# Lightfront Wave Functions of the nucleon

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In collaboration with:
M. Riberdy, J. Segovia and C.D. Roberts

# Definitions and Classification of LFWFs

- Lightfront quantization allows to expand hadrons on a Fock basis:

$$|P,\pi\rangle \propto \sum_\beta \Psi_\beta^{q\bar{q}}|q\bar{q}\rangle + \sum_\beta \Psi_\beta^{q\bar{q},q\bar{q}}|q\bar{q},q\bar{q}\rangle + \dots$$

$$|P,N\rangle \propto \sum_\beta \Psi_\beta^{qqq}|qqq\rangle + \sum_\beta \Psi_\beta^{qqq,q\bar{q}}|qqq,q\bar{q}\rangle + \dots$$

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- Non-perturbative physics is contained in the $N$-particles Lightfront-Wave Functions (LFWF) $\Psi^N$
- Schematically a distribution amplitude $\varphi$ is related to the LFWF through:

$$\varphi(x) \propto \int \frac{\mathrm{d}^2 k_\perp}{(2\pi)^2}\Psi(x,k_\perp)$$

S. Brodsky and G. Lepage, PRD 22, (1980)

- Since they describe hadron states, LFWFs can be used to compute matrix element of the type :

$$\langle p'|O(z_1,\ldots z_n)|p\rangle \to \sum_N \sum_{N'} \psi^*_{N'}\psi_N \langle q_1\ldots q_{N'}|O(z_1,\ldots,z_n)|q_1\ldots q_N\rangle$$

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- These matrix elements typically encode hadron structure properties (EFF, PDFs, GPDs, etc)
- Thus, we can compute hadron structure distributions, as a convolution (or overlap) of Lightfront Wave functions

M. Diehl *et al.*, Nucl. Phys B596 (2001)

LFWFs modelling techniques have been widely used on mesons

- Simple Algebraic Nakanishi models for the pion and computation of GPDs in the DGLAP region (parton number conserved)

for instance C. Mezrag *et al.*, Few Body Syst. 57 (2016) 9, 729-772

- Advanced modelling with and without Nakanishi parametrisations

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# From Mesons to Baryons

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We would like to extend all this to the baryon sector
and compute nucleon DVCS observable

$$\langle 0|O^{\alpha,\cdots}(\{z_1^-, z_{\perp 1}\}, \ldots, \{z_n^-, z_{\perp n}\})|P,\lambda\rangle\Big|_{z_i^+=0}$$

- Lightfront operator $O$ of given number of quark and gluon fields

$$\langle 0|O^{\alpha,\cdots}(\{z_1^-, z_{\perp 1}\}, \ldots, \{z_n^-, z_{\perp n}\})|P,\lambda\rangle\big|_{z_i^+=0} = \sum_j^n \tau_j^{\alpha,\cdots} N(P,\lambda) F_j(z_i)$$

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- Expansion in terms of scalar non-pertubative functions $F(z_i)$

# LFWFs: formal definitions

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Both mesons and baryons can (in principle) have multiple independent leading-twist LFWFs

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$$\langle 0|\bar{q} n_\mu \sigma^{\mu i}\gamma_5 q|\pi\rangle|_{z^+=0}$$

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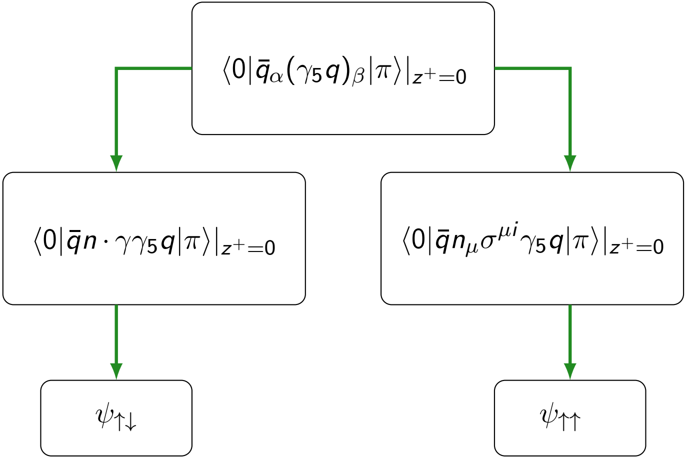

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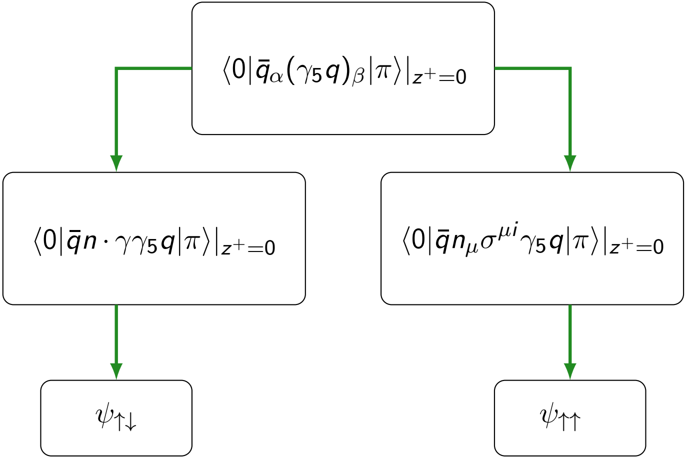


We can build one LFWFs with OAM projection 0, and one with OAM projection 1.

- In the nucleon case, the procedure applies with three quarks at leading Fock state:

$$\langle 0|\epsilon^{ijk} u^i_\alpha(z_1) u^j_\beta(z_2) d^k_\gamma(z_3)|P,\uparrow\rangle$$

# Nucleon LFWFs classification

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- The LFWFs carry different amount of OAM projections:

| states | $\langle\downarrow\downarrow\downarrow\|P,\uparrow\rangle$ | $\langle\downarrow\downarrow\uparrow\|P,\uparrow\rangle$ | $\langle\uparrow\downarrow\uparrow\|P,\uparrow\rangle$ | $\langle\uparrow\uparrow\uparrow\|P,\uparrow\rangle$ |
|---|---|---|---|---|
| OAM | 2 | 1 | 0 | -1 |
| LFWFs | $\psi^6$ | $\psi^3$, $\psi^4$ | $\psi^1$, $\psi^2$ | $\psi^5$ |

- Since the Faddeev wave function $\chi$ is given as:

$$\langle 0| T\left\{q_{\alpha_1}(z_1)q_{\alpha_2}(z_2)q_{\alpha_3}(z_3)\right\}|P,\lambda\rangle = \frac{1}{4} f_N N_\sigma(P,\lambda)$$

$$\times \int \prod_{j=1}^{3} \mathrm{d}^{(4)}k_j e^{-ik_j z_j} \delta^{(4)}(P - \sum_j k_j) \chi_{\alpha_1\alpha_2\alpha_3\sigma}(k_1,k_2,k_3),$$

# Relation with the Faddeev Wave function

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- one can get the LFWFs schematically through

$$\psi_i \Gamma_{\alpha_3'\sigma'} = \int \prod_{j=1}^{3} [\mathrm{d}k_j^-] \mathcal{P}_{i;\alpha_1\alpha_2\alpha_3\alpha_3'\sigma\sigma'} \chi_{\alpha_1\alpha_2\alpha_3\sigma}$$

where $\mathcal{P}_i$ are the relevant leading-twist and OAM projectors.

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where $\mathcal{P}_i$ are the relevant leading-twist and OAM projectors.

**Important**

The FWF allows a **consistent** derivation of the 6 leading-fock states LFWFs of the nucleon

Modelling the Faddeev wave Function

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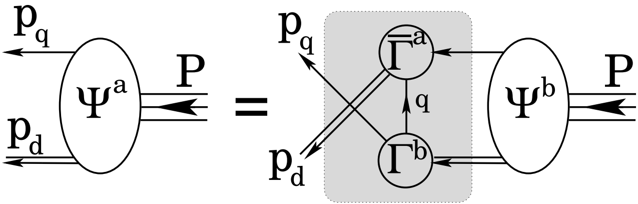

# Baryon and Diquarks

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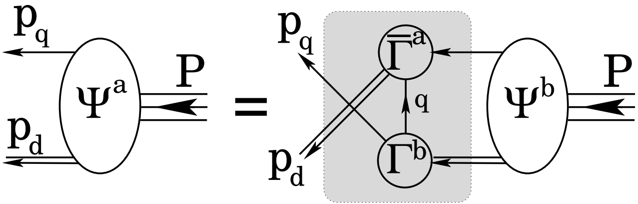


- Mostly two types of diquark are dynamically generated by the Faddeev equation:
  - Scalar diquarks,
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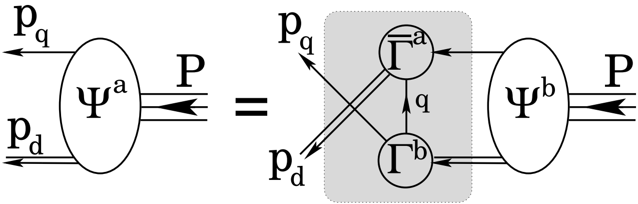


- Mostly two types of diquark are dynamically generated by the Faddeev equation:
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- In the following we build a model inspired by numerical solutions of the Faddeev equations

# Example with $\psi_1$ and $\psi_2$

- A single projector allows us to compute both $\psi_1$ and $\psi_2$:

$$\langle 0|\epsilon^{ijk}\left(u^i_\uparrow(z_1^-, z_{1\perp})C\not{n}u^j_\downarrow(z_2^-, z_{2\perp})\right)\not{n}d^k_\uparrow(z_3^-, z_{3\perp})|P,\uparrow\rangle$$
$$\rightarrow \psi_1(x_1, k_{1\perp}, x_2, k_{2\perp}) + \epsilon^{ij}k_i^1 k_j^2 \psi_2(x_1, k_{1\perp}, x_2, k_{2\perp})$$

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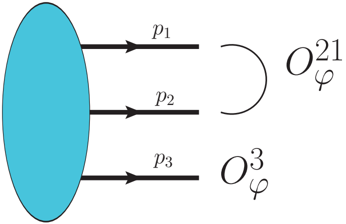
- We can apply it on the Faddeev wave function:



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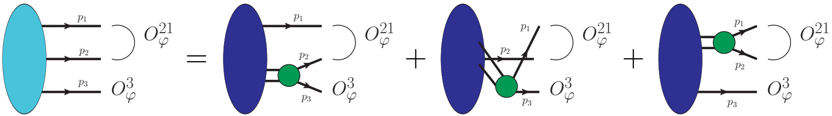

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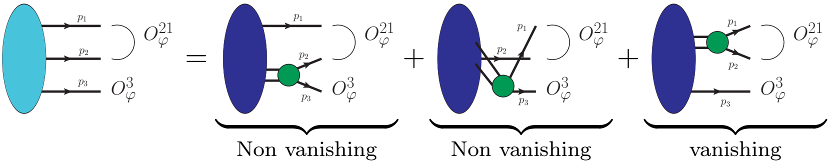
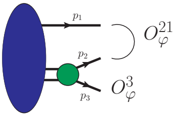



- The operator then selects the relevant component of the wave function.

Considering the diquark amplitude $\Gamma_0$ and the Quark-diquark amplitude $\mathcal{S}$, we choose the following tensorial structure:

$$\Gamma_0 \propto i\gamma_5 C, \quad \mathcal{S} \propto I$$




$$\propto \frac{\gamma_\nu}{4} \mathrm{Tr}\left[\gamma^\nu \not{n} L^\uparrow S(k_3) \Gamma^{0T} S^T(k_2) L^\downarrow \not{n}^T (C^\dagger)^T L^\uparrow S(k_1) \mathcal{S}\right] \Delta(K)$$

# Dirac Structure and Factorisation I

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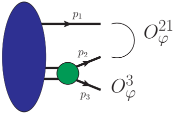


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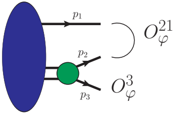


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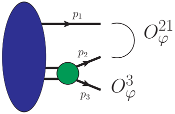


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Note that $\int \mathrm{d}^{(2)} k_{1\perp} \mathrm{d}^{(2)} k_{2\perp} \psi_1 = \varphi$, the nucleon DA.

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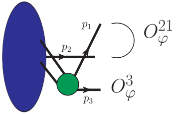


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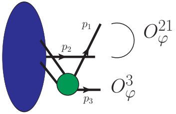


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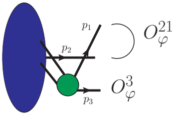
$$\propto \frac{\gamma_\nu}{4} \underbrace{\mathrm{Tr}\left[S(k_3)\Gamma^{0T} S^T(k_1) L^\uparrow C^\dagger \not{n} \gamma_\alpha\right]}_{\text{Diquark LFWF } \psi_{\uparrow\uparrow}} \underbrace{\mathrm{Tr}\left[\gamma^\nu \not{n} \gamma^\alpha L^\downarrow S(k_2) \mathcal{S}\right]}_{\text{Projection of the Faddeev WF}} \Delta(K)$$

# Dirac Structure and Factorisation II

Considering the diquark amplitude $\Gamma_0$ and the Quark-diquark amplitude $\mathcal{S}$, we choose the following tensorial structure:

$$\Gamma_0 \propto i\gamma_5 C, \quad \mathcal{S} \propto I$$




$$\propto \frac{\gamma_\nu}{4} \mathrm{Tr}\left[\gamma^\nu \not{n} L^\uparrow S(k_3) \Gamma^{0T} S^T(k_1) L^\uparrow \not{n}^T (C^\dagger)^T L^\downarrow S(k_2) \mathcal{S}\right] \Delta(K)$$

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$$\propto \epsilon^{\mu\nu\rho\sigma} n_\mu p_\nu k_{1\perp\rho} k_{2\perp\sigma} \psi_2(x_1, k_{1\perp}, x_2, k_{2\perp})$$

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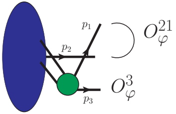


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Note that the antisymmetric structure guarantees that the contribution vanishes when integrated over the transverse momenta.

# Example with $\psi_3$ and $\psi_4$

- $\psi_3$ and $\psi_4$ are given by:

$$\langle 0|\epsilon^{ijk}\left(u_\uparrow^i(z_1^-,z_{1\perp})C\not{n}u_\downarrow^j(z_2^-,z_{2\perp})\right)\not{n}d_\uparrow^k(z_3^-,z_{3\perp})|P,\downarrow\rangle$$

$$\to\left(k_{1,\perp}\psi_3(x_1,k_{1\perp},x_2,k_{2\perp})+k_{2,\perp}\psi_4(x_1,k_{1\perp},x_2,k_{2\perp})\right)\gamma^\perp$$

Braun *et al.*, Nucl.Phys. B589 (2000)
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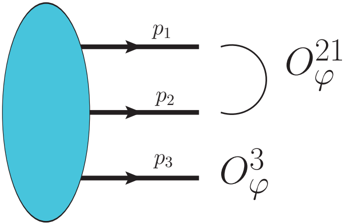
- We can apply it on the Faddeev wave function:



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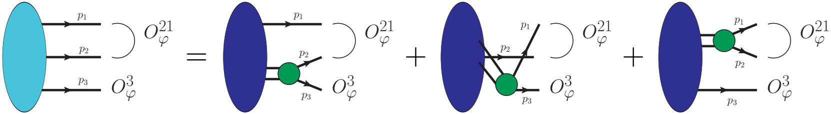

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$$\to (k_{1,\perp}\psi_3(x_1, k_{1\perp}, x_2, k_{2\perp}) + k_{2,\perp}\psi_4(x_1, k_{1\perp}, x_2, k_{2\perp}))\,\gamma^\perp$$

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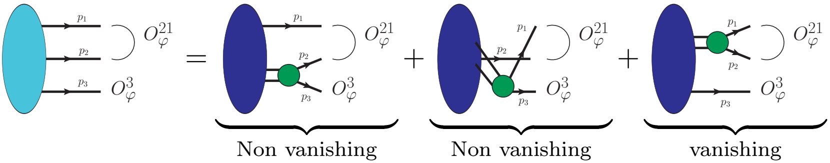
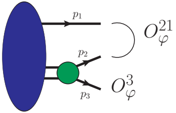



- The operator then selects the relevant component of the wave function.

Considering the diquark amplitude $\Gamma_0$ and the Quark-diquark amplitude $\mathcal{S}$, we choose the following tensorial structure:

$$\Gamma_0 \propto i\gamma_5 C, \quad \mathcal{S} \propto I$$




$$\propto \frac{\sigma_{\mu\nu}}{48}\mathrm{Tr}\left[\sigma^{\mu\nu}\not{n}L^{\uparrow}S(k_3){\Gamma^0}^T S^T(k_2)L^{\downarrow}\not{n}^T(C^{\dagger})^T L^{\uparrow}S(k_1)\mathcal{S}\Lambda^{+}\right]\Delta(K)$$

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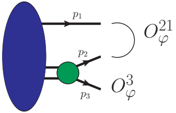


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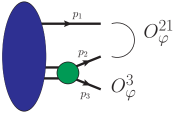


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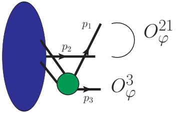


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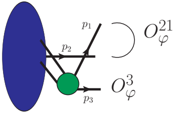


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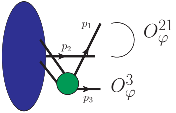


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$$\propto k_{1\perp\rho}\psi_3(x_1, k_{1\perp}, x_2, k_{2\perp}) + k_{2\perp\sigma}\psi_4(x_1, k_{1\perp}, x_2, k_{2\perp})$$

# Case of $\psi_5$ and $\psi_6$

- $\psi_5$ is connected to the matrix element:

$$\langle 0|\epsilon^{ijk}\left(u^i_\uparrow(z_1^-,z_{1\perp})Ci\sigma^{\nu\perp}n_\nu u^j_\uparrow(z_2^-,z_{2\perp})\right)\not{n}d^k_\uparrow(z_3^-,z_{3\perp})|P,\uparrow\rangle$$
$$\to k_{1,\perp}(\psi_5(x_1,k_{1\perp},x_2,k_{2\perp})-\psi_5(x_1,k_{1\perp},x_3,k_{3\perp}))+\dots$$

Braun *et al.*, Nucl.Phys. B589 (2000)
X. Ji *et al.*, Nucl.Phys. B652 (2003)

- $\psi_6$ is obtained projecting:

$$\langle 0|\epsilon^{ijk}\left(u^i_\downarrow(z_1^-,z_{1\perp})Ci\sigma^{\nu\perp}n_\nu u^j_\downarrow(z_2^-,z_{2\perp})\right)\not{n}d^k_\downarrow(z_3^-,z_{3\perp})|P,\uparrow\rangle$$
$$\to k_{1,\perp}k_{1,\perp'}\psi_6(x_1,k_{1\perp},x_2,k_{2\perp})+\dots$$

Braun *et al.*, Nucl.Phys. B589 (2000)
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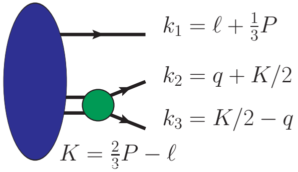
Scalar Diquark part of the nucleon

The diquark approach allows us to simplify a three-body system into two convoluted two-body systems. The strategy is thus:




1. Compute the virtuality ($K$) dependent LFWFs of the diquark by integrating over $q$
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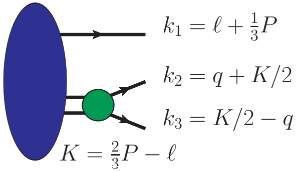


1. Compute the virtuality ($K$) dependent LFWFs of the diquark by integrating over $q$
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- For LFWFs, only the $q^-$ and $\ell^-$ momenta should be integrated
- For the Distribution Amplitude, we have to integrate also on $q_\perp$ and $\ell_\perp$.

# Mellin Moments

- Reminder : we use a model, not a solution of the Faddeev equation
- We do not integrate over the $q^-$ and $\ell^-$ directly because we work on Euclidean space. Instead, we work with Mellin moments of it:

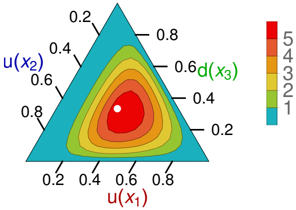
$$\langle x_1^m x_2^n \rangle(k_{1\perp}, k_{2\perp}) = \int_0^1 \mathrm{d}x_1 \int_0^{1-x_1} \mathrm{d}x_2 \; x_1^m x_2^n \psi(x_1, x_2, k_{1\perp}, k_{2\perp})$$

- For a general moment $\langle x_1^m x_2^n \rangle$, we change the variable in such a way to write down our moments as:

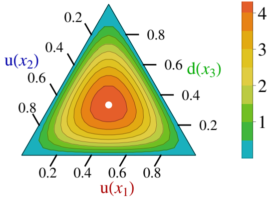
$$\langle x_1^m x_2^n \rangle(k_{1\perp}, k_{2\perp}) = \int_0^1 \mathrm{d}\alpha \int_0^{1-\alpha} \mathrm{d}\beta \; \alpha^m \beta^n f(\alpha, \beta, k_{1\perp}, k_{2\perp})$$

- $f$ is a complicated function involving the integration on 6 parameters
- Uniqueness of the Mellin moments of continuous functions allows us to identify $f$ and $\psi$

# Preliminary Results at DA level

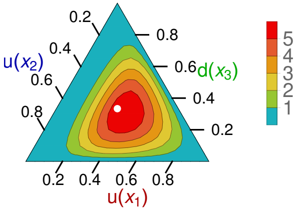



Scalar diquark Only




Asymptotic DA

- Typical symmetry in the pure scalar case

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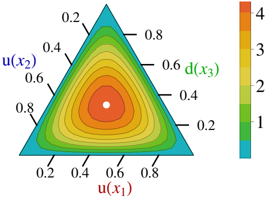


Asymptotic DA

- Typical symmetry in the pure scalar case
- Nucleon DA is skewed compared to the asymptotic one

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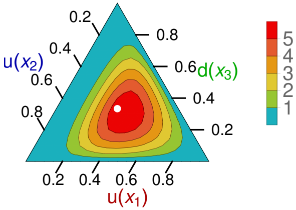


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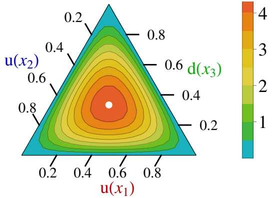


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- Typical symmetry in the pure scalar case
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  - Impact of the virtuality dependence of the diquark WF

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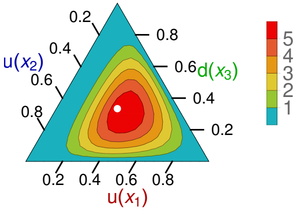


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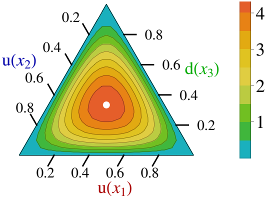


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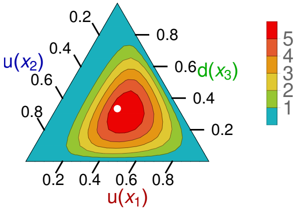


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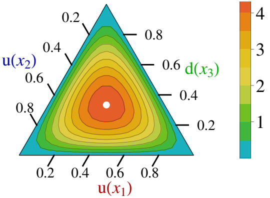


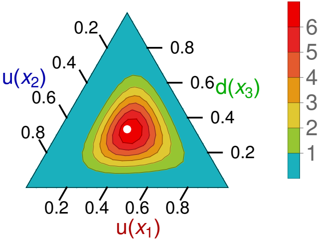

Asymptotic DA

- Typical symmetry in the pure scalar case
- Nucleon DA is skewed compared to the asymptotic one
- Deformation along the symmetry axis and orthogonally to it
  - Impact of the virtuality dependence of the diquark WF
- These properties are consequences of our quark-diquark picture
- Improvement in the modelling with respect to our previous work

C. Mezrag *et al.*, Phys.Lett. B783 (2018)

# Preliminary results for $\psi_1$

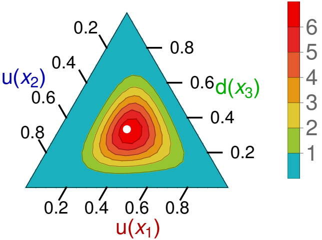
## Momentum dependence




$$\psi_1(x_1, x_2, q_\perp^2 = 0, \ell_\perp^2 = 0, \theta_{q\ell} = 0)/20$$

# Preliminary results for $\psi_1$

## Momentum dependence

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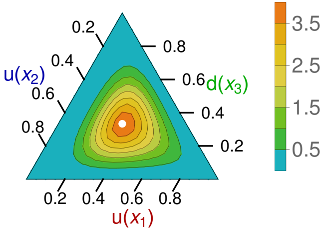
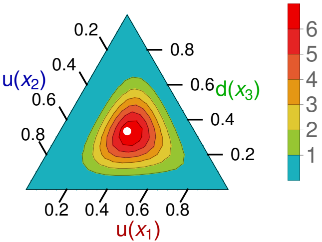



$$\psi_1(x_1, x_2, q_\perp^2 = 0.1 M_N^2, \ell_\perp^2 = 0, \theta_{q\ell} = 0)/10$$

# Preliminary results for $\psi_1$

## Momentum dependence

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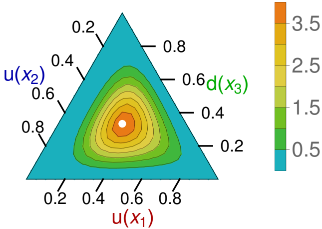
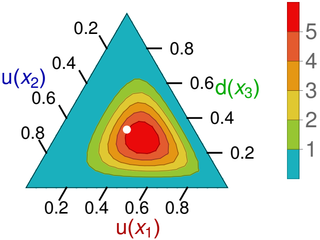


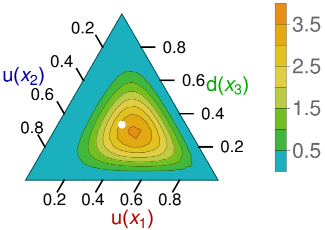

$$\psi_1(x_1, x_2, q_\perp^2 = 0.1 M_N^2, \ell_\perp^2 = 0, \theta_{q\ell} = 0)/10$$



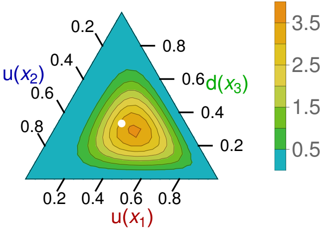

$$\psi_1(x_1, x_2, q_\perp^2 = 0, \ell_\perp^2 = 0.1 M_N^2, \theta_{q\ell} = 0)/3$$

# Preliminary results for $\psi_1$

## Angle dependence




$$\psi_1(x_1, x_2, q_\perp^2 = \ell_\perp^2 = 0.1M_N^2, \theta_{q\ell} = \pi/2)/3$$

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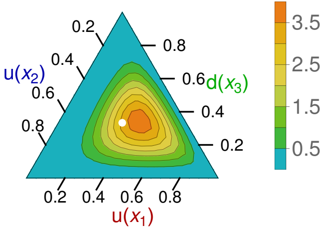
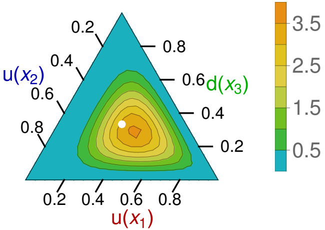



$$\psi_1(x_1, x_2, q_\perp^2 = \ell_\perp^2 = 0.1 M_N^2, \theta_{q\ell} = 0)/10$$

# Preliminary results for $\psi_1$

## Angle dependence

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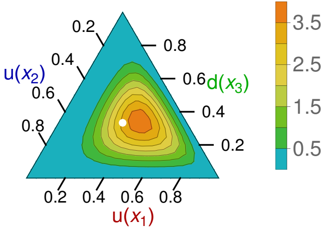
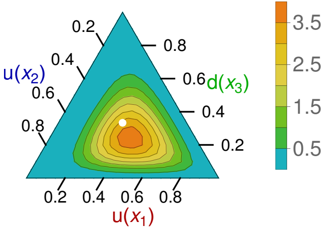



$$\psi_1(x_1, x_2, q_\perp^2 = \ell_\perp^2 = 0.1 M_N^2, \theta_{q\ell} = 0)/10$$




$$\psi_1(x_1, x_2, q_\perp^2 = \ell_\perp^2 = 0.1 M_N^2, \theta_{q\ell} = \pi)/3$$

# The case of $\psi_2$

The results for $\psi_2$ are the same, up to a permutation $(x_1, k_{1\perp}) \leftrightarrow (x_1, k_{1\perp})$ and a normalisation factor because :

- We have choosen a single Dirac structure for $\Gamma_0$ and $\mathcal{S}$, hence the same Nakanishi weights and parametrisation contribute.

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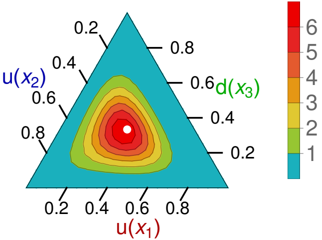
Adding tensorial structures or modifying the propagators will break this symmetry between $\psi_1$ and $\psi_2$.

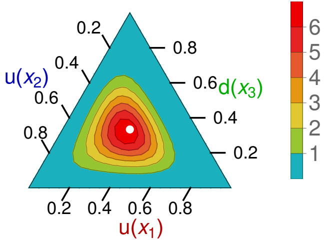
# The cases of $\psi_3 \ldots \psi_6$

- Pretty much the same analyses can be applied to the other LFWFs...
- ... but at the exception of small differences coming from kinematics
  $\Rightarrow$ one can get an additional power of $x_1$ or $x_2$
- ... and contributions of multiple diquark configurations.

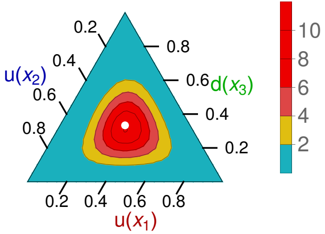
# Some illustrations



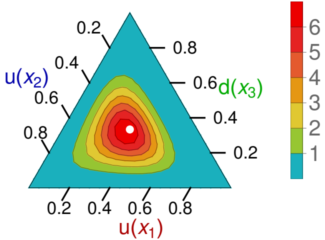

$$\psi_2(x_1, x_2, q_\perp^2 = 0, \ell_\perp^2 = 0)/20$$




$\psi_2(x_1, x_2, q_\perp^2 = 0, \ell_\perp^2 = 0)/20$




$\psi_3(x_1, x_2, q_\perp^2 = \ell_\perp^2 = 0)/20$

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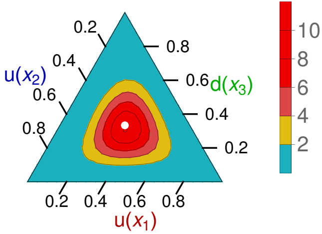


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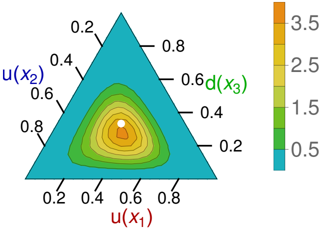


$\psi_4(x_1, x_2, q_\perp^2 = \ell_\perp^2 = 0)/20$

Toward the computation of GPDs

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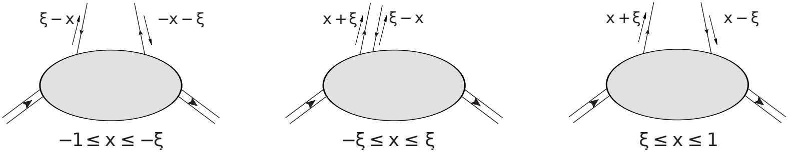


  - ★ $x$: average momentum fraction carried by the active parton
  - ★ $\xi$: skewness parameter $\xi \simeq \frac{x_B}{2-x_B}$
  - ★ $t$: the Mandelstam variable

- Generalised Parton Distributions (GPDs):
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  - are defined in terms of a non-local matrix element,

$$\frac{1}{2}\int \frac{e^{ixP^+z^-}}{2\pi}\langle P+\frac{\Delta}{2}|\bar{\psi}^q(-\frac{z}{2})\gamma^+\psi^q(\frac{z}{2})|P-\frac{\Delta}{2}\rangle \mathrm{d}z^-|_{z^+=0,z=0}$$
$$= \frac{1}{2P^+}\left[H^q(x,\xi,t)\bar{u}\gamma^+u + E^q(x,\xi,t)\bar{u}\frac{i\sigma^{+\alpha}\Delta_\alpha}{2M}u\right].$$

$$\frac{1}{2}\int \frac{e^{ixP^+z^-}}{2\pi}\langle P+\frac{\Delta}{2}|\bar{\psi}^q(-\frac{z}{2})\gamma^+\gamma_5\psi^q(\frac{z}{2})|P-\frac{\Delta}{2}\rangle \mathrm{d}z^-|_{z^+=0,z=0}$$
$$= \frac{1}{2P^+}\left[\tilde{H}^q(x,\xi,t)\bar{u}\gamma^+\gamma_5 u + \tilde{E}^q(x,\xi,t)\bar{u}\frac{\gamma_5\Delta^+}{2M}u\right].$$

D. Müller *et al.*, Fortsch. Phy. 42 101 (1994)
X. Ji, Phys. Rev. Lett. **78**, 610 (1997)
A. Radyushkin, Phys. Lett. **B380**, 417 (1996)

4 GPDs without helicity transfer + 4 helicity flip GPDs

# Generalised Parton Distributions

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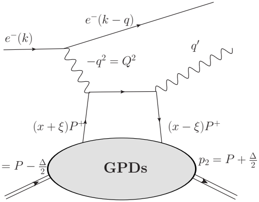
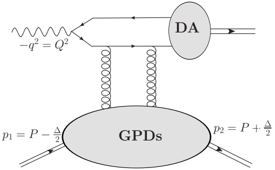
  - are related to PDF in the forward limit $H(x, \xi = 0, t = 0; \mu) = q(x; \mu)$
  - are universal, *i.e.* are related to the Compton Form Factors (CFFs) of various exclusive processes through convolutions

$$\mathcal{H}(\xi, t) = \int \mathrm{d}x \; C(x, \xi) H(x, \xi, t)$$



As Mentioned before, the GPDs can be computed as an overlap of LFWFs:

$$\langle P+\frac{\Delta}{2}|\bar{\psi}(-z/2)\gamma\cdot n\psi(z/2)|P-\Delta/2\rangle$$
$$=\sum\int \mathcal{D}(x,k_\perp)\Psi^*_{out}\Psi_{in}\langle qqq,out|\bar{\psi}(-z/2)\gamma\cdot n\psi(z/2)|qqq,in\rangle$$
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where $\mathcal{D}$ is the measure term.

# Overlap Representation of GPDs

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**Caveat**

This kind of formulæ are valid in the so-call DGLAP region only ($|x| > |\xi|$) where GPDs can be seen as a putting out and then in a quark or an antiquark.
We know how to get the inner region (up to the D-term)

N. Chouika *et al.*, EPJC 77 (2017) 12, 906

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**Results**

We have obtained preliminary results in the case of the PDF using $\psi_1$ only.

- Good news: there is no show stopper and the computation can be performed
- Bad news: the results are too preliminary for me to show them now

# *Summary*

# Summary and conclusion

## Achievements

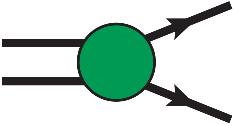
- **DSE compatible** framework for Nucleon LFWFs computations
- Based on the Nakanishi representation
- Improved models from the first exploratory work on PDA
- Relation between LFWFs and GPDs has been worked out
- Proof of concept up to PDF computations / no show stopper identified

## Work in progress/future work

- Finishing the computation for the 6 LFWFs
- Tackling the AV-diquark contributions
- Computations of GPDs
- Finally, compute experimental observables

Thank you for your attention

# Back up slides



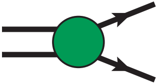
At all order of perturbation theory, one can write (Euclidean space):

$$\Gamma(k,P) = \mathcal{N}\int_0^\infty \mathrm{d}\gamma \int_{-1}^{1} \mathrm{d}z \frac{\rho_n(\gamma,z)}{(\gamma + (k+\frac{z}{2}P)^2)^n}$$

We use a "simpler" version of the latter as follow:

$$\tilde{\Gamma}(q,P) = \mathcal{N}\int_{-1}^{1} \mathrm{d}z \frac{\rho_n(z)}{(\Lambda^2 + (q+\frac{z}{2}P)^2)^n}$$

- We need to obtain the structure of the scalar diquark itself



$$= \mathcal{N}\int_{-1}^{1} \mathrm{d}z \frac{(1-z^2)}{(\Lambda_q^2 + (q+\frac{z}{2}K)^2)}$$

  - $q$ is the relative momentum between the quarks and $K$ the total diquark momentum
  - $\Lambda_q$ is a free parameter to be fit on DSE computations
  - $\rho(z,\gamma) = \rho(z) = 1 - z^2 \rightarrow$ we keep only the 0th degree coefficient in a Gegenbauer expansion of the Nakanishi weight

# Modelling the Scalar Diquark DA

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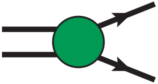

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- We couple this with a simple massive fermion propagator:

$$S(p) = \frac{-ip \cdot \gamma + M}{p^2 + M^2}$$

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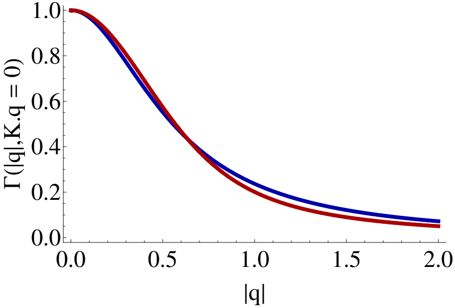
- Width of the diquark BSA $\Lambda_q = 3/5 M_N$ fitted on previous computations:




red curve from Segovia *et al.*,Few Body Syst. 55 (2014) 1185-1222

# Scalar Diquark DA

- From that we can compute the scalar diquark DA as:

$$\phi(x, q_\perp) \propto \int d^{(2)} q \delta\left(q \cdot n - xK \cdot n\right) \mathrm{Tr}\left[S \Gamma^{0\,T} S^T L^{\downarrow} C^{\dagger} n \cdot \gamma L^{\uparrow}\right]$$

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- We compute Mellin moments $\rightarrow$ avoid difficulties with lightcone in euclidean space
- Nakanishi representation $\rightarrow$ analytic treatments of singularities and analytic reconstruction of the function from the moment

$$\phi(x, q_\perp) = \int_x^1 \mathrm{d}u \int_0^x \mathrm{d}v \frac{F(u, v, x)}{\left(M^2_{\mathrm{eff}}(u, v, x, M^2, \Lambda^2) + (q_\perp^{\mathrm{eff}}(u, v, x, q_\perp, K_\perp))^2 + K^2\right)^2}$$

$F$, $M_{\mathrm{eff}}$ and $q_\perp^{\mathrm{eff}}$ are computed analytically

# First results for the diquark

- We present the first results at the level of the diquark DA
  - It depends on a single variable
  - It has been computed in the RL case

Y. Lu *et al.*, Eur.Phys.J.A 57 (2021) 4, 115

$\rightarrow$ we have a comparison point for our simple Nakanishi model.

# Analytic results

- In the specific case $M^2 = \Lambda_q^2$, the PDA can be analytically obtained:

$$\phi(x) \propto \frac{M^2}{K^2}\left[1 - \frac{M^2}{K^2}\frac{\ln\left[1+\frac{K^2}{M^2}x(1-x)\right]}{x(1-x)}\right]$$

C. Mezrag *et al.*, Springer Proc.Phys. 238 (2020) 773-781

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C. Mezrag *et al.*, Springer Proc.Phys. 238 (2020) 773-781

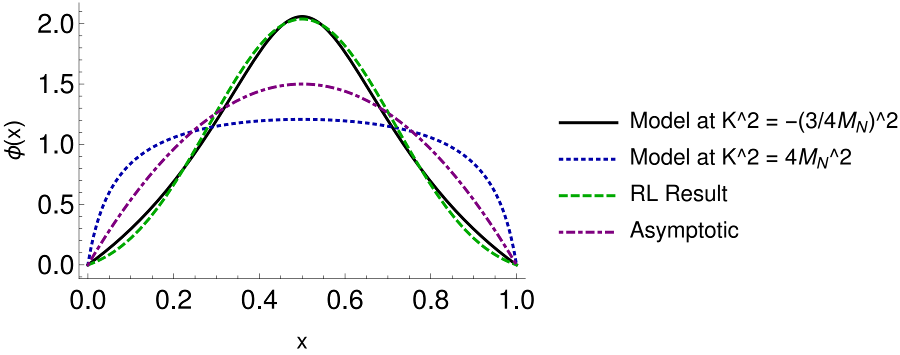
- Note that expanding the log, one get:

$$\phi(x) \propto \frac{1}{2}x(1-x) - \frac{1}{3}K^2/M^2x^2(1-x)^2 + \dots$$

so that:

  - at the end point the DA remains linearly decreasing (important impact on observable)
  - at vanishing diquark virtuality, one recovers the asymptotic DA

# Comparison with DSE results




RL results from Y. Lu *et al.*, Eur.Phys.J.A 57 (2021) 4, 115

# Limitations

- Complex plane singularities for large timelike virtualities

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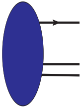
But overall, we expect to gain insights from this simple model

Quark-diquark amplitude

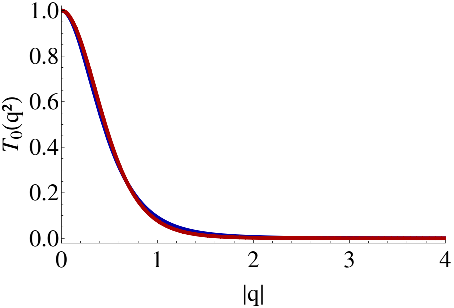
# Nucleon Quark-Diquark Amplitude

## Scalar diquark case

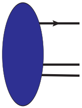


$$= \mathcal{N} \int_{-1}^{1} \mathrm{d}z \frac{(1-z^2)\tilde{\rho}(z)}{(\Lambda^2 + (\ell - \frac{1+3z}{6}P)^2)^3}, \quad \tilde{\rho}(z) = \prod_j (z - a_j)(z - \bar{a}_j)$$

Fits of the parameters through comparison to Chebychev moments:

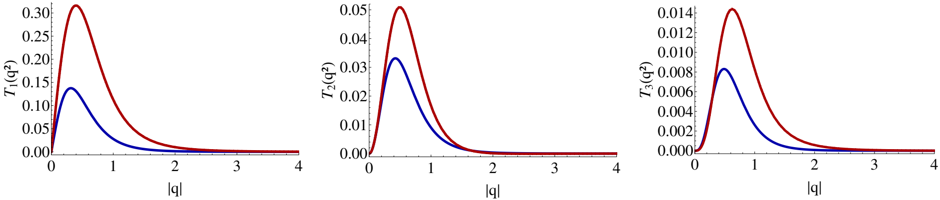



red curve from Segovia *et al.*,



$$= \mathcal{N}\int_{-1}^{1} \mathrm{d}z \frac{(1-z^2)\tilde{\rho}(z)}{(\Lambda^2 + (\ell - \frac{1+3z}{6}P)^2)^3}, \quad \tilde{\rho}(z) = \prod_j (z-a_j)(z-\bar{a}_j)$$

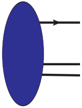
Fits of the parameters through comparison to Chebychev moments:




red curves from Segovia *et al.*,

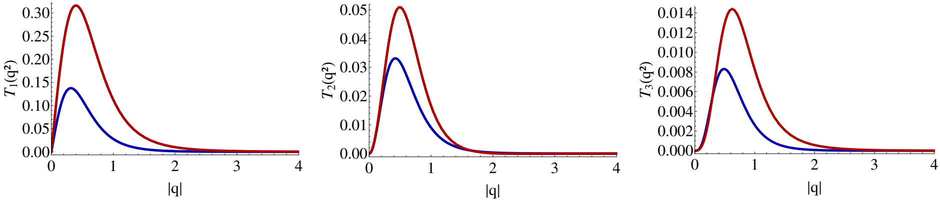
# Nucleon Quark-Diquark Amplitude

## Scalar diquark case



$$= \mathcal{N} \int_{-1}^{1} \mathrm{d}z \frac{(1-z^2)\tilde{\rho}(z)}{(\Lambda^2 + (\ell - \frac{1+3z}{6}P)^2)^3}, \quad \tilde{\rho}(z) = \prod_j (z - a_j)(z - \bar{a}_j)$$

Fits of the parameters through comparison to Chebychev moments:




red curves from Segovia *et al.*,

Modification of the $\tilde{\rho}$ Ansatz ? $\tilde{\rho}(z) \to \tilde{\rho}(\gamma, z)$?