# Can quantum statistics help distinguish Dirac from Majorana neuitrinos?

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# Neutrinos: Dirac or Majorana?

One of the most fundamental questions in neutrino physics!

No answer despite significant experimental effort.

Reason: $m_\nu$ very small compared to typical neutrino energies.

Dirac neutrinos: 4-component; possess conserved lepton number ($\nu \neq \bar{\nu}$).

$$\nu = \nu_L + \nu_R$$

Majorana neutrinos: 2-component; no conserved lepton number ($\nu = \bar{\nu}$).

$$\nu = \nu_L + (\nu_L)^c = \nu_L + \nu_R^c$$

Mass terms in Lagrangian:

$$-\mathcal{L}^D = m_\nu \bar{\psi}_L \psi_R + h.c.\,, \qquad -\mathcal{L}^M = \frac{m_\nu}{2} \bar{\psi}_L \psi_R^c + h.c.$$

In the limit $m_\nu \to 0$ LH and RH components of neutrino fields decouple $\Rightarrow$

Both D and M neutrinos become 2-component Weyl particles.
RH components of D. $\nu$s ($\nu_R$) are sterile in the SM $\Rightarrow$

In both D. and M. cases states that can be produced and absorbed by SM weak interactions are $\nu_L$ and their CPT conjugates $\nu_R^c = (\nu_L)^c$.

$\Rightarrow$ In SM differences between D. and M. $\nu$s disappear in the limit $m_\nu \to 0$

(Case, 1957; Li & Wilczek, 1982; Kayser & Shrock, 1982; Kayser, 1982)

[N.B.: Need not be true BSM, where $\nu_R$ may not be sterile!]

*In SM smallness of $m_\nu$ makes D. and M. neutrinos practically indistinguishable*

– "Practical Dirac-Majorana Confusion Theorem" (PDMCT)

(Kayser & Shrock, 1982; Kayser, 1982)

# Can quantum statistics help?

Suggestions in the literature: Processes of $\nu\bar{\nu}$ $(\nu\nu)$ production may be special.

In M. case final state neutrinos are identical and the matrix element must be antisymmetrized w.r.t. their interchange no matter how small $m_\nu$.

Not for D. neutrinos because $\nu$ and $\bar{\nu}$ are distinct!

Can this help tell D. and M. neutrinos apart?

Claims in the literature: M. neutrino antisymmetrization in pair-production processes leads to D/M differences that survive for arbitrarily small $m_\nu$, though disappear when $m_\nu$ vanishes exactly.

$\Rightarrow$ Non-smooth behavior in $m_\nu \to 0$ limit, very counterintuitive and unsettling!

$\diamondsuit$ Our analysis: these claims are erroneous.

# PDMCT: General arguments

Crucial to PDMCT: both CC and NC $\nu$ interactions in SM are purely chiral:

$$j^{\mu}_{\rm CC}(x) = \bar{l}(x)\gamma^{\mu}(1-\gamma_5)\nu_l(x), \qquad j^{\mu}_{\rm NC} = \bar{\nu}_l(x)\gamma^{\mu}(1-\gamma_5)\nu_l(x)$$

[Strictly speaking, NC is chiral only for D. $\nu$s, for M. $\nu$s it is purely axial-vector. But in the limit $m_\nu/E \to 0$ this makes no difference.]

A well known example (for CC processes): $\beta$-decay and inverse $\beta$-decay.

It is known that electron $\nu$s produced in $\beta^+$-decays, (e.g. solar $\nu_e$) are different from $\nu$s produced in $\beta^-$-decays (e.g. in reactors), usually called $\bar{\nu}_e$. There are reaction caused by $\nu_e$s but not by $\bar{\nu}_e$s and vice versa.

Can be easily explained if $\nu$s are D. particles and possess conserved lepton number $L$: $\nu_e$ and $\bar{\nu}_e$ are then electron $\nu$s and anti-$\nu$s, resp.ly $\Rightarrow$ selection rules are just a consequence of $L$-conservation.

This does not mean that we have exp. proof that $\nu$s are D. particles!

# CC processes

Chiral structure of weak currents means that leptons participate in CC weak interactions only by their LH chirality components and antileptons by their RH chirality components.

Chirality: not a good quantum number for $m \neq 0$ fermions but for relativistic particles nearly coincides with helicity, which ***is*** conserved; the difference between them is of the order of $m_\nu/E$. For $u$-type and $v$-type spinors:

$$u_L(p) \simeq u_-(p) + \mathcal{O}\Big(\frac{m_\nu}{2E}\Big)\,, \qquad v_R(p) \simeq v_+(p) + \mathcal{O}\Big(\frac{m_\nu}{2E}\Big)\,,$$
$$u_R(p) \simeq u_+(p) + \mathcal{O}\Big(\frac{m_\nu}{2E}\Big)\,, \qquad v_L(p) \simeq v_-(p) + \mathcal{O}\Big(\frac{m_\nu}{2E}\Big)\,.$$

[$u_{L,R} = P_{L,R}u$, $v_{L,R} = P_{R,L}v$, $\pm$ stand for positive and negative helicities $h$].

The chirality selection rules of CC weak interactions play essentially the same role for relativistic M. $\nu$s as lepton number conservation plays for D. $\nu$s.

# CC processes – contd.

E.g. what we call $\bar{\nu}_e$ is an electron antineutrino in the D. case and neutrino of nearly positive helicity if $\nu$s are M. particles.

The difference: chirality is only approximately conserved for relativ. $\nu$s with $m_\nu \neq 0$ while for D. $\nu$s lepton number $L$ is conserved exactly $\Rightarrow$ In the M. case processes like detection of solar $\nu$s via IBD on $p$s are not strictly forbidden but are strongly suppressed; suppression factors are $\mathcal{O}(m_\nu/(2E))^2 \lesssim 10^{-14}$.

# NC processes: a subtlety

For M. particles vector NC vanishes identically: $\bar{\psi}(x)\gamma^\mu\psi(x) \equiv 0 \quad \Rightarrow$
Neutrino NC is purely axial-vector:

$$\bar{\nu}(x)\gamma^\mu(1-\gamma_5)\nu(x) = -\bar{\nu}(x)\gamma^\mu\gamma_5\nu(x)$$

– i.e. the interaction is not chiral!

Still it can be shown that in the limit $m_\nu/E \to 0$ this does not make any difference – relativistic M. neutrinos participating in NC processes can be considered to be in states of definite chirality, just as D. neutrinos.

(Kayser & Shrock, 1982; Kayser, 1997; Hannestad, 1997; Hansen, 1997; Zralek, 1997; Czakon, Zralek & Gluza, 1999; EA & Trautner, 2024).

$\Rightarrow$ For Majorana neutrinos the role of (nearly) conserved lepton number is played by chirality, which is approximately conserved for relativistic $\nu$s. All effects of chirality violation are suppressed by powers of $m_\nu/E$.

# $\nu$ pair production: general arguments

M. $\nu$s born in pair-production processes (e.g. $\ell^+\ell^- \to \nu\nu$) are identical, no matter how small $m_\nu$. $\Rightarrow$ strictly speaking, the amplitude must always be antisymmetrized w.r.t. their interchange.

But: one can expect that with decreasing $m_\nu$ the *observable* effects of this antisymmetrization will decrease and will become unmeasurable for arbitrarily small $m_\nu$.

With decreasing $m_\nu$ the LH and RH components of M. neutrino field, $\nu_L$ and $(\nu_L)^c \equiv \nu_R^c$, become less strongly coupled to each other. In the limit $m_\nu/E \to 0$ they decouple and behave effectively as distinct particles, and the amplitude of their pair production need not be antisymmetrized.

Technically, this should manifest itself as the suppression of the observable effects of the antisymmetrization by positive powers of $m_\nu/E$.

# D/M differences due to quantum statistics?

Ma & Pantaleone, 1989; Kogo & Tsai, 1991; Hoefer & Sehgal, 1996:

In NC processes

$$e^+e^- \to Z^* \to \nu\bar{\nu}(\nu\nu)$$

diff. cross sections in D. and M. cases are different even in the limit $m_\nu \to 0$!
But: differences disappear if the final-state $\nu$s are not detected.

C.S. Kim *at al.* 2022, C.S. Kim, 2023: in second order CC processes

$$B^0 \to \mu^+\mu^-\nu_\mu\bar{\nu}_\mu(\nu_\mu\nu_\mu)$$

D. and M. differential decay rates differ in the $m_\nu \to 0$ limit even when the final-state neutrinos are not detected! (N.B.: special back-to-back kinematics was considered).

Results of Kim et al. are incorrect, based on a computational error.
Results of Ma et al: technically correct, but contain some questionable and confusing statements.

# $e^+e^- \to Z^* \to \nu\bar{\nu}(\nu\nu)$

Ma & Pantaleone et al.: the differential coss sections for D. and M $\nu$s are

$$\frac{d\sigma^D}{d\Omega} = \frac{\sigma_0}{2}\left[f_1(1+\cos^2\theta) + 2f_2\cos\theta\right](1-n_z)(1-n'_z) + \mathcal{O}(m_\nu/E)\,.$$

Coordinate choice: $\nu$ and $\bar{\nu}$ momenta point in the positive and negative directions of the $z$-axis (in c.m.s.). $\vec{n}$ and $\vec{n}'$: unit spin vectors in rest frames.

$$\frac{d\sigma^M}{d\Omega} = \frac{\sigma_0}{2}\beta^3\{f_1[(1+n_z n'_z)(1+\cos^2\theta) - (n_x n'_x - n_y n'_y)\sin^2\theta]$$
$$-2f_2(n_z+n'_z)\cos\theta\}\ .$$

Term $\propto (n_x n'_x - n_y n'_y)$ depends on transverse spin components, not suppressed by $m_\nu/E$! *[N.B.: It comes from antisymmetrization of M. $\nu$s]*.

Ma et al.: Since massless fermions cannot have transverse spin components, this term becomes unphysical for $m_\nu = 0$ and must be dropped in this limit.

# Resolving the problem

Presence of such term quite disturbing! Prescription of dropping this (or actually any) term by hand in the limit $m_\nu = 0$ unsatisfactory.

Way out: actually already hinted by Ma et al.
Term $\propto n_x n'_x - n_y n'_y$ may only have effect if *both* final-state $\nu$s are observed and their spins are measured. The summation over even one of the spins makes it vanish.

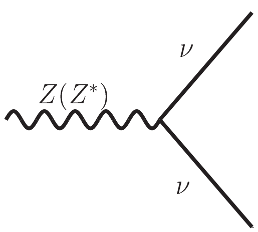
$\Rightarrow$ Observation process must be included into the consideration.
Ma et al.: mostly concentrated on the case of heavy $\nu$s. Considered their detection through decay.

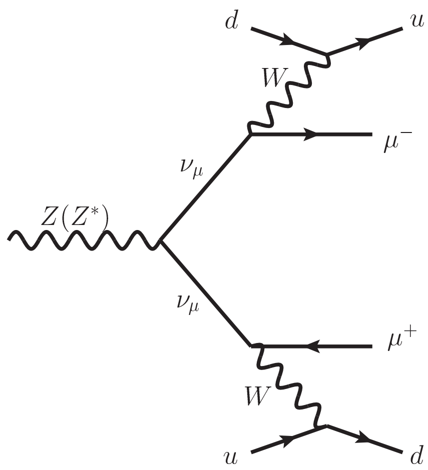
But: this cannot resolve the problem of non-smooth behavior in the $m_\nu \to 0$ limit (neutrinos become essentially stable).

# CC detection

⇒ One should consider detection processes which do not require finite $m_\nu$.
Within SM: only the processes related to neutrino gauge interactions.

I. CC processes



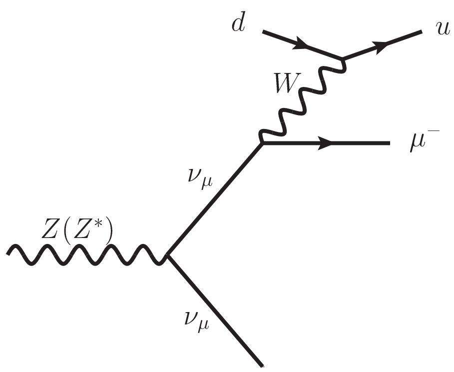




No neutrinos in the final state – nothing to be antisymmetrized!

Instructive to see how the anomalous term becomes inoperative:
Neutrino producing $\mu^-$ must be of predominantly negative-$h$ and that producing $\mu^+$ predominantly positive-$h$; in the limit $m_\nu \to 0$ they become pure helicity eigenstates ⇒ indistnguishability is lost.

# CC detection – contd.

It is broken even if only one of $\nu$s is detected!



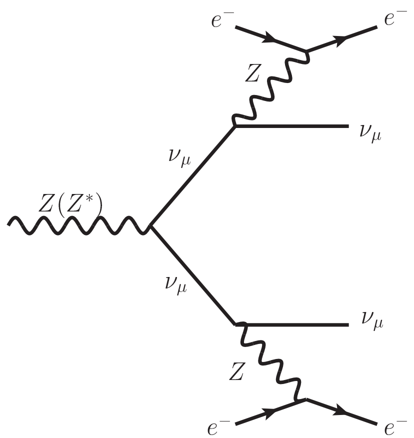

Would single out the predominant helicity of the detected $\nu$, and the other one would automatically have the opposite predominant helicity due to their entanglement: Matrix element $\langle \nu_{s_1}(p_1)\nu_{s_2}(p_2)|j^\mu_{\rm NC}(0)|0\rangle$ is

$$\frac{1}{\sqrt{2}}\left[\bar{u}_{s_1}(p_1)\gamma^\mu(1-\gamma_5)v_{s_2}(p_2)-\bar{u}_{s_1}(p_1)\gamma^\mu(1+\gamma_5)v_{s_2}(p_2)\right]$$

First term: $\nu$ of momentum $p_1$ is LH and that of momentum $p_2$ RH; second term corresponds to the opposite situation. Disentangled by detection of one $\nu$.

# NC detection

II. NC processes




*D. case.* It is not possible to find out on a case-by-case basis if a given detector has observed $\nu_\mu$ or $\bar{\nu}_\mu$; but, if one detects $\nu_\mu$, the other will detect $\bar{\nu}_\mu$ and vice versa. $\Rightarrow$ number of scattering events in both detectors in a simultaneous detection experiment is $\propto (\mathrm{d}\sigma^{\mathrm{D}}_{\nu_\mu e}/\mathrm{d}T + \mathrm{d}\sigma^{\mathrm{D}}_{\bar{\nu}_\mu e}/\mathrm{d}T)$.

*M. case.* For $m_\nu \to 0$ number of events in *each* detector $\propto (\mathrm{d}\sigma^{\mathrm{D}}_{\nu_\mu e}/\mathrm{d}T + \mathrm{d}\sigma^{\mathrm{D}}_{\bar{\nu}_\mu e}/\mathrm{d}T)$. This has to be multiplied by a factor of 2 for two detectors, but there is also a factor 1/2 because of the identical nature of the two $\nu$s in $e^+e^- \to Z^* \to \nu\nu$. The two factors compensate each other $\Rightarrow$ $D/M$ differences smoothly disappears in the limit $m_\nu \to 0$.

$\Rightarrow$ There are two possibilities:

1. Neutrinos produced in $e^+e^- \to Z^* \to \nu\nu$ reaction are not detected. The anomalous term in M. cross section is then averaged away because of the summation over the spins of the unobserved neutrinos.

2. Either one or both final-state $\nu$s are detected. The anomalous term which originates from the antisymmetrization procedure then does not appear at all.

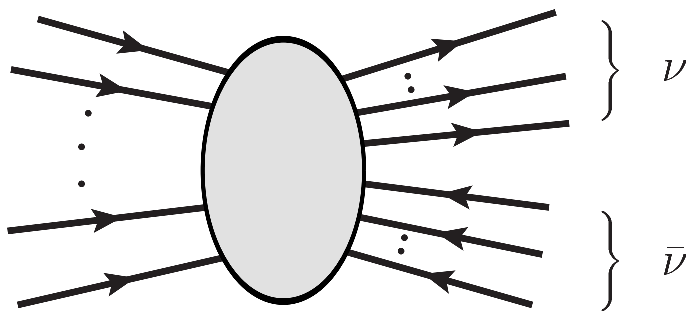
$$\Downarrow$$

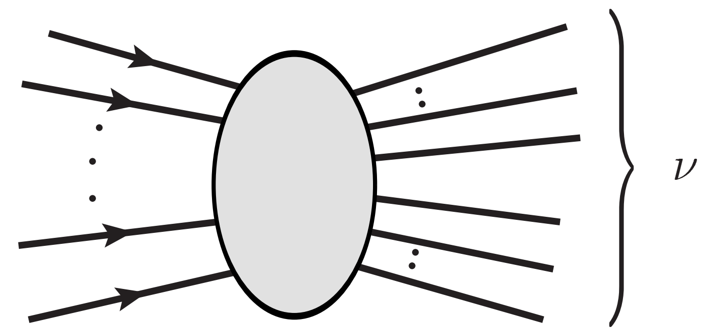
D/M differences in $e^+e^- \to Z^* \to \nu\bar{\nu}(\nu\nu)$ reaction that are not suppressed by powers of $m_\nu/E$ never appear, as far as observable quantities are concerned.

It is not necessary to drop any terms from the cross sections by hand in the case of exactly vanishing $m_\nu$.

# General analysis of quant. stat. effects

Antisymmetrization must be done for final-state neutrinos




D. case: among all $\nu$s and among all $\bar{\nu}$s

M. case: among all $\nu$s

D. case: neutrinos are born in LH and antineutrinos in RH chirality states.
M. case: up to corrections $\mathcal{O}(m_\nu/E)$ $\nu$s are also produced in states of definite chirality. For $m_\nu \to 0$ oppositie chirality contributions to amplitudes do not interfere $\Rightarrow$

To leading order in $m_\nu/E$ antisymmetrization should only be done in the M. case for the same neutrinos for which it should be performed in the D. case.

# Summary

- Claims that neutrino for pair-production processes there are D/M differences that do not disappear in the limit $m_\nu \to 0$ are incorrect.

- For $m_\nu/E \ll 1$ chirality plays for Majorana neutrinos essentially the same role as lepton number plays for Dirac neutrinos. All observable deviations from this rule are suppressed by powers of $m_\nu/E$.

- To leading order in $m_\nu/E$ antisymmetrization should only be done in the M. case for the same $\nu$s for which it should be carried out in the D. case. All effects of “additional” antisymmetrization related to their M. nature are suppressed at least as $m_\nu/E$.

- Within the SM quantum statistics does not lead to any exceptions to the Practical Dirac-Majorana Confusion Theorem.

# Backup slides

# NC-induced scattering

A subtlety: For M. particles vector NC vanishes identically: $\bar{\psi}(x)\gamma^\mu\psi(x) \equiv 0$

$\Rightarrow$ For M. $\nu$s NC is purely axial-vector:

$$\bar{\nu}(x)\gamma^\mu(g_V - g_A\gamma_5)\nu(x) = -g_A\bar{\nu}(x)\gamma^\mu\gamma_5\nu(x)$$

NC matrix element $\langle\nu(p')|j^\mu_{\rm NC}(0)|\nu(p)\rangle$ in the M. case:

$$\bar{u}(p')\gamma^\mu(g_V - g_A\gamma_5)u(p) - \bar{v}(p)\gamma^\mu(g_V - g_A\gamma_5)v(p') =$$
$$\bar{u}(p')\gamma^\mu(g_V - g_A\gamma_5)u(p) - \bar{u}(p')\gamma^\mu(g_V + g_A\gamma_5)u(p) = -2g_A\bar{u}(p')\gamma^\mu\gamma_5 u(p)\,.$$

In the D. case:

$$\bar{u}(p')\gamma^\mu(g_V - g_A\gamma_5)u(p) = -(g_V + g_A)\bar{u}(p')\gamma^\mu\gamma_5 u(p) + \mathcal{O}\left(\frac{m_\nu}{2E}\right)$$

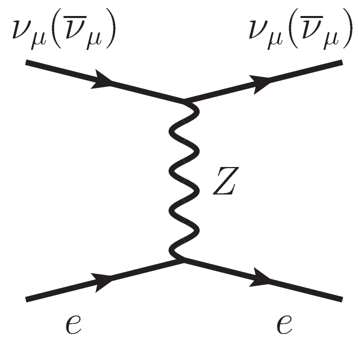
(taking into account that $\gamma_5 u \simeq -u$ for relativistic D. $\nu$s). In the SM $g_V = g_A$

$\Rightarrow$ results for D. and M. cases coincide up to terms $\mathcal{O}(m_\nu/2E)$

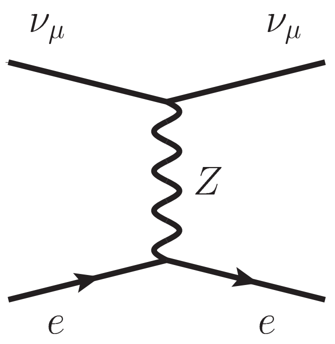
(Kayser & Shrock, 1982; Kayser, 1997; Hannestad, 1997; Hansen, 1997; Zralek, 1997; Czakon, Zralek & Gluza, 1999).

# Important ingredient in the argument:

Incident neutrinos and/or antineutrinos in NC scattering experiments were produced in CC processes (e.g. $\pi^\pm$ decays for $\nu_\mu e$ scattering expts.) and arrived in states of nearly definite chirality




D. $\nu$ case: $Z\nu\nu$ vertex $\gamma^\mu(1-\gamma_5)$




M. $\nu$ case: $Z\nu\nu$ vertex $\gamma^\mu\gamma_5$

D. case: $V-A$ interactions projects out states of definite chirality.
M. case: axial-vector interaction does not project out definite chirality, but does not flip it either: if incoming state is of LH (RH) chirality, so will be the outgoing state.

$\Rightarrow$ Chirality considerations play a crucial role for PDMCT in NC processes, just as it does in CC processes.

# Incident neutrinos from NC pair production?

What if incident neutrinos in NC experiment were produced in NC pair-production process? (has not been considered before).

In the M. the amplitude depends on the matrix element $\langle\nu_{s_1}(p_1)\nu_{s_2}(p_2)|j^\mu_{\rm NC}(0)|0\rangle$ given by

$$\frac{1}{\sqrt{2}}\left[\bar{u}_{s_1}(p_1)\gamma^\mu(1-\gamma_5)v_{s_2}(p_2)-\bar{u}_{s_1}(p_1)\gamma^\mu(1+\gamma_5)v_{s_2}(p_2)\right]$$

$$=-\sqrt{2}\bar{u}_{s_1}(p_1)\gamma^\mu\gamma_5 v_{s_2}(p_2)\,.$$

Pure axial-vector nature of the NC in the M. case is a result of coherent superposition of LH and RH contributions with equal weights.

$\Rightarrow$ In general, $\nu$s are produced in such processes in states of no definite chirality.

But: In the limit $m_\nu\to 0$ LH and RH contributions do not interfere. $\Rightarrow$ neutrinos born in NC processes can also be considered as being in states of definite chirality.

# NC detection: $\nu_\mu e \to \nu_\mu e$ scattering

*D. case.* Matrix elements for $\nu_\mu e$ and $\bar\nu_\mu e$ scattering, up to a const. factor:

$$\mathcal{M}^{\rm D}_{\nu_\mu e} = \mathcal{J}_\mu \left[\bar u_{s_1'}(p_1')\gamma^\mu(1-\gamma_5)u_{s_1}(p_1)\right],$$
$$\mathcal{M}^{\rm D}_{\bar\nu_\mu e} = \mathcal{J}_\mu \left[\bar v_{s_1}(p_1)\gamma^\mu(1-\gamma_5)v_{s_1'}(p_1')\right] = \mathcal{J}_\mu \left[\bar u_{s_1'}(p_1')\gamma^\mu(1+\gamma_5)u_{s_1}(p_1)\right].$$

$\mathcal{J}_\mu$: convolution of the electron NC matrix element $j_e^\nu$ and the $Z^0$-boson propagator $D^Z_{\nu\mu}$: $\mathcal{J}_\mu = j_e^\nu D^Z_{\nu\mu}$.
It is not possible to find out on a case-by-case basis if a given detector has observed $\nu_\mu$ or $\bar\nu_\mu$; but, if one detects $\nu_\mu$, the other will detect $\bar\nu_\mu$ and vice versa. $\Rightarrow$ number of scattering events in both detectors in a simultaneous detection experiment is $\propto (\mathrm{d}\sigma^{\rm D}_{\nu_\mu e}/\mathrm{d}T + \mathrm{d}\sigma^{\rm D}_{\bar\nu_\mu e}/\mathrm{d}T)$.

*M. case.* Matrix element of $\nu_\mu e$ scattering

$$\mathcal{M}^{\rm M}_{\nu_\mu e} = \mathcal{J}_\mu \left[\bar u_{s_1'}(p_1')\gamma^\mu(1-\gamma_5)u_{s_1}(p_1) - \bar v_{s_1}(p_1)\gamma^\mu(1-\gamma_5)v_{s_1'}(p_1')\right]$$
$$= \mathcal{J}_\mu \left[\bar u_{s_1'}(p_1')\gamma^\mu(1-\gamma_5)u_{s_1}(p_1) - \bar u_{s_1'}(p_1')\gamma^\mu(1+\gamma_5)u_{s_1}(p_1)\right].$$
$$\mathcal{M}^{\rm M}_{\nu_\mu e} = \mathcal{M}^{\rm D}_{\nu_\mu e} - \mathcal{M}^{\rm D}_{\bar\nu_\mu e}.$$

# NC detection – contd.

Squared matrix element of the process: $L_{\mu\nu}N^{\mu\nu}$ with $L_{\mu\nu} = \mathcal{J}_\mu \mathcal{J}_\nu^*$.

$N^{\mu\nu}$ depends on $\nu$ nature and on the process. In the D. case:

$$N^{(\mathrm{D})\mu\nu}_{\nu_\mu e} = \left[\bar{u}_{s_1'}(p_1')\gamma^\mu(1-\gamma_5)u_{s_1}(p_1)\right]\left[\bar{u}_{s_1}(p_1)\gamma^\nu(1-\gamma_5)u_{s_1'}(p_1')\right],$$

$$\begin{aligned} N^{(\mathrm{D})\mu\nu}_{\bar{\nu}_\mu e} &= \left[\bar{v}_{s_1}(p_1)\gamma^\mu(1-\gamma_5)v_{s_1'}(p_1')\right]\left[\bar{v}_{s_1'}(p_1')\gamma^\nu(1-\gamma_5)v_{s_1}(p_1)\right] \\ &= \left[\bar{u}_{s_1'}(p_1')\gamma^\mu(1+\gamma_5)u_{s_1}(p_1)\right]\left[\bar{u}_{s_1}(p_1)\gamma^\nu(1+\gamma_5)u_{s_1'}(p_1')\right]. \end{aligned}$$

In the M. case:

$$N^{(\mathrm{M})\mu\nu}_{\nu_\mu e} = N^{(\mathrm{D})\mu\nu}_{\nu_\mu e} + N^{(\mathrm{D})\mu\nu}_{\bar{\nu}_\mu e} - T^{\mu\nu}.$$

$T^{\mu\nu}$ comes from the interference of the two terms:

$$\begin{aligned} T^{\mu\nu} &= \left[\bar{u}_{s_1'}(p_1')\gamma^\mu(1-\gamma_5)u_{s_1}(p_1)\right]\left[\bar{u}_{s_1}(p_1)\gamma^\nu(1+\gamma_5)u_{s_1'}(p_1')\right] \\ &\quad + \left[\bar{u}_{s_1'}(p_1')\gamma^\mu(1+\gamma_5)u_{s_1}(p_1)\right]\left[\bar{u}_{s_1}(p_1)\gamma^\nu(1-\gamma_5)u_{s_1'}(p_1')\right] \\ &= -\,\mathrm{tr}\{\not{p}_1'\not{s}_1'\gamma^\mu\not{p}_1\not{s}_1\gamma^\nu - m_\nu\left(\not{p}_1'\not{s}_1'\gamma^\mu\gamma^\nu - \gamma^\mu\not{p}_1\not{s}_1\gamma^\nu\right)\gamma_5 - m_\nu^2\gamma^\mu\gamma^\nu\}. \end{aligned}$$

# NC detection – contd.

$s_1$ and $s'_1$ are the spin four-vectors of the incident and scattered neutrinos,

$$s_1^\mu = \left( \frac{\vec{p}_1 \cdot \vec{n}_1}{m_\nu}, \, \vec{n}_1 + \frac{(\vec{p}_1 \cdot \vec{n}_1)\vec{p}_1}{m_\nu(E_1 + m_\nu)} \right),$$

with $\vec{n}_1$ being the unit spin vector of the incoming neutrino in its rest frame, and similarly for $s_1'^\mu$.

Detection of $\nu$s produced in $e^+e^- \to Z^* \to \nu\nu$ achieved through the measurement of electron recoil in $\nu_\mu e$ scattering, i.e. the scattered neutrinos in the final state are not observed. Summation over the spin $s_1'$ has to be done:

$$\sum_{s'} T^{\mu\nu} = \mathrm{tr}\{ -m_\nu \gamma^\mu \not{p}_1 \not{s}_1 \gamma^\nu \gamma_5 + m_\nu^2 \gamma^\mu \gamma^\nu \}\,.$$

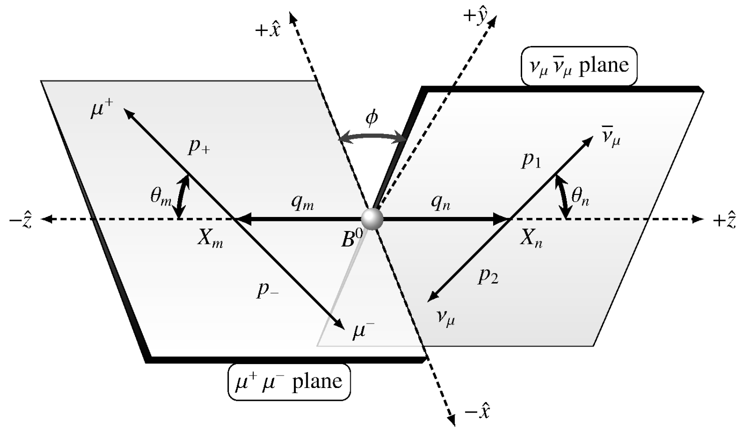
$\Rightarrow$ the interference term $T^{\mu\nu}$ is suppressed at least as $m_\nu/E$.

For negligibly small $m_\nu$ the number of events in one detector in the M. case is $\propto$ the sum of the D. neutrino cross sections of $\nu_\mu e$ and $\bar{\nu}_\mu e$ scattering events. This has to be multiplied by a factor of 2 for two detectors, but there is also a factor 1/2 because of the identical nature of the two neutrinos in $e^+e^- \to Z^* \to \nu\nu$. The two factors compensate each other $\Rightarrow$

The total number of events in the simultaneous neutrino detection experiment remains $\propto \left(\frac{\mathrm{d}\sigma^{\mathrm{D}}_{\nu_\mu e}}{\mathrm{d}T} + \frac{\mathrm{d}\sigma^{\mathrm{D}}_{\bar{\nu}_\mu e}}{\mathrm{d}T}\right)$ with the same proportionality factor as in the D. neutrino case.

$\Rightarrow$ Also in the case of $\nu$ detection through NC processes the difference between the cross sections for D. and M. neutrinos smoothly disappears in the limit $m_\nu \to 0$, without any need to drop any terms in the squared matrix element by hand.

# Kim et al.: $B^0 \to \mu^+\mu^-\nu_\mu\bar{\nu}_\mu(\nu_\mu\nu_\mu)$ decay




$\vec{q}_m = \vec{p}_+ + \vec{p}_-$, $\quad \vec{q}_n = \vec{p}_1 + \vec{p}_2$. B2b kinematics: $\vec{q}_m \to 0$, $\vec{q}_n \to 0$.

Kim et al.: $\cos\theta_m = \sin\theta$ (??) ($\theta$: the angle between $\mu$ and $\nu$ directions).

$$\frac{\mathrm{d}^3\Gamma^D_{\leftrightarrow}}{\mathrm{d}E_\mu^2\,\mathrm{d}\sin\theta} = \frac{G_F^4 |\mathbb{F}_a|^2\,(m_B - 2\,E_\mu)^4\,K_\mu}{512\,\pi^6\,m_B\,E_\mu}(E_\mu - K_\mu\,\cos\theta)^2,$$

$$\frac{\mathrm{d}^3\Gamma^M_{\leftrightarrow}}{\mathrm{d}E_\mu^2\,\mathrm{d}\sin\theta} = \frac{G_F^4 |\mathbb{F}_a|^2\,(m_B - 2\,E_\mu)^4\,K_\mu}{512\,\pi^6\,m_B\,E_\mu}\left(E_\mu^2 + K_\mu^2\cos^2\theta\right).$$

$\mathrm{d}\sin\theta \to \mathrm{d}\cos\theta$: Upon integration over $\mathrm{d}\cos\theta$ RHSs coincide – no D/M difference for $m_\nu \to 0$.