Unify Symmetries & Flavors

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& in progress

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Fundamental Puzzles in the SM:

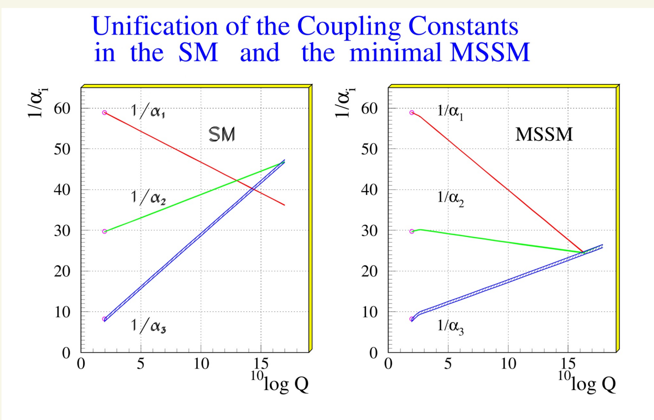
- $\mathcal{L} = \sqrt{-g}\Big[R - \frac{1}{4}(F^a_{\mu\nu})^2 - \frac{\theta}{8\pi}\alpha_s G^a_{\mu\nu}\tilde{G}^{\mu\nu,a} + \bar{\psi} i \not{D} \psi$

$+ Y_{ij}\psi_i\psi_j\Phi - |D_\mu \Phi|^2 - V(\Phi)\Big]$

- Strong CP: PQ mech'; PQ quality (M_{pl})
- Flavors: most puzzling, quarks & leptons are due to the SM Higgs Yukawa couplings.
- Higgs potential: why $m_H = 125\,\mathrm{GeV}$?
- A real GUT must answer these questions.

Background of GUT:

- 1974, Georgi – Glashow SU(5) GUT, $3\times[\bar{5}_F \oplus 10_F]$
- 1975, Fritzsch – Minkowski SO(10) GUT, $3\times 16_F$
- 1981, Dimopoulos & Georgi SUSY SU(5)



GUT unifies sym' & flavors

$$\bar{5}_F \supset \underbrace{(\bar{3}, 1, +\tfrac{1}{3})_F}_{d_R^c} \oplus \underbrace{(1, \bar{2}, -\tfrac{1}{2})_F}_{\ell_L}$$

$$10_F \supset \underbrace{(3, 2, +\tfrac{1}{6})_F}_{q_L} \oplus \underbrace{(\bar{3}, 1, -\tfrac{2}{3})_F}_{u_R^c} \oplus \underbrace{(1, 1, +1)_F}_{e_R^c}$$

Elements in GG '74 SU(5):

1) AF of the QCD, '73 Gross-Wilczek, Politzer

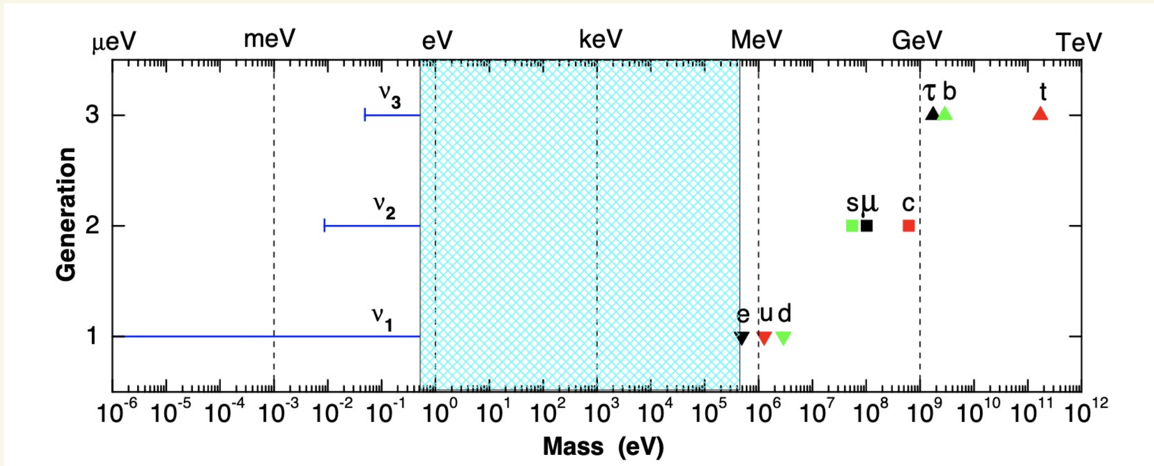
2) GG assumed the 2-generational fermions (u, d, ν_e, e) & (c, s, ν_μ, μ), charm was proposed by Glashow-Iliopoulos-Maiani (GIM) '70 & discovered as J/Ψ meson in late '74

3) '73 Kobayashi-Maskawa have proposed the 3^{rd}-generational quarks (t, b) [6-plet model]

Flavors are keys to def' the unified Sym'.

Flavor Sector:

邢志忠 (1909.09610)

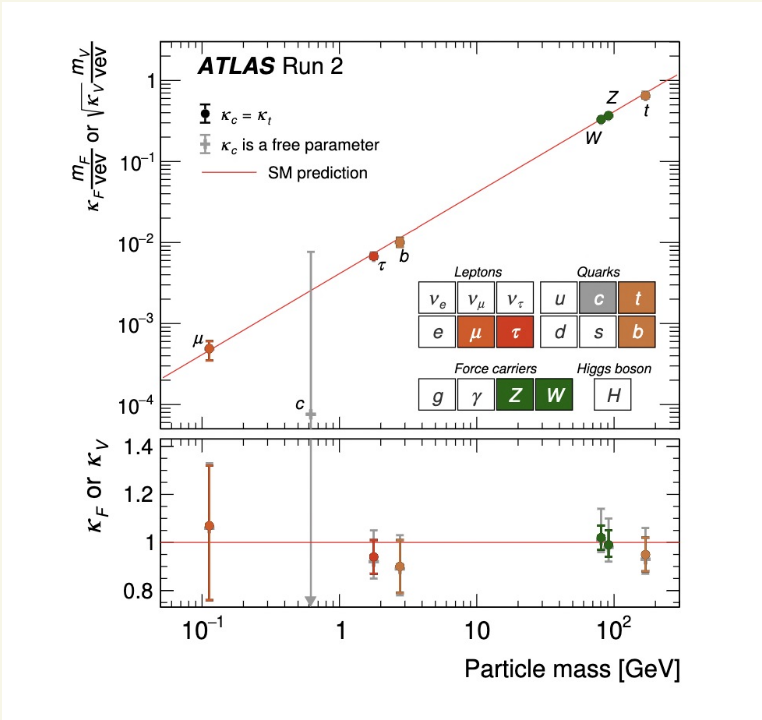


non-universal intragenerational mass splitting patterns:

$\Rightarrow$ non-universal SYMMETRY properties of SM fermions?

but universal γ_{m_f} in each generation, SU(5), SO(10) - -

Flavor Sector:



(2207.00092)

SM Higgs distinguishes flavors with hierarchical Yukawa interactions.

only $y_t \sim \mathcal{O}(1)$

杨振宁 [1980]

"SYMMETRY DICTATES INTERACTIONS."

Back to GUT:

- All flavor puzzles are simply to ask WHY/How $n_g = 3$?
- Poor imagination: $3 \times [\bar{5}_F \oplus 10_F]$ in $SU(5)$, or $3\times 16_F$ in $SO(10)$, or $3\times 27_F$ in E_6
- Three simple repetition is Nature's camouflage @ ~ 100 GeV.
- Nanopoulos richmen's view ['80]: all fermions ψ_i are in inequiv' irreps of GUT group G with

$$\sum_i \mathrm{Anom}(\psi_i) = 0$$

No $\underline{3\times[\,-\,-\,]}$ here? → an IAFFS

Georgi's 1979 proposal: flavor-unified $SU(N>5)$ theory

- Law I: $SU(3)_c$ is real (vectorial)

- Law II: $SU(2)_W \otimes U(1)_Y$ is complex (chiral)

- Law III: $\{f_L\}_{SU(N)} = \overline{\sum_k} m_k [N,k]_F$, $m_k = 0$ or 1

& $\overline{\sum} m_k \, \mathrm{Anom}([N,k]_F) = 0$ No repetition of any irrep

$\Rightarrow SU(11)$: $[11,4]_F \oplus [11,8]_F \oplus [11,9]_F \oplus [11,10]_F$ $\dim_F = 561$

$SO(2N)$ is impossible, e.g. $SO(14)$

$32_F = 16_F \oplus \overline{16_F}$ $n_g = 0$.

Georgi's Counting Rule: decompose irreps into the SU(5) irreps

$[N,1]_F = (N-5) \times \mathbf{1}_F \oplus 5_F$ and etc.

$\Rightarrow$ A set of $(\mathbf{1}_F, 5_F, 10_F, \overline{10}_F, \overline{5}_F)$

Anomaly-free: $\nu_{5_F} + \nu_{10_F} = \nu_{\overline{5}_F} + \nu_{\overline{10}_F}$

# of total generation: $n_g = \nu_{10_F} - \nu_{\overline{10}_F} = \nu_{\overline{5}_F} - \nu_{5_F}$

- In the SU(N), one has: $\nu_{10_F}[N,2]_F - \nu_{\overline{10}_F}[N,2]_F \equiv 1$

$\nu_{10_F}[N,3]_F - \nu_{\overline{10}_F}[N,3]_F = N-6$, and etc.

$\Rightarrow$ rank-2 SU(N) theory alone is insufficient!

Def: irreducible anomaly-free fermion set (IAFFS)

$$\sum_R m_R f_L(R), \quad m_R \in \mathbb{Z}, \quad \text{s.t.} \quad \sum_R m_R \,\text{Anom}(f_L(R)) = 0.$$

They satisfy 3 conditions:

(1) $GCD\{m_R\} = 1$;

(2) Not any of $f_L(R)$ can be removed, otherwise Gauge Anom $\neq 0$;

(3) No singlet, self-conjugate, adjoint fermions.

— Law III (CHEN): No simple repetition of any IAFFS in the GUT, & $n_g = 3$ (2209.11446, 2307.07921)

Examples:

1) One-generational SM: $(3,2,+\frac{1}{6})_F \oplus (\bar{3},1,-\frac{2}{3})_F \oplus (\bar{3},1,+\frac{1}{3})_F$

$\oplus (1,\bar{2},-\frac{1}{2})_F \oplus (1,1,+1)_F$, an LAFFS

2) SU(5): $3\times[\bar{5}_F \oplus 10_F]$, fails the Law-III.

3) SU(7) [Frampton]: $7\times \bar{7}_F \oplus 21_F \oplus 2\times 35_F$

Ans: This fails the Law-III, by rewriting fermions into

$$[3\times \bar{7}_F \oplus 21_F] \oplus \underbrace{[4\times \bar{7}_F \oplus 2\times 35_F]}_{\uparrow}$$

Simple repetition.

- Georgi's $SU(11)$:

$$330_F = 15\times 1_F \oplus 20\times 5_F \oplus 15\times 10_F \oplus 6\times \overline{10}_F \oplus \overline{5}_F$$

$$\Rightarrow 6\ (10_F, \overline{10}_F)\ \text{VLF} \ \&\ 1\ (5_F, \overline{5}_F)\ \text{VLF} \ \&\ 9\times 10_F \ \&\ 19\times 5_F$$

$$165_F = 20\times 1_F \oplus 10_F \oplus 6\times \overline{10}_F \oplus 15\times \overline{5}_F$$

$$\Rightarrow 1\ (10_F, \overline{10}_F)\ \text{VLF} \ \&\ 5\times \overline{10}_F \ \&\ 15\times \overline{5}_F$$

$$\overline{55}_F = 15\times 1_F \oplus 6\times \overline{5}_F \oplus \overline{10}_F$$

$$\overline{11}_F = 6\times 1_F \oplus \overline{5}_F$$

All 3 $\overline{5}_F$'s & 3 10_F's transform differently.

3 (d_R^c, l_L) & 3 (q_L, u_R^c, e_R^c) are all different

- Minimal Flavor-unified theory: SU(8), S. Barr ['08]

$\mathrm{Anom}(\bar{8}_F) = -1$ $\quad$ $\mathrm{Anom}(28_F) = 4$ $\quad$ $\mathrm{Anom}(56_F) = 5$

$\bar{8}_F = 3\times \mathbb{1}_F \oplus \bar{5}_F$

$28_F = 3\times \mathbb{1}_F \oplus 3\times \bar{5}_F \oplus \underbrace{10_F}$

$56_F = \mathbb{1}_F \oplus 3\times 5_F \oplus \underbrace{3\times 10_F} \oplus \underbrace{\overline{10}_F}$

$27\,\check{m}_L - 4\,\check{m}_R \Rightarrow 23\,\check{m}_L$ massless

$6\times (5_F, \bar{5}_F)$ VLF

$1\times (10_F, \overline{10}_F)$ VLF

$n_g = 3$

$\{f_L\}_{SU(8)} = \underline{[\bar{8}_F^{\lambda} \oplus 28_F] \oplus [\bar{8}_F^{\dot{\lambda}} \oplus 56_F]}$ $\to$ Two distinctive IAFFSs, $n_g = 3$

$\lambda = 3, \underline{\mathrm{IV}}, \underline{\mathrm{V}}, \underline{\mathrm{VI}}$: rank-2 IAFFS

$\dot{\lambda} = \dot{1}, \dot{2}, \underline{\dot{\mathrm{VII}}}, \underline{\dot{\mathrm{VIII}}}, \underline{\dot{\mathrm{IX}}}$: rank-3 IAFFS

only 3 (q_L, u_R^c, e_R^c) are different, 3 (d_R^c, ℓ_L) are same.

How to def' the B-L ? S. Weinberg, Wilczek-Zee [1979]

- $SU(5)$: $\bar{5}_F \oplus 10_F$ $\tilde{U}(1)_{\bar{\square}} \otimes \tilde{U}(1)_{\boxminus} \to$ non-anomalous $\tilde{U}(1)_{T_2}$

$$T_2(\bar{5}_F) = -3t_2 \quad T_2(10_F) = +t_2 \quad \text{s.t. } [SU(5)]^2 \cdot \tilde{U}(1)_{T_2} = 0$$

$$-\mathcal{L}_Y = \bar{5}_F\, 10_F\, 5_H^\dagger + 10_F\, 10_F\, 5_H + \text{H.c.} \qquad T_2(5_H) = -2t_2$$

- $B-L \equiv \tilde{a}_1 T_2 + \tilde{a}_2 Y$ with $\tilde{a}_1 = 1$ from the 't Hooft

anomaly matching (1980): $[\tilde{U}(1)_{T_2}]^3 = [\tilde{U}(1)_{B-L}]^3$

- $\tilde{U}(1)_{B-L}$-neutral: $10_F\, 10_F\, 5_H + \text{H.c.} \supset \underbrace{(3,2,+\tfrac{1}{6})_F}_{+1/3} \otimes \underbrace{(\bar{3},1,-\tfrac{2}{3})_F}_{-1/3} \otimes \underbrace{(1,2,+\tfrac{1}{2})_H}_{0.}$

$$(B-L)(1,2,+\tfrac{1}{2})_H = -2t_2 + \tfrac{1}{2}\tilde{a}_2 = 0 \Rightarrow \tilde{a}_2 = 4t_2$$

$$B-L \equiv T_2 + 4t_2 Y \quad \text{in the } SU(5).$$

• In the $SU(8)$:

$$\{f_L\} = [\,\overline{8_F}^{\lambda} \oplus 28_F\,] \oplus [\,\overline{8_F}^{\dot\lambda} \oplus 56_F\,]$$

Global Dimopoulos – Raby – Susskind (DRS) sym' of ['80]

$$[\widetilde{SU}(4)_\lambda \otimes \widetilde{U}(1)_\lambda \otimes \widetilde{U}(1)_{A_2}] \otimes [\widetilde{SU}(5)_{\dot\lambda} \otimes \widetilde{U}(1)_{\dot\lambda} \otimes \widetilde{U}(1)_{A_3}]$$

• Find the non-anomalous $\widetilde{U}(1)_\lambda \otimes \widetilde{U}(1)_{A_2} \to \widetilde{U}(1)_{T_2}$

& $\widetilde{U}(1)_{\dot\lambda} \otimes \widetilde{U}(1)_{A_3} \to \widetilde{U}(1)_{T_3}$

	$\overline{8_F}^{\lambda}$	28_F
T_2:	$-3t_2$	$+2t_2$

	$\overline{8_F}^{\dot\lambda}$	56_F
T_3:	$-3t_3$	$+t_3$

$\boxed{t_2 = t_3}$

in the end.

- Higgs sector of the SU(8):

$$-\mathcal{L}_Y = \bar{8}_F^{\lambda}\, 28_F\, \overline{8}_{H,\lambda} + 28_F\, 28_F\, 70_H + \cancel{56_F\, 56_F\, 28_H} \equiv 0$$

$$+ \bar{8}_F^{\dot\lambda}\, 56_F\, \overline{28}_{H,\dot\lambda} + \frac{1}{M_{pl}} 56_F\, 56_F (\overline{28}_{H,\dot\lambda})^{\dagger}\, 63_H + 28_F\, 56_F\, 56_H + \text{H.c.}$$

- $\{H\} = \overline{8}_{H,\lambda} \oplus \overline{28}_{H,\dot\lambda} \oplus 56_H \oplus 70_H \oplus \underset{\text{real}}{63_H}$

- SSB pattern: determined by Higgs irreps, L-F Li [1974]

- Any GUT must undergo a max SSB stage & is only by its adjoint Higgs & as high as possible for "proton decay"

$$SU(8) \xrightarrow{63_H} \mathcal{G}_{441} \qquad \langle 63_H \rangle = \frac{1}{4}\,\mathrm{diag}(-\mathbb{1}_4, +\mathbb{1}_4)\, v_U$$

- SSB pattern of the SU(8):

$$SU(8) \xrightarrow{63_H} \mathcal{G}_{441X_0} \xrightarrow{\overline{8}_{H,\lambda}} \mathcal{G}_{341X_1} \xrightarrow{\overline{8}_{H,\lambda},\ \overline{28}_{H,\dot{\lambda}}} \mathcal{G}_{331X_2}$$

$$\xrightarrow[56_H]{\overline{8}_{H,\lambda},\ \overline{28}_{H,\dot{\lambda}}} \mathcal{G}_{SM} \xrightarrow{70_H} SU(3)_c \otimes U(1)_{EM}.$$

- Decomposition rule: $8_H = (4, 1, -\frac{1}{4})_H \oplus (1, 4, +\frac{1}{4})_H$

$$\left.\begin{aligned} (\bar{4}, 1, +\tfrac{1}{4})_H &= (\bar{3}, 1, +\tfrac{1}{3})_H \oplus \underline{(1, 1, 0)_H} \\ (1, \bar{4}, -\tfrac{1}{4})_H &= (1, \bar{3}, -\tfrac{1}{3})_H \oplus \underline{(1, 1, 0)_H} \end{aligned}\right\}$$

singlets, can develop VEVs

— — - -

To decompose the Higgs fields to find the SSB directions, with the LieART (1912.10969)

- Decompositions of the SU(8) fermions

SU(8)	$\mathcal{G}_{441}$	$\mathcal{G}_{341}$	$\mathcal{G}_{331}$	$\mathcal{G}_{\rm SM}$
$\overline{\mathbf{8}_{\mathbf{F}}}^{\Lambda}$	$(\overline{\mathbf{4}},\mathbf{1},+\frac{1}{4})_{\mathbf{F}}^{\Lambda}$	$(\overline{\mathbf{3}},\mathbf{1},+\frac{1}{3})_{\mathbf{F}}^{\Lambda}$	$(\overline{\mathbf{3}},\mathbf{1},+\frac{1}{3})_{\mathbf{F}}^{\Lambda}$	$(\overline{\mathbf{3}},\mathbf{1},+\frac{1}{3})_{\mathbf{F}}^{\Lambda}\ :\ \mathcal{D}_R^{\Lambda\,c}$
		$(\mathbf{1},\mathbf{1},0)_{\mathbf{F}}^{\Lambda}$	$(\mathbf{1},\mathbf{1},0)_{\mathbf{F}}^{\Lambda}$	$(\mathbf{1},\mathbf{1},0)_{\mathbf{F}}^{\Lambda}\ :\ \check{\mathcal{N}}_L^{\Lambda}$
	$(\mathbf{1},\overline{\mathbf{4}},-\frac{1}{4})_{\mathbf{F}}^{\Lambda}$	$(\mathbf{1},\overline{\mathbf{4}},-\frac{1}{4})_{\mathbf{F}}^{\Lambda}$	$(\mathbf{1},\overline{\mathbf{3}},-\frac{1}{3})_{\mathbf{F}}^{\Lambda}$	$(\mathbf{1},\overline{\mathbf{2}},-\frac{1}{2})_{\mathbf{F}}^{\Lambda}\ :\ \mathcal{L}_L^{\Lambda}=(\mathcal{E}_L^{\Lambda},-\mathcal{N}_L^{\Lambda})^T$
				$(\mathbf{1},\mathbf{1},0)_{\mathbf{F}}^{\Lambda'}\ :\ \check{\mathcal{N}}_L^{\Lambda'}$
			$(\mathbf{1},\mathbf{1},0)_{\mathbf{F}}^{\Lambda''}$	$(\mathbf{1},\mathbf{1},0)_{\mathbf{F}}^{\Lambda''}\ :\ \check{\mathcal{N}}_L^{\Lambda''}$

SU(8)	$\mathcal{G}_{441}$	$\mathcal{G}_{341}$	$\mathcal{G}_{331}$	$\mathcal{G}_{\rm SM}$
$\mathbf{28}_{\mathbf{F}}$	$(\mathbf{6},\mathbf{1},-\frac{1}{2})_{\mathbf{F}}$	$(\mathbf{3},\mathbf{1},-\frac{1}{3})_{\mathbf{F}}$	$(\mathbf{3},\mathbf{1},-\frac{1}{3})_{\mathbf{F}}$	$(\mathbf{3},\mathbf{1},-\frac{1}{3})_{\mathbf{F}}\ :\ \mathfrak{D}_L$
		$(\overline{\mathbf{3}},\mathbf{1},-\frac{2}{3})_{\mathbf{F}}$	$(\overline{\mathbf{3}},\mathbf{1},-\frac{2}{3})_{\mathbf{F}}$	$(\overline{\mathbf{3}},\mathbf{1},-\frac{2}{3})_{\mathbf{F}}\ :\ t_R{}^c$
	$(\mathbf{1},\mathbf{6},+\frac{1}{2})_{\mathbf{F}}$	$(\mathbf{1},\mathbf{6},+\frac{1}{2})_{\mathbf{F}}$	$(\mathbf{1},\mathbf{3},+\frac{1}{3})_{\mathbf{F}}$	$(\mathbf{1},\mathbf{2},+\frac{1}{2})_{\mathbf{F}}\ :\ (\mathfrak{e}_R{}^c,\mathfrak{n}_R{}^c)^T$
				$(\mathbf{1},\mathbf{1},0)_{\mathbf{F}}\ :\ \check{\mathfrak{n}}_R^c$
			$(\mathbf{1},\overline{\mathbf{3}},+\frac{2}{3})_{\mathbf{F}}$	$(\mathbf{1},\overline{\mathbf{2}},+\frac{1}{2})_{\mathbf{F}}'\ :\ (\mathfrak{n}_R'{}^c,-\mathfrak{e}_R'{}^c)^T$
				$(\mathbf{1},\mathbf{1},+1)_{\mathbf{F}}\ :\ \tau_R{}^c$
	$(\mathbf{4},\mathbf{4},0)_{\mathbf{F}}$	$(\mathbf{3},\mathbf{4},-\frac{1}{12})_{\mathbf{F}}$	$(\mathbf{3},\mathbf{3},0)_{\mathbf{F}}$	$(\mathbf{3},\mathbf{2},+\frac{1}{6})_{\mathbf{F}}\ :\ (t_L,b_L)^T$
				$(\mathbf{3},\mathbf{1},-\frac{1}{3})_{\mathbf{F}}'\ :\ \mathfrak{D}_L'$
			$(\mathbf{3},\mathbf{1},-\frac{1}{3})_{\mathbf{F}}''$	$(\mathbf{3},\mathbf{1},-\frac{1}{3})_{\mathbf{F}}''\ :\ \mathfrak{D}_L''$
		$(\mathbf{1},\mathbf{4},+\frac{1}{4})_{\mathbf{F}}$	$(\mathbf{1},\mathbf{3},+\frac{1}{3})_{\mathbf{F}}''$	$(\mathbf{1},\mathbf{2},+\frac{1}{2})_{\mathbf{F}}''\ :\ (\mathfrak{e}_R''{}^c,\mathfrak{n}_R''{}^c)^T$
				$(\mathbf{1},\mathbf{1},0)_{\mathbf{F}}'\ :\ \check{\mathfrak{n}}_R'{}^c$
			$(\mathbf{1},\mathbf{1},0)_{\mathbf{F}}''$	$(\mathbf{1},\mathbf{1},0)_{\mathbf{F}}''\ :\ \check{\mathfrak{n}}_R''{}^c$

SU(8)	$\mathcal{G}_{441}$	$\mathcal{G}_{341}$	$\mathcal{G}_{331}$	$\mathcal{G}_{\rm SM}$
$\mathbf{56}_{\mathbf{F}}$	$(\mathbf{1},\overline{\mathbf{4}},+\frac{3}{4})_{\mathbf{F}}$	$(\mathbf{1},\overline{\mathbf{4}},+\frac{3}{4})_{\mathbf{F}}$	$(\mathbf{1},\overline{\mathbf{3}},+\frac{2}{3})_{\mathbf{F}}'$	$(\mathbf{1},\overline{\mathbf{2}},+\frac{1}{2})_{\mathbf{F}}'''\ :\ (\mathfrak{n}_R'''^c,-\mathfrak{e}_R'''^c)^T$
				$(\mathbf{1},\mathbf{1},+1)_{\mathbf{F}}'\ :\ e_R{}^c$ or $\mu_R{}^c$
			$(\mathbf{1},\mathbf{1},+1)_{\mathbf{F}}''$	$(\mathbf{1},\mathbf{1},+1)_{\mathbf{F}}''\ :\ \mathfrak{E}_R{}^c$
	$(\overline{\mathbf{4}},\mathbf{1},-\frac{3}{4})_{\mathbf{F}}$	$(\overline{\mathbf{3}},\mathbf{1},-\frac{2}{3})_{\mathbf{F}}'$	$(\overline{\mathbf{3}},\mathbf{1},-\frac{2}{3})_{\mathbf{F}}'$	$(\overline{\mathbf{3}},\mathbf{1},-\frac{2}{3})_{\mathbf{F}}'\ :\ u_R{}^c$ or $c_R{}^c$
		$(\mathbf{1},\mathbf{1},-1)_{\mathbf{F}}$	$(\mathbf{1},\mathbf{1},-1)_{\mathbf{F}}$	$(\mathbf{1},\mathbf{1},-1)_{\mathbf{F}}\ :\ \mathfrak{E}_L$
	$(\mathbf{4},\mathbf{6},+\frac{1}{4})_{\mathbf{F}}$	$(\mathbf{3},\mathbf{6},+\frac{1}{6})_{\mathbf{F}}$	$(\mathbf{3},\mathbf{3},0)_{\mathbf{F}}'$	$(\mathbf{3},\mathbf{2},+\frac{1}{6})_{\mathbf{F}}'\ :\ (u_L,d_L)^T$
				or $(c_L,s_L)^T$
				$(\mathbf{3},\mathbf{1},-\frac{1}{3})_{\mathbf{F}}'''\ :\ \mathfrak{D}_L'''$
			$(\mathbf{3},\overline{\mathbf{3}},+\frac{1}{3})_{\mathbf{F}}$	$(\mathbf{3},\overline{\mathbf{2}},+\frac{1}{6})_{\mathbf{F}}''\ :\ (\mathfrak{d}_L,-\mathfrak{u}_L)^T$
				$(\mathbf{3},\mathbf{1},+\frac{2}{3})_{\mathbf{F}}\ :\ \mathfrak{U}_L$
		$(\mathbf{1},\mathbf{6},+\frac{1}{2})_{\mathbf{F}}'$	$(\mathbf{1},\mathbf{3},+\frac{1}{3})_{\mathbf{F}}'$	$(\mathbf{1},\mathbf{2},+\frac{1}{2})_{\mathbf{F}}''''\ :\ (\mathfrak{e}_R''''^c,\mathfrak{n}_R''''^c)^T$
				$(\mathbf{1},\mathbf{1},0)_{\mathbf{F}}'''\ :\ \check{\mathfrak{n}}_R'''^{\,c}$
			$(\mathbf{1},\overline{\mathbf{3}},+\frac{2}{3})_{\mathbf{F}}''$	$(\mathbf{1},\overline{\mathbf{2}},+\frac{1}{2})_{\mathbf{F}}'''''\ :\ (\mathfrak{n}_R'''''^c,-\mathfrak{e}_R'''''^c)^T$
				$(\mathbf{1},\mathbf{1},+1)_{\mathbf{F}}'''\ :\ \mu_R{}^c$ or $e_R{}^c$
	$(\mathbf{6},\mathbf{4},-\frac{1}{4})_{\mathbf{F}}$	$(\mathbf{3},\mathbf{4},-\frac{1}{12})_{\mathbf{F}}'$	$(\mathbf{3},\mathbf{3},0)_{\mathbf{F}}''$	$(\mathbf{3},\mathbf{2},+\frac{1}{6})_{\mathbf{F}}'''\ :\ (c_L,s_L)^T$
				or $(u_L,d_L)^T$
				$(\mathbf{3},\mathbf{1},-\frac{1}{3})_{\mathbf{F}}''''\ :\ \mathfrak{D}_L''''$
			$(\mathbf{3},\mathbf{1},-\frac{1}{3})_{\mathbf{F}}'''''$	$(\mathbf{3},\mathbf{1},-\frac{1}{3})_{\mathbf{F}}'''''\ :\ \mathfrak{D}_L'''''$
		$(\overline{\mathbf{3}},\mathbf{4},-\frac{5}{12})_{\mathbf{F}}$	$(\overline{\mathbf{3}},\mathbf{3},-\frac{1}{3})_{\mathbf{F}}$	$(\overline{\mathbf{3}},\mathbf{2},-\frac{1}{6})_{\mathbf{F}}\ :\ (\mathfrak{d}_R{}^c,\mathfrak{u}_R{}^c)^T$
				$(\overline{\mathbf{3}},\mathbf{1},-\frac{2}{3})_{\mathbf{F}}''\ :\ \mathfrak{U}_R{}^c$
			$(\overline{\mathbf{3}},\mathbf{1},-\frac{2}{3})_{\mathbf{F}}'''$	$(\overline{\mathbf{3}},\mathbf{1},-\frac{2}{3})_{\mathbf{F}}'''\ :\ c_R{}^c$ or $u_R{}^c$

Flavor ID ?

$\check{\mathcal{N}}_L$ $\check{\mathfrak{n}}_R$: sterile neutrinos.

- Def' $\widetilde{U}(1)_{B-L}$ in the $SU(8)$:

(1) 't Hooft anomaly matching @ each SSB stage

(2) Higgs fields that can develop VEVs are $\widehat{U}(1)_T$ - neutral

- $SU(8) \to \mathcal{G}_{441X_0}$: $\mathcal{T}_2' = \mathcal{T}_2 - 4t_2 \mathcal{X}_0 \qquad \mathcal{T}_3' = \mathcal{T}_3 - 4t_3 \mathcal{X}_0$

$\mathcal{G}_{441} \to \mathcal{G}_{341X_1}$: $\mathcal{T}_2'' = \mathcal{T}_2' + 8t_2 \mathcal{X}_1 \qquad \mathcal{T}_3'' = \mathcal{T}_3' + 8t_3 \mathcal{X}_1$

$\mathcal{G}_{341} \to \mathcal{G}_{331X_2}$: $\mathcal{T}_2''' = \mathcal{T}_2'' \qquad \mathcal{T}_3''' = \mathcal{T}_3''$

$\mathcal{G}_{331} \to \mathcal{G}_{SM}$: $B-L = \mathcal{T}_2''' = \mathcal{T}_3'''$

$t_2 = t_3 \equiv t = +\frac{1}{4}$: universal $\widetilde{U}(1)_{B-L}$ for 3 generations

- Non-anomalous $\tilde{U}(1)_{T_2} \otimes \tilde{U}(1)_{T_3}$ charges

	$\overline{8_F}^{\lambda}$	28_F	$\overline{8}_{H,\lambda}$	56_H	70_H
$\mathcal{T}_2$:	$-3t_2$	$+2t_2$	$+t_2$	$-2t_2$	$-4t_2$

	$\overline{8_F}^{\dot\lambda}$	56_F	$\overline{28}_{H,\dot\lambda}$	56_H
$\mathcal{T}_3$:	$-3t_3$	$+t_3$	$+2t_3$	$-t_3$

- $28_F\, 56_F\, 56_H + H.c.$

$$56_H \supset \cdots \supset (1, \bar{3}, +\tfrac{2}{3})_H \oplus [(1, 3, +\tfrac{1}{3})_H^{\prime\prime} \oplus (1, \bar{3}, +\tfrac{2}{3})_H^{\prime\prime}]$$

	$(1, \bar{3}, +\frac{2}{3})_H$	$(1, 3, +\frac{1}{3})_H^{\prime\prime}$	$(1, \bar{3}, +\frac{2}{3})_H^{\prime\prime}$
$\mathcal{T}_2^{\prime\prime\prime}$:	$+t_2$	$+t_2$	$+t_2$
$\mathcal{T}_3^{\prime\prime\prime}$:	$+2t_3$	$+2t_3$	$+2t_3$
$\mathcal{P}^{\prime\prime\prime}$:	0	0	0

if $\mathcal{T}(56_H) = -3t$

✓

- 70_H only contains the EWSB components:

$$70_H \supset (4,\bar{4},+\tfrac{1}{2})_H \oplus (\bar{4},4,-\tfrac{1}{2})_H \supset \cdots$$

$$\supset \underbrace{(1,\bar{2},+\tfrac{1}{2})_H'''}_{B-L=0} \oplus \underbrace{(1,2,-\tfrac{1}{2})_H'''}_{B-L=-2} \Rightarrow (1,\bar{2},+\tfrac{1}{2})_H''' \text{ is}$$

the SM Higgs doublet

Yukawa: $28_{\bar{F}}\, 28_{\bar{F}}\, 70_H \supset \cdots \supset \underbrace{(\bar{3},1,-\tfrac{2}{3})_F}_{t_R{}^c} \otimes \underbrace{(3,2,+\tfrac{1}{6})_F}_{(t_L,b_L)^T} \otimes (1,\bar{2},+\tfrac{1}{2})_H'''$

@ tree-level: SM Higgs doublet of $(1,\bar{2},+\tfrac{1}{2})_H'''$ only gives the top quark mass.

$\Rightarrow$ $28_{\bar{F}}$ only contains the (t_L, b_L), $t_R{}^c$, $\tau_R{}^c$ (3rd generation)

1st/2nd generations of (u_L, d_L), (c_L, s_L), $u_R{}^c$, $c_R{}^c$, $e_R{}^c$, $\mu_R{}^c$ are in the 56_F

- up-type quark masses

$d \geq 5$ Op's: $\frac{1}{m_{pl}} 56_F\, 56_F\, \overline{28}_{H,i}\, 63_H \Rightarrow m_{u,u} \sim O(v_{341}) \cdot \frac{v_u}{m_{pl}}$

$\frac{1}{m_{pl}} 56_F\, 56_F\, \overline{28}_{H,i}\, 70_H + h.c.$

$$\supset \frac{1}{m_{pl}} \left[(\bar{4}, 1, -\tfrac{3}{4})_F \otimes (4, 6, +\tfrac{1}{4})_F \oplus \cancel{(6, 4, -\tfrac{1}{4})_F} \otimes (6, 4, -\tfrac{1}{4})_F \right]$$

$$\otimes (\bar{4}, \bar{4}, 0)_{H,i} \otimes (4, \bar{4}, +\tfrac{1}{2})_H + h.c. \supset \cdots$$

$$\supset \zeta_2 v_{EW} \cdot (c_L c_R^c + \cdots) + h.c.$$

$$(M_u)_{5\times5} \sim \frac{1}{\sqrt{2}} \begin{pmatrix} \begin{matrix} 0 & 0 & \zeta_1 \\ 0 & \zeta_2 & \zeta_2 \\ \zeta_2 & \zeta_1 & 1 \end{matrix} \times v_{EW} & \\ & \end{pmatrix}$$

$\zeta_1 \sim \frac{v_{441}}{m_{pl}} \quad \zeta_2 \sim \frac{v_{341}}{m_{pl}}$

- The flavor-unified $SU(N)$ theory contains sterile neutrinos, e.g., in $\overline{8}_F^{\lambda}$ & $\overline{8}_F^{\dot\lambda}$ of $SU(8)$. 27 copies of $\check{\mathcal{N}}_L^{\Lambda,\Lambda',\Lambda''}$

- only 4 right-handed sterile neutrinos from the 28_F & 56_F, they obtain Dirac masses with their left-handed $\check{\mathcal{N}}_L^{\Lambda,\Lambda',\Lambda''}$

- Masses of the remaining 23 $\check{\mathcal{N}}_L^{\Lambda,\Lambda',\Lambda''}$? W.o. any detail of the seesaw mech' and/or Weinberg Op', they cannot be massive above the EW scale, by 't Hooft anomaly matching s.t. $[\widetilde{U}(1)_{T_2/T_3}]^3 = \cdots = [\widetilde{U}(1)_{B-L}]^3$

- Summary

1. The Higgs sector of SU(8): $\bar{8}_{H,\lambda} \oplus \overline{28}_{H,\dot\lambda} \oplus 56_H \oplus 70_H \oplus \underline{63_H}$

$$V \supset \mu^2 |\Phi|^2 + \lambda |\Phi|^4 + \frac{1}{M_{pl}} \left(\varepsilon \cdot \bar{8}_{H,\lambda}^4 \right) \cdot 70_H + \cdots + H.c.$$

2. # of "massless" sterile neutrinos counted precisely.

3. Non-trivial embedding of three generational SM fermions

$\Rightarrow$ flavor non-universality under the heavy G_{441} & G_{341}

flavor-conserving neutral GBs, e.g. in the SU(7) toy model (2209.11446)

• Outlook

1. SU(8) is the only AF flavor-unified theory ($b_1 = -\frac{13}{2}$)

2. The next-minimal theory? Frampton's SU(9) [1980, 2009]:

$\cancel{9\times \bar{9}_F \oplus 84_F}$ since $\frac{1}{M_{pl}}\, 84_F\, 84_F\, 80_H\, 84_H$ leads to suppressed top quark Yukawa coupling

3. My proposal: $\underbrace{[5\times \bar{9}_F \oplus 36_F]}_{\text{rank-2 IAFFS}} \oplus \underbrace{[5\times \bar{9}_F \oplus 126_F]}_{\text{rank-4 IAFFS}}$ ($b_1 = +\frac{61}{6}$)

4. The ultimate goal: SM fermion masses & gauge coupling unification & pheno implication.