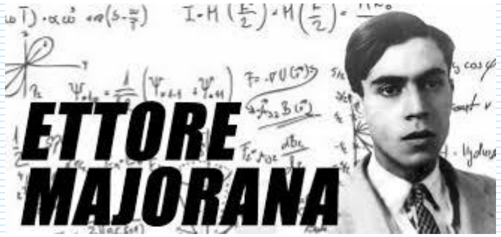
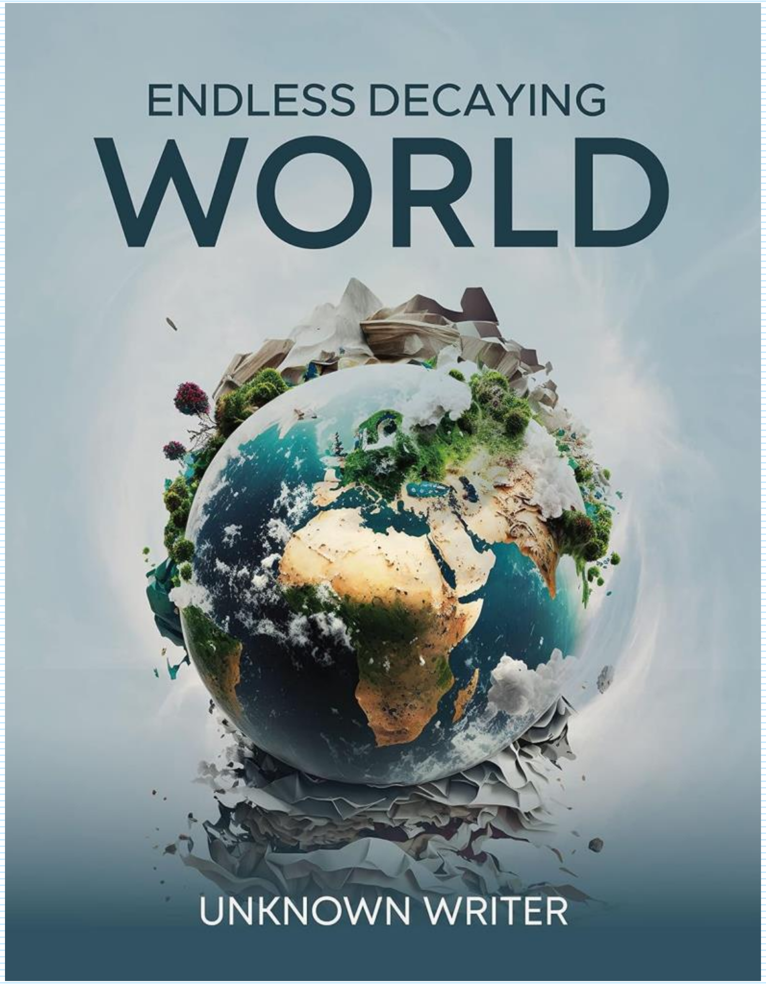
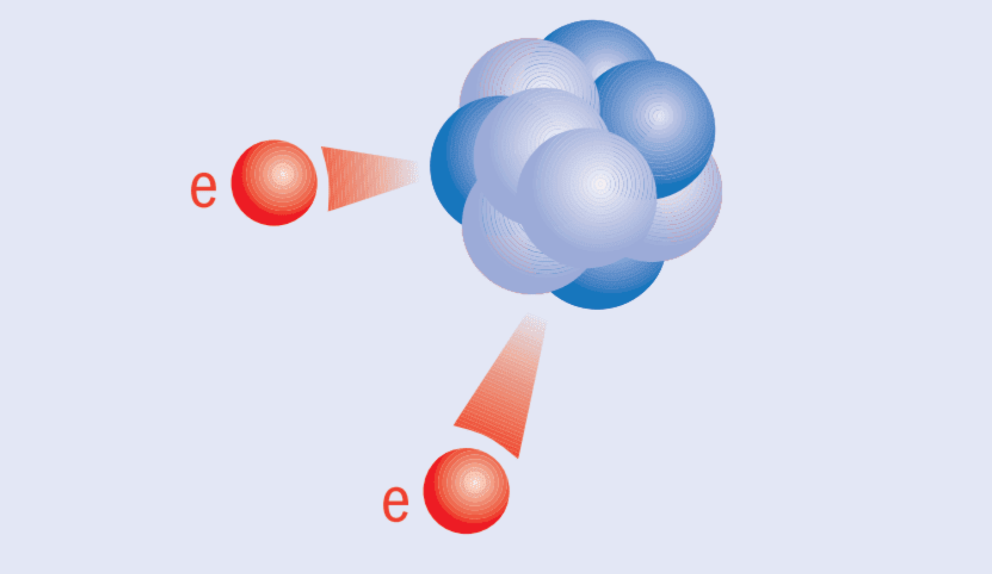
International Symposium on Neutrino Physics and Beyond
HKUST Jockey Club Institute for Advanced Study,
Hong Kong, China, Feb 18 – 21, 2024

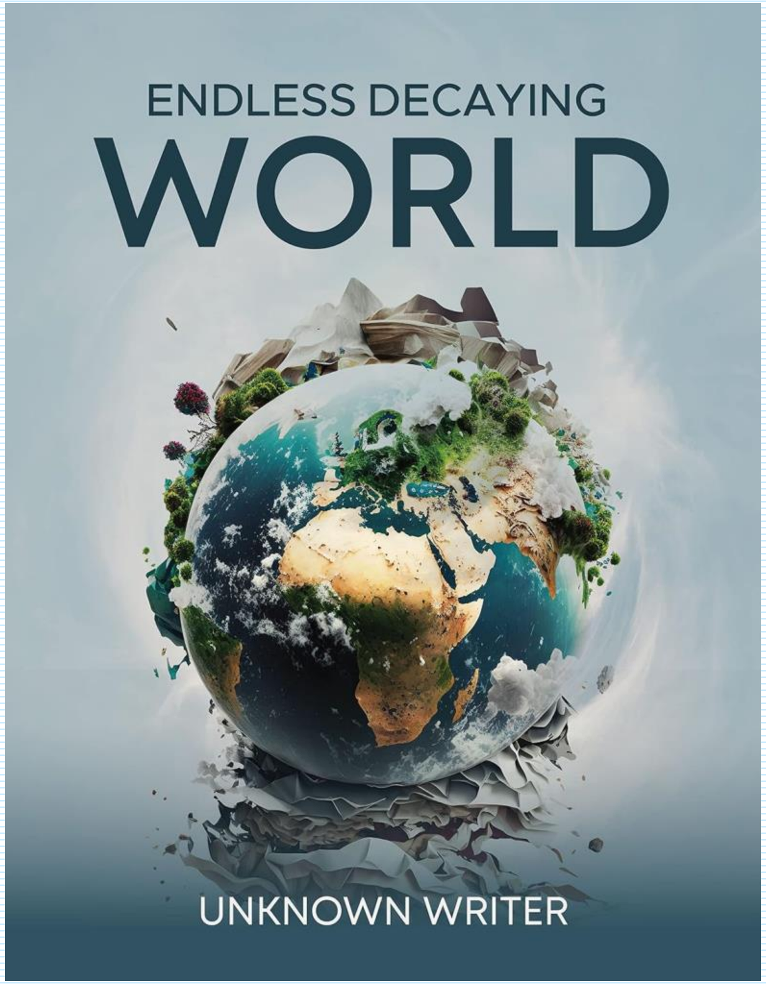


$\nu\bar{\nu}$ ARE NEUTRINOS THEIR OWN ANTIPARTICLES?

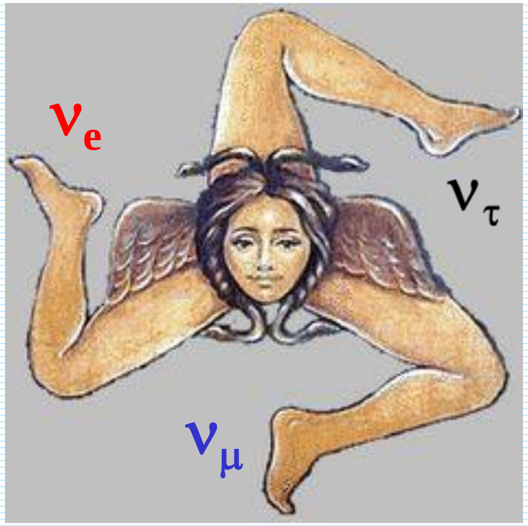




Review on $0\nu2\beta$ Decays
(unique decay)
Fedor Šimkovic



OUTLINE



I. Introduction

II. The *$0\nu\beta\beta$-decay is a particle physics problem*

(QCSS scenario, sterile ν, LR symmetric model, Quasi-Dirac ν, neutrino-antineutrino oscillations

III. The *$0\nu\beta\beta$-decay is a nuclear physics problem*

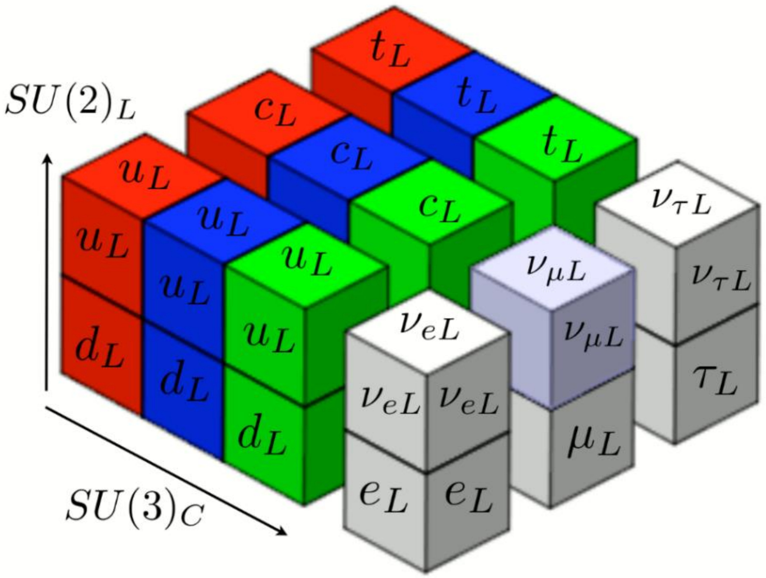
(status of NMEs calculation, contact term, g_A, supporting nuclear physics activities - $2\nu\beta\beta$-decay, muon capture, DCE heavy ion reactions)

IV. The *$0\nu\beta\beta$-decay is an atomic physics problem*

(electron exchange effect radiative corrections, atomic relaxation time, atomic overlap factor, etc.)

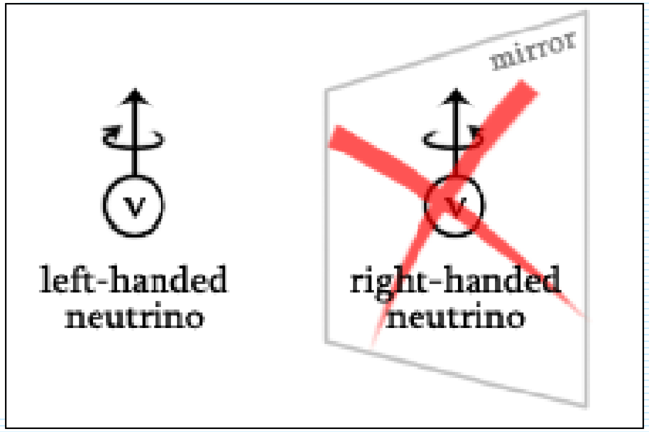
V. Outlook

Standard Model
(an astonishing successful theory,
based on few principles)



ν is a special particle in SM:

- It is the only fermion that does not carry electric charge (like γ, g, H^0)
- There are only left-handed ν's ($\nu_{eL}, \nu_{\mu L}, \nu_{\tau L}$)
- ν-mass can not be generated with any renormalizable coupling with the Higgs fields through SSB



ν's oscillations experiments
⇒ tiny neutrino masses (!)
⇒ Beyond SM physics (!)

SUPER K

SNO

KamLAND
Kamioka Liquid scintillator Anti-Neutrino Detector

DOUBLE CHOOZ

Daya Bay θ13

RENO ν θ13

T2K

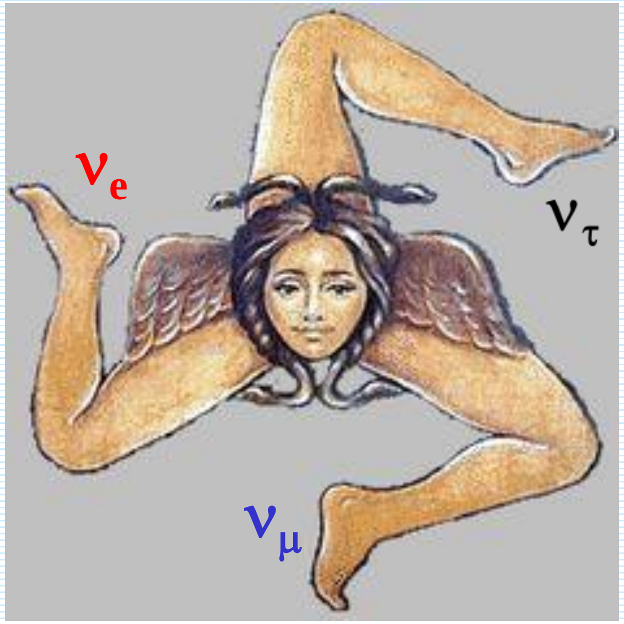
NOνA

, etc

Fundamental ν properties

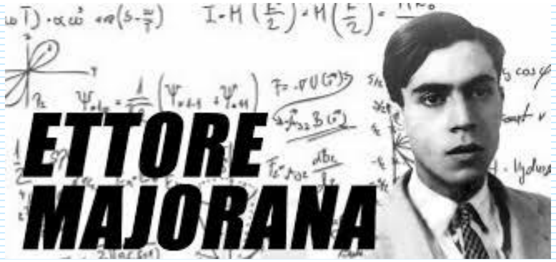
After 94/68 years we know

- 3 families of light (V-A) neutrinos: ν_e, ν_μ, ν_τ
- ν are massive: we know mass squared differences
- relation between flavor states and mass states (neutrino mixing)



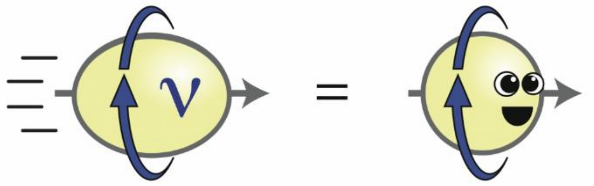
No answer yet

- Are ν Dirac or Majorana?
- Is there a CP violation in ν sector?
- Are neutrinos stable?
- What is the magnetic moment of ν?
- Sterile neutrinos?
- Statistical properties of ν? Fermionic or partly bosonic?



Currently main issue

Nature, Mass hierarchy,
CP-properties, sterile ν



The observation of neutrino oscillations has opened a new excited era in neutrino physics and represents a big step forward in our knowledge of neutrino properties

Majorana fermions

Ettore Majorana

Teoria simmetrica dell'elettrone e del positrone (A symmetric theory of electrons and positrons).

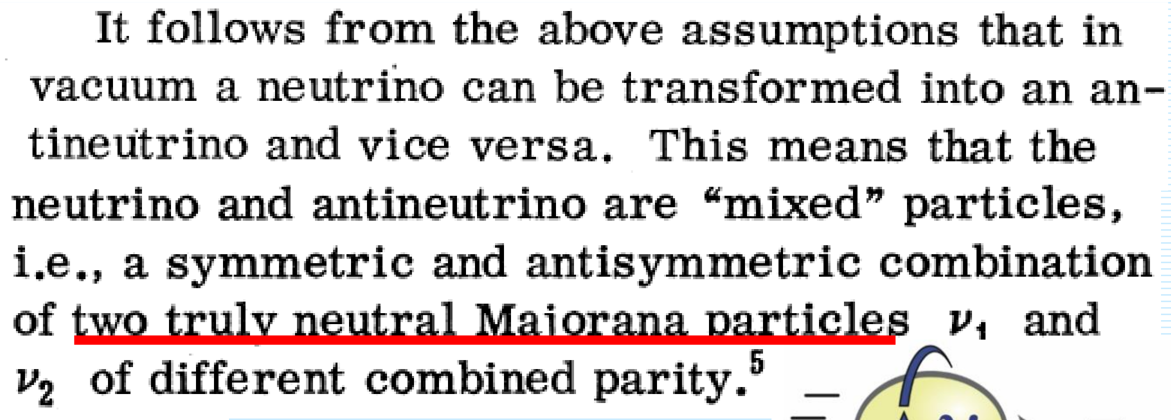
Il Nuovo Cimento, 14: 171–184, 1937.) 171

ν is its own antiparticle

Bruno Pontecorvo

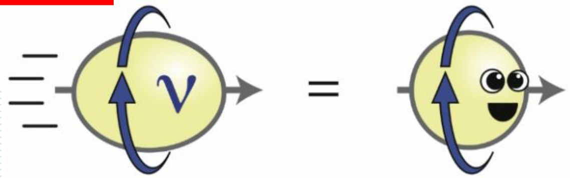
Inverse beta processes and nonconservation of lepton charge

Zhur. Eksptl'. i Teoret. Fiz. 34, 247 (1958)



It follows from the above assumptions that in vacuum a neutrino can be transformed into an antineutrino and vice versa. This means that the neutrino and antineutrino are "mixed" particles, i.e., a symmetric and antisymmetric combination of two truly neutral Majorana particles ν_1 and ν_2 of different combined parity.[5]

ν ↔ anti-ν oscillation



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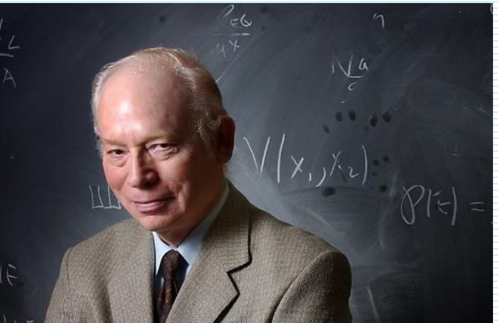
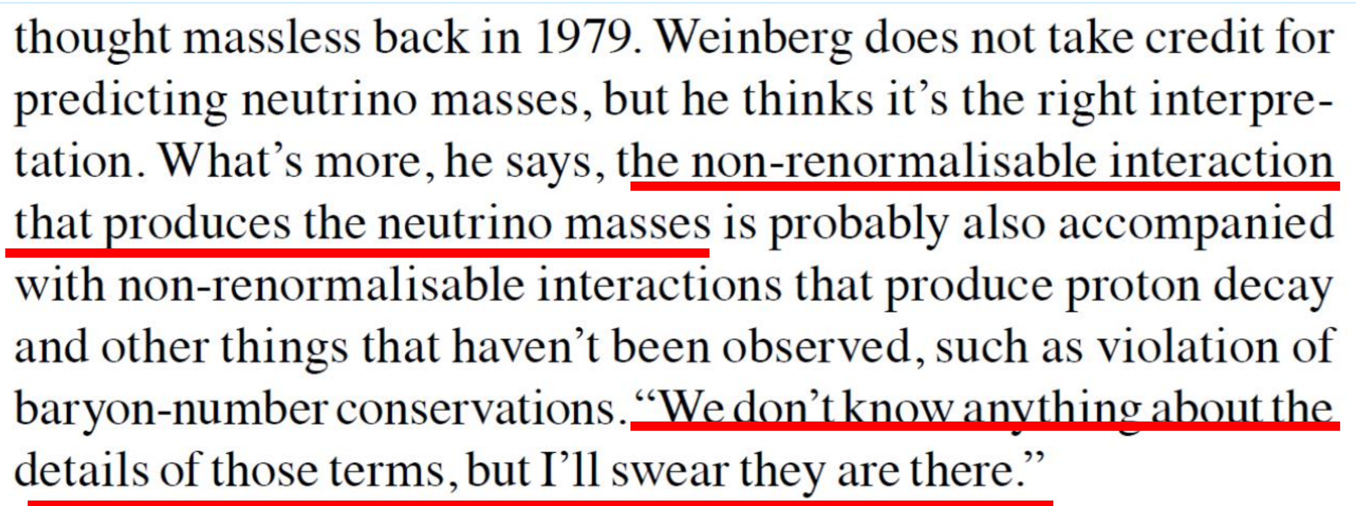


Steve Weinberg

ν-mass generation via d=5 eff. oper. related to unknown high energy scale (GUT?)



thought massless back in 1979. Weinberg does not take credit for predicting neutrino masses, but he thinks it's the right interpretation. What's more, he says, the non-renormalisable interaction that produces the neutrino masses is probably also accompanied with non-renormalisable interactions that produce proton decay and other things that haven't been observed, such as violation of baryon-number conservations. "We don't know anything about the details of those terms, but I'll swear they are there."

Neutrino Mass Term

Dirac

$$L^{D}_{\text{mass}} = -\sum_{\alpha\beta} \overline{\nu}_{\alpha R}\ M^{D}_{\alpha\beta}\ \nu_{\beta_L} + H.c.$$

$$= -\sum_{k=1}^{3} m_k \overline{\nu}_k \nu_k$$

$\alpha, \beta = e, \mu, \tau,$ $\quad V^{\dagger}\ M^{D}\ U = M^{D}_{\text{diag}}$

Majorana

$$L^{M}_{\text{mass}} = \frac{1}{2} \sum_{\alpha\beta} \nu^{T}_{\alpha L} C^{\dagger}\ M^{L}_{\alpha\beta}\ \nu_{\beta_L} + H.c.$$

$$= \frac{1}{2} \sum_{k=1}^{3} m_k \nu^{T}_{k} C^{\dagger} \nu_k$$

$\alpha, \beta = e, \mu, \tau$

$M^{L}_{\alpha\beta} = M^{L}_{\beta\alpha}$ $\quad U^{T}\ M^{M}\ U = M^{M}_{\text{diag}}$

Dirac-Majorana

$$L^{D+M}_{\text{mass}} = -\sum_{\alpha\beta} \overline{N}_{\alpha R}\ M^{D+M}_{\alpha\beta}\ N_{\beta_L} + H.c.$$

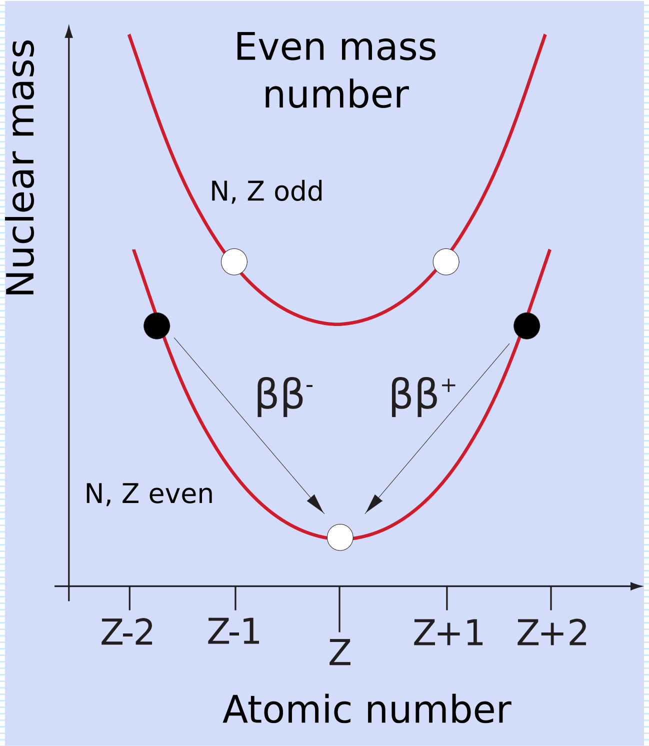
$$= -\sum_{k=1}^{6} m_k \overline{\nu}_k \nu_k$$

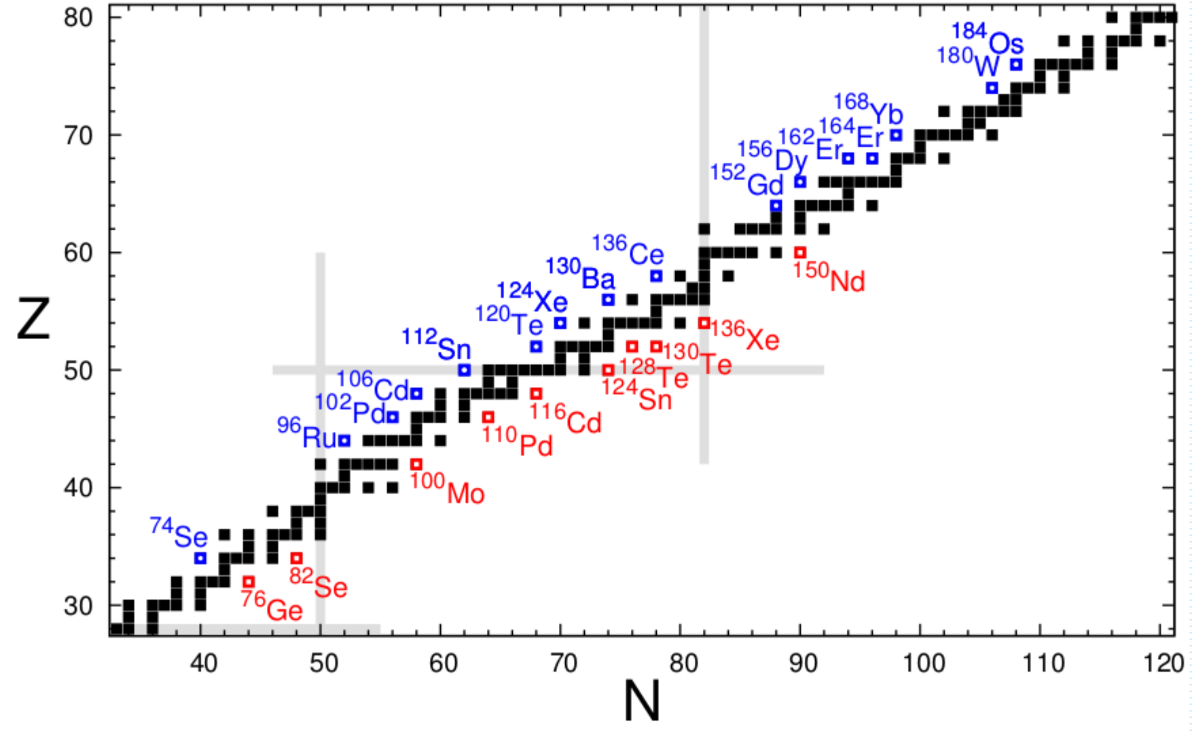
$N = \begin{pmatrix} \nu_L \\ \nu^{C}_{R} \end{pmatrix},$ $\quad \alpha, \beta = e, \mu, \tau, s_1, s_2, s_3$

$$U^{T}\ M^{D+M}\ U = M^{D+M}_{\text{diag}}$$

Nuclear double-β decay

(even-even nuclei, pairing int.)





Phys. Rev. 48, 512 (1935)

Two-neutrino double-β decay – LN conserved

$$(A,Z) \rightarrow (A,Z+2) + e^- + e^- + \nu_e + \nu_e$$

Goepert-Mayer – 1935. 1st observation in 1987

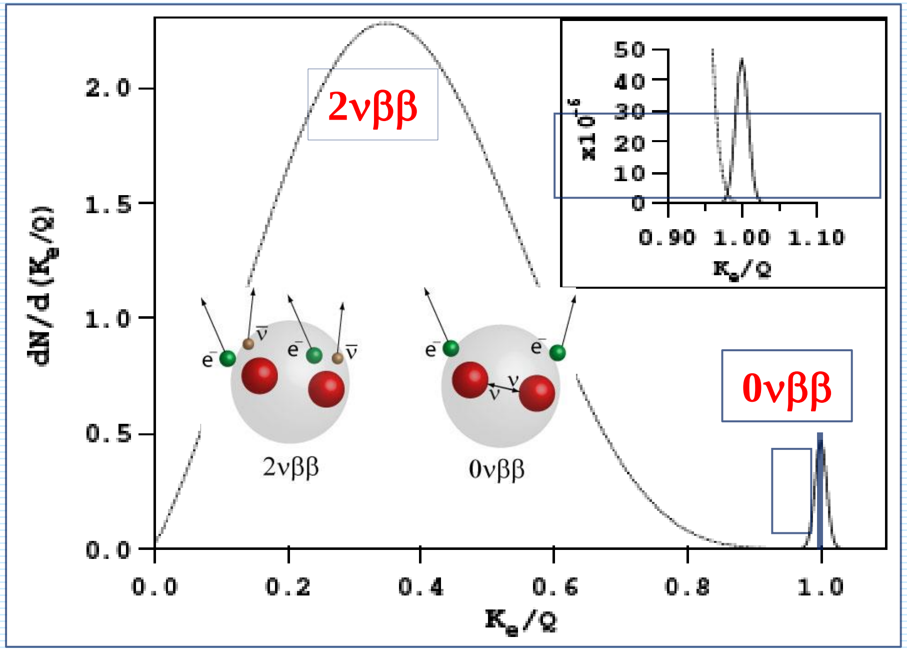
Nuovo Cim. 14, 322 (1937)

Phys. Rev. 56, 1184 (1939)

Neutrinoless double-β decay – LN violated

$$(A,Z) \rightarrow (A,Z+2) + e^- + e^- \quad \text{(Furry 1937)}$$

Not observed yet. Requires massive Majorana ν's



$0\nu\beta\beta$ decay isotopes and experiments

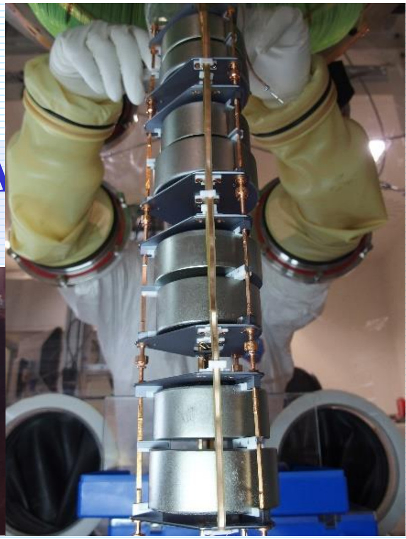
NvDEx
No neutrino-Double-beta Decay Experiment

High-pressure TPC chamber

Candidates	$Q_{\beta\beta}$(MeV)	N.A. (%)
$^{48}Ca\rightarrow^{48}Ti$	4.268	0.187
$^{76}Ge\rightarrow^{76}Se$	2.039	7.8
$^{82}Se\rightarrow^{82}Kr$	2.998	8.8
$^{96}Zr\rightarrow^{96}Mo$	3.356	2.8
$^{100}Mo\rightarrow^{100}Ru$	3.034	9.7
$^{110}Pd\rightarrow^{110}Cd$	2.017	11.7
$^{116}Cd\rightarrow^{116}Sn$	2.813	7.5
$^{124}Sn\rightarrow^{124}Te$	2.293	5.8
$^{130}Te\rightarrow^{130}Xe$	2.528	34.1
$^{136}Xe\rightarrow^{136}Ba$	2.458	8.9
$^{150}Nd\rightarrow^{150}Sm$	3.371	5.6

CANDLE
CaF scintillating crystal

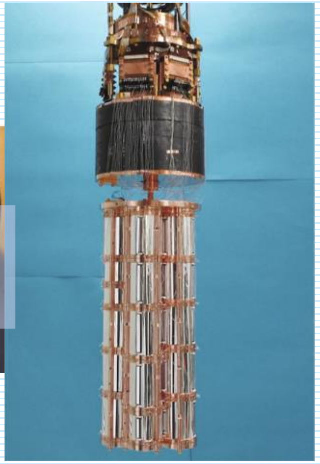
SuperNEMO
Se source foil

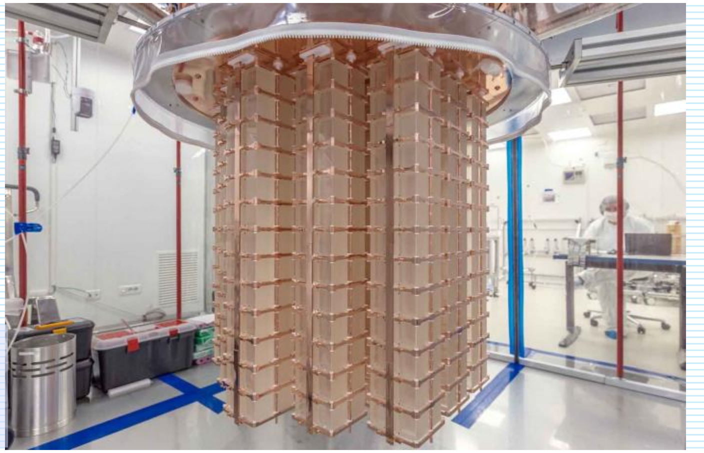


GERDA, MAJORANA
Ge crystal

CUPID-0
ZnSe scintillating crystal

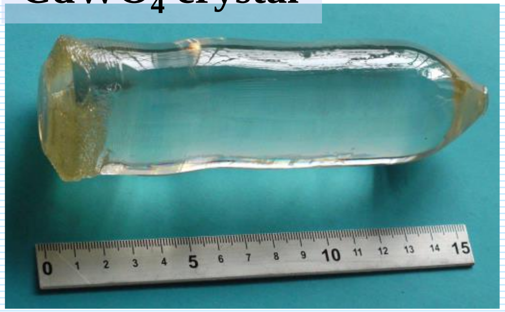
SuperNEMO
Se source foil



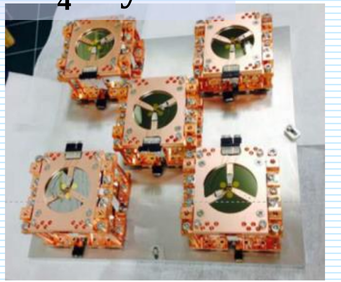


CUORE
TeO_2 crystal

Aurora
$CdWO_4$ crystal



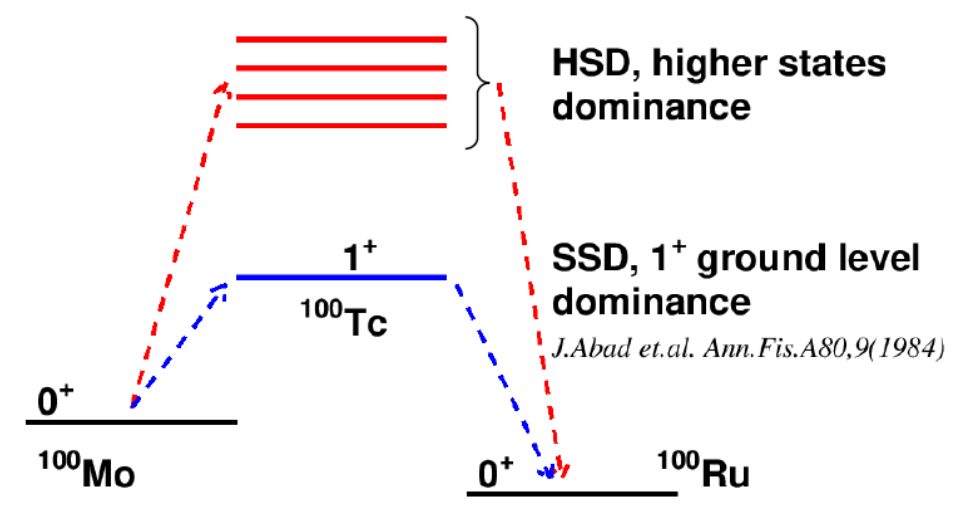
Amore
$CaMoO_4$ crystal



EXO, KamLAND-Zen
Liquid Xe

2νββ-decay

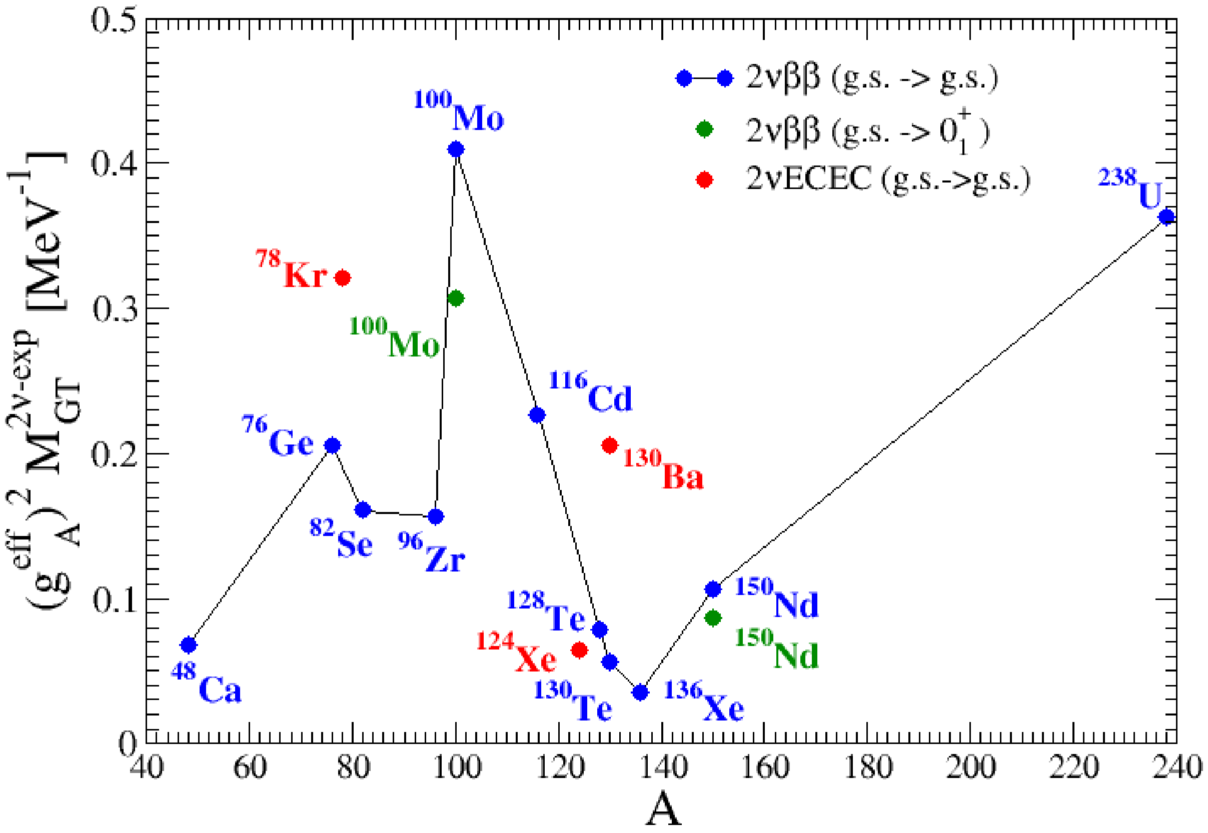
$$\left(T^{2\nu}_{1/2}\right)^{-1} \simeq g_A^4 \ \left|M^{2\nu}_{GT}\right|^2 \ G^{2\nu}_{01}$$



HSD, higher states dominance

SSD, 1⁺ ground level dominance

J.Abad et.al. Ann.Fis.A80,9(1984)



$$M^{2\nu}_{GT} = \sum_m \frac{< 0_f^+||\tau^+\sigma||1_m^+ >< 1_m^+||\tau^+\sigma||0_i^+ >}{E_m - E_i + \Delta}$$

There is no reliable calculation of the *2νββ-decay NMEs yet* - quenching of g_A, sensitivity to particle-particle int. of nuclear Hamiltonian

2νββ probes
New Beyond SM Physics

All 100 kg- and ton-class 0νββ experiments can also study a diverse range of exotic phenomena, e.g. through spectral distortion in $2\nu\beta\beta$.
Future searches will probe the $2\nu\beta\beta$ with high statistics about 10^5-10^6 events.

Common subjects:

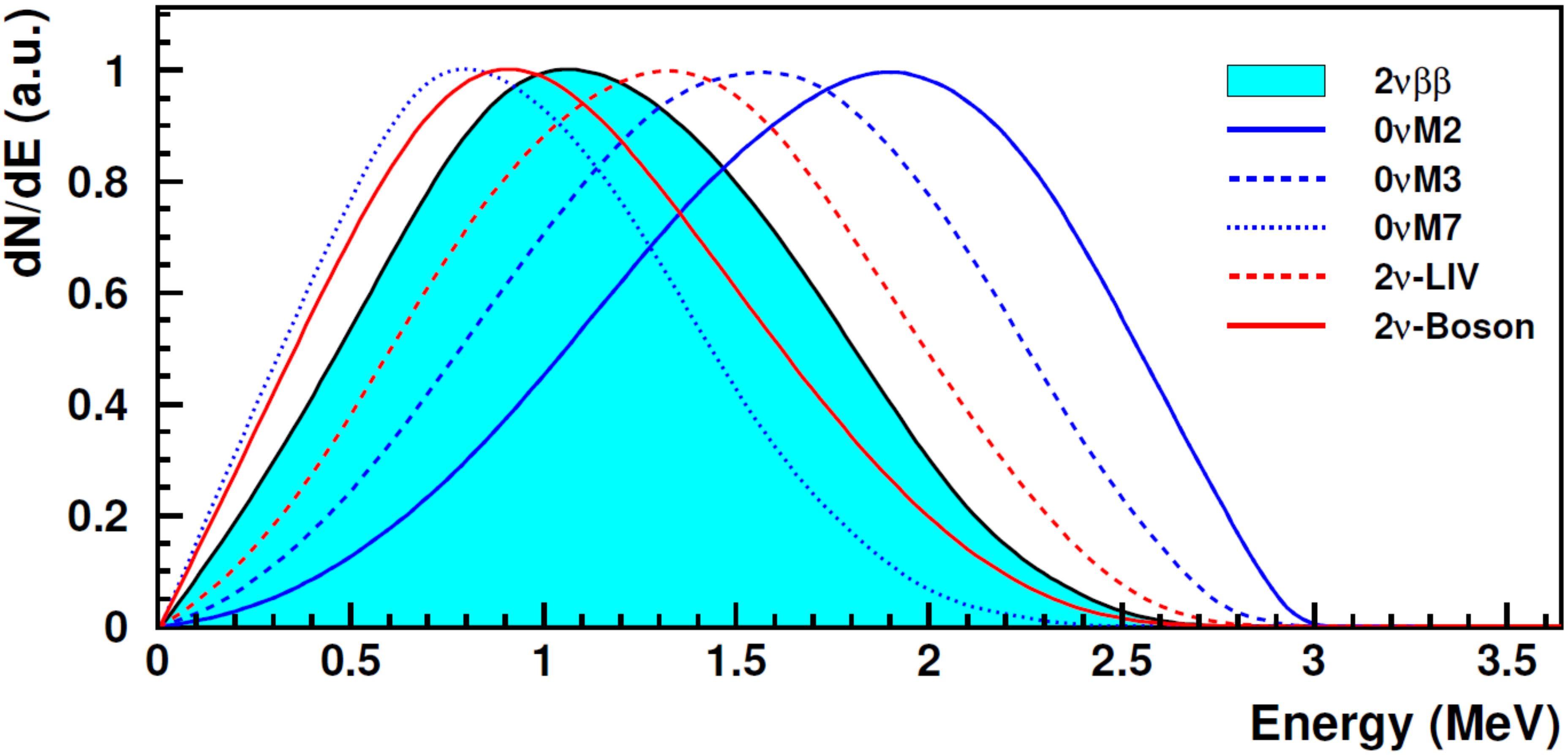
Majoron(s) emission
(partly)bosonic neutrinos,
Lorentz invariance violation

Recent subjects:

Lepton-number conserving right-handed currents
(PRL 125 (2020) 17, 171801)

Neutrino self-interactions
(PRD 102 (2020) 5, 051701)

Sterile neutrino and light fermion searches through energy end point
(PRD 103 (2021) 5, 055019;
PLB 815 (2021) 136127)



$$\frac{d\Gamma}{d\varepsilon_1 d\varepsilon_2} = C(Q-\varepsilon_1-\varepsilon_2)^n \; [p_1\varepsilon_1 F(\varepsilon_1)] \; [p_2\varepsilon_2 F(\varepsilon_2)]$$

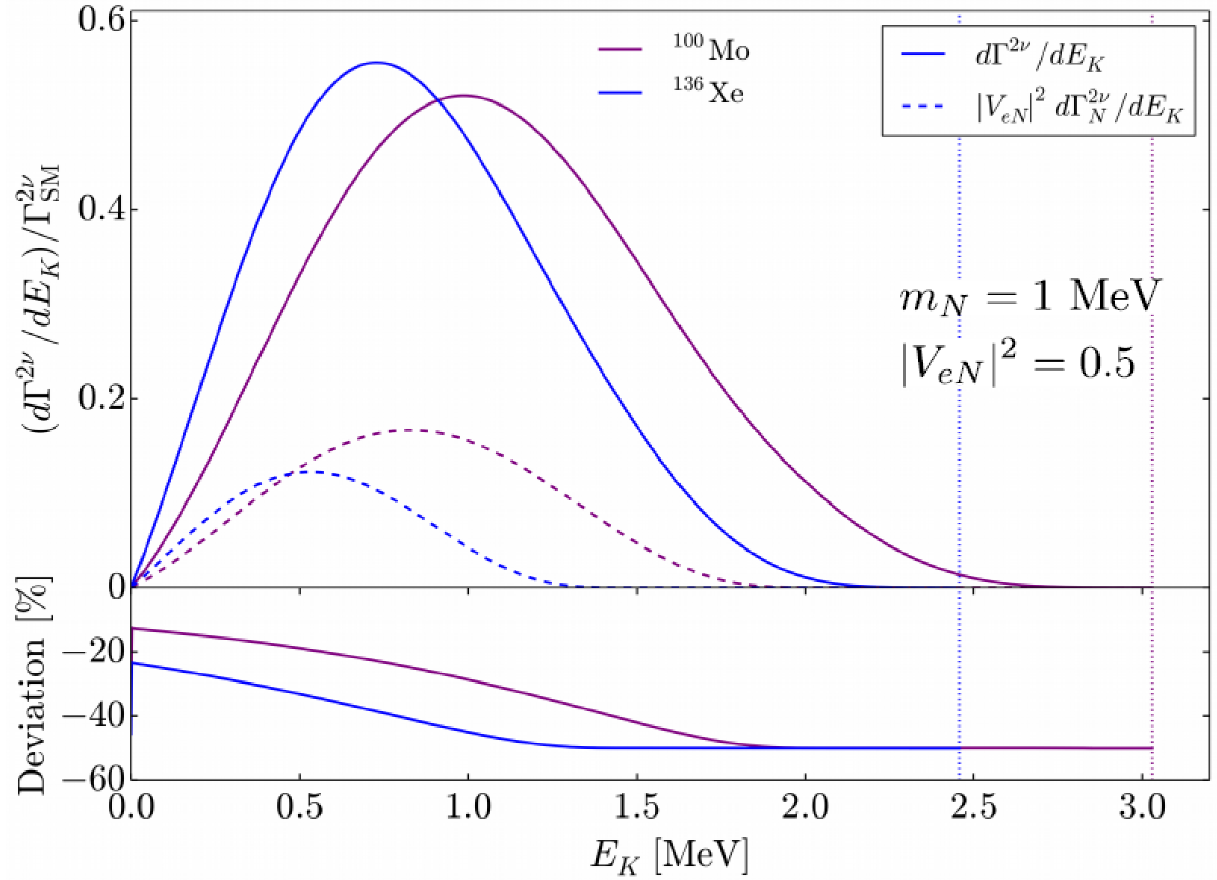
Spectral index n

Exotic 2νββ Sterile Neutrino (or HNL)

$$\mathcal{L} = \frac{G_F \cos\theta_C}{\sqrt{2}} \left[(1+\delta_{\rm SM}) j_L^\mu J_{L\mu} + V_{eN} j_L^{N\mu} J_{L\mu} + \epsilon_{LR} j_R^{N\mu} J_{L\mu} + \epsilon_{RR} j_R^{N\mu} J_{R\mu} \right] + \text{h.c.}$$

Consider either mixing of sterile ν with active ν, or right-handed currents

Phys.Rev.D 103 (2021) 055019



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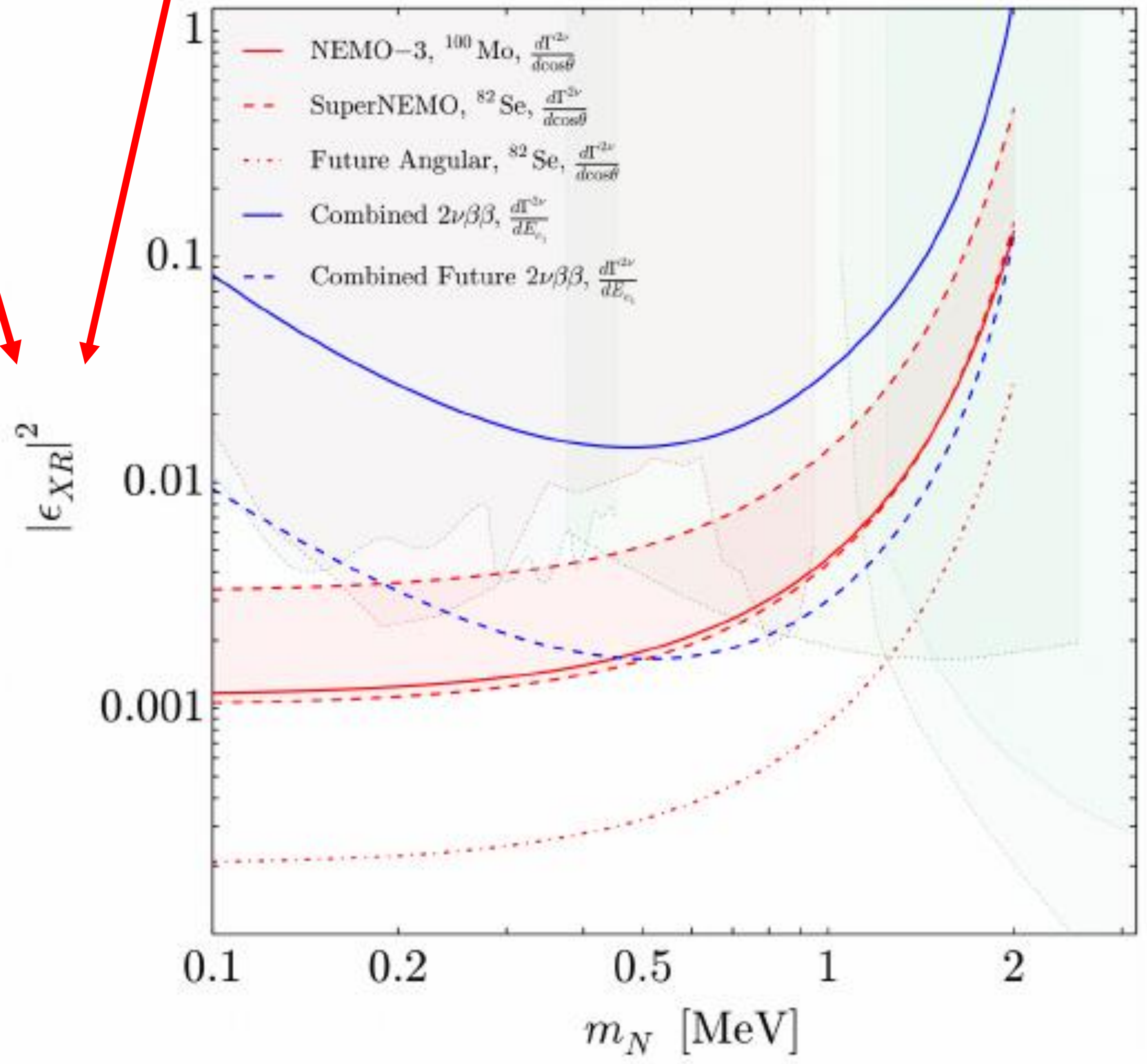
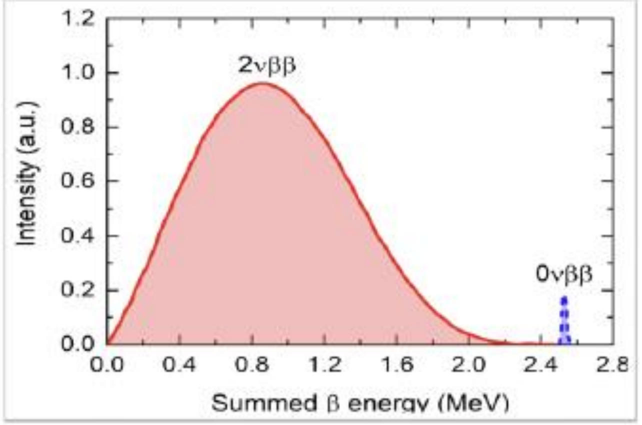
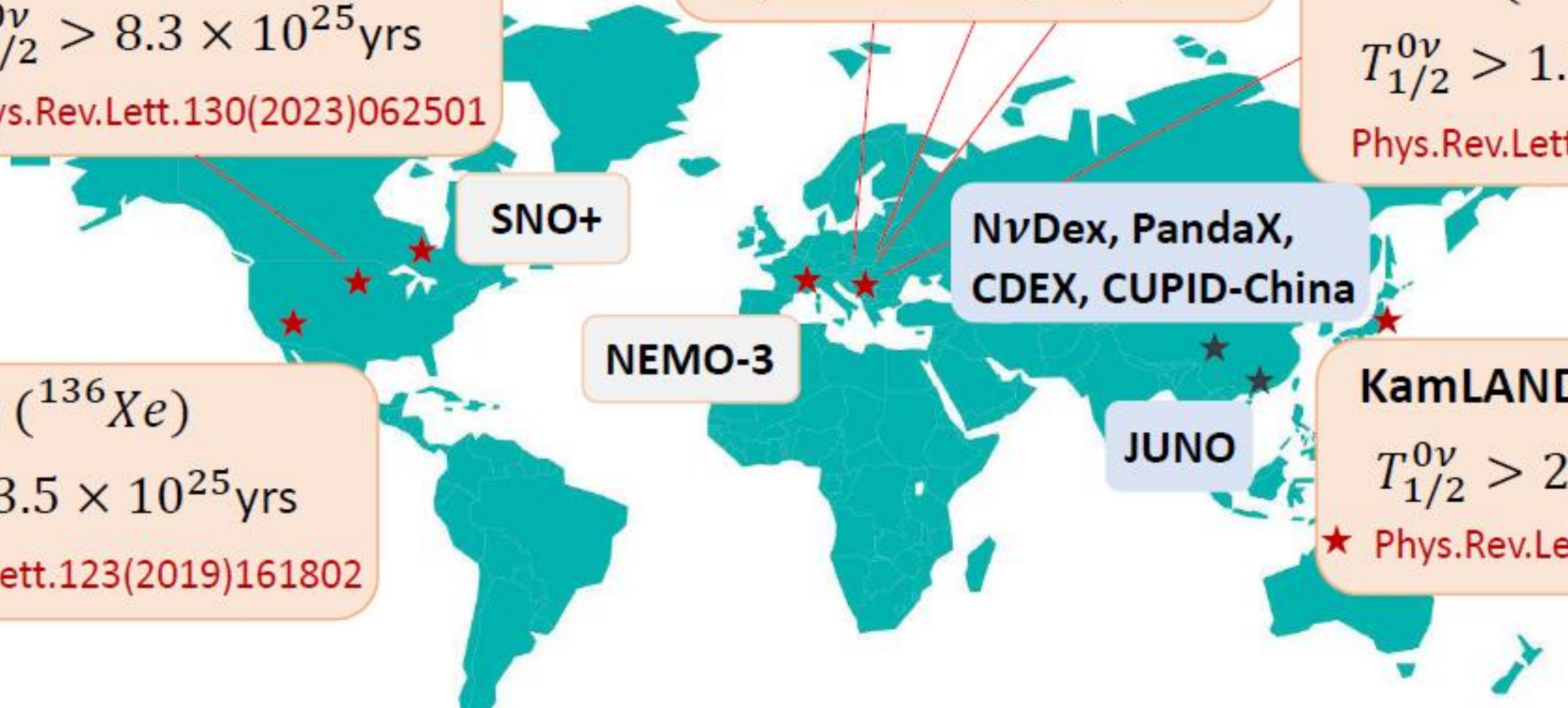


$0\nu\beta\beta$ experiments – a worldwide competition of ideas and underground physics technologies





GERDA (^{76}Ge)

$T_{1/2}^{0\nu} > 1.8 \times 10^{26}$yrs

Phys.Rev.Lett.125(2020)252502

GUORE (^{130}Te)

$T_{1/2}^{0\nu} > 2.2 \times 10^{25}$yrs

Nature 604(2022)53

CUPID-0 (^{82}Se)

$T_{1/2}^{0\nu} > 2.4 \times 10^{24}$yrs

Phys.Rev.Lett.120(2018)232502

MAJORANA (^{76}Ge)

$T_{1/2}^{0\nu} > 8.3 \times 10^{25}$yrs

Phys.Rev.Lett.130(2023)062501

CUPID (^{100}Mo)

$T_{1/2}^{0\nu} > 1.5 \times 10^{24}$yrs

Phys.Rev.Lett.126(2021)181802

NνDex, PandaX, CDEX, CUPID-China

EXO-200 (^{136}Xe)

$T_{1/2}^{0\nu} > 3.5 \times 10^{25}$yrs

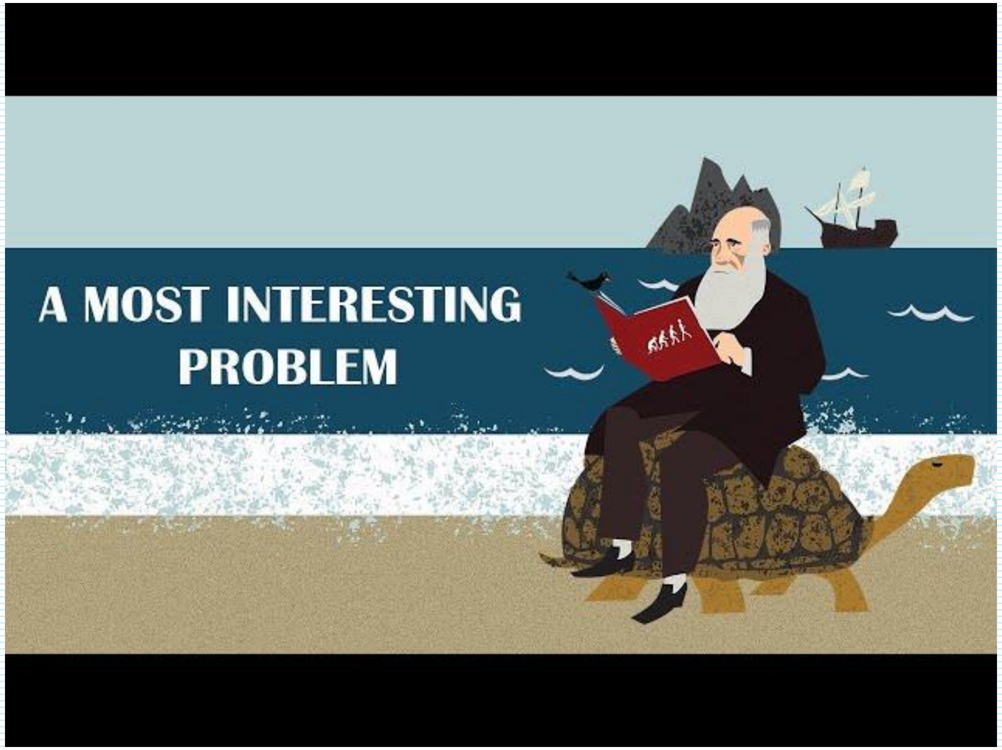
Phys.Rev.Lett.123(2019)161802

KamLAND-Zen (^{136}Xe)

$T_{1/2}^{0\nu} > 2.3 \times 10^{26}$yrs

★ Phys.Rev.Lett.130(2023)051801

0νββ is a particle physics problem



(A,Z) → (A,Z+2) + e⁻ + e⁻

0νββ-decay

(LNV at ≈GUT scale, exchange of three light ν)

$$\left(T^{0\nu}_{1/2}\right)^{-1} = \left|\frac{m_{\beta\beta}}{m_e}\right|^2 \underset{?}{g_A^4} \left|M^{0\nu}_{\nu}\right|^2 G^{0\nu}$$

Phase space factor well understood

$$m_{\beta\beta} = \left|c^2_{13}c^2_{12}e^{i\alpha_1}m_1 + c^2_{13}s^2_{12}e^{i\alpha_2}m_2 + s^2_{13}m_3\right|$$

NME must be evaluated using tools of nuclear theory

Constraint from cosmology

$\Sigma = m_1 + m_2 + m_3$

< 0.90 eV

< 0.26 eV (Planck coll.)

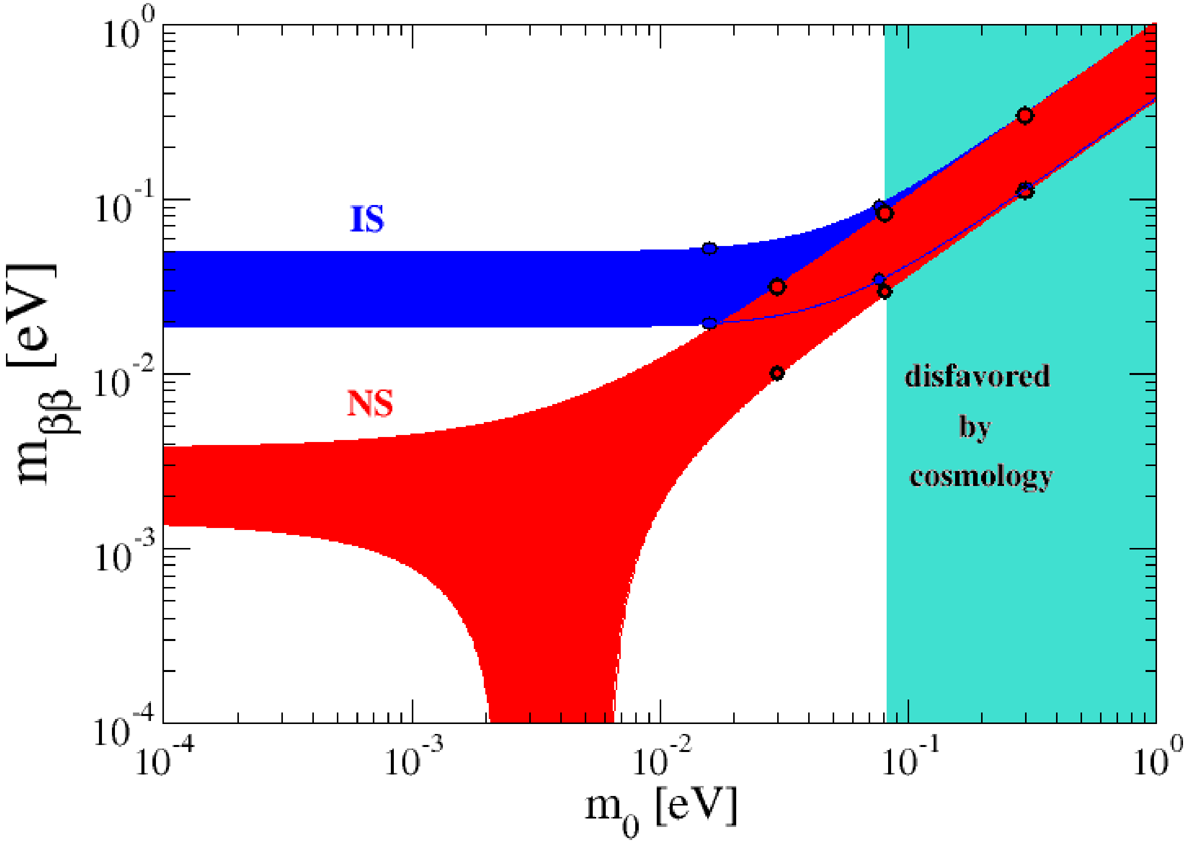
< 0.12 eV

Contrary, the constraint from 0νββ-decay (KLZ)

$m_{\beta\beta} < 0.036\text{-}0.156$ eV

implies

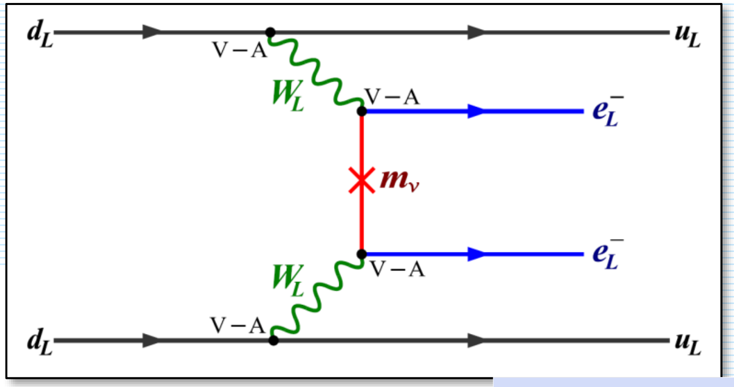
$\Sigma < 0.12$ eV

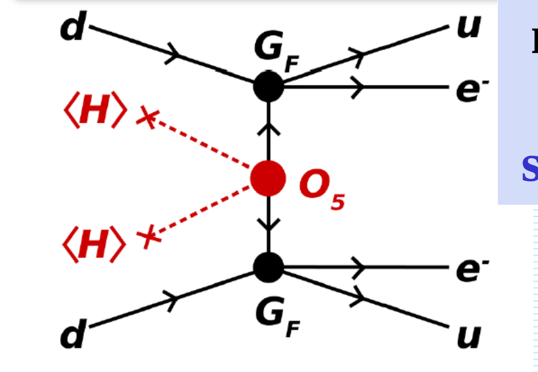


0νββ governed by exotic mechanisms

Light ν-mass mechanism can be strongly suppressed: $m_{\beta\beta} < 1$ meV

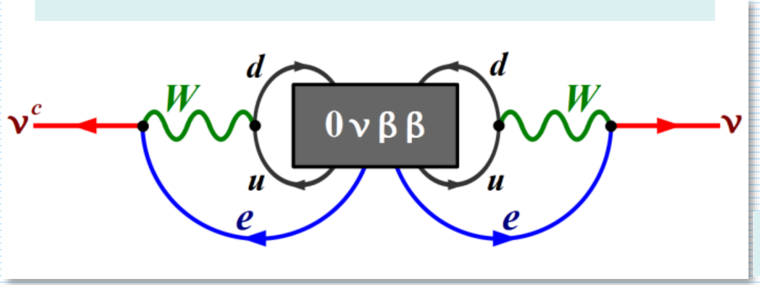
- It is not possible to discover 0νββ with **10-100 ton-class** experiment
- It should be a **subject of theory** to justify it
- There might be a dominance of other 0νββ mechanisms



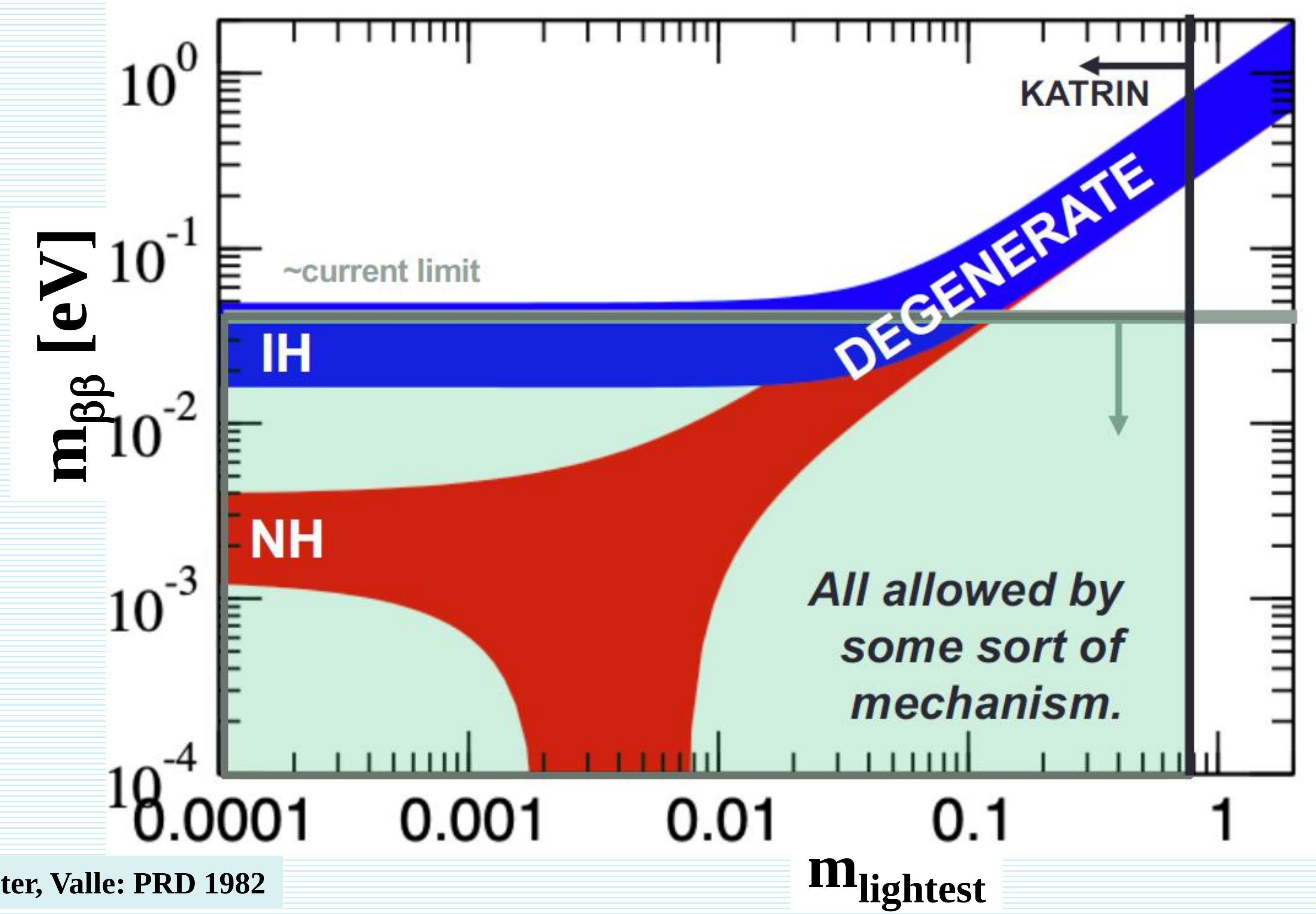


$m_{\beta\beta}$ mech. strongly suppressed

Any 0νββ mech. generates a small correction to ν-mass



Schechter, Valle: PRD 1982

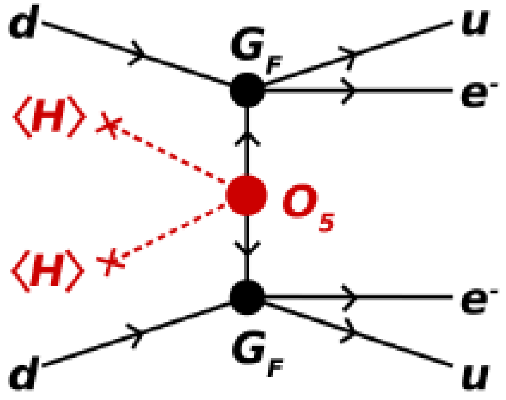


Beyond the SM physics

$$\mathcal{L} = \mathcal{L}_{SM}^{(4)} + \boxed{\frac{1}{\Lambda}\sum_i c_i^{(5)}\mathcal{O}_i^{(5)} + \frac{1}{\Lambda^2}\sum_i c_i^{(6)}\mathcal{O}_i^{(6)} + O(\frac{1}{\Lambda^3})}$$

Amplitude for
(A,Z)→(A,Z+2)+2e⁻
can be divided into:

mass mechanism: d=5

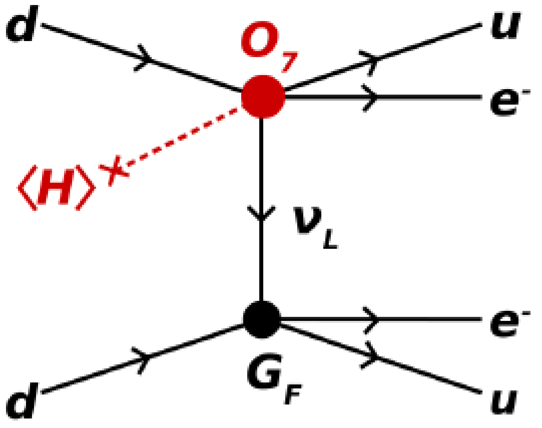


$$\mathcal{O}_W \propto \frac{c_{ij}}{\Lambda}(L_i H)(L_j H)$$

Weinberg, 1979

+

long range: d=7

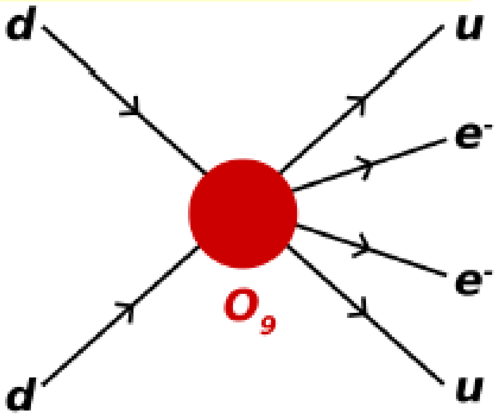


$$\mathcal{O}_2 \propto LLLe^c H$$
$$\mathcal{O}_3 \propto LLQd^c H$$
$$\mathcal{O}_4 \propto LL\bar{Q}\bar{u}^c H$$
$$\mathcal{O}_8 \propto L\bar{e}^c\bar{u}^c d^c H$$

Babu, Leung: 2001
de Gouvea, Jenkins: 2007

+

short range: d=9 (d=11)



$$\mathcal{O}_5 \propto LLQd^c HHH^\dagger$$
$$\mathcal{O}_6 \propto LL\bar{Q}\bar{u}^c HH^\dagger H$$
$$\mathcal{O}_7 \propto LQ\bar{e}^c\bar{Q}HHH^\dagger$$
$$\mathcal{O}_9 \propto LLLe^c Le^c$$
$$\mathcal{O}_{10} \propto LLLe^c Qd^c$$
$$\mathcal{O}_{11} \propto LLQd^c Qd^c$$
$$\cdots\cdots$$

Valle

Quark Condensate Seesaw Mechanism for Neutrino Mass

PRD 103, 015007 (2021).

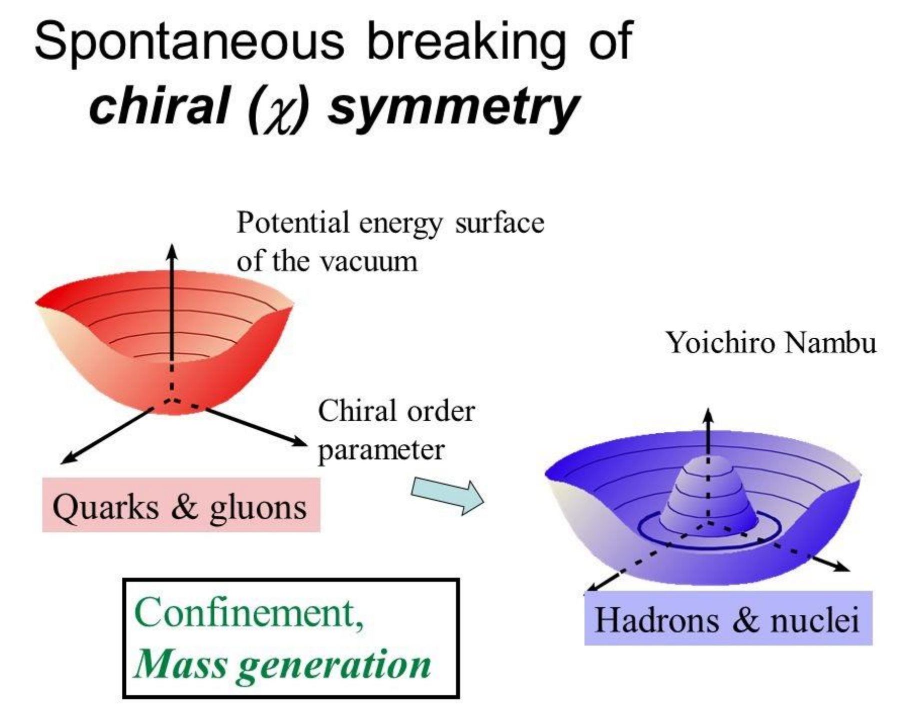
This operator contributes to the Majorana-neutrino mass matrix due to chiral symmetry breaking via the light-quark condensate.

The SM gauge-invariant effective operators

$$\mathcal{O}_7^{u,d} = \frac{\tilde{g}^{u,d}_{\alpha\beta}}{\Lambda^3}\, \overline{L^C_\alpha}\, L_\beta\, H \left\{ (\overline{Q}\, u_R),\ (\overline{d_R}\, Q) \right\}$$

After the EWSB and ChSB one arrives at the Majorana mass matrix of active neutrinos

$$\begin{aligned} m_{\alpha\beta}{}^{\nu} &= g_{\alpha\beta}\, v\, \frac{\langle \overline{q} q \rangle}{\Lambda^3} \\ &= g_{\alpha\beta}\, v \left(\frac{\omega}{\Lambda} \right)^3 \\ g_{\alpha\beta} &= g^u_{\alpha\beta} + g^d_{\alpha\beta}, \quad v/\sqrt{2} = \langle H^0 \rangle \\ \omega &= -\langle \overline{q} q \rangle^{1/3}, \quad \langle \overline{q} q \rangle^{1/3} \approx -283\ \mathrm{MeV} \end{aligned}$$



**Λ ~ a few TeV
we get the neutrino mass
in the sub-eV ballpark**

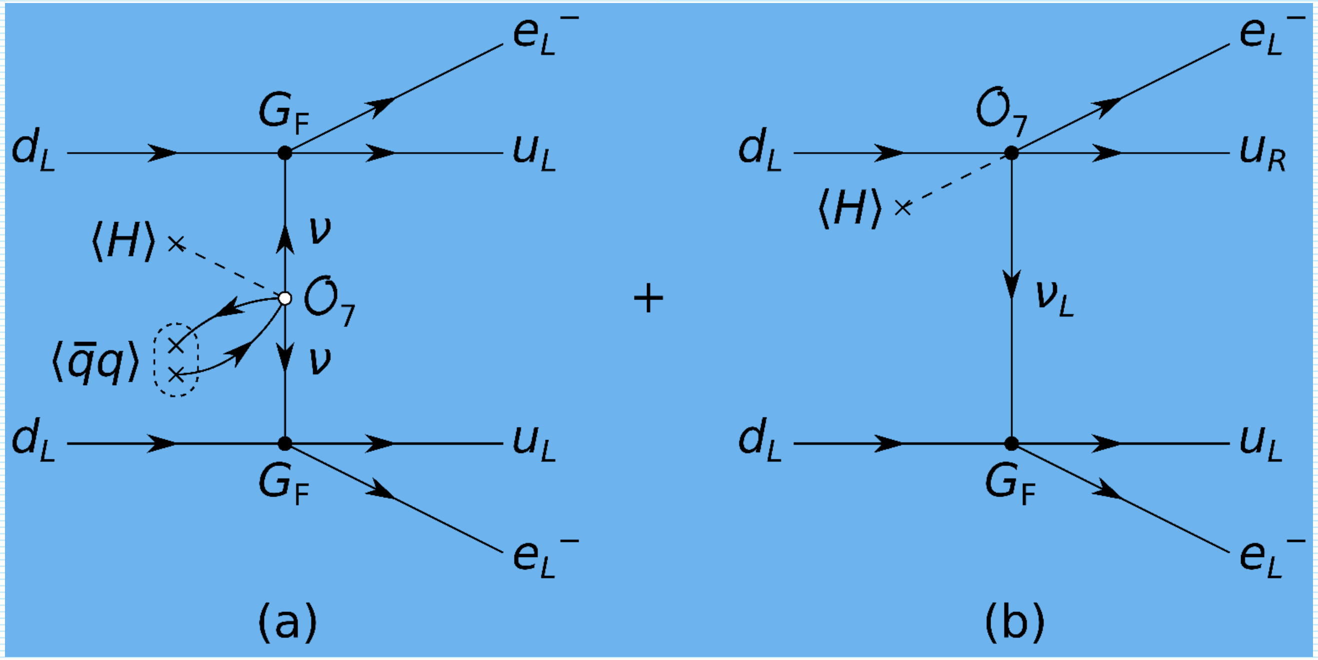
The genuine QCSS scenario
(predicts NH and ν-mass spectrum)

$$\mathcal{L}_7 = \frac{1}{\sqrt{2}} \sum_{\alpha\beta} \frac{v}{\Lambda^3} \overline{\nu^C_{\alpha L}} \nu_{\beta L} \left(g^u_{\alpha\beta} \overline{u_L} u_R + g^d_{\alpha\beta} \overline{d_R} d_L \right) + \text{H.c.}$$

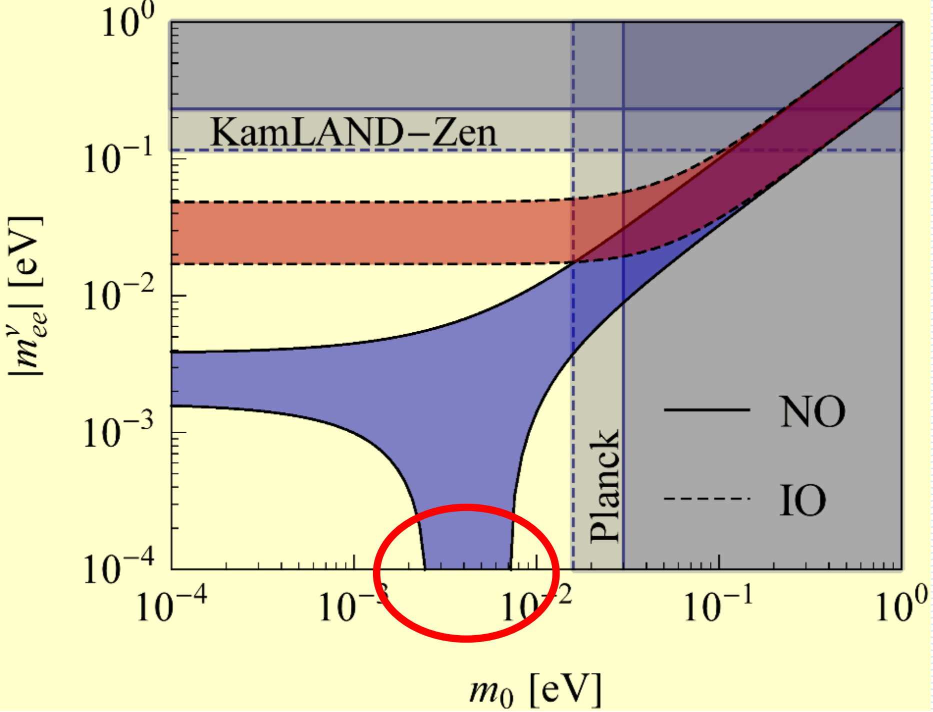
$$m^\nu_{\alpha\beta} = -\frac{g_{\alpha\beta}}{\sqrt{2}} v \frac{\langle \bar{q} q \rangle}{\Lambda^3} = \frac{g_{\alpha\beta}}{\sqrt{2}} v \left(\frac{\omega}{\Lambda} \right)^3$$

(a) PRL 112, 142503 (2014).

(b) PLB 453, 194 (1999).



(a) (b)



Neutrino spectrum (NH) !!!

2 meV < m_1 < 7 meV

9 meV < m_2 < 11 meV

50 meV < m_2 < 51 meV

Prediction for m_β

9 meV < m_b < 12 meV

Prediction for cosmology (Σ)

62 meV < $m_1 + m_2 + m_3$ < 69 meV

Majorana neutrino mass eigenstate N
with arbitrary mass m_N mixed with 3 active neutrinos (U_{eN})

Dominant contribution of N

$$[T^{0\nu}_{1/2}]^{-1} = G^{0\nu} g_{\rm A}^4 \left| \sum_{\rm N} \left(U^2_{e\rm N} m_{\rm N} \right) m_{\rm p}\, M'^{\,0\nu}(m_{\rm N}, g_{\rm A}^{\rm eff}) \right|^2$$

General case

$$M'^{\,0\nu}(m_{\rm N}, g_{\rm A}^{\rm eff}) = \frac{1}{m_{\rm p} m_{\rm e}} \frac{R}{2\pi^2 g_A^2} \sum_n \int d^3x\, d^3y\, d^3p$$
$$\times e^{i\mathbf{p}\cdot(\mathbf{x}-\mathbf{y})} \frac{\langle 0_F^+ | J^{\mu\dagger}(\mathbf{x}) | n \rangle \langle n | J_\mu^\dagger(\mathbf{y}) | 0_I^+ \rangle}{\sqrt{p^2 + m_N{}^2}\left(\sqrt{p^2 + m_N{}^2} + E_n - \frac{E_I - E_F}{2}\right)}$$

light ν exchange

$$M'^{0\nu}(m_{\rm N} \to 0, g_{\rm A}^{\rm eff}) = \frac{1}{m_{\rm p} m_{\rm e}} M'^{0\nu}_{\nu}(g_{\rm A}^{\rm eff})$$

$$M'^{0\nu}(m_{\rm N} \to \infty, g_{\rm A}^{\rm eff}) = \frac{1}{m_{\rm N}^2} M'^{0\nu}_{\rm N}(g_{\rm A}^{\rm eff})$$

heavy ν exchange

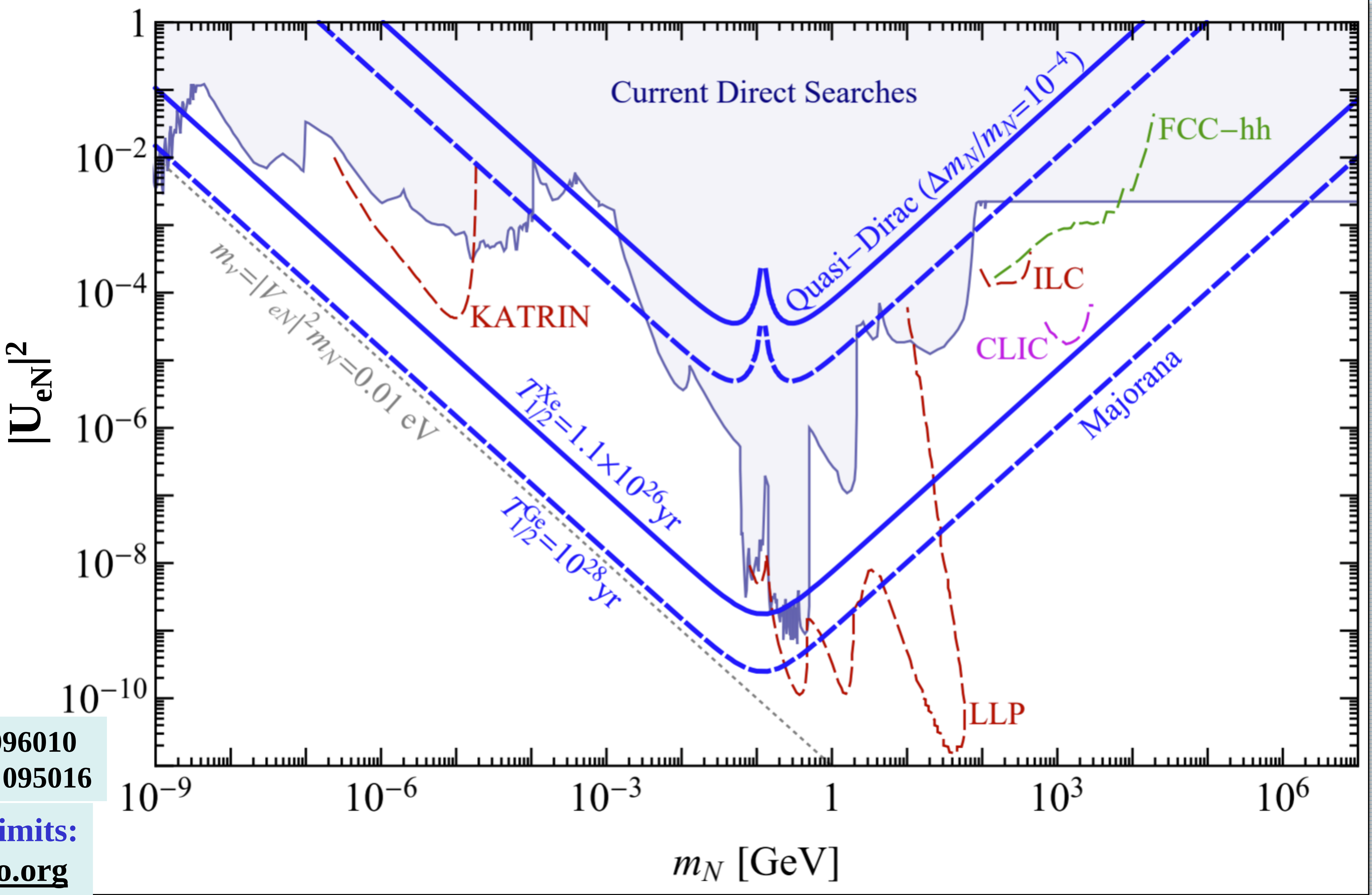
Particular cases

$$[T^{0\nu}_{1/2}]^{-1} = G^{0\nu} g_{\rm A}^4 \times$$
$$\times \begin{cases} \left| \frac{\langle m_\nu \rangle}{m_e} \right|^2 \left| M'^{0\nu}_{\nu}(g_{\rm A}^{\rm eff}) \right|^2 & \text{for } m_{\rm N} \ll p_{\rm F} \\ \left| \langle \frac{1}{m_{\rm N}} \rangle m_{\rm p} \right|^2 \left| M'^{0\nu}_{\rm N}(g_{\rm A}^{\rm eff}) \right|^2 & \text{for } m_{\rm N} \gg p_{\rm F} \end{cases}$$

or Simkovic

Sterile ν
(U_{en}, m_N)

Constraints from Direct Searches are less Stringent as those from 0νββ



PRD 90 (2014) 096010
PRD 102 (2020) 095016

Direct search limits:
sterile-neutrino.org

The 0νββ-decay within L-R symmetric theories

(D-M mass term, see-saw, V-A and V+A int., exchange of heavy neutrinos)

$$\mathcal{U} = \begin{pmatrix} U & S \\ T & V \end{pmatrix}$$

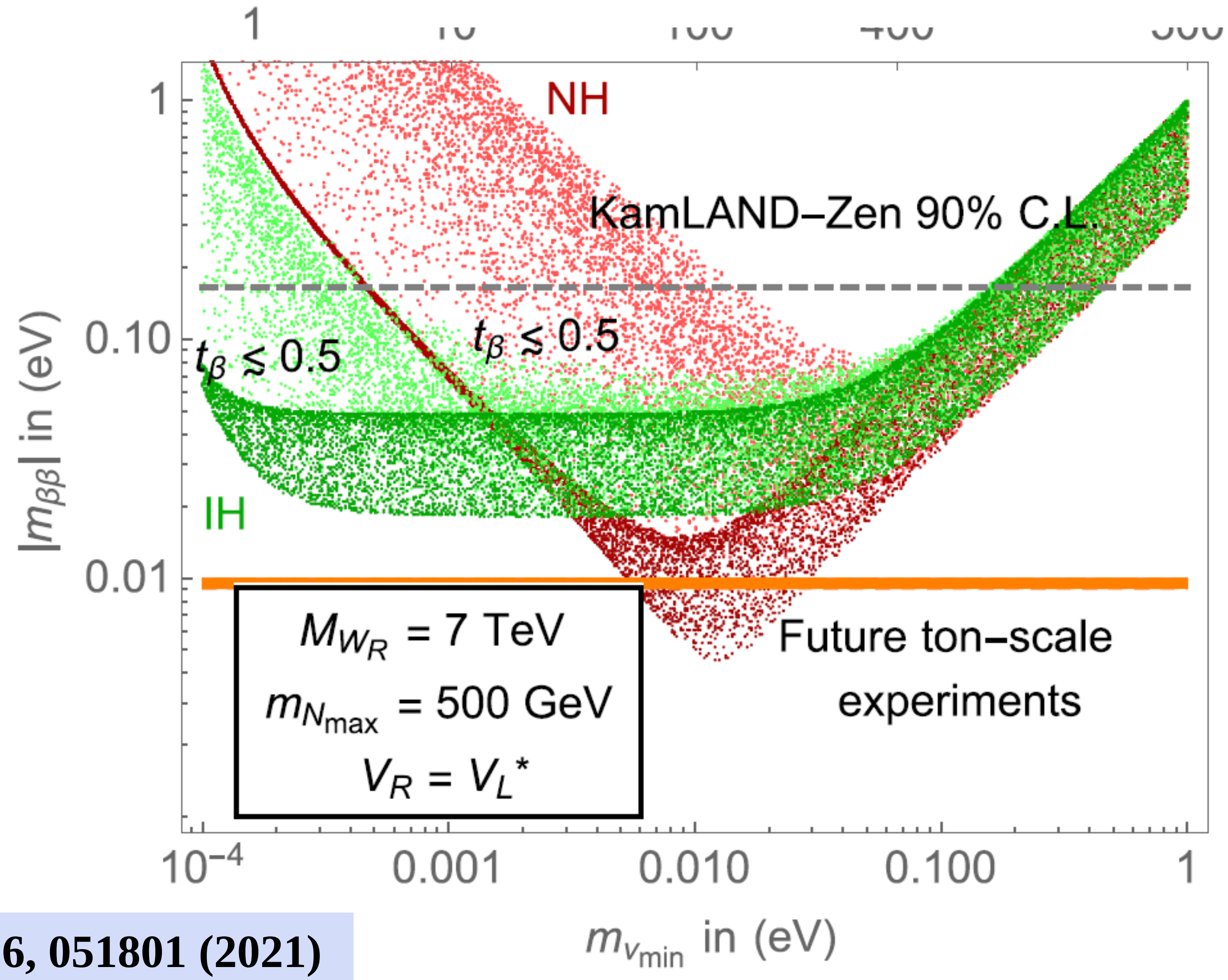
Mixing of 3 light and 3 heavy neutrinos:
15 angles + 15 phases

$$(T^{0\nu}_{1/2})^{-1} = G_{0\nu}\,\mathcal{M}^2_\nu |m_{\beta\beta}|^2$$
$$= G_{0\nu}\,\mathcal{M}^2_\nu (|m^{ee}_\nu|^2 + |m^{ee}_N|^2)$$

The most economical: **S=T= ζ 1**
(ζ - see-saw parameter)

The **mixing for heavy neutrinos,**
(V≡V$_R$= U$^+_{PMNS}$ with U = U$_{PMNS}$)
follows from the unitarity
of the whole 6x6 matrix ***U***

PRD 98, 015003 (2018)



2/19/2024

PRL 126, 051801 (2021)

Six Quasi-Dirac neutrinos and 0νββ-decay

Symmetry 12, 1310 (2020).

Dirac-Majorana mass term

$$\mathcal{L}_m = \frac{1}{2} \left(\overline{\nu_L} \quad \overline{\nu^c_R} \right) \mathcal{M} \begin{pmatrix} \nu^c_L \\ \nu_R \end{pmatrix} + h.c.$$

$$\mathcal{U}^T \; \tilde{\mathcal{M}} \; \mathcal{U} = \mathcal{M}$$

Diagonalization: 6x6 unitary mixing matrix (15 mixing angles plus 15 phases)

$$\mathcal{U} = \mathcal{X} \cdot \mathcal{A} \cdot \mathcal{S}$$

Product of 3 unitary matrices. A and S mix exclusively active and sterile neutrino flavors, each given by 3 angles and 3 phases.

$$\mathcal{A} \equiv \begin{pmatrix} U^T & 0 \\ 0 & 1 \end{pmatrix}$$

$$\mathcal{S} \equiv \begin{pmatrix} 1 & 0 \\ 0 & V^\dagger \end{pmatrix}$$

M_D - 3x3 complex matrix (18 real numb.)
$M_{L,R}$ - 3x3 symmetric matrix (12 real numb.)
(42 parameters)

$$\mathcal{M} = \begin{pmatrix} M_L & M_D \\ M_D^T & M_R \end{pmatrix}$$

$|M_{L,R}| \ll |M_D|$

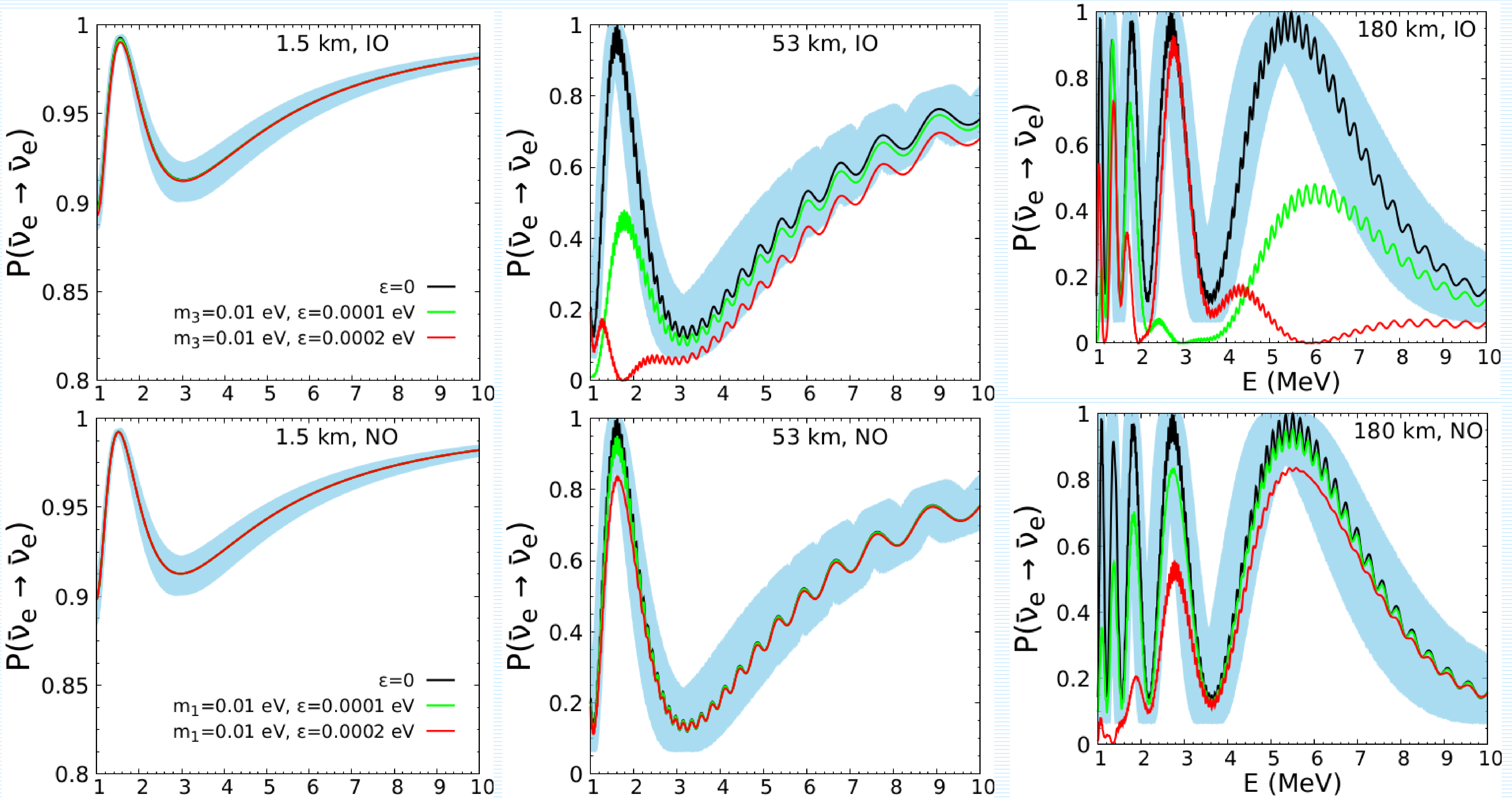
6 eigenvalues:
3 Dirac masses $m_{1,2,3}$, 3 Majorana mass splitting $\varepsilon_{1,2,3}$

$$m_i^{\pm} = \pm m_i + \epsilon_i$$

$$\mathcal{X} = \begin{pmatrix} 1 & X^\dagger \\ -X & 1 \end{pmatrix} + O(X^2)$$

X given by 9 angles and 9 phases, small parameters.

Quasi-Dirac neutrino oscillations at different distances



Quasi-Dirac ν a simplified scenario

(limits on neutrino masses)

$$\mathcal{U}_{\rm QD} = \frac{1}{\sqrt{2}}\begin{pmatrix} U & U \\ -V^* & V^* \end{pmatrix}$$

$$m_i^{\pm} = \pm m_i + \epsilon \qquad (\epsilon > 0)$$

The survival probability of electron antineutrinos

$$P_{\bar{\nu}_e\to\bar{\nu}_e}(\epsilon\neq 0) = P_{\bar{\nu}_e\to\bar{\nu}_e}(\epsilon = 0) - \frac{\epsilon^2 L^2}{E^2}\left[c_{13}^4 c_{12}^4 m_1^2 + c_{13}^4 s_{12}^4 m_2^2 + s_{13}^4 m_3^2\right]$$
$$-\frac{\epsilon^2 L^2}{4E^2}\left[4\, c_{13}^4 s_{12}^2 c_{12}^2 \Sigma m_{21}^2 \cos\frac{\Delta m_{21}^2 L}{2E} + 4 s_{13}^2 c_{13}^2 c_{12}^2 \Sigma m_{31}^2 \cos\frac{\Delta m_{31}^2 L}{2E}\right.$$
$$\left. +4\, s_{13}^2 c_{13}^2 s_{12}^2 \Sigma m_{32}^2 \cos\frac{\Delta m_{32}^2 L}{2E}\right] + \mathcal{O}(\epsilon^4)\,,$$

Tritium β-decay

$$m_\beta = \sqrt{m_1^2 c_{12}^2 c_{13}^2 + m_2^2 c_{13}^2 s_{12}^2 + m_3^2 s_{13}^2 + \epsilon^2}$$
$$= m_\beta^{(0)}\left(1 + \frac{1}{2}\left(\epsilon/m_\beta^{(0)}\right)^2 + \ldots\right)$$

Cosmology

$$\frac{1}{2}\sum_{i=1}^{6}\left|\tilde{\mathcal{M}}_{ii}\right| = \sum_{i=1}^{3} m_i$$

Restriction from Daya-Bay data (3σ):

Survival probabilities with non-zero ε are the same 3ν cases.

0νββ-decay

$$m_{\beta\beta} = \left[M_L\right]_{ee}$$
$$= \epsilon\, \left|c_{12}^2 c_{13}^2 + e^{2i\alpha_{21}} c_{13}^2 s_{12}^2 + e^{2i\alpha_{31}} s_{13}^2\right|$$

$$m_{\beta\beta} \lesssim 30 \text{ meV} \quad \text{for} \quad \text{NO}$$
$$\lesssim 1 \text{ meV} \quad \text{for} \quad \text{IO}$$

neutrino ↔ antineutrinos oscillations

Nuovo Cim. 14, 322 (1937)

Second order process with real intermediate neutrinos *($0\nu\beta\beta$ with real neutrinos)*

$$S + D \to \ell_\alpha^+ + \ell_\beta^+ + S' + D'$$

$$S \to S' + \ell_\alpha^+ + \nu_\alpha, \;\; \nu_\alpha \to \overline{\nu}_\beta, \;\; \overline{\nu}_\beta + D \to D' + \ell_\beta^+$$

Amplitude proportional to ν–mass

$$T_k^{\alpha\beta} = J_S^\mu(P_S', P_S) J_D^\nu(P_D', P_D) \times \\ \overline{v}(P_\alpha; \lambda_\alpha)\gamma_\mu(1-\gamma_5) m_k \gamma_\nu u(P_\beta; \lambda_\beta)$$

Oscillation probability

$$\mathcal{P}_{\alpha\bar{\beta}}^{\mathrm{QFT}}(E_\nu, L) \equiv \left|\langle \nu_\beta | \overline{\nu}_\alpha \rangle\right|^2 \\ = \left| \sum_{j=1}^{3} U_{\alpha j}^* U_{\beta j}^* \frac{m_j}{E_\nu} e^{-i m_j^2 L/(2E_\nu)} \right|^2$$

Replacement:

$$U_{\alpha k} \to U_{\alpha k}^*$$
$$U_{\beta m}^* \to U_{\beta m}$$

Particular process:

$$\pi^+ + p \to \mu^+ + e^+ + n$$

Production rate

$$\Gamma_{QFT}^{\pi^+ p} = \frac{1}{2\pi^2} G_\beta^2 \left(\frac{f_\pi}{\sqrt{2}}\right)^2 \frac{m_\mu^2}{m_\pi} E_\nu^2 \frac{P_{\nu_\mu \overline{\nu}_e}^{\mathrm{QFT}}(E_\nu, L)}{4\pi L^2} \left(g_V^2 + 3g_A^2\right) p_e E_e$$

Neutrino oscillations as a single Feynman diagram
J. Phys. G 51, 035202 (2024)

A connection of neutrino-antineutrino oscillation with 0νββ-decay

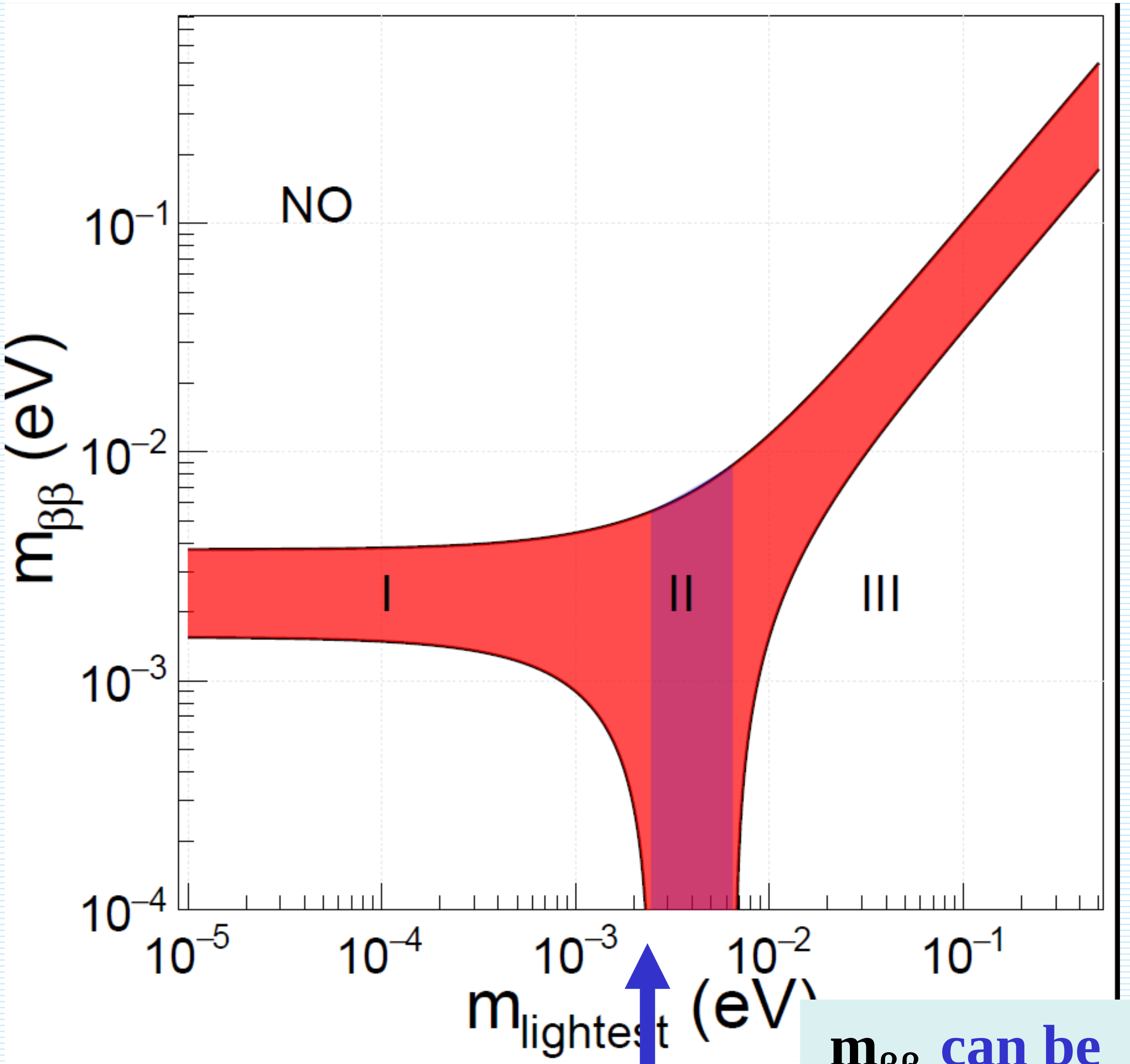
$$\mathbf{m^{L=0}_{ee} = m_{\beta\beta}}$$

$$\mathcal{P}^{\rm QFT}_{e\bar{e}}(E_\nu, L=0) \equiv |\langle \nu_e|\overline{\nu}_e\rangle|^2 = \left|\sum_{k=1}^{3} U^*_{ek}U^*_{ek}\frac{m_k}{E_\nu}\right|^2$$

$$= \frac{(m^{L=0}_{ee})^2}{E^2_\nu} = \frac{(m_{\beta\beta})^2}{E^2_\nu}$$

$$m_{\beta\beta} = |\rho_1 e^{2i\phi_1} + \rho_2 e^{2i\phi_2} + \rho_3|$$

$$\rho_1 = c^2_{12}c^2_{13}m_1,\ \ \rho_2 = s^2_{12}c^2_{13}m_2,\ \ \rho_3 = s^2_{13}m_3$$

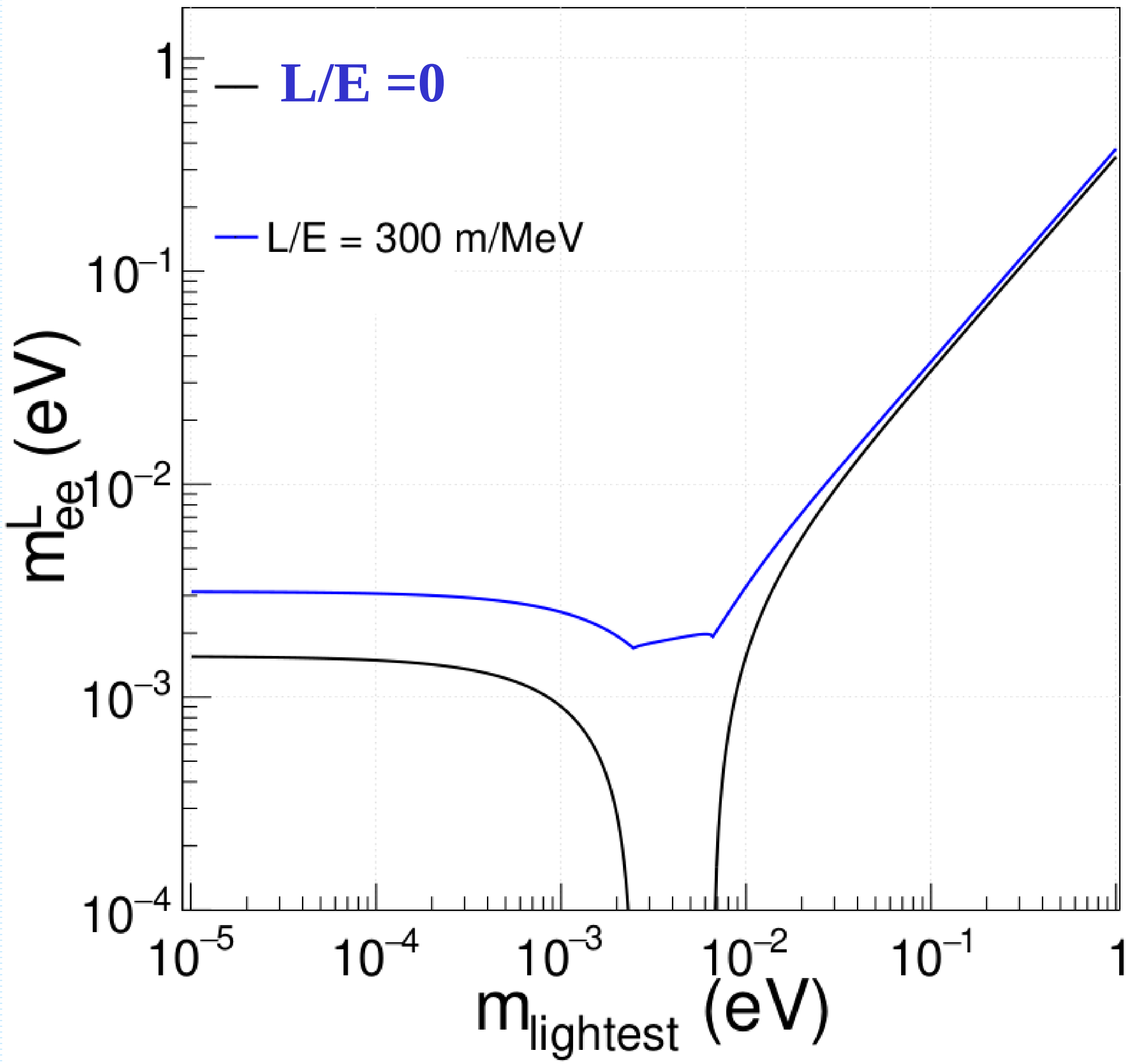


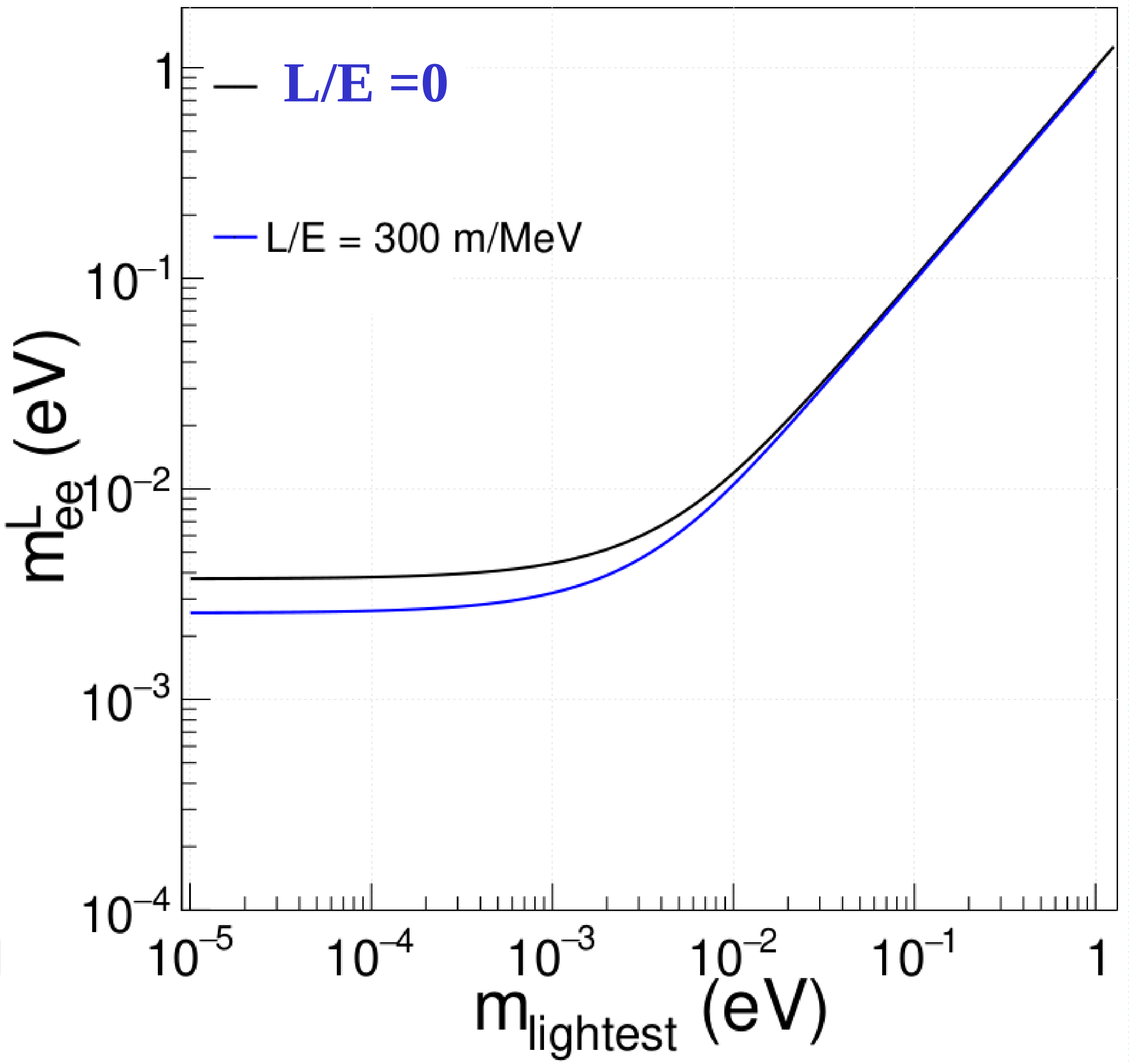
$$\min_{\phi_1,\phi_2} m_{\beta\beta} = \begin{cases} |\rho_2-\rho_3|-\rho_1, & \text{if } \rho_1 < |\rho_2-\rho_3| \quad : \text{ region I},\\ 0, & \text{if } |\rho_2-\rho_3| \le \rho_1 \le \rho_2+\rho_3 \quad : \text{ region II},\\ \rho_1-(\rho_2+\rho_3), & \text{if } \rho_2+\rho_3 < \rho_1 \quad : \text{ region III}. \end{cases}$$

$m_{\beta\beta}$ can be strongly \suppressed (?!)

Dependence of m^{L}_{ee} on $m_{lightest}$ and L/E

$m^{L=0}_{ee} = m_{\beta\beta}$





0νββ is a nuclear physics problem

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0νββ-decay NME status 2023

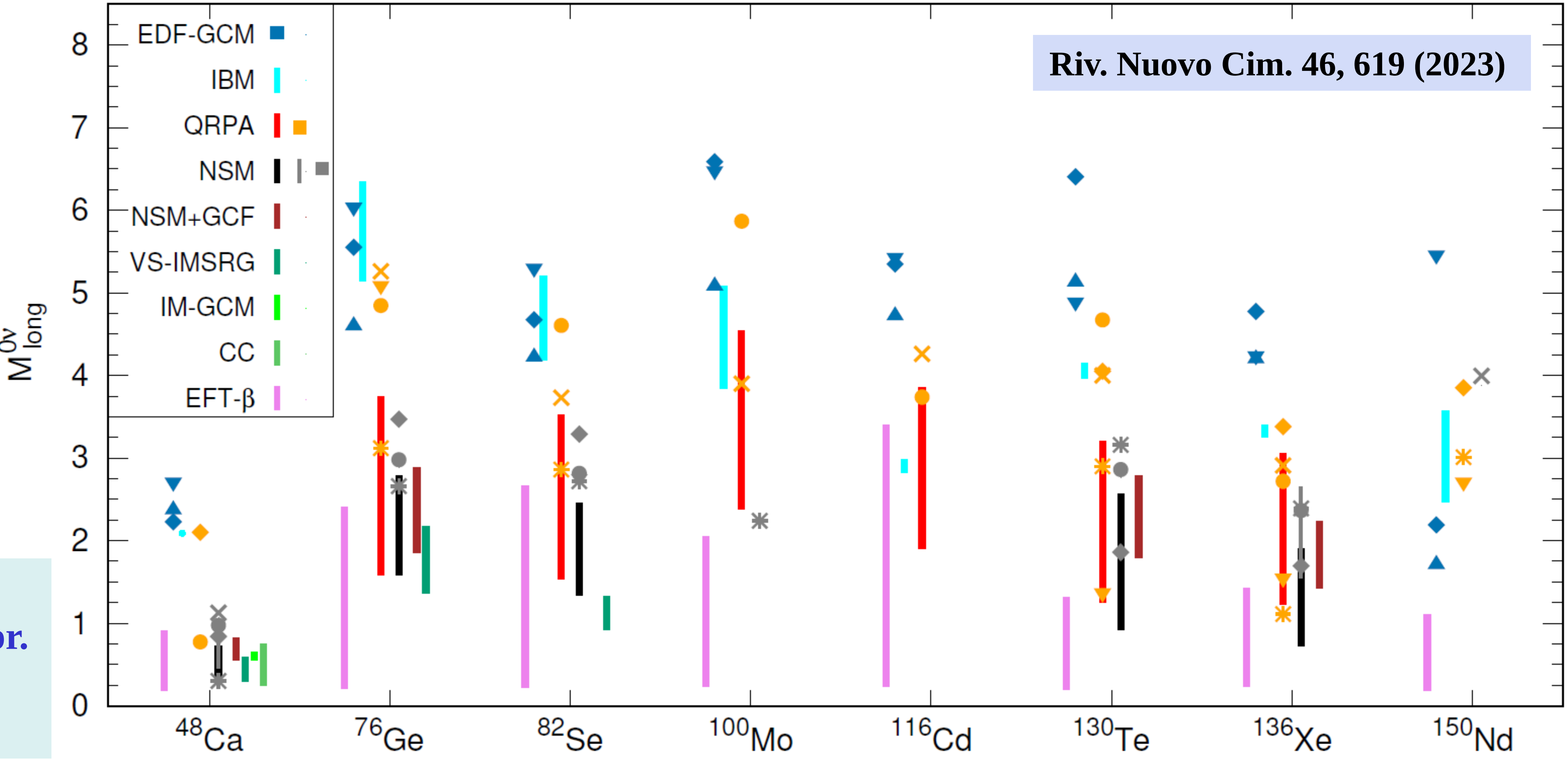
The nuclear w. f. of (A,Z), (A,Z+1)*, (A,Z+2) Many-body methods of choice:

The 0νββ nuclear transition operators (F, GT, and tensor type):

! Isospin, and spin-isospin symmetries ($M_{Fcl}\approx 0$, M_{GTcl} strongly suppressed):

All models missing essential physics

Impossible to assign rigorous uncertainties



Riv. Nuovo Cim. 46, 619 (2023)

Differencies:
Many-body appr.
Size of the m.s.
Residual int.

Chiral effective-field theory approach – contact term

PRL 126, 172002 (2021)

Assuming that the 0νββ process is mediated by a light-Majorana-neutrino exchange, a systematic analysis in chiral effective field theory shows that already at leading order a contact operator is required to ensure renormalizability of the amplitude for nn -> pp + ee process. Without the strong $^1S_0$ short range interaction (which appears universally in all nuclear potentials) there would be no need of contact term.

$$M^{0\nu} = -\frac{1}{g_A^2} M_F^{0\nu} + M_{GT}^{0\nu} + M_T^{0\nu} + 2\frac{m_\pi^2 \; \mathrm{g}_\nu^{\mathrm{NN}}}{g_A^2} M_{F,sd}^{0\nu}$$

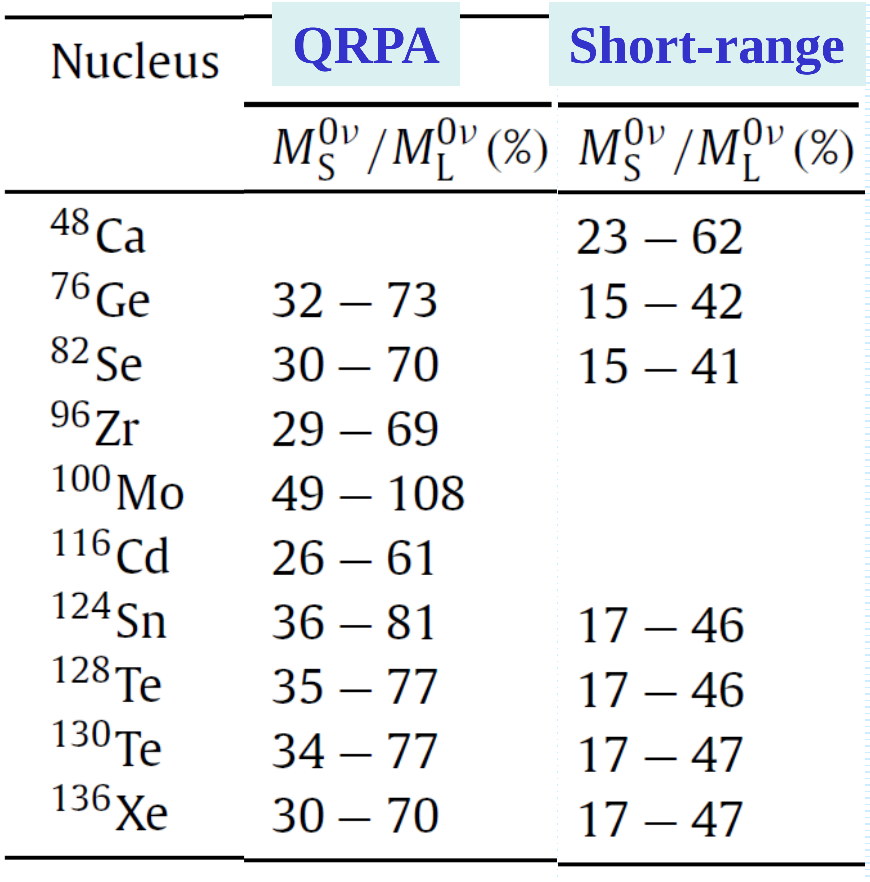
Long-range **Short-range**

Some questions:

A correspondence of the standard and the chiral field theory formalisms. Is the contact term involved in the standard mechanism (completeness …)?

What is the magnitude of the contact term NME? Can it be large? Justification with other phenomenology needed – pion and heavy-ion DCX, etc.

ic

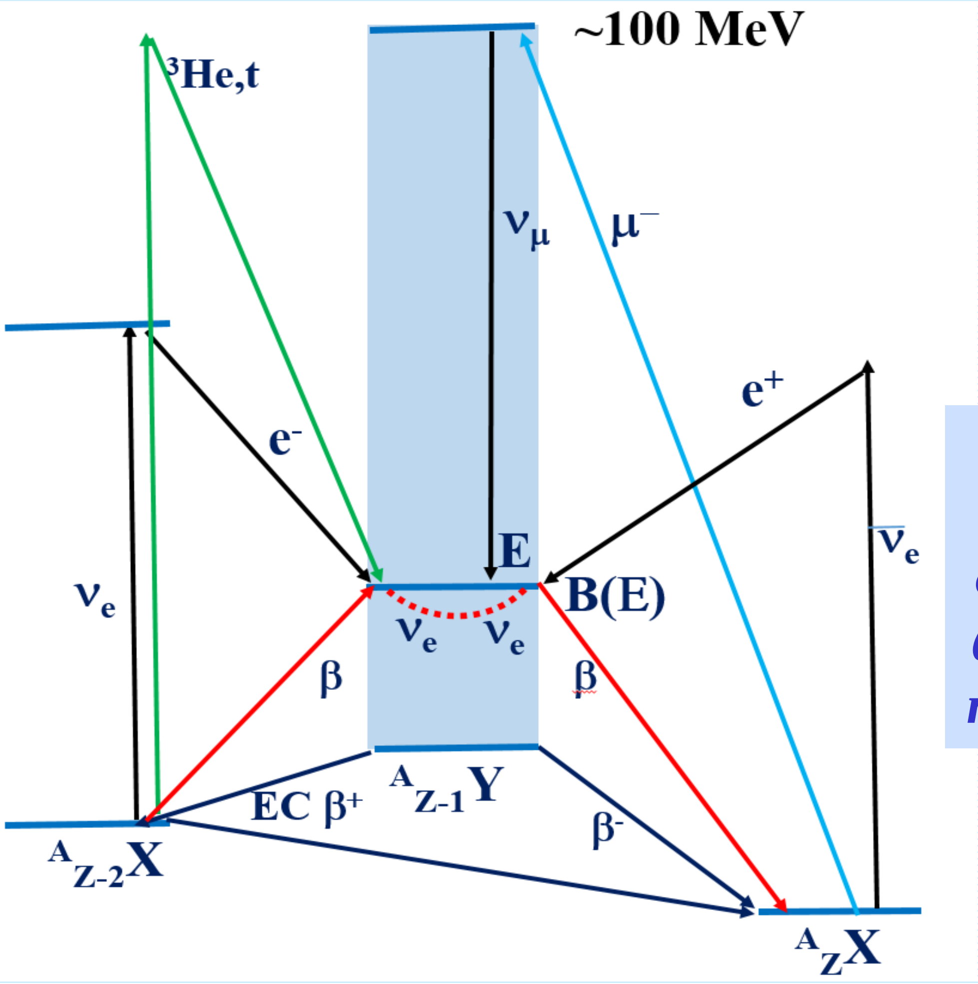


Nucleus	QRPA	Short-range
	$M_S^{0\nu}/M_L^{0\nu}$ (%)	$M_S^{0\nu}/M_L^{0\nu}$ (%)
^{48}Ca		23 – 62
^{76}Ge	32 – 73	15 – 42
^{82}Se	30 – 70	15 – 41
^{96}Zr	29 – 69	
^{100}Mo	49 – 108	
^{116}Cd	26 – 61	
^{124}Sn	36 – 81	17 – 46
^{128}Te	35 – 77	17 – 46
^{130}Te	34 – 77	17 – 47
^{136}Xe	30 – 70	17 – 47

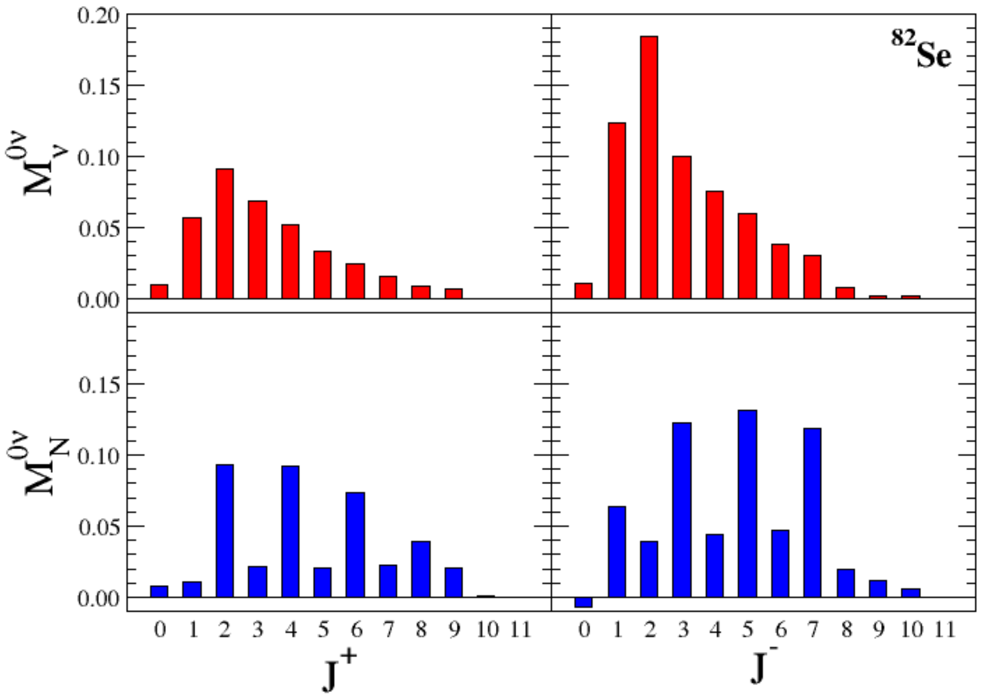
Supporting nuclear physics experiments

(Measurements still not conclusive for 0νββ NME)

- ✓β-decay, EC and 2νββ decay
- ✓μ-capture
- ✓(π^+, π^-), single charge exchange
- ✓ (^{3}He,t), (d,^{2}He), transfer reactions
- ✓γ-ray spectroscopy, γγ-decay
- ✓A promising experimental tool:
  Heavy-Ion Double Charge-Exchange



Multipole decomposition of light and heavy 0νββ-decay NMEs normalized to unity



Higher multiploles are populated mostly due large ν-momenta transfer

Fedor Simkovic

Improved description of the 0νββ–decay rate (a way to fix g_A eff)

Both 2νββ and 0νββ operators connect the same states.
Both change two neutrons into two protons.
Explaining 2νββ-decay is necessary but not sufficient

PRC 97, 034315 (2018).

$$M_{GT}^{K,L} \quad = \quad m_e \sum_n M_n \frac{E_n - (E_i + E_f)/2}{[E_n - (E_i + E_f)/2]^2 - \varepsilon^2_{K,L}}$$

$$\epsilon_K \quad = \quad (E_{e_2} + E_{\nu_2} - E_{e_1} - E_{\nu_1})/2$$
$$\epsilon_L \quad = \quad (E_{e_1} + E_{\nu_2} - E_{e_2} - E_{\nu_1})/2$$

Taylor expansion

$$\frac{\varepsilon_{K,L}}{E_n - (E_i + E_f)/2}$$

$$\epsilon_{K,L} \in (-\frac{Q}{2}, \frac{Q}{2})$$

$$M^{2\nu}_{GT-1} = \sum_n M_n \frac{1}{(E_n - (E_i + E_f)/2}$$

$$M^{2\nu}_{GT-3} = \sum_n M_n \frac{4\ m_e^3}{(E_n - (E_i + E_f)/2)^3}$$

We get

$$\left[T^{2\nu\beta\beta}_{1/2}\right]^{-1} \simeq \left(g_A^{\mathrm{eff}}\right)^4 \left|M^{2\nu}_{GT-3}\right|^2 \frac{1}{|\xi^{2\nu}_{13}|^2} \left(G^{2\nu}_0 + \xi^{2\nu}_{13} G^{2\nu}_2\right)$$

$$\xi^{2\nu}_{13} \quad = \quad \frac{M^{2\nu}_{GT-3}}{M^{2\nu}_{GT-1}}$$

$$\xi^{2\nu}_{15} \quad = \quad \frac{M^{2\nu}_{GT-5}}{M^{2\nu}_{GT-1}}$$

The g_A^{eff} can be deterimed with measured half-life and ratio of NMEs and calculated NME dominated by transitions through low lying states of the intermediate nucleus ***(ISM)***

The running sum of the 2νββ–decay NMEs (QRPA)

$$M^{2\nu}_{GT-1} = \sum_n M_n \frac{1}{(E_n - (E_i + E_f)/2}$$

$$M^{2\nu}_{GT-3} = \sum_n M_n \frac{4\ m_e^3}{(E_n - (E_i + E_f)/2)^3}$$

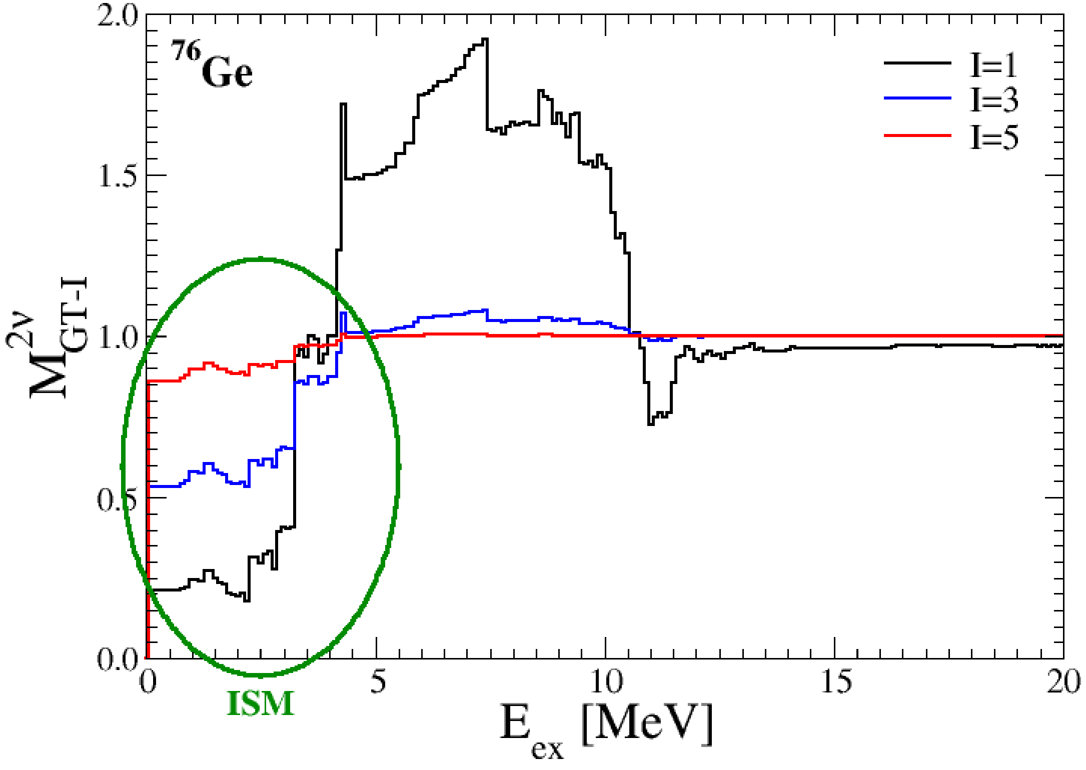
$$\xi^{2\nu}_{13} = \frac{M^{2\nu}_{GT-3}}{M^{2\nu}_{GT-1}}$$

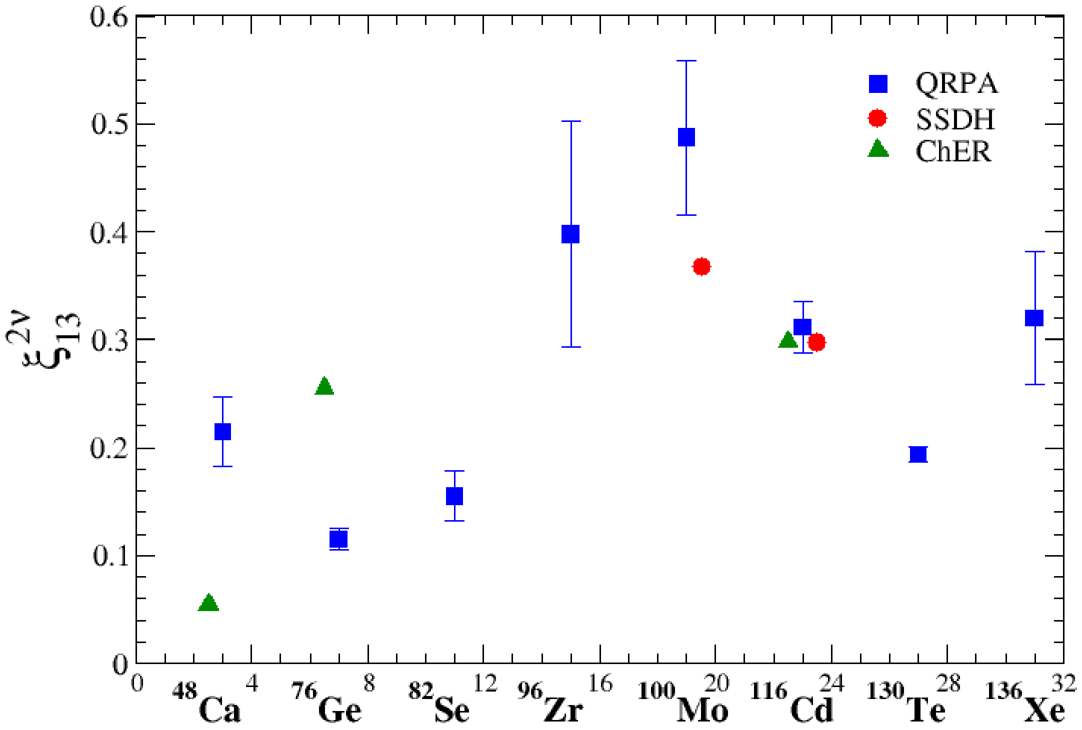
$$\xi^{2\nu}_{15} = \frac{M^{2\nu}_{GT-5}}{M^{2\nu}_{GT-1}}$$

ξ_{13} tell us about importance of higher lying states of int. nucl.

HSD: ξ_{13}=0

ξ_{13}≈1 (large)
Possible due to a large cancellation of contributions through lower and higher lying states of (A,Z+1).





ξ_{13} can be determined phenomenologically from the shape of energy distributions of emitted electrons

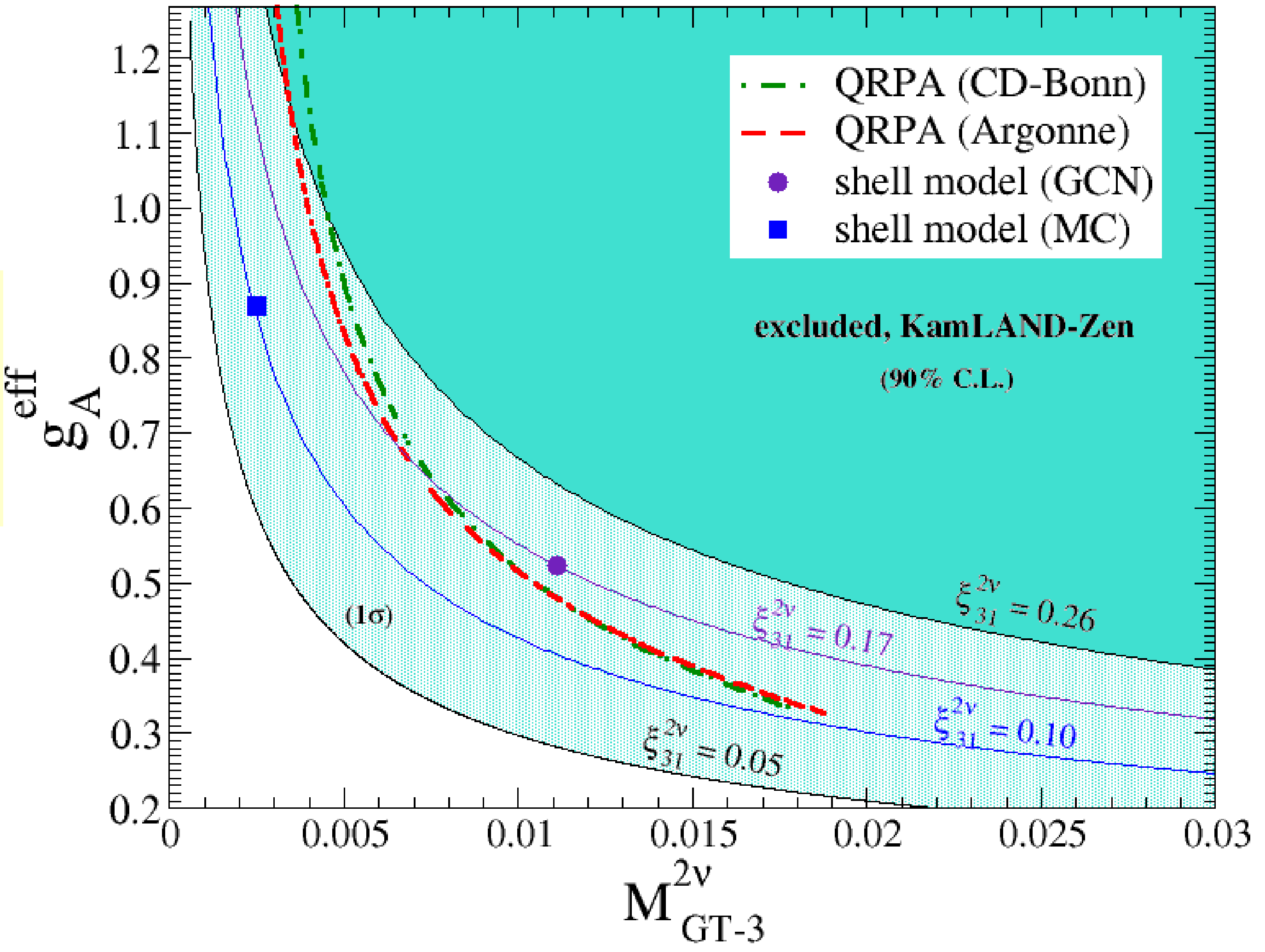
F.Š., Šmotlák, Semenov, J. Phys. G, 27, 2233, 2001

KamLAND-Zen Exp. : $\xi_{13}<0.26$ (^{136} Xe)

ξ_{13} can be determined phenomenologically from the shape of energy distributions of emitted electrons

The g_A^{eff} can be deterimed with measured half-life, ratio of NMEs ξ_{31} and calculated NME, dominated by transitions through low lying states of the intermediate nucleus.

$$\left(g_A^{\rm eff}\right)^2 = \frac{1}{\left|M^{2\nu}_{GT-3}\right|}\frac{\left|\xi^{2\nu}_{13}\right|}{\sqrt{T^{2\nu-exp}_{1/2}\left(G^{2\nu}_0+\xi^{2\nu}_{13}G^{2\nu}_2\right)}}$$

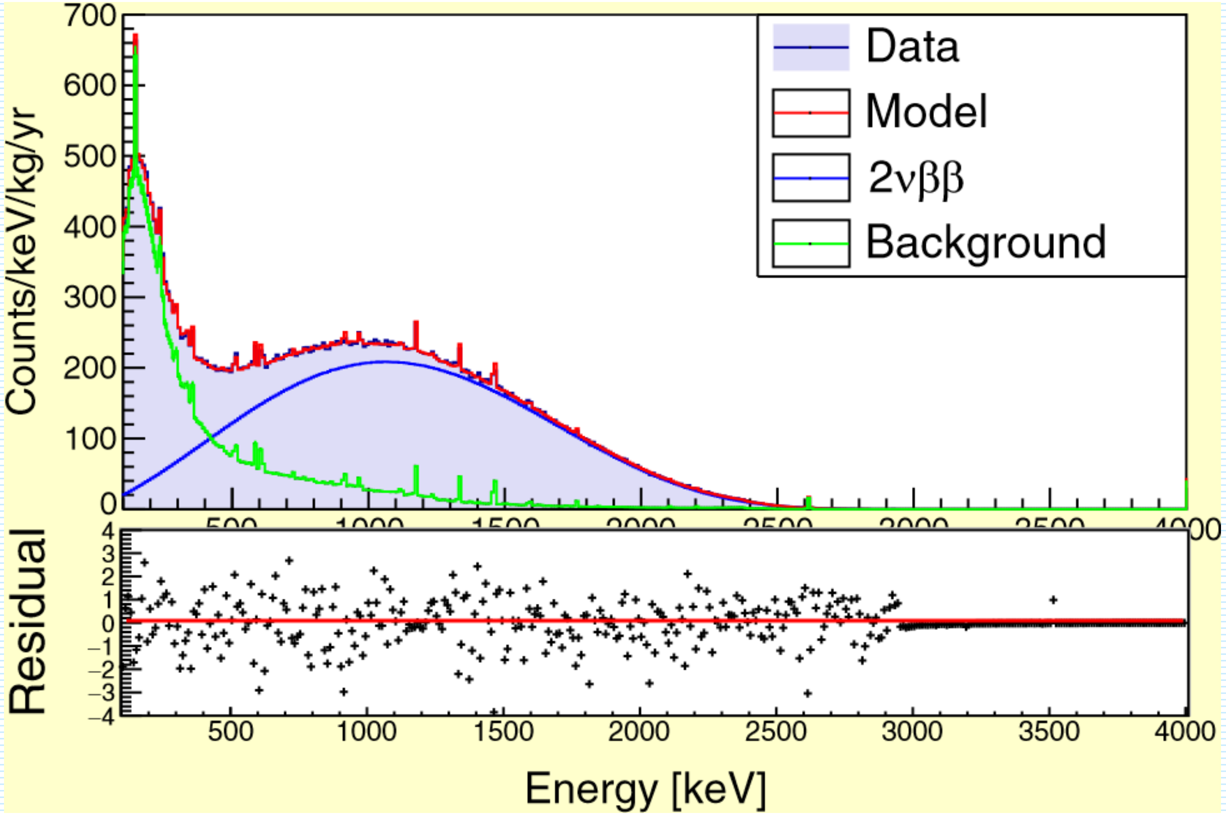


M_{GT-3} have to be calculated by nuclear theory - ISM

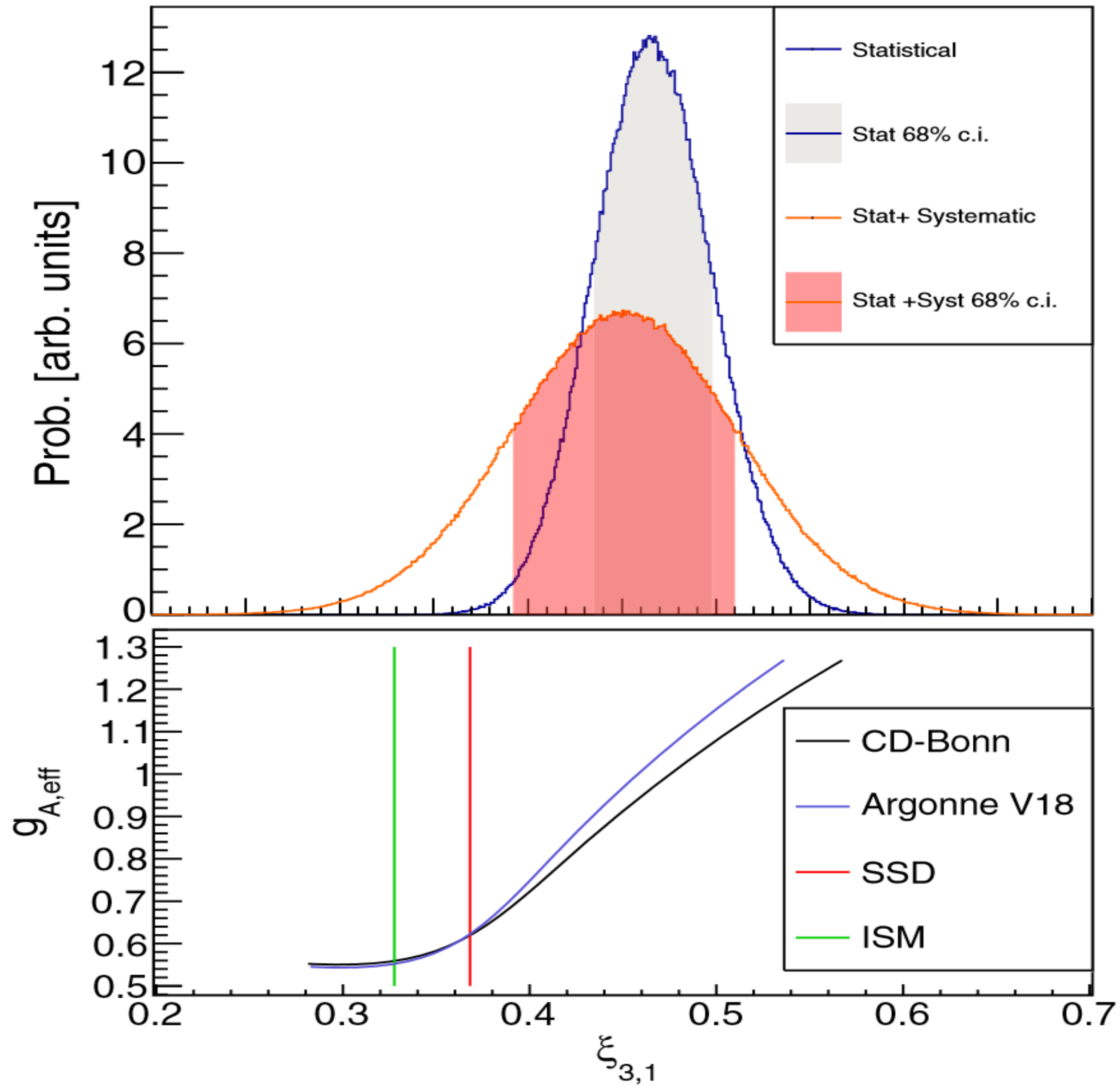
KamLAND-Zen Coll. (+J. Menendez, F.Š.), Phys.Rev.Lett. 122, 192501 (2019)

CUPID-Mo Exp. : ξ_{13}=0.45±0.03 (stat) ±0.05 (syst) (^{100} Mo)

CUPID-Mo Coll. (+F.Š.), Phys.Rev.Lett. 131, 162501 (2023)



$T_{1/2}$ = [7.07±0.02(stat)±0.11(syst)] x 10^{18} yr

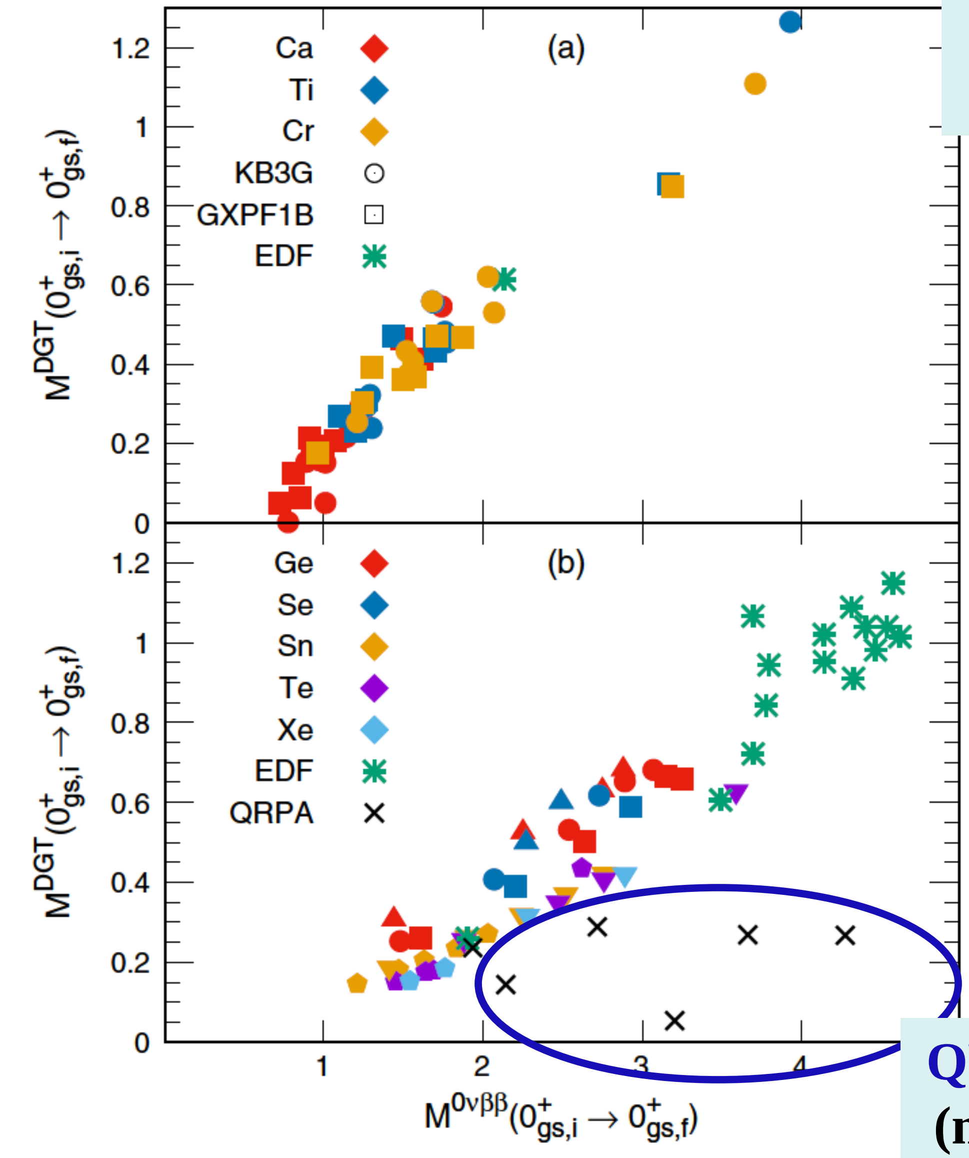


ξ_{51}/ξ_{31} = 0.364-0.368 (QRPA), 0.367 (SSD), 0.349 (ISM)

g_A^{eff} (pnQRPA) = 1.0 ±0.1(stat)±0.2(syst)

g_A^{eff} (ISM) = 1.11 ±0.03(stat)±0.05(syst)

ISM, EDF: $M^{0\nu} \propto M^{2\nu}_{GT\text{-}cl}$



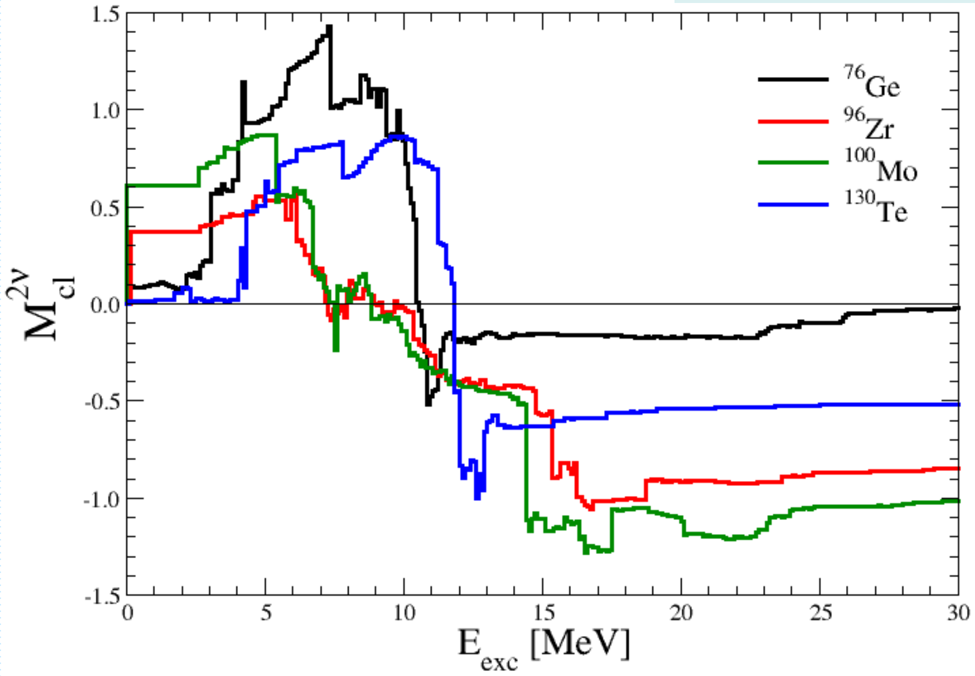
PRL 120, 142502 (2018)

Assumption exploited by the EFT-β approach for the calculation of 0νββ NMEs

M^{DGT} – only 1^+
$M^{0\nu}$ - contribution from many J^{π} (!)

ISM versus QRPA

PRC 83, 015502 (2011)



QRPA – no proportionality between $M^{0\nu}$ and $M^{2\nu}_{GT\text{-}cl}$ (modest quenching of g_A)

Favored by ξ_{13} (^{100}Mo) =0.45±0.03 > ξ^{SSD}_{13}

Measurement of GT strength via μ-capture

$$\mu^-_b + (A,Z) \rightarrow (A,Z-1) + \nu_\mu$$

Contradicting results:

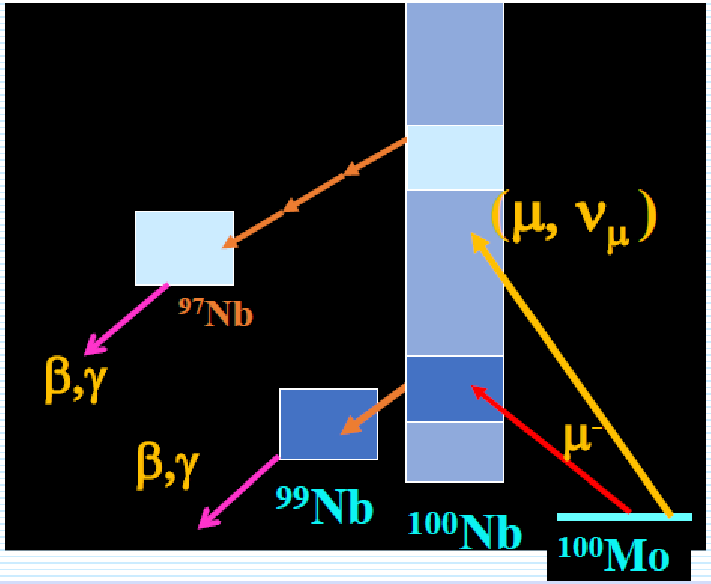
- **Strong quenching ($g_A \approx 0.6$)**

PRC 100, 014619 (2019)

- **Weak quenching ($g_A \approx 1.1$)**

PRC 74, 024326 (2006)
PRC 79, 054323 (2009)

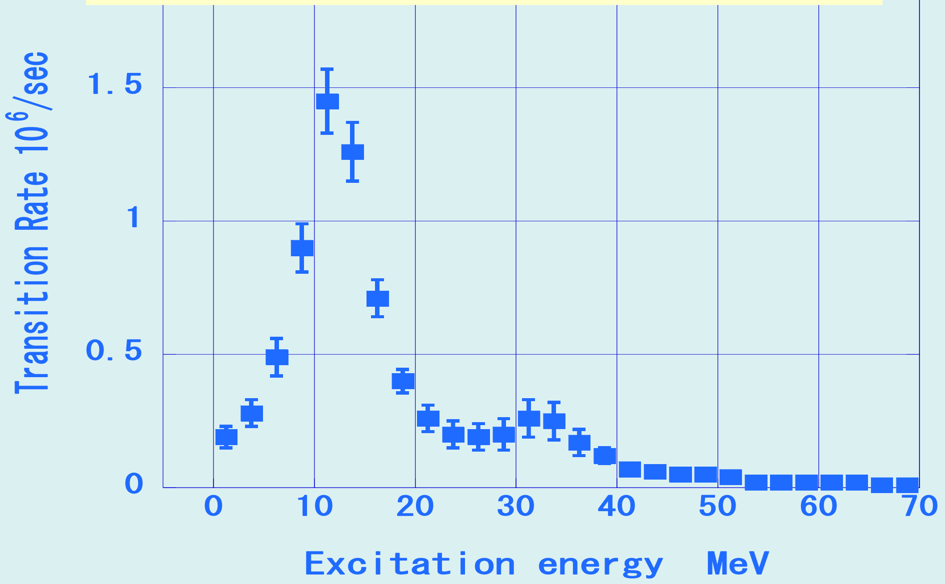
J-PARC 3-50 GeV p, ν, μ



⇒ Small basis nuclear structure calculations (NSM, IBM) are disfavored. ⇒

I. Hashim H. Ejiri , MXG16, PR C 97 2018

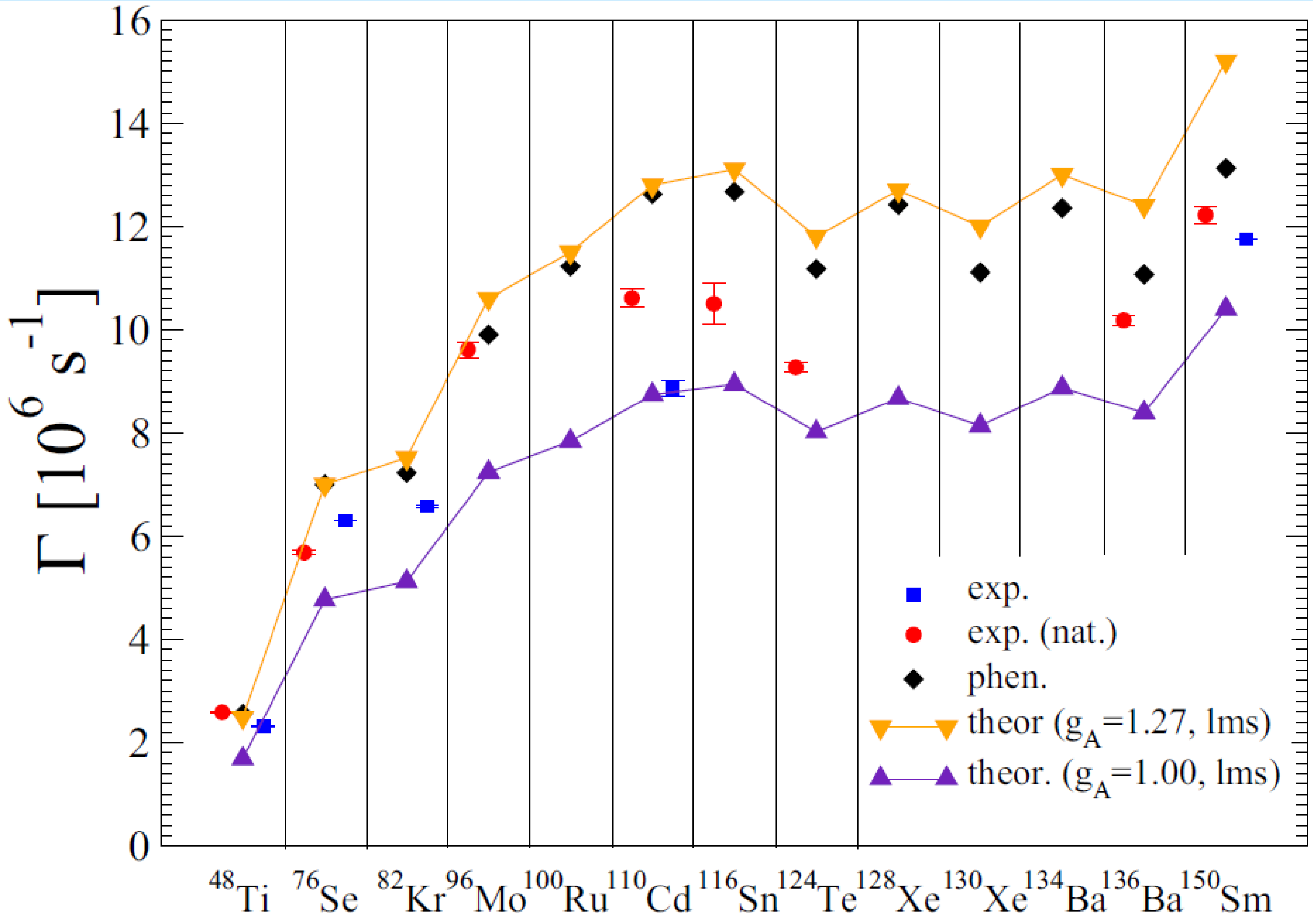
Momentum transfer q~80 MeV



Fedor S

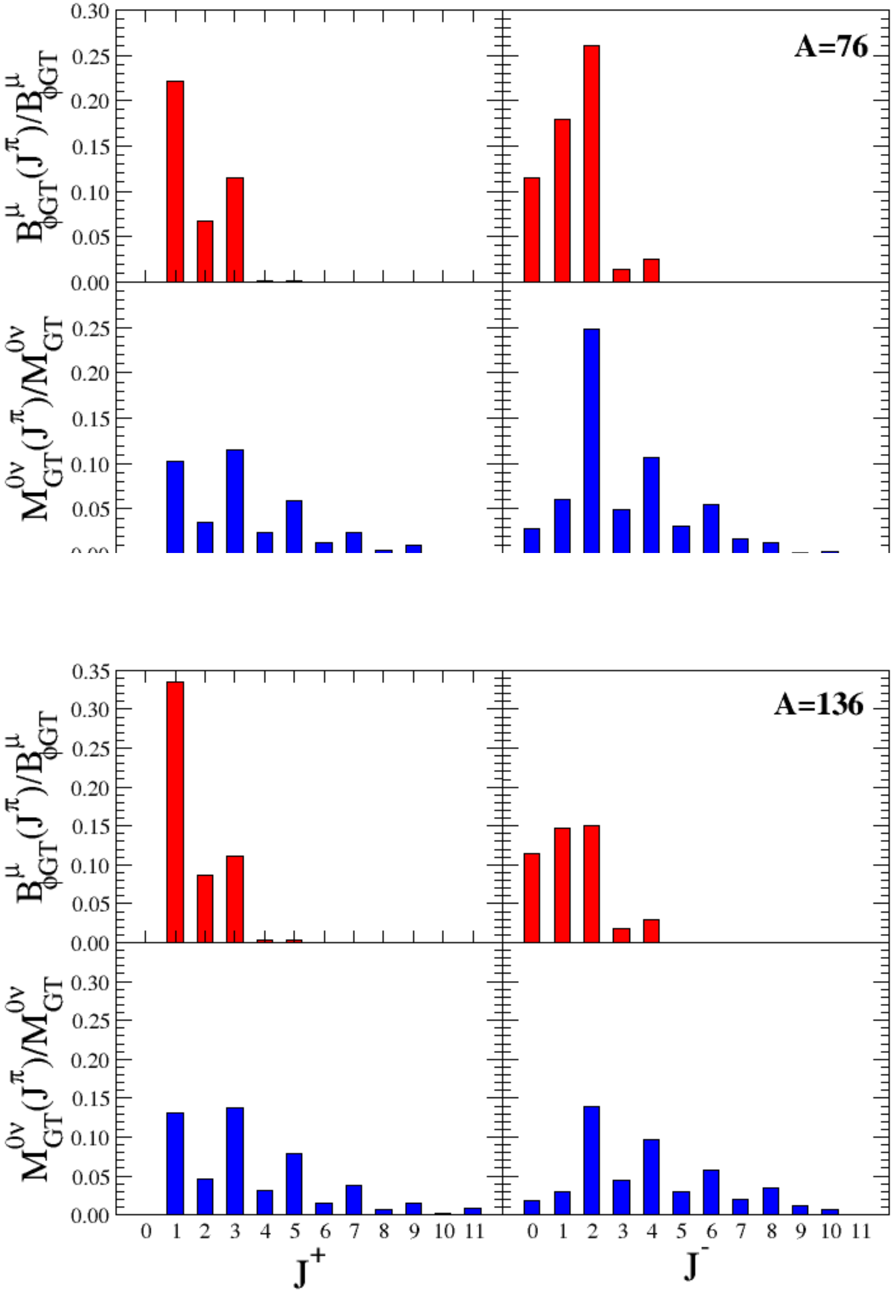
Muon capture rates evaluated within QRPA

In agreement with soft quenching ($g_A \approx 1.1$)

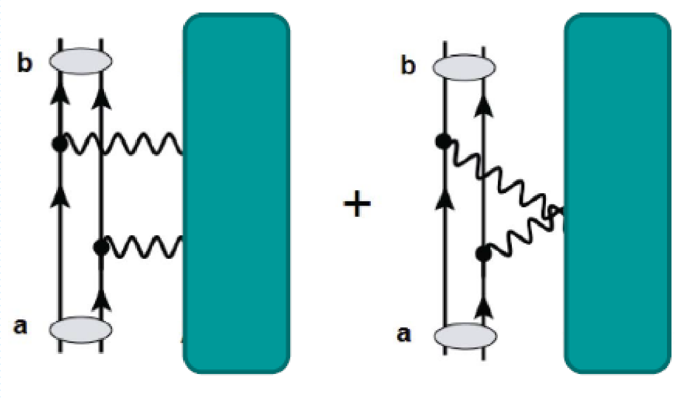


PRC 102, 034301 (2020).

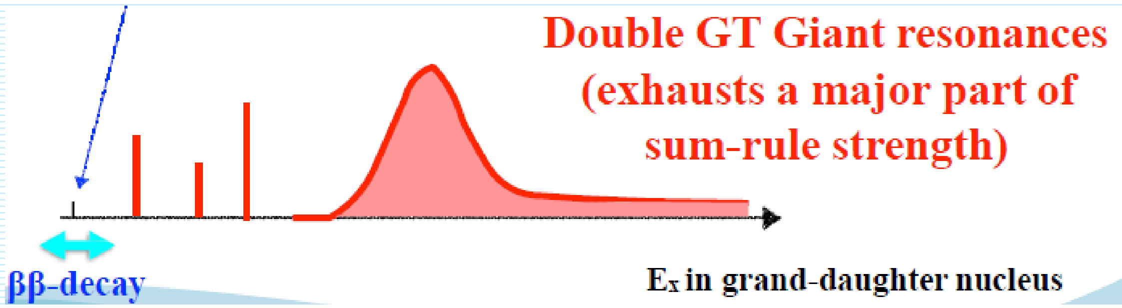
Experiment Monument at PSI will study contributions from all multipoles



Multipole decomposition of B^{μ}_{GT} and $M^{0\nu}_{GT}$

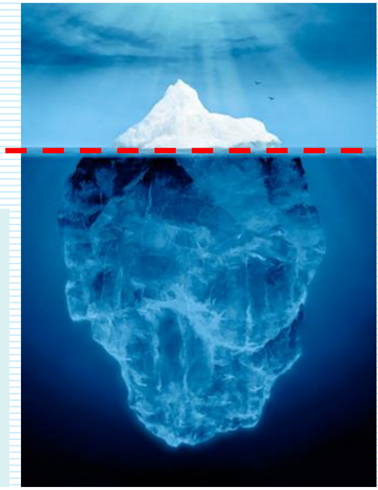


Heavy-ion DCE as surrogate processes of ββ-decay



- ✓ Induced by strong interaction
- ✓ Sequential nucleon transfer mechanism 4th order: Kinematical matching
- ✓ Meson exchange mechanism 1st or 2nd order
- ✓ Possibility to go in both directions
- ✓ Low cross section

Tiny amount of DGT strenght for low lying states

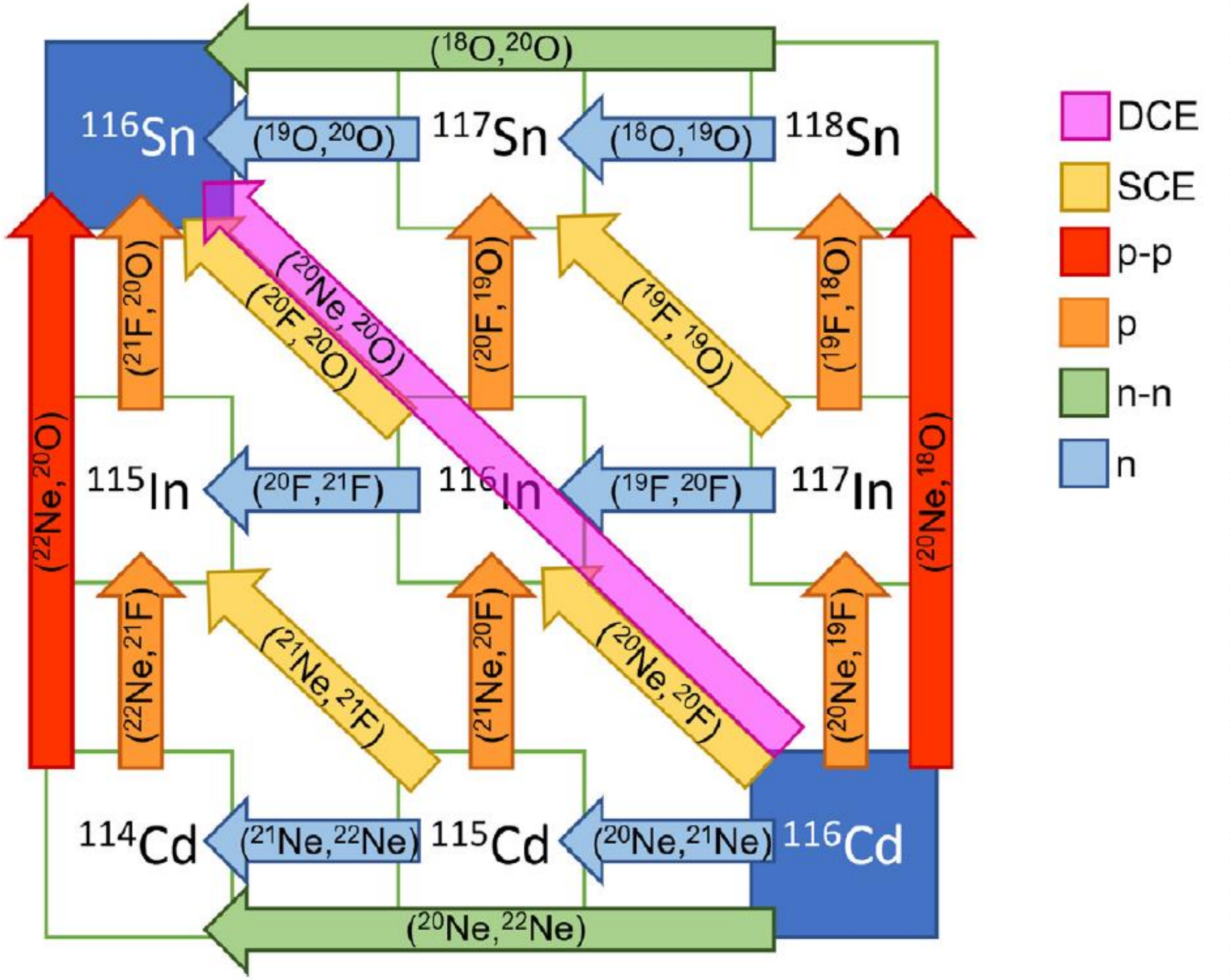


Sum rule almost exhausted by DGT Giant Mode, still not observed

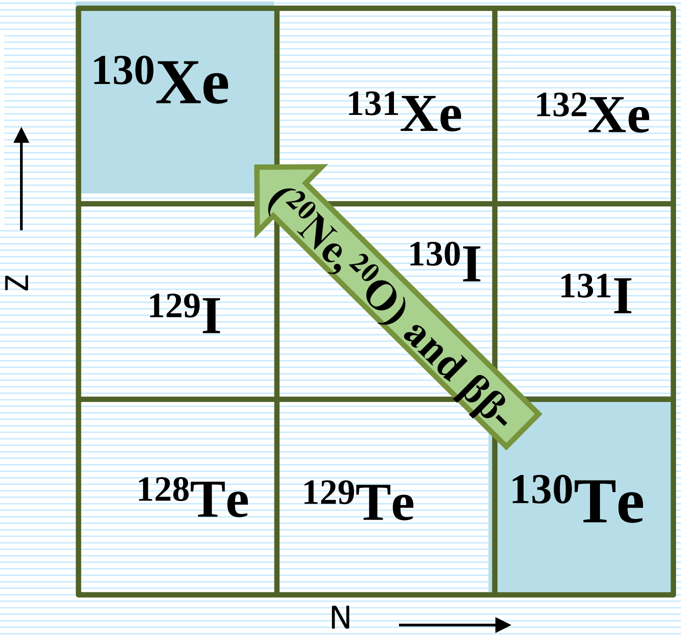
RIKEN

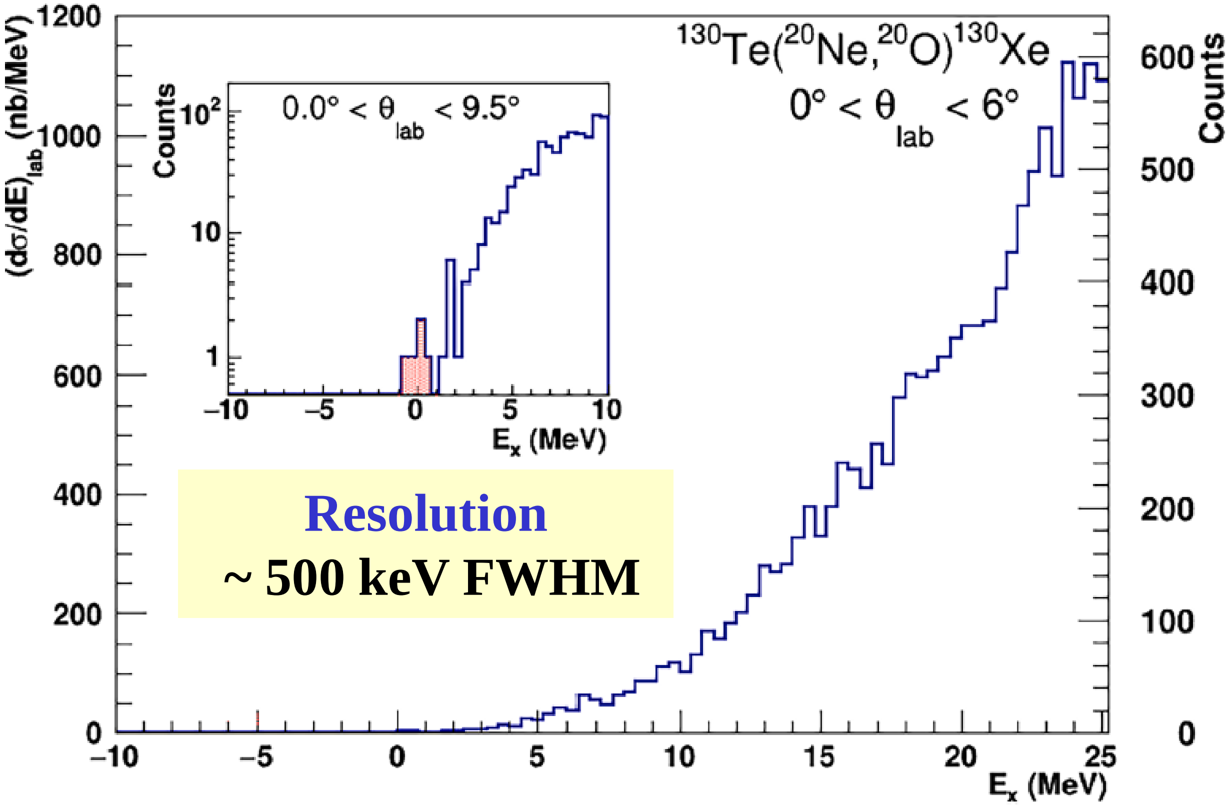
RCNP

Future: INFN-LN



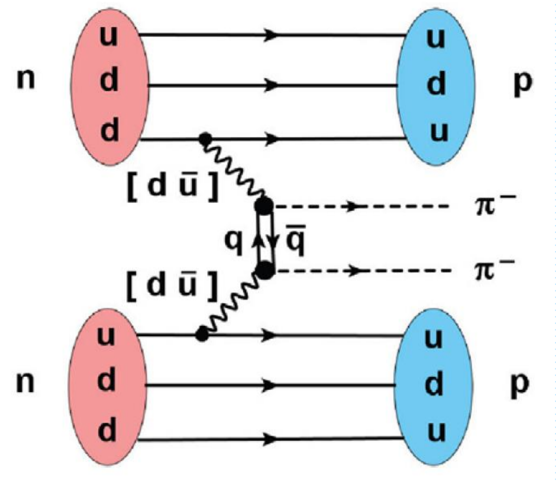
The ^{130}Te(^{20}Ne,^{20}O)^{130}Xe DCE reaction

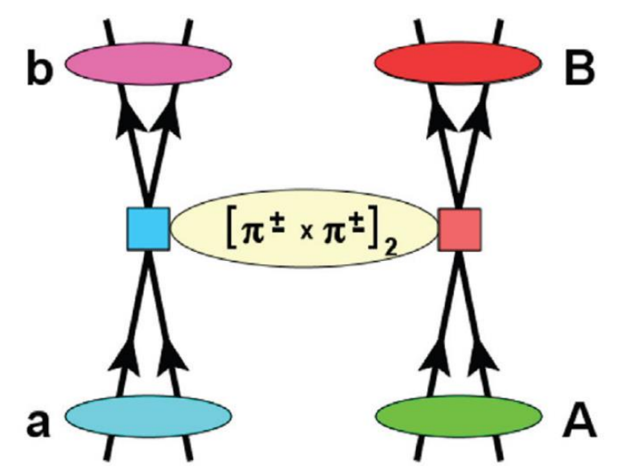




Resolution
~ 500 keV FWHM

- g.s. → g.s. **transition can be isolated**
- **Absolute cross section measured**





State (MeV)	Counts	Absolute coss section (nb)	Cross section 95% limit (nb)
g.s. (0^+) + 2^+ (536 keV)	5	13	[3--18]

Analysis of cross-section sensitivity < 0.1 nb in the Region Of Interest

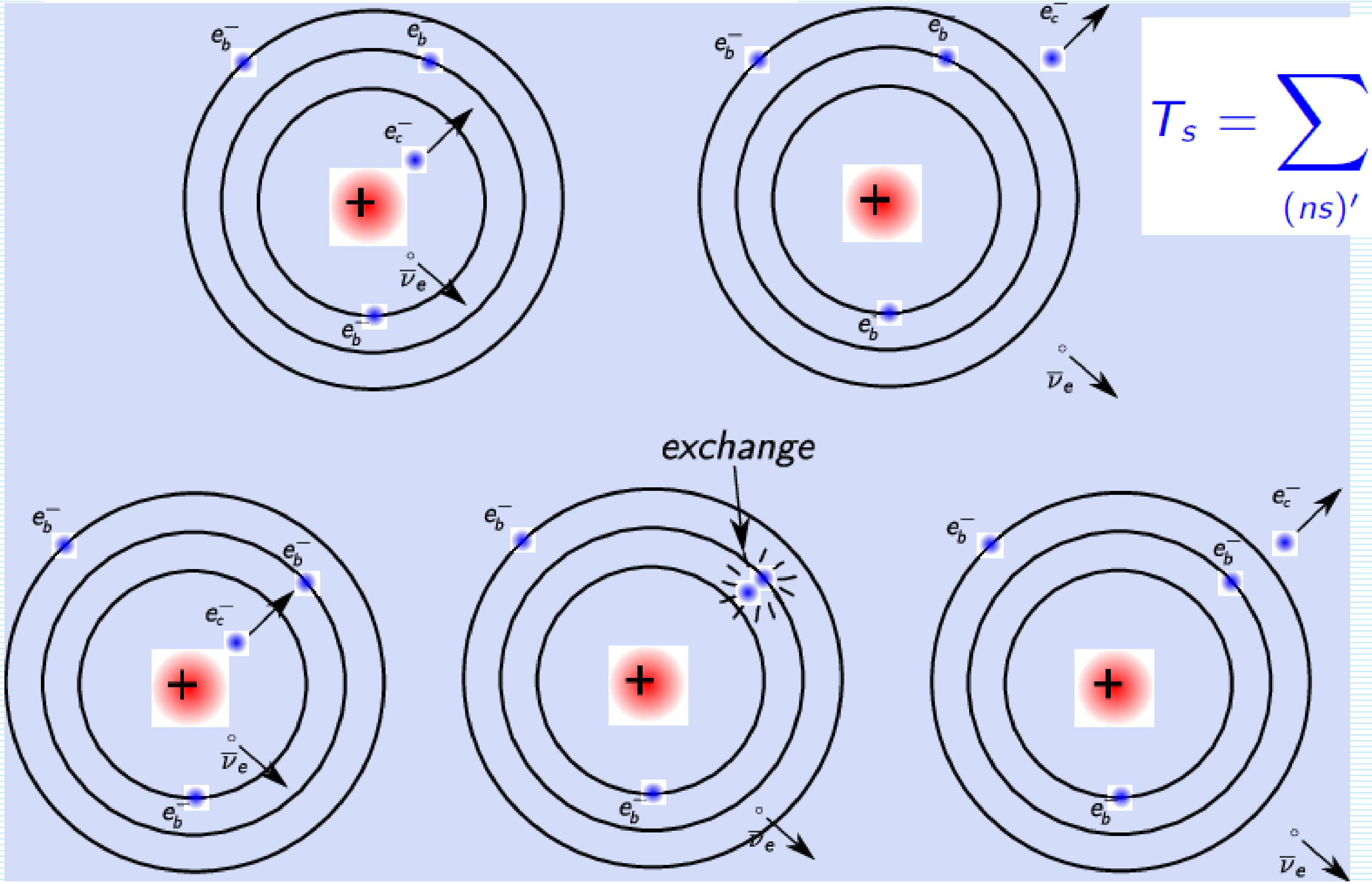
$0\nu\beta\beta$ is an atomic physics problem

Atomic effects in β-decay
(electron exchange effect)

$$\frac{d\Gamma}{dE_e} \Rightarrow \frac{d\Gamma}{dE_e} \times \left[1 + \eta^T(E_e)\right]$$

$$\eta^T(E_e) = f_s(2T_s + T_s^2) + (1 - f_s)(2T_{\bar{p}} + T_{\bar{p}}^2) = \eta_s(E_e) + \eta_{\bar{p}}(E_e)$$

Overlap of (A, Z) bound and (A, Z+1) continuum e-states



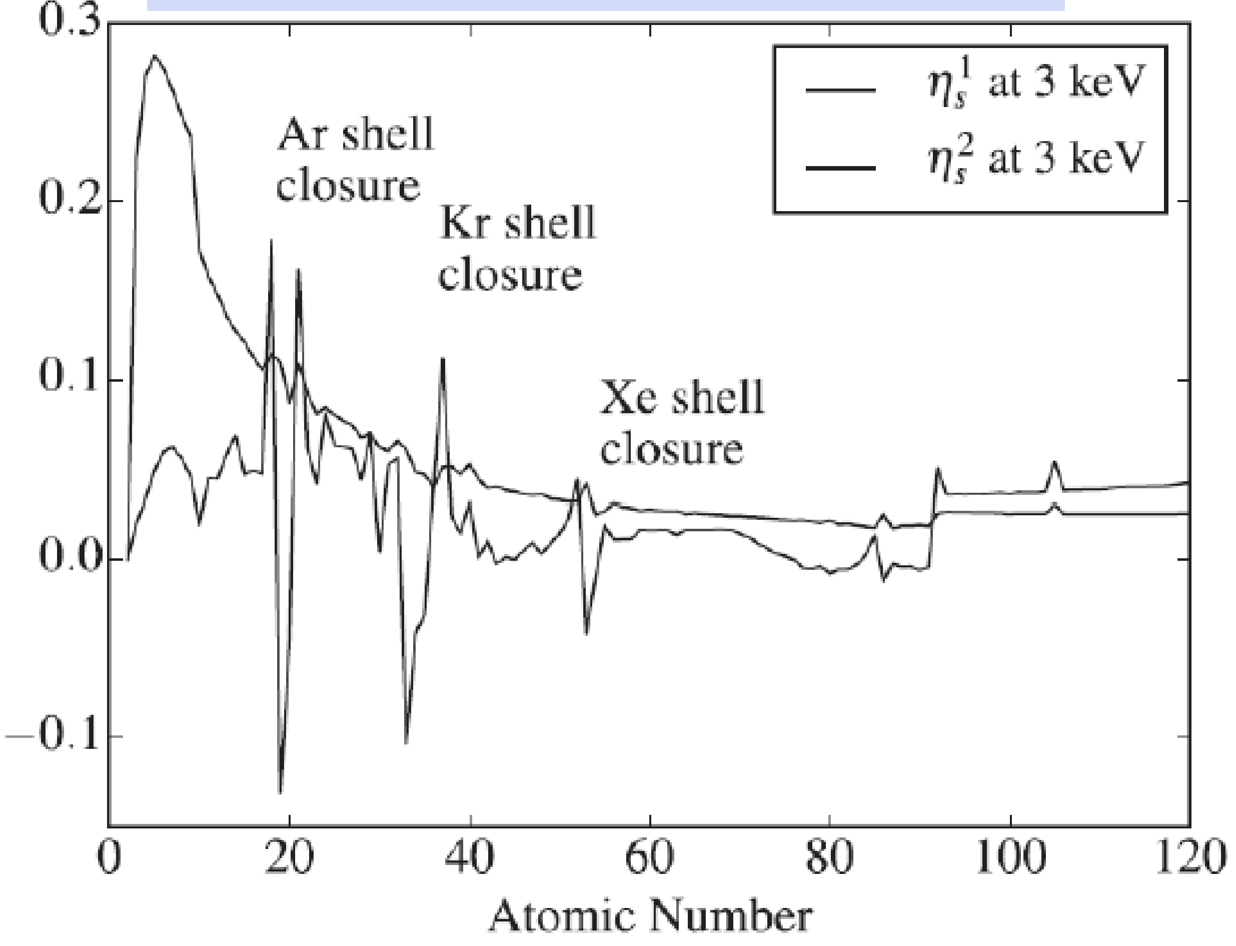
$$T_s = \sum_{(ns)'} T_{ns} = -\sum_{(ns)'} \frac{\left\langle \psi'_{E_e s} \middle| \psi_{ns} \right\rangle}{\langle \psi'_{ns} | \psi_{ns} \rangle} \frac{g'_{n,-1}(R)}{g'_{-1}(E_e, R)}$$

$$f_s = \frac{g'^2_{-1}(E_e, R)}{g'^2_{-1}(E_e, R) + f'^2_{+1}(E_e, R)}$$

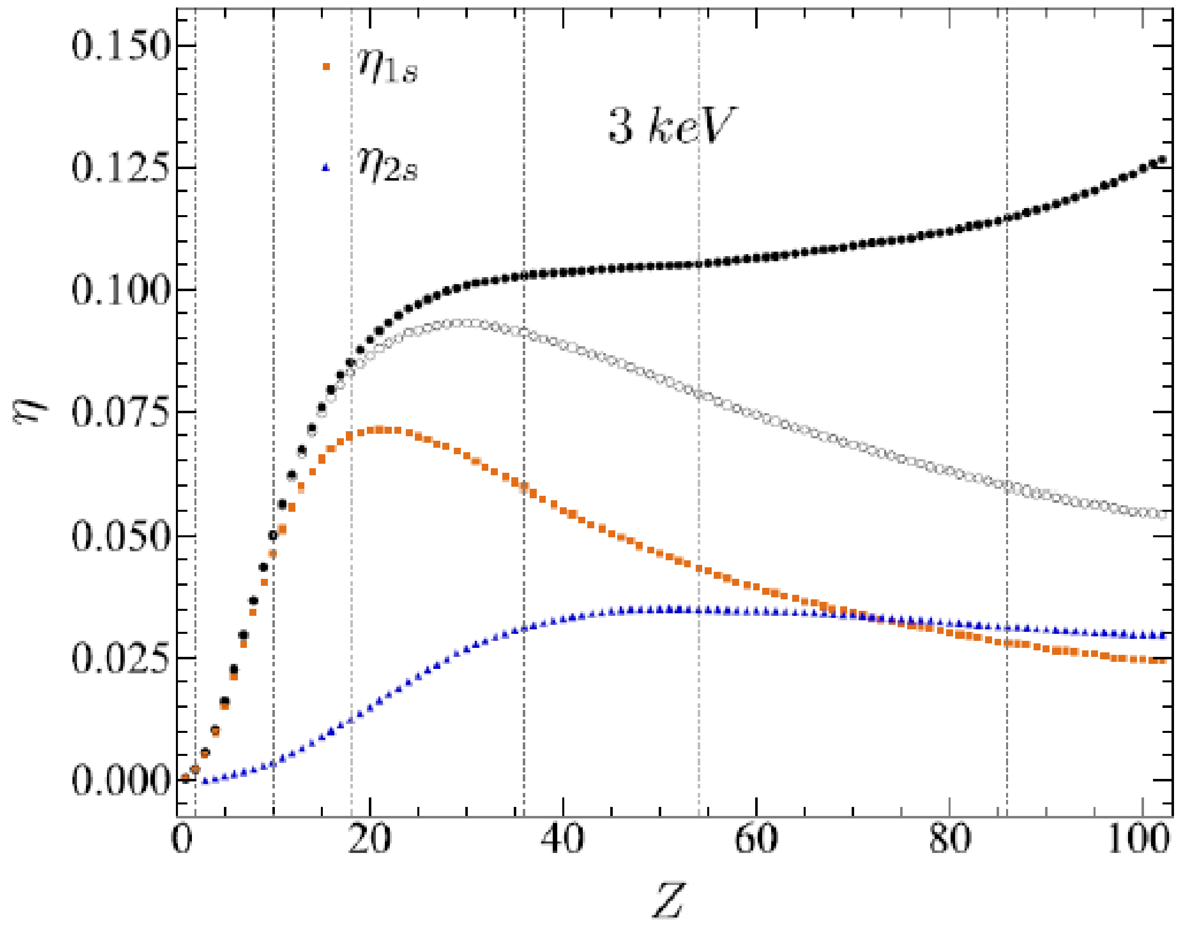
Transition	L	|ΔJ|	Δπ
allowed	0	0,1	0
first-forbidden	1	0,1,2	1
second-forbidden	2	1,2,3	0
third-forbidden	3	2,3,4	1
fourth-forbidden	4	3,4,5	0

Orthogonalization of bound and continuum states

Old results: without orthogonalization



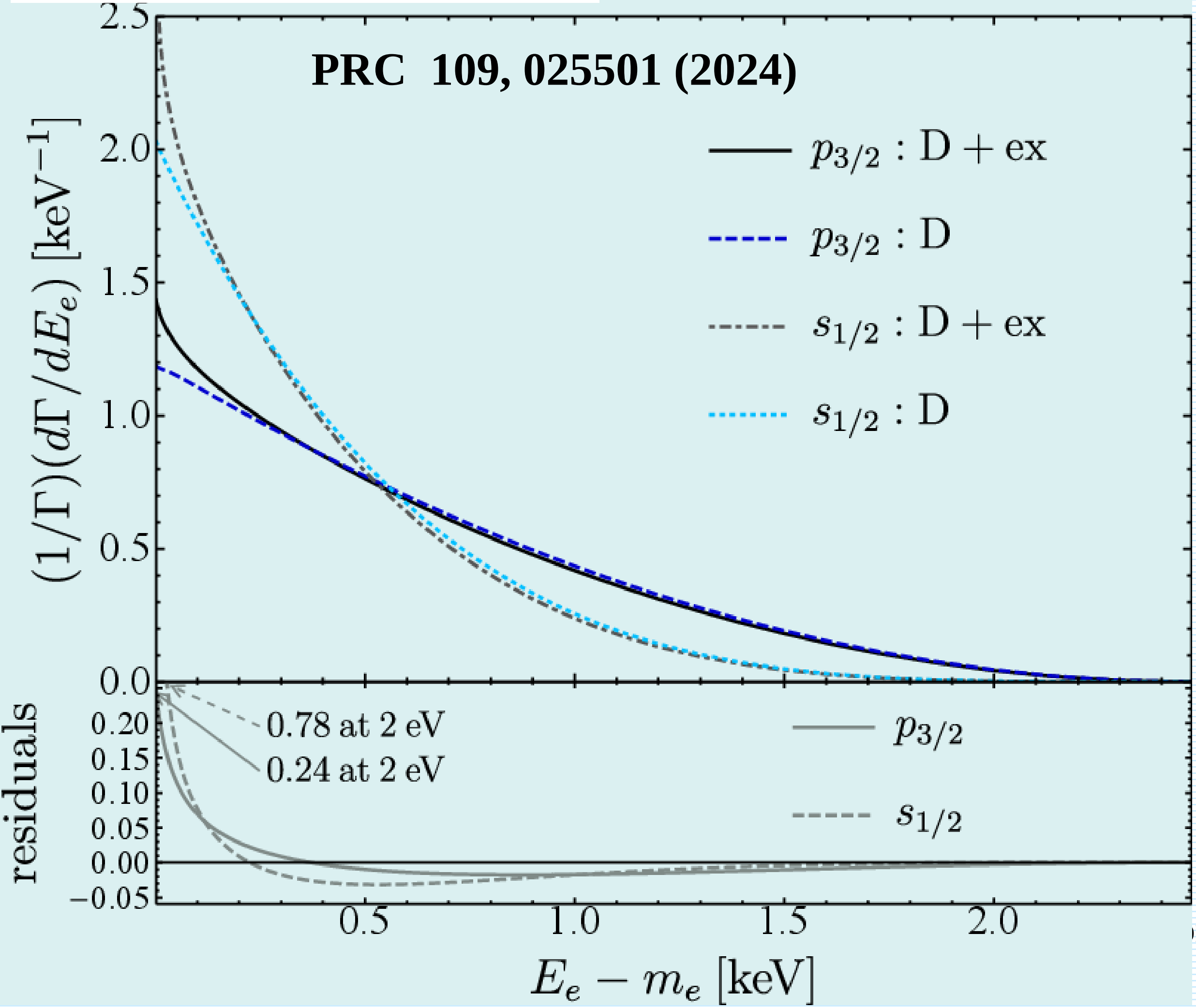
with orthogonalization



PRC 107, 025501 (2023)

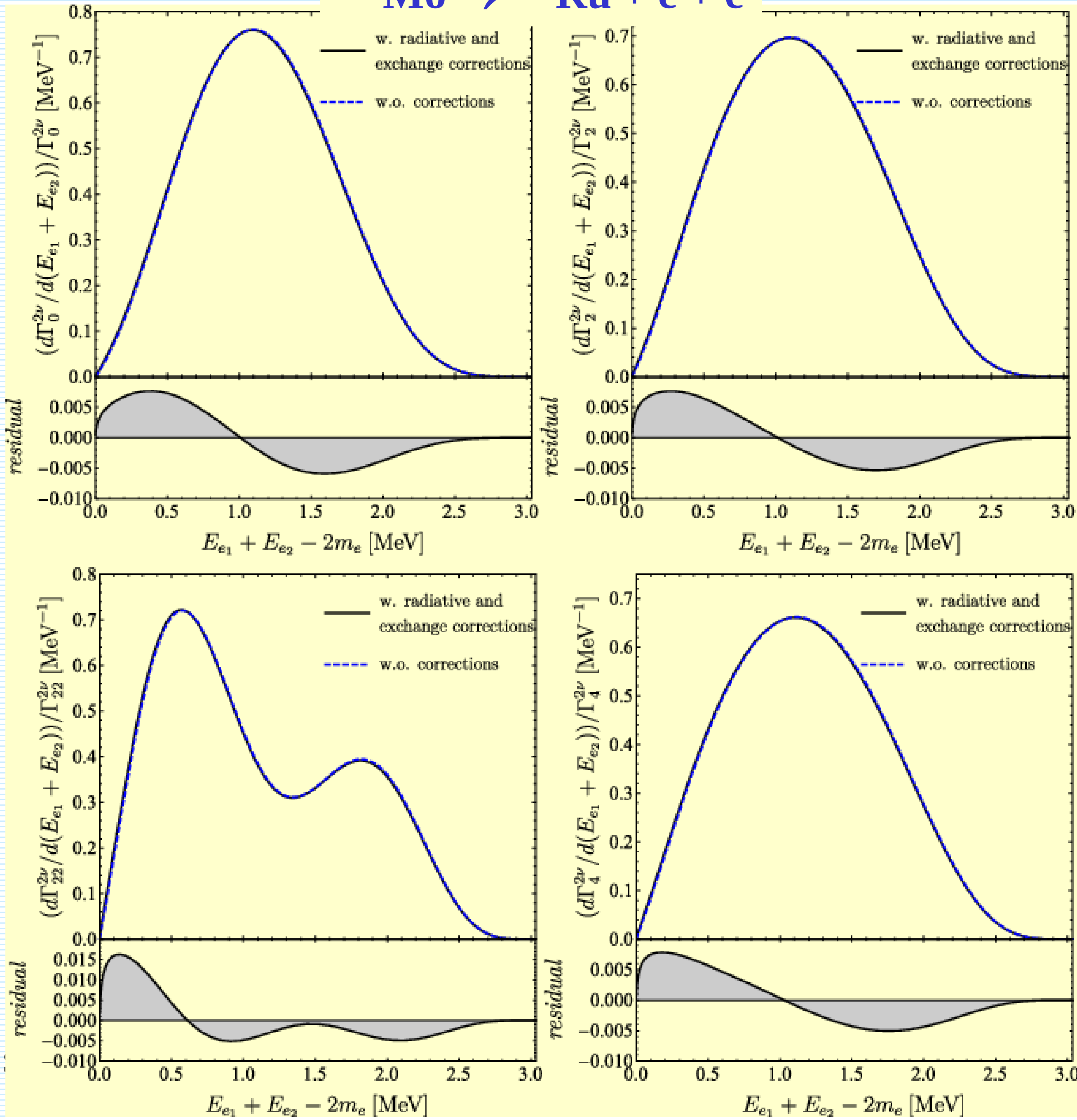
Electron exchange effect and radiative corrections

$^{187}_{85}\mathrm{Re} \rightarrow {}^{187}_{86}\mathrm{Os} + e^- + \bar{\nu}_e$



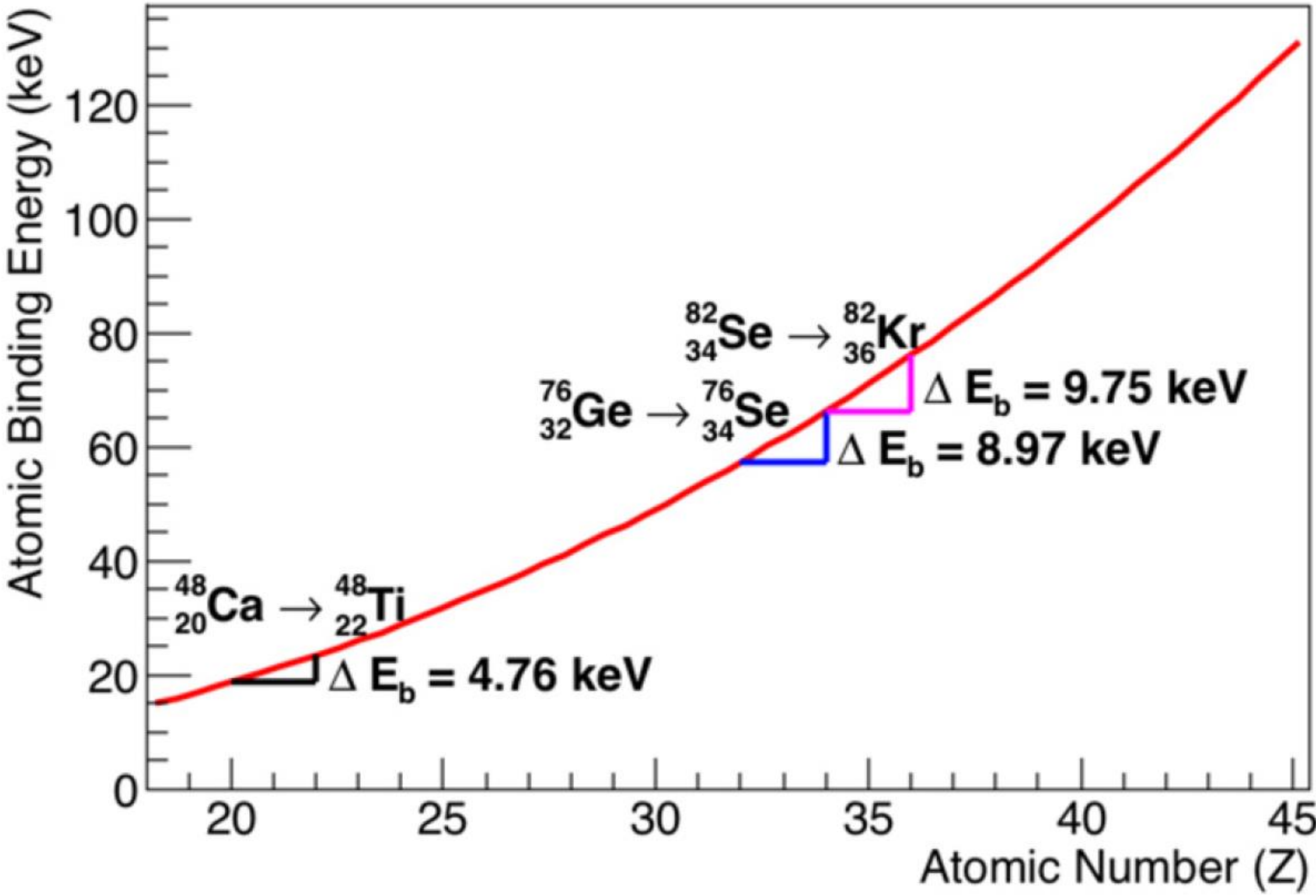
PRC 109, 025501 (2024)

$^{100}\mathrm{Mo} \rightarrow {}^{100}\mathrm{Ru} + e + e$



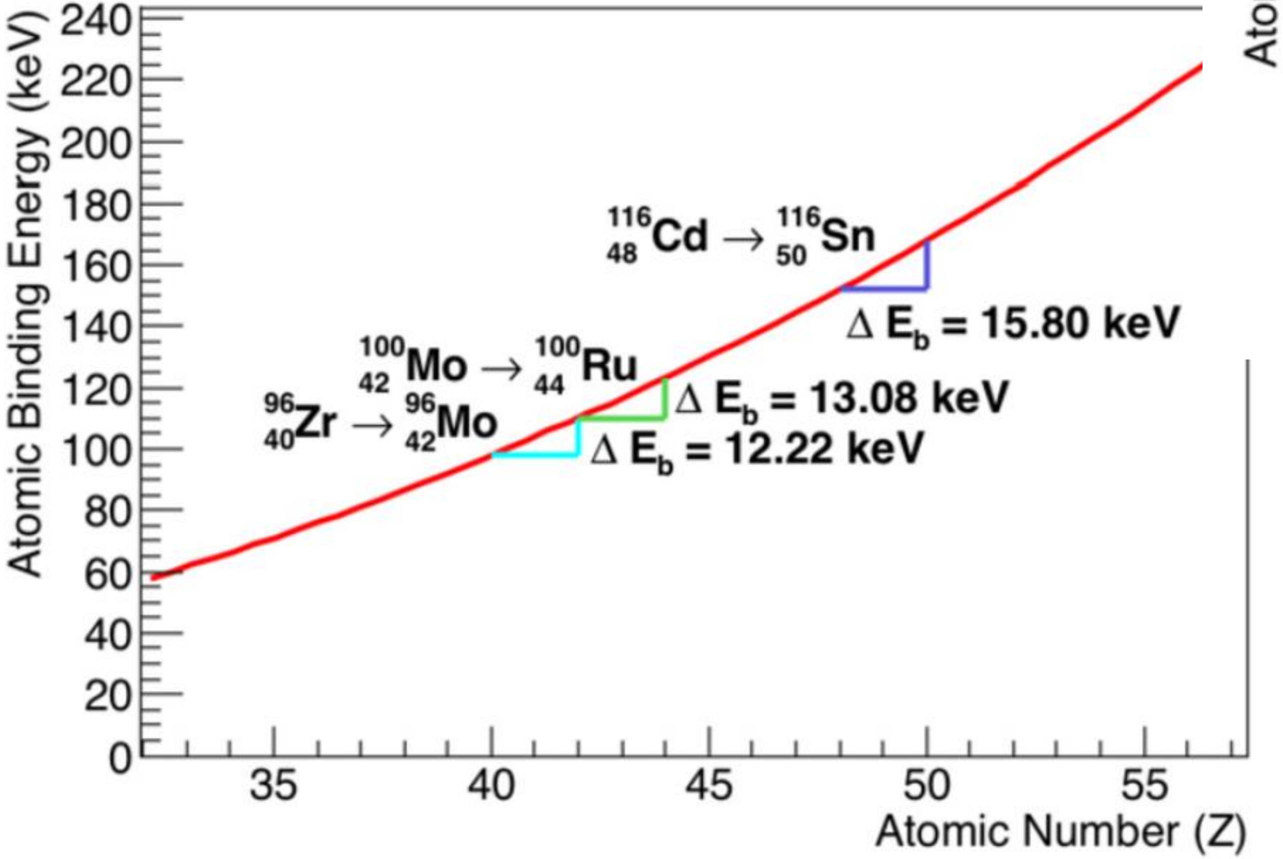
The relaxation time of the atomic orbits can be very long (who knows?).

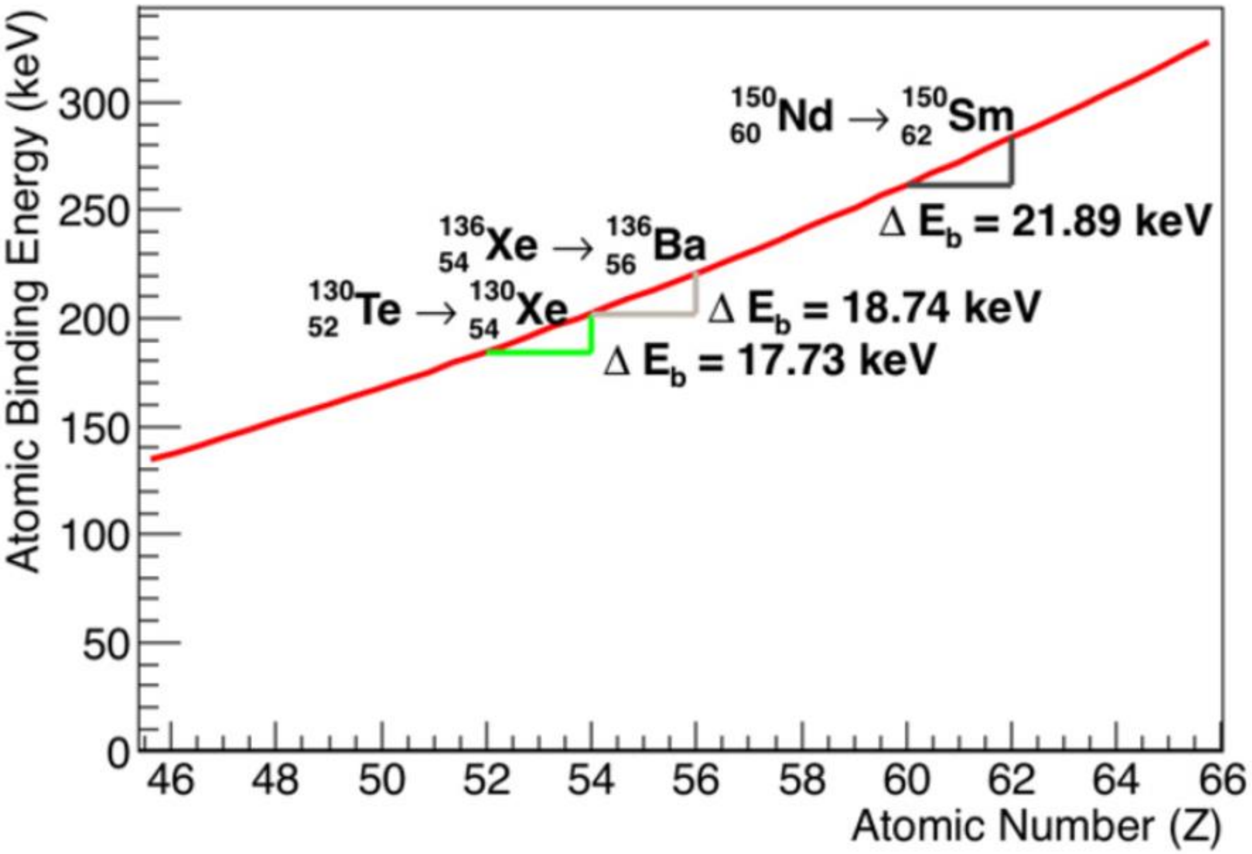
0νββ decay happens quickly in picoseconds, causing the atomic structure to be unable to respond to the nuclear charge change. The daughter atom becomes a double-ionized ion, surrounded by atomic electrons from the mother atom. It is reasonable to assume that the release of the atomic binding energy after **the 0νββ decay** may not come within the detection time window for the energy deposition from the two ejected beta particles.



NPA 1032, 122623 (2023)

Electron binding energy as a function of atomic number **Z**





Overlap in electron shells in double beta decay

(assuming no problem with a relaxation of atomic orbits)

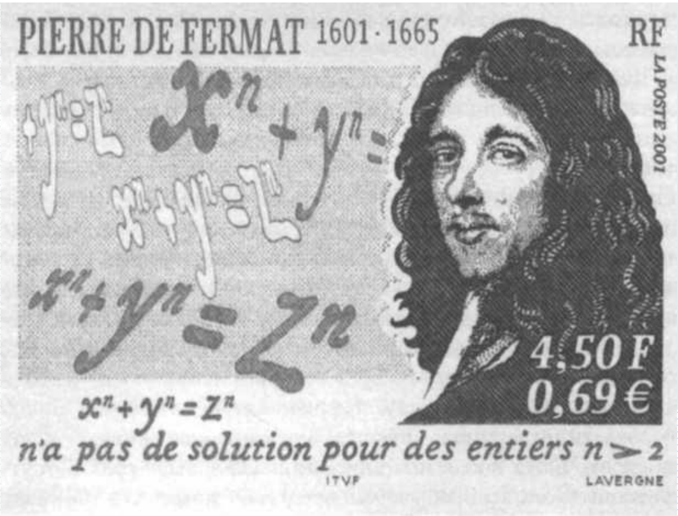
The β and double-β decay channels, which are not accompanied by excitation of the electron shells, are suppressed due to the nonorthogonality of the electron wave functions of the parent and daughter atoms. The effect is sensitive to the contribution of the outer electron shells. Since valence electrons participate in chemical bonding and collectivize in metals, the decay rates of the unstable nuclides are modified when they are embedded in a host material. Core electrons are less affected by the environment, and their overlap amplitudes are more stable.

Overlap amplitude	$_{32}$Ge (4) $[\mathrm{Ar}]3d^{10}4s^24p^2$	$_{36}$Kr (8) $[\mathrm{Ar}]3d^{10}4s^24p^6$	$_{42}$Mo (6) $[\mathrm{Kr}]4d^55s^1$	$_{52}$Te (6) $[\mathrm{Kr}]4d^{10}5s^25p^4$	$_{54}$Xe (8) $[\mathrm{Kr}]4d^{10}5s^25p^6$	$_{67}$Ho (3) $[\mathrm{Xe}]4f^{11}6s^2$
K_Z	$6.2 \cdot 10^{-3}$	0.89	0.56	$1.4 \cdot 10^{-4}$	0.22	0.53
$K_Z^{\text{core shells}}$	0.26	0.90	0.58	0.069	0.36	0.64

It's excellent that 0νββ has been studied using various isotopes in different environments.

$$\left(T^{0\nu}_{1/2}\right)^{-1} = \left|\frac{m_{\beta\beta}}{m_e}\right|^2 \; |K(Z)|^2 \; g_A^4 \; \left|M^{0\nu}\right|^2 \; G^{0\nu}$$

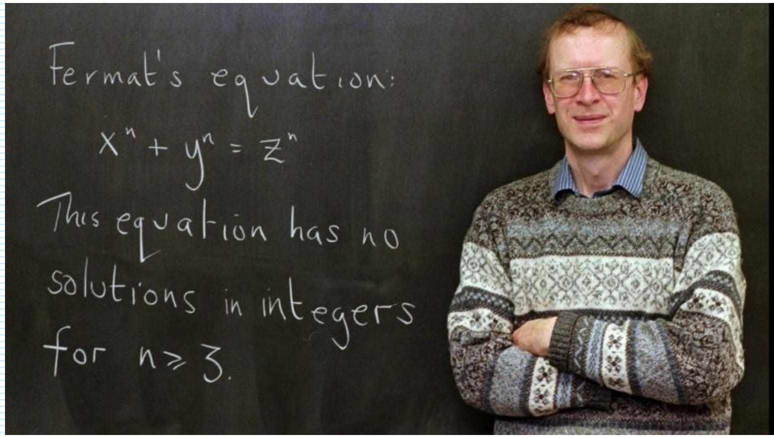
Eur. Phys. J. A 56, 16 (2020)



Around 1637, Pierre de Fermat wrote in the margin of a book that the more general equation

$$a^n + b^n = c^n$$

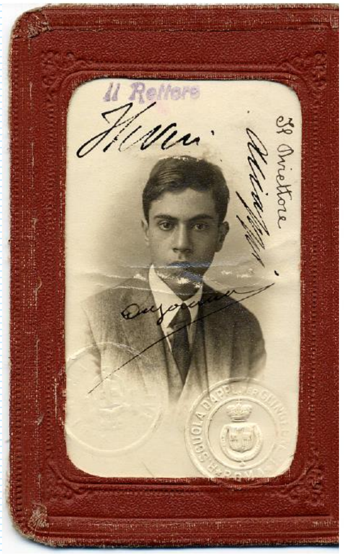
had no solutions in positive integers if n is an integer greater than 2.



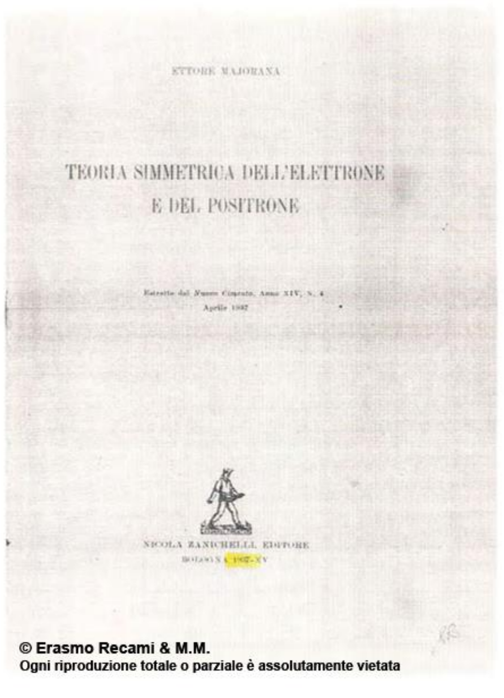
After **358** years

The proof was published by Andrew Wiles in 1995.

Some long-standing tasks of humanity …



1937



ETTORE MAJORANA

TEORIA SIMMETRICA DELL'ELETTRONE
E DEL POSITRONE

Estratto dal Nuovo Cimento, Anno XIV, N. 4
Aprile 1937

NICOLA ZANICHELLI EDITORE
BOLOGNA 1937-XV

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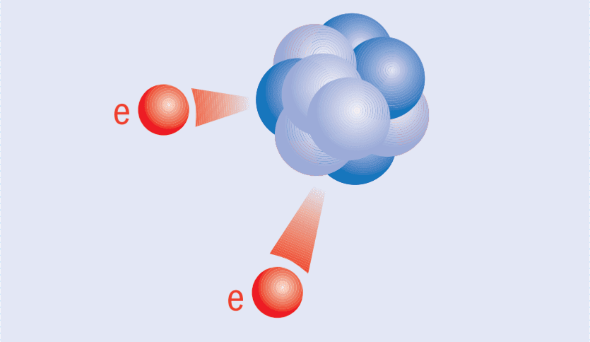
Ogni riproduzione totale o parziale è assolutamente vietata

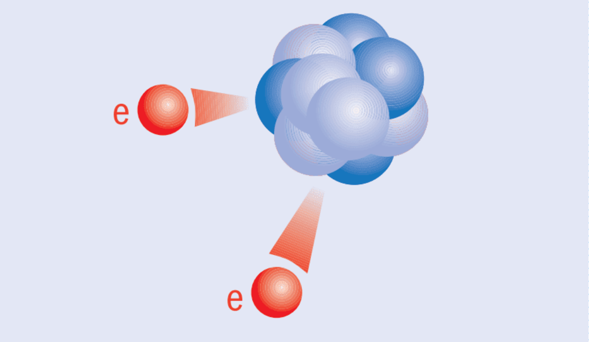
After
85
years

n-ton-class $0\nu\beta\beta$ exp.
with discovery potential
KamLAND-Zen 800
SNO+
LEGEND
nEXO
NEXT
CUPID
etc

After
?
years

If $m_{\beta\beta} < 1$ meV, what technology is needed for observation of $0\nu\beta\beta$?






Time flies when you are having fun.

Albert Einstein