Pseudoscalar mesons & Emergent Mass

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Pseudoscalar Mesons and Emergent Mass

arXiv:2403.00629

Universidad de Huelva

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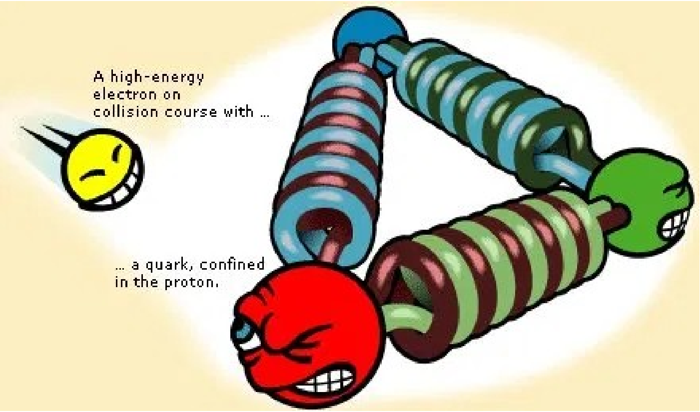
QCD: Emergent Phenomena

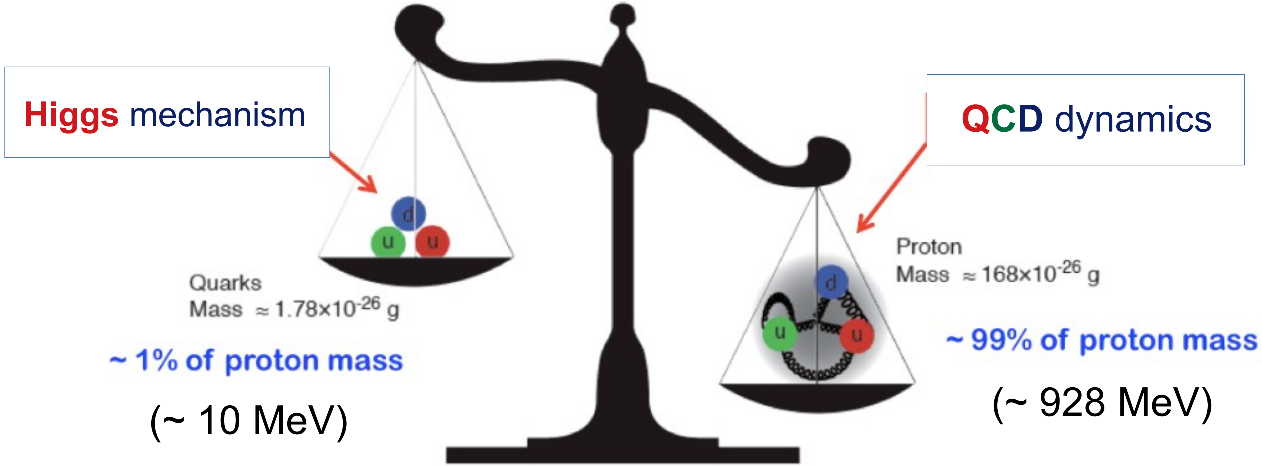
- QCD is characterized by two **emergent** phenomena: confinement and dynamical generation of mass (DGM).

$$\mathcal{L}_{\rm QCD} = \sum_{j=u,d,s,\ldots} \bar{q}_j[\gamma_\mu D_\mu + m_j]q_j + \frac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu},$$
$$D_\mu = \partial_\mu + ig\frac{1}{2}\lambda^a A^a_\mu\,,$$
$$G^a_{\mu\nu} = \partial_\mu A^a_\nu + \partial_\nu A^a_\mu - \underline{g f^{abc} A^b_\mu A^c_\nu},$$

- Quarks and gluons not *isolated* in nature.
  - Formation of colorless bound states: "**Hadrons**"
  - **1-fm scale** size of hadrons?

- Emergence of hadron masses (EHM) from QCD **dynamics**





QCD: Emergent Phenomena

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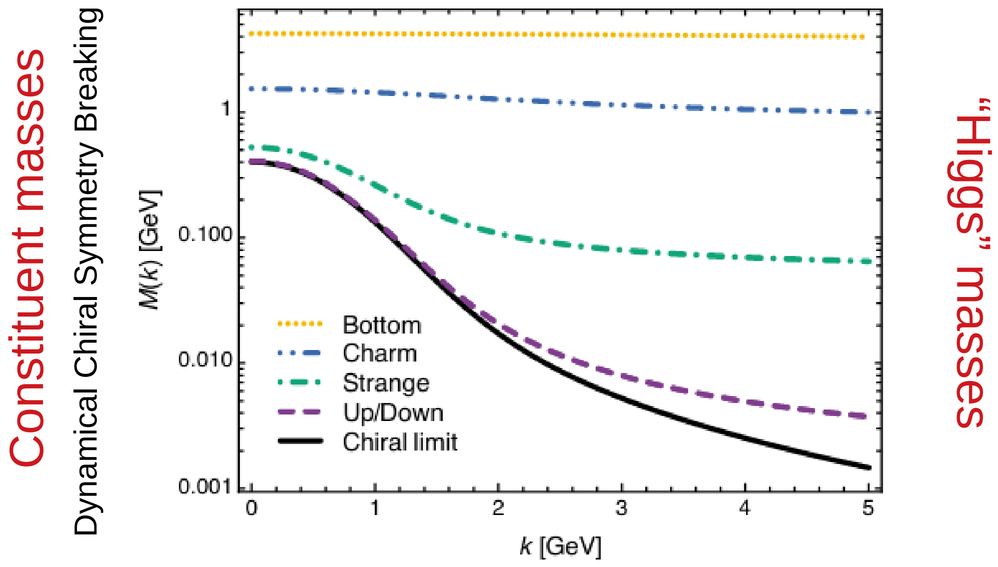
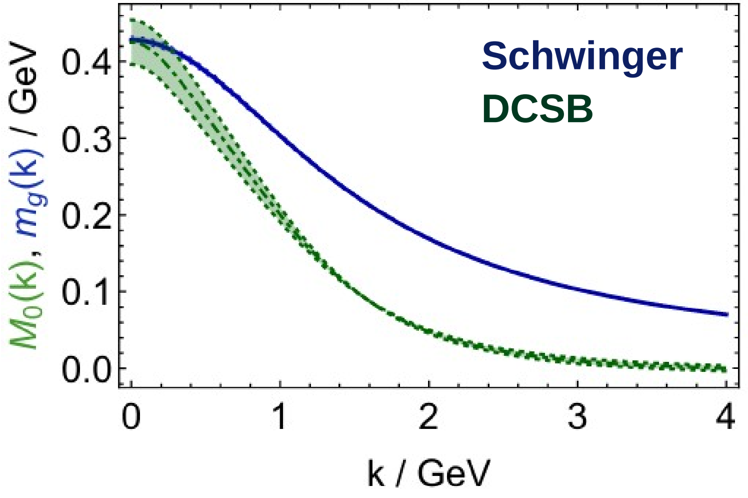
Can we trace them down to fundamental d.o.f?

◆ Emergence of hadron masses (EHM) from QCD **dynamics**



$$S_f^{-1}(p) = Z_f^{-1}(p^2)(i\gamma\cdot p + \mathbf{M_f(p^2)})$$



Gluon and quark *running masses*

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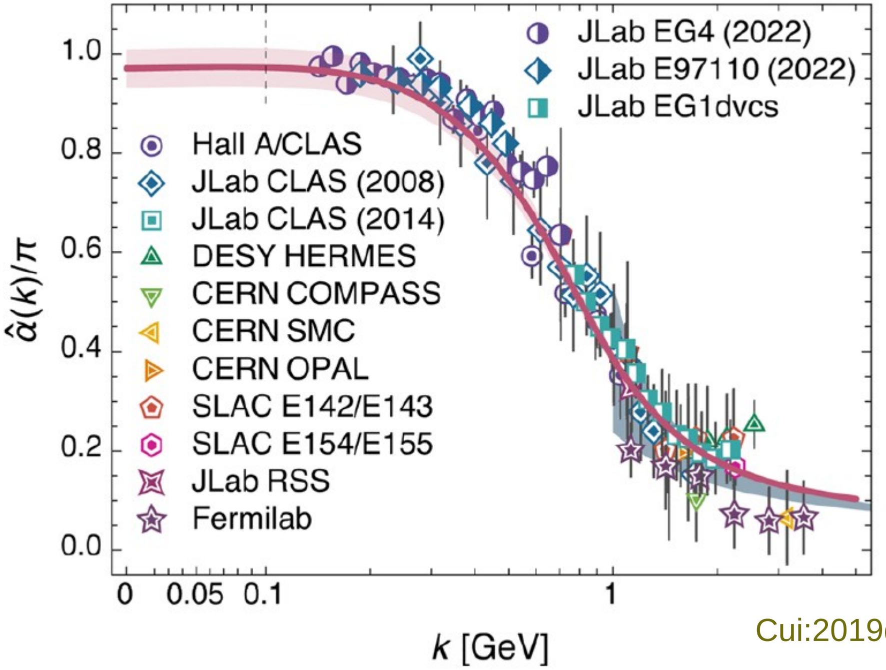
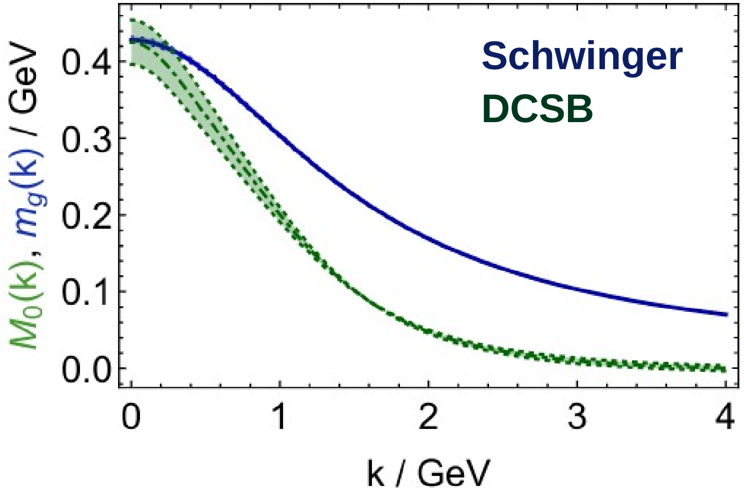


Cui:2019dwv



Gluon and quark *running masses*

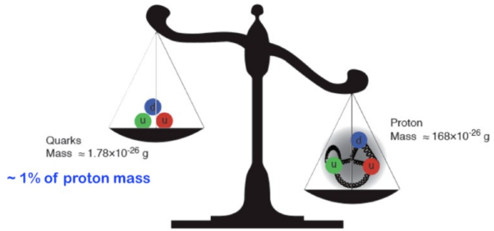
Mass **Budgets**

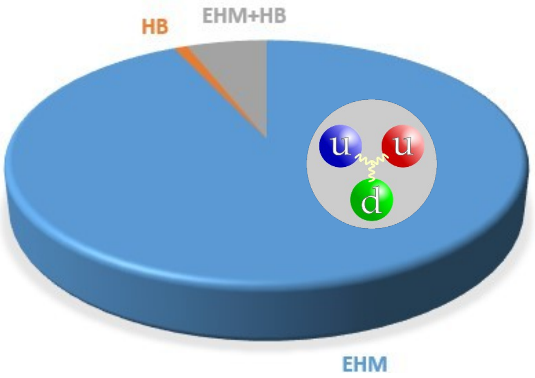
$$M_{u/d} \approx 0.3\,\text{GeV}$$

- What is the origin of **EHM**?

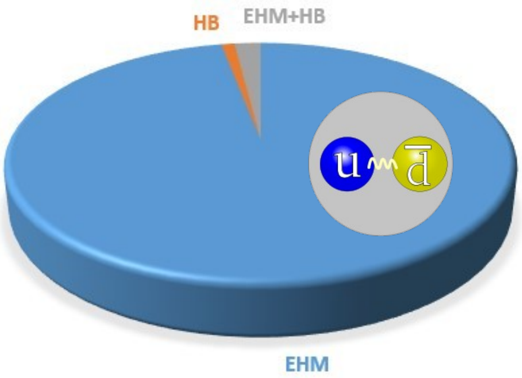
  … its connection with *e.g.* **confinement** and **DCSB**?

- **Most** of the **mass** in the visible universe is contained within **nucleons**
  - Which remain **pretty massive** whether there is Higgs mechanism or not...





Proton mass budget



Rho meson mass budget

$$m_p = 0.938\,\text{GeV} \approx 2M_u + M_d$$

$$m_\rho = 0.775\,\text{GeV} \approx M_u + M_d$$

Proton and **rho** meson mass budgets are practically **identical**

Mass **Budgets**

$$m_s/m_u \sim 20$$
$$f_K/f_\pi \sim M_s/M_u \sim 1.2$$

- What is the origin of **EHM**?

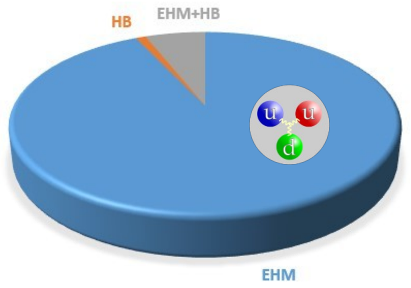
  … its connection with e.g. **confinement** and **DCSB**?

- What is the role of **NG-bosons**?

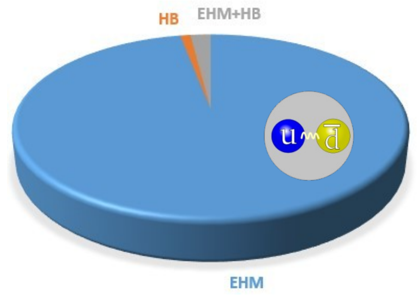
◆ **Pion and Kaon** would be **massless** in the absence of **Higgs** mass generation.

→ And **structurally alike**.

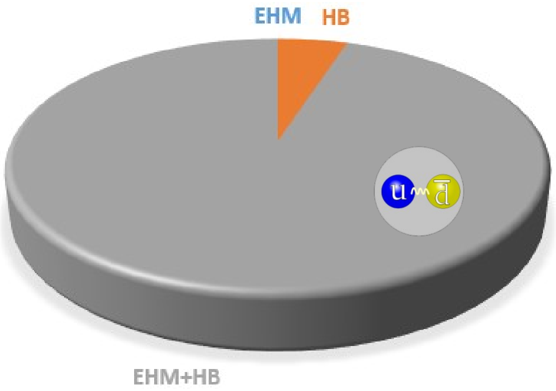
$$m_\pi = 0.14\,\mathrm{GeV} \neq M_u + M_d$$
$$m_K = 0.49\,\mathrm{GeV} \neq M_u + M_s$$



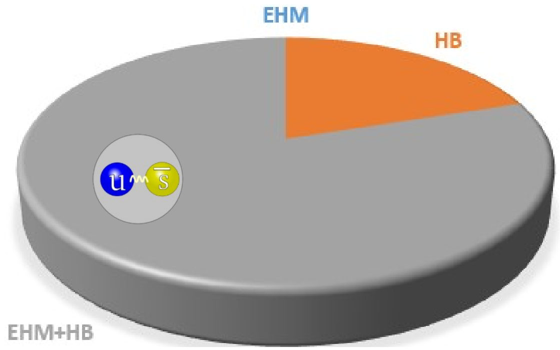
Proton mass budget



Rho meson mass budget



Pion mass budget



Kaon mass budget

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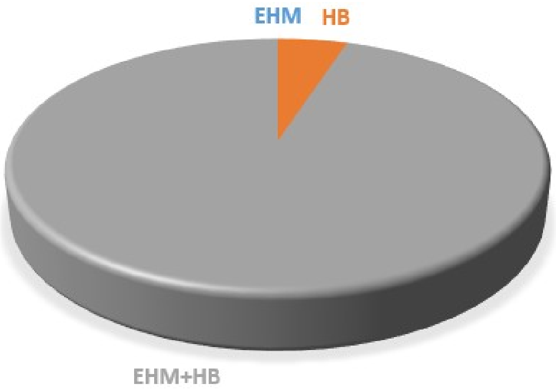
Pion and Kaon

- Both quark-antiquark **bound-states** and **NG bosons**
  - Their mere **existence** is connected with **mass** generation in the **SM**



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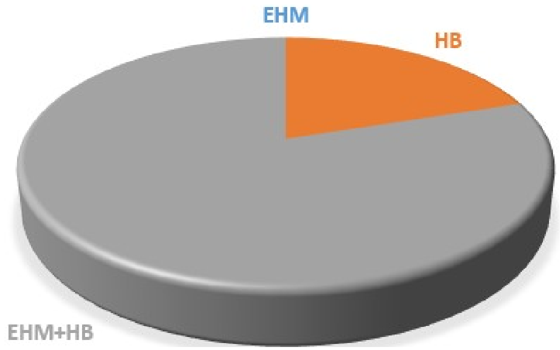

Kaon mass budget

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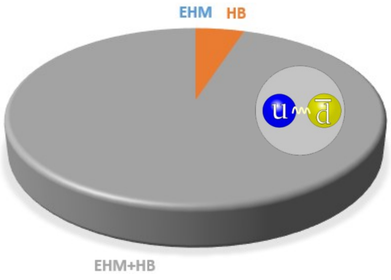
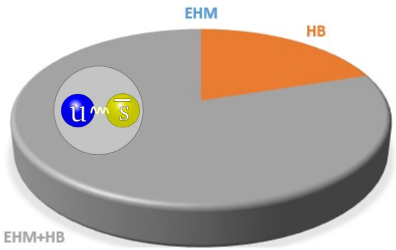
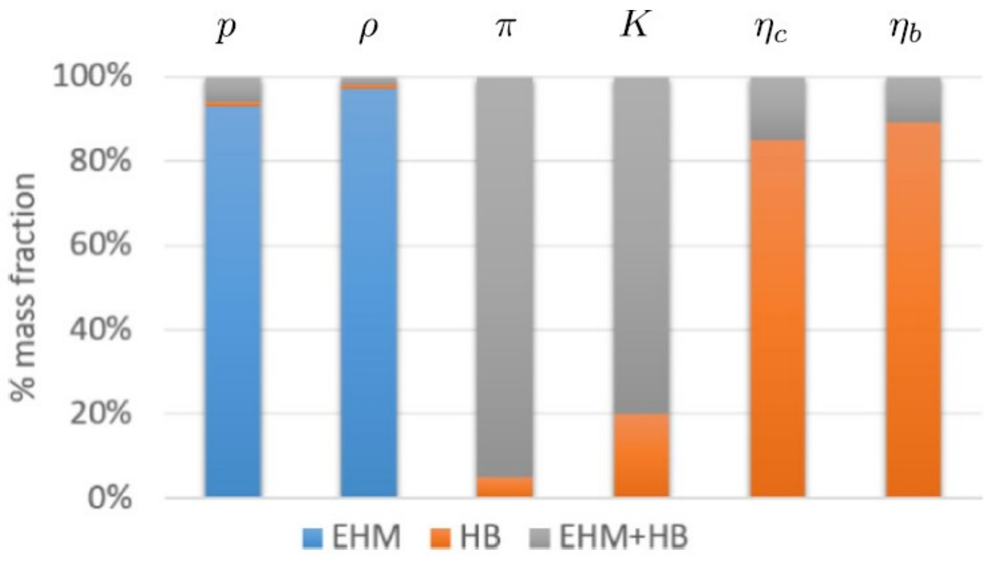


Pion mass budget



Kaon mass budget



- The scrutiny of ***all*** pseudoscalars sheds light on the role of **Mass Sources** on the hadron **structural properties**.

L. Chang *et al.*,
Phys.Rev.Lett. 110 (2013) 13, 132001

Valence-quark distribution amplitudes (DAs)

$$f_M \phi_M^q(x) = \mathrm{tr} \int_{dk} \delta_n^x(k_M) \gamma_5 \gamma \cdot n \chi_M(k_-, P)$$

Light-front momentum fraction

Written in terms of **BSWF**

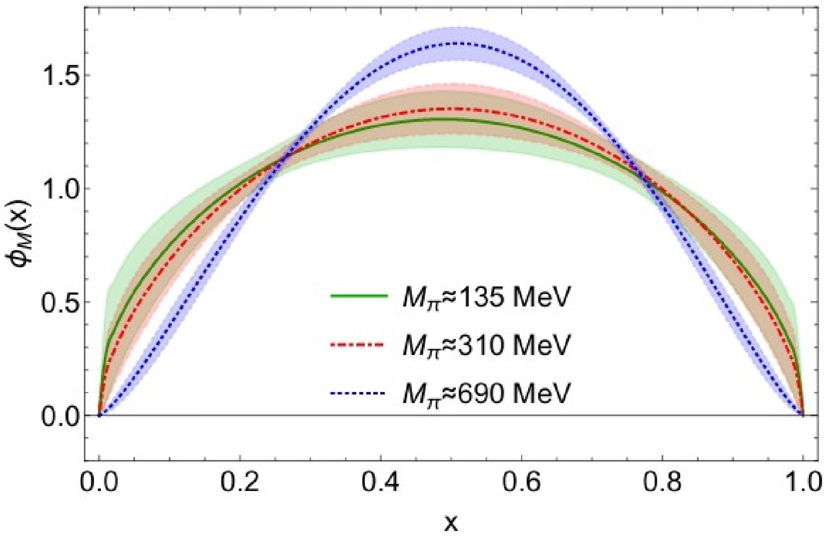
- **1-dimensional** projection of the **light-front wavefunction**.
- Clear **probe of EHM**, related with hard **exclusive processes**, etc.

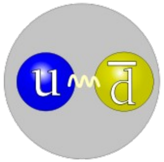
π-K **DAs**

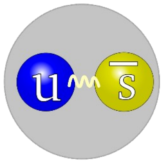
$m_s/m_u \sim 20$

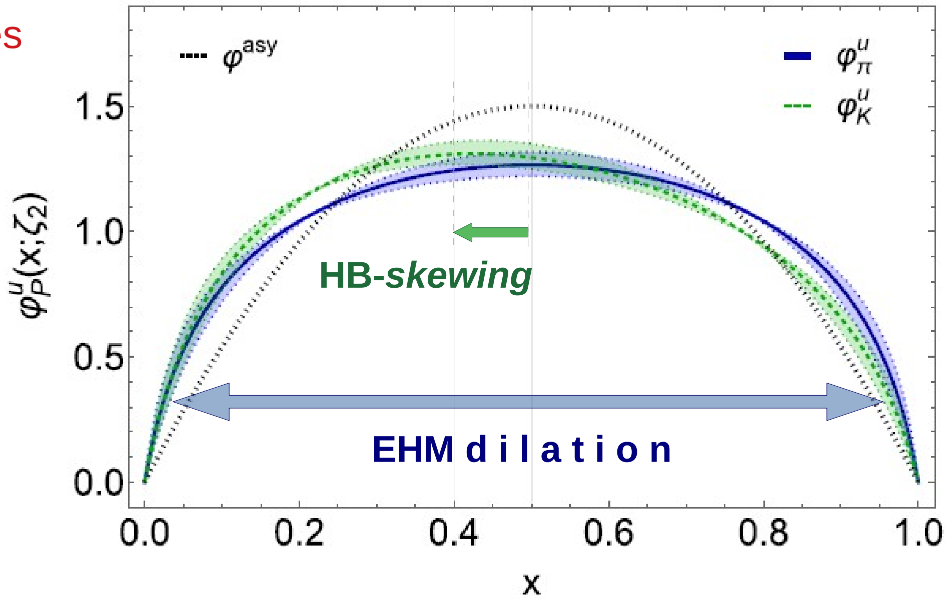
$f_K/f_\pi \sim M_s/M_u \sim 1.2$

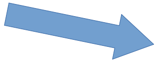
- **Broad** and concave **DAs**.   @ real-life scales
- **Mild** skewing in **Kaon**: **strong** & **weak** interplay.











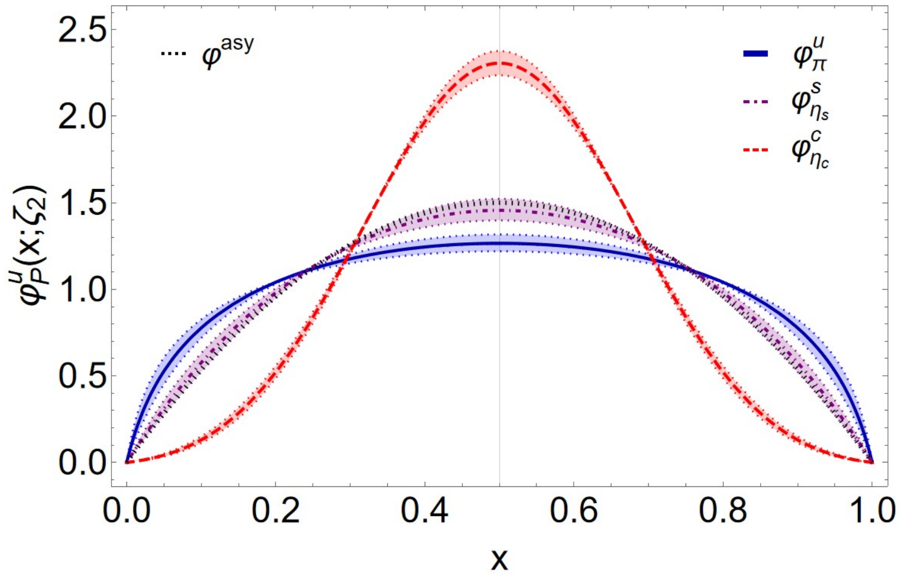
- Lattice QCD supports those findings:

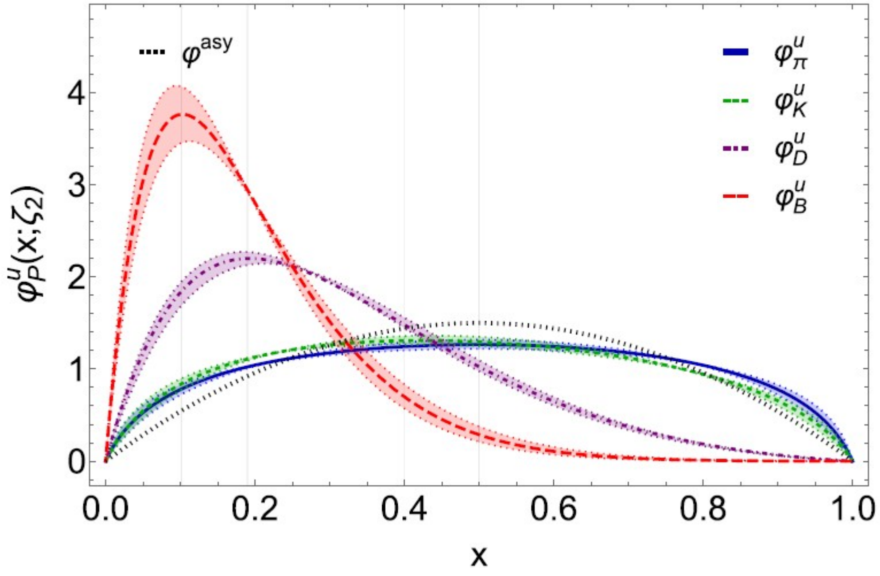
Zhang:2020gaj   Bali:2019dqc   Segovia:2013eca

Pointwise form of the **PDA**!.

‘Heavy’ mesons **DAs**

- In systems with heavy quarks, the **DAs** become **narrow**.



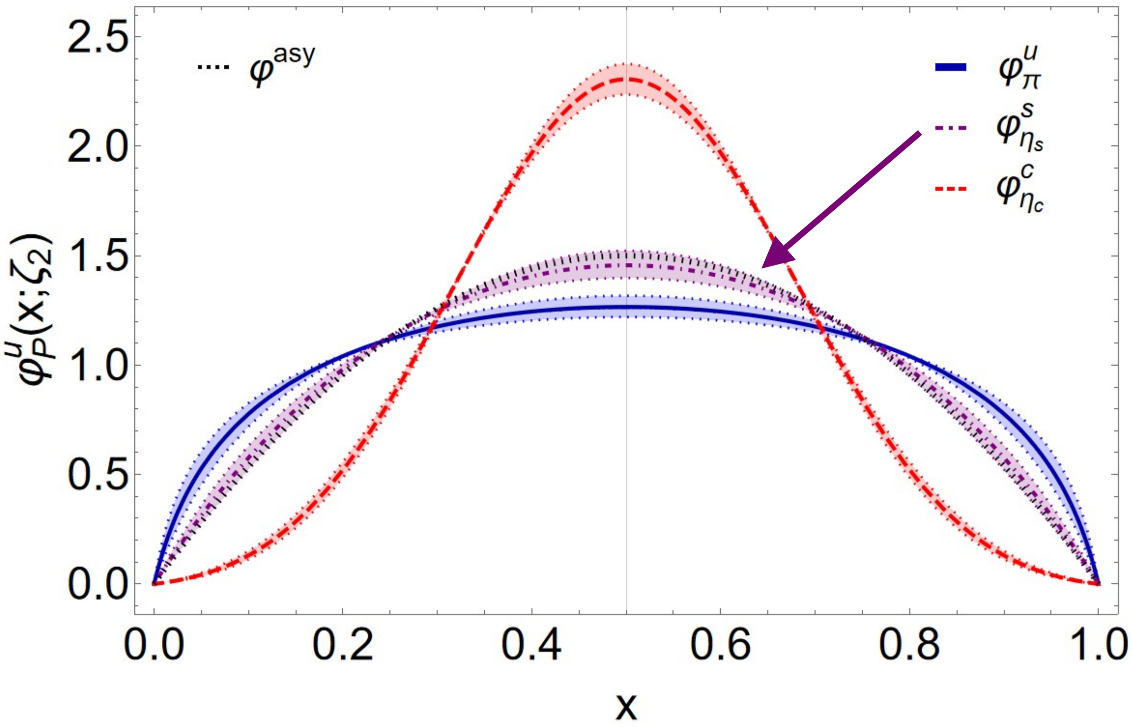


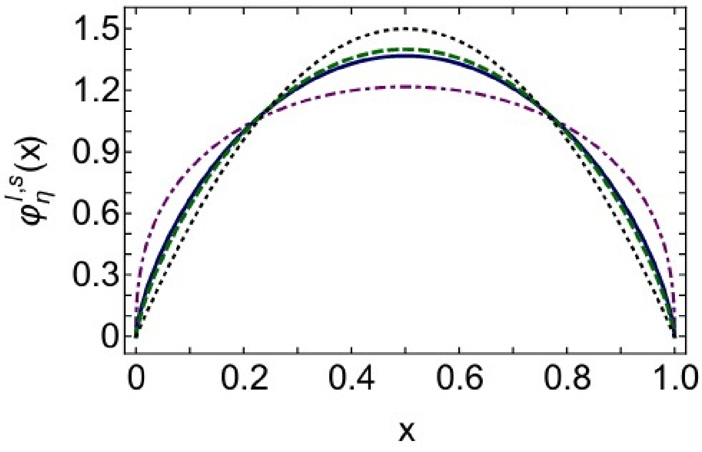
- Unlike the Kaon, heavy-light systems **DAs** are markedly ***skewed***
  - The **peaks** located at: $x_{\max}^{\pi,K,D,B} = 0.5,\, 0.4,\, 0.18,\, 0.1$

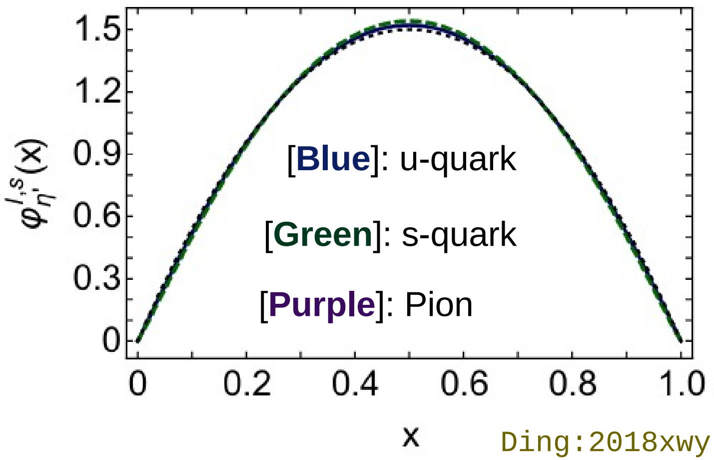
Binosi:2018rht

Drawing **boundaries**

- Systems with ss-bar components draw the line between **strong** and **weak** mass generation being dominant.



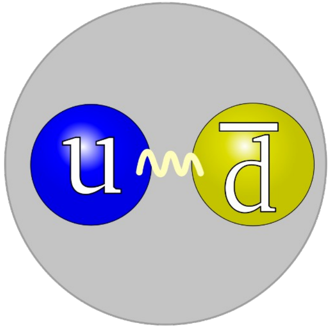




Ding:2018xwy

M. Ding *et al.*, PRD 101 5, 054014 (2020)
Z-F Cui *et al.*, EPJC 80 11, 1064 (2021)

Distribution functions (DFs)

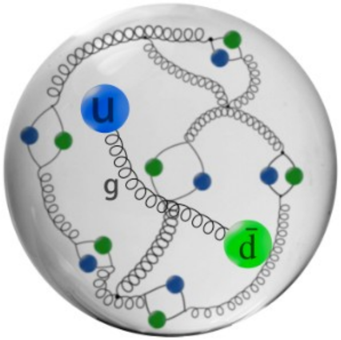


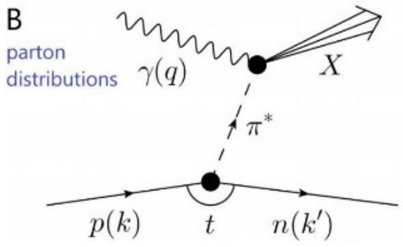
ζ_H

$\zeta > \zeta_H$

Resolution Scale

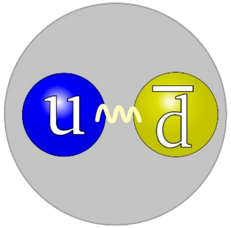
Evolution equations





- Yields *e.g.* information on **momentum** distribution.
- Evolution disentangles **valence**, **sea** and **gluon** contributions.

π-K DFs: hadronic scale



$(M_u = M_d)$

- **Fully-dressed valence quarks** (quasiparticles)

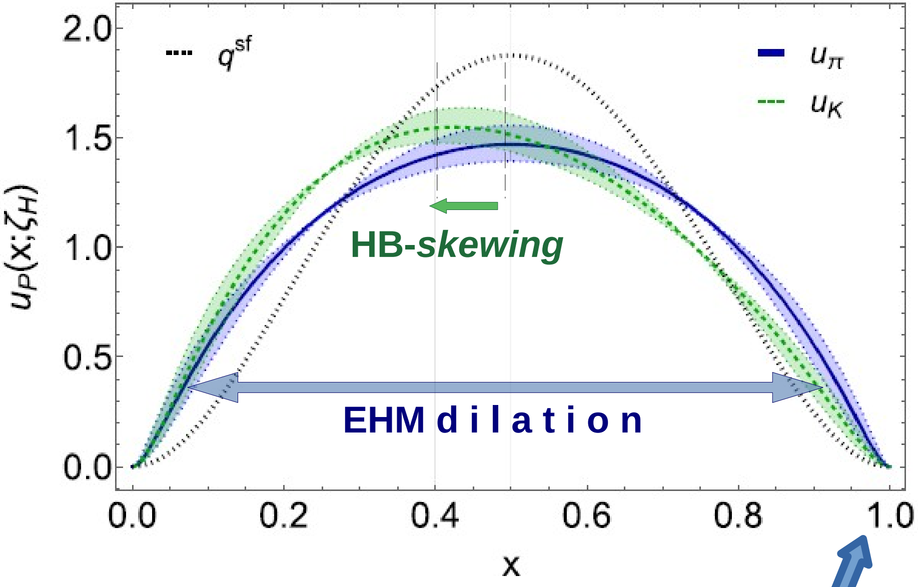
ζ_H : hadronic scale

- At this scale, **all properties** of the hadron are contained within their valence quarks.
  - Equally massive quarks symmetric distributions and equitable distribution of momentum fraction:

$$<x>^u_\pi = 0.5\,,\; u_\pi(x;\zeta_H) = u_\pi(1-x;\zeta_H)$$

- The **kaon** distributions are only-shifted by a **few-percentage**.

$$<x>^u_K = 0.48\,,\; <x>^{\bar{s}}_K = 0.52$$

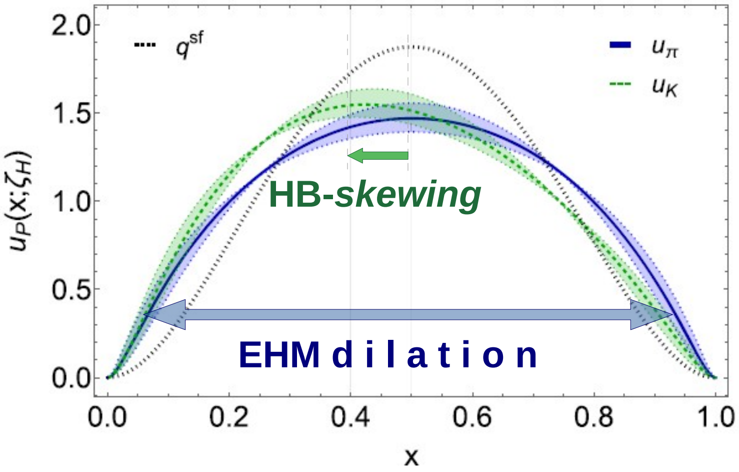


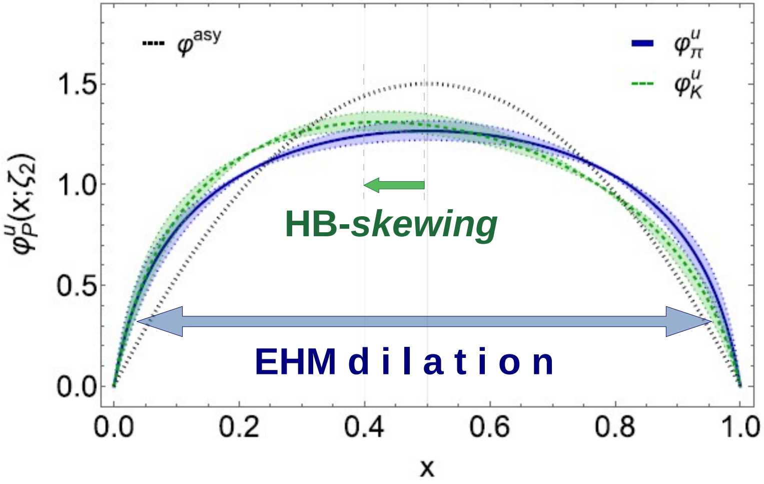
Endpoint behavior is a reflection of the **underlying interaction**

$$1/(k^2)^\beta \to (1-x)^{2\beta}$$

Farrar:1975yb Berger:1979du Holt:2010vj

π-K DFs: hadronic scale

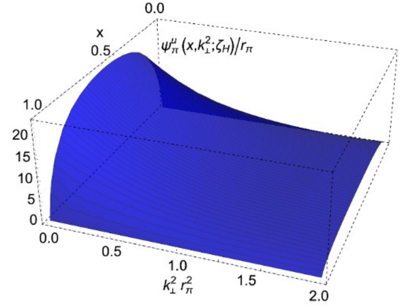


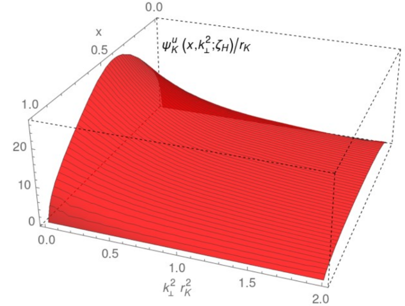


- As the **DAs**, the **π-K DFs** are **dilated**.
- The **bridge** between **DA** and **DF** is the **light-front wavefunction**:

$$f_P \varphi^u_P(x,\zeta_{\mathcal{H}}) = \int \frac{dk_\perp^2}{16\pi^3} \psi^u_P\left(x, k_\perp^2; \zeta_{\mathcal{H}}\right)$$

$$u^P(x;\zeta_{\mathcal{H}}) = \int \frac{d^2k_\perp}{16\pi^3} \left|\psi^u_P\left(x, k_\perp^2; \zeta_{\mathcal{H}}\right)\right|^2$$





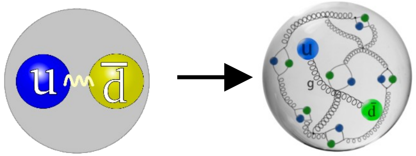
- Such connection suggests:

$$u^P(x;\zeta_H) \sim \left[\varphi^u_P(x;\zeta_H)\right]^2$$

  - Which is a fair approximation for integrated quantities of NG bosons

Raya:2021zrz

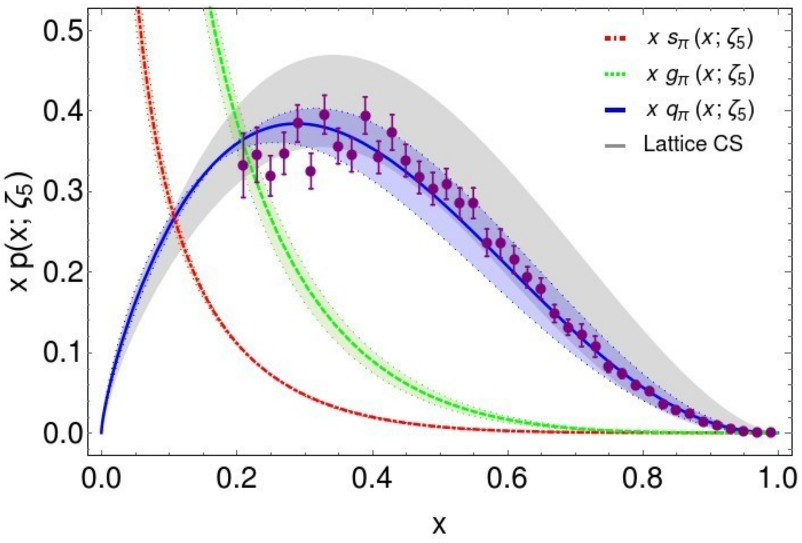
Pion DFs: Lattice & Experiment



- At **5.2 GeV**, the experimental scale, our predictions matches that from Aicher *et al.*

Aicher:2010cb

$$<x_{\text{gluon}}> = 0.45(1)\ ,\ <x_{\text{sea}}> = 0.14(2)$$



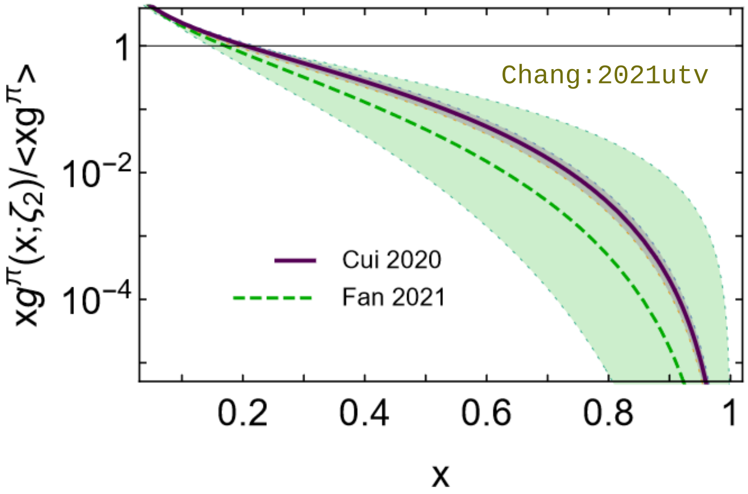
- An agreement with novel **lattice** "Cross Section" results is also obtained.

Sufian:2019bol

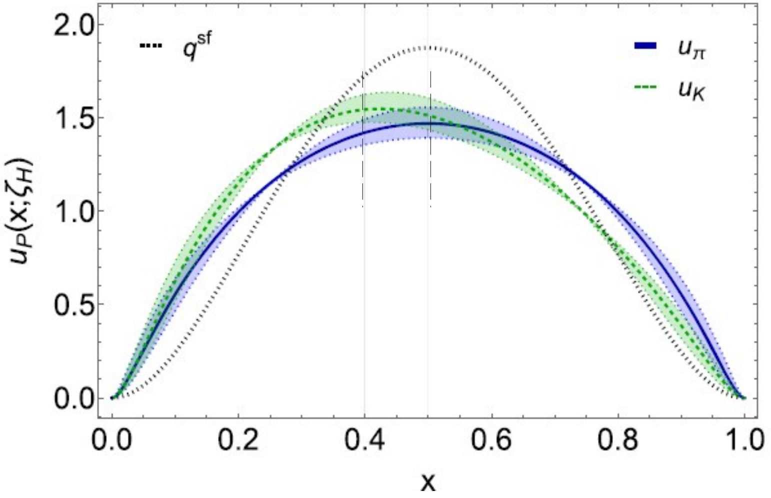
- At **2 GeV**, the **valence DF** shows agreement with lattice moments:

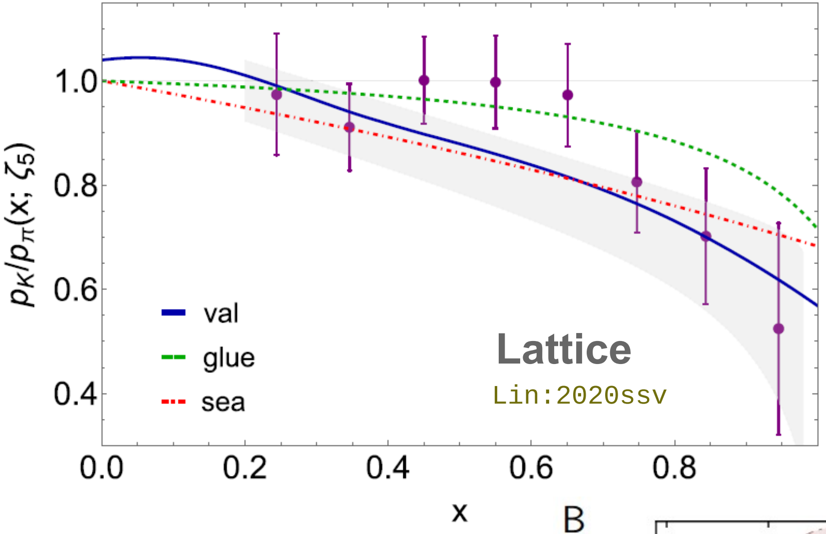
ζ_2	$\langle x\rangle_u^\pi$	$\langle x^2\rangle_u^\pi$	$\langle x^3\rangle_u^\pi$
Ref. [34]	0.24(2)	0.09(3)	0.053(15)
Ref. [35]	0.27(1)	0.13(1)	0.074(10)
Ref. [36]	0.21(1)	0.16(3)	
Herein	0.24(2)	0.098(10)	0.049(07)

- The **Gluon DF** profiles matches **lattice** expectations:

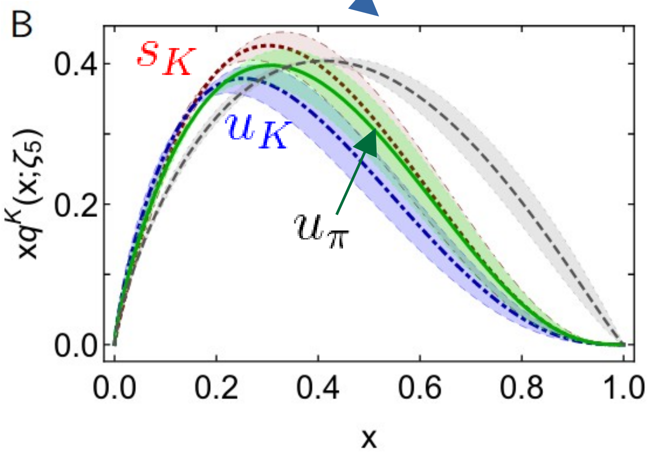


Kaon DFs: Lattice & Experiment





- Ratio is good but **too forgiving!**
- Besides, there are only **few data points**



➢ At the hadronic scale ζ_H, one has:

$$<x>^u_K = 0.48\,,\ <x>^{\bar{s}}_K = 0.52$$

➢ **Above $\zeta > \zeta_H$**, there is slightly more valence content in K.

Massiveness of the s-quark is considered in the evolution equations.

$$<\mathbf{x}>^{\text{val}}_\pi = 0.41(4)$$

$$<\mathbf{x}>^{\text{val}}_K = 0.43(4)$$

$$\zeta = 5.2\,\text{GeV}$$

Pion vs Proton

Y. Lu *et al.*
Phys.Lett.B 830 (2022) 137130

- The (nearly) massless **pion DFs** differs vastly from the massive **proton**. For instance:

  - **Counting rules** entail large-x behaviors **(1-x)²** and **(1-x)³** for the **pion** and **proton**, respectively.

  - The **momentum fractions** at ζ_H: $(M_u = M_d)$

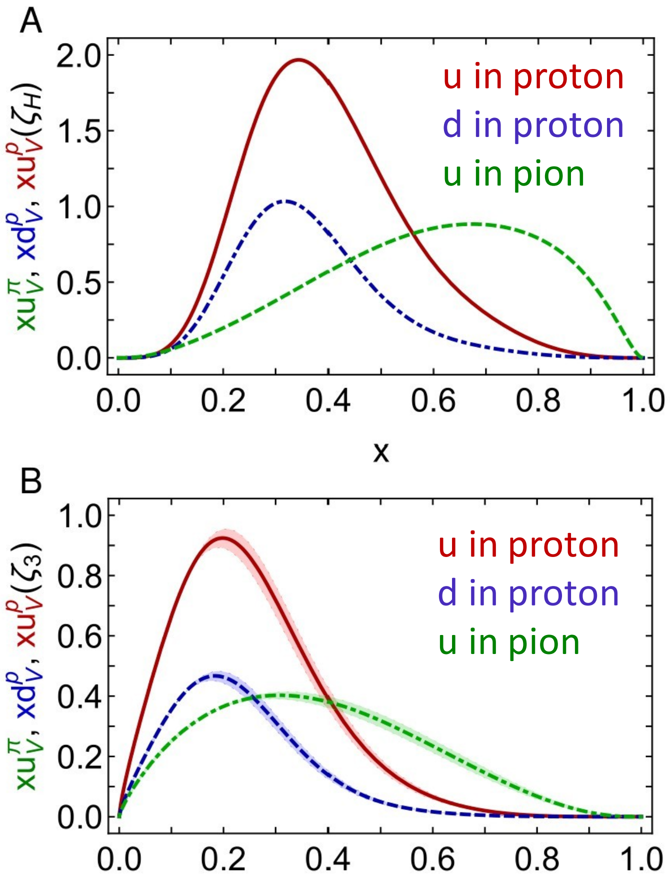
$$\langle x\rangle_{u_p}^{\zeta_{\mathcal{H}}} = 0.687\,,\ \langle x\rangle_{d_p}^{\zeta_{\mathcal{H}}} = 0.313\,,\ \langle x\rangle_{u_\pi}^{\zeta_{\mathcal{H}}} = 0.5$$

$$\Rightarrow u_V\,(x) \neq 2d_V(x)$$

EHM induced diquark correlations inside the proton:

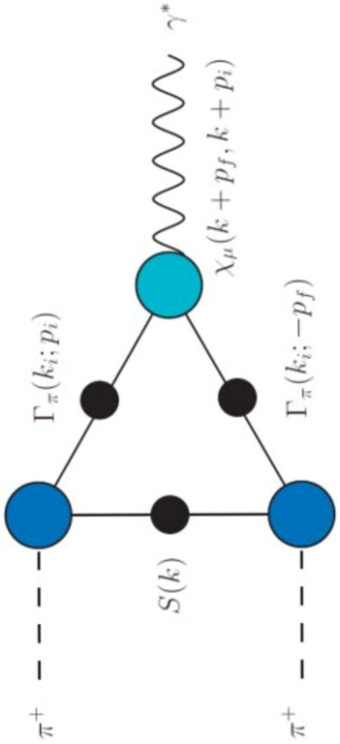
→ No equitable distribution of momentum!

  - Differences are **preserved** after evolution. →



L. Chang *et al.*,
Phys.Rev.Lett. 111 (2013) 14, 141802

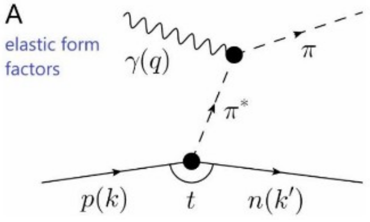
Electromagnetic Elastic Form Factors (EFFs)



$$K_\mu F_M(Q^2) = N_c \mathrm{tr} \int_{dk} \chi_\mu(k+p_f, k+p_i)\Gamma_M(k_i;p_i)S(k)\gamma_M(k_f;-p_f)$$

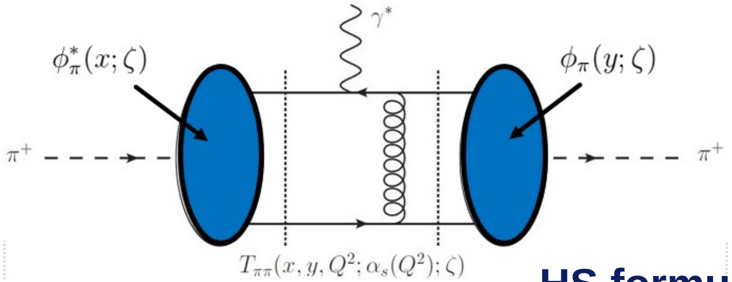
All can be written in terms of **propagators** and **vertices**

- Gives information on **momentum**/**charge** distribution.
- **Pion EFF** highly relevant for contemporary physics.



Elastic Form Factors

- Clear probe of the hadron's structure.
  - ➔ Structure manifests in $F(Q^2)$!= constant
- **Connected** with the **DA**:

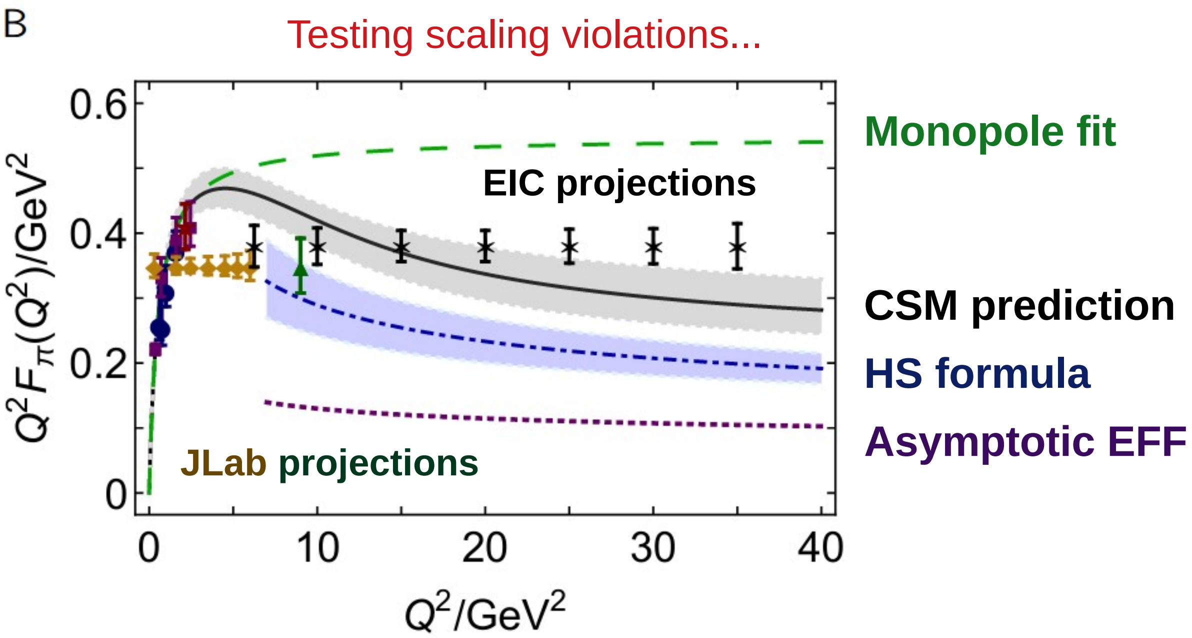


HS formula

(at sufficiently large Q^2)

- ➔ **Factorization** is a proof of the validity of **QCD itself.**

Testing scaling violations...



$$\exists Q_0 > \Lambda_{\rm QCD} \mid Q^2 F_\pi(Q^2) \overset{Q^2>Q_0^2}{\approx} 16\pi\alpha_s(Q^2) f_\pi^2 w_\varphi^2,$$

$$w_\varphi = \frac{1}{3}\int_0^1 dx\, \frac{1}{x}\varphi_\pi(x)$$ ← **PDA**

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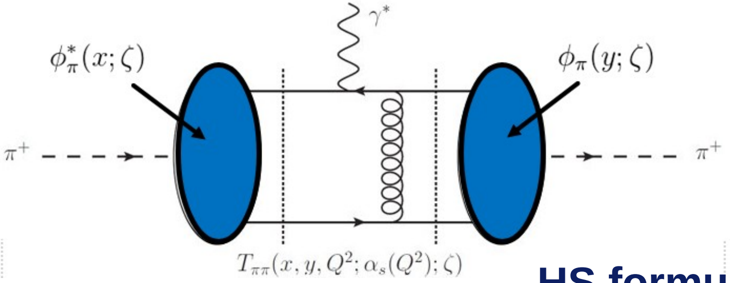

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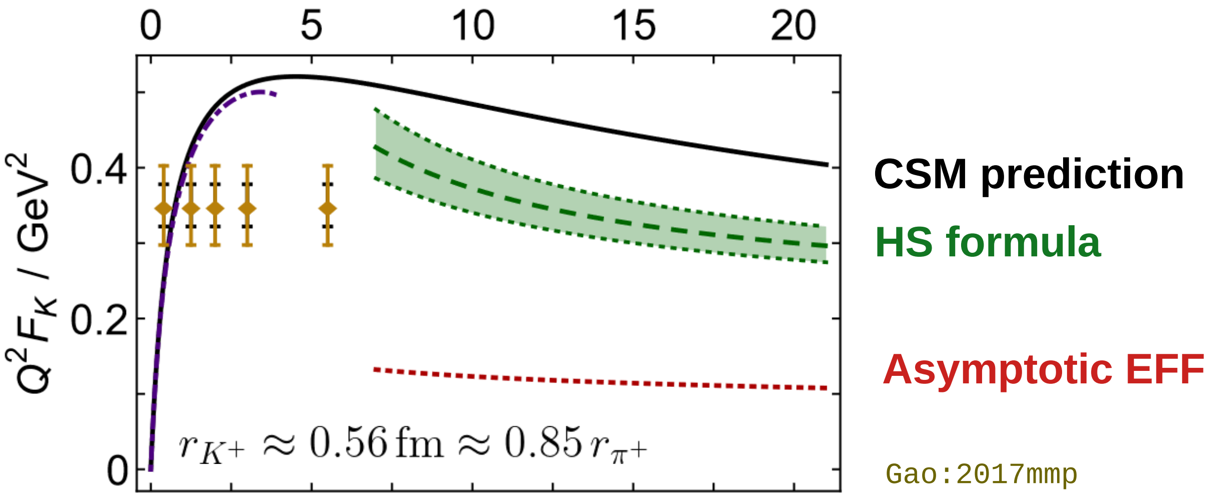

Gao:2017mmp

$$\exists Q_0 > \Lambda_{\rm QCD} \mid Q^2 F_\pi(Q^2) \overset{Q^2>Q_0^2}{\approx} 16\pi\alpha_s(Q^2) f_\pi^2 w_\varphi^2,$$

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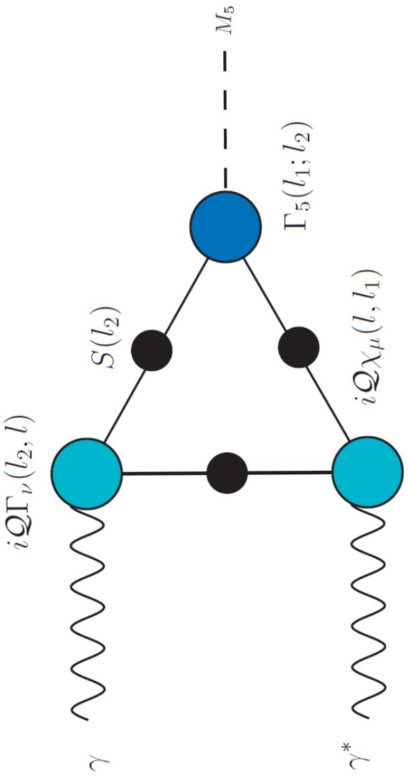
Two-photon Transition Form Factors (TFFs)

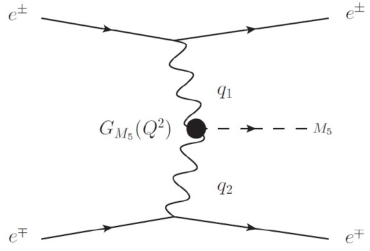


$$T_{\mu\nu}(k_1,k_2) = \epsilon_{\mu\nu\alpha\beta}k_{1\alpha}k_{2\beta}G_{M_5}(k_1^2, k_1\cdot k_2, k_2^2) \ ,$$

$$T_{\mu\nu}(k_1,k_2) = \mathrm{tr}\int \frac{d^4l}{(2\pi)^4} iQ\chi_\mu(l,l_1)\Gamma_{M_5}(l_1,l_2)S(l_2)iQ\Gamma_\nu(l_2,l)$$

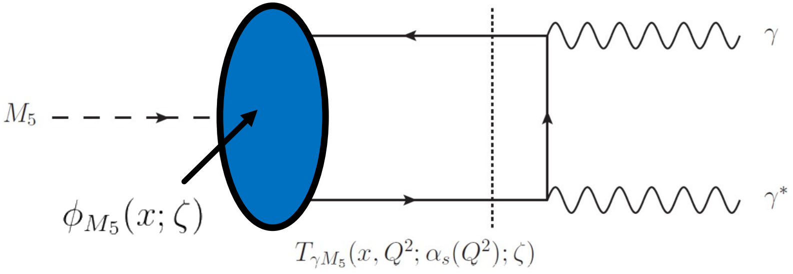
All can be expressed in terms of **propagators** and **vertices**

- Gives information on **momentum**/**charge** distribution.
- **Pion TFF** highly relevant for contemporary physics.



Two-photon TFFs

- Clear probe of the hadron's structure.
  - Structure manifests in $G(Q^2)$!= constant
- **Connected** with the **PDA**:



HS formula

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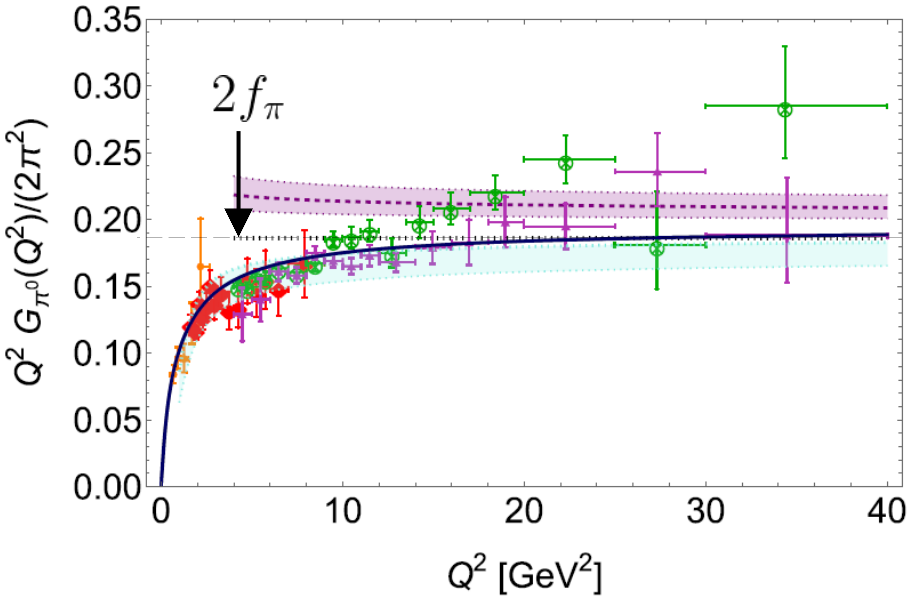


HS formula

CSM prediction

Asymptotic TFF

$$\exists \tilde{Q}_0 > \Lambda_{\rm QCD} | Q^2 G_5(Q^2) \overset{Q^2 \gtrsim \tilde{Q}_0^2}{\sim} f_5\, w_\phi$$

$$w_\varphi = \frac{1}{3}\int_0^1 dx\, \frac{1}{x}\varphi_\pi(x)$$ ← **PDA**

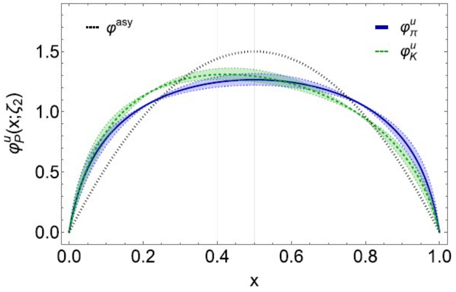
Two-photon TFFs

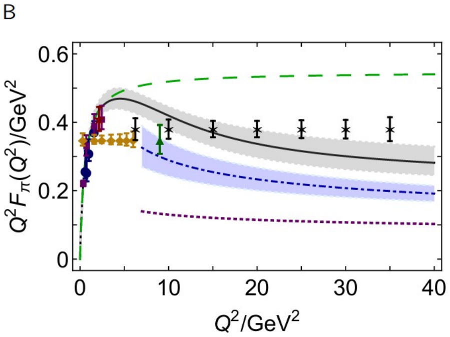
KR, L. Chang, A. Bashir *et al.*,
Phys.Rev.D 93 (2016) 7, 074017

- The **CSM prediction** satisfies the **Abelian anomaly**, $2f_\pi^0 G_{\pi^0}^0(Q^2=0)=1$

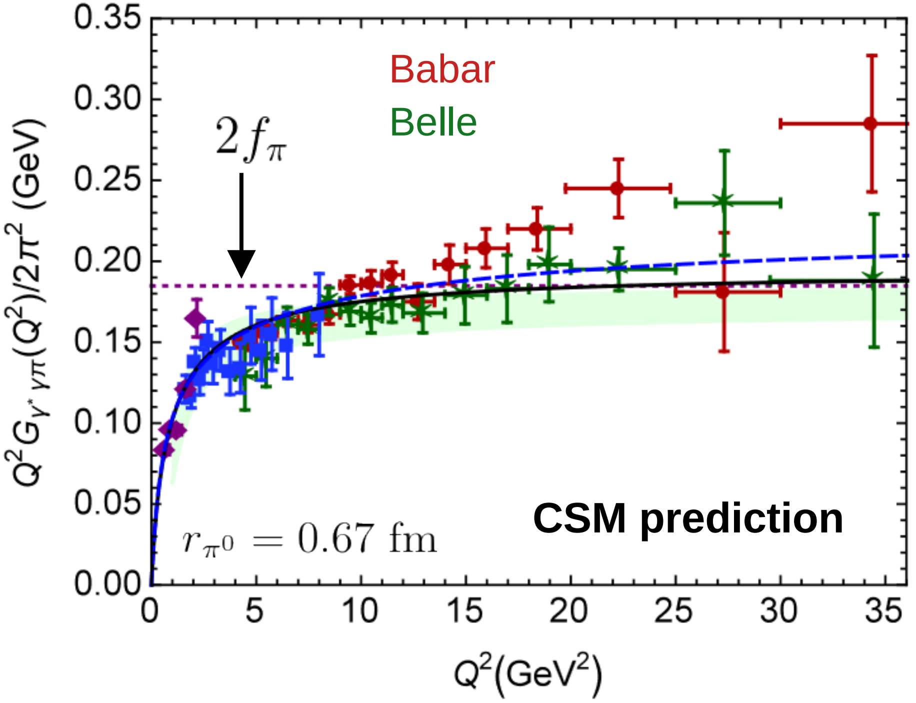
  … while faithfully recovering the **asymptotic limit**.

- A **dilated+concave DA**, at the hadronic scale, connects both pion **EFF** and **TFF**.





- Precise **agreement** with ***all*** experimental data; except for Babar at large **Q²**.

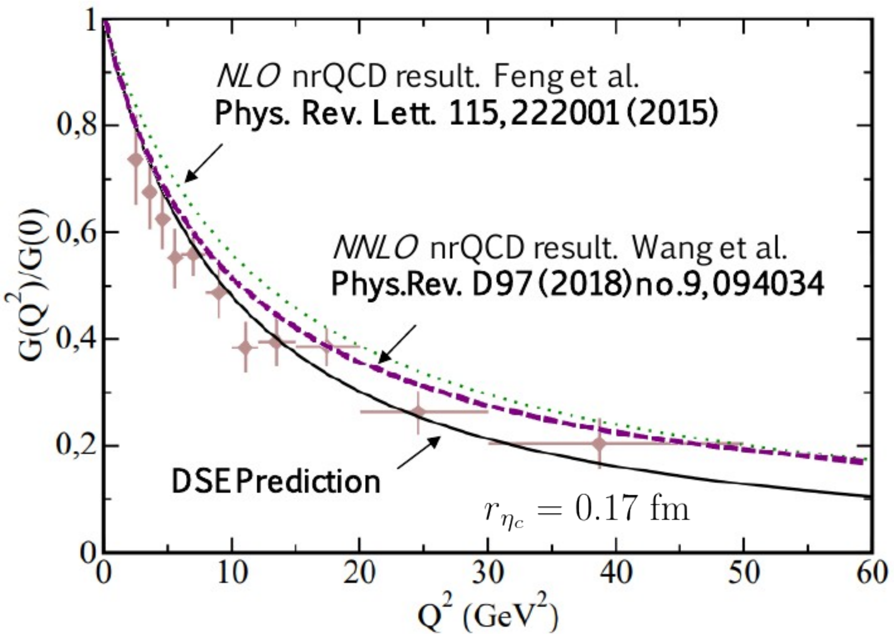


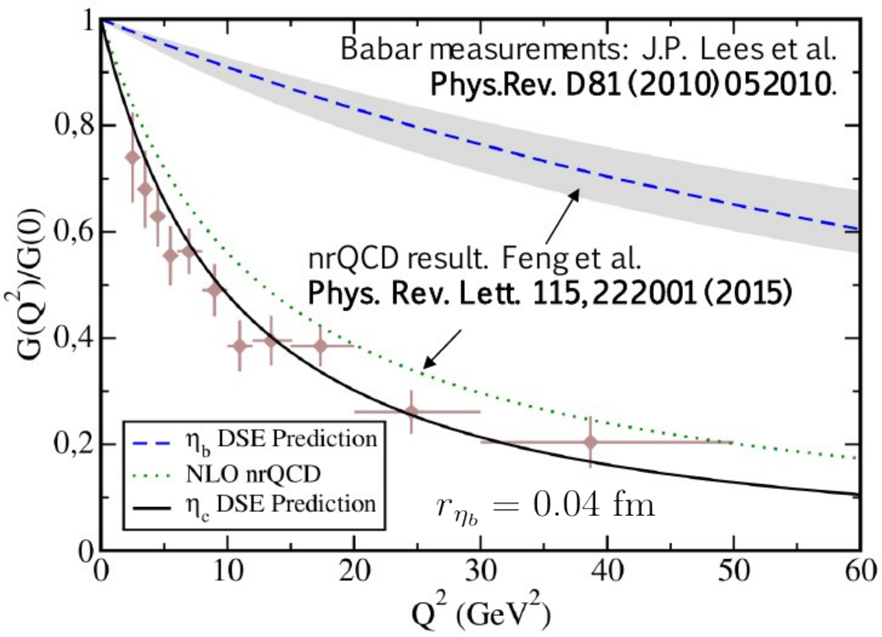
$$\exists \tilde{Q}_0 > \Lambda_{QCD} | Q^2 G_5(Q^2) \overset{Q^2>\tilde{Q}_0^2}{\sim} f_5\, w_\phi$$

Two-photon TFFs

KR, M. Ding *et al.*
Phys.Rev.D 95 (2017) 7, 074014

- ***All*** two-photon **TFFs** involving ground-state neutral pseudoscalars are within reach:
  - Invariably, **agreement** with the **experimental** data is found, with the exception of the large-Q^2 Babar data for the pion.





Two-photon TFFs

M. Ding, KR *et al.*
Phys.Rev.D 95 (2017) 7, 074014

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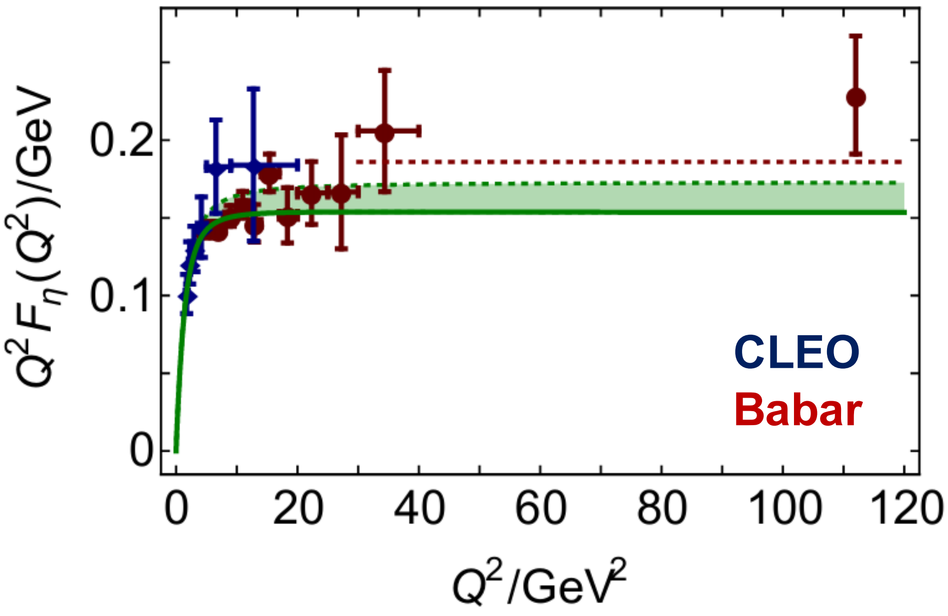
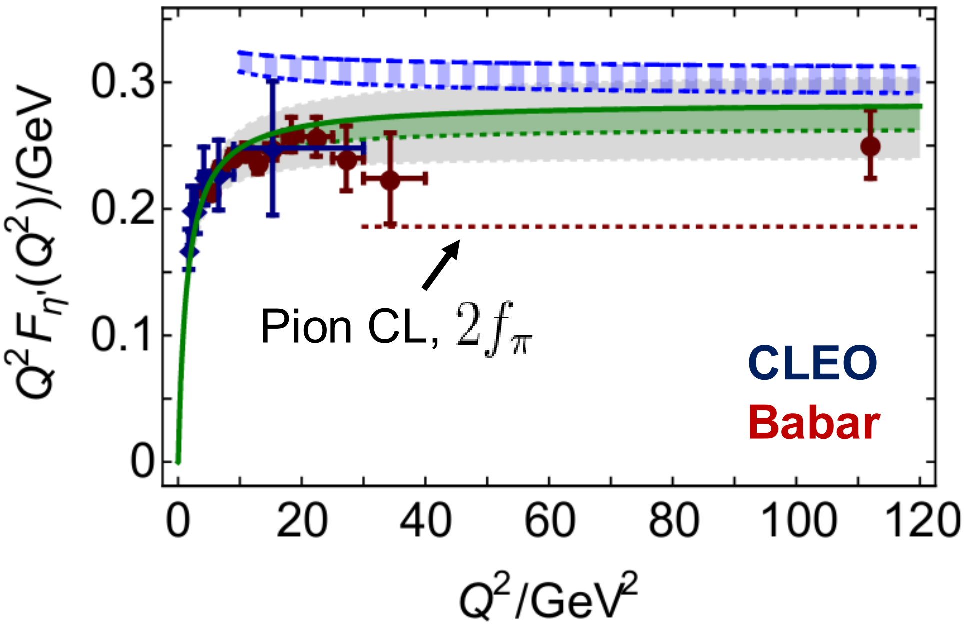


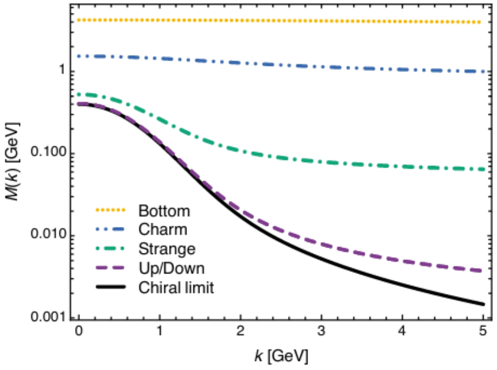


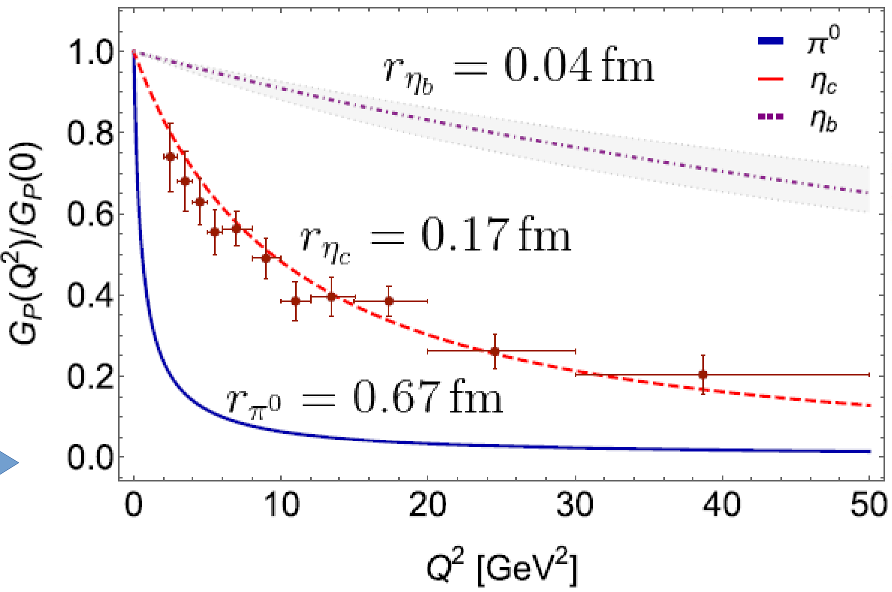
Two-photon **TFFs**

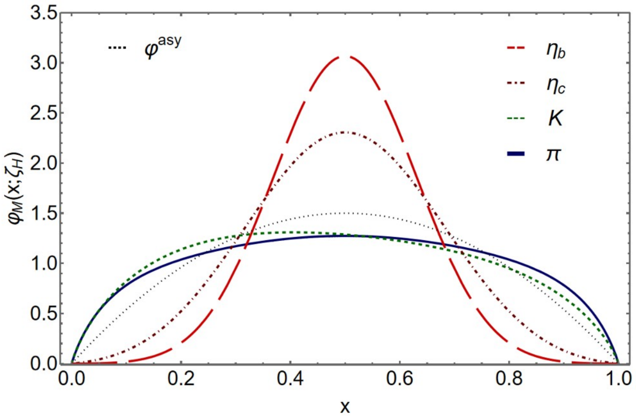
KR, A. Bashir, P. Roig
Phys.Rev.D 101 (2020) 7, 074021

- ***All*** two-photon **TFFs** involving ground-state neutral pseudoscalars are within reach:
  - Invariably, **agreement** with the **experimental** data is found, with the exception of the **large-Q²** Babar data for the pion.

- Clearly, the shape of **M(k)** echoes in **TFFs** and **DAs.**

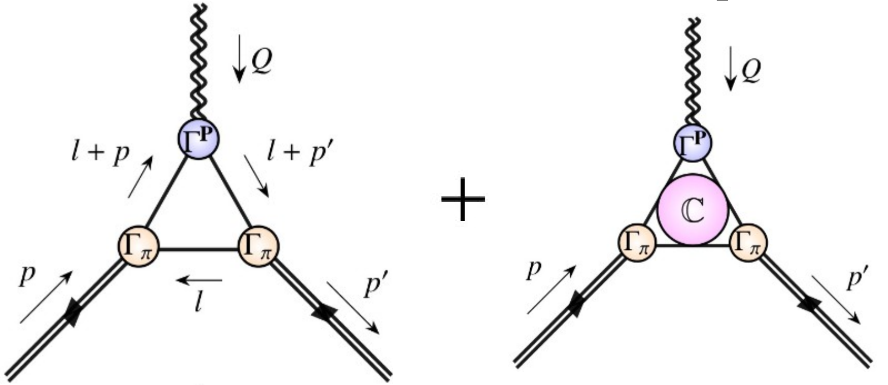






Yin-Zhen Xu *et al.*,
Eur.Phys.J.C 84 (2024) 2, 191

Gravitational Form Factors (GFFs)



- Gives information on **mass/pressure** distribution.
- A first step towards **nucleon GFFs**

Gravitational form factors

- The expectation value of the energy-momentum tensor (**EMT**) in the **pion** defines the **gravitational form factors**:

$$\Lambda_{\mu\nu}(P,Q) = 2P_\mu P_\nu \theta_2(Q^2) + \frac{1}{2}\left(Q^2 g_{\mu\nu} - Q_\mu Q_\nu\right)\theta_1(Q^2) + 2m_\pi^2 g_{\mu\nu}\bar{c}(Q^2)$$

- Where **symmetry principles** entail:

$$\theta_2^\pi(0) = 1$$

Momentum conservation

$$\theta_1^\pi(0) \overset{m_\pi^2=0}{=} 1$$

Soft-pion theorem

$$\bar{c}^\pi(Q^2) \equiv 0$$

EMT conservation

(Other pseudoscalars are defined in analogy)

Polyakov:2018zvc

GFFs: Setting the stage

Yin-Zhen Xu *et al.*,
Eur.Phys.J.C 84 (2024) 2, 191

- The matrix element can be expressed in terms of **propagators** and **vertices**:

$$\Lambda_{\mu\nu}(P,Q) = N_c \int_{dk} \mathrm{Tr}\left[\Gamma_\pi\left(k-\frac{Q}{4}, P-\frac{Q}{2}\right) S\left(k-\frac{P}{2}\right) \Gamma_\pi\left(k+\frac{Q}{4}, P+\frac{Q}{2}\right) S\left(k+\frac{P}{2}+\frac{Q}{2}\right) \underbrace{\Gamma_{\mu\nu}\left(k+\frac{P}{2}, Q\right)} S\left(k+\frac{P}{2}-\frac{Q}{2}\right)\right]$$

\+ beyond **I.A.**

Quark-tensor vertex (QTV): Interaction with a **spin-2** probe

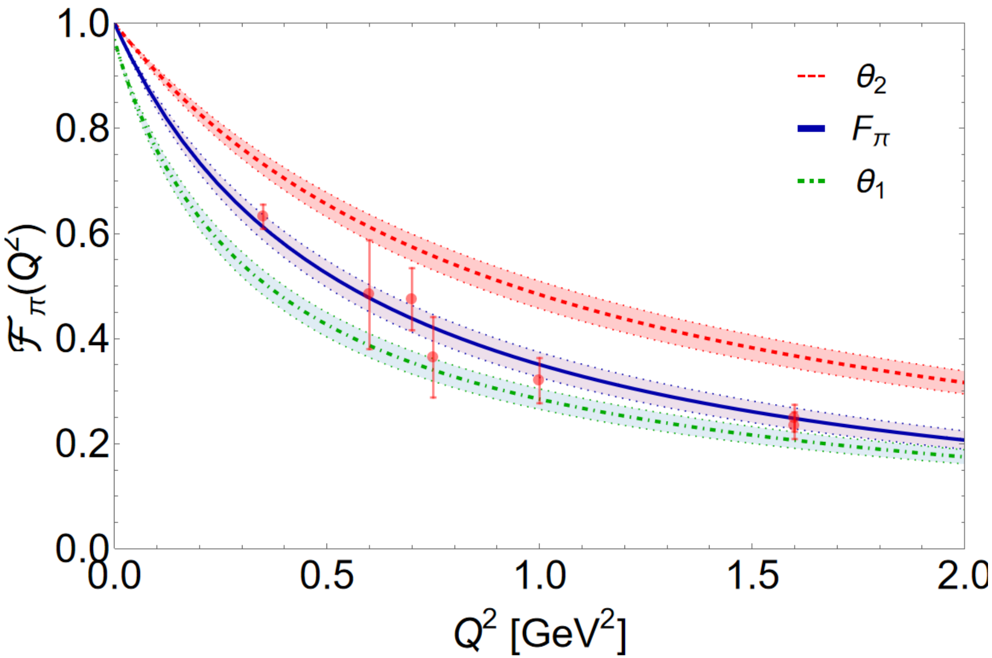
- Partially constrained by its **Ward-Green-Takahashi** identity:

$$iQ_\mu \Gamma^{\mu\nu}(P,Q) = P_i^\nu S^{-1}(P_f) - P_f^\nu S^{-1}(P_i)$$

- Those pieces escaping the **WGTI**, encode **scalar** and **tensor** meson poles.

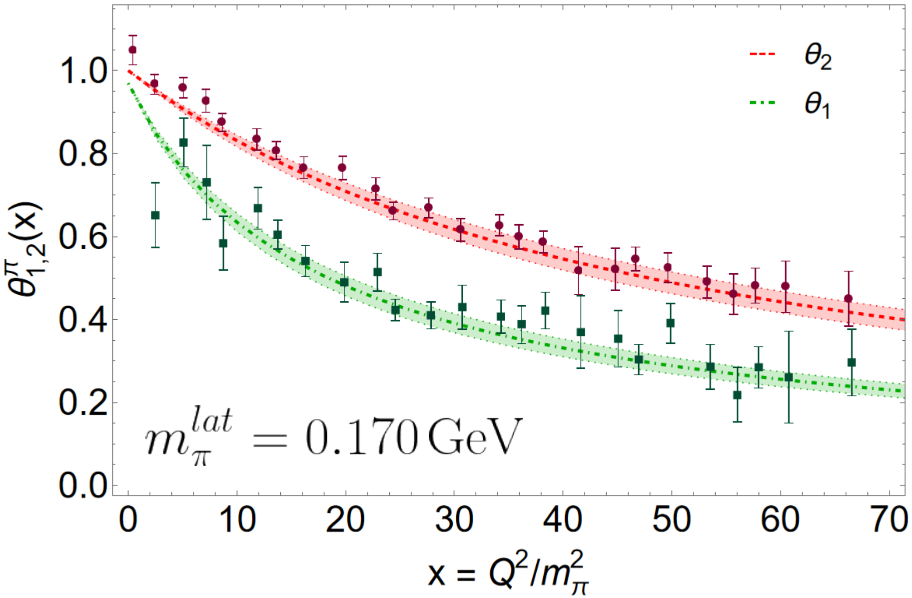
π-K **GFFs**

- It is found that: $r^{\pi}_{\theta_1}$ (pressure) $>$ r^{π}_{F} (charge) $>$ $r^{\pi}_{\theta_2}$ (mass)



$$r_{\theta_1}/r_F \approx r_F/r_{\theta_2} \approx 0.75$$

- Agreement with **Lattice QCD** is also obtained: Hackett:2023nkr

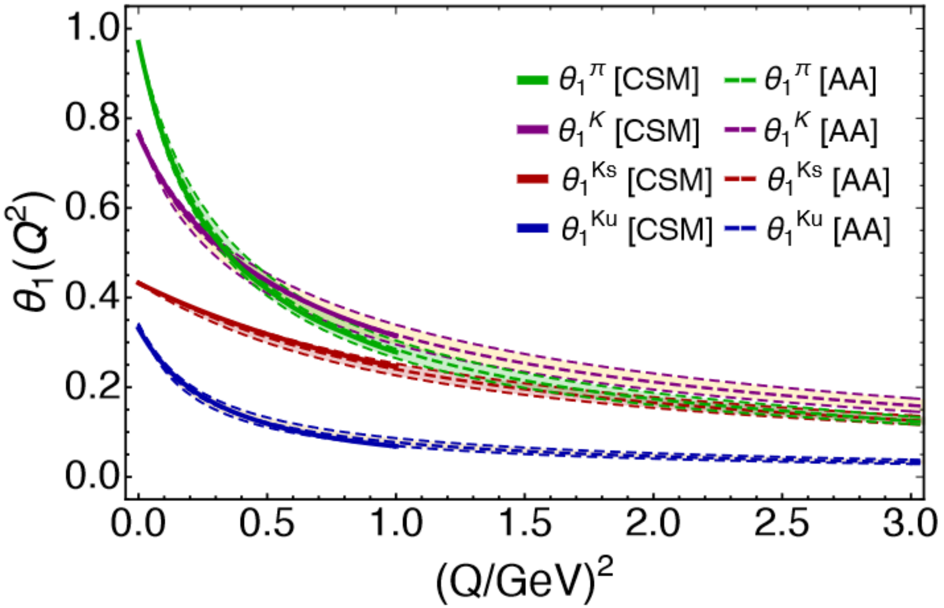


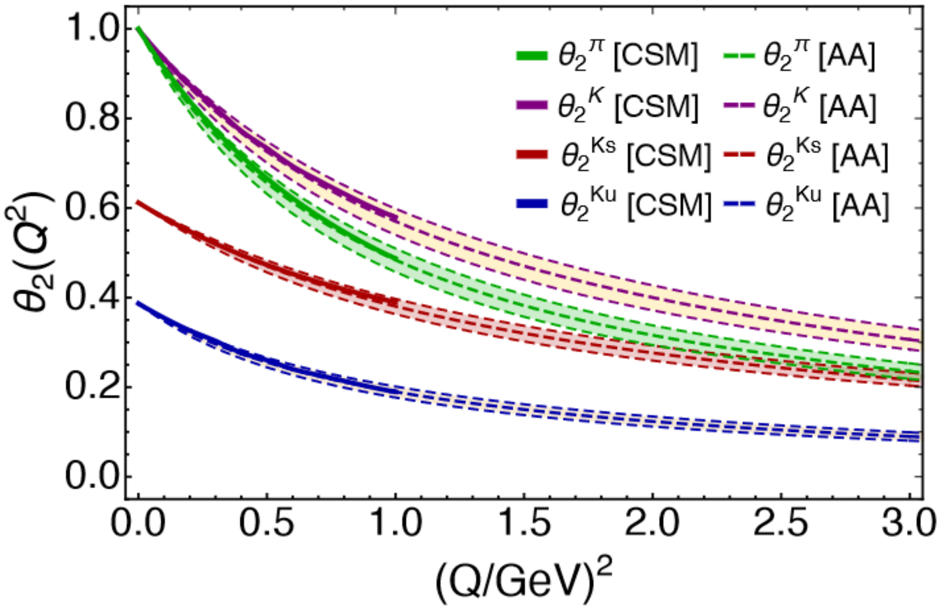
π-K GFFs

- The **Kaon**, albeit more **compressed**, exhibit similar patters.

$$\bar{r}_K/\bar{r}_\pi = 0.85(6)$$

$$(\approx f_\pi/f_K \sim M_u/M_s)$$





- In addition to:

$$\theta_1^{\pi,K}(0) = 0.97, 0.77$$

(ChPT predicts: Polyakov:2018zvc

$$\theta_1^{\pi,K}(0) = 0.97(1), 0.77(10)$$)

π-K: Pressure profiles

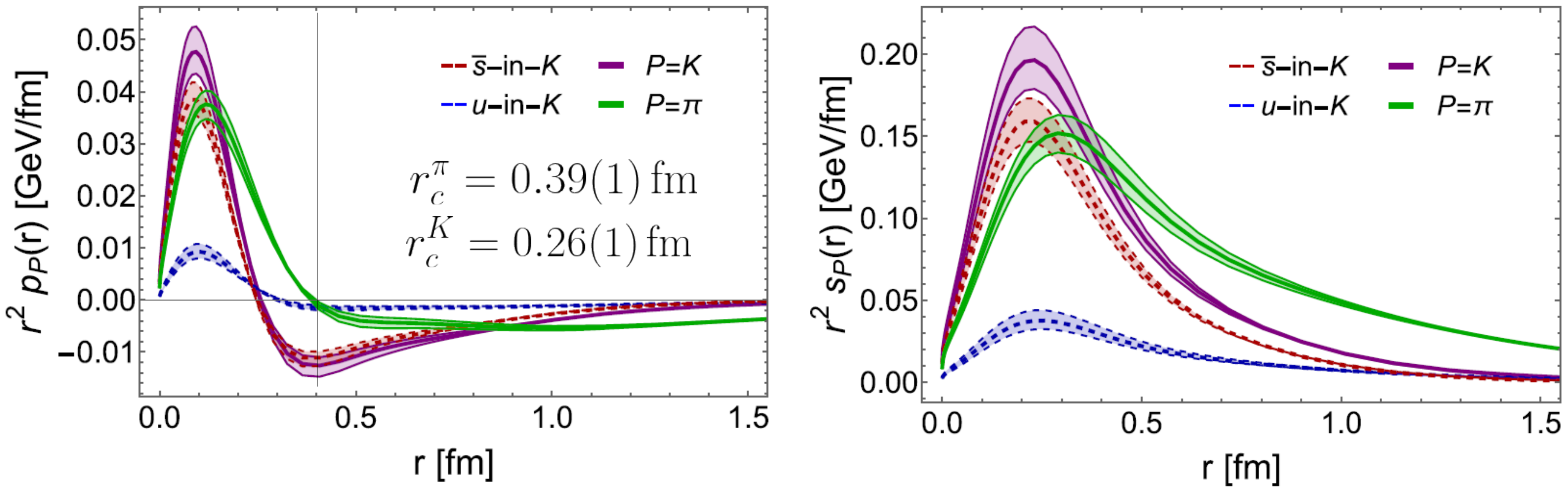
$$p_K^u(r) = \frac{1}{6\pi^2 r}\int_0^\infty d\Delta\, \frac{\Delta}{2E(\Delta)}\, \sin(\Delta r)[\Delta^2 \theta_1^{K_u}(\Delta^2)]\,,$$

$$s_K^u(r) = \frac{3}{8\pi^2}\int_0^\infty d\Delta\, \frac{\Delta^2}{2E(\Delta)}\, j_2(\Delta r)\,[\Delta^2 \theta_1^{K_u}(\Delta^2)]\,,$$

"Pressure" Quark attraction/repulsion

CONFINEMENT

"Shear" *Deformation* QCD forces



KR, *et al.*,
Chin.Phys.C 46 (2022) 1, 013105

Light-front wave functions (LFWF)

"One ring to rule them all"

$$\psi_{\mathrm{M}}^{q}\left(x,k_{\perp}^{2}\right)=\mathrm{tr}\int_{dk_{\parallel}}\delta_{n}^{x}(k_{\mathrm{M}})\gamma_{5}\gamma\cdot n\,\chi_{\mathrm{M}}(k_{-},P)$$

Bethe-Salpeter wave function

- **Intrinsic** of the hadron's nature.
- Yields a **variety** of **distributions**.

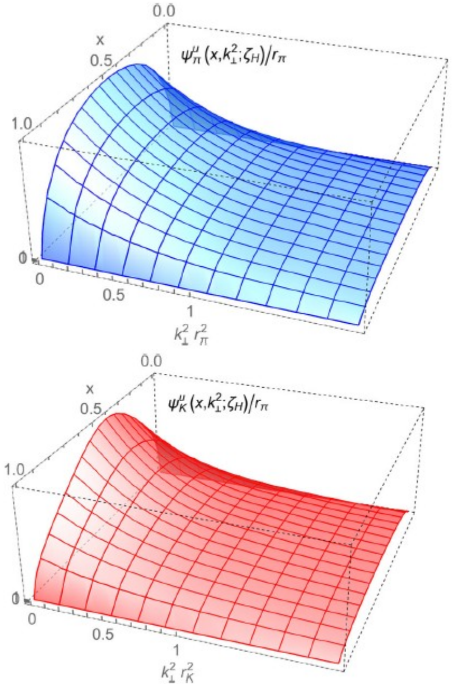
LFWFs: Connecting the dots

The idea: Connect ***everything*** through the **LFWF.**

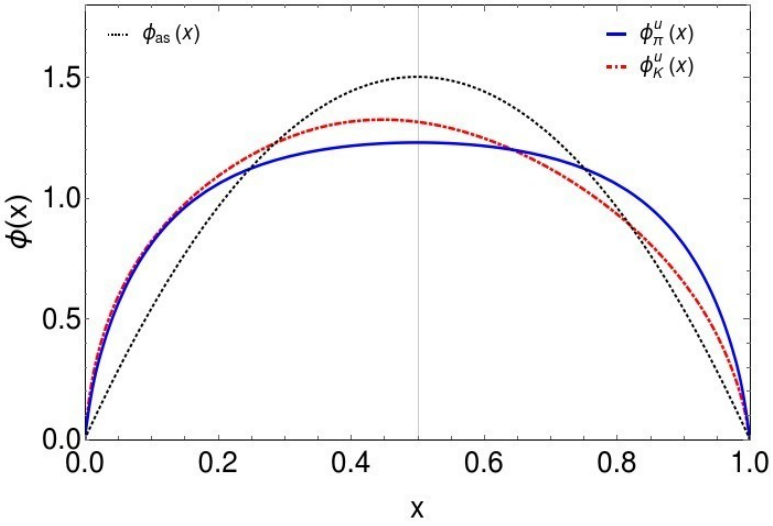
LFWFs → PDAs

LFWFs → TMDs $\sim \psi^* \psi$

$$f_{\mathsf{P}} \varphi_{\mathsf{P}}^{u}(x, \zeta_{\mathcal{H}}) = \int \frac{dk_\perp^2}{16\pi^3} \psi_{\mathsf{P}}^{u}\left(x, k_\perp^2; \zeta_{\mathcal{H}}\right)$$



- **1-D projection** of the hadron's **LFWF**.



$\int dx$ $\int dk_\perp$

$t = 0, \xi = 0$

LFWFs: Connecting the dots

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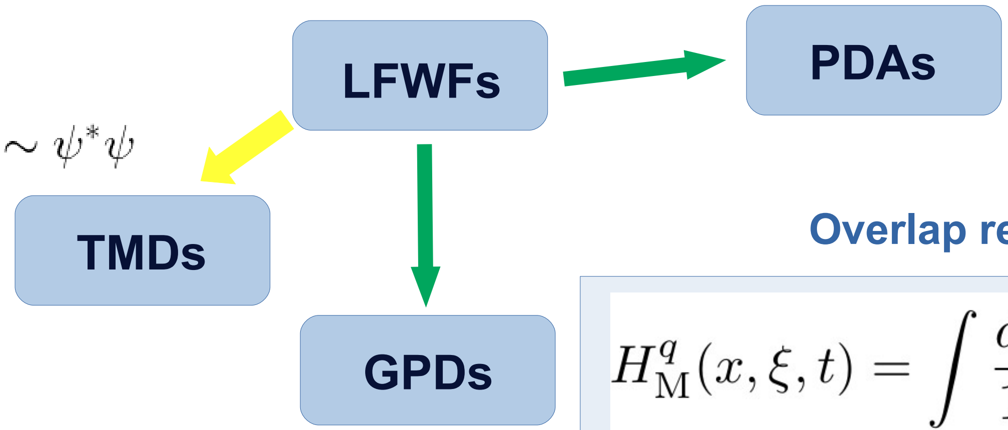


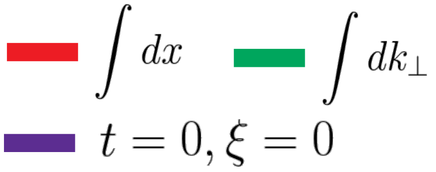
Overlap representation:

$$H_{\rm M}^q(x,\xi,t) = \int \frac{d^2 k_\perp}{16\pi^3} \psi_{\rm M}^{q*}\left(x^-, (\mathbf{k}_\perp^-)^2\right) \psi_{\rm M}^{q}\left(x^+, (\mathbf{k}_\perp^+)^2\right)$$

Sufficient to sketch charge, mass and IPS distributions...

- ✓ ***Valid*** in the **DGLAP** region
- ✓ ***Positivity*** fulfilled
- ✓ Can be ***extended*** to the **ERBL** region

Chavez:2021llq



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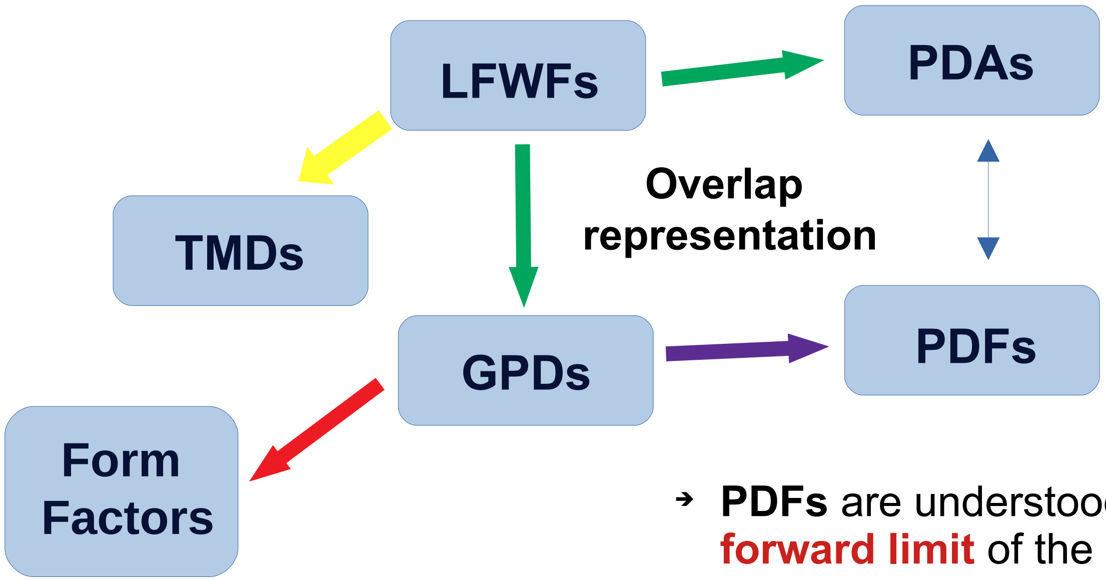


$$f_{\mathsf{P}}\varphi_{\mathsf{P}}^{u}(x,\zeta_{\mathcal{H}}) = \int \frac{dk_\perp^2}{16\pi^3}\psi_{\mathsf{P}}^{u}\left(x,k_\perp^2;\zeta_{\mathcal{H}}\right)$$

$$u^{\mathsf{P}}(x;\zeta_{\mathcal{H}}) = \int \frac{d^2k_\perp}{16\pi^3}\left|\psi_{\mathsf{P}}^{u}\left(x,k_\perp^2;\zeta_{\mathcal{H}}\right)\right|^2$$

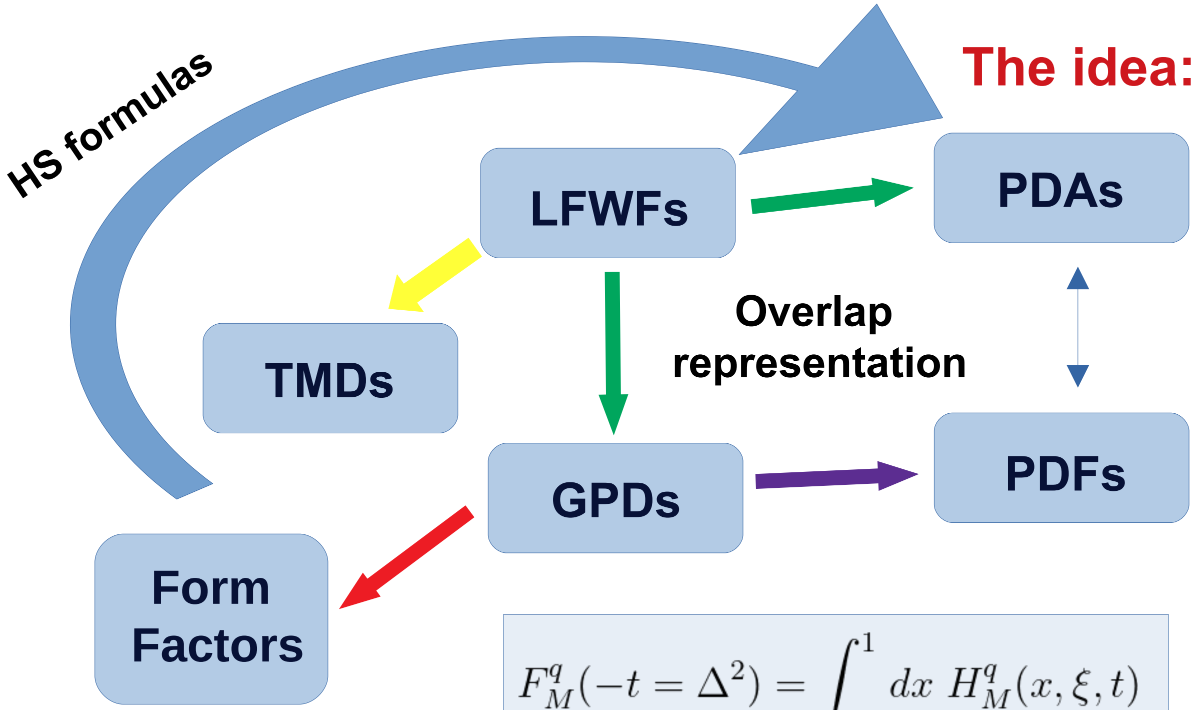
- **PDFs** are understood as the **forward limit** of the **GPD**

$$q(x) = H(x,0,0)$$

Red: $\int dx$ Green: $\int dk_\perp$

Purple: $t=0, \xi=0$

LFWFs: Connecting the dots



The idea: Connect ***everything*** through the **LFWF.**

$$f_{\mathsf{P}}\varphi_{\mathsf{P}}^{u}(x,\zeta_{\mathcal{H}}) = \int \frac{dk_\perp^2}{16\pi^3}\psi_{\mathsf{P}}^{u}\left(x,k_\perp^2;\zeta_{\mathcal{H}}\right)$$

PDFs are understood as the **forward limit** of the **GPD**

$$q(x) = H(x,0,0)$$

$$u_M(x;\zeta_H) \sim \int d^2k_\perp \psi^*\psi$$

Form factors are expressed as **sum rules** of the **GPD**

$$F_M^q(-t=\Delta^2) = \int_{-1}^{1} dx\; H_M^q(x,\xi,t)$$

Electromagnetic FF

$$J_M(t,\xi) = \int_{-1}^{1} dx\; xH_M(x,\xi,t) = \Theta_2^M(t) - \xi^2\Theta_1^M(t)$$

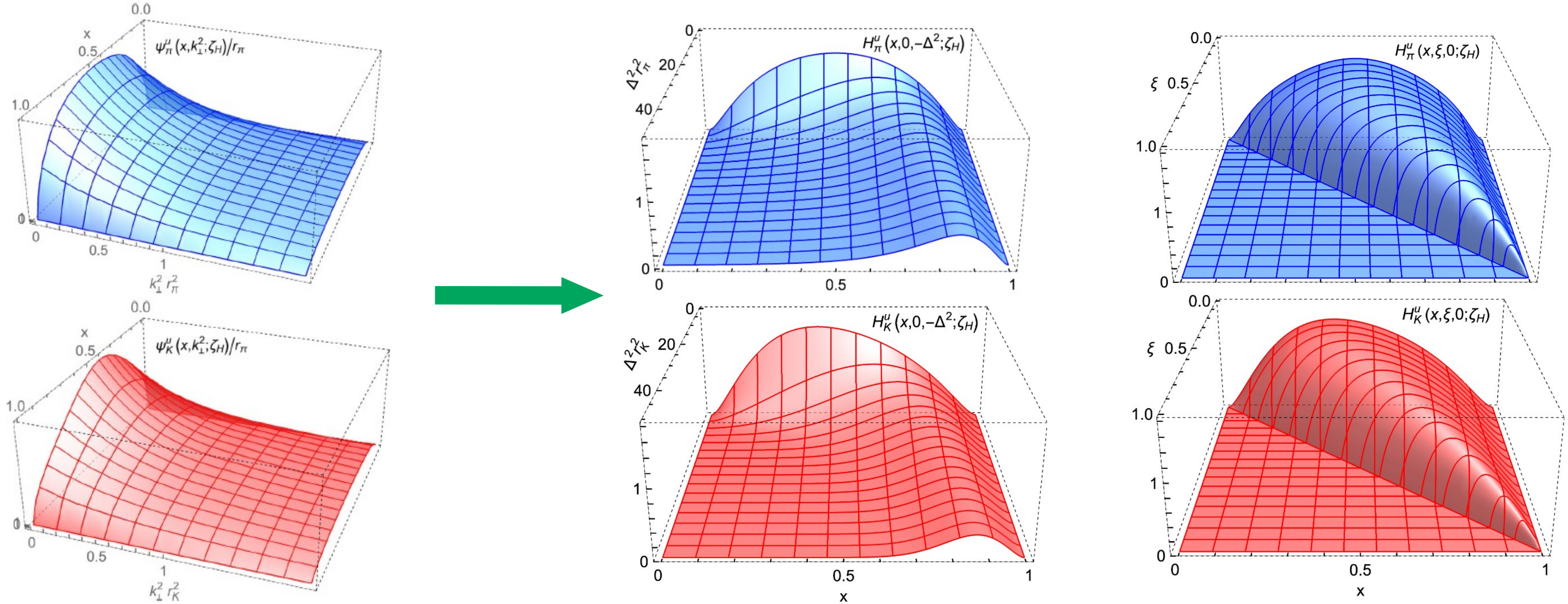
Gravitational FFs

Red: $\int dx$ — Green: $\int dk_\perp$ — Purple: $t=0, \xi=0$

π-K: LFWFs & GPDs

LFWFs → GPDs

$$H_{\mathrm{M}}^{q}(x,\xi,t) = \int \frac{d^2 k_\perp}{16\pi^3} \psi_{\mathrm{M}}^{q*}\left(x^-, (\mathbf{k}_\perp^-)^2\right) \psi_{\mathrm{M}}^{q}\left(x^+, (\mathbf{k}_\perp^+)^2\right)$$



Impact parameter space GPDs

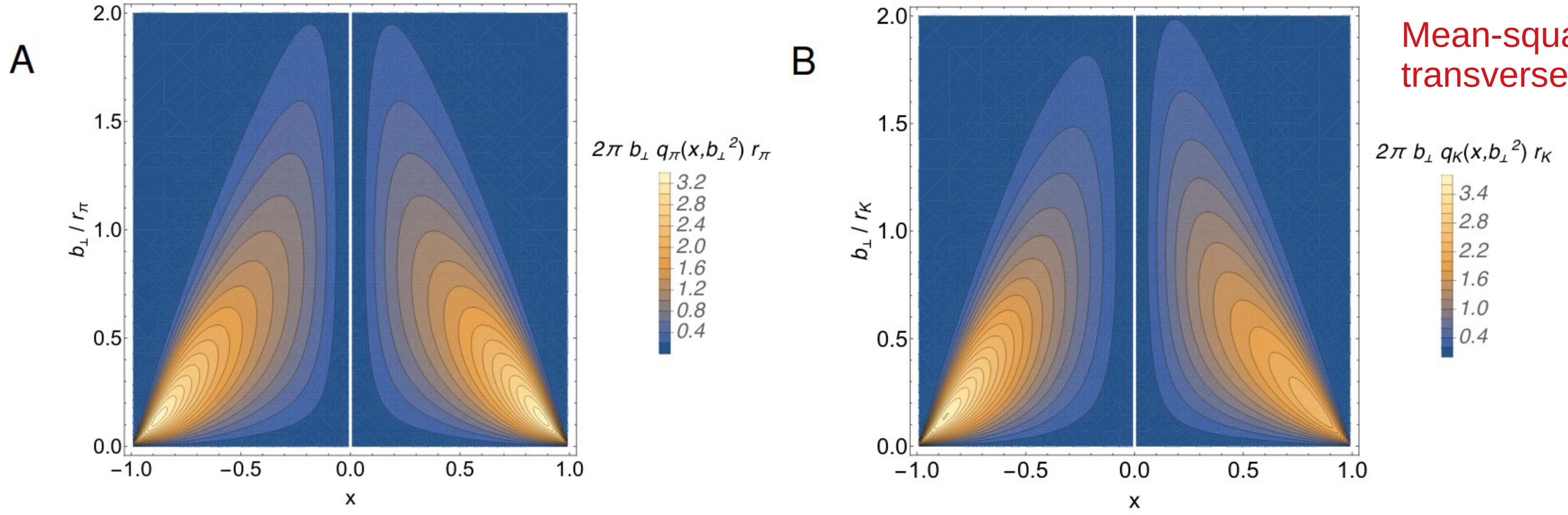
Algebraic derivation!

$$u^{\mathrm{P}}(x, b_\perp^2; \zeta_{\mathcal{H}}) = \int_0^\infty \frac{d\Delta}{2\pi} \Delta J_0(b_\perp \Delta)\, H_{\mathrm{P}}^u(x, 0, -\Delta^2; \zeta_{\mathcal{H}})$$

$$\langle b_\perp^2(\zeta_{\mathcal{H}})\rangle_u^\pi = \boxed{\frac{2}{3}} r_\pi^2 = \langle b_\perp^2(\zeta_{\mathcal{H}})\rangle_{\bar{d}}^\pi, \approx [r_\pi^{\theta_2}]^2$$

$$\langle b_\perp^2(\zeta_{\mathcal{H}})\rangle_u^K = 0.71 r_K^2 , \langle b_\perp^2(\zeta_{\mathcal{H}})\rangle_{\bar{s}}^K = 0.58 r_K^2 .$$

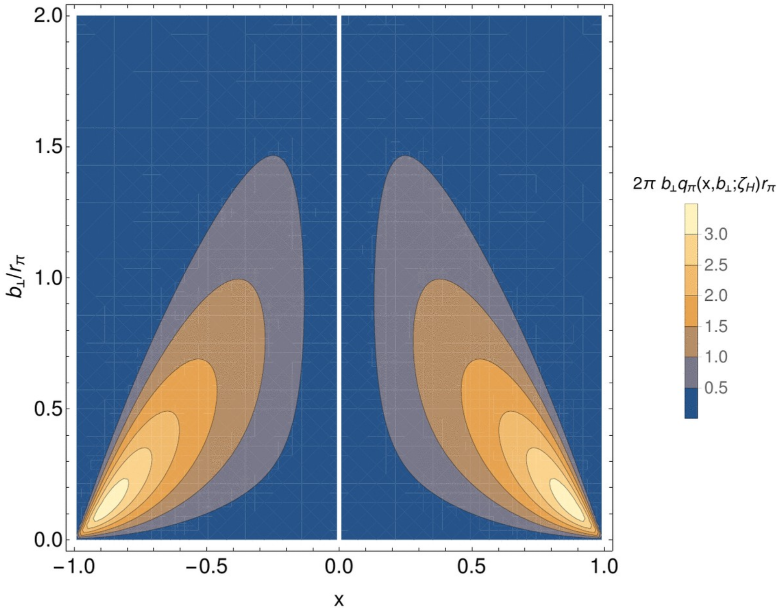
Mean-square transverse extent



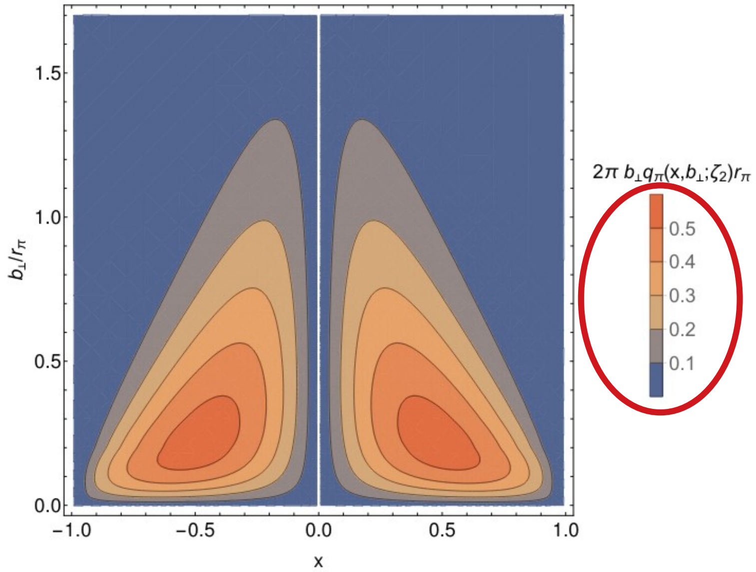
- Likelihood of finding a valence-**quark** with **momentum fraction x**, at **position b**.

Evolved IPS-GPD: Pion Case

$$u^{\mathsf{P}}(x, b_\perp^2; \zeta_{\mathcal{H}}) = \int_0^\infty \frac{d\Delta}{2\pi} \Delta J_0(b_\perp \Delta)\, H_{\mathsf{P}}^u(x, 0, -\Delta^2; \zeta_{\mathcal{H}})$$



$$\zeta_H \to \zeta = 2\ \text{GeV}$$



- **Likelihood** of finding a parton with LF momentum **x** at transverse position **b**

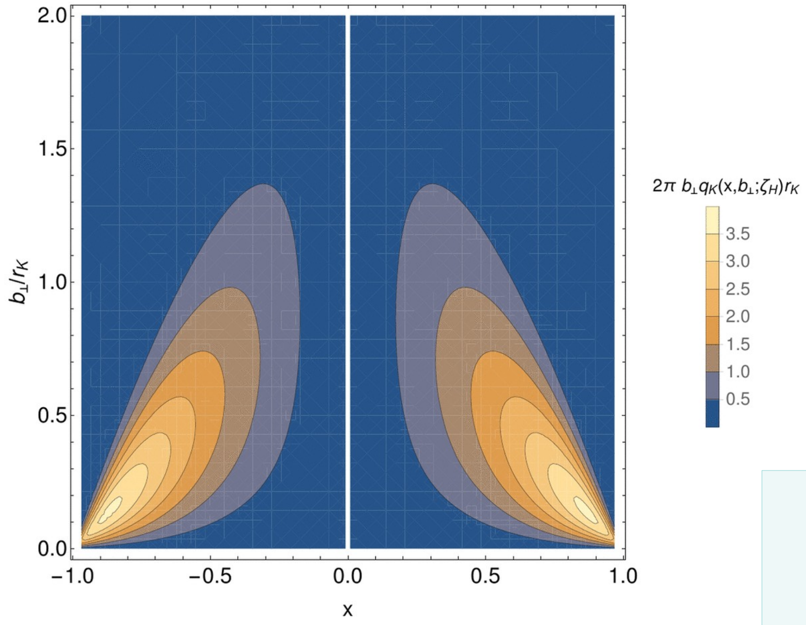
- Peaks **broaden** and **maximum drifts:**

$$\max: \;\; 3.29 \to 0.55$$

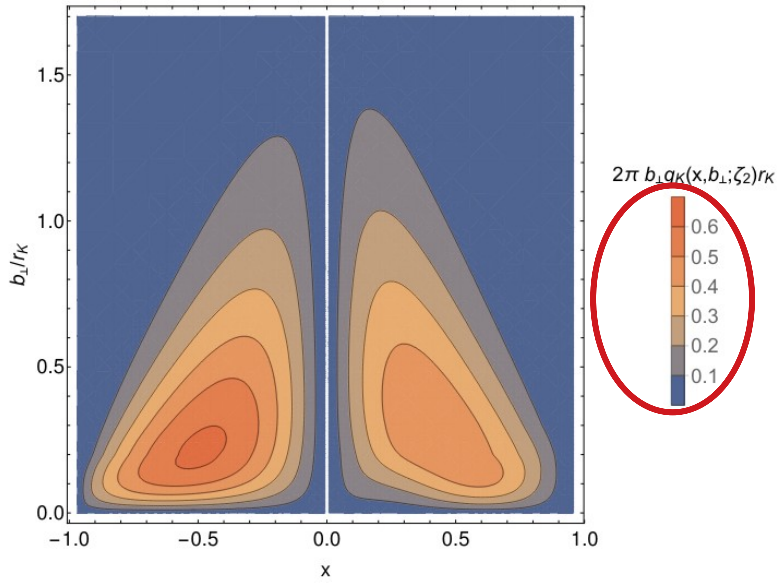
$$(|x|, b) = (0.88, 0.13) \to (0.47, 0.23)$$

Evolved IPS-GPD: Kaon Case

$$u^{\mathsf{P}}(x, b_\perp^2; \zeta_{\mathcal{H}}) = \int_0^\infty \frac{d\Delta}{2\pi} \Delta J_0(b_\perp \Delta)\, H_{\mathsf{P}}^u(x, 0, -\Delta^2; \zeta_{\mathcal{H}})$$



$\zeta_H \to \zeta = 2$ GeV



$$\max_{(s,u)} : (3.61, 2.38) \to (0.61, 0.49)$$
$$(x, b)_u = (0.84, 0.17) \to (0.41, 0.28)$$
$$(x, b)_s = (-0.87, 0.13) \to (-0.48, 0.22)$$

- **Likelihood** of finding a parton with LF momentum **x** at transverse position **b**

Distributions: Charge & Mass

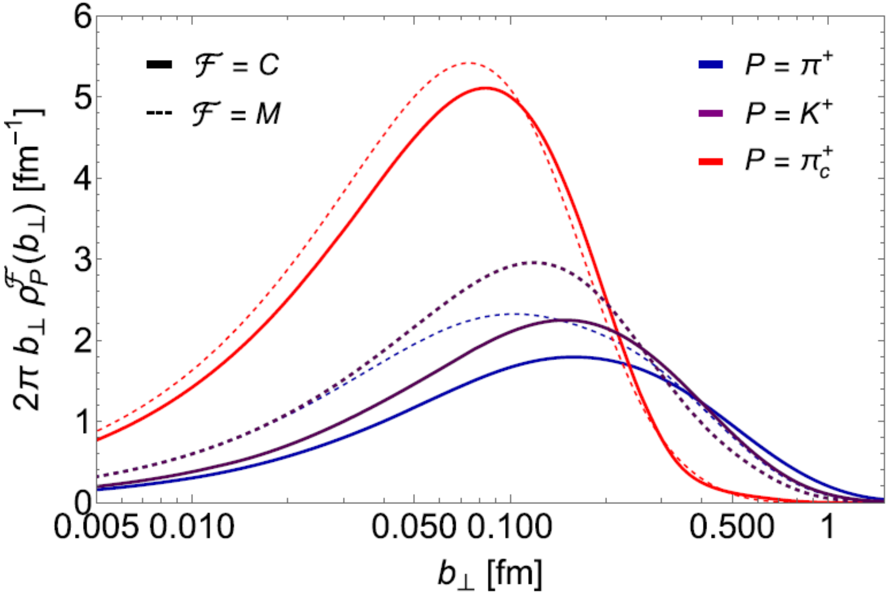
$$\rho_{\rm P}(b) = \frac{1}{2\pi}\int_0^\infty d\Delta\,\Delta J_0(\Delta\, b) F_{\rm P}(\Delta^2)$$

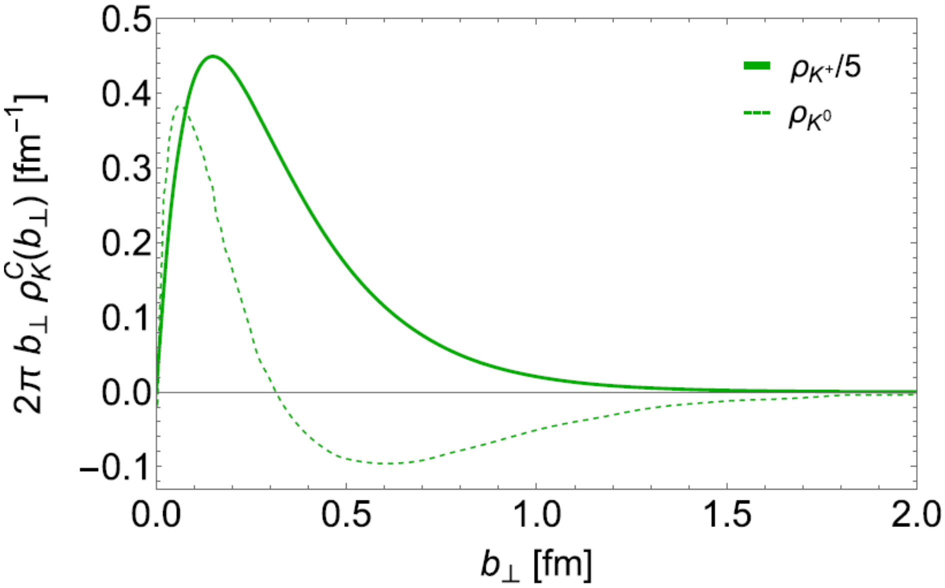
$$F_{\rm P}^{E}(\Delta^2) \rightarrow \rho_{\rm P}^{E}(b)$$

$$\theta_2^{\rm P}(\Delta^2) \rightarrow \rho_{\rm P}^{M}(b)$$

- **Intuitively**, we expect the meson to be localized at a **finite space**.
- **Charge** effect span over a **larger domain** than **mass** effects.

More **massive** hadron → More **compressed**





GPDs: Empirical determination

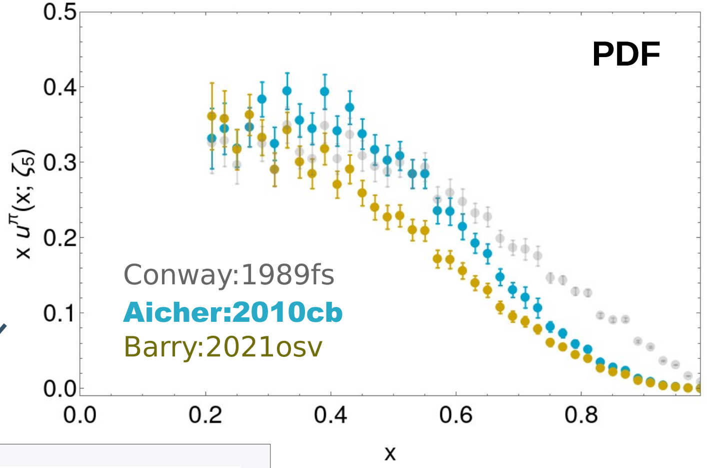
Pion GPD: Empirical determination

- The connection of **GPDs** with PDFs and EFFs enable us to use existing data on those quantities to **reconstruct** the **pion GPD**.

- Using a chi^2-based probabilistic selection procedure, an ensemble of **representations** for the **pion GPD** is generated.

$$u^\pi(x;\zeta_{e/l})$$

All orders evolution



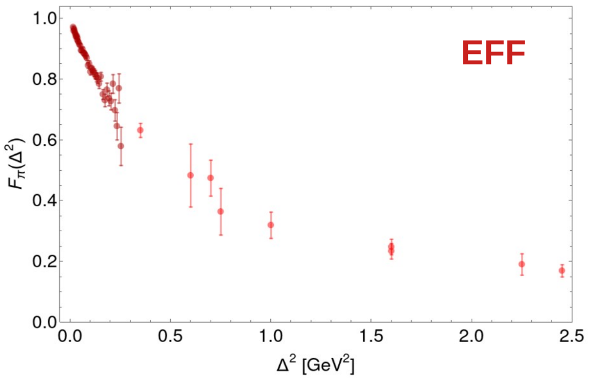
$$H_\pi^u(x,\xi,-\Delta^2;\zeta_H) = \theta(x_-)\sqrt{u^\pi(x_-;\zeta_H)u^\pi(x_+;\zeta_H)}\,\Phi^\pi(z^2;\zeta_H)$$

Factorized LFWF

$$H_\pi^u(x,\xi,-\Delta^2;\zeta_H) \sim \int_{k_\perp} \psi^*\psi$$

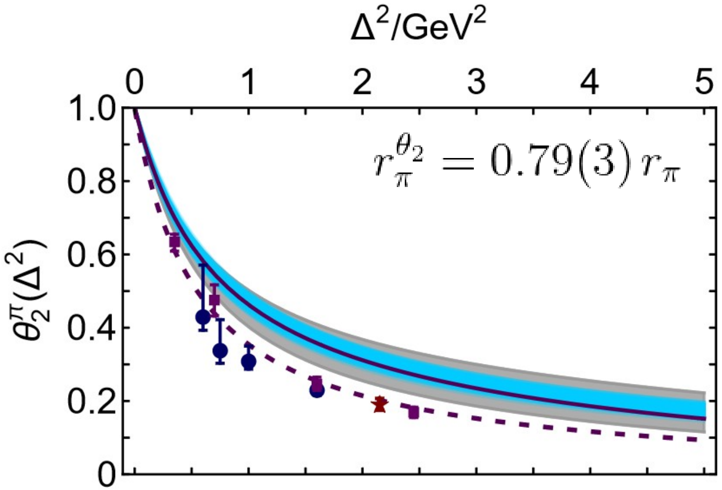
Sum rule

$$F_\pi(\Delta^2) = \int_0^1 dx\, H_\pi^u(x,0,-\Delta^2)$$



Pion GPD: Empirical determination

- The connection of **GPDs** with PDFs and EFFs enable us to use existing data on those quantities to **reconstruct** the **pion GPD**.
- Using a chi^2-based probabilistic selection procedure, an ensemble of **representations** for the **pion GPD** is generated.
- The produced ensemble turns out to be in **agreement** with previous **CSM predictions**.



- Proving, once again, that θ_2 is harder than the EFF:

  i.e. the **mass distribution** is **more compact** than the **charge** one.

The physical boundaries:

$$\frac{1}{\sqrt{2}} \leq r_\pi^{\theta_2}/r_\pi \leq 1$$

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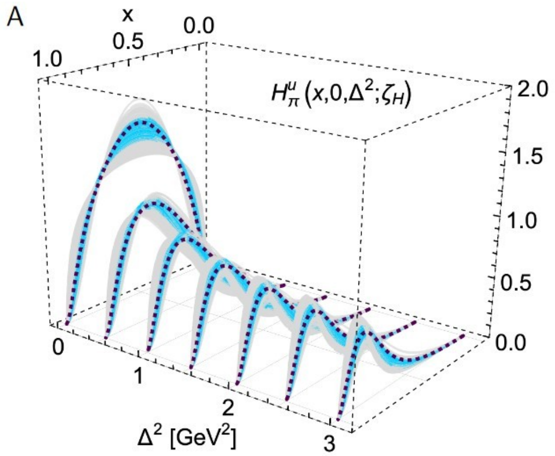
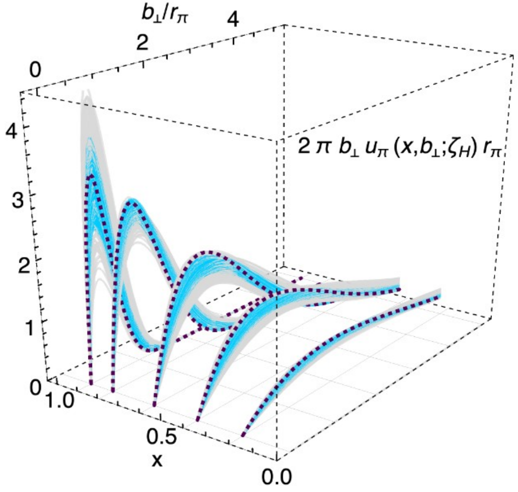
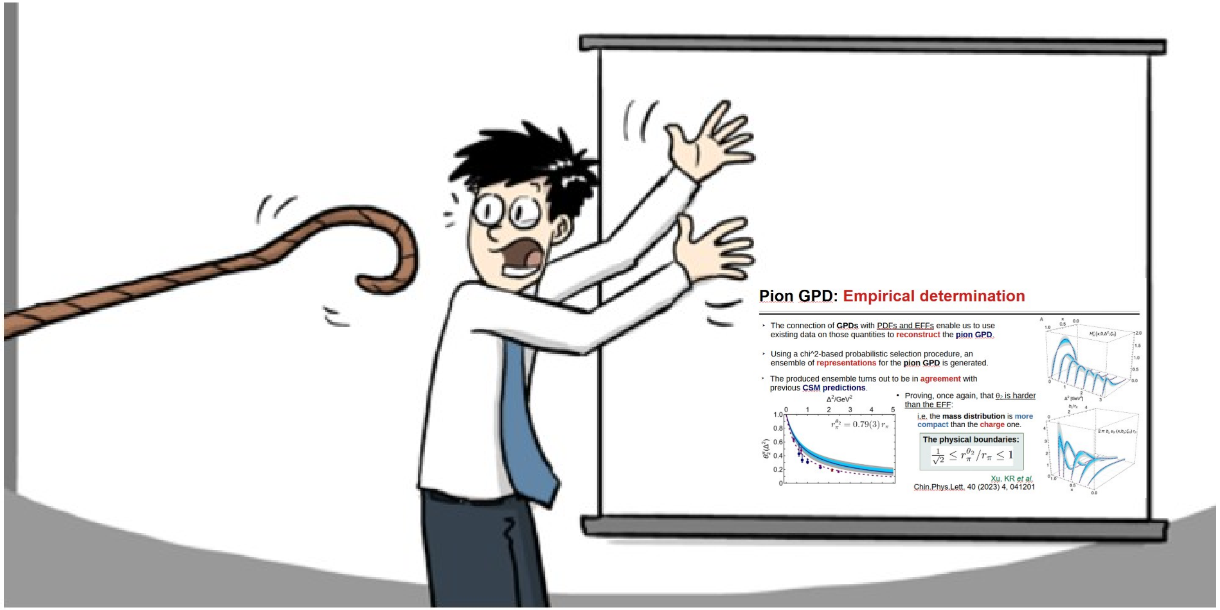




Final **Highlights**



Final **Highlights**

- The **emergent phenomena** in **QCD** produces unique outcomes:
  - The degrees-of-freedom are not directly accessible, we get to observe hadrons (**confinement**).
  - Through their own mechanisms, **dynamical mass generation** is present in both **matter** and **gauge** sectors of QCD; the later yielding a running **coupling** that saturates at infrared momenta.

- **Pseudoscalar** mesons are an ideal platform to inquire on these facets of **QCD**:
  - Their mere **existence and properties** are connected with the **mass generation** in the Standard Model and, potentially, **confinement**.
  - Modern facilities are **capable** to address the properties of **NG bosons** and it's connection with QCD's emergent phenomena.

- Theory has evolved to the point where **all sorts of** parton **distributions** within pseudoscalar mesons are **within reach**.

➔ Many of them connected via **LFWF**