



# Neutrino mass generation: connections with other aspects of particle physics

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1. Introduction
2. Radiative models: connections with flavour and collider physics
3. Seesaw scale = Peccei-Quinn scale
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# 1. Introduction

The discovery of neutrino masses is the discovery of New Physics (NP).

But exactly what that physics is remains unknown.

Pure type-I seesaw model: NP at an inaccessibly high scale.

Great feature: connection with baryogenesis via leptogenesis.  
That doesn't improve the discovery prospects, unfortunately.  
I know there are other games one can play, e.g. seesaw = PQ!

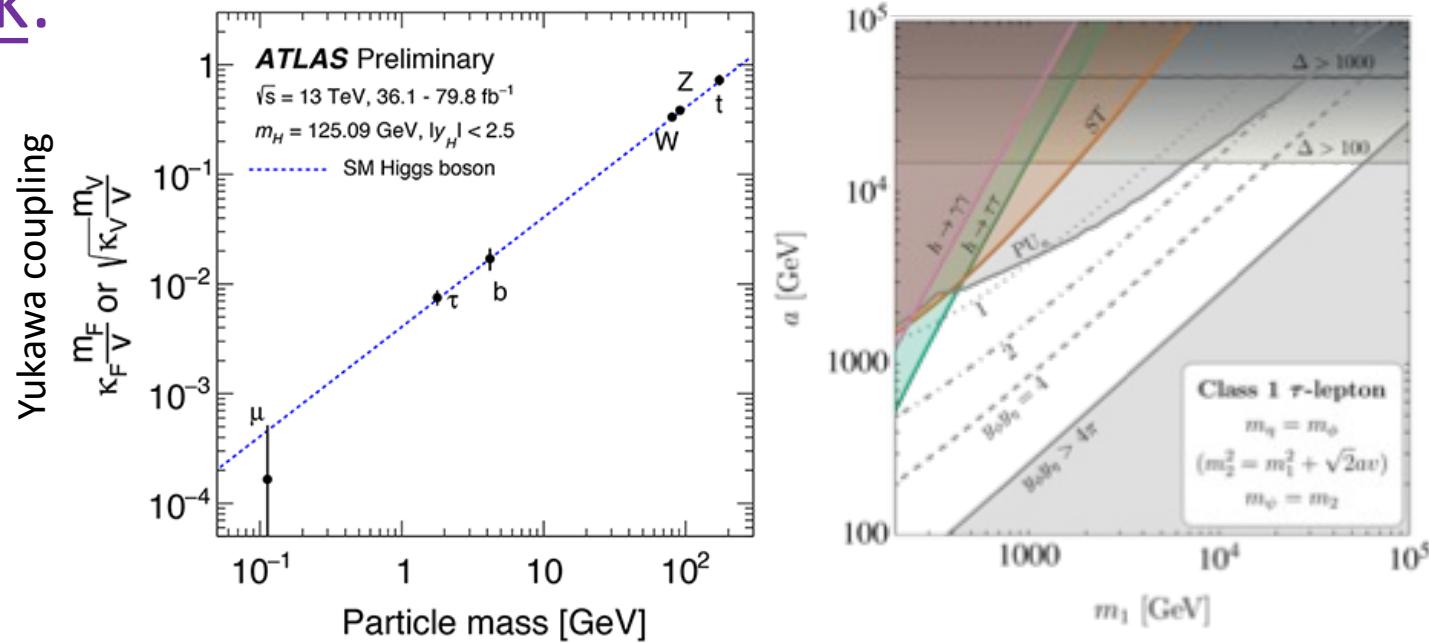
For some scenarios, connections with discoverable NP could signpost us to the origin of neutrino masses.

A driver of neutrino mass model building: the tiny  $m_\nu$ .

Two common approaches: (i) high seesaw scale  
(ii) loop-level, radiative models

## 2. Radiative models: connections with flavour and collider physics

Side remark:



Despite the leftmost plot, not yet proven that even the b and  $\tau$  masses must have SM origin.

Radiative origin also works!

Baker, Cox, RV 2021a, 2021b

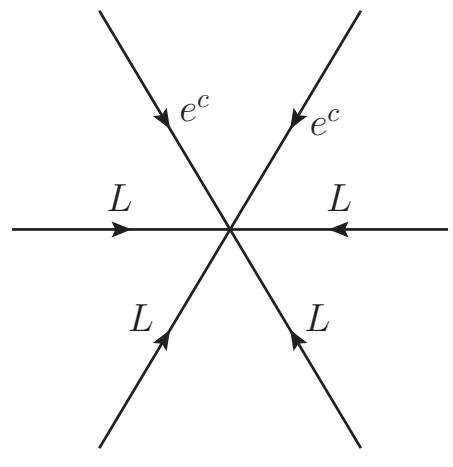
# Systematic model-building and classification procedure (Majorana case)

$\Delta L = 2$  effective operator  $\rightarrow$  open it up aka UV complete  $\rightarrow$  neutrino self-energy and mass

They occur at mass dimension 5, 7, 9, 11, ...

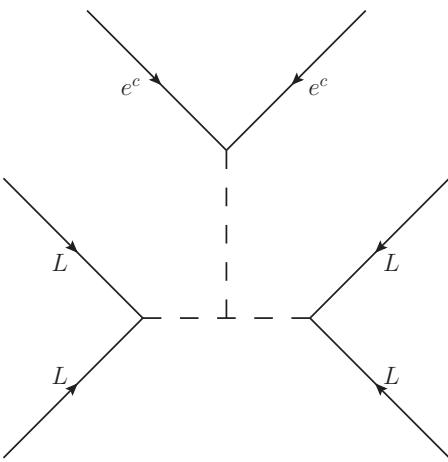
Babu, Leung 2001; de Gouvêa, Jenkins 2008; Melbourne group (many papers); Valencia group: Hirsch, Cepedello+ (many papers)

## Historic example: Zee-Babu model

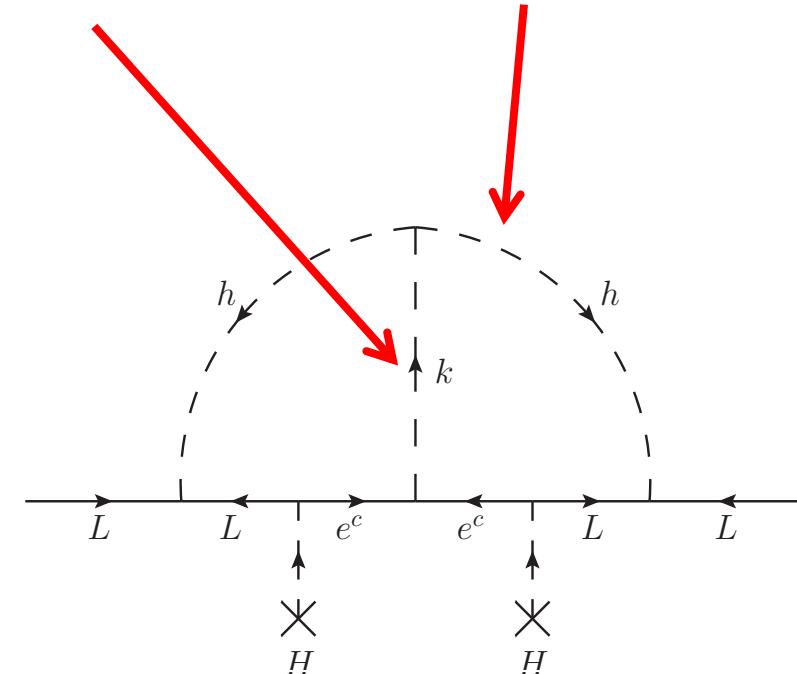


$O_9 = LLL e^c L e^c$

Effective op



Opening it up



2-loop v mass diagram

Doubly-charged  
scalar k

Singly-charged  
scalar h

The exotics k and h can be searched for at the LHC.

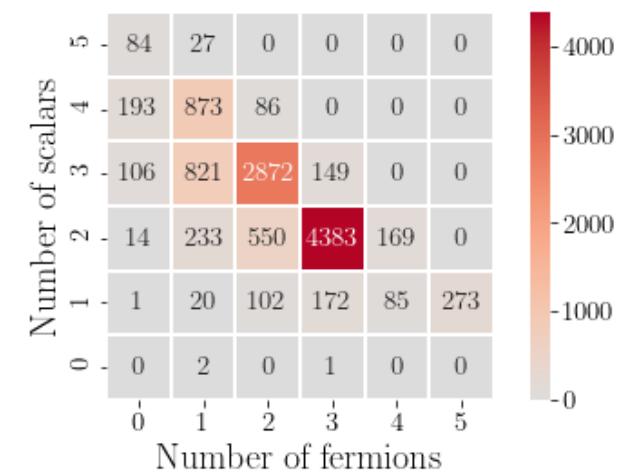
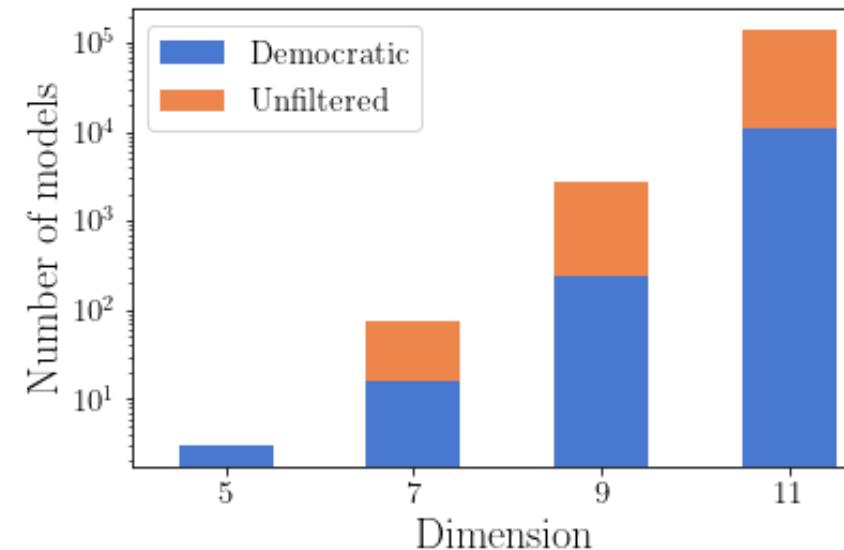
The following discussion is based on J. Gargalionis and RV, JHEP 01, 074 (2021)  
“Exploding operators for Majorana neutrino masses and beyond”

J. Gargalionis (2020), “neutrinomass” at <https://github.com/johngarg/neutrinomass>  
Full database at <https://doi.org/10.5281/zenodo.4054618>

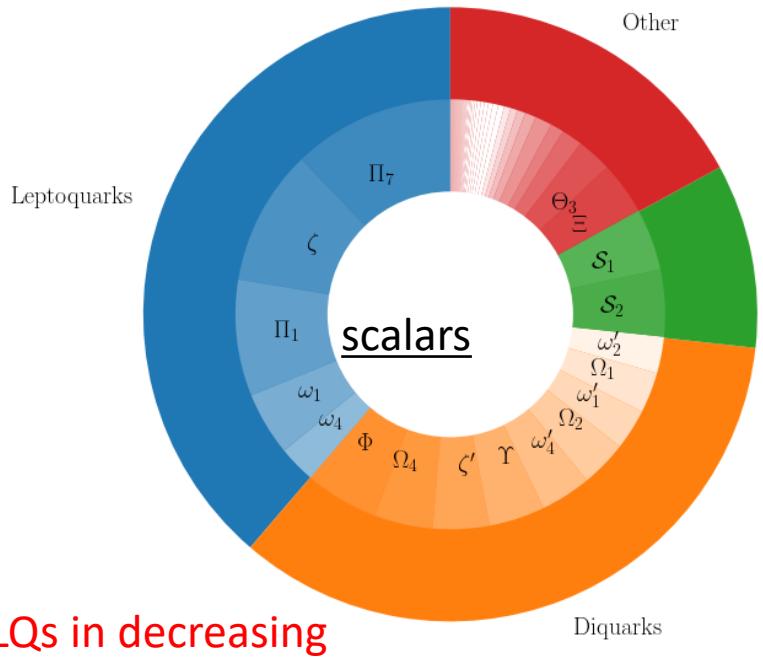
Assumptions: exotics are scalars, vector-like or Majorana fermions only, up to dim=11.

### How many models?

11,216 genuine models!



Most models have 5 exotics



LQs in decreasing order of frequency

$$\Pi_7 = R_2 \sim (3, 2, \frac{7}{6})$$

$$\zeta = S_3 \sim (\bar{3}, 3, \frac{1}{3})$$

$$\Pi_1 = \tilde{R}_2 \sim (3, 2, \frac{1}{6})$$

$$\omega_1 = S_1 \sim (\bar{3}, 1, \frac{1}{3})$$

$$\omega_4 = \tilde{S}_1 \sim (\bar{3}, 1, \frac{4}{3})$$

## Quantum numbers of the exotic species

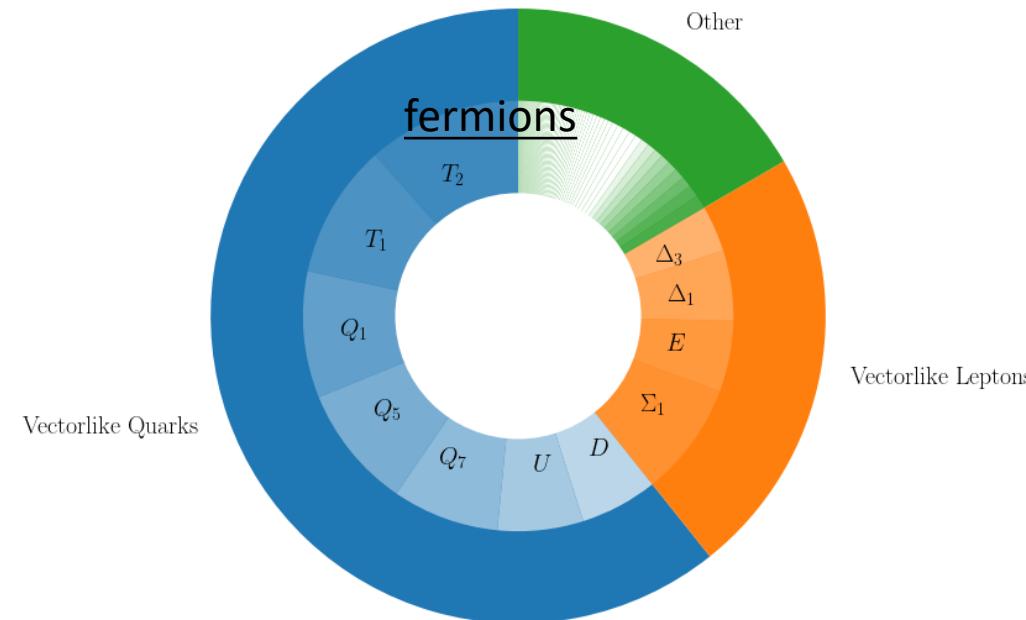
Name	$N$	$E$	$\Delta_1$	$\Delta_3$	$\Sigma$	$\Sigma_1$	
Irrep	$(\mathbf{1}, \mathbf{1}, 0)$	$(\mathbf{1}, \mathbf{1}, 1)$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$	$(\mathbf{1}, \mathbf{2}, \frac{3}{2})$	$(\mathbf{1}, \mathbf{3}, 0)$	$(\mathbf{1}, \mathbf{3}, 1)$	
Name	$U$	$D$	$Q_1$	$Q_5$	$Q_7$	$T_1$	$T_2$
Irrep	$(\mathbf{3}, \mathbf{1}, \frac{2}{3})$ , $(\bar{\mathbf{3}}, \mathbf{1}, \frac{1}{3})$	$(\bar{\mathbf{3}}, \mathbf{1}, \frac{1}{3})$	$(\mathbf{3}, \mathbf{2}, \frac{1}{6})$	$(\mathbf{3}, \mathbf{2}, -\frac{5}{6})$	$(\mathbf{3}, \mathbf{2}, \frac{7}{6})$	$(\bar{\mathbf{3}}, \mathbf{3}, \frac{1}{3})$	$(\mathbf{3}, \mathbf{3}, \frac{2}{3})$

$S_1, S_3, R_2$  (were?) of great interest for B-meson anomalies.  $R_{D(*)}$  anomaly still alive ...

$S_1, R_2$  of great interest for g-2 of muon and electron.

Symbol	$(SU(3)_c, SU(2)_L, U(1)_Y)$	Interactions	$F = 3B + L$
$S_3$	$(\bar{\mathbf{3}}, \mathbf{3}, 1/3)$	$\bar{Q}^C L$	-2
$R_2$	$(\mathbf{3}, \mathbf{2}, 7/6)$	$\bar{u}_R L, \bar{Q} e_R$	0
$\tilde{R}_2$	$(\mathbf{3}, \mathbf{2}, 1/6)$	$\bar{d}_R L$	0
$\tilde{S}_1$	$(\bar{\mathbf{3}}, \mathbf{1}, 4/3)$	$\bar{d}_R^C e_R$	-2
$S_1$	$(\bar{\mathbf{3}}, \mathbf{1}, 1/3)$	$\bar{Q}^C L, \bar{u}_R^C e_R$	-2

Table from Doršner+ 2020



## Economical and testable models: less than 4 exotic multiplets and $\Lambda < 100$ TeV

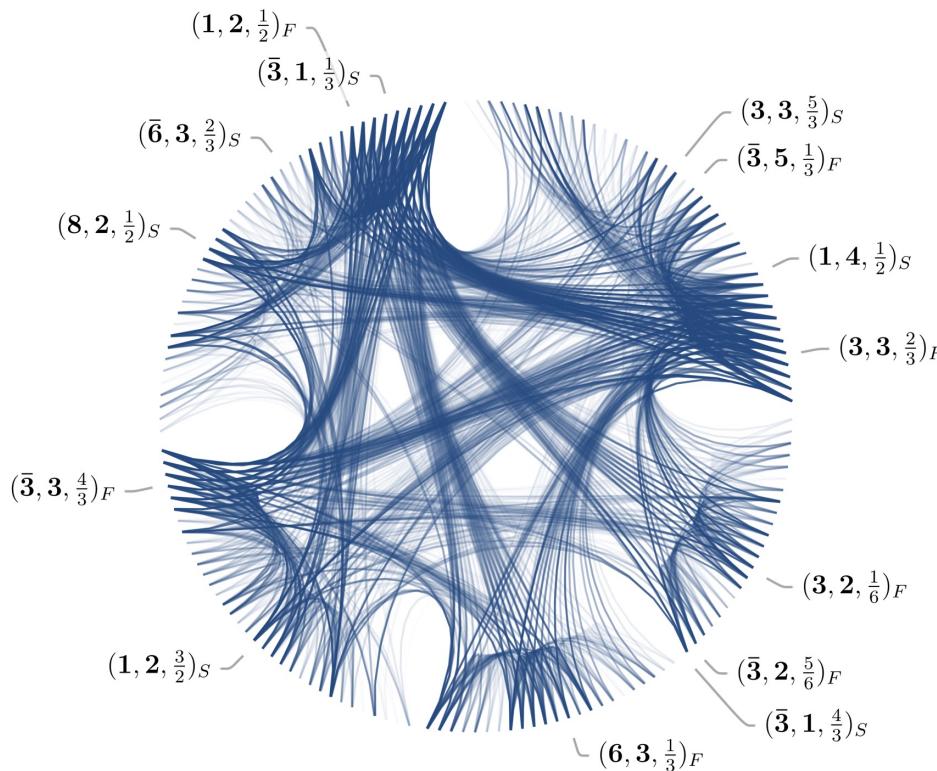
Field content	Operators	$\Lambda$ [TeV]	Dominant?	
$(3, 2, \frac{1}{6})_S, (3, 2, \frac{7}{6})_F$	$8, D15$	15	Y	The only previously known model. Y. Cai+ 2015 Klein, Lindner, Ohmer 2019
$(1, 2, \frac{1}{2})_F, (1, 1, 1)_S, (1, 2, \frac{3}{2})_S$	$62b$	16	N	
$(\bar{3}, 2, \frac{5}{6})_S, (3, 2, \frac{1}{6})_F, (3, 2, \frac{1}{6})_S$	$8'$	1	N	
$(\bar{3}, 1, \frac{1}{3})_S, (\bar{6}, 2, \frac{1}{6})_S, (3, 2, \frac{1}{6})_F$	$24f$	89	N	
$(\bar{3}, 3, \frac{1}{3})_F, (\bar{6}, 2, \frac{1}{6})_S, (3, 2, \frac{1}{6})_S$	$24d$	89	N	
$(\bar{3}, 2, \frac{5}{6})_S, (1, 2, \frac{3}{2})_F, (3, 2, \frac{1}{6})_S$	$8'$	1	N	
$(\bar{3}, 3, \frac{1}{3})_F, (\bar{6}, 4, \frac{1}{6})_S, (3, 2, \frac{1}{6})_S$	$24f$	89	N	
$(\bar{3}, 1, \frac{1}{3})_F, (\bar{6}, 2, \frac{1}{6})_S, (3, 2, \frac{1}{6})_S$	$24d$	89	N	
$(\bar{6}, 2, \frac{7}{6})_F, (8, 2, \frac{1}{2})_S, (3, 2, \frac{1}{6})_S$	20	0.8	Y	
$(6, 1, \frac{4}{3})_S, (6, 1, \frac{1}{3})_F, (3, 2, \frac{1}{6})_S$	20	0.8	Y	
$(6, 2, \frac{5}{6})_S, (3, 2, \frac{1}{6})_F, (3, 2, \frac{1}{6})_S$	$50a, b$	10	Y	
$(\bar{6}, 2, \frac{1}{6})_S, (\bar{3}, 2, \frac{5}{6})_F, (3, 2, \frac{1}{6})_S$	$50a, b$	10	Y	

Dominant contribution is from loop-level exotic-only completion of Weinberg operator.

The Y models have upper bounds on New Physics scale in the range 0.8-15 TeV.

Note: Sextets can be replaced by anti-triplets.

# Which exotics often occur together?



Each point on circumference is an exotic field.  
Lines between points indicate those fields occur together.  
The darker the colour, the more often a pairing occurs.

Rank	Edge
1	$(3, 3, \frac{2}{3})_F, (3, 4, \frac{1}{6})_S$
2	$(3, 2, \frac{1}{6})_S, (3, 2, \frac{1}{6})_F$
3	$(3, 3, \frac{2}{3})_S, (3, 2, \frac{7}{6})_S$
4	$(3, 2, \frac{7}{6})_F, (3, 2, \frac{1}{6})_S$
5	$(3, 3, \frac{2}{3})_F, (3, 4, \frac{7}{6})_F$
6	$(\bar{3}, 3, \frac{1}{3})_S, (3, 4, \frac{1}{6})_S$
7	$(3, 2, \frac{1}{6})_F, (3, 3, \frac{2}{3})_S$
8	$(\bar{3}, 3, \frac{4}{3})_F, (\bar{3}, 2, \frac{5}{6})_F$
9	$(3, 2, \frac{1}{6})_S, (3, 3, \frac{2}{3})_S$
10	$(3, 2, \frac{7}{6})_S, (\bar{3}, 2, \frac{5}{6})_F$

The ten most common pairings.

### 3. Seesaw scale = Peccei-Quinn scale

Solution of strong-CP problem by PQ mechanism => axion DM for PQ scale of  $10^{10} - 10^{11}$  GeV.

Seesaw formula  $M = \lambda^2 \frac{v^2}{m_\nu} \sim \lambda^2 10^{14}$  GeV gives  $M \sim 10^{10-11}$  GeV for  $\lambda \sim 10^{-2, -1.5}$   
(like  $m_b/v$  and  $m_\tau/v$ ).

Modify type-1 seesaw model through  $\mathcal{L} \supset h \overline{(\nu_R)^c} \nu_R S_{\text{PQ}} + h.c.$  with  $M \sim h \langle S_{\text{PQ}} \rangle$ .

And leptogenesis, of course.

Langacker, Peccei, Yanagida (1986)  
Shin (1987)  
Fukugita, Yanagida (1986)

And old idea, but a bit neglected. Recent developments: SMASH model (KSVZ extension)

Salvio (2015, 2019)

Ballesteros, Redondo, Ringwald, Tamarit (2017a, 2017b, 2019)

Explain: strong-CP, neutrino masses, baryogenesis, dark matter.

VISHv and SMASH added inflationary cosmology via non-minimal coupling to gravity.

**VISHv model (DFSZ extension/variant)**

Sopov, RV (2022)

Sopov, Tamarit, RV (work in progress)

building on RV, Davies, Joshi (1988) & Clarke, RV (2016)

What would be evidence for VISHv model?

Axion dark matter.

Two Higgs doublets.

Flavour-changing Higgs processes (to solve domain wall problem).

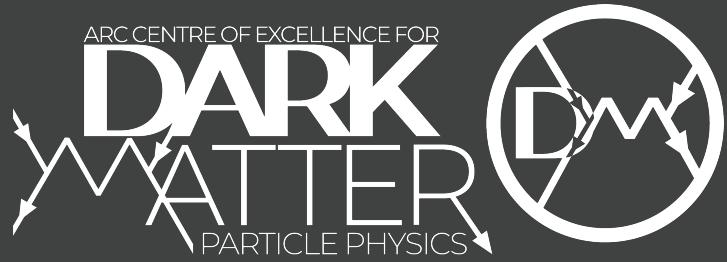
Continued excellent agreement with inflationary observables.

Lack of evidence for alternative  $m_\nu$  and baryogenesis mechanisms!

## 4. To take home

- We know neutrinos have tiny masses, but not what Lagrangian<sup>1</sup> to write in the textbooks.
- Whatever it is, it is New Physics.
- Radiative models have much new physics; connections with B-meson and g-2 anomalies.
- Type-1 seesaw scale could also be Peccei-Quinn scale; multiple connections.

<sup>1</sup> I really really want to know this.  
I really really want to know what dark matter is too.



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# Back up slides

# High-scale seesaw models

Interesting, well-known fact: lowest non-renormalisable SM effective operator is the Weinberg operator

L = LH lepton doublet

H = Higgs doublet

$\lambda$  = dimensionless coupling

M = new  $\Delta L=2$  physics scale

$$\frac{\lambda}{M} LLHH$$

⇒ Majorana neutrinos

$$m_\nu \sim \lambda \frac{v^2}{M}$$

seesaw formula

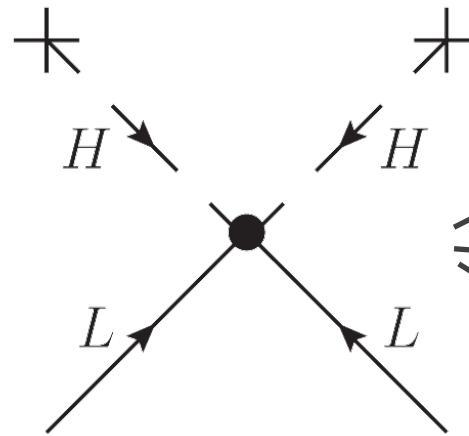
$m_\nu \ll v$  when  $M \gg v$  i.e. seesaw effect when L-violation scale very high

$$m_\nu \sim 0.1 \text{ eV}, \quad v \sim 10^2 \text{ GeV} \implies M \sim 10^{14} \text{ GeV}$$

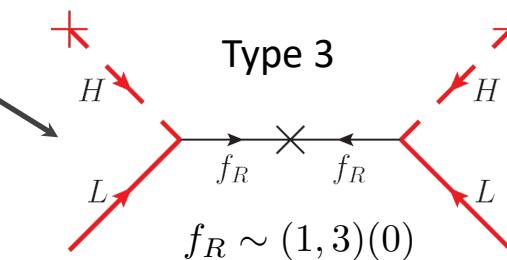
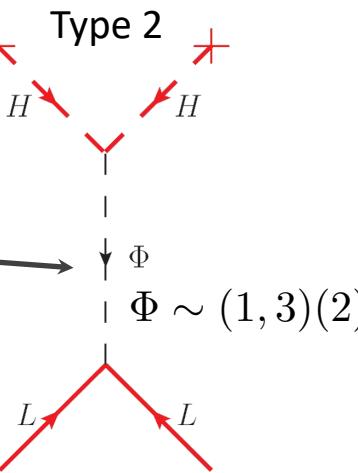
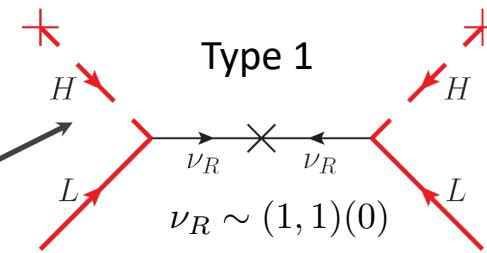
In its pure form, the seesaw scale is very high.  
Testability is very low.

## Type-1,2,3 seesaw models:

“Open up” LLHH in all minimal, tree-level ways.



Advantage of effective operator approach to constructing models is that you don't miss any.



Minkowski 1977  
Yanagida 1979  
Gell-Mann, Ramond, Slansky 1979  
Mohapatra, Senjanovic 1980

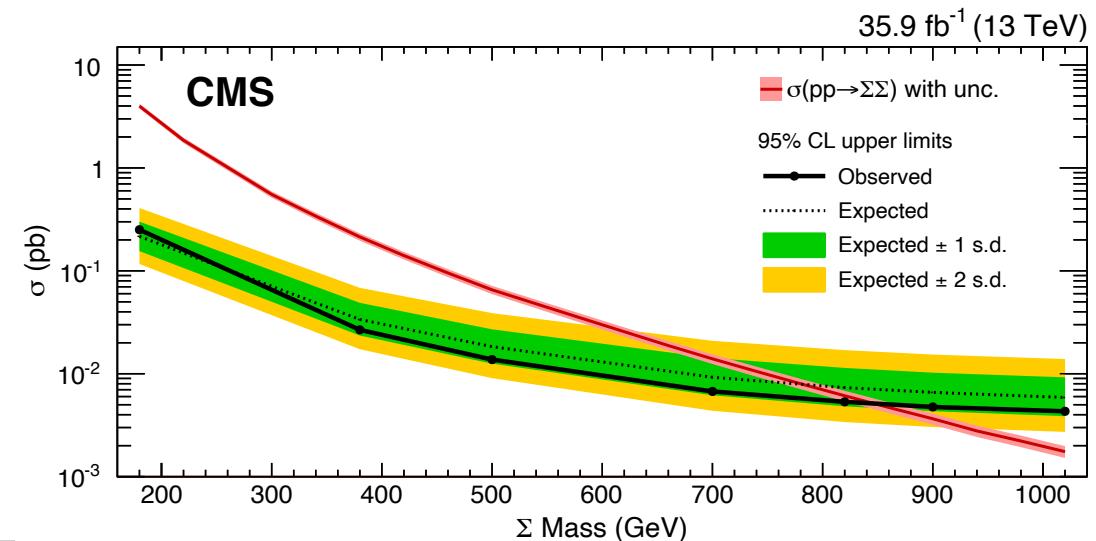
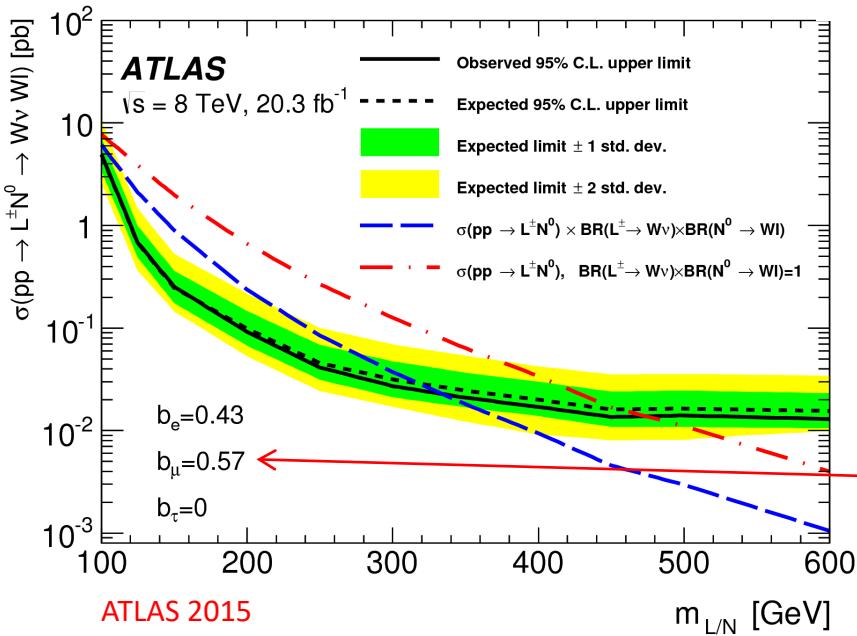
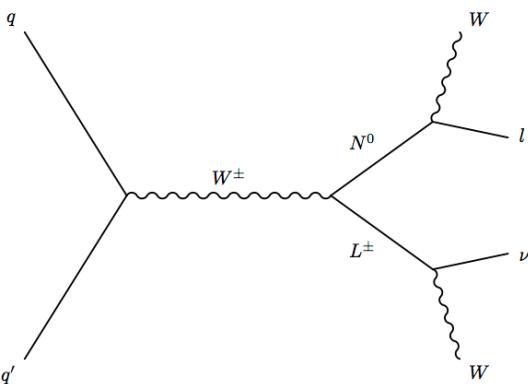
Magg, Wetterich 1980  
Schechter, Valle 1980  
Cheng, Li 1980  
Lazarides, Shafi, Wetterich 1981  
Wetterich 1981  
Mohapatra, Senjanovic 1981

Foot, Lew, He, Joshi 1989

## Example of search and bound: type 3 seesaw.

$$f_R \sim (1,3)(0) = (L^+, N^0, L^-)$$

Heavy lepton EW pair production.



Flavour democratic BR choice

Benchmark BR

There are many operators  
up to mass dimension 11 ...

Labels	Operator	Models	Filtered	Loops	$\Lambda$ [TeV]
1	$L^i L^j H^k H^l \cdot \epsilon_{ik} \epsilon_{jl}$	3	3	0	$6 \cdot 10^{11}$
2	$L^i L^j L^k \bar{e} H^l \cdot \epsilon_{ik} \epsilon_{jl}$	8	2	1	$4 \cdot 10^7$
3a	$L^i L^j Q^k \bar{d} H^l \cdot \epsilon_{ij} \epsilon_{kl}$	9	2	2	$2 \cdot 10^5$
3b	$L^i L^j Q^k d H^l \cdot \epsilon_{ik} \epsilon_{jl}$	14	5	1	$9 \cdot 10^7$
4a	$L^i L^j \tilde{Q}^k \bar{u}^\dagger H^l \cdot \epsilon_{ik} \epsilon_{jl}$	5	0	1	$4 \cdot 10^9$
4b	$L^i L^j \tilde{Q}^k \bar{u}^\dagger H^l \cdot \epsilon_{ij} \epsilon_{kl}$	4	2	2	$10 \cdot 10^6$
5a	$L^i L^j Q^k \bar{d} H^l H^m H^n \cdot \epsilon_{il} \epsilon_{jn} \epsilon_{km}$	790	36	2	$6 \cdot 10^5$
5b	$\mathcal{O}_1 \cdot Q^i \bar{d} \tilde{H}^j \cdot \epsilon_{ij}$	492	14	1,2	$6 \cdot 10^5$
5c	$\mathcal{O}_{3a} \cdot H^i \tilde{H}^j \cdot \epsilon_{ij}$	509	0	2,3	$1 \cdot 10^3$
5d	$\mathcal{O}_{3b} \cdot H^i \tilde{H}^j \cdot \epsilon_{ij}$	799	16	1,2	$6 \cdot 10^5$
6a	$L^i L^j \tilde{Q}^k \bar{u}^\dagger H^l H^m \tilde{H}^n \cdot \epsilon_{il} \epsilon_{jn} \epsilon_{km}$	289	14	2	$2 \cdot 10^7$
6b	$\mathcal{O}_1 \cdot \tilde{Q}^i \bar{u}^\dagger \tilde{H}^j \cdot \epsilon_{ij}$	177	0	1,2	$2 \cdot 10^7$
6c	$\mathcal{O}_{4a} \cdot H^i \tilde{H}^j \cdot \epsilon_{ij}$	262	0	1,2	$2 \cdot 10^7$
6d	$\mathcal{O}_{4b} \cdot H^i \tilde{H}^j \cdot \epsilon_{ij}$	208	0	2,3	$6 \cdot 10^4$
7	$L^i \bar{e}^\dagger Q^j \tilde{Q}^k H^l H^m H^n \cdot \epsilon_{il} \epsilon_{jm} \epsilon_{kn}$	240	15	2	$2 \cdot 10^5$
8	$L^i \bar{e}^\dagger u^\dagger d H^j \cdot \epsilon_{ij}$	5	1	2,3	$2 \cdot 10^1$
9	$L^i L^j L^k L^l \bar{e} \bar{e} \cdot \epsilon_{ik} \epsilon_{jl}$	14	1	2	$3 \cdot 10^3$
10	$L^i L^j L^k \bar{e} Q^l \bar{d} \cdot \epsilon_{ik} \epsilon_{jl}$	50	1	2	$6 \cdot 10^3$
11a	$L^i L^j Q^k \bar{Q}^l \bar{d} \bar{d} \cdot \epsilon_{ij} \epsilon_{kl}$	48	0	3	$4 \cdot 10^1$
11b	$L^i L^j Q^k \bar{Q}^l \bar{d} \bar{d} \cdot \epsilon_{ik} \epsilon_{jl}$	72	16	2	$1 \cdot 10^4$
12a	$L^i L^j \tilde{Q}^k \tilde{Q}^l \bar{u}^\dagger u^\dagger \cdot \epsilon_{ik} \epsilon_{jl}$	19	0	2	$2 \cdot 10^7$
12b	$L^i L^j \tilde{Q}^k \tilde{Q}^l \bar{u}^\dagger u^\dagger \cdot \epsilon_{ij} \epsilon_{kl}$	17	4	3	$6 \cdot 10^4$
13	$L^i L^j L^k \bar{e} \tilde{Q}^l \bar{u}^\dagger \cdot \epsilon_{ik} \epsilon_{jl}$	12	0	2	$2 \cdot 10^5$
14a	$L^i L^j Q^k \tilde{Q}^l \bar{u}^\dagger \bar{d} \cdot \epsilon_{ij} \epsilon_{kl}$	29	1	3	$1 \cdot 10^3$

Labels	Operator	Models	Filtered	Loops	$\Lambda$ [TeV]
14b	$L^i L^j Q^k \tilde{Q}^l \bar{u}^\dagger \bar{d} \cdot \epsilon_{ik} \epsilon_{jl}$	43	1	2	$6 \cdot 10^5$
15	$L^i L^j L^k \tilde{Q}^l \bar{u}^\dagger \bar{d} \cdot \epsilon_{ik} \epsilon_{jl}$	12	1	3	$1 \cdot 10^3$
16	$L^i L^j \bar{e} \bar{e}^\dagger \bar{u}^\dagger \bar{d} \cdot \epsilon_{ij}$	13	1	3	$1 \cdot 10^3$
17	$L^i L^j \bar{u}^\dagger \bar{d} \bar{d}^\dagger \cdot \epsilon_{ij}$	18	12	3	$1 \cdot 10^3$
18	$L^i L^j \bar{u} \bar{u}^\dagger \bar{u}^\dagger \bar{d} \cdot \epsilon_{ij}$	22	8	3	$1 \cdot 10^3$
19	$L^i \bar{e}^\dagger Q^j \bar{u}^\dagger \bar{d} \bar{d} \cdot \epsilon_{ij}$	27	0	3,4	$2 \cdot 10^{-1}$
20	$L^i \bar{e}^\dagger \tilde{Q}^j \bar{u}^\dagger \bar{u}^\dagger \bar{d} \cdot \epsilon_{ij}$	27	3	3,4	$8 \cdot 10^{-1}$
21a	$L^i L^j L^k \bar{e} \bar{Q}^l H^m H^n \cdot \epsilon_{il} \epsilon_{jm} \epsilon_{kn}$	3943	1	2,3	$2 \cdot 10^3$
21b	$L^i L^j L^k \bar{e} \bar{Q}^l H^m H^n \cdot \epsilon_{ik} \epsilon_{jm} \epsilon_{ln}$	4080	4	3	$2 \cdot 10^3$
22a	$L^i L^j L^k \bar{e} \bar{e}^\dagger H^m H^n \cdot \epsilon_{il} \epsilon_{jm} \epsilon_{kn}$	726	0	2	$2 \cdot 10^7$
22b	$\mathcal{O}_2 \cdot \tilde{L}^i \bar{e}^\dagger H^j \epsilon_{ij}$	931	0	2	$2 \cdot 10^7$
23a	$L^i L^j L^k \bar{Q}^l \bar{d}^\dagger H^m H^n \cdot \epsilon_{il} \epsilon_{jm} \epsilon_{kn}$	780	0	2,3	$4 \cdot 10^1$
23b	$\mathcal{O}_2 \cdot \tilde{Q}^i \bar{d}^\dagger H^j \cdot \epsilon_{ij}$	969	0	2,3	$4 \cdot 10^1$
24a	$L^i L^j Q^k \bar{Q}^l \bar{d} \bar{d} H^m \tilde{H}^n \cdot \epsilon_{il} \epsilon_{jn} \epsilon_{km}$	9613	193	3	$9 \cdot 10^1$
24b	$L^i L^j Q^k \bar{Q}^l \bar{d} \bar{d} H^m \tilde{H}^n \cdot \epsilon_{im} \epsilon_{jn} \epsilon_{kl}$	6058	110	3	$9 \cdot 10^1$
24c	$\mathcal{O}_{3a} \cdot Q^i \bar{d} \tilde{H}^j \cdot \epsilon_{ij}$	6022	34	3,4	1
24d	$\mathcal{O}_{3b} \cdot Q^i \bar{d} \tilde{H}^j \cdot \epsilon_{ij}$	9616	211	2,3	$9 \cdot 10^1$
24e	$\mathcal{O}_{11a} \cdot H^i \tilde{H}^j \cdot \epsilon_{ij}$	3834	18	3,4	1
24f	$\mathcal{O}_{11b} \cdot H^i \tilde{H}^j \cdot \epsilon_{ij}$	5915	131	2,3	$9 \cdot 10^1$
25a	$L^i L^j Q^k \bar{Q}^l \bar{d} \bar{d} H^m H^n \cdot \epsilon_{im} \epsilon_{jn} \epsilon_{kl}$	5960	151	2,3	$4 \cdot 10^3$
25b	$\mathcal{O}_{3a} \cdot Q^i \bar{u} \tilde{H}^j \cdot \epsilon_{ij}$	5913	9	3,4	10
25c	$\mathcal{O}_{3b} \cdot Q^i \bar{u} \tilde{H}^j \cdot \epsilon_{ij}$	14036	470	2,3	$4 \cdot 10^3$
26a	$L^i L^j L^k \bar{L}^l \bar{e}^\dagger Q^l \bar{d} H^m H^n \cdot \epsilon_{ik} \epsilon_{jm} \epsilon_{ln}$	1600	0	3	$4 \cdot 10^1$
26b	$L^i L^j L^k \bar{L}^l \bar{e}^\dagger Q^l \bar{d} H^m H^n \cdot \epsilon_{im} \epsilon_{jn} \epsilon_{kl}$	1040	0	2,3	$4 \cdot 10^1$
26c	$\mathcal{O}_{3a} \cdot \tilde{L}^i \bar{e}^\dagger H^j \cdot \epsilon_{ij}$	1149	0	3	$4 \cdot 10^1$
26d	$\mathcal{O}_{3b} \cdot \tilde{L}^i \bar{e}^\dagger H^j \cdot \epsilon_{ij}$	1797	0	2,3	$4 \cdot 10^1$
27a	$L^i L^j Q^k \bar{Q}^l \bar{d} \bar{d}^\dagger H^m H^n \cdot \epsilon_{ik} \epsilon_{jm} \epsilon_{ln}$	3851	164	2	$2 \cdot 10^7$
27b	$L^i L^j Q^k \bar{Q}^l \bar{d} \bar{d}^\dagger H^m H^n \cdot \epsilon_{im} \epsilon_{jn} \epsilon_{kl}$	2226	74	2	$2 \cdot 10^7$
27c	$\mathcal{O}_{3a} \cdot \tilde{Q}^i \bar{d}^\dagger H^j \cdot \epsilon_{ij}$	2469	33	3	$6 \cdot 10^4$
27d	$\mathcal{O}_{3b} \cdot Q^i \bar{d}^\dagger H^j \cdot \epsilon_{ij}$	3443	165	2	$2 \cdot 10^7$
28a	$L^i L^j Q^k \bar{Q}^l \bar{u}^\dagger \bar{d} H^m \tilde{H}^n \cdot \epsilon_{il} \epsilon_{jn} \epsilon_{km}$	4038	64	3	$4 \cdot 10^3$
28b	$L^i L^j Q^k \bar{Q}^l \bar{u}^\dagger \bar{d} H^m \tilde{H}^n \cdot \epsilon_{im} \epsilon_{jn} \epsilon_{kl}$	4103	0	3,4	10
28c	$L^i L^j Q^k \bar{Q}^l \bar{u}^\dagger \bar{d} H^m \tilde{H}^n \cdot \epsilon_{ik} \epsilon_{jn} \epsilon_{lm}$	4305	123	3	$4 \cdot 10^3$
28d	$\mathcal{O}_{3a} \cdot \tilde{Q}^i \bar{u} \tilde{H}^j \cdot \epsilon_{ij}$	2749	7	3,4	10
28e	$\mathcal{O}_{3b} \cdot Q^i \bar{u} \tilde{H}^j \cdot \epsilon_{ij}$	4304	90	2,3	$4 \cdot 10^3$
28f	$\mathcal{O}_{4a} \cdot Q^i \bar{d} \tilde{H}^j \cdot \epsilon_{ij}$	4039	74	2,3	$4 \cdot 10^3$
28g	$\mathcal{O}_{4b} \cdot Q^i \bar{d} \tilde{H}^j \cdot \epsilon_{ij}$	2748	14	3,4	10

4 pages omitted

Labels	Operator	Models	Filtered	Loops	$\Lambda$ [TeV]
D8c	$L^i L^j Q^k \tilde{Q}^l (DH)^m H^n \cdot \epsilon_{ik} \epsilon_{jl} \epsilon_{mn}$	25	0	2	$10 \cdot 10^6$
D8d	$L^i L^j Q^k \tilde{Q}^l (DH)^m H^n \cdot \epsilon_{im} \epsilon_{jk} \epsilon_{ln}$	53	11	1	$4 \cdot 10^9$
D8e	$L^i L^j Q^k \tilde{Q}^l (DH)^m H^n \cdot \epsilon_{im} \epsilon_{jl} \epsilon_{kn}$	44	6	1	$4 \cdot 10^9$
D8f	$L^i L^j Q^k \tilde{Q}^l (DH)^m H^n \cdot \epsilon_{im} \epsilon_{jn} \epsilon_{kl}$	30	5	1	$4 \cdot 10^9$
D8g	$L^i L^j Q^k \tilde{Q}^l (DH)^m H^n \cdot \epsilon_{ij} \epsilon_{km} \epsilon_{ln}$	35	7	2	$10 \cdot 10^6$
D8h	$L^i L^j Q^k \tilde{Q}^l (DH)^m H^n \cdot \epsilon_{ij} \epsilon_{kn} \epsilon_{lm}$	35	7	2	$10 \cdot 10^6$
D8i	$L^i L^j Q^k \tilde{Q}^l (DH)^m H^n \cdot \epsilon_{ij} \epsilon_{kl} \epsilon_{mn}$	16	3	2	$10 \cdot 10^6$
D9a	$L^i L^j \bar{d} \bar{d}^\dagger (DH)^k H^l \cdot \epsilon_{ik} \epsilon_{jl}$	30	5	1	$4 \cdot 10^9$
D9b	$L^i L^j \bar{d} \bar{d}^\dagger (DH)^k H^l \cdot \epsilon_{ij} \epsilon_{kl}$	16	4	2	$10 \cdot 10^6$
D10a	$(DL)^i L^j \bar{u}^\dagger \bar{d} H^k \tilde{H}^l \cdot \epsilon_{il} \epsilon_{jk}$	56	13	2,3	$1 \cdot 10^3$
D10b	$(DL)^i L^j \bar{u}^\dagger \bar{d} H^k \tilde{H}^l \cdot \epsilon_{ij} \epsilon_{kl}$	36	7	2,3	$1 \cdot 10^3$
D10c	$(DL)^i L^j \bar{u}^\dagger \bar{d} H^k \tilde{H}^l \cdot \epsilon_{ik} \epsilon_{jl}$	56	13	2,3	$1 \cdot 10^3$
D11	$(DL)^i L^j (D\bar{u}^\dagger) (D\bar{d}) \cdot \epsilon_{ij}$	—	—	2,3	$1 \cdot 10^3$
D12a	$L^i L^j \bar{u} \bar{u}^\dagger (DH)^k H^l \cdot \epsilon_{ik} \epsilon_{jl}$	30	5	1	$4 \cdot 10^9$
D12b	$L^i L^j \bar{u} \bar{u}^\dagger (DH)^k H^l \cdot \epsilon_{ij} \epsilon_{kl}$	16	4	2	$10 \cdot 10^6$
D13a	$(DL)^i L^j \tilde{Q}^k (\bar{d} \bar{u}^\dagger) H^l \cdot \epsilon_{ij} \epsilon_{kl}$	4	2	2	$10 \cdot 10^6$
D13b	$(DL)^i L^j \tilde{Q}^k (\bar{d} \bar{u}^\dagger) H^l \cdot \epsilon_{ik} \epsilon_{jl}$	5	0	1	$4 \cdot 10^9$
D14a	$L^i \bar{e}^\dagger Q^j \tilde{d} (DH)^k H^l \cdot \epsilon_{ik} \epsilon_{jl}$	53	0	2	$6 \cdot 10^3$
D14b	$L^i \bar{e}^\dagger Q^j \tilde{d} (DH)^k H^l \cdot \epsilon_{il} \epsilon_{jk}$	53	0	2	$6 \cdot 10^3$
D14c	$L^i \bar{e}^\dagger Q^j \tilde{d} (DH)^k H^l \cdot \epsilon_{ij} \epsilon_{kl}$	27	0	2	$6 \cdot 10^3$
D15	$(DL)^i \bar{e}^\dagger \bar{d} (D\bar{u}^\dagger) H^l \cdot \epsilon_{ij}$	5	1	2,3	$2 \cdot 10^1$
D16a	$L^i \bar{e}^\dagger \tilde{Q}^j \bar{u}^\dagger (DH)^k H^l \cdot \epsilon_{ik} \epsilon_{jl}$	58	8	2	$2 \cdot 10^5$
D16b	$L^i \bar{e}^\dagger \tilde{Q}^j \bar{u}^\dagger (DH)^k H^l \cdot \epsilon_{il} \epsilon_{jk}$	58	8	2	$2 \cdot 10^5$
D16c	$L^i \bar{e}^\dagger \tilde{Q}^j \bar{u}^\dagger (DH)^k H^l \cdot \epsilon_{ij} \epsilon_{kl}$	27	4	2	$2 \cdot 10^5$
D17	$\bar{e}^\dagger \bar{e}^\dagger \bar{d} \bar{d} (DH)^k H^l \cdot \epsilon_{ij}$	16	7	3,4	$2 \cdot 10^{-1}$
D18a	$(DL)^i L^j H^k H^l (DH)^m \tilde{H}^n \cdot \epsilon_{ik} \epsilon_{jm} \epsilon_{ln}$	53	1	0,1	$4 \cdot 10^9$
D18b	$(DL)^i L^j H^k H^l (DH)^m \tilde{H}^n \cdot \epsilon_{ik} \epsilon_{jl} \epsilon_{mn}$	53	1	0,1	$4 \cdot 10^9$
D18c	$(DL)^i L^j H^k H^l (DH)^m \tilde{H}^n \cdot \epsilon_{im} \epsilon_{jn} \epsilon_{kn}$	53	1	0,1	$4 \cdot 10^9$
D18d	$(DL)^i L^j H^k H^l (DH)^m \tilde{H}^n \cdot \epsilon_{ij} \epsilon_{km} \epsilon_{ln}$	24	1	1,2	$10 \cdot 10^6$
D18e	$(DL)^i L^j H^k H^l (DH)^m \tilde{H}^n \cdot \epsilon_{in} \epsilon_{jl} \epsilon_{km}$	34	0	1	$4 \cdot 10^9$
D18f	$(DL)^i L^j H^k H^l (DH)^m \tilde{H}^n \cdot \epsilon_{il} \epsilon_{jn} \epsilon_{km}$	34	0	1	$4 \cdot 10^9$
D19a	$(D^2 L)^i L^j (D^2 H)^k H^l \cdot \epsilon_{ij} \epsilon_{kl}$	1	0	1	$2 \cdot 10^9$
D19b	$(D^2 L)^i L^j (D^2 H)^k H^l \cdot \epsilon_{il} \epsilon_{jk}$	3	3	0	$6 \cdot 10^{11}$
D19c	$(D^2 L)^i L^j (D^2 H)^k H^l \cdot \epsilon_{ik} \epsilon_{jl}$	3	3	0	$6 \cdot 10^{11}$
D20	$L^i \bar{e}^\dagger H^j H^k H^l (DH)^m \tilde{H}^n \cdot \epsilon_{ij} \epsilon_{jm} \epsilon_{kn}$	129	0	1,2	$2 \cdot 10^5$
D21	$(DL)^i (D\bar{e}^\dagger) H^j H^k (DH)^l \cdot \epsilon_{ik} \epsilon_{jl}$	2	0	1	$4 \cdot 10^7$
D22	$\bar{e}^\dagger \bar{e}^\dagger (DH)^j H^k H^l \cdot \epsilon_{ik} \epsilon_{jl}$	9	0	2	$3 \cdot 10^3$

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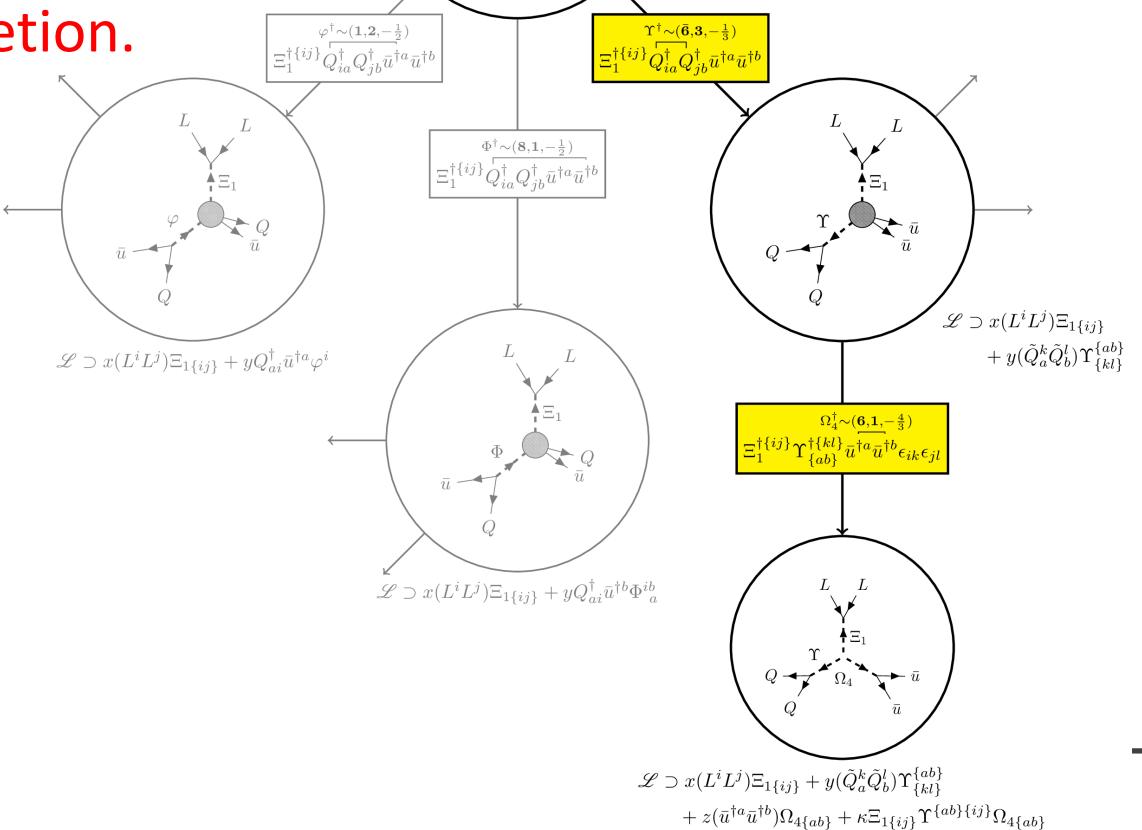
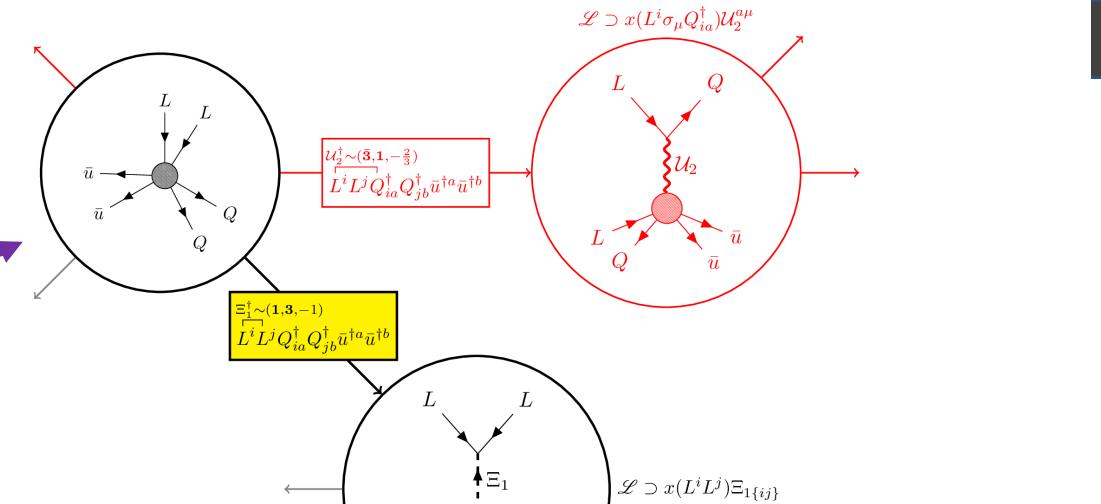
Example: opening of  $\mathcal{O}_{12a} = L^i L^j \tilde{Q}^k \tilde{Q}^\ell \bar{u}^\dagger \bar{u}^\dagger \epsilon_{ik} \epsilon_{jl}$

2-component notation:  $\tilde{Q}^i \equiv \epsilon^{ij} Q_j^\dagger$     $\tilde{Q} \bar{u}^\dagger$  = colour singlet

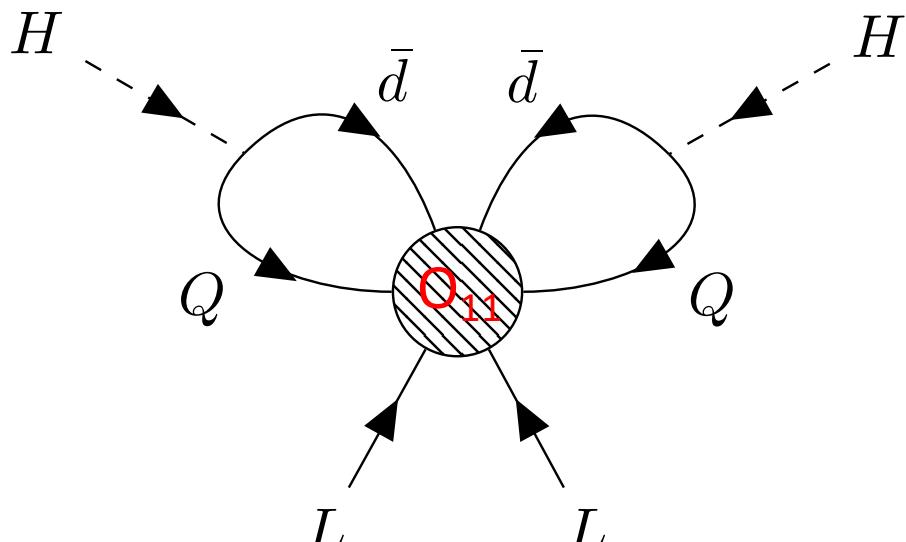
Effective operator

The yellow boxes track one pathway to opening  $\mathcal{O}_{12a}$  at tree level, producing a scalar-only completion.

The starts of other pathways are indicated in light grey.



Example: closing  $\mathcal{O}_{11} = LLQQ\bar{d}\bar{d}$  into a Majorana neutrino mass diagram.



$\mathcal{O}_{11} \rightarrow LLHH$  at 2-loop order

Opening up  $\mathcal{O}_{11}$  at tree level produces a 2-loop contribution to neutrino mass. The virtual states consist of both exotics and SM fields.

This enables the systematic construction of radiative neutrino mass models.

Tiny  $\nu$  mass due to (i) large exotic masses, (ii)  $1/16\pi^2$  loop factors, and (iii) products of mag.<1 coupling constants.

## Filtering

Opening up an operator produces **sets of exotic field content**, defining **unfiltered models**.

These fields then implicitly define the **most general renormalisable Lagrangian**.

If necessary, baryon-number conservation is imposed. Fields with the same SM quantum numbers but different B are considered different.

It often happens that the resulting interactions generate the **largest\*** contribution to neutrino masses from an **effective operator that is different from that used to derive the field content**.

The resulting models are then re-tagged against the dominant operator, giving **filtered models**.

\* We assume that there are no special parameter hierarchies when deriving the neutrino mass. This is called **democratic filtering** in the paper.

Our analysis looked at tree-level openings only. Some of our filtered models may produce exotic-only, loop-level contributions to the Weinberg operator that are larger than the closure of the dominant operator: use Valencia group's results to do more filtering.

# Models with exotic scalars and fermions only.

The tree-level seesaw models

Some operators produce no filtered models

unfiltered

Scale of new physics assuming O(1) exotic couplings

Labels	Operator	Models	Filtered	Loops	$\Lambda$ [TeV]
1	$L^i L^j H^k H^l \cdot \epsilon_{ik} \epsilon_{jl}$	3	3	0	$6 \cdot 10^{11}$
2	$L^i L^j L^k \bar{e} H^l \cdot \epsilon_{ik} \epsilon_{jl}$	8	2	1	$4 \cdot 10^7$
3a	$L^i L^j Q^k \bar{d} H^l \cdot \epsilon_{ij} \epsilon_{kl}$	9	2	2	$2 \cdot 10^5$
3b	$L^i L^j Q^k \bar{d} H^l \cdot \epsilon_{ik} \epsilon_{jl}$	14	5	1	$9 \cdot 10^7$
4a	$L^i L^j \tilde{Q}^k u^\dagger H^l \cdot \epsilon_{ik} \epsilon_{jl}$	5	0	1	$4 \cdot 10^9$
4b	$L^i L^j \tilde{Q}^k \bar{u}^\dagger H^l \cdot \epsilon_{ij} \epsilon_{kl}$	4	2	2	$10 \cdot 10^6$
5a	$L^i L^j Q^k \bar{d} H^l H^m H^n \cdot \epsilon_{il} \epsilon_{jn} \epsilon_{km}$	790	36	2	$6 \cdot 10^5$
5b	$\mathcal{O}_1 \cdot Q^i \bar{d} \tilde{H}^j \cdot \epsilon_{ij}$	492	14	1,2	$6 \cdot 10^5$
5c	$\mathcal{O}_{3a} \cdot H^i \tilde{H}^j \cdot \epsilon_{ij}$	509	0	2,3	$1 \cdot 10^3$
5d	$\mathcal{O}_{3b} \cdot H^i \tilde{H}^j \cdot \epsilon_{ij}$	799	16	1,2	$6 \cdot 10^5$
6a	$L^i L^j \tilde{Q}^k \bar{u}^\dagger H^l H^m \tilde{H}^n \cdot \epsilon_{il} \epsilon_{jn} \epsilon_{km}$	289	14	2	$2 \cdot 10^7$
6b	$\mathcal{O}_1 \cdot \tilde{Q}^i \bar{u}^\dagger \tilde{H}^j \cdot \epsilon_{ij}$	177	0	1,2	$2 \cdot 10^7$
6c	$\mathcal{O}_{4a} \cdot H^i \tilde{H}^j \cdot \epsilon_{ij}$	262	0	1,2	$2 \cdot 10^7$
6d	$\mathcal{O}_{4b} \cdot H^i \tilde{H}^j \cdot \epsilon_{ij}$	208	0	2,3	$6 \cdot 10^4$
7	$L^i \bar{e}^\dagger Q^j \tilde{Q}^k H^l H^m H^n \cdot \epsilon_{il} \epsilon_{jm} \epsilon_{kn}$	240	15	2	$2 \cdot 10^5$
8	$L^i \bar{e}^\dagger \bar{u}^\dagger \bar{d} H^j \cdot \epsilon_{ij}$	5	1	2,3	$2 \cdot 10^1$
9	$L^i L^j L^k L^l \bar{e} \bar{e} \cdot \epsilon_{ik} \epsilon_{jl}$	14	1	2	$3 \cdot 10^3$
10	$L^i L^j L^k \bar{e} Q^l \bar{d} \cdot \epsilon_{ik} \epsilon_{jl}$	50	1	2	$6 \cdot 10^3$
11a	$L^i L^j Q^k \tilde{Q}^l \bar{d} \bar{d} \cdot \epsilon_{ij} \epsilon_{kl}$	48	0	3	$4 \cdot 10^1$
11b	$L^i L^j Q^k \tilde{Q}^l \bar{d} \bar{d} \cdot \epsilon_{ik} \epsilon_{jl}$	72	16	2	$1 \cdot 10^4$
12a	$L^i L^j \tilde{Q}^k \tilde{Q}^l \bar{u}^\dagger \bar{u}^\dagger \cdot \epsilon_{ik} \epsilon_{jl}$	19	0	2	$2 \cdot 10^7$
12b	$L^i L^j \tilde{Q}^k \tilde{Q}^l \bar{u}^\dagger \bar{u}^\dagger \cdot \epsilon_{ij} \epsilon_{kl}$	17	4	3	$6 \cdot 10^4$
13	$L^i L^j L^k \bar{e} \tilde{Q}^l \bar{u}^\dagger \cdot \epsilon_{ik} \epsilon_{jl}$	12	0	2	$2 \cdot 10^5$
14a	$L^i L^j Q^k \tilde{Q}^l \bar{u}^\dagger \bar{d} \cdot \epsilon_{ij} \epsilon_{kl}$	29	1	3	$1 \cdot 10^3$
:					

Subtlety to do with the higher-dim Weinberg ops

Zee-Babu model