3rd Workshop on Neutrino Theories and Phenos in JUNO (NTPJ-3)

Grand Unification, Neutrino Masses, and Lepton Flavour Violation

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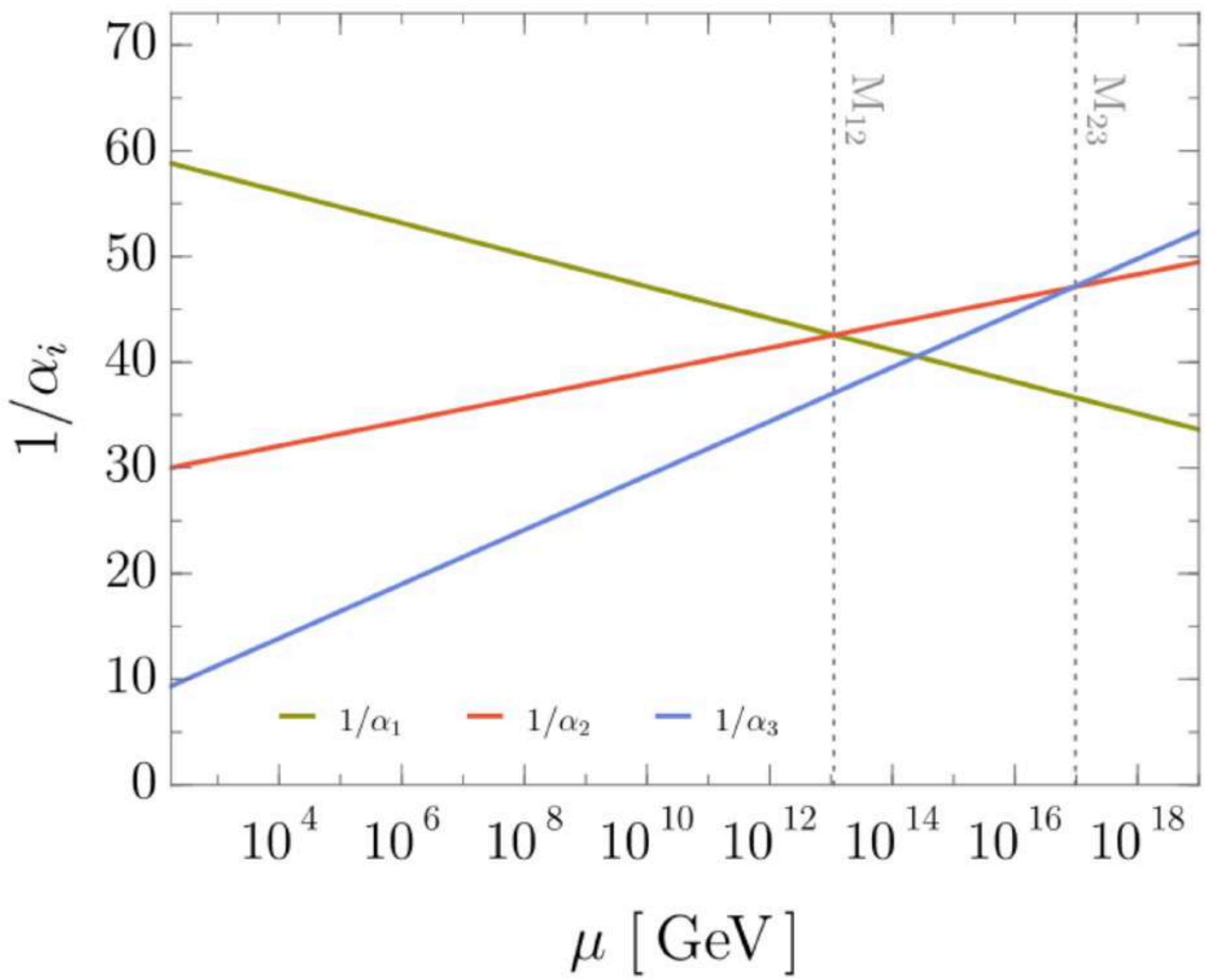
mainly based on LC, Xiyuan Gao arXiv:2206.10682

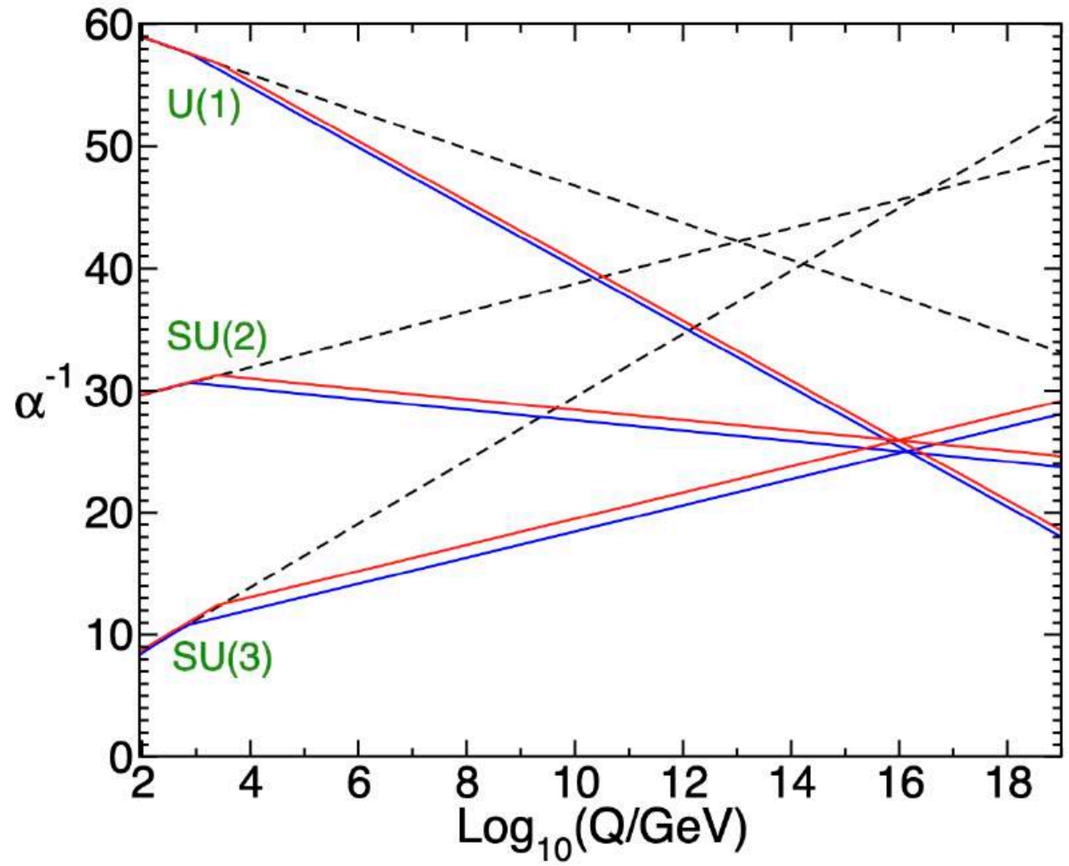
Grand Unification very appealing paradigm (fermions unified in simple GUT irreps, charge quantisation, anomaly cancellation, *p* decay…)

SM gauge couplings show a *tendency* to unification but do not quite unify:



⇒ gauge coupling unification requires new fields at intermediate scales (often close to the EW scale)

This is the case of (and still one of the strongest motivations for) the MSSM



- What about more *minimal* solutions (giving up on naturalness)?
- Can the fields triggering unification be the same fields related to open problems of the SM that require intermediate-scale new physics?
- Yes! And the most obvious example is neutrino masses: many refs starting from Dorsner Perez '05, Bajc Senjanovic '06 ("Seesaw at LHC")

Minimal SU(5)

In the minimal Georgi-Glashow SU(5) model one only introduces the irreps:

$\mathbf{\overline{5}}_F \quad \mathbf{10}_F \quad \mathbf{24}_H \quad \mathbf{5}_H$

Georgi Glashow '74

~~SU(5)~~ SM Higgs (EWSB)

$$SU(5) \xrightarrow{v_{24}} SU(3)_c \times SU(2)_L \times U(1)_Y \xrightarrow{v_5} SU(3)_c \times U(1)_{em}$$

SM fermions:

$$\psi_{\mathbf{\overline{5}}} = (D_R)^c\,(\mathbf{\overline{3}},\mathbf{1},1/3) \oplus L_L\,(\mathbf{1},\mathbf{2},-1/2)\,,$$
$$\psi_{\mathbf{10}} = Q_L\,(\mathbf{3},\mathbf{1},1/6) \oplus (U_R)^c\,(\mathbf{\overline{3}},\mathbf{1},-2/3) \oplus (E_R)^c\,(\mathbf{1},\mathbf{1},1)\,,$$

Three major problems of the model:

(1) Field content does not actually achieve gauge coupling unification

(2) Predicts “wrong” relations among lepton and down-quark masses:

$$-\mathcal{L}_{\text{Yukawa}} = Y_u \epsilon_{ijklm} \overline{\psi^{ij}_{\mathbf{10}}} (\psi^{kl}_{\mathbf{10}})^c \phi^{m*}_{\mathbf{5}} + Y_{d\ell} \phi^i_{\mathbf{5}} \overline{\psi^{ij}_{\mathbf{10}}} (\psi^j_{\mathbf{\overline{5}}})^c$$

⇨ $m_d = m_e\,,\ m_s = m_\mu\,,\ m_b = m_\tau$ (at the GUT scale)

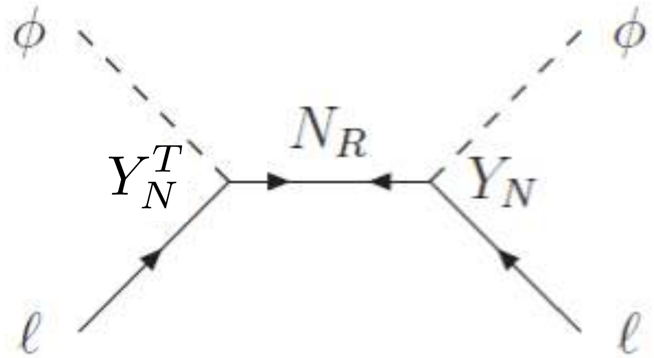
(3) Neutrino are massless just like in the Standard Model

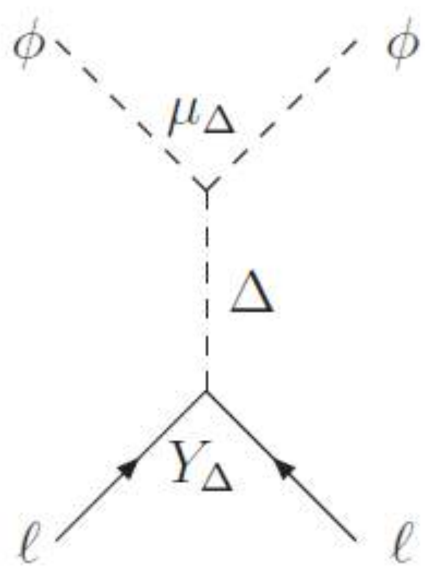
Can the 3 shortcomings of minimal SU(5) be fixed *simultaneously*?
Let's start adding neutrino mass terms:

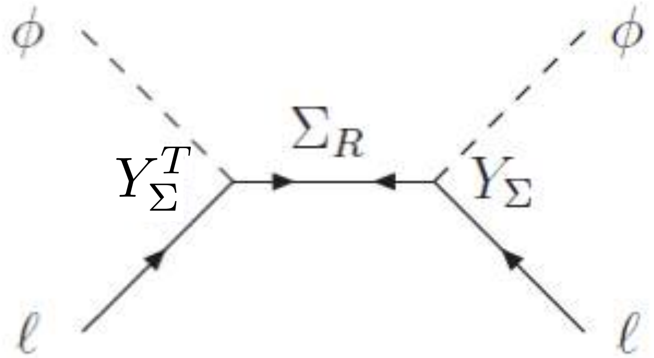
- Dirac: $\mathcal{L} \supset -(Y_\nu)_{ij}\, \overline{\nu}_{R\,i}\, \widetilde{\Phi}^\dagger\, L_{L\,j} + \text{h.c.} \quad \Longrightarrow \quad (m_\nu^D)_{ij} = \frac{v}{\sqrt{2}}(Y_\nu)_{ij}.$
  - At least 2 RH (i.e. sterile) neutrinos are introduced
  - Lepton number (*L*) is conserved
  - *L*-conservation actually needs to be enforced to prevent $M_R \bar{\nu}_R^c \nu_R$
  - Requires $Y_\nu \lesssim 10^{-12}$ (10^7 times smaller than the electron Yukawa)
  - SM singlets: gauge coupling unification *not affected*

- Majorana: $\mathcal{L} \supset \frac{C_{ij}}{\Lambda} \left(\overline{L^c_{L\,i}}\, \tau_2 \Phi\right) \left(\Phi^T \tau_2 L_{L\,j}\right) + \text{h.c.} \quad \Longrightarrow \quad (m_\nu^M)_{ij} = \frac{C_{ij} v^2}{\Lambda}$ Weinberg '79
  - Effective dimension-5 operator (only one of that order in the SMEFT)
  - $\Delta L = 2 \ \Rightarrow$ Lepton Number Violation
  - Naturally explain smallness of neutrino masses (if $\Lambda \gg v$)
  - Requires an UV completion at Λ (that is, indicates a *new physics* scale) that might potentially aid gauge coupling unification

Three ways of generating the Weinberg operator at the tree level:







Type I

Heavy fermionic singlets (RH neutrinos)

Minkowski, Gell-Mann, Ramond, Slansky, Yanagida, Glashow, Mohapatra, Senjanovic, ...

Type II

Heavy scalar triplet

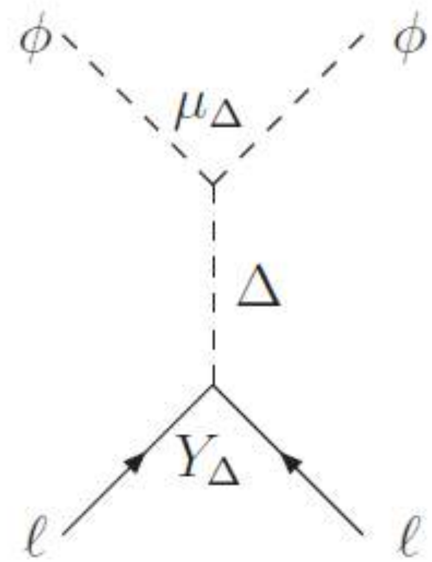
Magg, Wetterich, Lazarides, Shafi, Mohapatra, Senjanovic, Schecter, Valle, ...

Type III

Heavy fermionic triplets

Foot, Lew, He, Joshi, Ma, Roy, Hambye et al., Bajc et al., Dorsner, Fileviez-Perez, ...

We choose the most predictive (for spectrum & LFV) option:



Type I

Heavy fermionic singlets (RH neutrinos)

Type II

Heavy scalar triplet

Type III

Heavy fermionic triplets

Scalar SU(2) triplet (hypercharge Y=1)

$$\Delta = \begin{pmatrix} \Delta^-/\sqrt{2} & \Delta^0 \\ \Delta^{--} & -\Delta^-/\sqrt{2} \end{pmatrix}$$

$$\mathcal{L}_{\text{Type II}} \supset Y_\Delta^{\alpha\beta} \overline{L}_\alpha \Delta i\sigma_2 L^c_\beta + \mu_\Delta H^T i\sigma_2 \Delta H + \text{h.c.} \Longrightarrow m_\nu = Y_\Delta \frac{\mu_\Delta v^2}{M_\Delta^2}$$

L-breaking term $[L(\Delta) = -2]$

Type II seesaw can address problems (1) and (3) of minimal SU(5)
Only one additional scalar representation is needed:

$$\mathbf{15}_H \; : \qquad \phi_{\mathbf{15}} = \Delta\,(\mathbf{1},\mathbf{3},1) \oplus \widetilde{R_2}\,(\mathbf{3},\mathbf{2},1/6) \oplus S\,(\mathbf{6},\mathbf{1},-2/3)$$

Type II triplet ← Δ; $\widetilde{R_2}$ → a scalar leptoquark (LQ)

Three possible ways to fix problem (2), *i.e.* the fermion mass relations:

Model 1: add non-renormalisable operators

Ellis Gallard '79
Dorsner Perez '05

$$-\mathcal{L}_{\rm Yukawa} \supset \frac{Y'_u}{\Lambda}\epsilon_{ijklm}\overline{\psi^{ij}_{\mathbf{10}}}(\psi^{kl}_{\mathbf{10}})^c\phi^{mn}_{\mathbf{24}}\phi^{n*}_{\mathbf{5}} + \frac{Y''_u}{\Lambda}\epsilon_{ijklm}\overline{\psi^{ij}_{\mathbf{10}}}(\psi^{kn}_{\mathbf{10}})^c\phi^{mn}_{\mathbf{24}}\phi^{l*}_{\mathbf{5}}$$
$$+\boxed{\frac{Y'_{d\ell}}{\Lambda}\phi^i_{\mathbf{5}}\overline{\psi^{ij}_{\mathbf{10}}}\phi^{jk}_{\mathbf{24}}(\psi^k_{\bar{\mathbf{5}}})^c} + \frac{Y''_{d\ell}}{\Lambda}\phi^i_{\mathbf{5}}\phi^{ij}_{\mathbf{24}}\overline{\psi^{jk}_{\mathbf{10}}}(\psi^k_{\bar{\mathbf{5}}})^c + \text{h.c.}\,,$$

no new fields!

$$\Lambda \gg v_{24} \quad [e.g.\ \Lambda \sim M_{\rm Planck}]$$

$$\Rightarrow \quad M_d = \frac{v_5}{\sqrt{2}}\left(Y_{d\ell} \boxed{+\frac{v_{24}}{\Lambda}Y'_{d\ell}} - \frac{3v_{24}}{2\Lambda}Y''_{d\ell}\right), \qquad M_\ell = \frac{v_5}{\sqrt{2}}\left(Y^T_{d\ell} \boxed{-\frac{3v_{24}}{2\Lambda}Y'^T_{d\ell}} - \frac{3v_{24}}{2\Lambda}Y''^T_{d\ell}\right)$$

Type II seesaw can address problems (1) and (3) of minimal SU(5)
Only one additional scalar representation is needed:

$$\mathbf{15}_H \; : \qquad \phi_{\mathbf{15}} = \Delta\,(\mathbf{1},\mathbf{3},1) \oplus \widetilde{R_2}\,(\mathbf{3},\mathbf{2},1/6) \oplus S\,(\mathbf{6},\mathbf{1},-2/3)$$

Type II triplet ← Δ ; $\widetilde{R_2}$ → a scalar leptoquark (LQ)

Three possible ways to fix problem (2), *i.e.* the fermion mass relations:

Model 2: add a scalar $\mathbf{45}_H$: Dorsner Mocioiu '07

$$\phi_{\mathbf{45}} = \varphi_8(\mathbf{8},\mathbf{2},1/2) \oplus \varphi_{\overline{6}}(\overline{\mathbf{6}},\mathbf{1},-1/3) \oplus \varphi_3^T(\mathbf{3},\mathbf{3},-1/3) \oplus$$
$$\varphi_3^D(\overline{\mathbf{3}},\mathbf{2},-7/6) \oplus \varphi_3^S(\mathbf{3},\mathbf{1},-1/3) \oplus \varphi_{\overline{3}}^S(\overline{\mathbf{3}},\mathbf{1},4/3) \oplus H_2(\mathbf{1},\mathbf{2},1/2)$$

H_2: 2nd Higgs doublet

$$-\mathcal{L}_{\text{Yukawa}} \supset Y_u \epsilon_{ijklm} \overline{\psi^{ij}_{\mathbf{10}}} (\psi^{kl}_{\mathbf{10}})^c \phi^{m*}_{\mathbf{5}} + Y_{d\ell} \phi^i_{\mathbf{5}} \overline{\psi^{ij}_{\mathbf{10}}} (\psi^j_{\overline{\mathbf{5}}})^c$$
$$+ Y'_u \epsilon_{ijklm} \overline{\psi^{ij}_{\mathbf{10}}} (\psi^{nk}_{\mathbf{10}})^c \phi^{lmn*}_{\mathbf{45}} + \boxed{Y'_{d\ell} \phi^{ijk}_{\mathbf{45}} \overline{\psi^{ij}_{\mathbf{10}}} (\psi^k_{\overline{\mathbf{5}}})^c} + \text{h.c.}$$

$$\Longrightarrow \quad M_d = \frac{1}{\sqrt{2}}\left(v_5 Y_{d\ell} \boxed{+ 2 v_{45} Y'_{d\ell}}\right), \quad M_\ell = \frac{1}{\sqrt{2}}\left(v_5 Y^T_{d\ell} \boxed{- 6 v_{45} Y'^T_{d\ell}}\right)$$

Type II seesaw can address problems (1) and (3) of minimal SU(5)

Only one additional scalar representation is needed:

$$\mathbf{15}_H \; : \qquad \phi_{\mathbf{15}} = \Delta\,(\mathbf{1},\mathbf{3},1) \oplus \widetilde{R_2}\,(\mathbf{3},\mathbf{2},1/6) \oplus S\,(\mathbf{6},\mathbf{1},-2/3)$$

Type II triplet ← Δ $\qquad$ $\widetilde{R_2}$ → a scalar leptoquark (LQ)

Three possible ways to fix problem (2), *i.e.* the fermion mass relations:

Model 3: add vector-like fermions (**5**+**5** or **10**+**10**) Dorsner Fajfer Mustac '14

For example:

$$\psi^v_{\overline{\mathbf{5}}} = D^c_V(\overline{\mathbf{3}},\mathbf{1},1/3) \oplus L_V(\mathbf{1},\mathbf{2},-1/2)\,,$$
$$\psi^v_{\mathbf{5}} = D_V(\mathbf{3},\mathbf{1},-1/3) \oplus L^c_V(\mathbf{1},\mathbf{2},1/2)\,,$$

$$-\mathcal{L}_{\text{Yukawa}} \supset \begin{pmatrix} \overline{\psi^\alpha_{\overline{\mathbf{5}}}} & \overline{\psi^v_{\overline{\mathbf{5}}}} \end{pmatrix} \begin{pmatrix} Y^{\alpha\beta}_{d\ell}\phi^*_{\mathbf{5}} & M^\alpha_5 + \lambda^\alpha_5 \phi_{\mathbf{24}} \\ Y'^{\beta}_{d\ell}\phi^*_{\mathbf{5}} & M^V_5 + \lambda^V_5 \phi_{\mathbf{24}} \end{pmatrix} \begin{pmatrix} (\psi^\beta_{\mathbf{10}})^c \\ \psi^v_{\mathbf{5}} \end{pmatrix} + \text{h.c.}$$

$$\to \begin{pmatrix} \overline{(D^\alpha_R)^c} & \overline{D^c_V} \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} v_5 Y^{\alpha\beta}_{d\ell} & M^\alpha_5 + \lambda^\alpha_5 v_{24} \\ \frac{1}{\sqrt{2}} v_5 Y'^{\beta}_{d\ell} & M^V_5 + \lambda^V_5 v_{24} \end{pmatrix} \begin{pmatrix} (D^\beta_L)^c \\ D^c_V \end{pmatrix} +$$

$$\begin{pmatrix} \overline{E^\alpha_L} & \overline{E'_V} \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} v_5 Y^{\alpha\beta}_{d\ell} & M^\alpha_5 - \frac{3}{2}\lambda^\alpha_5 v_{24} \\ \frac{1}{\sqrt{2}} v_5 Y'^{\beta}_{d\ell} & M^V_5 - \frac{3}{2}\lambda^V_5 v_{24} \end{pmatrix} \begin{pmatrix} E^\beta_R \\ E'_V \end{pmatrix},$$

Intermediate-scale fields and gauge coupling unification

Fields that might have mass below the GUT scale and affect the running of g_i:

All models							
Field	$SU(5)$	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	b_3^I	b_2^I	b_1^I
ϱ_3	$\phi_{\mathbf{24}}$	$\mathbf{1}$	$\mathbf{3}$	0	0	1/3	0
ϱ_8	$\phi_{\mathbf{24}}$	$\mathbf{8}$	$\mathbf{1}$	0	1/2	0	0
Δ	$\phi_{\mathbf{15}}$	$\mathbf{1}$	$\mathbf{3}$	1	0	2/3	3/5
$\widetilde{R}_2$	$\phi_{\mathbf{15}}$	$\mathbf{3}$	$\mathbf{2}$	1/6	1/3	1/2	1/30
S	$\phi_{\mathbf{15}}$	$\mathbf{6}$	$\mathbf{1}$	-2/3	5/6	0	8/15

Model 2							
Field	$SU(5)$	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	b_3^I	b_2^I	b_1^I
φ_8	$\phi_{\mathbf{45}}$	$\mathbf{8}$	$\mathbf{2}$	1/2	2	4/3	4/5
$\varphi_{\overline{6}}$	$\phi_{\mathbf{45}}$	$\overline{\mathbf{6}}$	$\mathbf{1}$	-1/3	5/6	0	2/15
φ_3^T	$\phi_{\mathbf{45}}$	$\mathbf{3}$	$\mathbf{3}$	-1/3	1/2	2	1/5
$\varphi_{\overline{3}}^D$	$\phi_{\mathbf{45}}$	$\overline{\mathbf{3}}$	$\mathbf{2}$	-7/6	1/3	1/2	49/30
φ_3^S	$\phi_{\mathbf{45}}$	$\mathbf{3}$	$\mathbf{1}$	-1/3	1/6	0	1/15
$\varphi_{\overline{3}}^S$	$\phi_{\mathbf{45}}$	$\overline{\mathbf{3}}$	$\mathbf{1}$	4/3	1/6	0	16/15
H_2	$\phi_{\mathbf{45}}$	$\mathbf{1}$	$\mathbf{2}$	1/2	0	1/6	1/10

Model 3							
Field	$SU(5)$	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	b_3^I	b_2^I	b_1^I
L_V	$\psi^v_{\mathbf{5}}$	$\mathbf{1}$	$\mathbf{2}$	-1/2	0	1/3	1/5
D_V^c	$\psi^v_{\mathbf{5}}$	$\overline{\mathbf{3}}$	$\mathbf{1}$	1/3	1/3	0	2/15
L_V^c	$\psi^v_{\overline{\mathbf{5}}}$	$\mathbf{1}$	$\mathbf{2}$	1/2	0	1/3	1/5
D_V	$\psi^v_{\overline{\mathbf{5}}}$	$\mathbf{3}$	$\mathbf{1}$	-1/3	1/3	0	2/15
Q_V	$\psi^v_{\mathbf{10}}$	$\mathbf{3}$	$\mathbf{2}$	1/6	2/3	1	1/15
U_V^c	$\psi^v_{\mathbf{10}}$	$\overline{\mathbf{3}}$	$\mathbf{1}$	-2/3	1/3	0	8/15
E_V^c	$\psi^v_{\mathbf{10}}$	$\mathbf{1}$	$\mathbf{1}$	1	0	0	2/5
Q_V^c	$\psi^v_{\overline{\mathbf{10}}}$	$\overline{\mathbf{3}}$	$\mathbf{2}$	-1/6	2/3	1	1/15
U_V	$\psi^v_{\overline{\mathbf{10}}}$	$\mathbf{3}$	$\mathbf{1}$	2/3	1/3	0	8/15
E_V	$\psi^v_{\overline{\mathbf{10}}}$	$\mathbf{1}$	$\mathbf{1}$	-1	0	0	2/5

$$\alpha_{\rm GUT}^{-1} = \alpha_i^{-1}(m_Z) - \frac{b_i^{\rm eff}}{2\pi}\ln\left(\frac{M_{\rm GUT}}{m_Z}\right), \quad b_i^{\rm eff} \equiv b_i^{\rm SM} + \sum_I b_i^I r_I\,, \quad r_I \equiv \frac{\ln(M_{\rm GUT}/M_I)}{\ln(M_{\rm GUT}/m_Z)} \subset [0,1]\,,$$

⇨ gauge coupling unification constraint (on fields' masses M_I):

$$\frac{b_2^{\rm eff} - b_3^{\rm eff}}{b_1^{\rm eff} - b_2^{\rm eff}} = \frac{\alpha_2^{-1}(m_Z) - \alpha_3^{-1}(m_Z)}{\alpha_1^{-1}(m_Z) - \alpha_2^{-1}(m_Z)} = \frac{5\sin^2\theta_w - 5\alpha_{em}/\alpha_s}{3 - 8\sin^2\theta_w} = 0.717 \pm 0.002$$

Giveon Hall Sarid '91

As well known, SU(5) gauge bosons X^μ, Y^μ $(\overline{\mathbf{3}}, \mathbf{2}, \pm 5/6)$ induce p decay

We set $M_{X,Y} \equiv M_{\rm GUT}$, that is, the scale where the coupling unify:

$$\ln\left(\frac{M_{\rm GUT}}{m_Z}\right) = \frac{6\pi - 16\pi\sin^2\theta_w}{5\alpha_{em}(b_1^{\rm eff} - b_2^{\rm eff})}$$

⇩

$$\Gamma(p\to\pi^0\ell_i^+) = \frac{\pi\, m_p\,\alpha^2_{\rm GUT}}{2M^4_{\rm GUT}} A^2 \Big\{ \left|(V_1)_{11}(V_3)_{1i}\langle\pi^0|(ud)_R u_L|p\rangle\right|^2 + \left|\left[(V_1)_{11}(V_2)_{i1} + (V_1V^*_{\rm CKM})_{11}(V_2V^T_{\rm CKM})_{i1}\right]\langle\pi^0|(ud)_L u_L|p\rangle\right|^2 \Big\},$$

$$\Gamma(p\to K^0\ell_i^+) = \frac{\pi\, m_p\,\alpha^2_{\rm GUT}}{2M^4_{\rm GUT}} \left(1-\frac{m_K^2}{m_p}\right)^2 A^2 \Big\{ \left|(V_1)_{11}(V_3)_{2i}\langle K^0|(us)_R u_L|p\rangle\right|^2 + \left|\left[(V_1)_{11}(V_2)_{i2} + (V_1V^*_{\rm CKM})_{12}(V_2V^T_{\rm CKM})_{i1}\right]\langle K^0|(us)_L u_L|p\rangle\right|^2 \Big\},$$

$$\Gamma(p\to\pi^+\overline{\nu}) = \frac{\pi\, m_p\,\alpha^2_{\rm GUT}}{2M^4_{\rm GUT}} A^2 |(V_1V_{\rm CKM})_{11}\langle\pi^+|(du)_R d_L|p\rangle|^2,$$

$$\Gamma(p\to K^+\overline{\nu}) = \frac{\pi\, m_p\,\alpha^2_{\rm GUT}}{2M^4_{\rm GUT}} \left(1-\frac{m_K^2}{m_p}\right)^2 A^2 \Big\{ \left|(V_1V_{\rm CKM})_{11}\langle K^+|(us)_R d_L|p\rangle\right|^2 + \left|(V_1V_{\rm CKM})_{12}\langle K^+|(ud)_R s_L|p\rangle\right|^2 \Big\}.$$

Nath Perez '06

Mode	Limit (years)
$p\to\pi^0 e^+$	$> 2.4\times 10^{34}$
$p\to\pi^0 \mu^+$	$> 1.6\times 10^{34}$
$p\to K^0 e^+$	$> 1.0\times 10^{33}$
$p\to K^0 \mu^+$	$> 3.6\times 10^{33}$
$p\to\pi^+\overline{\nu}$	$> 3.9\times 10^{32}$
$p\to K^+\overline{\nu}$	$> 5.9\times 10^{33}$

SuperKamiokande '05-'22

Prediction affected by the (unknown) flavour rotations of RH fermions:

$$V_f^\dagger M_f V_f' = M_f^{\rm diag}$$

$$V_1 \equiv V_u'^\dagger V_u^*\,, \quad V_2 \equiv V_\ell'^\dagger V_d^*\,, \quad V_3 \equiv V_\ell^\dagger V_d'^*\,.$$

But summing over neutrino flavours, the neutrino modes less uncertain: only unknown is V_1 that is =**1** in minimal SU(5) and whenever the up-quark mass matrix is symmetric (and ≈**1** if RH rotations are small)

→ $p \to K/\pi\,\bar\nu$ give conservative, (almost) model-independent bounds!

$$\Gamma(p \to K^0 \ell_i^+) = \frac{\pi\, m_p\, \alpha_{\rm GUT}^2}{2M_{\rm GUT}^4}\left(1-\frac{m_K^2}{m_p}\right)^2 A^2 \Big\{ \left|(V_1)_{11}(V_3)_{2i}\langle K^0|(us)_R u_L|p\rangle\right|^2 + \left|\left[(V_1)_{11}(V_2)_{i2} + (V_1 V_{\rm CKM}^*)_{12}(V_2 V_{\rm CKM}^T)_{i1}\right]\langle K^0|(us)_L u_L|p\rangle\right|^2 \Big\},$$

$$\Gamma(p \to \pi^+\bar\nu) = \frac{\pi\, m_p\, \alpha_{\rm GUT}^2}{2M_{\rm GUT}^4} A^2 |(V_1 V_{\rm CKM})_{11}\langle \pi^+|(du)_R d_L|p\rangle|^2\,,$$

$$\Gamma(p \to K^+\bar\nu) = \frac{\pi\, m_p\, \alpha_{\rm GUT}^2}{2M_{\rm GUT}^4}\left(1-\frac{m_K^2}{m_p}\right)^2 A^2 \Big\{ \left|(V_1 V_{\rm CKM})_{11}\langle K^+|(us)_R d_L|p\rangle\right|^2 + \left|(V_1 V_{\rm CKM})_{12}\langle K^+|(ud)_R s_L|p\rangle\right|^2 \Big\}.$$

Nath Perez '06

$p \to \pi^0 e^+$	$> 2.4 \times 10^{34}$
$p \to \pi^0 \mu^+$	$> 1.6 \times 10^{34}$
$p \to K^0 e^+$	$> 1.0 \times 10^{33}$
$p \to K^0 \mu^+$	$> 3.6 \times 10^{33}$
$p \to \pi^+ \bar\nu$	$> 3.9 \times 10^{32}$
$p \to K^+ \bar\nu$	$> 5.9 \times 10^{33}$

SuperKamiokande '05-'22

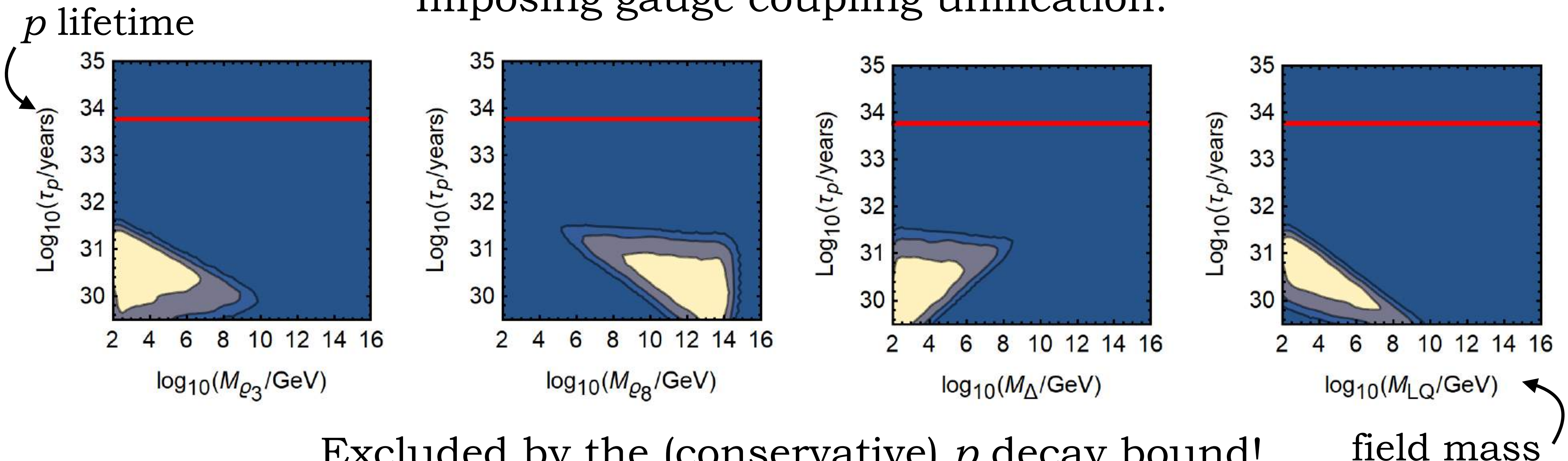
Fit of the models' spectrum

To reduce the number of parameters in the fit, we set at the GUT scale the masses of the fields that tend to reduce $M_{\rm GUT}$ (reducing p lifetime) or directly contribute to p decay (e.g. several leptoquarks from $\mathbf{45}_H$)

Model 1: only 4 parameters, the masses of

$\varrho_3\,(\mathbf{1},\mathbf{3},0)$, $\varrho_8\,(\mathbf{8},\mathbf{1},0)$ from $\mathbf{24}_H$ $\quad \Delta\,(\mathbf{1},\mathbf{3},1)$, $\widetilde{R}_2\,(\mathbf{3},\mathbf{2},1/6)$ from $\mathbf{15}_H$

Imposing gauge coupling unification:

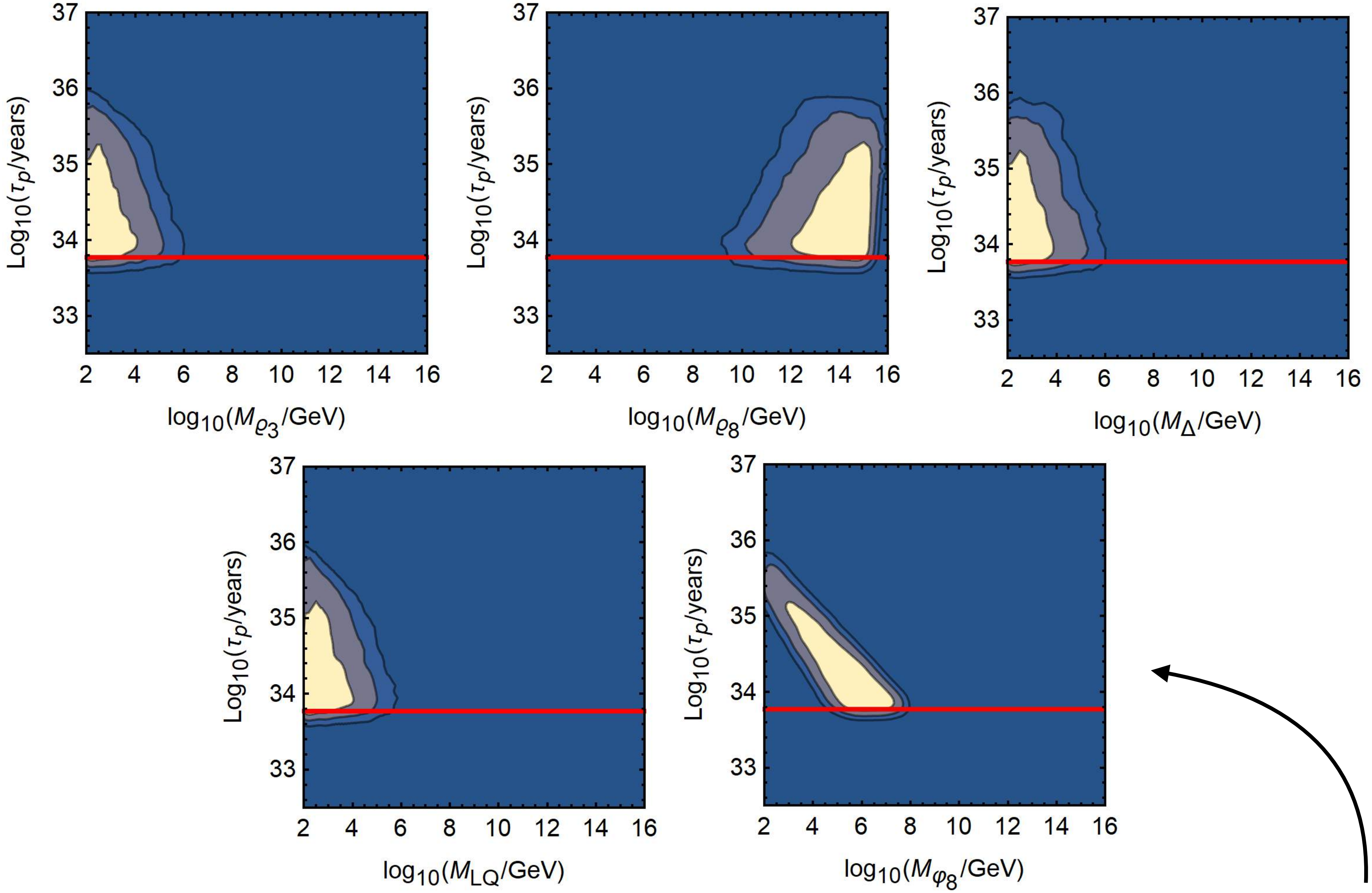


Excluded by the (conservative) p decay bound!

(virtually impossible to have a flavour structure of the fermion mass matrices that can *simultaneously* suppress $p \to K/\pi\,\ell$, $p \to K/\pi\,\nu$)

Model 2: previous 4 parameters + $\varphi_8\,(\mathbf{8},\mathbf{2},1/2)$ [and $H_2(\mathbf{1},\mathbf{2},1/2)$] from $\mathbf{45}_H$

Imposing gauge coupling unification & p-lifetime bound:



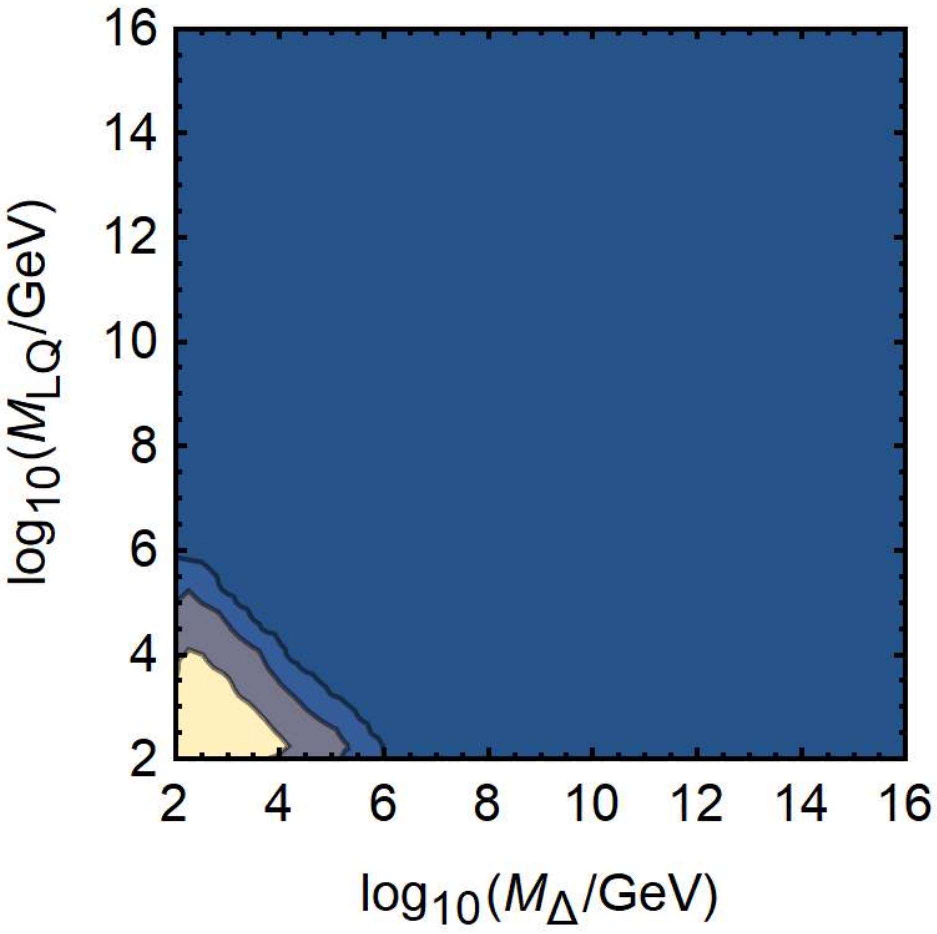
$\Rightarrow$ $\tau_p \lesssim 10^{35\,(36)}$ years at 1σ (2σ)

anti-correlated to the colour octet mass!

Model 2: previous 4 parameters + $\varphi_8\,(\mathbf{8},\mathbf{2},1/2)$ [and $H_2(\mathbf{1},\mathbf{2},1/2)$] from $\mathbf{45}_H$

4 particles have masses bounded from above

In particular, the type II triplet and the LQ in $\mathbf{15}_H$:



$M_\Delta,\ M_{\rm LQ} \lesssim 10\ (100)\ \text{TeV at } 1\sigma\ (2\sigma)$!!!

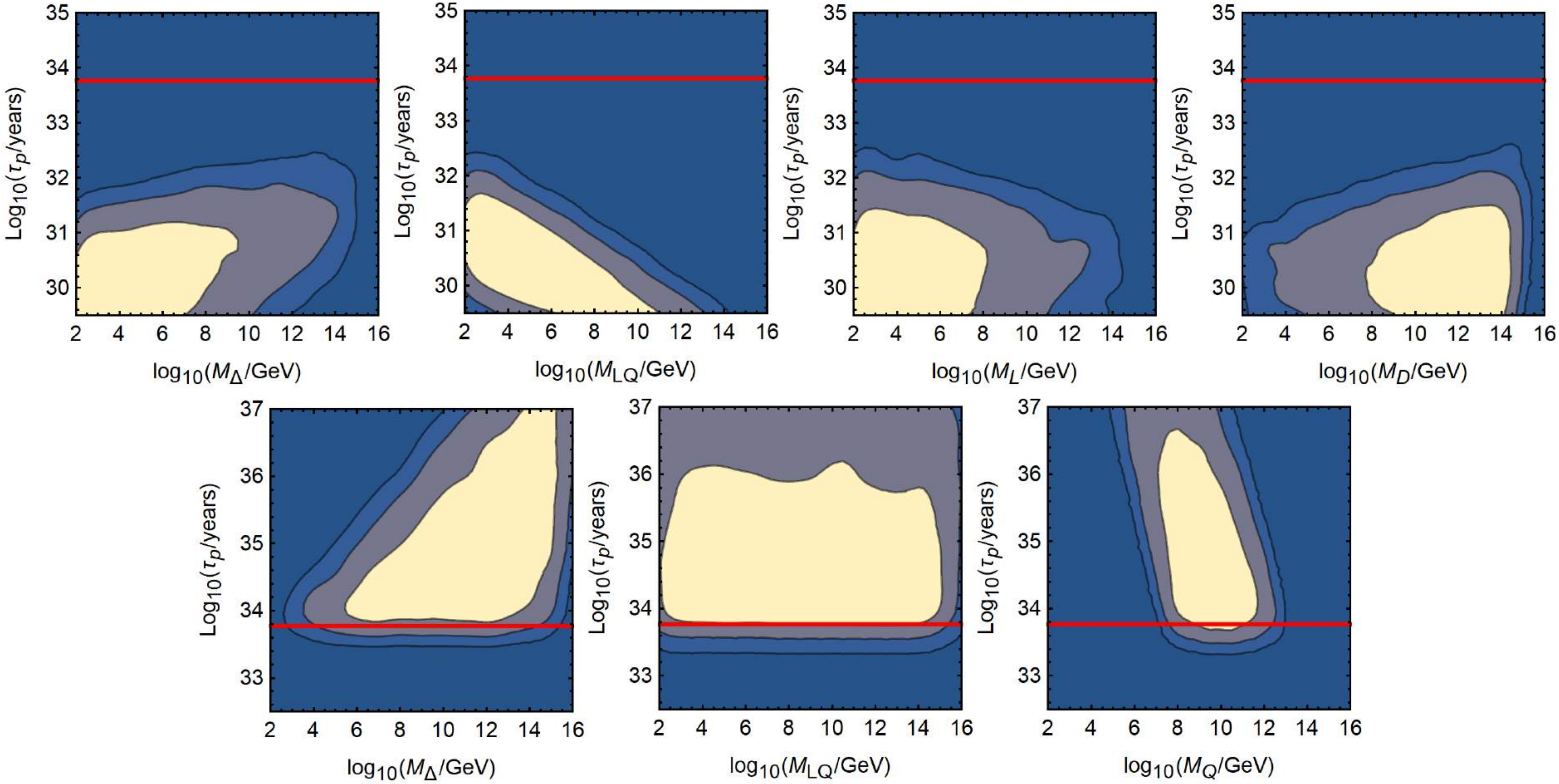
[prediction rather robust, even if the number of parameters is increased, see backup slide]

ted to the

et mass!

Model 3: previous 4 parameters + the masses of

$$L_V + L_V^c,\ D_V + D_V^c \text{ from } \mathbf{5}_F + \bar{\mathbf{5}}_F,\ \text{ or } Q_V + Q_V^c \text{ from } \mathbf{10}_F + \bar{\mathbf{10}}_F$$



Either excluded or not predictive!

The most minimal model requires relatively light triplet and LQ from $\mathbf{15}_H$

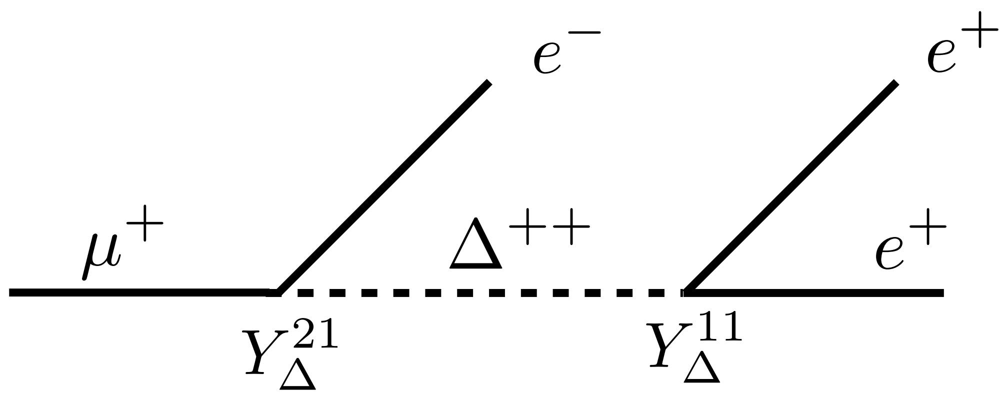
$$-\mathcal{L}_{\rm Yukawa} \supset Y_{15}^{\alpha\beta}\,\overline{\psi_{\mathbf{\bar 5}}}_{\alpha}\phi^*_{\mathbf{15}}\psi^c_{\mathbf{\bar 5}\,\beta} + {\rm h.c.} \;\to\; Y_{\Delta}^{\alpha\beta}\,\overline{L_L}_{\alpha}\Delta i\sigma_2 L^c_{L\,\beta} + Y_{LQ}^{\alpha\beta}\,\overline{D_R}_{\alpha}\widetilde{R_2}L_{L\,\beta} + {\rm h.c.}$$

$$\Delta = \begin{pmatrix} \Delta^-/\sqrt{2} & \Delta^0 \\ \Delta^{--} & -\Delta^-/\sqrt{2} \end{pmatrix}, \qquad \widetilde{R_2}^T = \left(\widetilde{R_2}^{2/3}, \widetilde{R_2}^{-1/3}\right)$$

still probably beyond the reach of colliders: how to test it then?

⇨ The most powerful way is searching for LFV decays

The type-II triplet effects are well known, in particular tree-level processes like:



$$\mathrm{BR}(\mu \to eee) = \frac{1}{4G_F^2 M_\Delta^4}\left|Y_\Delta^{21}\right|^2\left|Y_\Delta^{11}\right|^2$$

see e.g. Abada et al. '07

Flavour structure of the couplings directly linked to neutrino parameters:

$$Y_\Delta = \frac{M_\Delta^2}{\mu_\Delta\, v^2}\,U^*_{\rm PMNS}\, m_\nu^{\rm diag}\, U^\dagger_{\rm PMNS}$$

but overall effect depends on the unknown L-breaking term μ_Δ

The most minimal model requires relatively light triplet and LQ from $\mathbf{15}_H$

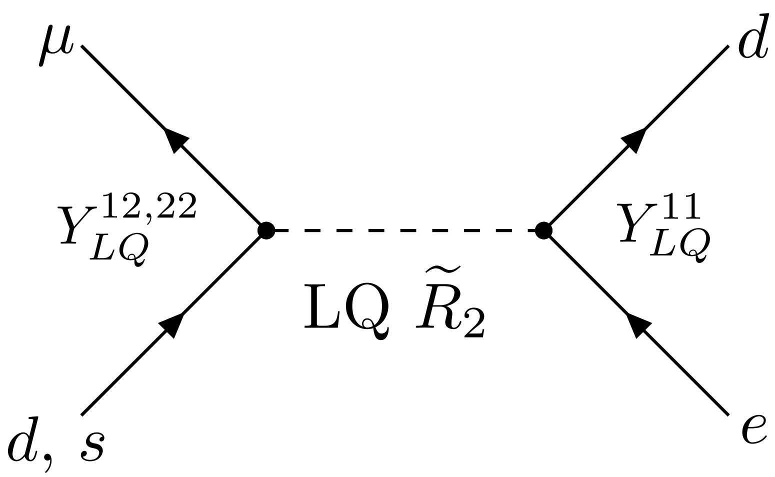
$$-\mathcal{L}_{\text{Yukawa}} \supset Y_{15}^{\alpha\beta}\, \overline{\psi_{\mathbf{5}}}_{\alpha} \phi_{\mathbf{15}}^{*} \psi_{\mathbf{5}\,\beta}^{c} + \text{h.c.} \;\rightarrow\; Y_{\Delta}^{\alpha\beta}\, \overline{L_L}_{\alpha} \Delta i\sigma_2 L_{L\,\beta}^{c} + Y_{LQ}^{\alpha\beta}\, \overline{D_R}_{\alpha} \widetilde{R}_2 L_{L\,\beta} + \text{h.c.}$$

$$\Delta = \begin{pmatrix} \Delta^{-}/\sqrt{2} & \Delta^{0} \\ \Delta^{--} & -\Delta^{-}/\sqrt{2} \end{pmatrix}, \qquad \widetilde{R}_2^{\,T} = \left(\widetilde{R}_2^{\,2/3}, \widetilde{R}_2^{\,-1/3}\right)$$

still probably beyond the reach of colliders: how to test it then?

The leptoquark mediates tree-level processes such as

see e.g. Dorsner et al. '16



$$\text{CR}(\mu\, N \to e\, N) = \frac{m_\mu^5}{4\Gamma_{\text{capt}}\, M_{LQ}^4} \left(V^{(p)} + 2V^{(n)}\right)^2 \left|Y_{LQ}^{21}\right|^2 \left|Y_{LQ}^{11}\right|^2$$

$$\text{BR}(K_L \to \mu e) = \frac{m_K \tau_{K_L}}{256\pi} \frac{m_\mu^2 f_{K_L}^2}{M_{LQ}^4} \left(1 - \frac{m_\mu^2}{m_{K_L}^2}\right)^2 \left|Y_{LQ}^{12} Y_{LQ}^{12\,*} + Y_{LQ}^{11} Y_{LQ}^{22\,*}\right|^2$$

The peculiarity of our GUT model is that the effects mediated by the LQ are also *controlled by the neutrino mass matrix*, due to the GUT relation:

$$Y_\Delta = Y_{LQ}/\sqrt{2} = Y_{15}\,, \qquad \text{[GUT scale]} \qquad \Rightarrow \qquad Y_{LQ} \approx 2.1\, Y_\Delta\,, \qquad \text{[TeV scale]}$$

(to very good approx., the RGEs do not affect the flavour structure)

Correlations among LFV processes

Δ-mediated

$$\mathrm{BR}(\mu \to eee) \simeq 1.1 \times 10^{-12} \left(\frac{10\ \mathrm{TeV}}{M_\Delta}\right)^4 \left(\frac{\left|Y_\Delta^{21}\right|^2 \left|Y_\Delta^{11}\right|^2}{0.05^4}\right),$$

$$\mathrm{BR}(\mu \to e\gamma) \simeq 3.6 \times 10^{-13} \left(\frac{10\ \mathrm{TeV}}{M_\Delta}\right)^4 \left(\frac{\left|\sum_\beta Y_\Delta^{2\beta\,*} Y_\Delta^{1\beta}\right|^2}{0.4^4}\right),$$

LQ-mediated

$$\mathrm{CR}(\mu\,\mathrm{Au} \to e\,\mathrm{Au}) \simeq 2.4 \times \mathrm{CR}(\mu\,\mathrm{Al} \to e\,\mathrm{Al}) \simeq 7.3 \times 10^{-13} \left(\frac{10\ \mathrm{TeV}}{M_{LQ}}\right)^4 \left(\frac{\left|Y_{LQ}^{21}\right|^2 \left|Y_{LQ}^{11}\right|^2}{0.02^4}\right),$$

$$\mathrm{BR}(K_L \to \mu e) \simeq 3.2 \times 10^{-12} \left(\frac{10\ \mathrm{TeV}}{M_{LQ}}\right)^4 \left(\frac{\left|Y_{LQ}^{12} Y_{LQ}^{12\,*} + Y_{LQ}^{11} Y_{LQ}^{22\,*}\right|^2}{0.04^4}\right),$$

$$\mathrm{BR}(K^+ \to \pi^+ \mu^+ e^-) \simeq 1.2 \times 10^{-11} \left(\frac{10\ \mathrm{TeV}}{M_{LQ}}\right)^4 \left(\frac{\left|Y_{LQ}^{21}\right|^4}{0.15^4}\right),$$

$$\mathrm{BR}(K^+ \to \pi^+ \mu^- e^+) \simeq 6.2 \times 10^{-10} \left(\frac{10\ \mathrm{TeV}}{M_{LQ}}\right)^4 \left(\frac{\left|Y_{LQ}^{11} Y_{LQ}^{22\,*}\right|^2}{0.4^4}\right).$$

Δ-mediated

$$\mathrm{BR}(\mu \to eee) \simeq 1.1 \times 10^{-12} \left(\frac{10\ \mathrm{TeV}}{M_\Delta}\right)^4 \left(\frac{\left|Y_\Delta^{21}\right|^2 \left|Y_\Delta^{11}\right|^2}{0.05^4}\right),$$

$$\mathrm{BR}(\mu \to e\gamma) \simeq 3.6 \times 10^{-13} \left(\frac{10\ \mathrm{TeV}}{M_\Delta}\right)^4 \left(\frac{\left|\sum_\beta Y_\Delta^{2\beta\,*} Y_\Delta^{1\beta}\right|^2}{0.4^4}\right),$$

same dependence on couplings!

LQ-mediated

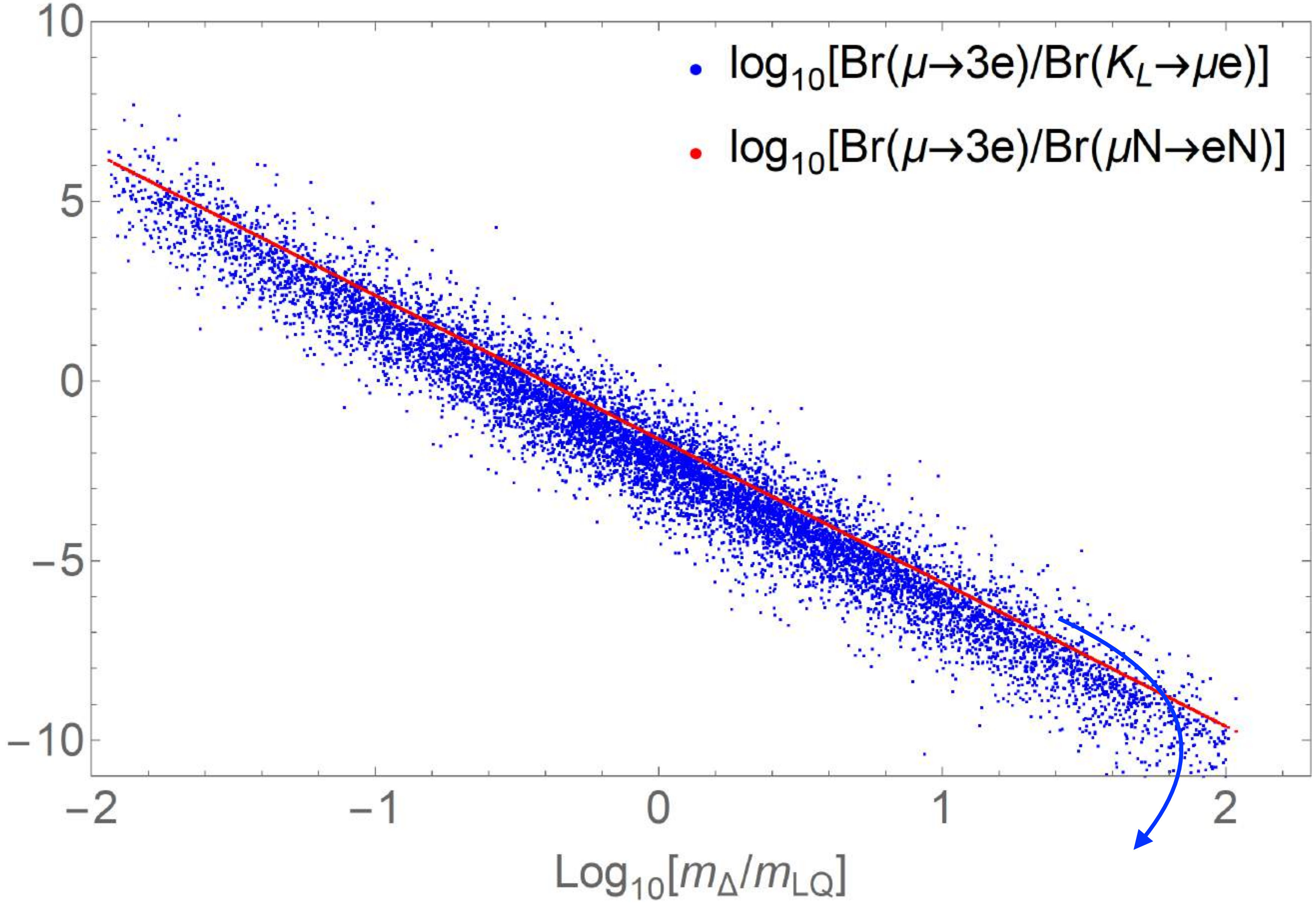
$$\mathrm{CR}(\mu\,\mathrm{Au} \to e\,\mathrm{Au}) \simeq 2.4 \times \mathrm{CR}(\mu\,\mathrm{Al} \to e\,\mathrm{Al}) \simeq 7.3 \times 10^{-13} \left(\frac{10\ \mathrm{TeV}}{M_{LQ}}\right)^4 \left(\frac{\left|Y_{LQ}^{21}\right|^2 \left|Y_{LQ}^{11}\right|^2}{0.02^4}\right),$$

Ratios of BRs only depends on mass ratios:

$$\mathrm{BR}(\mu \to eee) \simeq 0.0021 \left(\frac{M_{LQ}}{M_\Delta}\right)^4 \mathrm{CR}(\mu\,\mathrm{Au} \to e\,\mathrm{Au}) \simeq 0.0049 \left(\frac{M_{LQ}}{M_\Delta}\right)^4 \mathrm{CR}(\mu\,\mathrm{Al} \to e\,\mathrm{Al})$$

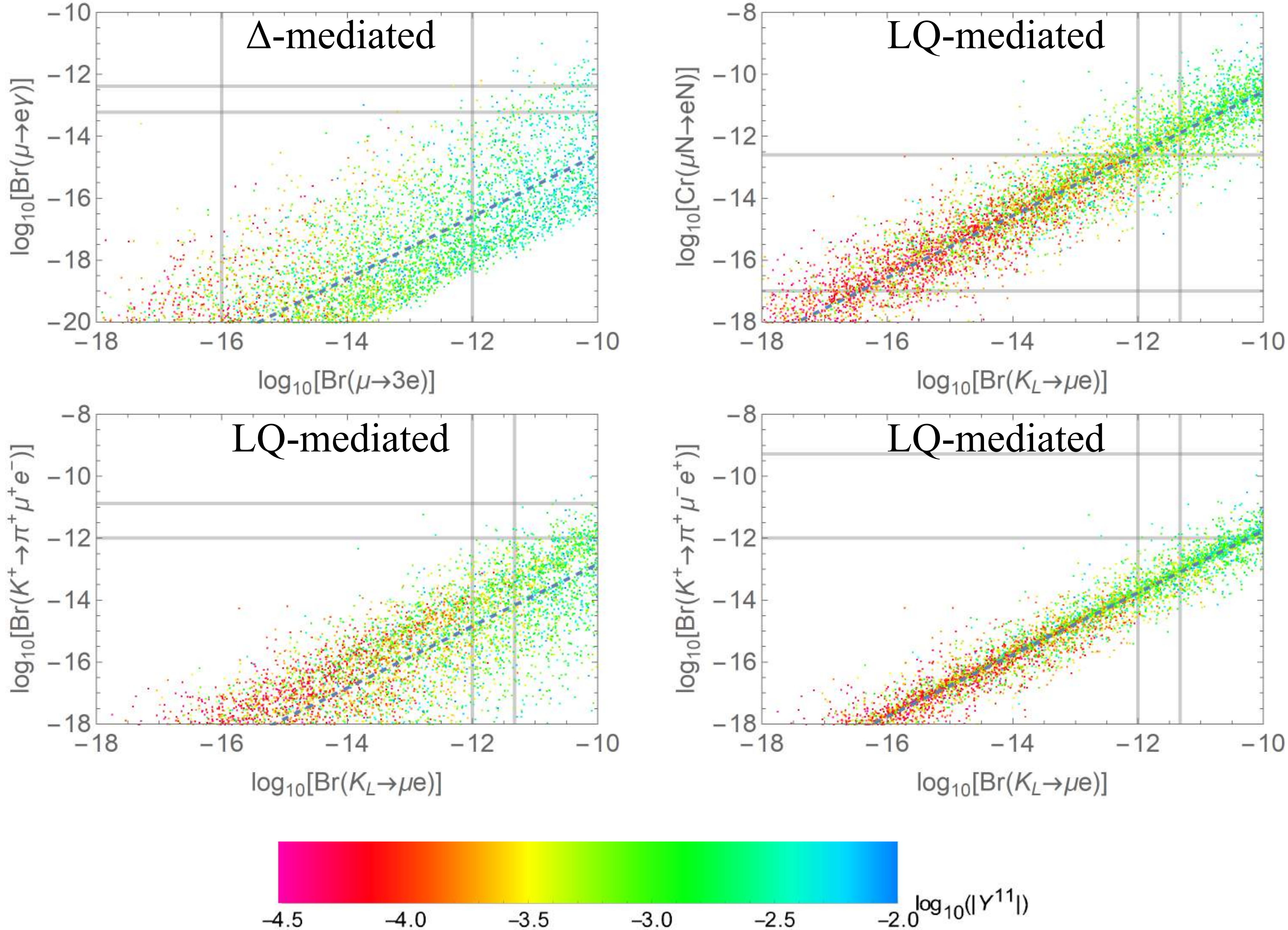
⇨ additional information on the mass spectrum to be confronted with unification and p-decay constraints: possible handle to test the underlying GUT structure!

Neglecting loop-suppressed contributions:



spread due to dependence on different combinations of the couplings (hence uncertainties on neutrino parameters, cf. e.g. Esteban et al. '20)

Correlations among decays induced by the same field

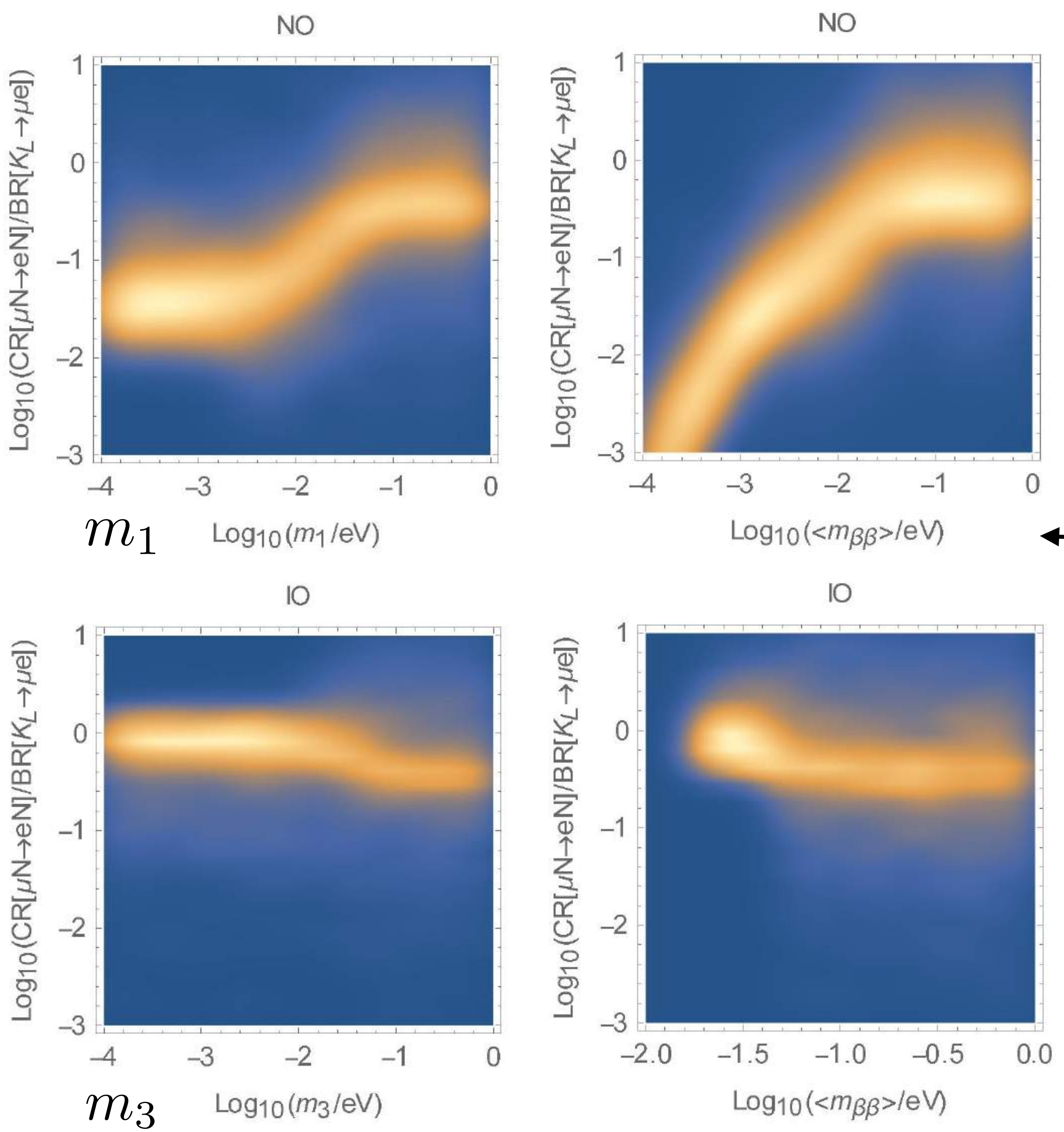


Spread only due to couplings dependence: ratio of BRs of LFV processes sensitive to neutrino mass parameters (including *unmeasured* ones)!

Sensitivity of $\mathrm{CR}(\mu\,\mathrm{Al} \to e\ \mathrm{Al})/\mathrm{BR}(K_L \to \mu e)$ to neutrino mass scale

Normal Ordering:

Inverted Ordering:

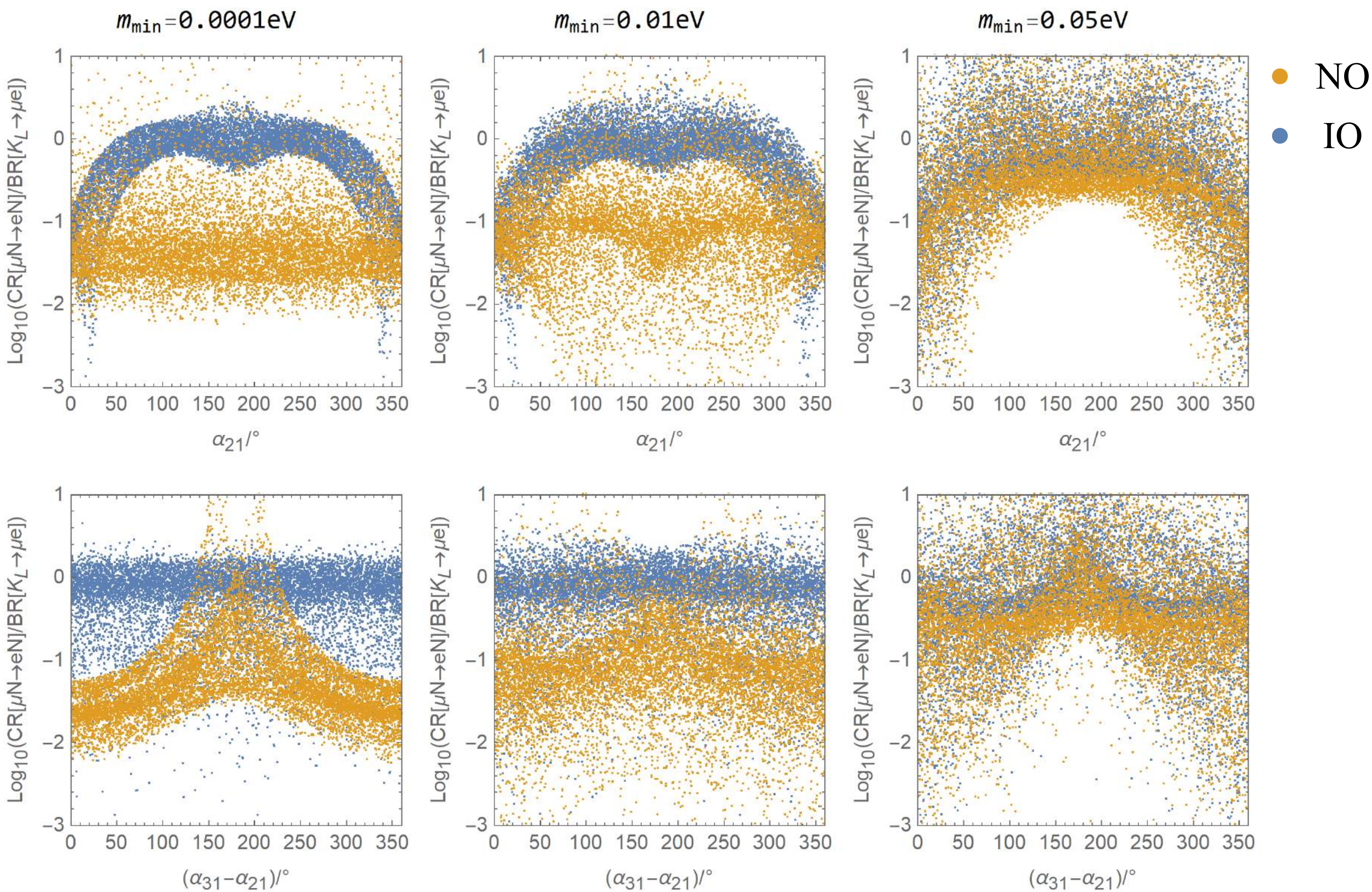


$\langle m_{\beta\beta}\rangle \equiv |\sum_i U_{ei}^2 m_i|$

For instance, $\mathrm{BR}(K_L \to \mu e) \gtrsim 10 \times \mathrm{CR}(\mu N \to eN)$ ⇨ IO disfavoured and $m_1,\ \langle m_{\beta\beta}\rangle \lesssim 10^{-2}$ eV

Sensitivity of $\mathrm{CR}(\mu\,\mathrm{Al} \to e\,\mathrm{Al})/\mathrm{BR}(K_L \to \mu e)$ to Majorana phases

$$U_{\mathrm{PMNS}} = U_{\mathrm{DIRAC}} \cdot P\,, \;\; P \equiv \mathrm{diag}(1, e^{-i\alpha_{21}/2}, e^{-i\alpha_{31}/2})$$

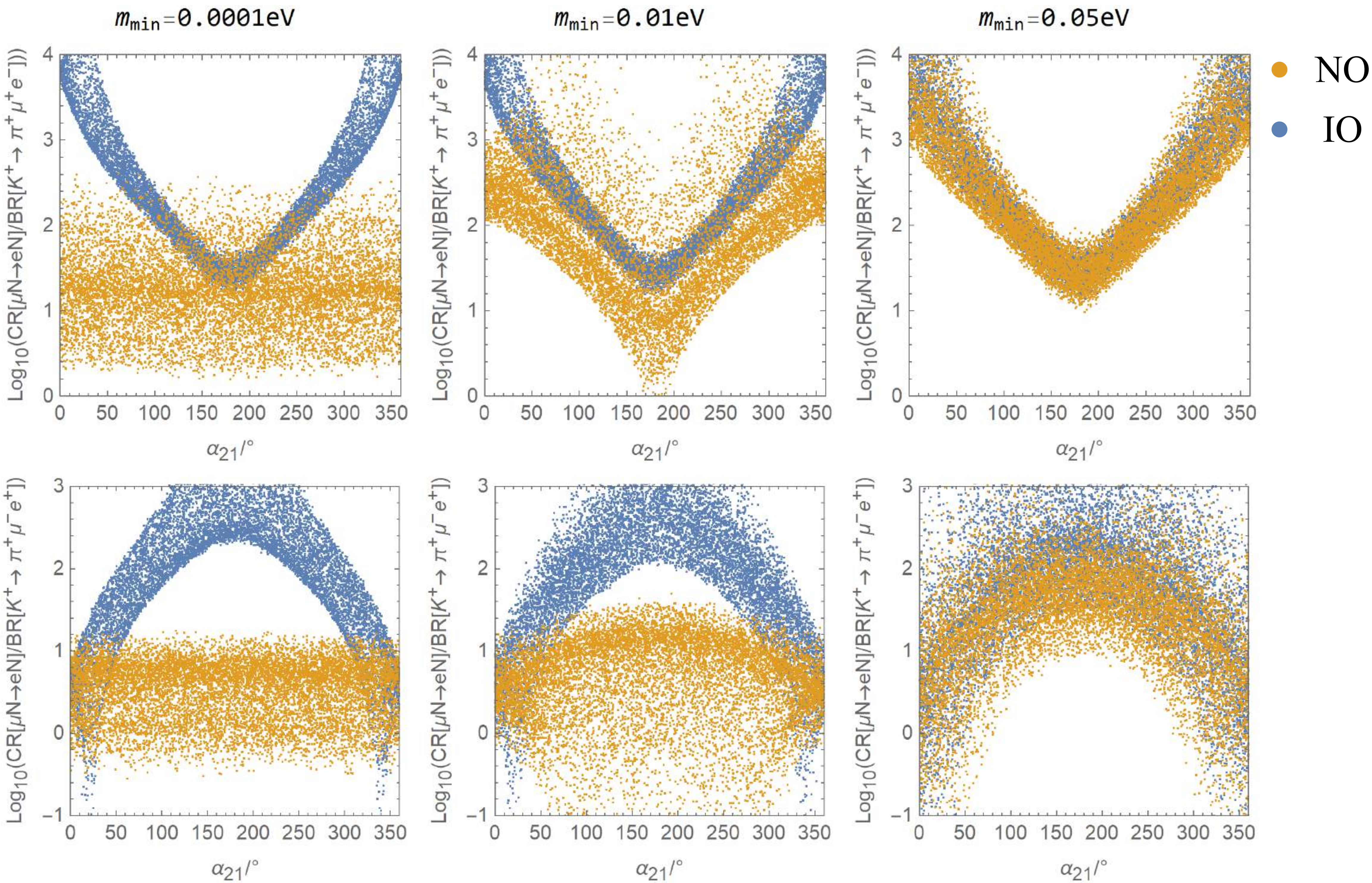


Sensitivity of $\mathrm{CR}(\mu\,\mathrm{Al} \to e\ \mathrm{Al})/\mathrm{BR}(K^+ \to \pi^+\mu^\pm e^\mp)$ to Majorana phases

$$U_{\mathrm{PMNS}} = U_{\mathrm{DIRAC}} \cdot P\,, \;\; P \equiv \mathrm{diag}(1, e^{-i\alpha_{21}/2}, e^{-i\alpha_{31}/2})$$

μ^+e^- :

μ^-e^+ :



Minimal GUT models typically predict low-energy/ intermediate-scale new fields to trigger unification

Type II seesaw is the most economical neutrino mass model that can induce unification and evade *p*-decay bounds

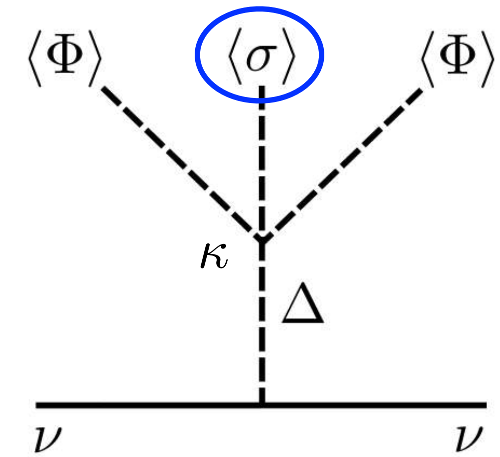
The type II triplet and its SU(5) partner are expected in the 1-100 TeV range and can thus induce sizeable LFV rates

Processes mediated by the triplet and by the leptoquark are highly correlated and could give information on their masses (and point to an underlying GUT origin)

The flavour structure of the couplings are dictated by the neutrino mass matrix, hence ratios of different LFV obs can be sensitive to neutrino parameters difficult to measure

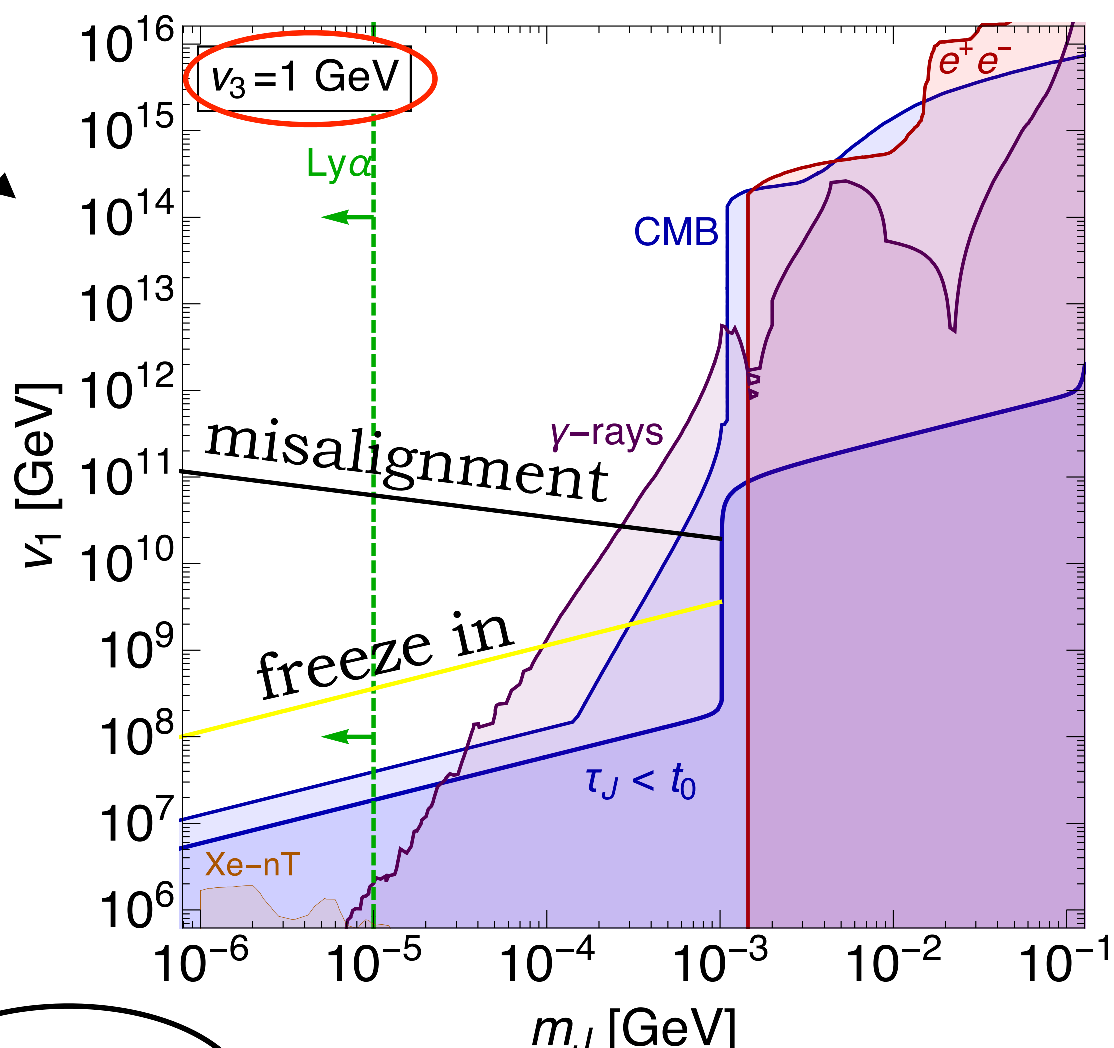
Bonus track

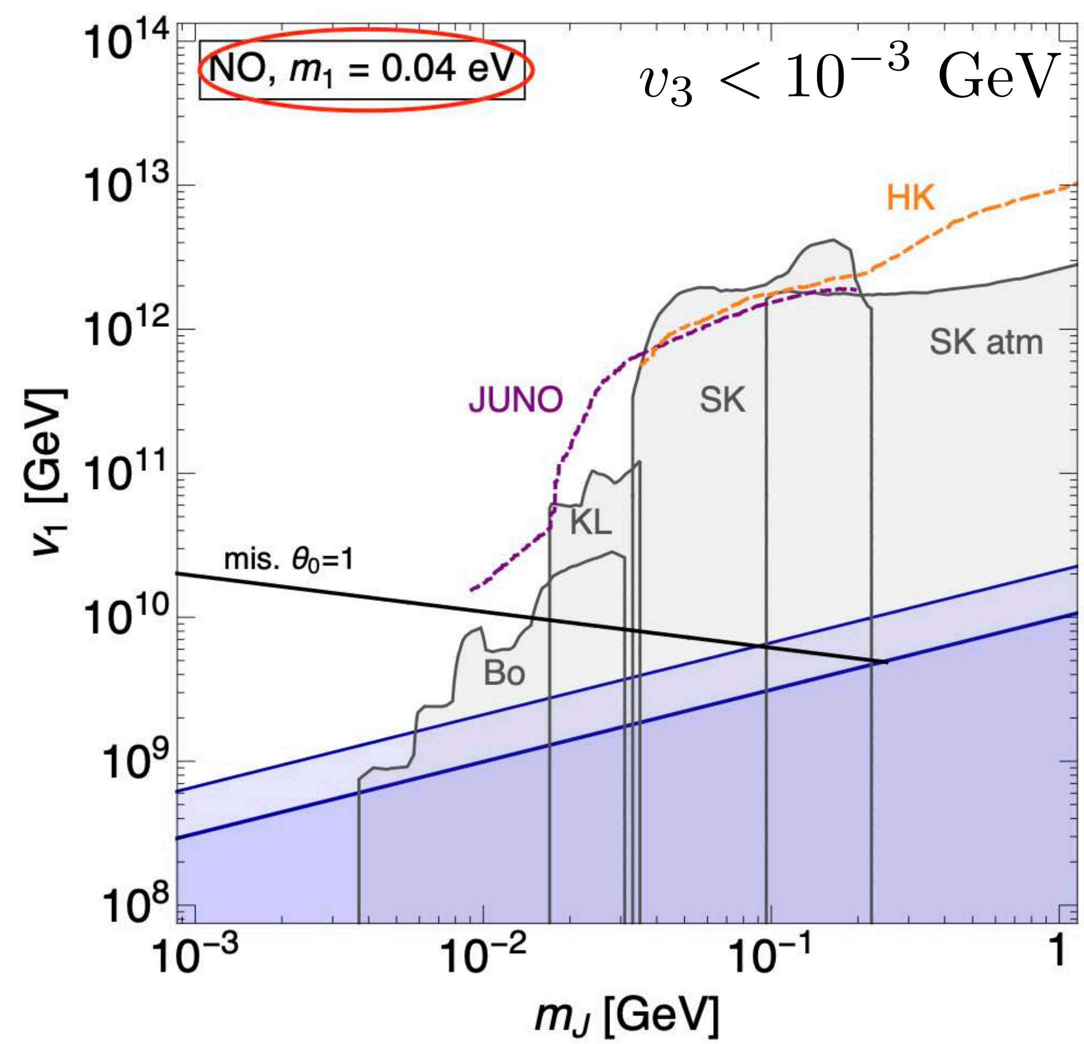
Type II seesaw can also provide an excellent dark matter candidate if lepton number is *spontaneously* broken



the type II majoron:

L-breaking scale





majoron mass

cf. Biggio LC Ota Zanchini arXiv:2304.12527

谢谢大家!

Thank you!

Additional slides

Adding more parameters to Model 2 fit

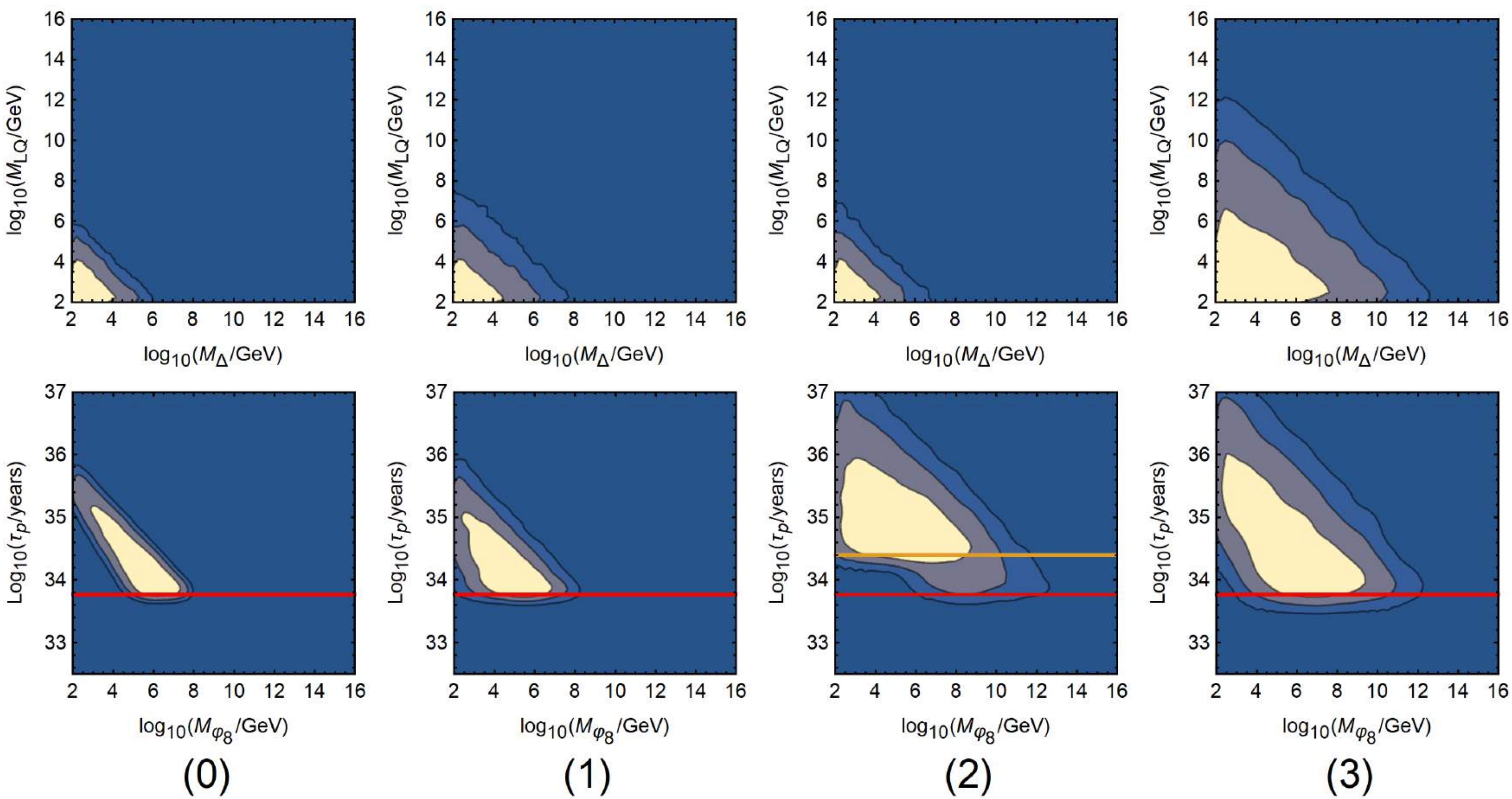
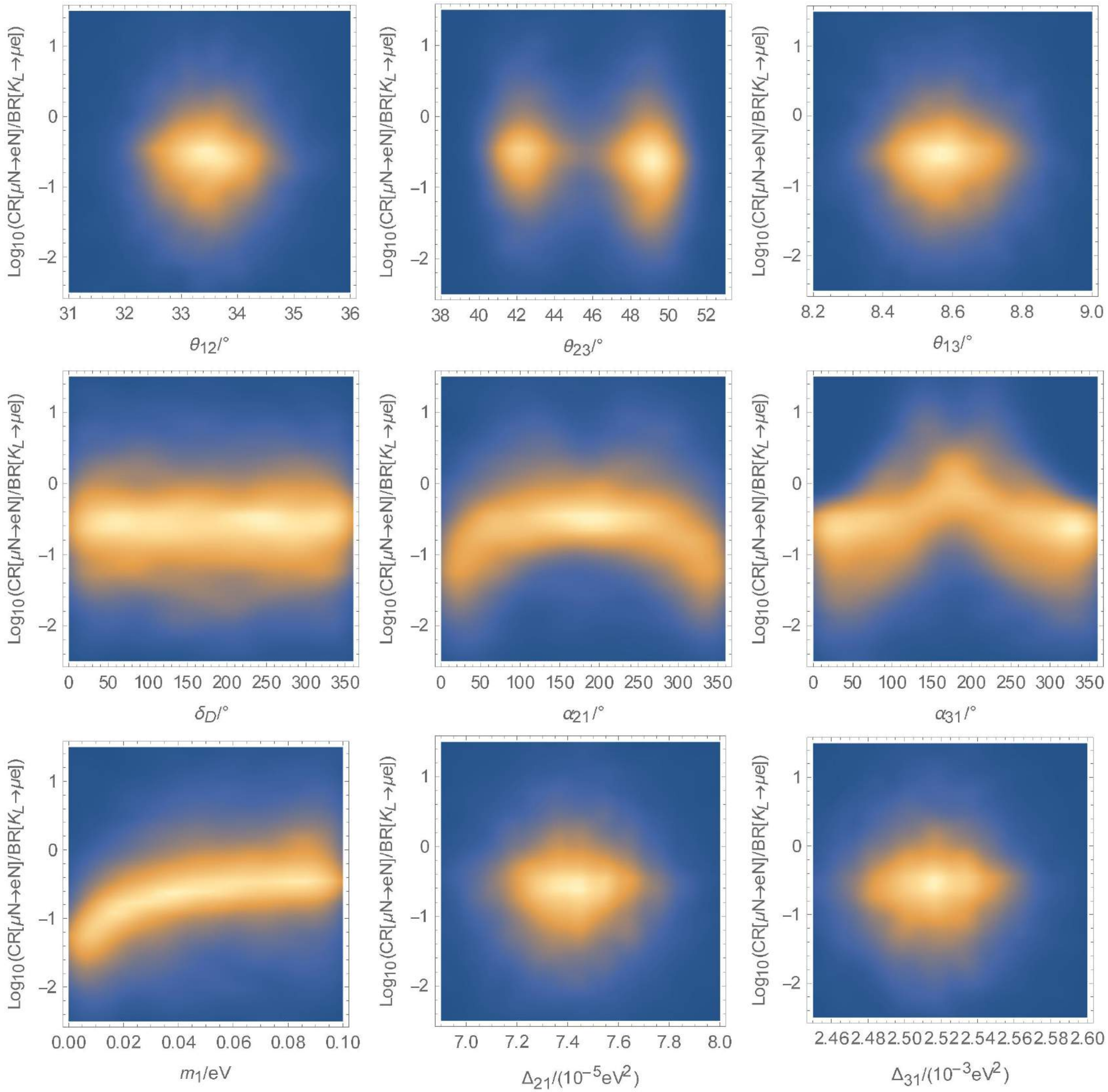


Figure 3: Impact on M_Δ, M_{LQ}, and the $M_{\varphi_8} - \tau_p$ correlation of relaxing the assumptions of the Model 2 fit. From left to right: (0) minimal setup as in Figure 2, (1) 2HDM, (2) generic flavour mixing, (3) p-decay mediators. See the text for details.

Experimental limits on LFV processes

Observable	90% CL upper limit	Future sensitivity
$\mathrm{BR}(\mu^+ \to e^+\gamma)$	4.2×10^{-13} [62]	6×10^{-14} [63]
$\mathrm{BR}(\mu^+ \to e^+e^-e^+)$	1.0×10^{-12} [64]	10^{-16} [65]
$\mathrm{CR}(\mu^- N \to e^- N)$	7.0×10^{-13} $(N = \mathrm{Au})$ [66]	6×10^{-17} $(N = \mathrm{Al})$ [67, 68]
$\mathrm{BR}(K_L \to \mu^\pm e^\mp)$	4.7×10^{-12} [69]	$\sim 10^{-12}$ [70]
$\mathrm{BR}(K_L \to \pi^0\mu^+e^-)$	7.6×10^{-11} [71]	$\sim 10^{-12}$ [70]
$\mathrm{BR}(K^+ \to \pi^+\mu^+e^-)$	1.3×10^{-11} [72]	$\sim 10^{-12}$ [70]
$\mathrm{BR}(K^+ \to \pi^+\mu^-e^+)$	5.2×10^{-10} [73]	$\sim 10^{-12}$ [70]

Dependence of $\mathrm{CR}(\mu\,\mathrm{Al} \to e\,\mathrm{Al})/\mathrm{BR}(K_L \to \mu e)$ on neutrino parameters



Dependence of $\mathrm{BR}(\mu \to e\gamma)/\mathrm{BR}(\mu \to eee)$ on neutrino parameters