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# Phenomenological tests of supersymmetric $SO(10)$ grand unified theories

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# Contents



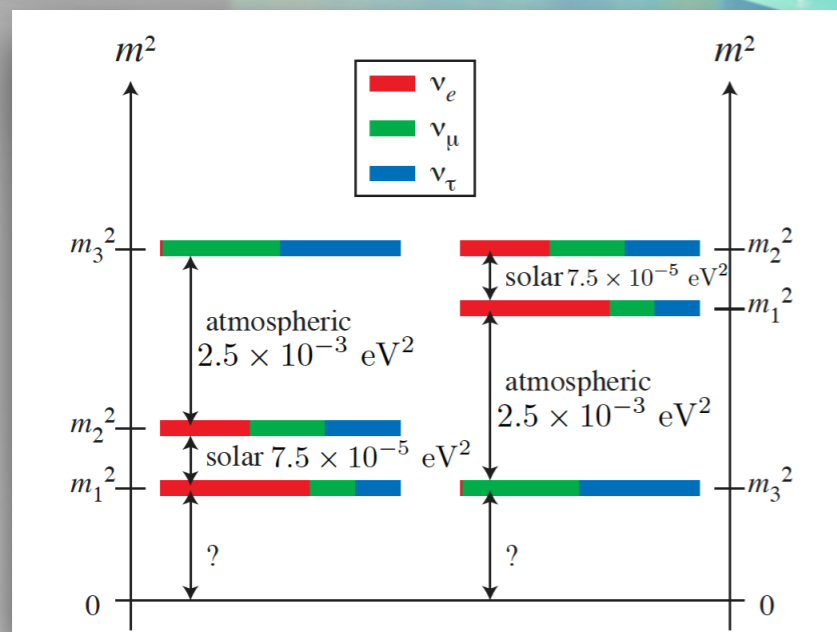
- Motivation
- Model framework
- Phenomenology
  - fermion mass and mixing in SO(10) SUSY GUT
  - leptogenesis and  $0\nu\beta\beta$
  - proton decay
  - gravitational wave
- Result

# Motivation

$$SU(3)_c \times SU(2)_L \times U(1)_Y$$



$$SO(10)$$



# Motivation

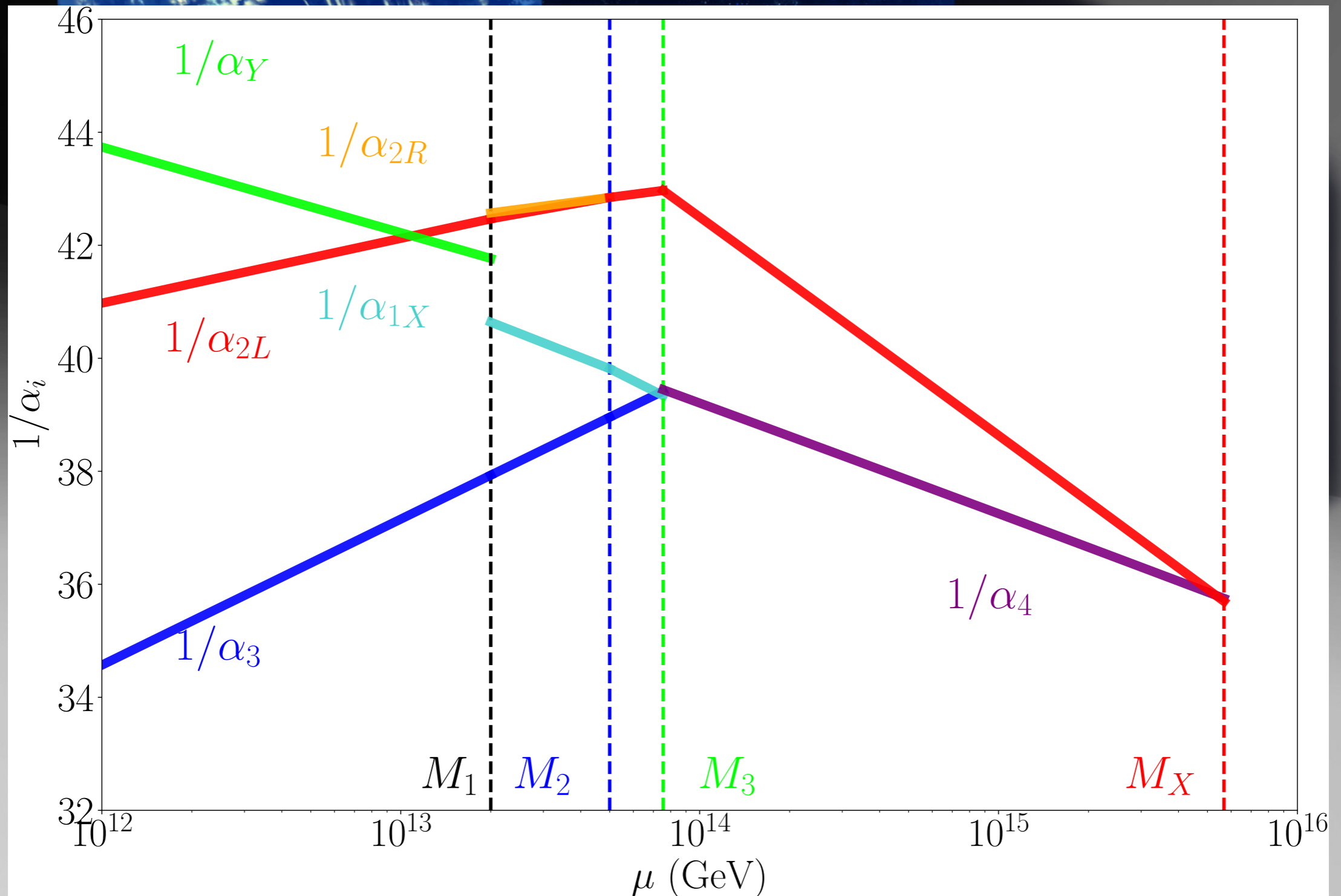
- Fermion masses and mixing are highly correlated in the framework of  $SO(10)$
- $U(1)_{B-L}$  gauge symmetry can appear as an intermediate symmetry and its breaking scale is associated with RH neutrino masses
- Limit on proton life time put constraint on the  $SO(10)$  breaking scale

Neutrino Physics

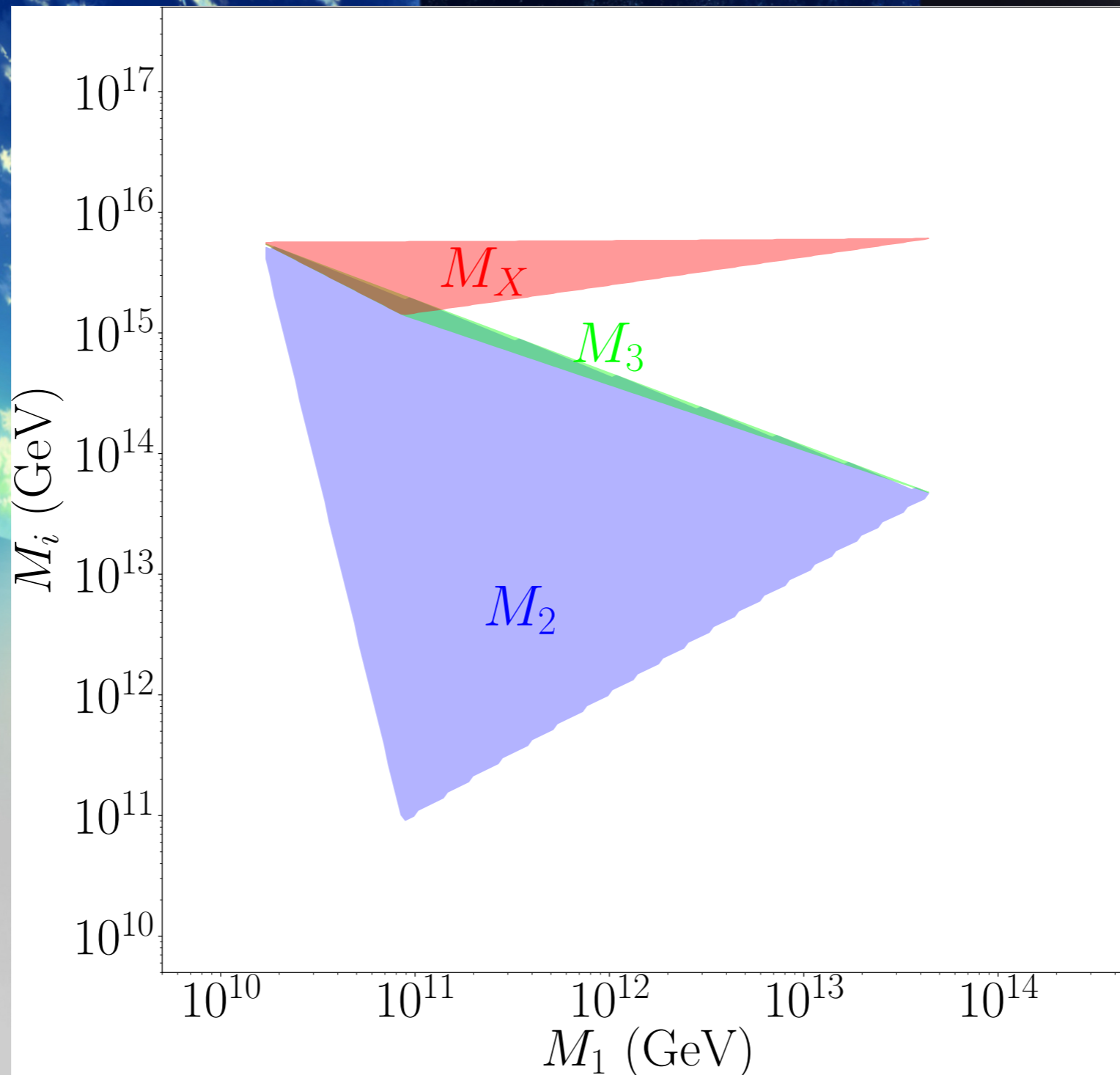
GW

Proton decay

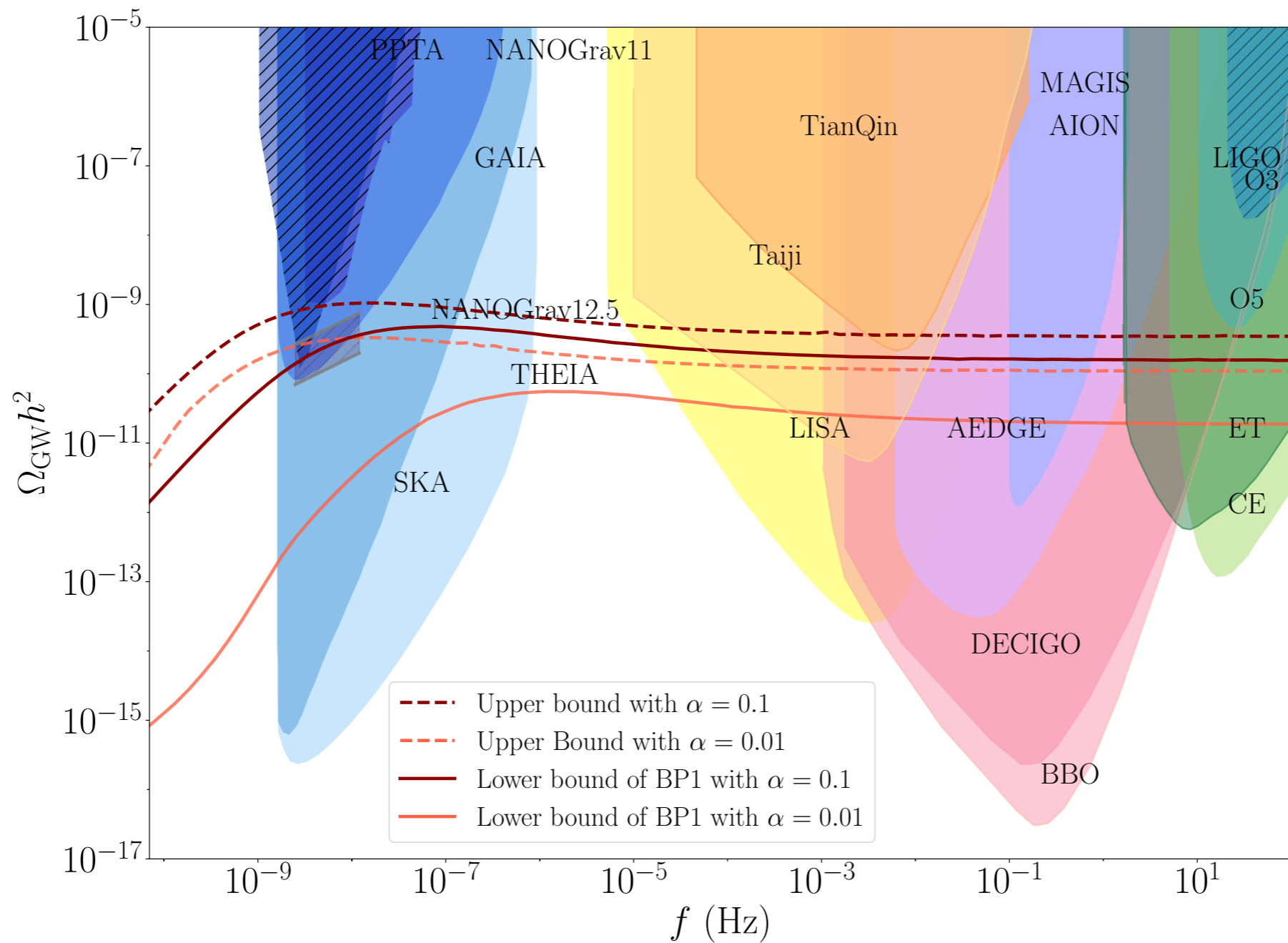
# Non-SUSY SO(10)



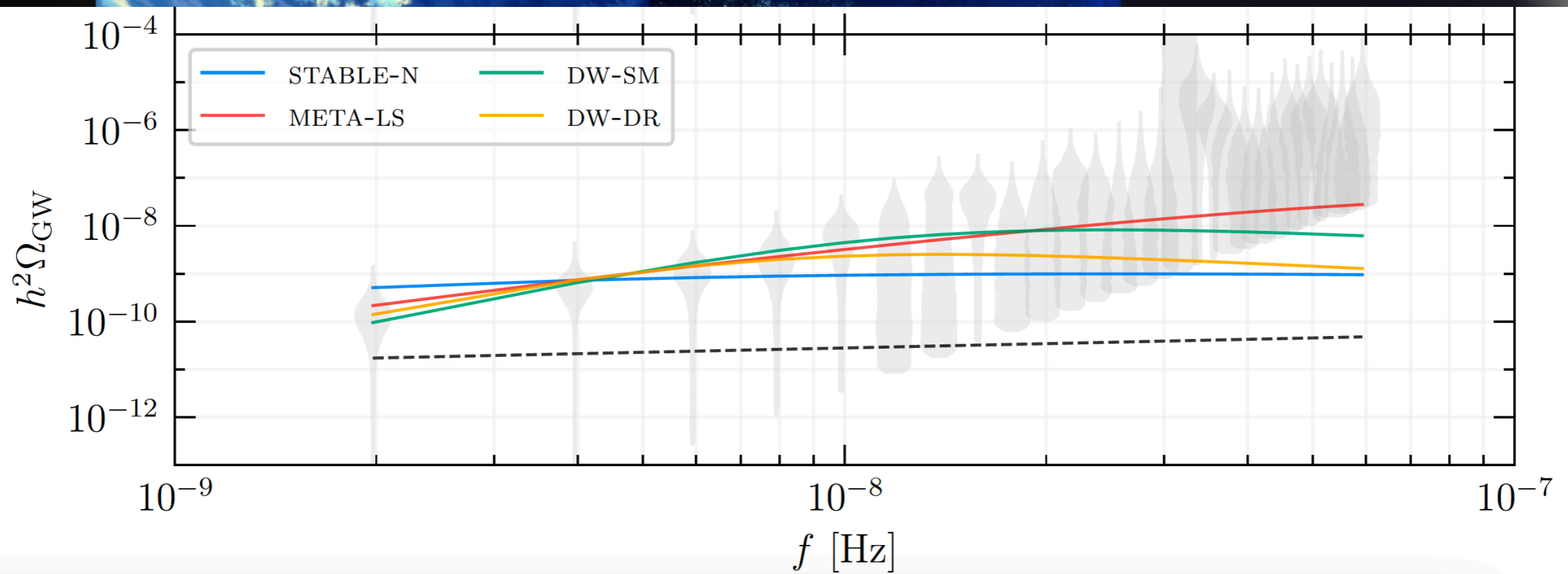
# Non-SUSY SO(10)



# GW signal in non-SUSY SO(10)



# NANOGrav 15







model framework

# Symmetry breaking of SO(10)

We consider a specific breaking chain of SUSY SO(10)

$$SO(10) \times SUSY$$

$$45 \downarrow \text{broken at } M_{GUT}$$

$$G_{LRSM} \equiv SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SUSY$$

$$\overline{126} \downarrow \text{broken at } M_{B-L}$$

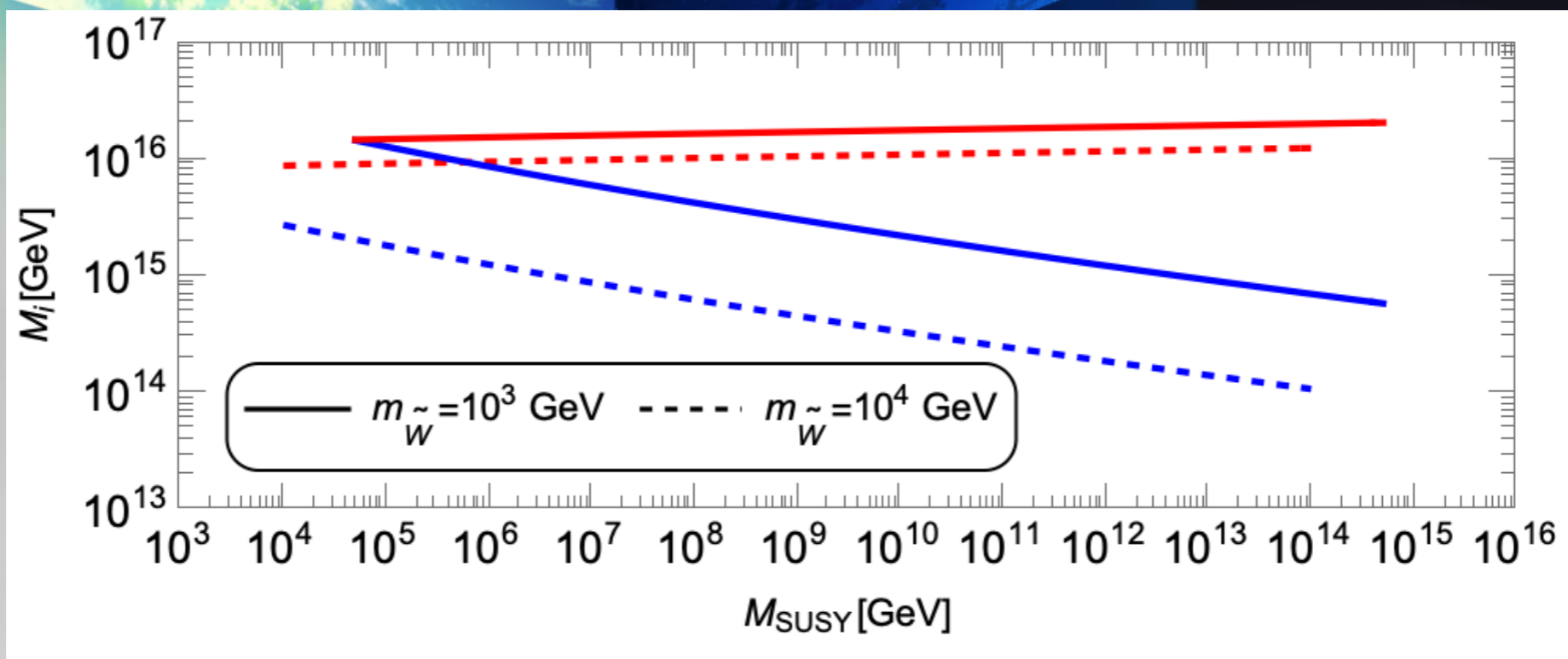
$$G_{MSSM} \equiv SU(3)_c \times SU(2)_L \times U(1)_Y \times SUSY$$

$$\downarrow \text{broken at } M_{SUSY}$$

$$G_{SM} \equiv SU(3)_c \times SU(2)_L \times U(1)_Y$$

# Gauge Unification

- Split supersymmetry
  - $M_{\text{SUSY}}$ : mass of sfermions
  - $M_{\tilde{W}}$ : mass of gauginos and higgsinos





# Phenomenology

# Fermion mass and mixing

- To fit the fermion mass and mixing, we consider three Higgs multiplets in the Yukawa sector

$$Y_{10}^* \mathbf{16} \cdot \mathbf{16} \cdot \mathbf{10} + Y_{126}^* \mathbf{16} \cdot \mathbf{16} \cdot \overline{\mathbf{126}} + Y_{120}^* \mathbf{16} \cdot \mathbf{16} \cdot \mathbf{120} + \text{h.c.}$$

- CP symmetry at GUT scale — real Yukawa couplings
- The up, down, neutrino, charged lepton Yukawa couplings are Hermitian and can be parameterised as

$$Y_u = h + r_2 f + i r_3 h', \quad Y_d = r_1 (h + f + i h'),$$

$$Y_\nu = h - 3r_2 f + i c_\nu h', \quad Y_e = r_1 (h - 3f + i c_e h')$$

- $h, f$  are real symmetric and  $h'$  is real antisymmetric
- $r_1, r_2, r_3, c_e, c_\nu$  are all real parameters ( $r_3 = 0$ )
- The neutrino mass matrix is determined by  $M_\nu = m_0 Y_\nu f^{-1} Y_\nu$

# Fitting the parameters

- Choose a basis where the up-type quark Yukawa matrix is diagonalised

$$Y_u = \text{diag}\{\eta_u y_u, \eta_c y_c, \eta_t y_t\}$$

$$Y_d = P_a V_{\text{CKM}} \text{diag}\{\eta_d y_d, \eta_s y_s, \eta_b y_b\} V_{\text{CKM}}^\dagger P_a^*$$

- $P_a = \text{diag}\{e^{ia_1}, e^{ia_2}, 1\}$  contains 2 phases
- $\eta_q$  are signs that cannot be determined by the real-orthogonal transformation
- By fixing the quark Yukawa couplings and CKM mixing, the matrices  $h, f$  and  $h'$  can be solved as

$$h = -\frac{Y_u}{r_2 - 1} + \frac{r_2 \text{Re}Y_d}{r_1(r_2 - 1)} \quad f = \frac{Y_u}{r_2 - 1} - \frac{\text{Re}Y_d}{r_1(r_2 - 1)} \quad h' = i \frac{\text{Im}Y_d}{r_1}$$

# Fitting the parameters

$$Y_e = -\frac{4r_1}{r_2 - 1} Y_u + \frac{r_2 + 3}{r_2 - 1} \text{Re}Y_d + ic_e \text{Im}Y_d$$

$$M_\nu = m_0 \left( \frac{8r_2(r_2 + 1)}{r_2 - 1} Y_u - \frac{16r_2^2}{r_1(r_2 - 1)} \text{Re}Y_d + \frac{r_2 - 1}{r_1} (r_1 Y_u + ic_\nu \text{Im}Y_d) (r_1 Y_u - \text{Re}Y_d)^{-1} (r_1 Y_u - ic_\nu \text{Im}Y_d) \right)$$

- 7 remaining free parameters:  $\{a_1, a_2, r_1, r_2, c_e, c_\nu, m_0\}$
- Undetermined signs:  $\eta_q$
- 8 observables:  $\{y_e, y_\mu, y_\tau, \Delta m_{21}^2, \Delta m_{31}^2, \theta_{12}, \theta_{13}, \theta_{23}\}$

# Leptogenesis

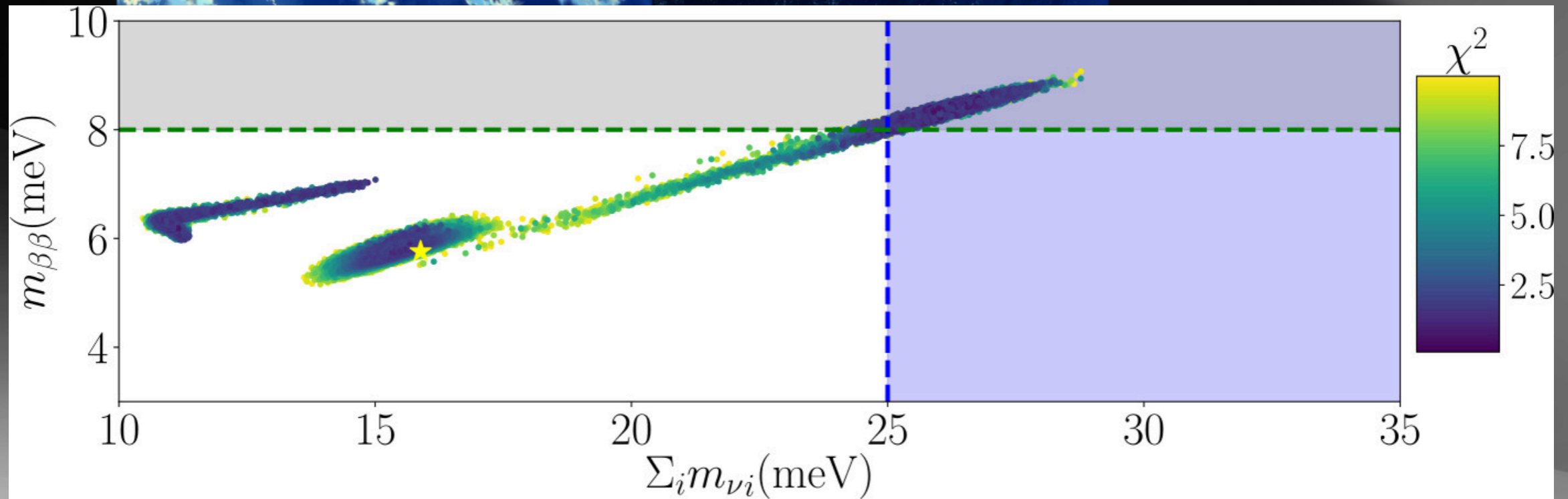
- **ULYSSES** and the associated “3DME” code (which accounts for the decays and washout of all three RH neutrinos)
- Example benchmark point

Inputs	$a_1$ 35, 40°	$a_2$ 221.27°	$c_\nu$ -1.49	$m_0$ 44.24 meV	$(\eta_u, \eta_c, \eta_t; \eta_d, \eta_s, \eta_b)$ (-, +, +; +, -, -)
Outputs	$\theta_{13}$ 8.66°	$\theta_{12}$ 33.19°	$\theta_{23}$ 44.14°	$\delta$ 131.57°	$m_1$ 5.29 meV
$(\chi^2 = 8.22)$	$m_{\beta\beta}$ 5.76 meV		$M_{N_1}$ $8.18 \cdot 10^{11}$ GeV	$M_{N_2}$ $1.53 \cdot 10^{12}$ GeV	$M_{N_3}$ $4.67 \cdot 10^{13}$ GeV

- Baryon-to-photon ratio  $\eta_B \sim 6.16 \times 10^{-10}$



# Neutrinoless Double Beta Decay



- The lightest neutrino mass:  $5 \lesssim m_{\nu_1} \text{ (meV)} \lesssim 10$
- Partly testable for the next generation of  $\nu 0\beta\beta$  and CMB experiments

# Proton Decay

- Pion channel
  - Dimension-6 operators

$$\begin{aligned} & (\overline{u_R^c} \gamma^\mu Q_\alpha) (\overline{d_R^c} \gamma_\mu L_\beta), (\overline{u_R^c} \gamma^\mu Q_\alpha) (\overline{e_R^c} \gamma_\mu Q_\beta), \\ & (\overline{d_R^c} \gamma^\mu Q_\alpha) (\overline{u_R^c} \gamma_\mu L_\beta), (\overline{d_R^c} \gamma^\mu Q_\alpha) (\overline{\nu_R^c} \gamma_\mu Q_\beta) \end{aligned}$$

HK

- Kaon channel
  - dimension-5 operators

$$Q_\alpha^a Q_\beta^b Q_\gamma^c L_l, U_\alpha^{Ca} D_\beta^{Cb} U_\gamma^{Cc} E_\delta^C$$

Juno

$$\tau \propto M_{\text{SUSY}}^2 / m_{\widetilde{W}}^2$$

# Proton Decay

$$\frac{\epsilon_{abc}}{M_T} \left( C_{\alpha\beta\gamma\delta}^L Q_\alpha^a Q_\beta^b Q_\gamma^c L_l + C_{[\alpha\beta\gamma]\delta}^R U_\alpha^{Ca} D_\beta^{Cb} U_\gamma^{Cc} E_\delta^C \right)$$

$$\begin{aligned} C_{\alpha\beta\gamma\delta}^R = & (Y_{10})_{\alpha\beta} (Y_{10})_{\gamma\delta} + x_1 (Y_{\overline{126}})_{\alpha\beta} (Y_{\overline{126}})_{\gamma\delta} + x_2 (Y_{120})_{\alpha\beta} (Y_{120})_{\gamma\delta} \\ & + x_3 (Y_{10})_{\alpha\beta} (Y_{\overline{126}})_{\gamma\delta} + x_4 (Y_{\overline{126}})_{\alpha\beta} (Y_{10})_{\gamma\delta} + x_5 (Y_{\overline{126}})_{\alpha\beta} (Y_{120})_{\gamma\delta} \\ & + x_6 (Y_{120})_{\alpha\beta} (Y_{\overline{126}})_{\gamma\delta} + x_7 (Y_{10})_{\alpha\beta} (Y_{120})_{\gamma\delta} + x_8 (Y_{120})_{\alpha\beta} (Y_{10})_{\gamma\delta} \\ & + x_9 (Y_{\overline{126}})_{\alpha\delta} (Y_{120})_{\beta\gamma} + x_{10} (Y_{120})_{\alpha\delta} (Y_{120})_{\beta\gamma} \end{aligned}$$

$$\begin{aligned} C_{\alpha\beta\gamma\delta}^L = & (Y_{10})_{\alpha\beta} (Y_{10})_{\gamma\delta} + x_1 (Y_{\overline{126}})_{\alpha\beta} (Y_{\overline{126}})_{\gamma\delta} - x_3 (Y_{10})_{\alpha\beta} (Y_{\overline{126}})_{\gamma\delta} \\ & - x_4 (Y_{\overline{126}})_{\alpha\beta} (Y_{10})_{\gamma\delta} + y_5 (Y_{\overline{126}})_{\alpha\beta} (Y_{120})_{\gamma\delta} + y_7 (Y_{10})_{\alpha\beta} (Y_{120})_{\gamma\delta} \\ & + y_9 (Y_{120})_{\alpha\gamma} (Y_{\overline{126}})_{\beta\delta} + y_{10} (Y_{120})_{\alpha\gamma} (Y_{120})_{\beta\delta} \end{aligned}$$

$$Y_{10} = \frac{h}{V_{11}}, \quad Y_{\overline{126}} = -f \frac{v_u^2}{m_0 v_S}, \quad Y_{120} = i h' \frac{c_\nu}{4V_{13}}$$

# GW from cosmic strings

- GW relic density depends on the string tension  $G\mu$

$$G\mu \simeq \frac{1}{2(\alpha_{2R}(M_{B-L}) + \alpha_{1X}(M_{B-L}))} \frac{M_{B-L}^2}{M_{\text{pl}}^2}$$

- In SUSY GUT

$$M_{B-L} \sim M_{\text{GUT}}$$

⇓

$$\alpha_{2R}(M_{B-L}) \simeq \alpha_{1X}(M_{B-L}) \simeq \alpha_{\text{GUT}}(M_{\text{GUT}})$$

⇓

$$G\mu \simeq \frac{1}{4\alpha_{\text{GUT}}(M_{\text{GUT}})} \frac{M_{B-L}^2}{M_{\text{pl}}^2}$$

# GW from cosmic strings

- Decay rate of cosmic strings into monopoles

$$\Gamma_d = \frac{\mu}{2\pi} e^{-\pi\kappa}, \quad \kappa = \frac{m^2}{\mu}$$

- Monopole mass  $m = \frac{M_V}{\alpha} f_m$

- $\mu \simeq \frac{1}{\alpha_{\text{GUT}}} M_{B-L}^2, m = \frac{M_{\text{GUT}}}{\alpha_{\text{GUT}}} \Rightarrow \sqrt{\kappa} \simeq \alpha_{\text{GUT}}^{-1/2} \frac{M_{\text{GUT}}}{M_{B-L}}$

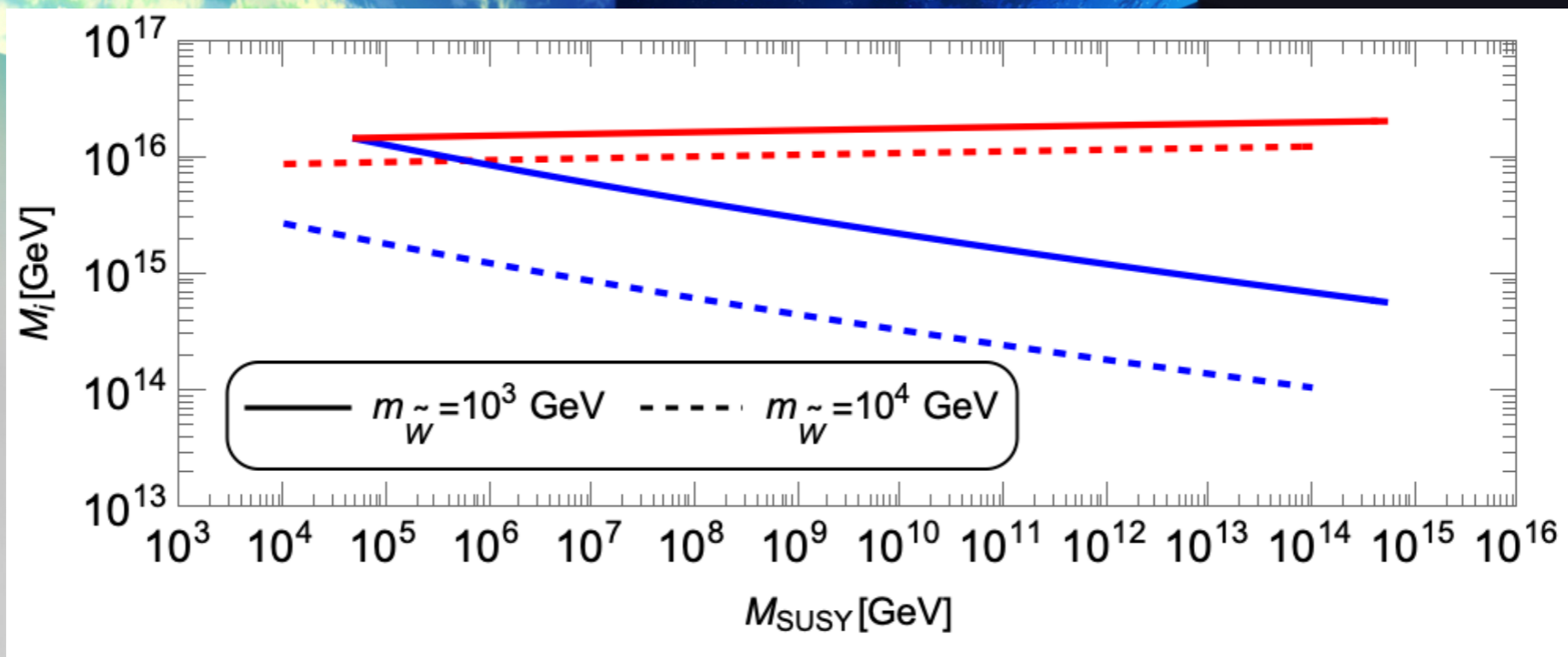
- Stability of cosmic strings

- Stable:  $M_{B-L} \ll M_{\text{GUT}} \quad (\sqrt{\kappa} > 9)$

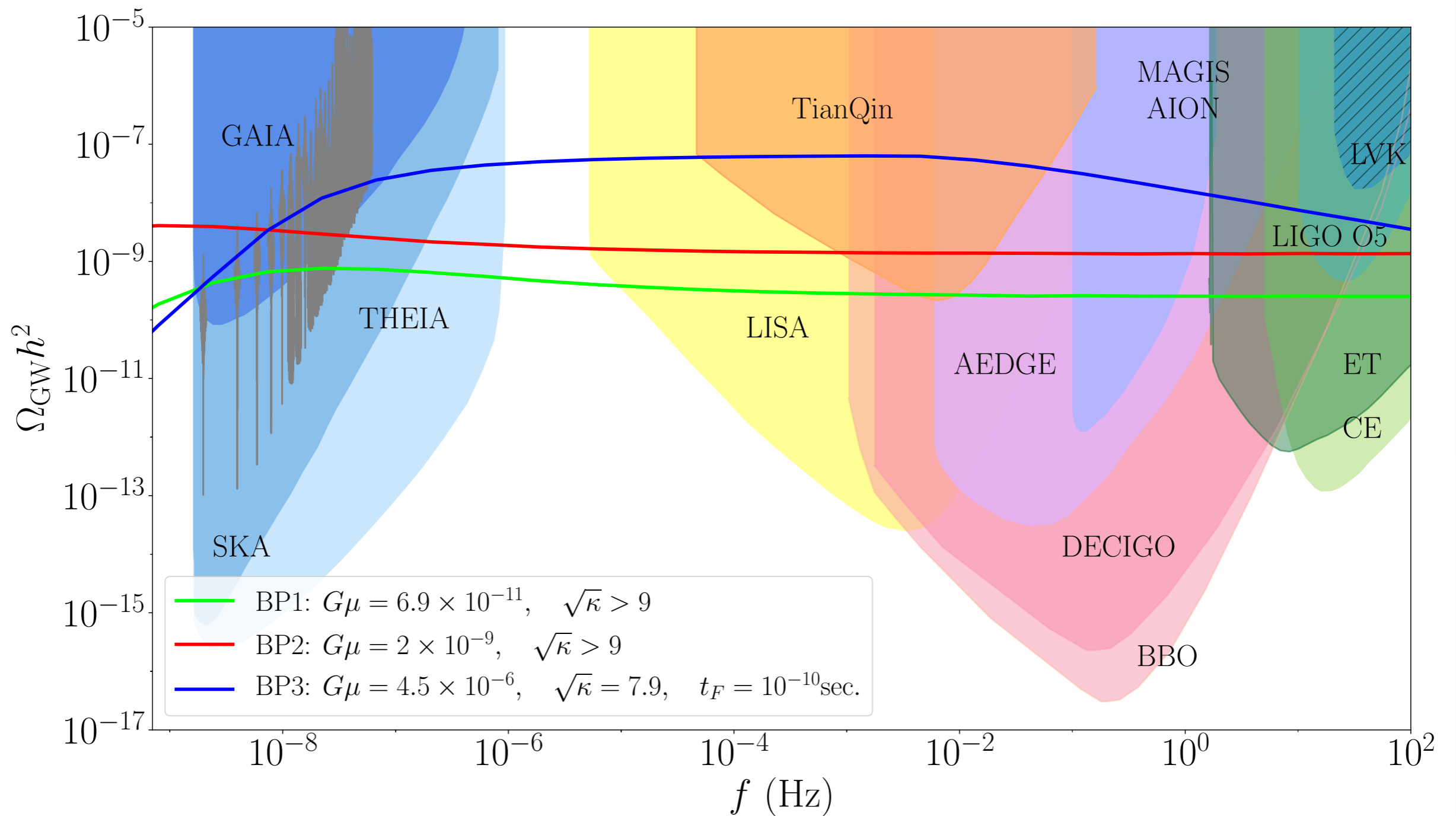
- Metastable:  $M_{B-L} \lesssim M_{\text{GUT}} \quad (\sqrt{\kappa} < 9)$

# GW from cosmic strings

- Small  $M_{\text{SUSY}}$ , small  $\sqrt{\kappa}$ , metastable string
- Small  $M_{\tilde{W}}$ , small  $\sqrt{\kappa}$ , metastable string



# Gravitational Waves

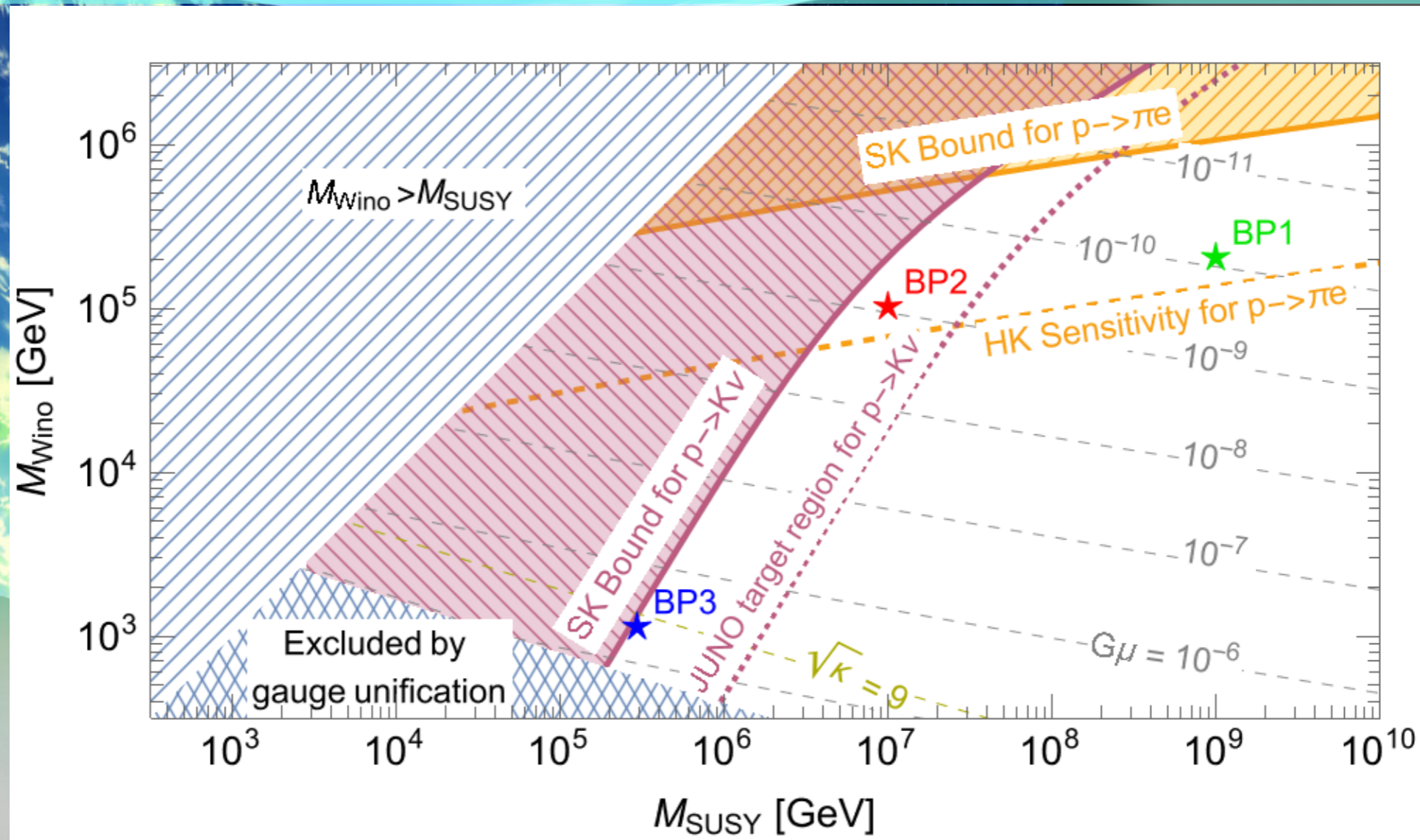




# Results



# Predictions of Benchmark Points



	HK sensitivity	JUNO target	NANOGrav15
BP1	testable	no signal	consistent
BP2	testable	targeted	inconsistent
BP3	no signal	targeted	support



Thank you