q,r,p)P_{\alpha'\alpha}(q)P_{\mu'\mu}(r)P_{\nu'\nu}(p)\mathrm{I}\Gamma^{\alpha\mu\nu}(q,r,p)}{\Gamma_0^{\alpha'\mu'\nu'}(q,r,p)P_{\alpha'\alpha}(q)P_{\mu'\mu}(r)P_{\nu'\nu}(p)\Gamma_0^{\alpha\mu\nu}(q,r,p)}$$

with $P_{\mu\nu}(q) := g_{\mu\nu} - q_\mu q_\nu / q^2$

$$\mathrm{I}\Gamma^{\alpha\mu\nu}(q,r,p) = \underbrace{\Gamma^{\alpha\mu\nu}(q,r,p)}_{\text{pole-free}} + \underbrace{V^{\alpha\mu\nu}(q,r,p)}_{\text{poles}}$$

Given that the poles are longitudinally coupled:

$$P_{\alpha\alpha'}(q)P_{\mu\mu'}(r)P_{\nu\nu'}(p)V^{\alpha\mu\nu}(q,r,p) = 0$$

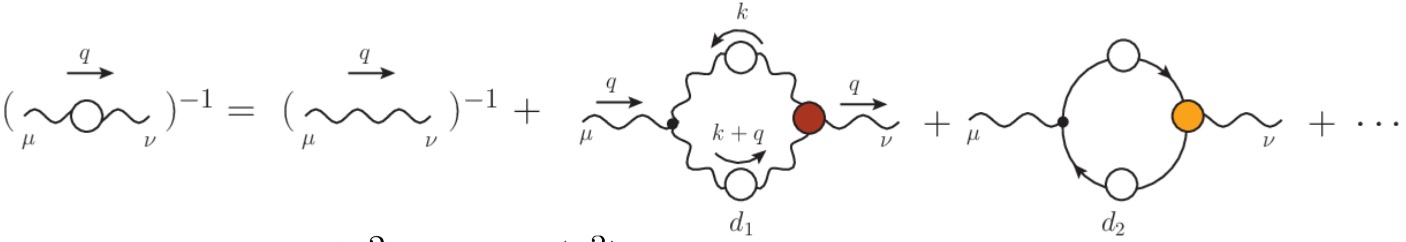
$$\mathcal{A}(q,r,p) = \frac{\Gamma_0^{\alpha'\mu'\nu'}(q,r,p)P_{\alpha'\alpha}(q)P_{\mu'\mu}(r)P_{\nu'\nu}(p)\Gamma^{\alpha\mu\nu}(q,r,p)}{\Gamma_0^{\alpha'\mu'\nu'}(q,r,p)P_{\alpha'\alpha}(q)P_{\mu'\mu}(r)P_{\nu'\nu}(p)\Gamma_0^{\alpha\mu\nu}(q,r,p)}$$

Lattice extracts the pole-free part of the vertex.

A. Athenodorou, D. Binosi, P. Boucaud, F. De Soto, J. Papavassiliou, J. Rodriguez-Quintero and S. Zafeiropoulos, Phys. Lett. B 761, 444-449 (2016).
A. C. Aguilar, F. De Soto, M. N. F., J. Papavassiliou and J. Rodríguez-Quintero, Phys. Lett. B 818, 136352 (2021).

Gluon self-interaction is dominant in gluon mass generation

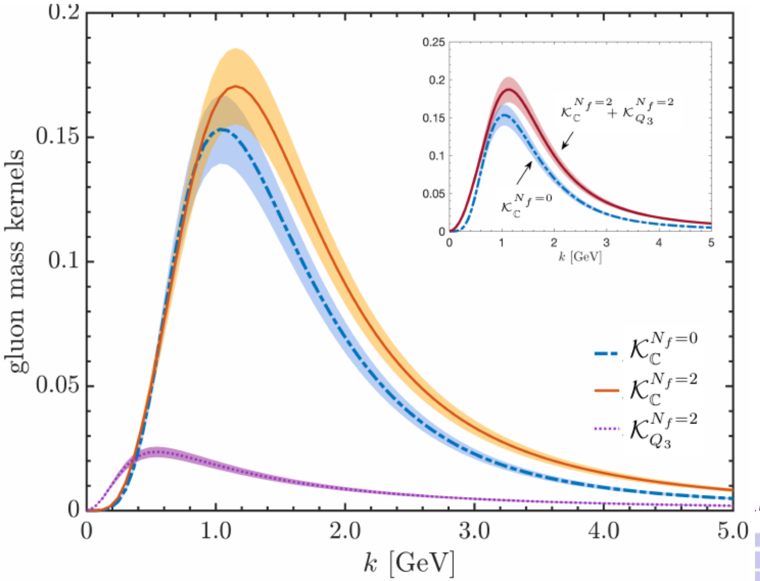
From the gluon SDE:

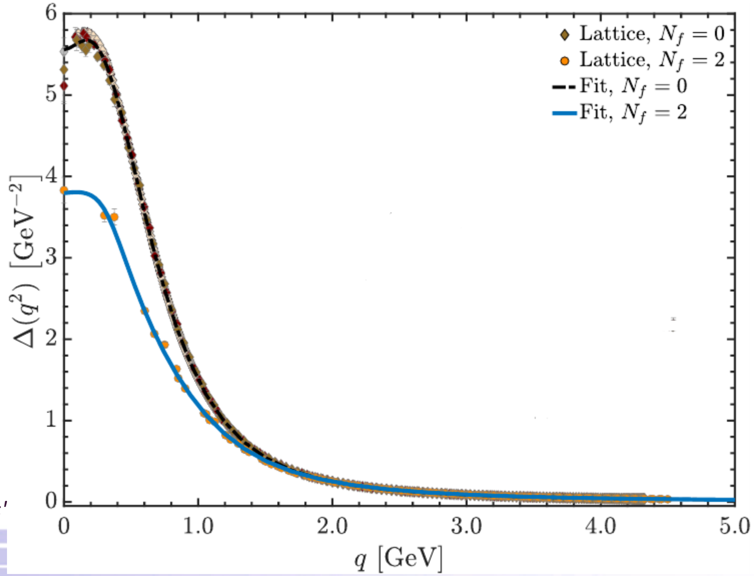


one finds an expression for the mass in terms of $\mathbb{C}(p^2)$ and $Q_3(p^2)$:

$$m^2 = \int_0^\infty dy \boxed{\mathcal{K}_{\mathbb{C}}^{N_f}(y)} + \int_0^\infty dy \boxed{\mathcal{K}_{Q_3}^{N_f}(y)}$$

three-gluon **quark-gluon**





- Unquenched gluon mass is larger, in agreement with lattice.
- Three-gluon is the biggest contribution.
- Gluon self-interaction drives the Schwinger mechanism in QCD.

Results for $Q_3(p^2)$

We are in position to compute $\lambda_1^\star(p^2)$ and then obtain $Q_3(p^2)$ from the **WI displacement**

$$\lambda_1^\star(p^2) = F(0)A(p^2)\left\{\left[1 + 4p^2 K_4(p^2)\right] - 2K_1(p^2)\mathcal{M}(p^2)\right\}$$

$$Q_3(p^2) = \lambda_1^\star(p^2) - \lambda_1(p^2)$$

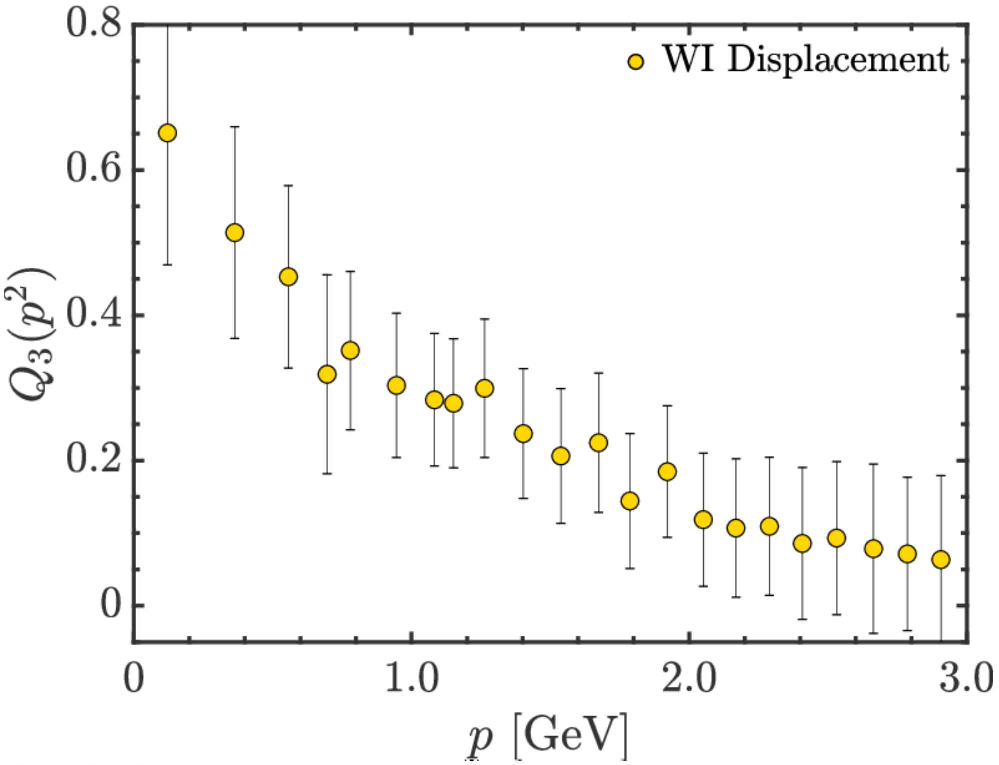
- $Q_3(p^2)$ obtained is clearly nonzero.
- Define the **null hypothesis**,

$$Q_3(p^2) = Q_3^0(p^2) := 0$$

***p*-value of null hypothesis is very small:**

$$P_{Q_3^0} = \int_{\chi^2=119}^{\infty} \chi^2_{\mathrm{PDF}}(18, x)dx = 6.5 \times 10^{-17}$$

- Excludes the null hypothesis at the 8σ level.



A. C. Aguilar, M. N. F., D. Ibañez and J. Papavassiliou, arXiv:2308.16297

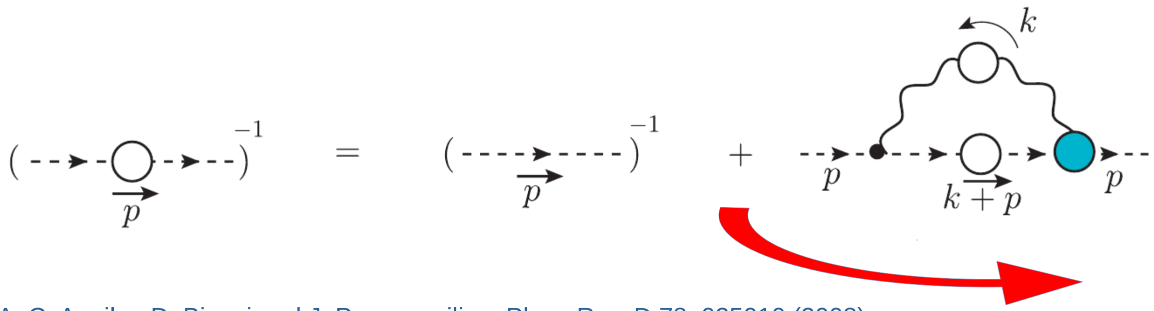
Indirect signals: Finite ghost dressing function

The generation of a gluon mass gap leaves distinctive imprints in other Green's functions. For example:

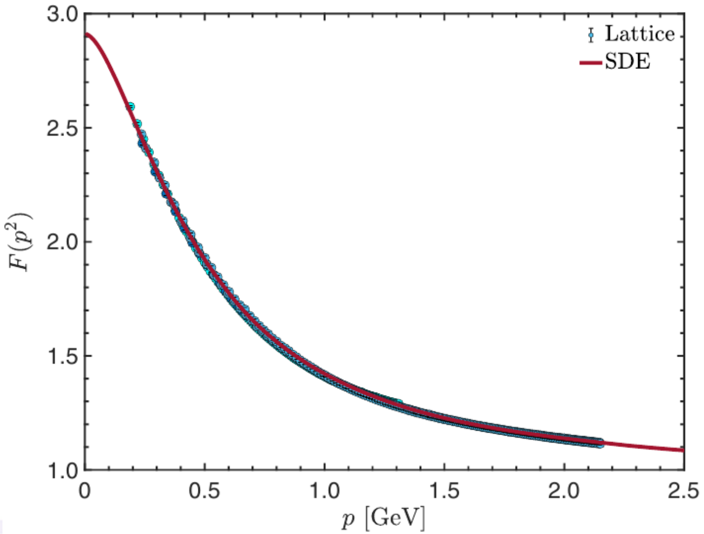
- The Schwinger mechanism leaves the **ghost propagator,** $D(q^2)$, **massless.**
- But its **dressing function**, $F(q^2)$, given by

$$\boxed{D(q^2) = \frac{iF(q^2)}{q^2}}$$

  becomes IR finite.



A. C. Aguilar, D. Binosi and J. Papavassiliou, Phys. Rev. D 78, 025010 (2008).
M. R. Pennington and D. J. Wilson, Phys. Rev. D **84**, 119901 (2011).
A. C. Aguilar, C. O. Ambrosio, F. De Soto, M. N. F., et al, Phys. Rev. D 104 no.5, 054028, (2021).



More examples in:

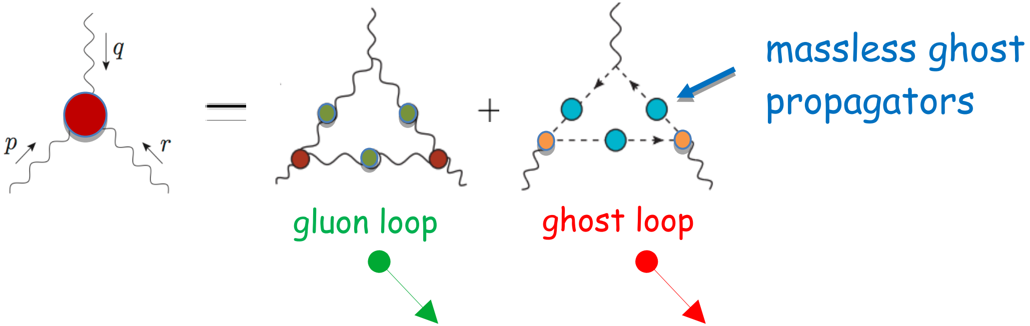
M. N. F. and J. Papavassiliou, Particles **6**, no.1, 312-363 (2023).

Indirect signals: IR divergence of three-gluon vertex

Three-gluon vertex in the IR exhibits suppression and zero crossing

A. C. Aguilar, D. Binosi, D. Ibañez, J. Papavassiliou, Phys. Rev. D 89, no. 8, 085008 (2014).
G. Eichmann, R. Williams, R. Alkofer, M. Vujinovic, Phys. Rev. D89,105014 (2014).
A. G. Duarte, O. Oliveira and P. J. Silva, Phys. Rev. D 94, no.7, 074502 (2016).
A. C. Aguilar, M. N. F., J. Papavassiliou and L. R. Santos, Eur. Phys. J. C **83**, no.6, 549 (2023).
A. K. Cyrol, L. Fister, M. Mitter, J. M. Pawlowski, N. Strodthoff, Phys. Rev. D94, 054005 (2016)
R. Williams, C. S. Fischer, and W. Heupel, Phys. Rev. D 93, no. 3, 034026 (2016)
M. Q. Huber, Phys. Rev. D 101, 114009 (2020).

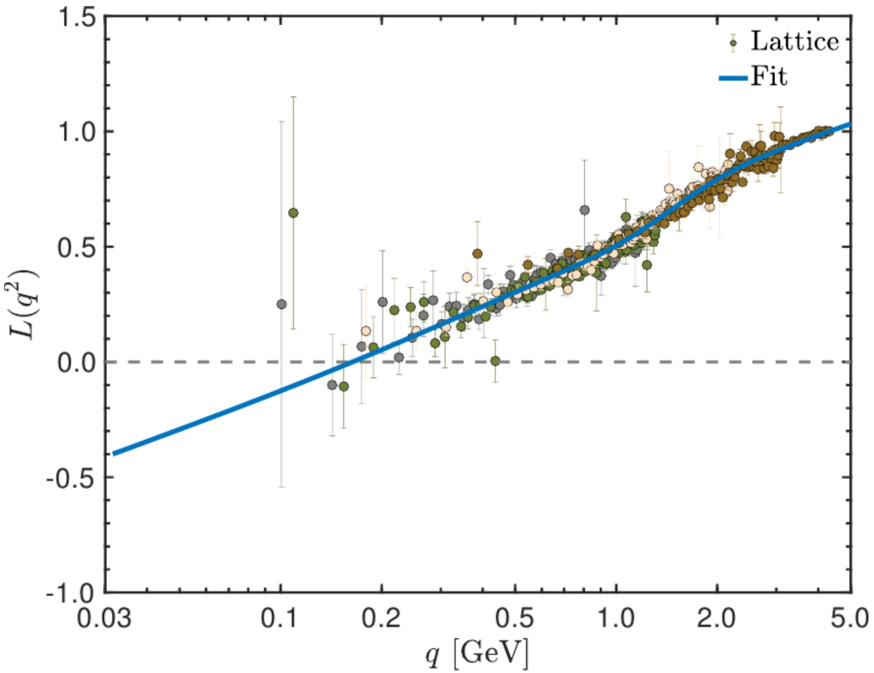
Within the Schwinger mechanism, the infrared behavior of the classical form factor of the three-gluon vertex is characterized by the interplay between two types of logarithms:



$$L_{\text{sg}}(r^2) = a + b\ln\left(\frac{r^2+m^2}{\mu^2}\right) + c\ln\left(\frac{r^2}{\mu^2}\right)$$

"protected" "unprotected"

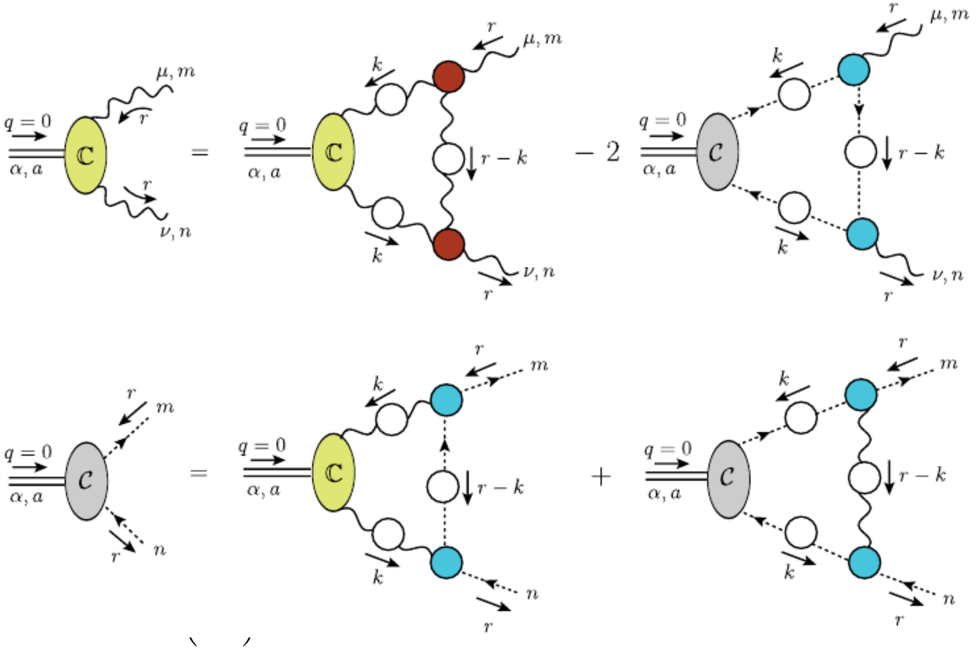
- In the IR, $L_{\text{sg}}(r^2) \to -\infty$, logarithmically.
- Explains IR suppression and zero-crossing

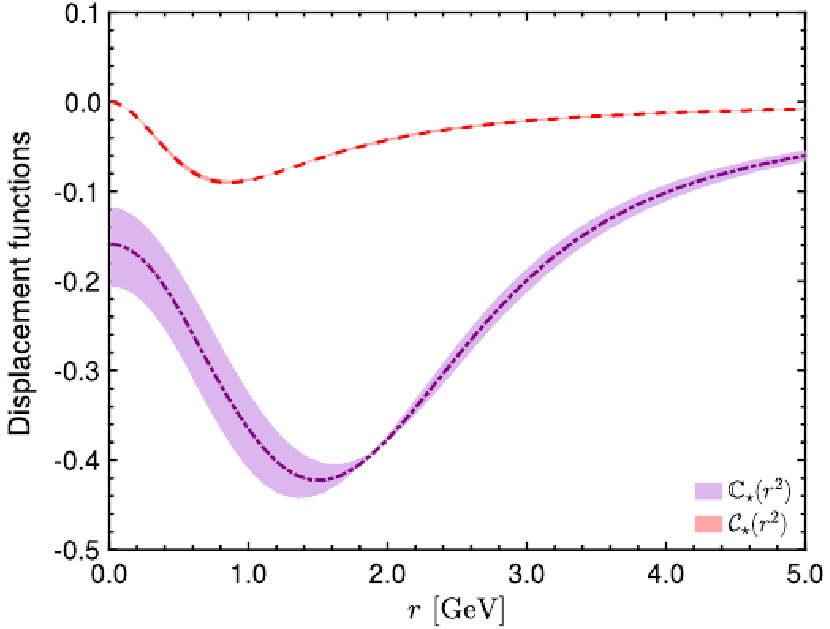


A. C. Aguilar, F. De Soto, M.N.F., J. Papavassiliou, J. Rodriguez-Quintero, Phys. Lett. B818 (2021) 136352

Pole of the ghost-gluon vertex

The Schwinger-Dyson equation for the displacement amplitude $\mathbb{C}(r^2)$ can be coupled to a pole also in **ghost-gluon vertex**



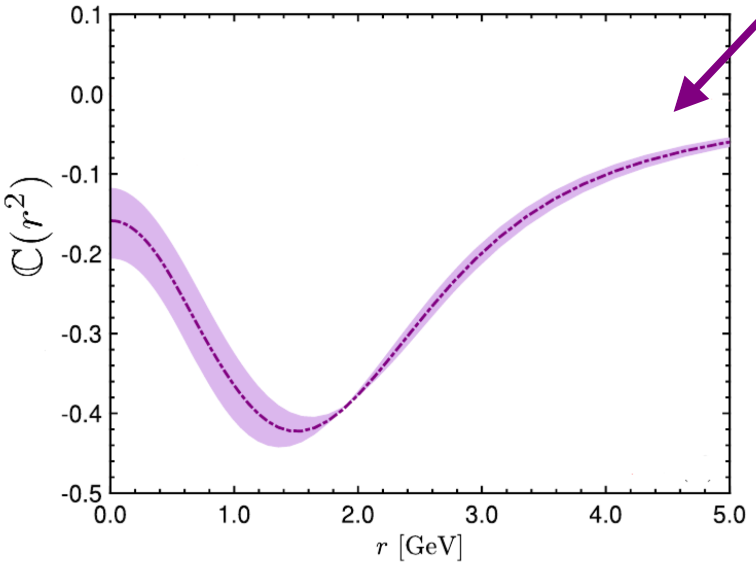


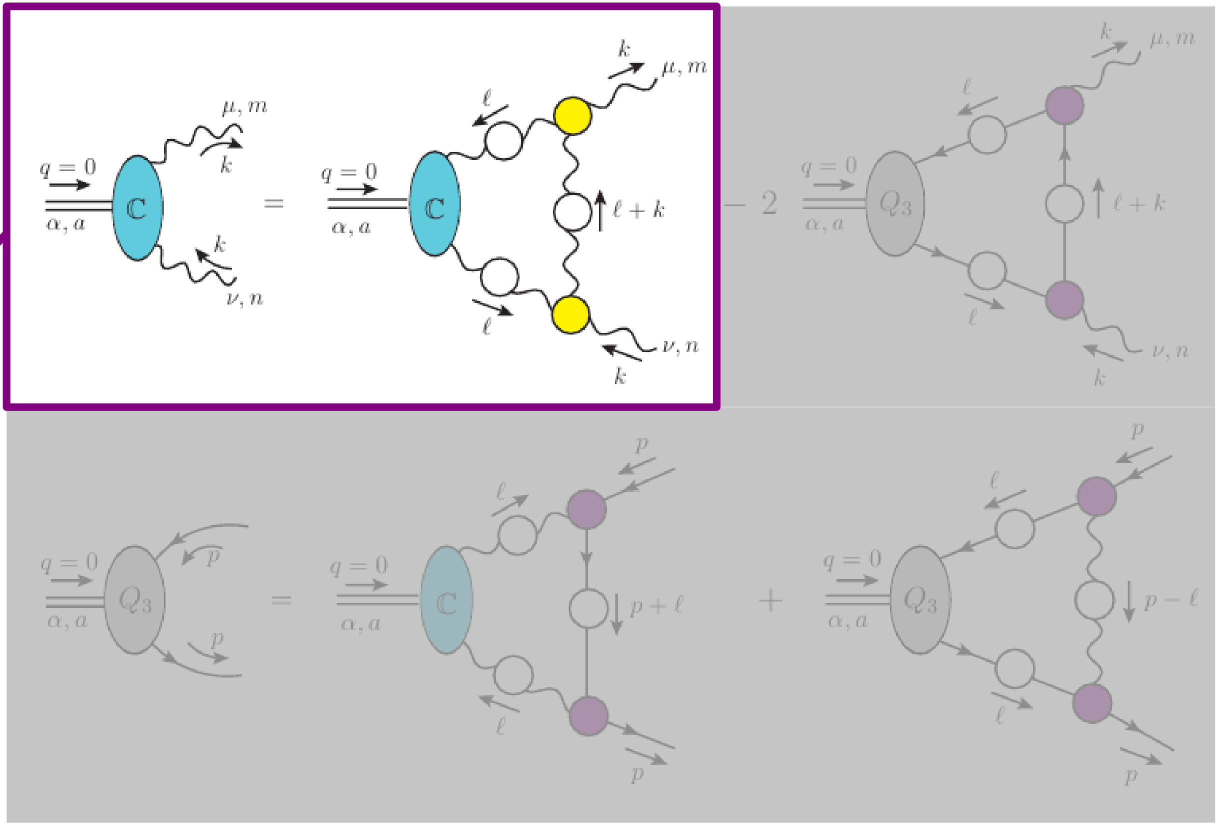
Effect on $\mathbb{C}(r^2)$ is negligible because ghost-gluon pole amplitude, $\mathcal{C}(r^2)$, is subleading.

A. C. Aguilar, et al, Eur. Phys. J. C **78**, no.3, 181 (2018).
A. C. Aguilar, M. N. F. and J. Papavassiliou, Phys. Rev. D **105**, no.1, 014030 (2022).

Schwinger mechanism with dynamical quarks

- The three-gluon pole BSE has solutions even if quarks are neglected.

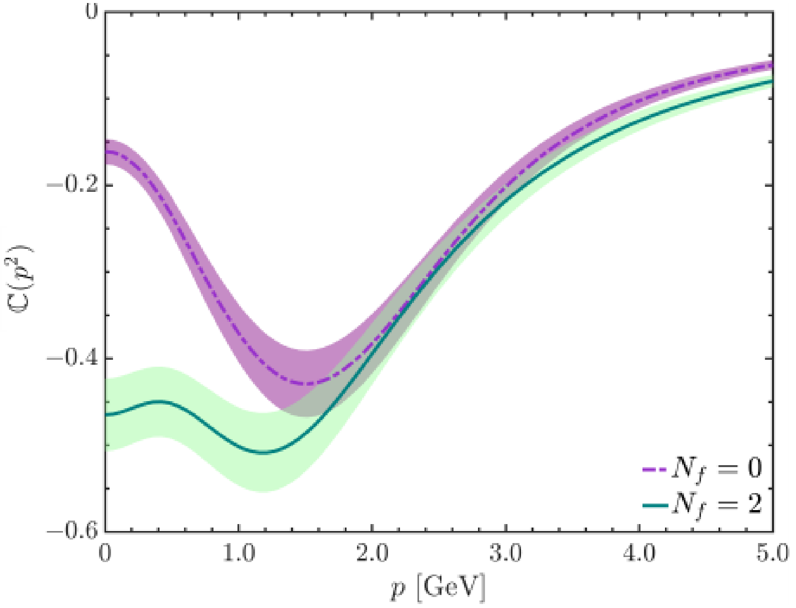


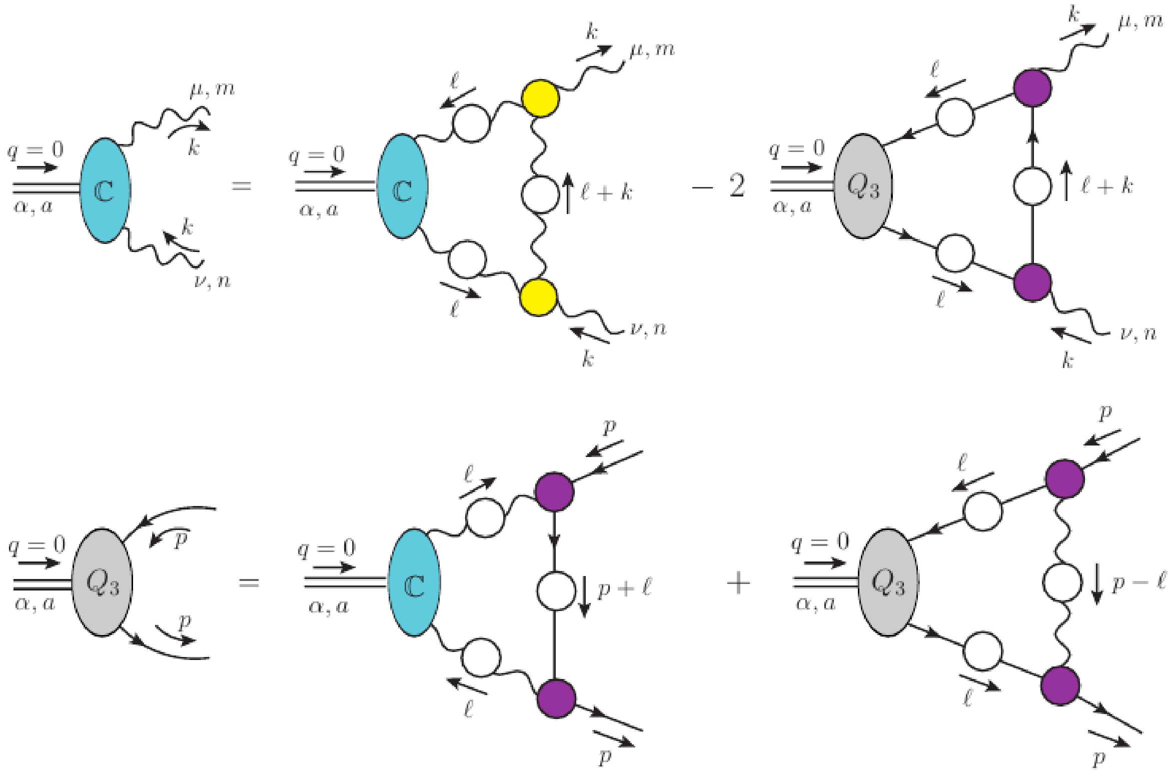


A. C. Aguilar, M. N. F. and J. Papavassiliou, Phys. Rev. D 105, no.1, 014030 (2022).

Schwinger mechanism with dynamical quarks

- The three-gluon pole BSE has solutions even if quarks are neglected.
- Turning on quarks, changes the three-gluon BSE amplitude only in the deep IR.

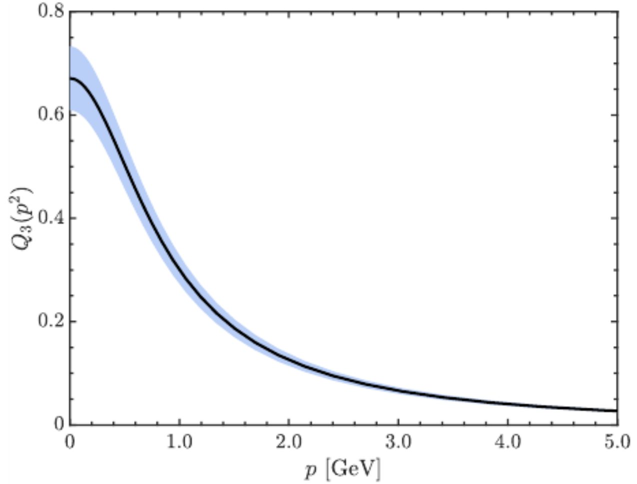


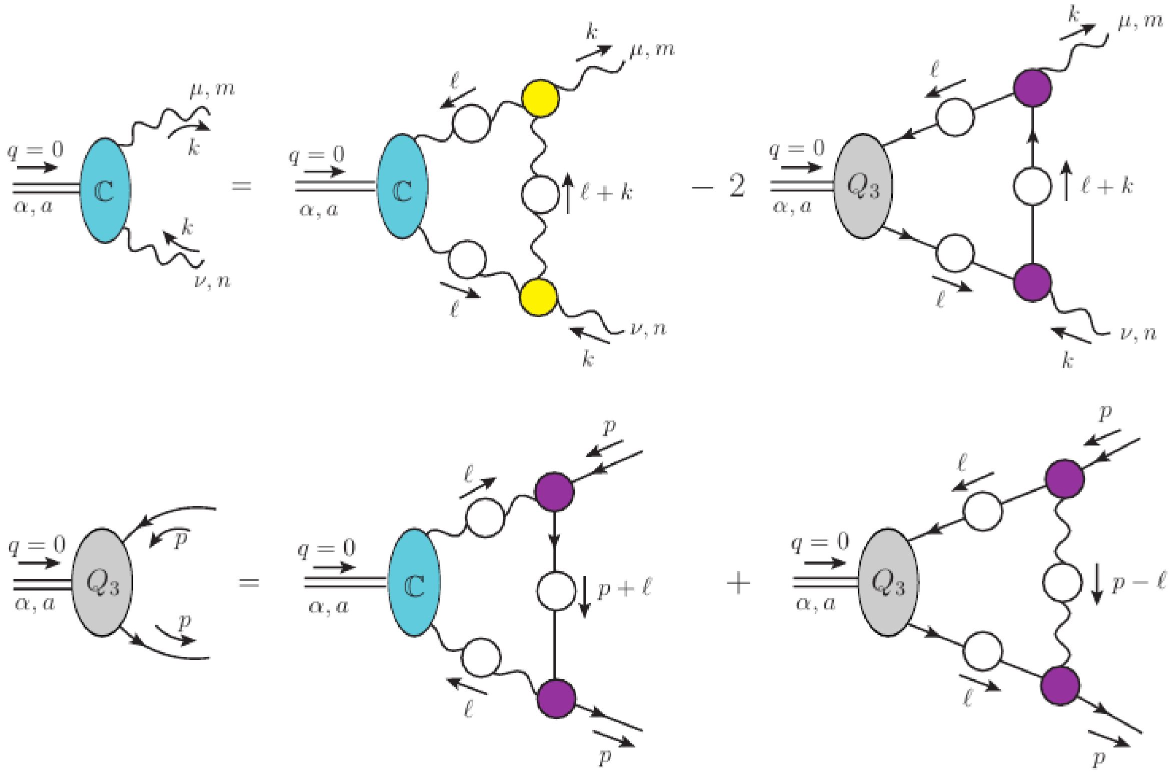


A. C. Aguilar, M. N. F. and J. Papavassiliou, Phys. Rev. D 105, no.1, 014030 (2022).

Schwinger mechanism with dynamical quarks

- The three-gluon pole BSE has solutions even if quarks are neglected.
- Turning on quarks, changes the three-gluon BSE amplitude only in the deep IR.
- But also generates a **pole in the quar-gluon vertex**, with amplitude *$Q_3(p^2)$.*





A. C. Aguilar, M. N. F. and J. Papavassiliou, Phys. Rev. D 105, no.1, 014030 (2022).

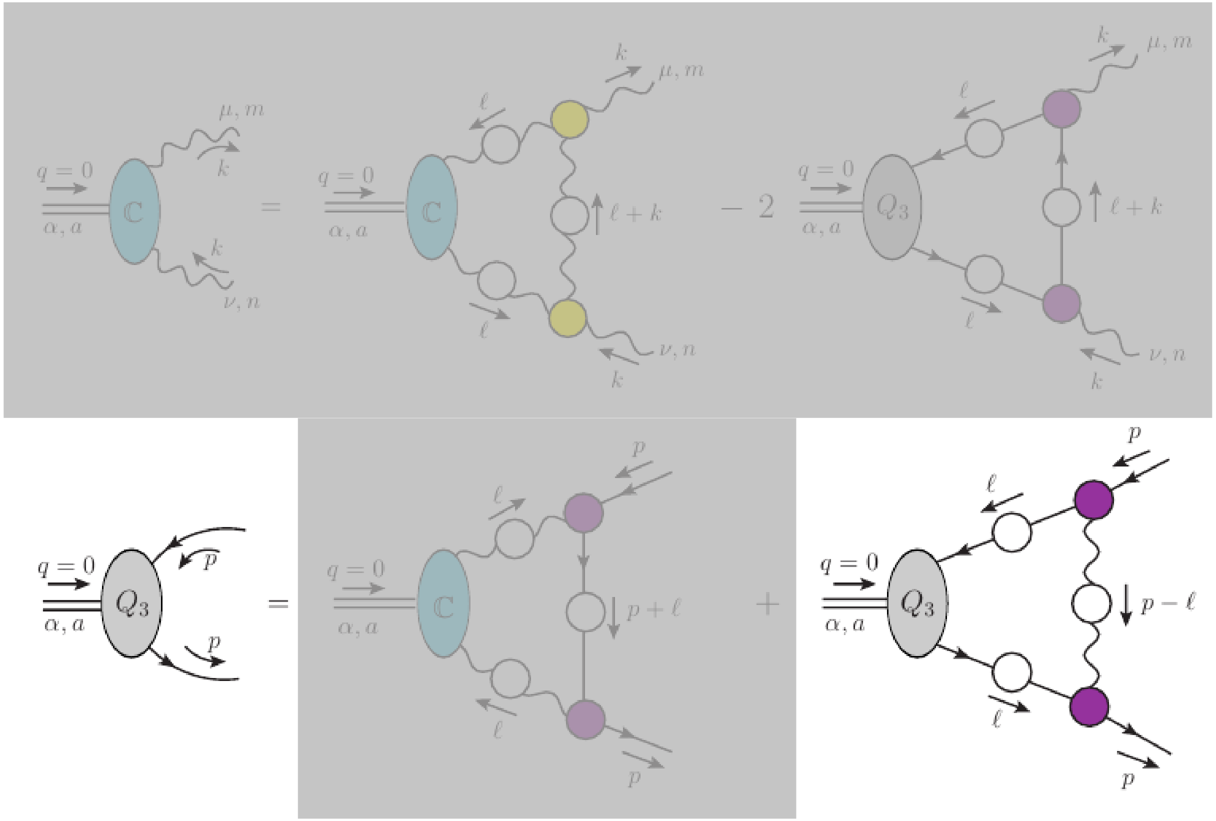
Gluon self-interaction is dominant in gluon mass generation

- The three-gluon pole BSE has solutions even if quarks are neglected.

- Turning on quarks, changes the three-gluon BSE amplitude only in the deep IR.
-
- But also generates a **pole in the quar-gluon vertex**, with amplitude $Q_3(p^2)$.

- **But turning off the three-gluon pole, no solution is found!**

Gluon self-interaction drives gluon mass generation



A. C. Aguilar, M. N. F. and J. Papavassiliou, Phys. Rev. D 105, no.1, 014030 (2022).

Ward identity displacement of the quark-gluon vertex

$$q^\alpha \mathrm{I\Gamma}_\alpha(q,p_2,-p_1) = F(q^2)[S^{-1}(p_1)H(q,p_2,-p_1) - \overline{H}(-q,p_1,-p_2)\,S^{-1}(p_2)]$$

$q \to 0$

Isolate classical tensor structure

Ward identity

$$\lambda_1(p^2) = F(0)A(p^2)\left\{\left[1+4p^2\boxed{K_4(p^2)}\right] - 2\boxed{K_1(p^2)}\mathcal{M}(p^2)\right\} - Q_3(p^2)$$

Partial derivative of the quark-ghost kernel

$$\left.\frac{\partial H(q,p,-q-p)}{\partial q^\mu}\right|_{q=0} = \gamma_\mu K_1(p^2) + 4p_\mu \not{p} K_2(p^2) + 2p_\mu K_3(p^2) + 2\widetilde{\sigma}_{\mu\nu}p^\nu K_4(p^2)$$

A. C. Aguilar, D. Binosi, D. Ibañez and J. Papavassiliou, Phys. Rev. D **90**, no.6, 065027 (2014).
A. C. Aguilar, M. N. F., D. Ibañez and J. Papavassiliou, Eur. Phys. J. C 83, no.10, 967 (2023).

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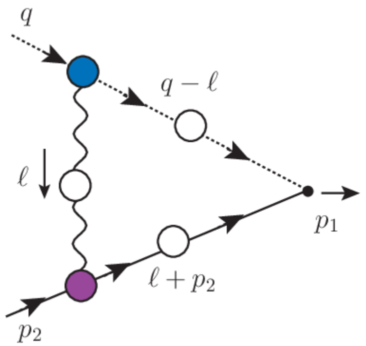
$q \to 0$ Isolate classical tensor structure

Ward identity

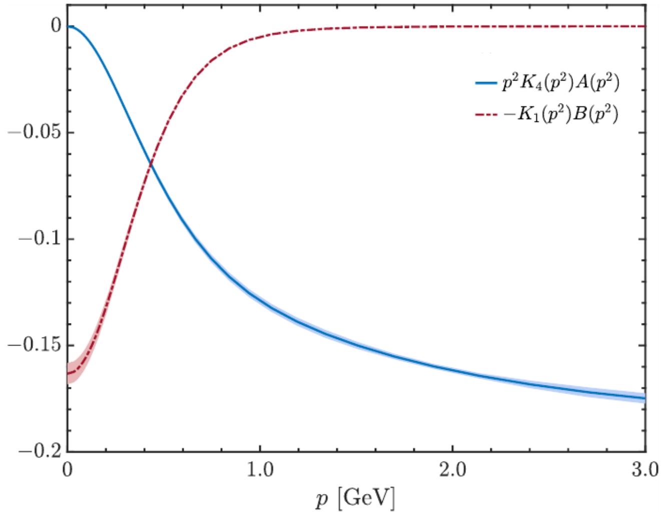
$$\lambda_1(p^2) = F(0)A(p^2)\left\{\left[1+4p^2\boxed{K_4(p^2)}\right] - 2\boxed{K_1(p^2)}\mathcal{M}(p^2)\right\} - Q_3(p^2)$$

Computed through a lattice driven Schwinger-Dyson analysis

$$H^a(q,p_2,-p_1) = -g\frac{\lambda^a}{2} +$$



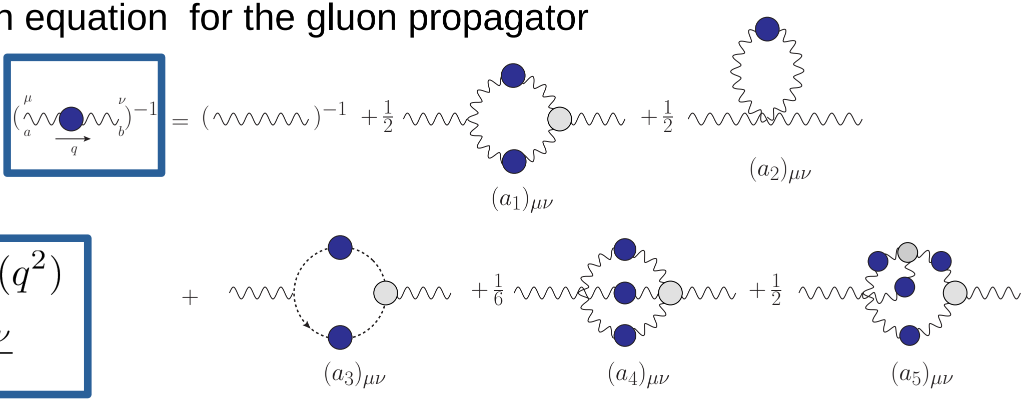
$+ \ldots$



Seagull cancellation

- The **gluon mass generation must occur without violating gauge symmetry**.

- Recalling the Schwinger-Dyson equation for the gluon propagator



$$\Delta_{\mu\nu}(q) = -iP_{\mu\nu}(q)\Delta(q^2)$$
$$P_{\mu\nu}(q) := g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}$$

It can be shown that

Gauge symmetry + Regular vertices at *q^2 = 0* $\longrightarrow$ $\Delta^{-1}(0) = 0$

★ **The key to generate gluon mass is to have massless poles, longitudinaly coupled to the gluon momenta, in the vertices of QCD.**

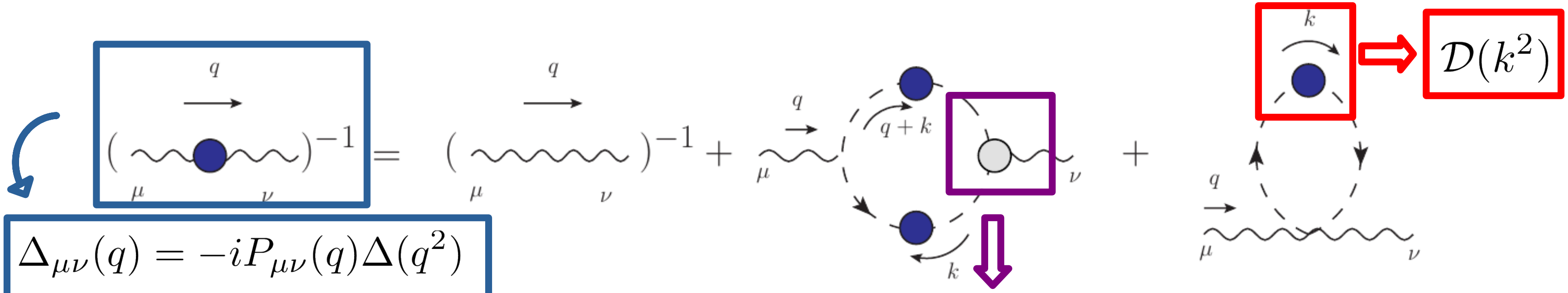
A. C. Aguilar and J. Papavassiliou, JHEP **12**, 012 (2006).
A. C. Aguilar, D. Ibanez, V. Mathieu and J. Papavassiliou, Phys. Rev. D **85**, 014018 (2012).
A. C. Aguilar, D. Binosi, C. T. Figueiredo and J. Papavassiliou, Phys. Rev. D **94**, no.4, 045002 (2016).
A. C. Aguilar, D. Binosi and J. Papavassiliou, Front. Phys. (Beijing) **11**, no.2, 111203 (2016).
C. Eichmann, J. M. Pawlowski and J. M. Silva, Phys. Rev. D **104**, no.11, 114016 (2021).

Seagull cancellation

To understand **how gauge fields can become massive by the Schwinger mechanism**, let us first recall how gauge symmetry ***usually*** implies their masslessness.

A. C. Aguilar, D. Binosi, C. T. Figueiredo and J. Papavassiliou, Phys. Rev. D **94**, no.4, 045002 (2016).
A. C. Aguilar, D. Binosi and J. Papavassiliou, Front. Phys. (Beijing) **11**, no.2, 111203 (2016).

To this end, consider the **Schwinger-Dyson equation** for the scalar QED **photon propagator**



$$\Delta_{\mu\nu}(q) = -iP_{\mu\nu}(q)\Delta(q^2)$$

$$P_{\mu\nu}(q) := g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}$$

$$\Gamma_\nu(q, k, -q-k)$$

At q = 0, we obtain:

$$\Delta^{-1}(0) = \frac{2ie^2}{d}\int_k \mathcal{D}^2(k^2)k^\mu\Gamma_\mu(0,k,-k) - 2ie^2\int_k \mathcal{D}(k^2)$$

Seagull cancellation

Now, **gauge symmetry** implies the **Ward identity**:

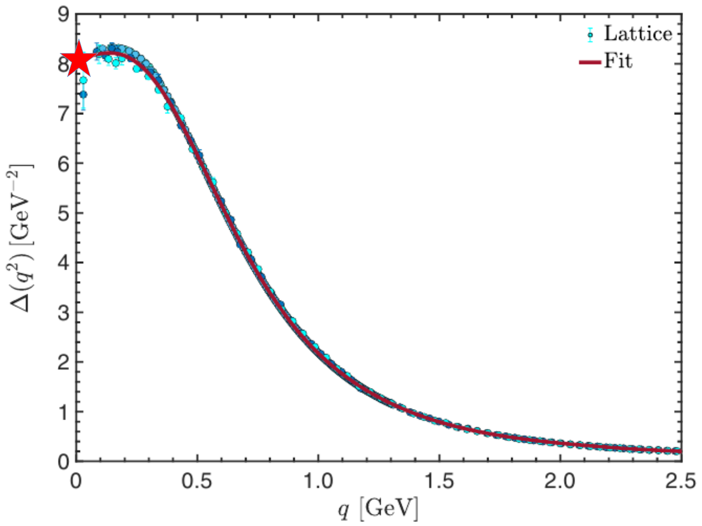
$$q^\mu \Gamma_\mu(q,r,p) = \mathcal{D}^{-1}(p^2) - \mathcal{D}^{-1}(r^2) \xrightarrow{\;q\,=\,0\;} \Gamma_\mu(0,r,-r) = \frac{\partial \mathcal{D}^{-1}(r^2)}{\partial r^\mu}$$

$$\Delta^{-1}(0) = \frac{2ie^2}{d}\int_k \mathcal{D}^2(k^2)k^\mu\Gamma_\mu(0,k,-k) - 2ie^2\int_k \mathcal{D}(k^2)$$

$$\Delta^{-1}(0) = -\frac{4ie^2}{d}\boxed{\left[\int_k k^2\frac{\partial \mathcal{D}^{-1}(k^2)}{\partial k^2} + \frac{d}{2}\int_k \mathcal{D}(k^2)\right] = 0}$$

Seagull identity (integration by parts in *d* dimensions).

Then, how can we have saturation?



A. C. Aguilar and J. Papavassiliou, JHEP **12**, 012 (2006).
A. C. Aguilar, D. Binosi, C. T. Figueiredo and J. Papavassiliou, Phys. Rev. D **94**, no.4, 045002 (2016).
A. C. Aguilar, D. Binosi and J. Papavassiliou, Front. Phys. (Beijing) **11**, no.2, 111203 (2016).

Evading the seagull cancellation

Suppose the vertex has a **pole at *q*=0, coupled longitudinally to *q***, i.e.

A. C. Aguilar and J. Papavassiliou, JHEP **12**, 012 (2006).
A. C. Aguilar, D. Binosi, C. T. Figueiredo and J. Papavassiliou, Phys. Rev. D **94**, no.4, 045002 (2016).

$$\Gamma_\mu(q,r,p) \to \mathrm{I}\!\Gamma_\mu(q,r,p) = \boxed{\frac{q_\mu}{q^2}C(q,r,p)} + \Gamma_\mu(q,r,p)$$

Does not contribute explicitly to $\Delta(q^2)$ because it is longitudinal.

$$\Delta^{-1}(0) = \frac{2ie^2}{d}\int_k \mathcal{D}^2(k^2)k^\mu\Gamma_\mu(0,k,-k) - 2ie^2\int_k \mathcal{D}(k^2)$$

However, now the regular part satisfies a "displaced" Ward identity:

A. C. Aguilar, M. N. F. and J. Papavassiliou, Phys. Rev. D **105**, no.1, 014030 (2022).

$$\Gamma_\mu(0,r,-r) = \frac{\partial\mathcal{D}^{-1}(k^2)}{\partial k^\mu} - 2r_\mu\mathcal{C}(r^2)$$

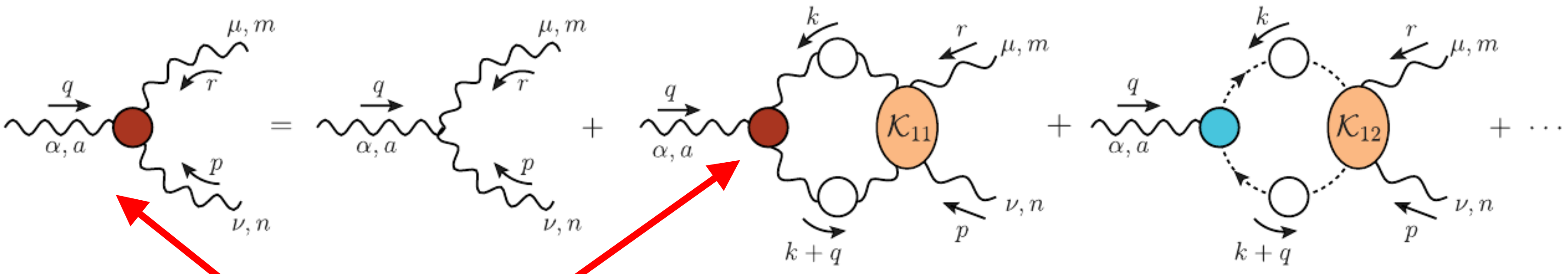
$$\mathcal{C}(r^2) := \left[\frac{\partial C(q,r,p)}{\partial p^2}\right]_{q=0}$$

Displacement amplitude

$$\Delta^{-1}(0) = -\frac{4ie^2}{d}\int_k k^2\mathcal{D}^2(k^2)\mathcal{C}(k^2)$$

Derivation of the Schwinger pole Bethe-Salpeter equation

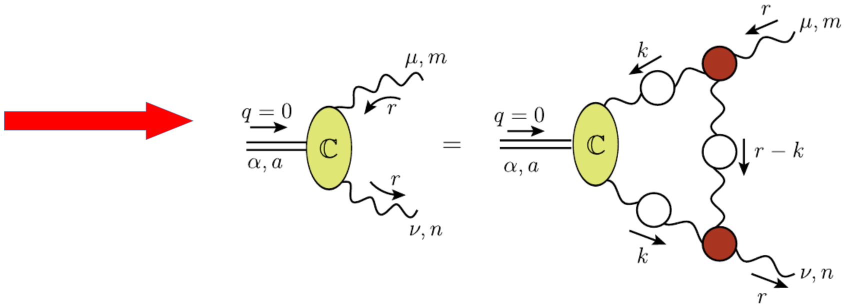
We start with the Schwinger-Dyson (or more generaly nPI) equation for the vertex and assume the presence of a massless pole:



$$\mathrm{I\!\Gamma}_{\alpha\mu\nu}(q,r,p) = \Gamma_{\alpha\mu\nu}(q,r,p) + \frac{q_\alpha}{q^2} g_{\mu\nu} C_1(q,r,p) + \ldots$$

Now multiply by q^2 and take $q = 0$. Only terms containing poles remain:

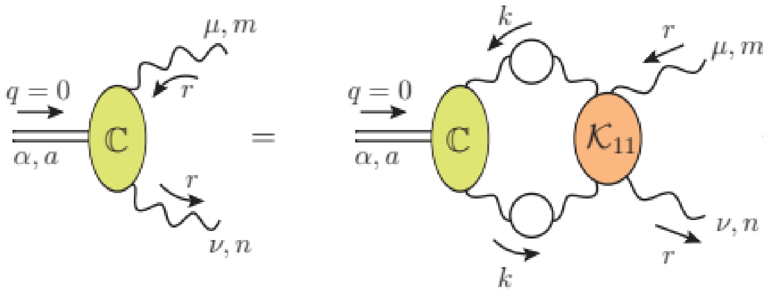
- **Inhomogeneous Schwinger-Dyson** equation becomes a **Homogeneous Bethe-Salpeter** equation.



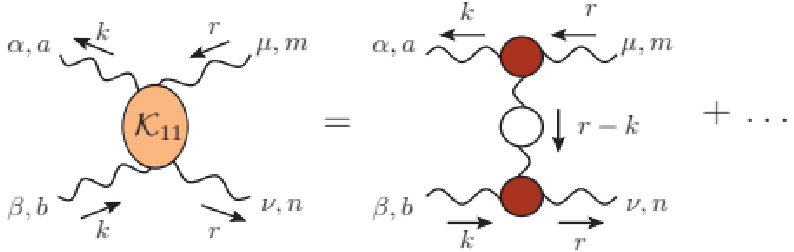
A. C. Aguilar, et al, Eur. Phys. J. C **78**, no.3, 181 (2018).
A. C. Aguilar, M. N. F. and J. Papavassiliou, Phys. Rev. D **105**, no.1, 014030 (2022).

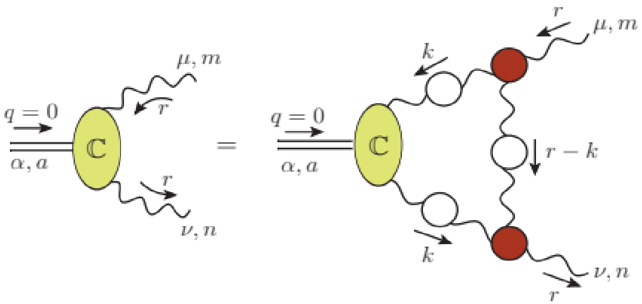
One-gluon exchange approximation

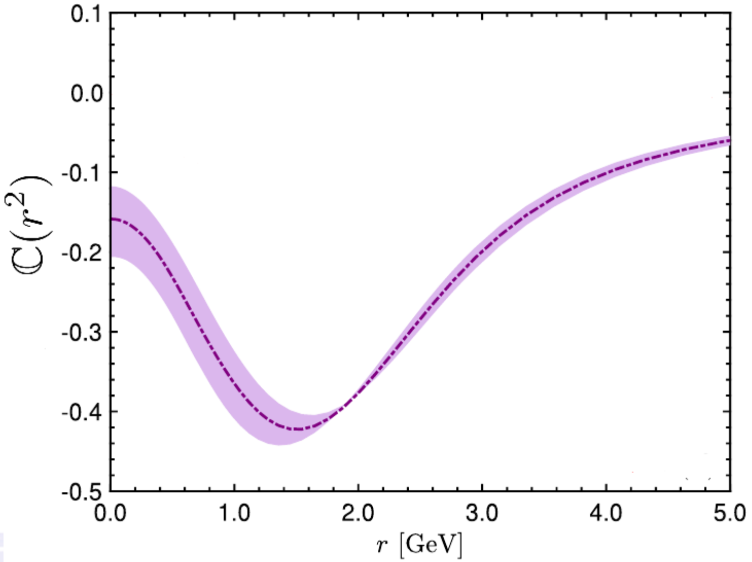
From the Bethe-Salpeter equation, we can



Truncation







A. C. Aguilar, M. N. F. and J. Papavassliou, Phys. Rev. D 105, no.1, 014030 (2022).

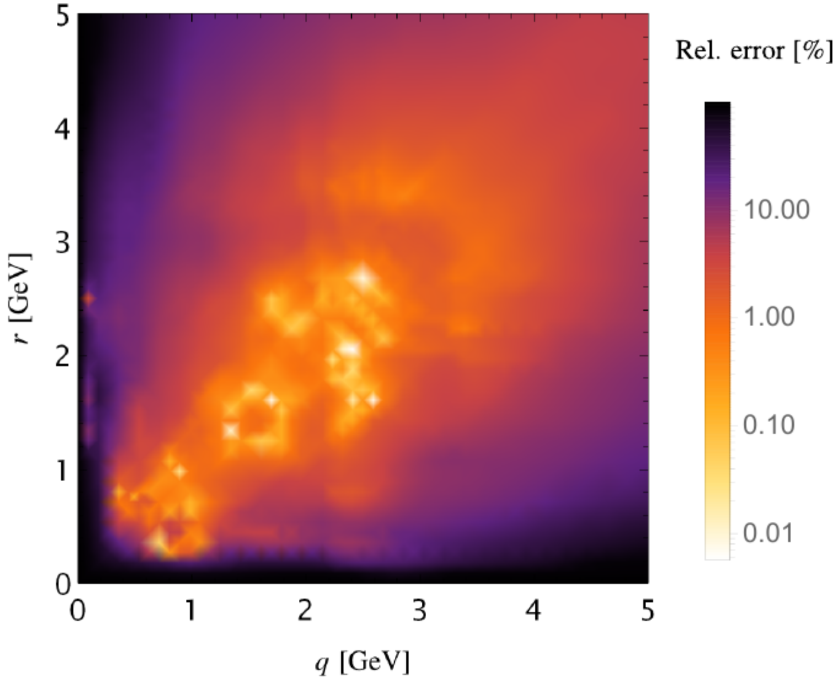
Method 2): Planar degeneracy

To quantify the accuracy of the approximation it is convenient to define

$$\overline{\mathcal{I}}_{\mathcal{W}}(q^2,r^2,p^2) := \frac{\overline{\mathcal{I}}_{\mathcal{W}}(q^2,r^2,p^2)}{\overline{\mathcal{I}}^0_{\mathcal{W}}(q^2,r^2,p^2)} \xrightarrow{\text{Planar degeneracy}} \overline{\mathcal{I}}_{\mathcal{W}}(q^2,r^2,p^2) \approx L_{\rm sg}(s^2)$$

Then we can measure the relative difference between $L_{\rm sg}(s^2)$ and $\overline{\mathcal{I}}_{\mathcal{W}}(q^2,r^2,p^2)$

- Approximation is accurate to within 1% near the diagonal.
- And within 10% for most of the kinematics.
- The measured error can then be propagated to the $\mathcal{W}(r^2)$



Results for $\mathcal{W}(r^2)$

We use the **planar degeneracy approximation** to obtain the **central curve**.

Errors are propagated from known error of the planar degeneracy approximation.

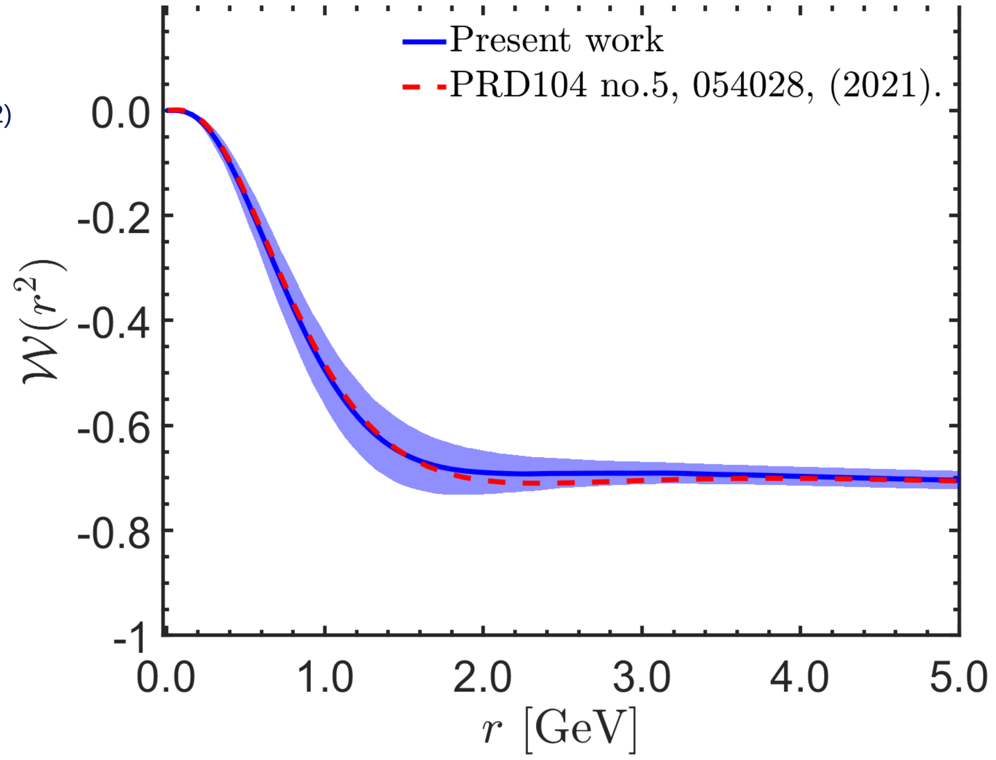
- **Result agrees well with previous calculation**

A. C. Aguilar, M. N. F. and J. Papavassliou, Phys. Rev. D 105, no.1, 014030 (2022)

- Previous result employed a particular *Ansatz* for three-gluon vertex.

- **New result stringently constrained by lattice simulation of the three-gluon vertex.**

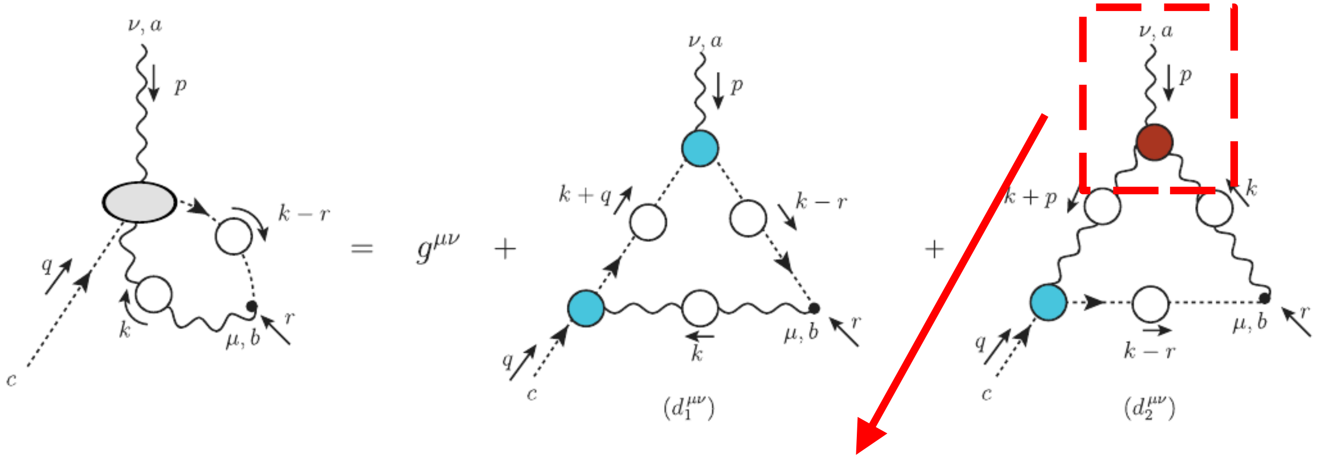
Impact of three-gluon vertex under control

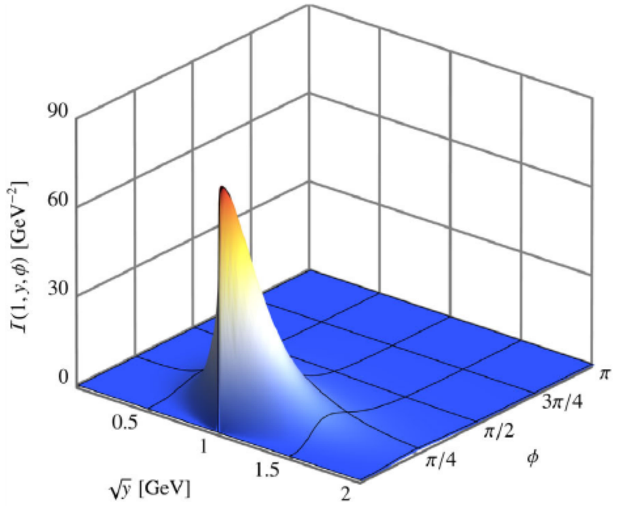


A. C. Aguilar, F. De Soto, M. N. F., J. Papavassiliou, F. Pinto-Gómez, C. D. Roberts, J. Rodríguez-Quintero, in preparation

Truncation error

The full Schwinger-Dyson equation for $\mathcal{W}(r^2)$ is





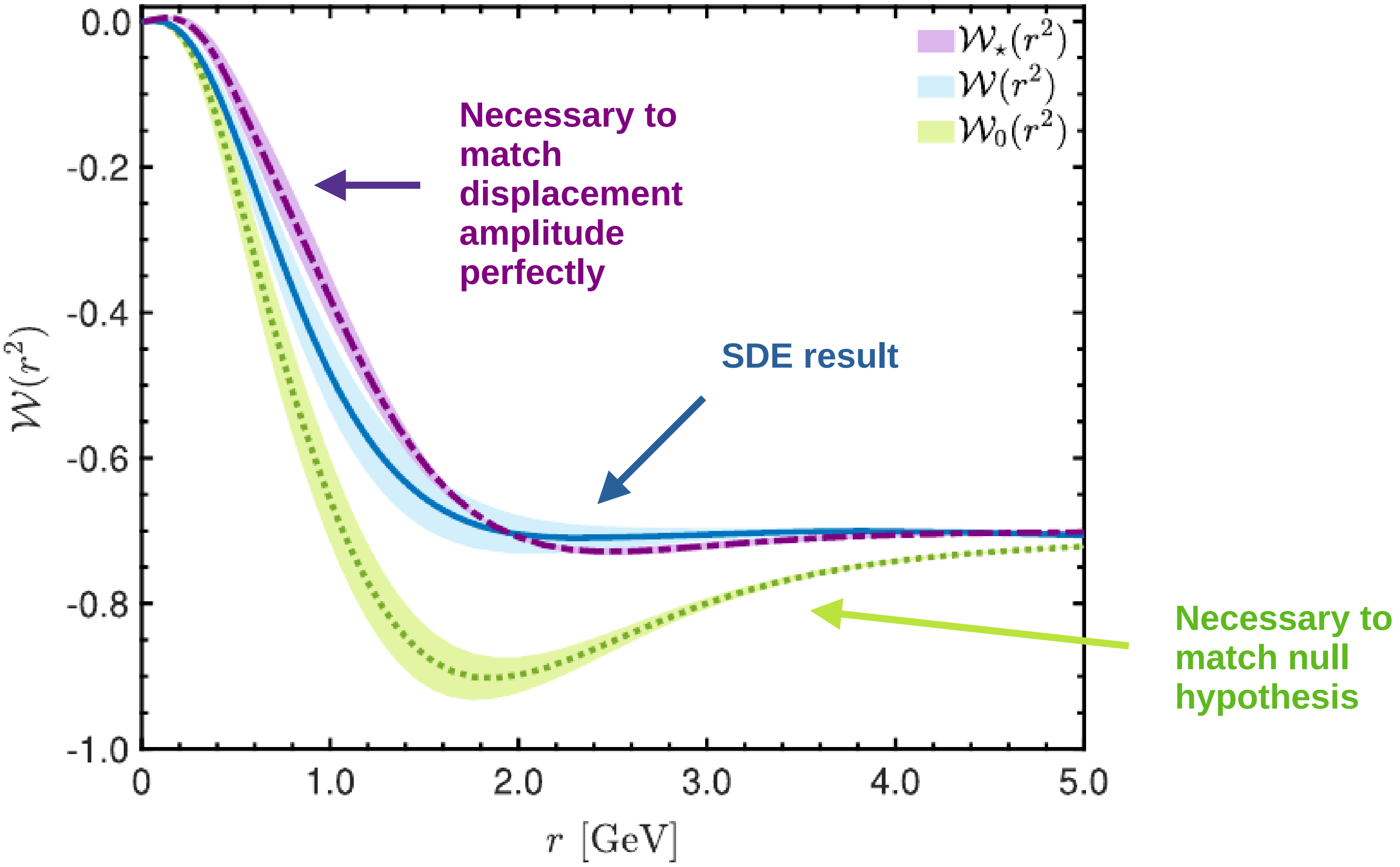
- Three-gluon vertex is a complicated object, with 14 tensor structures.

A. C. Aguilar, M. N. F., C. T. Figueiredo and J. Papavassiliou, Phys. Rev. D **99**, no.9, 094010 (2019).
J. S. Ball and T. W. Chiu, Phys. Rev. D 22, 2550 (1980). [erratum: Phys. Rev. D **23**, 3085 (1981)].

- But $\mathcal{W}(r^2)$ integrand is sharply peaked, and is sensitive only to the particular projection $L_{\rm sg}(r^2)$ which is well determined by **lattice simulations.**

A. C. Aguilar, M. N. F. and J. Papavassiliou, Phys. Rev. D **105**, no.1, 014030 (2022).

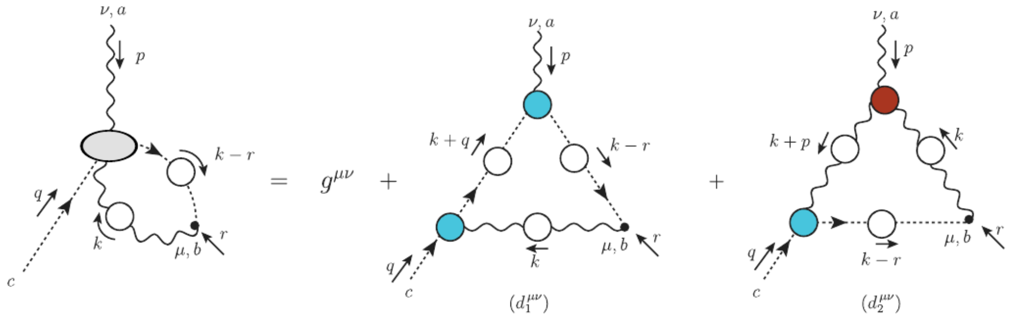
Truncation error

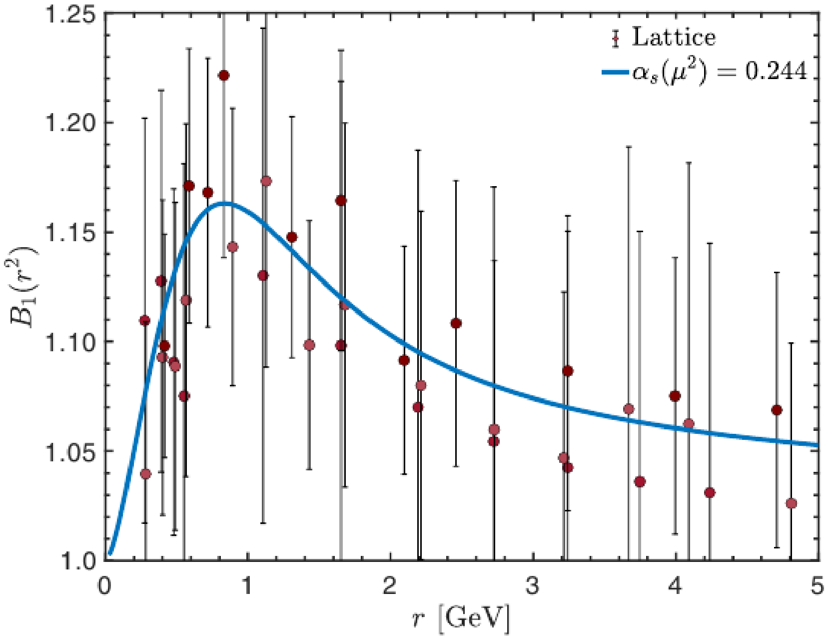


A. C. Aguilar, M. N. F. and J. Papavassiliou, Phys. Rev. D **105**, no.1, 014030 (2022).

Truncation error

The same truncation used to determine $\mathcal{W}(r^2)$, reproduces the available lattice data for the ghost-gluon vertex:





A. C. Aguilar, M. N. F. and J. Papavassiliou, Phys. Rev. D **105**, no.1, 014030 (2022).

Lattice data from: A. C. Aguilar, et al Phys. Rev. D **104**, no.5, 054028 (2021).

Inputs

The parametrizations to lattice data used were of the form:

$$\Delta^{-1}(r^2) = r^2\left[\frac{d}{1+(r^2/\kappa^2)}\ln\left(\frac{r^2}{\mu^2}\right) + A^\delta(r^2)\right] + \nu^2 R(r^2),$$

$$F^{-1}(r^2) = A^\gamma(r^2) + R(r^2),$$

where

$$A(r^2) := 1 + \omega\ln\left(\frac{r^2+\eta^2(r^2)}{\mu^2+\eta^2(r^2)}\right),$$

$$\eta^2(r^2) = \frac{\eta_1^2}{1+r^2/\eta_2^2},$$

$$R(r^2) = \frac{b_0 + b_1^2 r^2}{1+(r^2/b_2^2)+(r^2/b_3^2)^2} - \frac{b_0 + b_1^2\mu^2}{1+(\mu^2/b_2^2)+(r^2/b_3^2)^2}.$$

A. C. Aguilar, M. N. F. and J. Papavassiliou, Phys. Rev. D **105**, no.1, 014030 (2022).