Neutrino Oscillation, Seesaw Mass and Matter-Animatter Asymmetry

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1. Neutrino Oscillation and Chiral Oscillation

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3. Neutrino and Matter-Antimatter Asymetry

1. Neutrino Oscillation and Chiral Oscillation

An old story with a new twist

Equation of motion for a neutrino in free space

$$(i\partial\!\!\!/ - m)\psi = 0\,, \quad i\partial\!\!\!/\psi_L - m\psi_R = 0\,, \quad i\partial\!\!\!/\psi_R - m\psi_L = 0\,,$$

$$\psi(t,\mathbf{x}) = U(t)\psi(0)e^{i\mathbf{p}\cdot\mathbf{x}}$$

$$\psi_L = \tfrac{1-\gamma_5}{2}\psi\,, \quad \psi_R = \tfrac{1+\gamma_5}{2}\psi\,, \quad \psi = \psi_L + \psi_R\,.$$

$$U = e^{-iHt}\,, \quad H = \gamma^0\boldsymbol{\gamma}\cdot\mathbf{p} + m\gamma^0 = \boldsymbol{\alpha}\cdot\mathbf{p} + m\beta$$

How left-handed and right-handed are entangeled in free space?

In chiral representation: $\gamma^0 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix},\ \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix},\ \gamma^5 = \begin{pmatrix} -I & 0 \\ 0 & I \end{pmatrix}$ $\quad U(t) = e^{-iHt} = \cos(Et) - i\dfrac{\boldsymbol{\alpha}\cdot\mathbf{p} + m\beta}{E}\sin(Et)$

$$\psi^h(t,\mathbf{x}) = U(t)\psi^h(0)e^{i\mathbf{p}\cdot\mathbf{x}} = \psi^h(0)e^{-i(Et-\mathbf{p}\cdot\mathbf{x})}\,, \quad \psi^h(0) = \frac{1}{\sqrt{2E}}\begin{pmatrix} \sqrt{E - h\cdot p}\; u^h \\ \sqrt{E + h\cdot p}\; u^h \end{pmatrix}$$

$\psi(0)^\dagger\psi(0) = 1.$ $h = \pm 1$ - helicity, $\mathbf{p}\cdot\boldsymbol{\sigma}u^h = (h\cdot p)u^h$, $\mathbf{p} = (p_x, p_y, p_z) = p(\cos\phi\sin\theta, \sin\phi\sin\theta, \cos\theta)$.

$$u^{h=+1} = \begin{pmatrix} \cos(\theta/2) \\ \sin(\theta/2)e^{i\phi} \end{pmatrix}\,, \quad u^{h=-1} = \begin{pmatrix} -\sin(\theta/2)e^{-i\phi} \\ \cos(\theta/2) \end{pmatrix}\,,$$

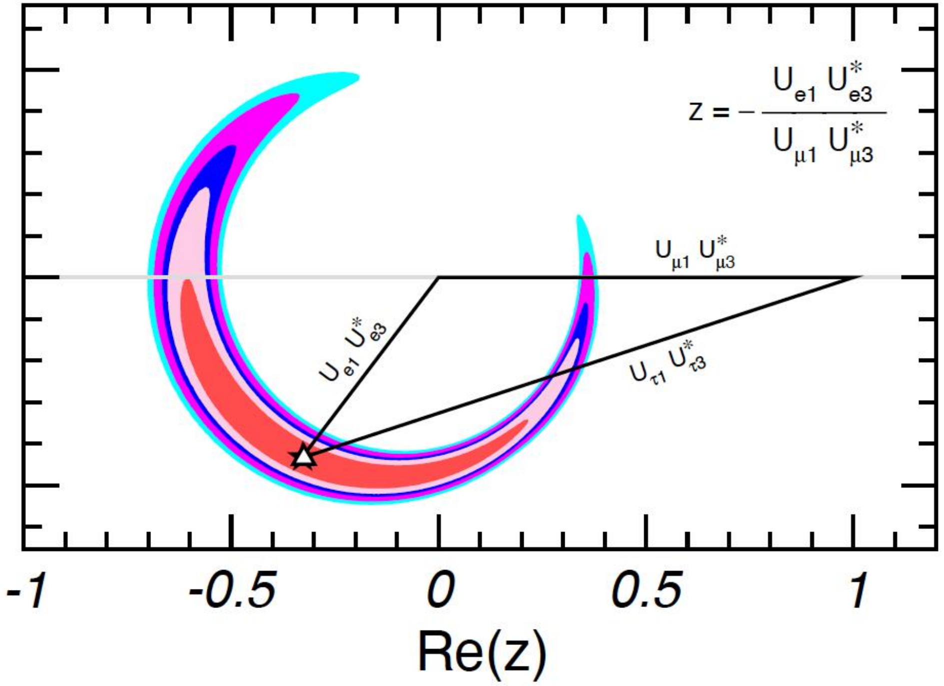
Oscilation probability from i to k for Dirac neutrinos:

$$P(\psi_i \to \psi_k) = |<\psi(0)_k|\psi(t)_i>|^2 = |\sum V_{ij}V_{kj}^* e^{-i(E_j t - \mathbf{p}\cdot\mathbf{x})}|^2$$

Neutrino oscillations with 3 generations PDG

$$L = -\frac{g}{\sqrt{2}}\overline{U}_L \gamma^\mu V_{\rm CKM} D_L W^+_\mu - \frac{g}{\sqrt{2}}\overline{E}_L \gamma^\mu U_{\rm PMNS} N_L W^-_\mu + H.C. \,,$$

$$\begin{pmatrix} c_{12}\,c_{13} & s_{12}\,c_{13} & s_{13}\,e^{-i\delta_{\rm CP}} \\ -s_{12}\,c_{23} - c_{12}\,s_{13}\,s_{23}\,e^{i\delta_{\rm CP}} & c_{12}\,c_{23} - s_{12}\,s_{13}\,s_{23}\,e^{i\delta_{\rm CP}} & c_{13}\,s_{23} \\ s_{12}\,s_{23} - c_{12}\,s_{13}\,c_{23}\,e^{i\delta_{\rm CP}} & -c_{12}\,s_{23} - s_{12}\,s_{13}\,c_{23}\,e^{i\delta_{\rm CP}} & c_{13}\,c_{23} \end{pmatrix}.$$



PDG 2023

Table 14.7: 3ν oscillation parameters obtained from different global analyses of neutrino data. In all cases, the numbers labeled as NO (IO) are obtained assuming NO (IO), *i.e.*, relative to the respective local minimum. SK-ATM makes reference to the tabulated χ^2 map from the Super-Kamiokande analysis of their data in Ref. [97].

	Ref. [185] w/o SK-ATM		Ref. [185] w SK-ATM		Ref. [186] w SK-ATM		Ref. [187] w SK-ATM	
NO	Best Fit Ordering		Best Fit Ordering		Best Fit Ordering		Best Fit Ordering	
Param	bfp $\pm1\sigma$	3σ range	bfp $\pm1\sigma$	3σ range	bfp $\pm1\sigma$	3σ range	bfp $\pm1\sigma$	3σ range
$\frac{\sin^2\theta_{12}}{10^{-1}}$	$3.10^{+0.13}_{-0.12}$	$2.75 \to 3.50$	$3.10^{+0.13}_{-0.12}$	$2.75 \to 3.50$	$3.04^{+0.14}_{-0.13}$	$2.65 \to 3.46$	$3.20^{+0.20}_{-0.16}$	$2.73 \to 3.79$
$\theta_{12}/^\circ$	$33.82^{+0.78}_{-0.76}$	$31.61 \to 36.27$	$33.82^{+0.78}_{-0.76}$	$31.61 \to 36.27$	$33.46^{+0.87}_{-0.88}$	$30.98 \to 36.03$	$34.5^{+1.2}_{-1.0}$	$31.5 \to 38.0$
$\frac{\sin^2\theta_{23}}{10^{-1}}$	$5.58^{+0.20}_{-0.33}$	$4.27 \to 6.09$	$5.63^{+0.18}_{-0.24}$	$4.33 \to 6.09$	$5.51^{+0.19}_{-0.80}$	$4.30 \to 6.02$	$5.47^{+0.20}_{-0.30}$	$4.45 \to 5.99$
$\theta_{23}/^\circ$	$48.3^{+1.2}_{-1.9}$	$40.8 \to 51.3$	$48.6^{+1.0}_{-1.4}$	$41.1 \to 51.3$	$47.9^{+1.1}_{-4.0}$	$41.0 \to 50.9$	$47.7^{+1.2}_{-1.7}$	$41.8 \to 50.7$
$\frac{\sin^2\theta_{13}}{10^{-2}}$	$2.241^{+0.066}_{-0.065}$	$2.046 \to 2.440$	$2.237^{+0.066}_{-0.065}$	$2.044 \to 2.435$	$2.14^{+0.09}_{-0.07}$	$1.90 \to 2.39$	$2.160^{+0.083}_{-0.069}$	$1.96 \to 2.41$
$\theta_{13}/^\circ$	$8.61^{+0.13}_{-0.13}$	$8.22 \to 8.99$	$8.60^{+0.13}_{-0.13}$	$8.22 \to 8.98$	$8.41^{+0.18}_{-0.14}$	$7.9 \to 8.9$	$8.45^{+0.16}_{-0.14}$	$8.0 \to 8.9$
$\delta_{\rm CP}/^\circ$	222^{+38}_{-28}	$141 \to 370$	221^{+39}_{-28}	$144 \to 357$	238^{+41}_{-33}	$149 \to 358$	218^{+38}_{-27}	$157 \to 349$
$\frac{\Delta m^2_{21}}{10^{-5}\ {\rm eV}^2}$	$7.39^{+0.21}_{-0.20}$	$6.79 \to 8.01$	$7.39^{+0.21}_{-0.20}$	$6.79 \to 8.01$	$7.34^{+0.17}_{-0.14}$	$6.92 \to 7.91$	$7.55^{+0.20}_{-0.16}$	$7.05 \to 8.24$
$\frac{\Delta m^2_{32}}{10^{-3}\ {\rm eV}^2}$	$2.449^{+0.032}_{-0.030}$	$2.358 \to 2.544$	$2.454^{+0.029}_{-0.031}$	$2.362 \to 2.544$	$2.419^{+0.035}_{-0.032}$	$2.319 \to 2.521$	2.424 ± 0.03	$2.334 \to 2.524$
IO	$\Delta\chi^2 = 6.2$		$\Delta\chi^2 = 10.4$		$\Delta\chi^2 = 9.5$		$\Delta\chi^2 = 11.7$	
$\frac{\sin^2\theta_{12}}{10^{-1}}$	$3.10^{+0.13}_{-0.12}$	$2.75 \to 3.50$	$3.10^{+0.13}_{-0.12}$	$2.75 \to 3.50$	$3.03^{+0.14}_{-0.13}$	$2.64 \to 3.45$	$3.20^{+0.20}_{-0.16}$	$2.73 \to 3.79$
$\theta_{12}/^\circ$	$33.82^{+0.78}_{-0.76}$	$31.61 \to 36.27$	$33.82^{+0.78}_{-0.75}$	$31.62 \to 36.27$	$33.40^{+0.87}_{-0.81}$	$30.92 \to 35.97$	$34.5^{+1.2}_{-1.0}$	$31.5 \to 38.0$
$\frac{\sin^2\theta_{23}}{10^{-1}}$	$5.63^{+0.19}_{-0.26}$	$4.30 \to 6.12$	$5.65^{+0.17}_{-0.22}$	$4.36 \to 6.10$	$5.57^{+0.17}_{-0.24}$	$4.44 \to 6.03$	$5.51^{+0.18}_{-0.30}$	$4.53 \to 5.98$
$\theta_{23}/^\circ$	$48.6^{+1.1}_{-1.5}$	$41.0 \to 51.5$	$48.8^{+1.0}_{-1.2}$	$41.4 \to 51.3$	$48.2^{+1.0}_{-1.4}$	$41.8 \to 50.9$	$47.9^{+1.0}_{-1.7}$	$42.3 \to 50.7$
$\frac{\sin^2\theta_{13}}{10^{-2}}$	$2.261^{+0.067}_{-0.064}$	$2.066 \to 2.461$	$2.259^{+0.065}_{-0.065}$	$2.064 \to 2.457$	$2.18^{+0.08}_{-0.07}$	$1.95 \to 2.43$	$2.220^{+0.074}_{-0.076}$	$1.99 \to 2.44$
$\theta_{13}/^\circ$	$8.65^{+0.13}_{-0.12}$	$8.26 \to 9.02$	$8.64^{+0.12}_{-0.13}$	$8.26 \to 9.02$	$8.49^{+0.15}_{-0.14}$	$8.0 \to 9.0$	$8.53^{+0.14}_{-0.15}$	$8.1 \to 9.0$
$\delta_{\rm CP}/^\circ$	285^{+24}_{-26}	$205 \to 354$	282^{+23}_{-25}	$205 \to 348$	247^{+26}_{-27}	$193 \to 346$	281^{+23}_{-27}	$202 \to 349$
$\frac{\Delta m^2_{21}}{10^{-5}\ {\rm eV}^2}$	$7.39^{+0.21}_{-0.20}$	$6.79 \to 8.01$	$7.39^{+0.21}_{-0.20}$	$6.79 \to 8.01$	$7.34^{+0.17}_{-0.14}$	$6.92 \to 7.91$	$7.55^{+0.20}_{-0.16}$	$7.05 \to 8.24$
$\frac{\Delta m^2_{32}}{10^{-3}\ {\rm eV}^2}$	$-2.509^{+0.032}_{-0.032}$	$-2.603 \to -2.416$	$-2.510^{+0.030}_{-0.031}$	$-2.601 \to -2.419$	$-2.478^{+0.035}_{-0.033}$	$-2.577 \to -2.375$	$-2.50\pm^{+0.04}_{-0.03}$	$-2.59 \to -2.39$

What type of neutrinos are produced in weak interaction produced?

What neutrino state is produced in weak interaction? Let us use $\pi^- \to \mu^- + \bar{\nu}$ as example

$$M(\pi^- \to \mu^- \bar{\nu}) \sim < 0|\bar{u}\gamma^\mu \frac{1-\gamma_5}{2} d|\pi^- > \bar{\mu}\gamma_\mu \frac{1-\gamma_5}{2}\nu$$

$$\sim i f_\pi P_\pi^\mu \bar{\mu}\gamma_\mu (1-\gamma_5)\nu = i f_\pi \bar{\mu}(m_\nu \frac{1+\gamma_5}{2} + m_\mu \frac{1-\gamma_5}{2})\nu \,.$$

The neutrino participating the weak interaction and produced is in a state of $\psi(0)$

$$\psi(0) = (m_\nu \frac{1+\gamma_5}{2} + m_\mu \frac{1-\gamma_5}{2})\nu$$

$$= \frac{1}{\sqrt{2E(m_\nu^2 + m_\mu^2)}} \left(m_\nu \begin{pmatrix} 0 \\ \sqrt{E-P}u_+ + \sqrt{E+P}u_- \end{pmatrix} + m_\mu \begin{pmatrix} \sqrt{E+P}u_+ + \sqrt{E-P}u_- \\ 0 \end{pmatrix} \right) \,.$$

Relativistic case, $E >> m_\nu : \psi(0) \approx \begin{pmatrix} u_+ \\ 0 \end{pmatrix}$, Non-relativistic case, $E \sim m_\nu : \psi(0) \approx \begin{pmatrix} (u_- + u_+)/\sqrt{2} \\ 0 \end{pmatrix}$.

Impact on neutrino oscillations?

Neutrino Chiral Oscillation

Xiao-Gang He, Zhong-Lv Huang, Ming-Wei Le, arXiv: 2307.12561

In the SM neutrinos are produced by W and/or Z interactions.

At production $t=0$ point, they are left-handed and normalized, $\psi^h_L(0)=\sqrt{\frac{2E}{E-h\cdot p}}\frac{1-\gamma_5}{2}\psi^h(0)$.

$$\psi^h_L(t)=\sqrt{\frac{2E}{E-h\cdot p}}e^{-iHt}\frac{1-\gamma^5}{2}\psi^h(0)=\psi^h_L(t)=\sqrt{\frac{2E}{E-h\cdot p}}\left(e^{-iEt}\frac{1-\gamma_5}{2}\psi^h(0)-i\frac{m}{E}\sin(Et)\left[\beta,\frac{1-\gamma_5}{2}\right]\psi^h(0)\right).$$

Used
$$U(t)=e^{-iHt}=\cos(Et)-i\frac{\boldsymbol{\alpha}\cdot\mathbf{p}+m\beta}{E}\sin(Et)$$

$$\psi^{h\dagger}_L\psi^h_L(t)=\left(\cos(Et)+i\frac{h\cdot p}{E}\sin(Et)\right)e^{i\mathbf{p}\cdot\mathbf{L}}\ ,\quad \psi^{h\dagger}_R\psi^h_L(t)=\left(-i\frac{m}{E}\sin(Et)\right)e^{i\mathbf{p}\cdot\mathbf{L}}$$

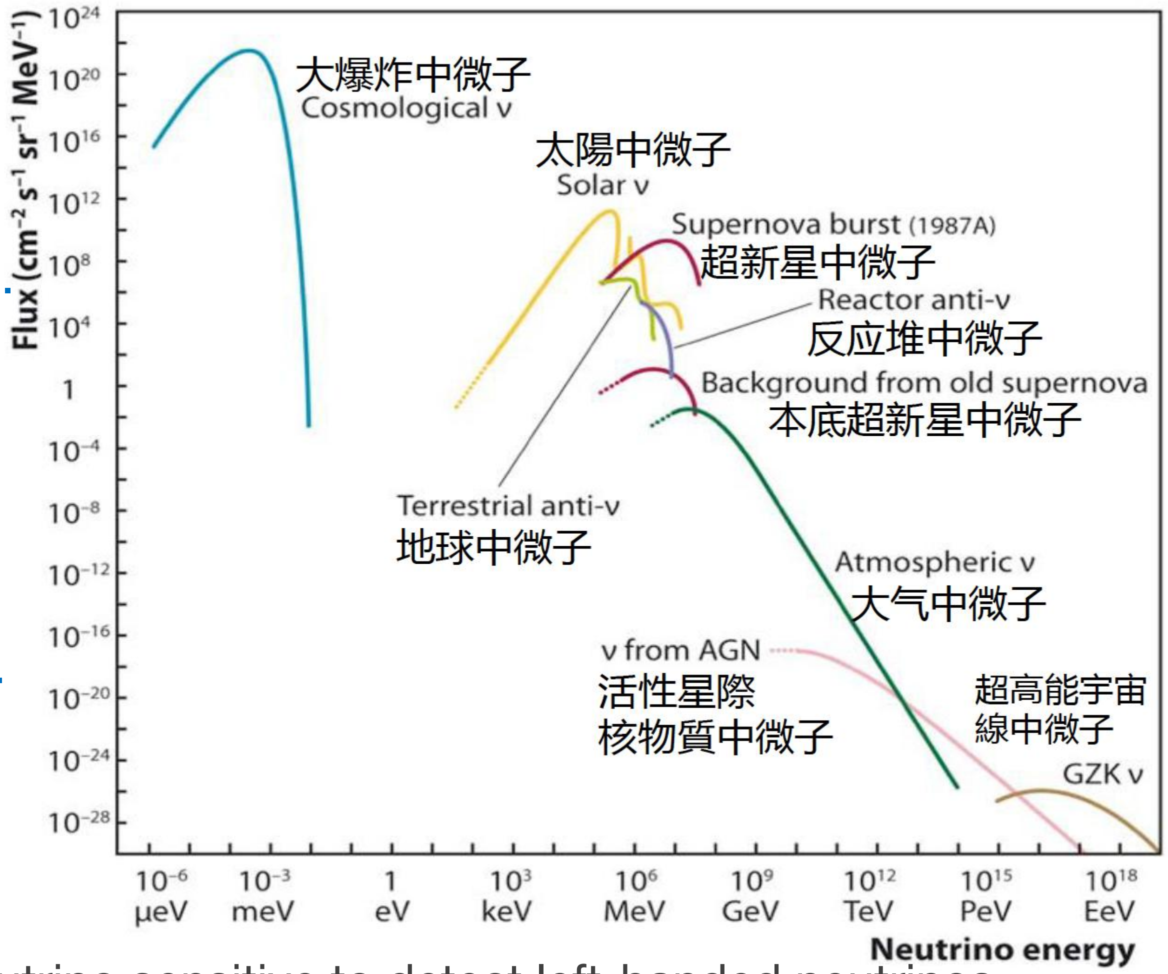
$$P(\nu^h_L\to\nu^h_L)=|\psi^{h\dagger}_L\psi^h_L(t)|^2=1-\frac{m^2}{E^2}\sin^2(Et)\ ,\quad P(\nu^h_L\to\nu^h_R)=|\psi^{h\dagger}_R\psi^h_L(t)|^2=\frac{m^2}{E^2}\sin^2(Et)$$

Left-handed neutrinos oscillated into right-handed ones!

$$P(\nu^h_{Li}\to\nu^h_{Lk})=|V_{ij}V^*_{kj}(\cos(E_jt)+i\frac{h\cdot p}{E_j}\sin(E_jt))|^2\ ,\quad P(\nu^h_{Li}\to\nu^h_{Rk})=|-iV_{ij}V^*_{kj}\frac{m_j}{E_j}\sin(E_jt)|^2\psi^{h\dagger}_R\psi^h_L(t)|^2$$

S-F Ge & P Pasquini, PLB811(2020)135961; V Bittencourt, A. Bernardini & M. Blasone, EPJC81 (2021)411.

Chiral oscillation probability extremely small for relativistic neutrinos because the suppresion factor: m^2/E^2.



Numerically vailid using Dirac neutrinos for neutrino oscillations.

Current usual neutrino oscillation measurements OK!, DUNE, Hyper-K, JUNO. NOv, T2K, ...OK!

Neutrino chiral scillation period can of order ps.

Similar for charged lepton, but the oscillation period is

Et = 2π, t <2π/me = 1.3×10^{-21}s too short. Time averaged effect.

Effect large for non-relativistic background cosmic nutrinos.

What might be the observable mechanism?

Detection at x = l is also through weak interaction, for Dirac neutrino sensitive to detect left-handed neutrinos and the right-handed neutrinos will practically not detectable.

Disappearance of neutrino measurement $P(\nu_L^h \to \nu_L^h)$

If neutrinos are Majorana ones, because the charge conjugated neutrinos are right-handed ones, left-handed to right-handed neutrino appearance processes can also be measured.

Disappearance neutrino measurement $P(\nu_L^h \to \nu_L^h)$ Also anit-neutrino appearance $P(\nu_L^h \to (\nu_L^h)^c)$

Neutrino Oscillation in Matter

When neutrinos travel in matter, due to interaction of neutrions with matter mediated by W and Z, the Lagrangian is modified   MSW Effect

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m)\psi - j^\mu \bar{\psi}\gamma_\mu \frac{1-\gamma_5}{2}\psi \ .$$

j^μ is the matter current which neutrino can interact.
In the rest frame of the homogeneous, isotropic, unpolarized electrical neutrality medium, $j^\mu = (\rho, \vec{0})$
$\rho = \sqrt{2}G_F\left(N_e\delta_{\alpha e} - \frac{1}{2}N_n\right)$. N_e, N_n number density of electron and neutron, $\delta_{\alpha e}$ are zero for ν_μ and ν_τ.

$$\left(i\partial\!\!\!/ - m - \rho\gamma_0\frac{1-\gamma_5}{2}\right)\psi = 0 \qquad H = \mathbf{p}\cdot\boldsymbol{\alpha} + m\beta + \rho\frac{1-\gamma_5}{2} = \begin{pmatrix} \rho - \mathbf{p}\cdot\boldsymbol{\sigma} & m \\ m & \mathbf{p}\cdot\boldsymbol{\sigma} \end{pmatrix} = \begin{pmatrix} \rho - h\cdot p & m \\ m & h\cdot p \end{pmatrix}$$

The eigenvalues of H are: $E_1 = \frac{\rho}{2} + E_h\ ,\ \ E_2 = \frac{\rho}{2} - E_h,\ E_h = \sqrt{m^2 + (h\cdot p - \rho/2)^2},$

$$\psi_1 = \frac{1}{\sqrt{2E_h}}\begin{pmatrix} \sqrt{E_h - (h\cdot p - \frac{\rho}{2})}\, u^h \\ \sqrt{E_h + (h\cdot p - \frac{\rho}{2})}\, u^h \end{pmatrix}\ ,\quad \psi_2 = \frac{1}{\sqrt{2E_h}}\begin{pmatrix} \sqrt{E_h + (h\cdot p - \frac{\rho}{2})}\, u^h \\ -\sqrt{E_h - (h\cdot p - \frac{\rho}{2})}\, u^h \end{pmatrix}$$

The evolution for a given wave function entering the media with momentum $\vec{p}$ at $t = 0$ is

$$\psi_L^h(t) = e^{-iHt}\psi_L^h(0) = e^{-\frac{i}{2}\rho t}\begin{pmatrix} \cos(E_h t) + i\frac{h\cdot p - \frac{\rho}{2}}{E_h}\sin(E_h t) & -i\frac{m}{E_h}\sin(E_h t) \\ -i\frac{m}{E_h}\sin(E_h t) & \cos(E_h t) - i\frac{h\cdot p-\frac{\rho}{2}}{E_h}\sin(E_h t)\end{pmatrix}\begin{pmatrix} u^h \\ 0\end{pmatrix},$$

$$P(\psi_L^h \to \psi_L^h) = 1 - \frac{m^2}{E_h^2}\sin^2(E_h t)\ , \quad P(\psi_L^h \to \psi_R^h) = \frac{m^2}{E_h^2}\sin^2(E_h t).$$

Similar as that for free space, but dependent on matter density via $E_h = \sqrt{m^2 + (h\cdot p - \rho/2)^2}$.

There is a resonant enhanced chiral oscillation at $h \cdot p - \rho/2 = 0$!

From the above, one can easily recover the usual matter oscillation formalism with $J_L^\mu = (\rho, 0)$ in the relativistic case $p \gg \quad m \gg \rho$. Keeping the leading effect in this limit, one obtains,

$$H_{\text{eff}} = p + \frac{M^\dagger M}{2p} - h\cdot\rho\ . \tag{13}$$

Then we can find that matter effect would influence the contribution from mixing angle and mass square. Note that for $h = -1$ helicity, it is the usual leading order neutrino oscillation in matter effective Hamiltonian which can cause matter induced MSW resonant effect. But for $h = +1$ or $\rho < 0$, the matter effects are different.

Majorana and Seesaw neutrinos chiral oscillation

The general seesaw neutrino Lagrangian in matter propagation

$$\mathcal{L} = \bar{\nu}_L i\partial\!\!\!/ \nu_L + \bar{N}_R i\partial\!\!\!/ N_R - \frac{1}{2}\left(\begin{pmatrix}\bar{\nu}_L^c & \bar{N}_R\end{pmatrix}\begin{pmatrix}M_L & M_D^T \\ M_D & M_R\end{pmatrix}\begin{pmatrix}\nu_L \\ N_R^c\end{pmatrix} + \text{h.c.}\right) - \begin{pmatrix}\bar{\nu}_L & \bar{N}_R^c\end{pmatrix}\begin{pmatrix}j_L^\mu & j_{RL}^\mu \\ j_{RL}^{\mu\dagger} & j_R\end{pmatrix}\gamma_\mu\begin{pmatrix}\nu_L \\ N_R^c\end{pmatrix}$$

$$= \bar{\psi}_L i\partial\!\!\!/ \psi_L - \frac{1}{2}\left(\bar{\psi}_L^c \mathcal{M}\psi_L + \text{h.c.}\right) - \bar{\psi}_L J^\mu \gamma_\mu \psi_L$$

For homogeneous, isotropic, unpolarized electrical neutrality matter medium at rest, only j_L^0 is non-zero

$$j_L^0 = \begin{pmatrix}\rho_e & 0 & 0 \\ 0 & \rho_\mu & 0 \\ 0 & 0 & \rho_\tau\end{pmatrix} = \begin{pmatrix}\sqrt{2}G_F\left(N_e - \frac{1}{2}N_n\right) & 0 & 0 \\ 0 & -\frac{G_F}{\sqrt{2}}N_n & 0 \\ 0 & 0 & -\frac{G_F}{\sqrt{2}}N_n\end{pmatrix}.$$

In terms of the mass eigenstate $\psi_L = V\psi_L^m$, the Lagrangian is

$$\mathcal{L} = \frac{1}{2}\left(\bar{\psi}^m(i\partial\!\!\!/ - \widehat{M})\psi^m\right) - \bar{\psi}^m \widetilde{J}^\mu \gamma_\mu \frac{1-\gamma_5}{2}\psi^m\,, \quad \psi^m = \psi_L^m + (\psi_L^m)^c\,, \quad \widetilde{J}^\mu = V^\dagger J^\mu V.$$

$$(i\partial\!\!\!/ - \widehat{M})\psi^m - \widetilde{J}^\mu \gamma_\mu \frac{1-\gamma_5}{2}\psi^m + (\widetilde{J}^\mu)^* \gamma_\mu \frac{1+\gamma_5}{2}\psi^m = 0\,.$$

$$H = \begin{pmatrix} \boldsymbol{\alpha}\cdot\mathbf{p} & 0 \\ 0 & \boldsymbol{\alpha}\cdot\mathbf{p} \end{pmatrix} + \begin{pmatrix} \beta\widehat{M}_l & 0 \\ 0 & \beta\widehat{M}_h \end{pmatrix} + V^\dagger \begin{pmatrix} j_L^\mu & j_{RL}^{\mu\dagger} \\ j_{RL}^\mu & -j_R^{\mu T} \end{pmatrix} V \gamma^0\gamma_\mu \frac{1-\gamma^5}{2} - \left(V^\dagger \begin{pmatrix} j_L^\mu & j_{RL}^{\mu\dagger} \\ j_{RL}^\mu & -j_R^{\mu T} \end{pmatrix} V \right)^* \gamma^0\gamma_\mu \frac{1+\gamma^5}{2}$$

Because the off-diagonal interaction, difficulty to get U(t). For just one light and one heavy neutrinos, can get a closed analytic expression.

Let $\widehat{M}_l = m$ and $\widehat{M}_h = M$, similar to Dirac case $j^\mu = (\rho, \vec{0})$ and $j_{RL}^\mu = j_R^\mu = 0$,

$$\widetilde{J}^\mu\gamma_\mu = V^\dagger J^\mu V \gamma_\mu = \begin{pmatrix} \frac{\rho}{2}(1+\cos 2\theta) & \frac{\rho}{2}e^{i\eta}\sin 2\theta \\ \frac{\rho}{2}e^{-i\eta}\sin 2\theta & \frac{\rho}{2}(1-\cos 2\theta) \end{pmatrix} \gamma_0 \,.$$

$$H = \begin{pmatrix} \frac{\rho}{2}(1+\cos 2\theta) - \mathbf{p}\cdot\boldsymbol{\sigma} & m & \frac{\rho}{2}e^{i\eta}\sin 2\theta & 0 \\ m & \mathbf{p}\cdot\boldsymbol{\sigma} - \frac{\rho}{2}(1+\cos 2\theta) & 0 & -\frac{\rho}{2}e^{-i\eta}\sin 2\theta \\ \frac{\rho}{2}e^{-i\eta}\sin 2\theta & 0 & \frac{\rho}{2}(1-\cos 2\theta) - \mathbf{p}\cdot\boldsymbol{\sigma} & M \\ 0 & -\frac{\rho}{2}e^{i\eta}\sin 2\theta & M & \mathbf{p}\cdot\boldsymbol{\sigma} - \frac{\rho}{2}(1-\cos 2\theta) \end{pmatrix}$$

Majorana phase effects on chiral oscillation

One light and one heavy example: $\widehat{M} = \begin{pmatrix} m & 0 \\ 0 & M \end{pmatrix}$, $V = \begin{pmatrix} V_{a1} & V_{a2} \\ V_{s1} & V_{s2} \end{pmatrix} = \begin{pmatrix} \cos\theta & e^{i\eta}\sin\theta \\ -\sin\theta & e^{i\eta}\cos\theta \end{pmatrix}$.

A left-handed neutrino produced at t =0

$$\nu_L^h = \begin{pmatrix} V_{a1}^* \begin{pmatrix} u^h \\ 0 \end{pmatrix} \\ V_{a2}^* \begin{pmatrix} u^h \\ 0 \end{pmatrix} \end{pmatrix}, \quad N_R^h = \begin{pmatrix} V_{s1} \begin{pmatrix} 0 \\ u^h \end{pmatrix} \\ V_{s2} \begin{pmatrix} 0 \\ u^h \end{pmatrix} \end{pmatrix}, \quad (\nu_L^h)^c = \begin{pmatrix} V_{a1} \begin{pmatrix} 0 \\ u^h \end{pmatrix} \\ V_{a2} \begin{pmatrix} 0 \\ u^h \end{pmatrix} \end{pmatrix}, \quad (N_R^h)^c = \begin{pmatrix} V_{s1}^* \begin{pmatrix} u^h \\ 0 \end{pmatrix} \\ V_{s2}^* \begin{pmatrix} u^h \\ 0 \end{pmatrix} \end{pmatrix}. \quad (2$$

$$P(\nu_L^h \to \nu_L^h) = \left(\cos^2\theta\cos(E_m t) + \sin^2\theta\cos(E_M t)\right)^2 + p^2\left(\frac{\cos^2\theta}{E_m}\sin(E_m t) + \frac{\sin^2\theta}{E_M}\sin(E_M t)\right)^2$$

$$P(\nu_L^h \to (N_R^h)^c) = \frac{\sin^2 2\theta}{4}\left((\cos(E_m t) - \cos(E_M t))^2 + p^2\left(\frac{\sin(E_m t)}{E_m} - \frac{\sin(E_M t)}{E_M}\right)^2\right)$$

$$P(\nu_L^h \to (\nu_L^h)^c) = \frac{m^2}{(E_m)^2}\cos^4\theta\sin^2(E_m t) + \frac{mM}{2E_m E_M}\sin^2 2\theta\cos 2\eta\sin(E_m t)\sin(E_M t) + \frac{M^2}{(E_M)^2}\sin^4\theta\sin^2(E_M t)$$

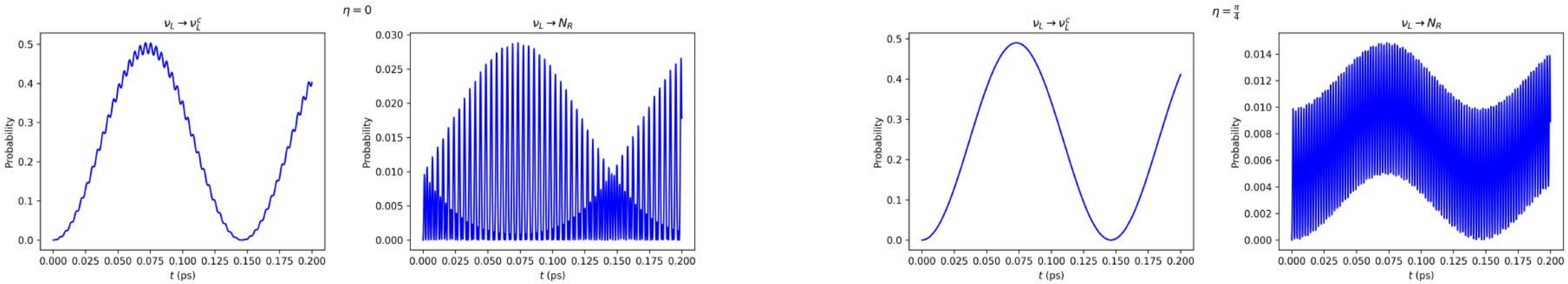
$$P(\nu_L^h \to N_R^h) = \frac{\sin^2 2\theta}{4}\left(\frac{m^2}{(E_m)^2}\sin^2(E_m t) - \frac{2mM}{E_m E_M}\cos 2\eta\sin(E_m t)\sin(E_M t) + \frac{M^2}{(E_M)^2}\sin^2(E_M t)\right).$$

Majorana phase show up in the chiral oscillation explicitly!

Majorana phases do not show up in ordinary oscillation.

This also applies to Type II seesaw with two generations!

Chiral oscillation too fast, sees time averaged effects
Majorana phase η effects averaged out!



(a)$\eta = 0$ (b)$\eta = \frac{\pi}{4}$

FIG. 1: Neutrino chiral rotation in last two equation in Eq.(23) in non-relativistic regime. The initial state is an active left-handed neutrino. The input parameters used are: $p = 0.01\text{eV}$, $m = 0.01\text{eV}$, $M = 1\text{eV}$, and $\sin\theta = \sqrt{m/M} = 0.1$, and the Majorana phase takes two different values $\eta = 0, \pi/4$.

Time averaged Majorana phase effects

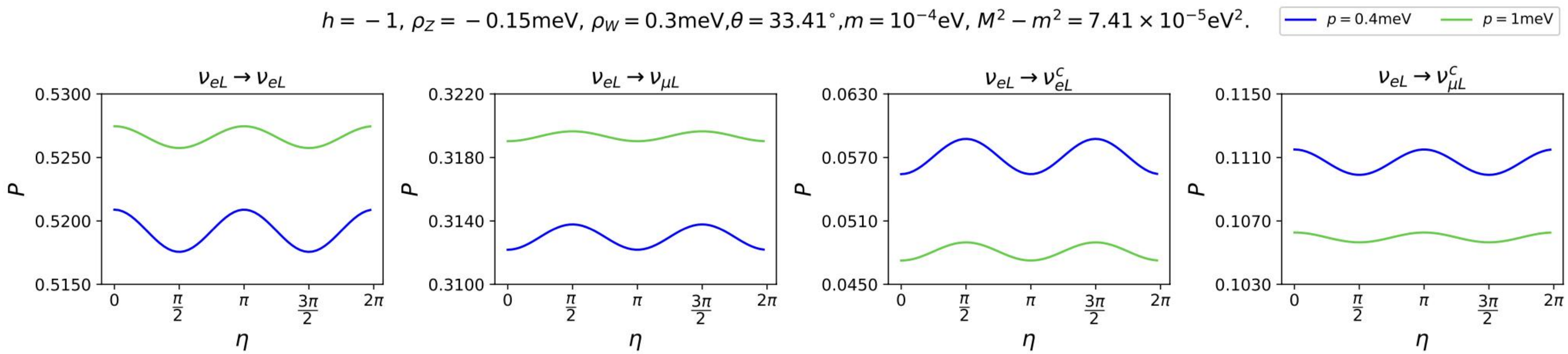


Figure 2: The time averaged Majorana phase η effects on probabilities in two active Majorana relic neutrinos case for different neutrino momentum (0.4, 1.0) meV. In the vertical axis, P is the time averaged probabilities in matter. The lighter neutrino mass is fixed to be 0.1 meV. The values of mixing angle $\theta = \theta_{12}$ and mass square difference $M^2 - m^2 = \Delta m^2_{21}$ are from [1]. For the matter densities, we use $\rho_W = 0.3$meV and $\rho_Z = -0.15$meV. Such a mater density could be found in neutron star cluster [12].

Measurement of chiral oscillation effects very challenging for All

Ptolemy may leads to a new era for low energy detection!

2. Neutrino Mass and Seesaw Mechanism

Theoretical Models for Neutrino masses

In the minimal SM: Gauge group: $SU(3)_C \times SU(2)_L \times U(1)_Y$

$$G(8,1)\ (0), W(1,3)\ (0), B\ (1,1)(0)\ ,$$

$$Q_L = \begin{pmatrix} U_L \\ D_L \end{pmatrix} (3,2)(1/6)\ ,\ \ U_R\ (3,1)(2/3)\ ,\ \ D_R\ (3,1)(-1/3)\ ,$$

$$L_l = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} (1,2)(-1/2)\ ,\ \ E_R\ (1,1)(-1)\ ,$$

$$H = \begin{pmatrix} h^+ \\ (v + h^0)/\sqrt{2} \end{pmatrix} (1,2,1/2)\ ,\ \ \text{v - vev of Higgs}\ .$$

Quark and charged lepton masses are from the following Yukawa couplings

$$\bar{Q}_L \tilde{H} U_R\ ,\ \ \bar{Q}_L H D_R\ ,\ \ \bar{L}_L H E_R\ .$$

Nothing to pair up with $L_L(\nu_L)$. In minimal SM, neutrinos are massless!

Extensions needed: Give neutrino masses and small ones!

To have Dirac mass, need to introduce right handed neutrinos v_R: (1,1)(0)

Dirac neutrino mass term

$$L = -\bar{L}_L Y_\nu \tilde{H} \nu_R + H.C\ , \to \quad -\bar{\nu}_L m_\nu \nu_R\ , \to m_\nu = \frac{v}{\sqrt{2}} Y_\nu$$

$$m_{\nu_e} < 0.3 \text{ eV}, \to \quad Y_{\nu_e}/Y_e < 10^{-5}, \text{ very much fine tuned!}$$

Neutrino oscillation --
evidence for non-zero mass!

Minimal SM does allow neutrino mass at renormalization level -- Weinberg dimension five operator: νν HH/Λ

–– Neutrinos are necessarily Majorana particles.

Introduce right handed neutrinos to have Dirac mass, but fine tuned hierarchy to have small masses.

More natural smaller neutrino masses?

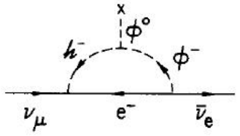
Some theoretical models for small neutrino masses

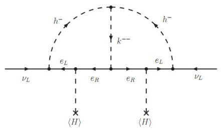
Loop generated neutrino masses:

The Zee Model(1980); Zee-Babu Model (zee 1980; Babu, 1988

Other loop models: Babu&He; E. Ma; Mohapatra et al; Geng et al

Seesaw models





Type I: Introduce singlet right-handed neutrinos

P. Minkoviski 1977, Yanagida (1979)...

$$L = \bar{\nu}_L (Y_\nu v/\sqrt{2})\nu_R + \bar{\nu}^c_R M_R \nu_R/2$$

$$M_\nu = \begin{pmatrix} 0 & Y_\nu v/\sqrt{2} \\ Y^T_\nu v/\sqrt{2} & M_R \end{pmatrix} \qquad m_\nu \approx Y^2_\nu v^2/M_R\,, \qquad M_N \approx M_R$$

Type II: Introduce singlet neutrinos

W. Konetschny and W. Kummer, 1977..., Schechter and J. W. F. Valle (1980)

Once the neutral component in Δ gets the vev, $<\delta^0> = v_\Delta/\sqrt{2}$

Type III: Introduce triplet leptons

Type-III: Introduce triplet lepton representations Σ: (1,3,0)) (Foot, Lew, He and Joshi, 1989).

Allowing one type new field to the SM to get neutrino masses

Only 3 types, they are the 3 seesaw models. (Ernest Ma (1998))

Loop naturally obtain small masses by loop factors: Dirac and Majorana masses possible

Seesaw mechanis realize -- Weinberg dimension five operator:

*For type I, III seesaw m v ~ v^2/Λ

$\Lambda = M_R \rightarrow m_\nu \sim m_e\, v/M_R$

Minimal model: two heavy right handed neutrinos prediting: one zero light neutrion mass

*For type II seesaw:

$m_\nu \sim Y v_\Delta \sim Y\, v^2\, (m_\Delta/M^2_\Delta \sim 1/M_\Delta\,)$

Explain why neutrinos have small mass, but not what values, not mass hierachy, and not answer mixing pattern...!

More hints from Data...

Experimental colleqgues will you?

The minimal Type I and III Seesaw models

3 left-handed neutrinos, and 2 right-handed neutrinos: mass, mixing, CP in oscillation.. Enough!

Y = 3X2 matrix, M = 2X2

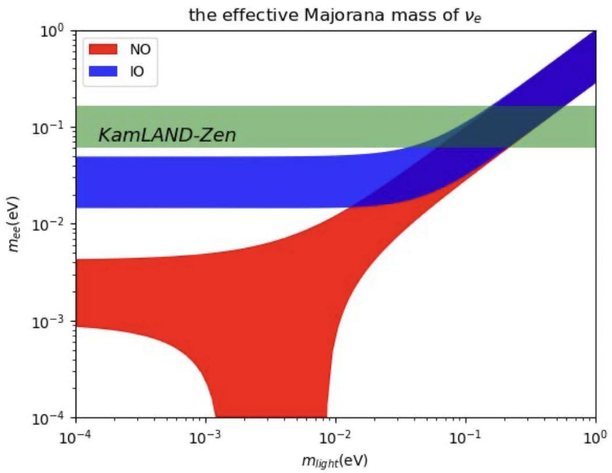
$$-\frac{1}{2}\left[\bar{\nu}_R M \nu_R^c + 2\bar{L}_L Y H \nu_R + H.C.\right] \qquad \begin{pmatrix} 0 & Y^* v/\sqrt{2} \\ Y^\dagger v/\sqrt{2} & M \end{pmatrix} \qquad m_\nu = -\frac{v^2}{2} Y^* M^{-1} Y^\dagger$$

m_ν is a 3X3 symmetric matric, butrank 2, predicts one zero. No control on NH and IH, nor mxing pattern

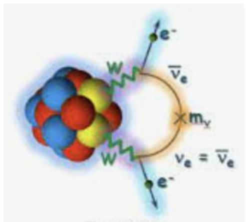
NH: m1=0, m2~ 8.6x10^{-3}eV, m3~0.05eV; IH: m1~0.0492eV, m2 ~0.05eV, m3 = 0

Type I,II,III Seesaw models matrix elements allowed ranges

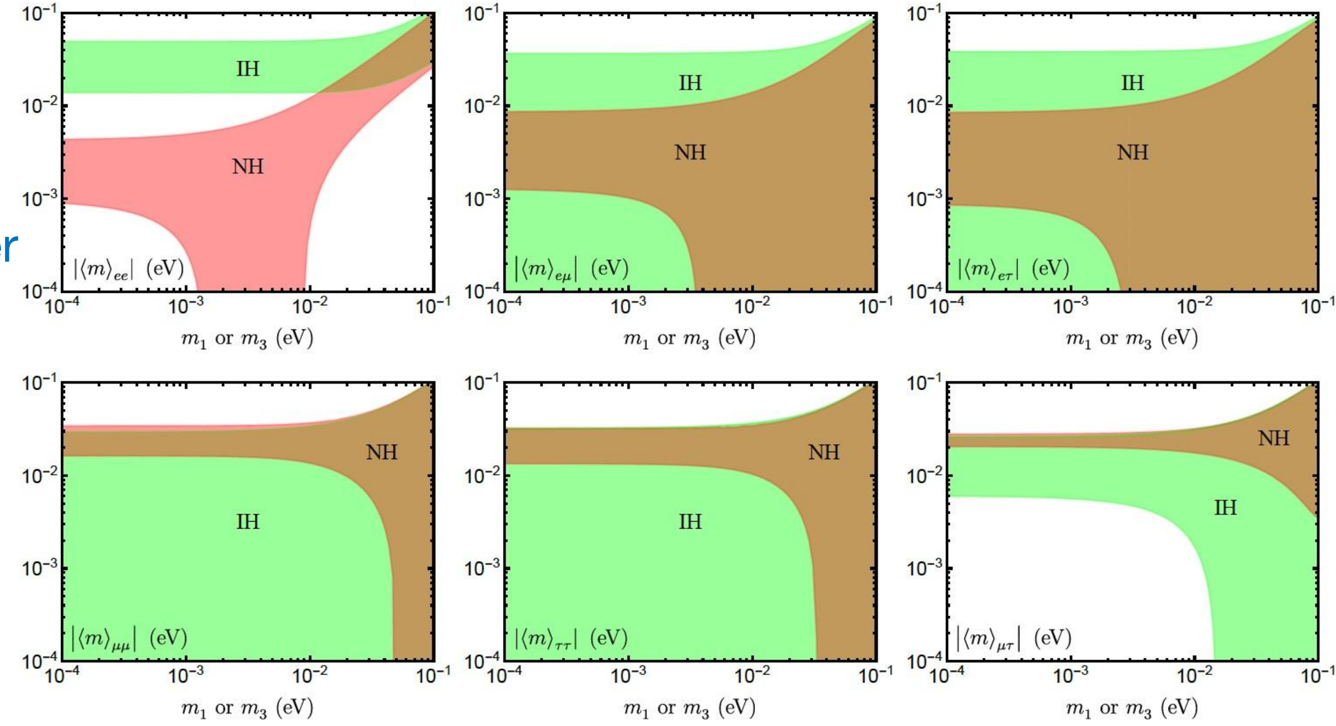
from mixing pattern and mass-squred differences XG He, ZL Huang and MW Li



Very the nature of whether neutrinos are Dirac or Majorana particle.



Chances for neutrinoless double beta decay experiments!



No prediction for matrix pattern! Needs more theory helps.

Mixing pattern

Historically from small mixing angles to large mixing angles, Bimaximal, Tribimaximal...?

Seesaw models have nothing to say! Need more theoretical considerations!

Additional symmetries, A_4 is one of the interesting ones!

Natural $\theta_{23} = \pi/4$, $\delta_{CP} = -\pi/2$ solution! (very well approximate inverted hierarchy case)

Assuming neutrinos are Majorana particles,

$$L = -\frac{1}{2}\bar{\nu}_L m_\nu \nu_L^C$$
$$m_\nu = V_{PMNS}\hat{m}_\nu V^T_{PMNS} ,$$

$\hat{m}_\nu = diag(m_1, m_2, m_3)$ with $m_i = |m_i| exp(i\alpha_i)$.

With $\delta = -\pi/2$ and $\theta_{23} = \pi/4$, m_ν has the following form

$$m_\nu = \begin{pmatrix} a & c+i\beta & -(c-i\beta) \\ c+i\beta & d+i\gamma & \tilde{b} \\ -(c-i\beta) & \tilde{b} & d-i\gamma \end{pmatrix} ,$$

μ-τ complex conjugate symmetry

Grimus&Lavoura (2004), Gui-Jun Ding et al., Petcov, Xing …

Particle contents and their transformation properties under standard $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge and A_4 family symmetry properties

$$l_L : (1,2,-1)(3) , \quad l_R : (1,1,-2)(1+1''+1') ,$$
$$\phi : (1,2,-1)(1) , \quad \Phi : (1,2,-1)(3) ,$$
$$\Delta^{0,','' } : (1,3,-2)(1+1'+1'') , \quad \chi : (1,3,-2)(3) .$$

If the structure of the vacuum expectation value (vev) is of the form $<\Phi_{1,2,3}>= v^\Phi_{1,2,3} = v^\Phi$, $<\chi_i>= v^\chi_i$, $<\phi>= v_\phi$, and $<\Delta^{0,','' }>= v_\Delta^{0,',''}$,

$$M_l = U_l \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix} , \quad M_\nu = \begin{pmatrix} w_1 & x & y \\ x & w_2 & z \\ y & z & w_3 \end{pmatrix} ,$$

In M_l diagonal basis, the neutrino mass matrix is given by the right form!

Type I and III in a similar fashion. two fold ambiguities: $\delta_{CP} = -(+)\pi/2$

X-G He, Chin. J. Phys 53, 100101(2015); X-G He and G-N Li, Phys. Lett. B750,620(2015); E Ma, Phys. Rev. D92, 051301(2015).

More data for CP violation and mixing needed!

3. Neutrino and Matter-Antimatter Asymmetry

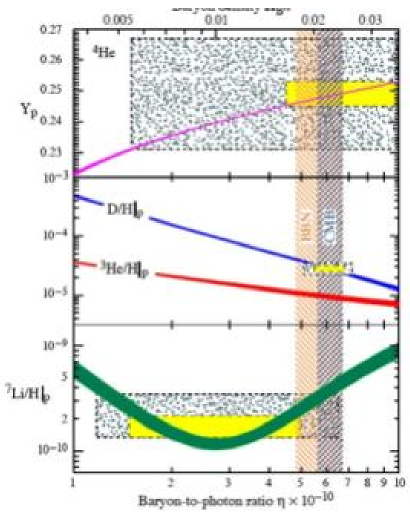
What else seesaw models are good for? Leptogenesis! ...

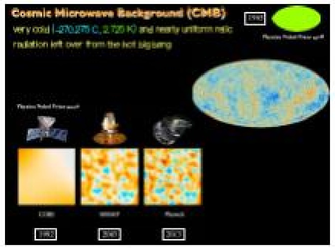
Seesaw Provids Sakharov 3 conditions (1967) for Baryon Asymmetry .

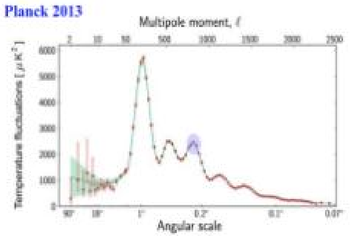
Big-Bang: start with initally equal number of matter antimatter, through annihilation... $n_B/n_\gamma \sim 10^{-20}$

Observations, nucleothesis and CMB, $n_B/n_\gamma \sim 6\times10^{-10}$

10 orders larger than expected! Needs explanation!!!

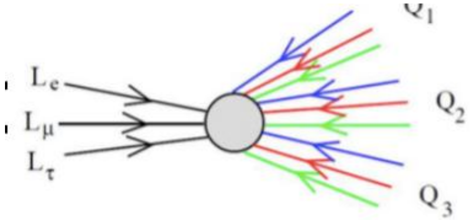


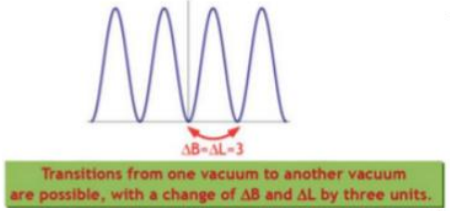




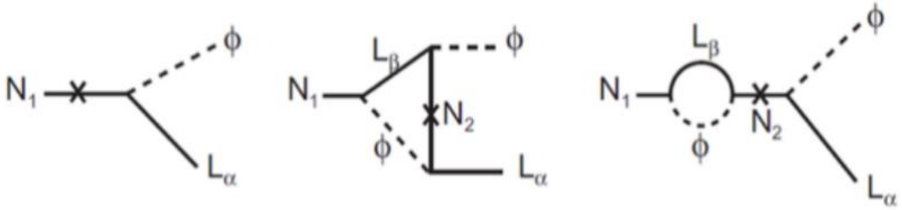
Type I and III seesaw examsples (Fukugita and Yanagida 1986)

1. Baryon number violation: Majorna neutrino mass violate lepton number by 2 units, SMSphelaron effect translates lepton violation to baryon number violation.





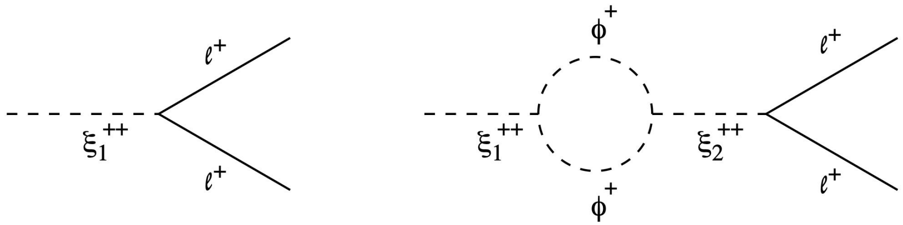
2. CP and P violations: Seesaw mass terms provide sufficiently large CP and P violating source other than that from SM.



3. Non-thermal equilibrium enviroment: Heavy enutrino decay when 1 and 2 occured.

Type-II seesaw eaxmple (E Ma and U. Sarkar, 1998)

Needs two triplet scalars to provide large CP violation.



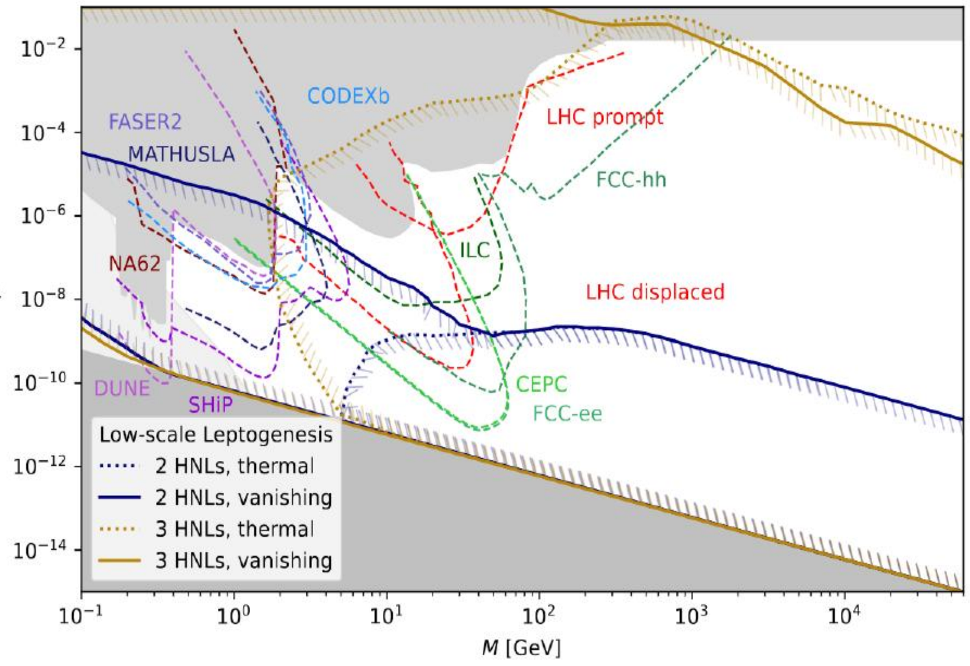
Early leptogenesis concentrated on large heavy neutrino mass of order larger than 10^9GeV.

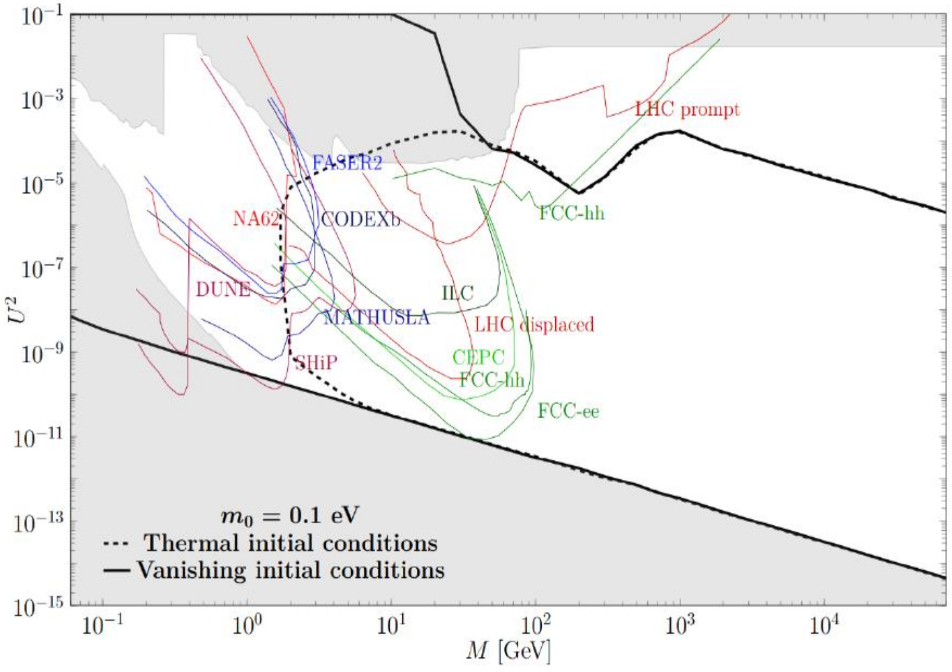
Really need that high? How to probe it? Usually taken to be grand unification scale (SO(10) model requires so) Not possible to probe the heavy degrees of freedom directly.

Lower seesaw scale? $m_\nu \sim m_\tau^2/M$, M~ 6×10^7 TeV; $m_\nu \sim m_e^2/M$, M~ 5 TeV

May be testable at colliders for heavy degrees of freedom!

Leptogenesis at TeV scae? Yes. Resonant leptogenesis (Pilaftsis and Underwood (2004)). Degenerate heavy neutrino, large rate asymmetry (Y. Georis, arXiv: 24–1.04840...)





2 heavy neutrinos are enough for leptogenesis

Frampto, Glashow and Yanagida

For Type iiI, there is an associated charged particle, a TeV mass, may be probed at colliders.

Occam Razor: no need the 3rd neutrino for leptogenesis, and make use of it to address some other problem-- plays the role of Dark Matter!

Left 2 (3) heavy neutrinos blue (orange) curves (m_0=0), right m_0 = 0.1 eV.

Dark matter from seesaw

Making one of the right-handed neutrino light, KeV ~ MeV, with small mixing, it can play the role of dark matter (Dodelson and Windrow (1994), Asaka, Blanchet, Rubakov, Smirnov(1997), Shaposhnikov(2005), Akhmedov, He and Liao (2010))

$$\Omega_{\nu_R} h^2 \sim 0.1 \frac{\sum_l |R_{l1}|^2}{10^{-8}} \left(\frac{M_1}{3 \text{ keV}} \right)^2$$

Taking ν_{R1} to be the "dark matter" neutrino, Rl1 to be the mixing between active lth neutrino

$$\nu_{R_1} \to \nu + 2\bar{\nu} \text{ and } \nu_{R_1} \to 2\nu + \bar{\nu}. \qquad \tau_{\nu_{R1}} = 5. \times 10^{26} \text{s} \left(\frac{1 \text{ keV}}{M_1} \right)^5 \frac{10^{-8}}{\sum_l |R_{l1}|^2}$$

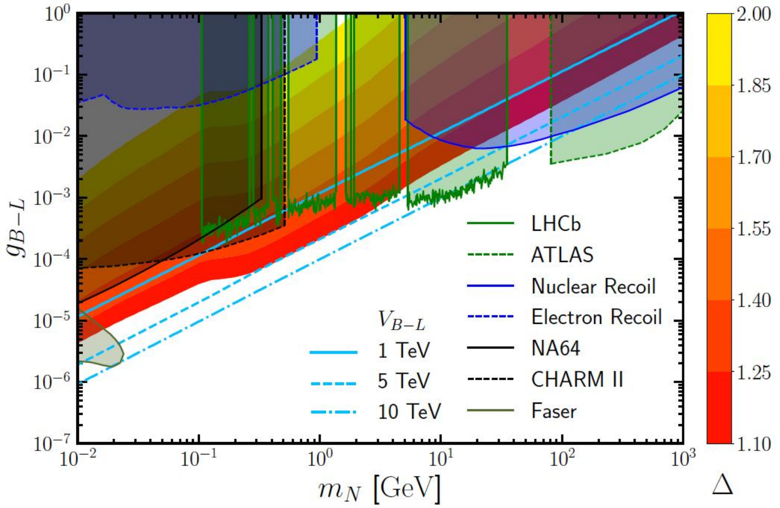
Light "heavy" neutrino. Type III seesaw not possible because associated with a charged light particle!

Recently Cheng, Sheng and Yanagida (arXiv: 2312.15637), proposed a model with symmetry for such a possibility.

$$\mathcal{L} = \frac{i}{2} \bar{N}_i \gamma^\mu \partial_\mu N_i + \left(\lambda_{i\alpha} \bar{N}_i L_\alpha H - \frac{1}{2} M_{Ri} \bar{N}_i^c N_i + \text{ h.c.} \right) - \frac{1}{2} g_{B-L} \bar{N}_i \gamma^\mu \gamma_5 N_i A'_\mu + g_{B-L} Q_{B-L} \bar{f} \gamma^\mu f A'_\mu . \quad (1)$$

A' the gauged B-L boson, Z_2 symmetry makes $\lambda_{3\alpha}=0$.

If the B–L breaking sacel is sufficiently low _ 10 TeV, dark matter in thermal bath produce the relic density



Neutrino oscillation, masses and mixing are intimately related to many big questions about our universe. There are many theoretical ideas in answering these questions. But all need to be tested by experiments.

JUNO and all other experiments, we are eagerly waiting what you will bring us.

Thank you for Listening