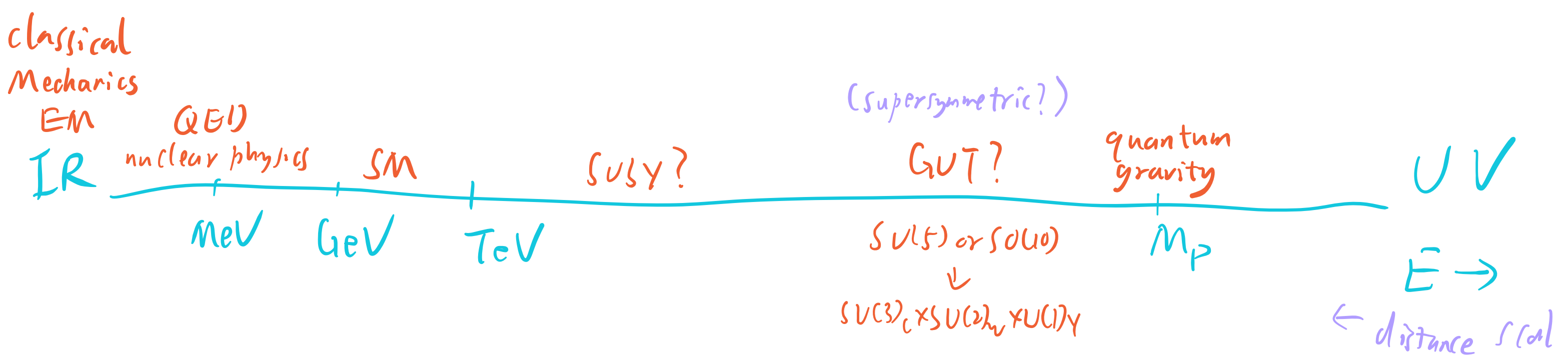
SMEFT phenomenology

Why EFT?

(we think) Every theory is an effective theory!

natural units: 1 unit $[E] \sim [L]^{-1}$



A more fundamental theory will appear at a higher energy / smaller scale.

Or maybe there is some ultimate theory? → stops somewhere?

Quantum gravity ⟸ space time quantized?

notion of energy / distance breaks down?

Key ingredient: locality (many definitions, here it means)

Measurements at large distance / low energy should not be sensitive to the physics at small distance / high energy.

Engineers don't need to learn QFT to build bridges / car!

classical Mechanics is not wrong, it's a low energy effective theory.

why SMEFT ?

- SM is incomplete (gravity, dark matter, matter anti-matter asymmetry ......)
- There must be BSM New physics

  but we don't know what it is. (some people think they know ....)

  light particle
  very weak coupling
  ~~SMEFT~~

  heavy particle ($m \gg v$)
  SMEFT

- bottom up approach

  Be agnostic about the UV physics and try to systematically parameterize its effects at low energies.

  write down all possibilities

  $\Rightarrow$ write down a basis (eliminate redundant operators)

  $\Rightarrow$ all wilson coefficients are free parameters
  to be measured by experiments (pheno)

- top down approach

New physics model $\xrightarrow[\text{integrating out heavy new particles}]{\text{matching}}$ SMEFT

Wilson coefficients depend on parameters of the New physics model.

SMEFT is still useful! match & running .....

useful refs.

Manohar's lectures on EFT 1804.05863 or TASI 2022

Skiba's TASI lecture notes (a bit old) 1006.2142

Warsaw basis 1008.4884

Higgs+EW SMEFT Pomarol et al. 1308.1879 (Barcelona)

SMEFTsim 3.0 Ilaria Brivio 2012.11343 (MadGraph package)

SMEFT at work Isidori et al. 2303.16922

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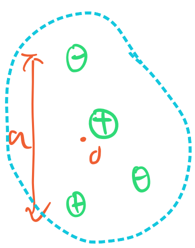
example 1 Multipole Expansion in Electrostatics



$a \sim$ scale of the charge distribution

$\vec{r}$. $V(\vec{r})$ electric potential

locality: For $r \gg a$, this looks like a point charge!

$$V(\vec{r}) = \frac{1}{r} \sum_{l,m} b_{lm} \frac{1}{r^l} \underbrace{Y_{lm}(\theta, \varphi)}_{\text{spherical harmonics}}$$

$$= \frac{1}{r} \sum_{\substack{l,m \\ =0,1,\dots}} c_{lm} \left(\frac{a}{r}\right)^l Y_{lm} \qquad b_{lm} \equiv c_{lm} a^l$$

c_{lm} s are dimensionless parameters (usually of order 1).

<u>seperation of scales</u> $\left(\frac{a}{r} \ll 1\right)$ $\Rightarrow$ The expansion is useful,

i.e. we only need to keep a few terms in the l expansion to get a good approximation (more terms $\Rightarrow$ better accuracy)

	distance	energy	
IR	r	$E \sim 1/r$	collider energy (or v)
UV	a	$\Lambda \sim 1/a$	scale of new physics
expansion parameter	$\frac{a}{r}$	$\frac{E}{\Lambda}$	(or $\frac{v}{\Lambda}$)

- There is no precise definition of a. One could only measure the combination $b_{lm} = c_{lm} a^l$. $\left(\frac{c}{\Lambda^n}\right)$

- If we know the charge distribution (UV theory), we can do the expansion to find out all the b_{lm} (c_{lm}). This is called matching.

  If we don't know ...... we can treat all c_{lm} s as free parameters and try to measure them experimentally.

- After truncating the series (throwing away terms with $l > l_{max}$) there are a finite number of parameters.

  If we make enough measurements we can constrain all parameters. (C_{lm})

- To precisely determine the values of C_{lm} we can either
  - make very precise measurement at large r (low energy)
  - make measurements at small r (high energy)

energy vs. precision (or both!)

Important aspects for colliders.

If $r \approx a$, the expansion breaks down!

multiple scales

High energy is always good, but EFT may not be valid!

$$r \gg a \gg d$$

$$(\Lambda_{EW} \ll \Lambda_{SUSY} \ll \Lambda_{GUT})$$

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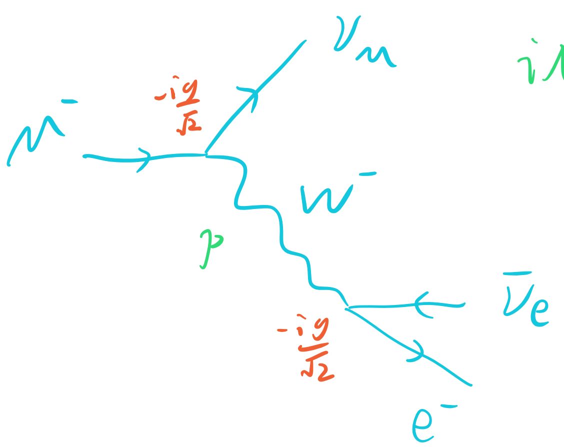
example 2 Fermi's theory

muon decay



$$i\mathcal{M} = \left(\frac{-ig}{\sqrt{2}}\right)^2 (\bar{\nu}_\mu \gamma^\mu \mu_L)(\bar{e}_L \gamma^\nu \nu_e) \cdot \frac{-ig_{\mu\nu}}{p^2 - M_W^2}$$

(ignore W width since $p^2 \ll M_W^2$)

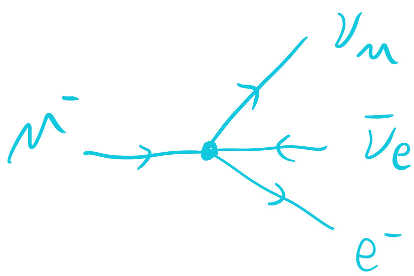
For $p^2 \ll M_W^2$, we can expand the operator

$$\frac{1}{p^2 - M_W^2} = -\frac{1}{M_W^2}\left(1 + \frac{p^2}{M_W^2} + \frac{p^4}{M_W^4} + \cdots\right)$$

Keeping only the 1st term we have

$$i\mathcal{M} = \frac{-ig^2}{2M_W^2}(\bar{\nu}_\mu \gamma^\mu \mu_L)(\bar{e}_L \gamma_\mu \nu_e) + \mathcal{O}\left(\frac{1}{M_W^4}\right)$$

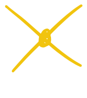
4-fermion contact interaction



which can be produced by the local Lagrangian

$$\mathcal{L} = \underbrace{-\frac{g^2}{2M_W^2}(\bar{\nu}_\mu \gamma^\mu \mu_L)(\bar{e}_L \gamma_\mu \nu_e)}_{\text{dimension-6 operator}} + \mathcal{O}\left(\frac{1}{M_W^4}\right)$$

dimension-6 operator, what Fermi wrote down.

EFT:  $\sim \frac{E^2}{\Lambda^2}$ $(\Lambda \sim M_W)$, breaks down at large E!

If we keep more terms in the Lagrangian we'll generate higher dimensional operators, e.g. the $\frac{1}{M_W^4}$ term corresponds to dim-8 operators.

(Higher dimensional operators may look very complicated, it's actually much easier to use on-shell amplitudes!)

This is the simplest example of the (amplitude) matching between the full model (SM) and the low-energy effective field theory (Fermi's theory).

- For $p^2 \ll M_W^2$, the 4F operator gives a very good approximation of the full theory. This is the case for muon decay. ($p^2 < m_\mu^2$ $\quad \frac{m_\mu^2}{M_W^2} \sim 10^{-6}$)

- The coefficient of the 4F operator is $-\frac{g^2}{2M_W^2} \sim \frac{1}{v^2}$.
Measuring muon decay only tells us the value of v
(or $G_F \equiv \frac{1}{\sqrt{2}\, v^2}$) but not M_W, which depends on g.

- M_W is the scale at which the EFT breaks down!

$$\frac{1}{p^2 - M_W^2} = -\frac{1}{M_W^2}\left(1 + \frac{p^2}{M_W^2} + \frac{p^4}{M_W^4} + \cdots\right)$$

breaks down at $p^2 \sim M_W^2$!

In our world, $g \simeq 0.65$.

If g is $\begin{cases} \text{very small}, & W, Z \text{ would be much lighter} \\ \text{very large}, & \text{〃 〃 〃 heavier.} \end{cases}$

(but if $g \gtrsim 4\pi$, the theory becomes non-perturbative)

- In this simple example, if we also measure the dim-8 coefficient $\left(\sim \frac{g^2}{m_W^4}\right)$ we can derive the W mass. In more complicated cases (with multiple heavy particles) it is in general not possible.

- global fron ....

SMEFT

$$\mathcal{L}_{SMEFT} = \mathcal{L}_{SM} + \sum_i \frac{C_i^{(5)}}{\Lambda} O_i^{(5)} + \sum_i \frac{C_i^{(6)}}{\Lambda^2} O_i^{(6)} + \sum_i \frac{C_i^{(7)}}{\Lambda^3} O_i^{(7)} + \dots$$

$[\mathcal{L}] = 4, \quad [O^{(n)}] = n, \quad [\Lambda] = 1$

A good expansion if $\Lambda \gg v, E$

v: 246 GeV

E: energy scale of the experiment

Note: each O_i needs to be invariant under Lorentz and gauge transformations $\underline{SU(3) \times SU(2) \times U(1)}$

HEFT: only $SU(3) \times U(1)$. (linear vs. non-linear ...)

dim 5 : only 1 type of operators $\sim LLHH$ (Weinberg operator)

HW: write down the exact form of the Weinberg operator

neutrino majorana mass

$$\mathcal{L} \sim \frac{c}{\Lambda} LLHH \rightarrow c\frac{v^2}{\Lambda}\nu\nu \qquad \left.\begin{array}{l} c \sim 1 \\ \Lambda \sim \Lambda_{GUT} \end{array}\right\} \Rightarrow m_\nu \sim 10^{-2}\,eV$$

Seesaw mechanism !

$\cancel{B}$ $\cancel{L}$ effects are usually strongly constrained (e.g. proton decay).

Assuming B, L are conserved around the TeV scale

$$\mathcal{L}_{SMEFT} = \mathcal{L}_{SM} + \sum_i \frac{C_i^{(6)}}{\Lambda^2} O_i^{(6)} + \sum_i \frac{C_i^{(8)}}{\Lambda^2} O_i^{(8)} + \cdots\cdots$$

use Integration by parts, e.o.m, ... (Field redefinition) to eliminate redundant operators...

How many independent parameters do we have ?

	1 generation		3 generations	
				Manohar et al.
dim-6	76	1008.4884	2499	1312.2014
dim-8	895		36971	2005.00008 理论所组 2005.00059 Murphy

Hilbert series : Murayama et al.
1512.03433

Is 2499 too many parameters?

Warsaw basis 1008.4884

- first to write down a complete d6 basis
- try to eliminate operators with more derivatives in favor of operators with more fields.

Buchmüller & Wyler almost did it in 1986.... (why no one completed it in 24 years!)

X^3		φ^6 and $\varphi^4 D^2$		$\psi^2\varphi^3$	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	Q_φ	$(\varphi^\dagger\varphi)^3$	$Q_{e\varphi}$	$(\varphi^\dagger\varphi)(\bar l_p e_r \varphi)$
$Q_{\widetilde G}$	$f^{ABC} \widetilde G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$Q_{\varphi\Box}$	$(\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$	$Q_{u\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p u_r \widetilde\varphi)$
Q_W	$\varepsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$Q_{\varphi D}$	$(\varphi^\dagger D^\mu\varphi)^\star (\varphi^\dagger D_\mu\varphi)$	$Q_{d\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p d_r \varphi)$
$Q_{\widetilde W}$	$\varepsilon^{IJK} \widetilde W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$				

4+3+3

$X^2\varphi^2$		$\psi^2 X\varphi$		$\psi^2\varphi^2 D$	
$Q_{\varphi G}$	$\varphi^\dagger\varphi\, G^A_{\mu\nu} G^{A\mu\nu}$	Q_{eW}	$(\bar l_p \sigma^{\mu\nu} e_r)\tau^I \varphi W^I_{\mu\nu}$	$Q_{\varphi l}^{(1)}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar l_p \gamma^\mu l_r)$
$Q_{\varphi\widetilde G}$	$\varphi^\dagger\varphi\, \widetilde G^A_{\mu\nu} G^{A\mu\nu}$	Q_{eB}	$(\bar l_p \sigma^{\mu\nu} e_r)\varphi B_{\mu\nu}$	$Q_{\varphi l}^{(3)}$	$(\varphi^\dagger i \overleftrightarrow{D}^I_\mu \varphi)(\bar l_p \tau^I \gamma^\mu l_r)$
$Q_{\varphi W}$	$\varphi^\dagger\varphi\, W^I_{\mu\nu} W^{I\mu\nu}$	Q_{uG}	$(\bar q_p \sigma^{\mu\nu} T^A u_r)\widetilde\varphi\, G^A_{\mu\nu}$	$Q_{\varphi e}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar e_p \gamma^\mu e_r)$
$Q_{\varphi\widetilde W}$	$\varphi^\dagger\varphi\, \widetilde W^I_{\mu\nu} W^{I\mu\nu}$	Q_{uW}	$(\bar q_p \sigma^{\mu\nu} u_r)\tau^I \widetilde\varphi\, W^I_{\mu\nu}$	$Q_{\varphi q}^{(1)}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar q_p \gamma^\mu q_r)$
$Q_{\varphi B}$	$\varphi^\dagger\varphi\, B_{\mu\nu} B^{\mu\nu}$	Q_{uB}	$(\bar q_p \sigma^{\mu\nu} u_r)\widetilde\varphi\, B_{\mu\nu}$	$Q_{\varphi q}^{(3)}$	$(\varphi^\dagger i \overleftrightarrow{D}^I_\mu \varphi)(\bar q_p \tau^I \gamma^\mu q_r)$
$Q_{\varphi\widetilde B}$	$\varphi^\dagger\varphi\, \widetilde B_{\mu\nu} B^{\mu\nu}$	Q_{dG}	$(\bar q_p \sigma^{\mu\nu} T^A d_r)\varphi\, G^A_{\mu\nu}$	$Q_{\varphi u}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar u_p \gamma^\mu u_r)$
$Q_{\varphi WB}$	$\varphi^\dagger\tau^I\varphi\, W^I_{\mu\nu} B^{\mu\nu}$	Q_{dW}	$(\bar q_p \sigma^{\mu\nu} d_r)\tau^I \varphi\, W^I_{\mu\nu}$	$Q_{\varphi d}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar d_p \gamma^\mu d_r)$
$Q_{\varphi\widetilde WB}$	$\varphi^\dagger\tau^I\varphi\, \widetilde W^I_{\mu\nu} B^{\mu\nu}$	Q_{dB}	$(\bar q_p \sigma^{\mu\nu} d_r)\varphi\, B_{\mu\nu}$	$Q_{\varphi ud}$	$i(\widetilde\varphi^\dagger D_\mu \varphi)(\bar u_p \gamma^\mu d_r)$

8 × 3

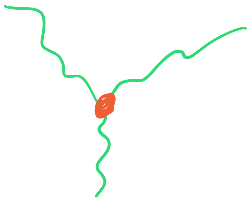
briefly explain each type of operators ...

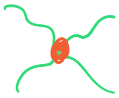
$(\bar LL)(\bar LL)$		$(\bar RR)(\bar RR)$		$(\bar LL)(\bar RR)$	
Q_{ll}	$(\bar l_p\gamma_\mu l_r)(\bar l_s\gamma^\mu l_t)$	Q_{ee}	$(\bar e_p\gamma_\mu e_r)(\bar e_s\gamma^\mu e_t)$	Q_{le}	$(\bar l_p\gamma_\mu l_r)(\bar e_s\gamma^\mu e_t)$
$Q_{qq}^{(1)}$	$(\bar q_p\gamma_\mu q_r)(\bar q_s\gamma^\mu q_t)$	Q_{uu}	$(\bar u_p\gamma_\mu u_r)(\bar u_s\gamma^\mu u_t)$	Q_{lu}	$(\bar l_p\gamma_\mu l_r)(\bar u_s\gamma^\mu u_t)$
$Q_{qq}^{(3)}$	$(\bar q_p\gamma_\mu \tau^I q_r)(\bar q_s\gamma^\mu \tau^I q_t)$	Q_{dd}	$(\bar d_p\gamma_\mu d_r)(\bar d_s\gamma^\mu d_t)$	Q_{ld}	$(\bar l_p\gamma_\mu l_r)(\bar d_s\gamma^\mu d_t)$
$Q_{lq}^{(1)}$	$(\bar l_p\gamma_\mu l_r)(\bar q_s\gamma^\mu q_t)$	Q_{eu}	$(\bar e_p\gamma_\mu e_r)(\bar u_s\gamma^\mu u_t)$	Q_{qe}	$(\bar q_p\gamma_\mu q_r)(\bar e_s\gamma^\mu e_t)$
$Q_{lq}^{(3)}$	$(\bar l_p\gamma_\mu \tau^I l_r)(\bar q_s\gamma^\mu \tau^I q_t)$	Q_{ed}	$(\bar e_p\gamma_\mu e_r)(\bar d_s\gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar q_p\gamma_\mu q_r)(\bar u_s\gamma^\mu u_t)$
		$Q_{ud}^{(1)}$	$(\bar u_p\gamma_\mu u_r)(\bar d_s\gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar q_p\gamma_\mu T^A q_r)(\bar u_s\gamma^\mu T^A u_t)$
		$Q_{ud}^{(8)}$	$(\bar u_p\gamma_\mu T^A u_r)(\bar d_s\gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar q_p\gamma_\mu q_r)(\bar d_s\gamma^\mu d_t)$
				$Q_{qd}^{(8)}$	$(\bar q_p\gamma_\mu T^A q_r)(\bar d_s\gamma^\mu T^A d_t)$
$(\bar LR)(\bar RL)$ and $(\bar LR)(\bar LR)$		B-violating			
Q_{ledq}	$(\bar l_p^j e_r)(\bar d_s q_t^j)$	Q_{duq}	~~$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(q_s^{\gamma j})^T C l_t^k\right]$~~		
$Q_{quqd}^{(1)}$	$(\bar q_p^j u_r)\varepsilon_{jk}(\bar q_s^k d_t)$	Q_{qqu}	~~$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(u_s^\gamma)^T C e_t\right]$~~		
$Q_{quqd}^{(8)}$	$(\bar q_p^j T^A u_r)\varepsilon_{jk}(\bar q_s^k T^A d_t)$	Q_{qqq}	~~$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jn}\varepsilon_{km}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$~~		
$Q_{lequ}^{(1)}$	$(\bar l_p^j e_r)\varepsilon_{jk}(\bar q_s^k u_t)$	Q_{duu}	~~$\varepsilon^{\alpha\beta\gamma}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(u_s^\gamma)^T C e_t\right]$~~		
$Q_{lequ}^{(3)}$	$(\bar l_p^j \sigma_{\mu\nu} e_r)\varepsilon_{jk}(\bar q_s^k \sigma^{\mu\nu} u_t)$				

5+7+8

5

= 59 operators

1)  (transverse) anomalous triple gauge coupling aTGC

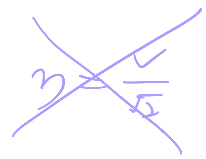
 & (quartic GC) aQGC

2) $\varphi \leftrightarrow H$ $\quad |H|^6 \to$  h^3 modifies h^3 & h^4 couplings

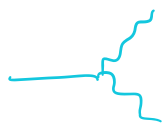
$(\partial |H|^2)^2$ modify $\partial_\mu h \partial^\mu h$, h wave function renormalization

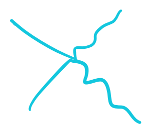
shift Higgs couplings

$|H^\dagger D^\mu H|^2$ modifies m_W ....

3) $\psi^2 \varphi^3 \to$ modify Yukawa coupling (relation between m & y) 

4) $|H|^2 V_{\mu\nu} V^{\mu\nu}$ <

$h V_{\mu\nu} V^{\mu\nu}$ 

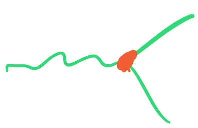
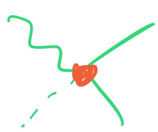
$hh V_{\mu\nu} V^{\mu\nu}$ 

different from $h Z^\mu Z_\mu$ $\quad h W^\mu W_\mu$

5) $H \to V$ dipole 

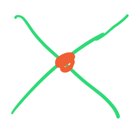
real magnetic

imaginary electric

6) $H \to V$  $\Longleftrightarrow$ 

modifies SM $Vf\bar{f}$ coupling — contact interaction

7) 4f interaction



1 generation:

59 operators

- 17 non hermitian ⇒ complex coefficient
- 42 hermitian ⇒ real coefficient

$42 + 17 \times 2 = 76$ parameters

3 generations: 2499 parameters!

(many of them are 4f operators)

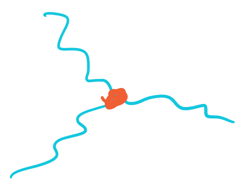
In other bases, we sometimes keep operators with more derivatives.

e.g. $O_{2W} = -\frac{1}{2}(D^{\mu} W^{a}_{\mu\nu})^2$ $\quad O_{2B} = -\frac{1}{2}(\partial^{\mu} B_{\mu\nu})^2$

useful in describing universal contributions to 4f interactions.

$O_{HW} = ig(D^{\mu}H)^{\dagger}\sigma^{a}(D^{\nu}H)W^{a}_{\mu\nu}$

$O_{HB} = ig'(D^{\mu}H)^{\dagger}(D^{\nu}H)B_{\mu\nu}$



useful for describing (longitudinal) anomalous triple gauge couplings (aTGCs)

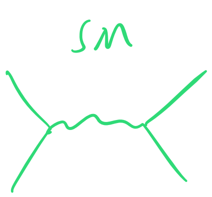
Phenomenology

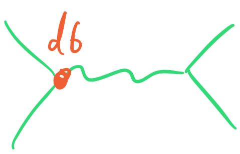
$$\mathcal{L}_{SMEFT} \longrightarrow \overset{\text{amplitude}}{\mathcal{M}} \longrightarrow \overset{\text{observable}}{\frac{d\sigma}{d\Omega}, \frac{dP}{d\Omega}}$$

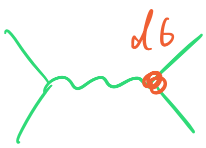
expand in terms of $\frac{1}{\Lambda}$

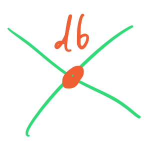
$$\mathcal{L} = \mathcal{L}_{SM} + \mathcal{L}_6 + \mathcal{L}_8 + \cdots$$

$\mathcal{M}$ $q\bar{q} \to l^+ l^-$

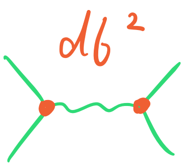


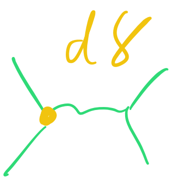
+ 

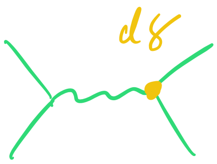
+ 

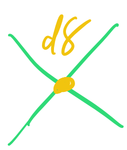
+ 

+ .... $\frac{1}{\Lambda^2}$

+ 

+ 

+ 

+ 

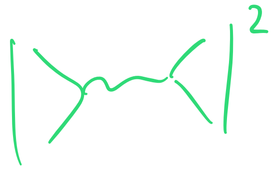
+ ... $\frac{1}{\Lambda^4}$

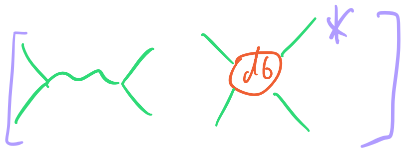
+ ......

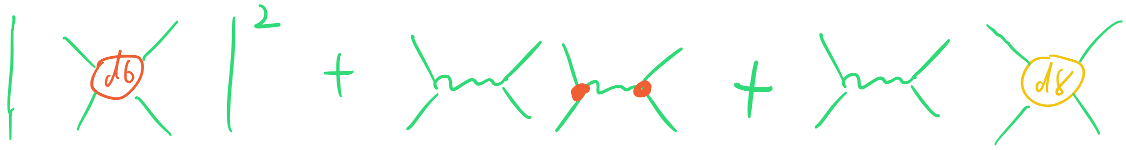
(d6^2 & d8 are formally indistinguishable)

Higher dimensional operators can contribute to m, Γ, which appears in dinominators $\frac{1}{p^2-m^2-im\Gamma}$. we don't consider it here (or just expand?)

$\sigma \sim |\mathcal{M}|^2$



$+ 2\mathrm{Re}$  $\frac{1}{\Lambda^2}$

$+$  $\frac{1}{\Lambda^4}$

$+ \cdots\cdots$

We can truncate σ at

$\frac{1}{\Lambda^2}$ is a very good approximation if $E \ll \Lambda$ (collider energy), $v \ll \Lambda$

$(1+x)^2 \simeq 1+2x$ if x is very small!

$\frac{1}{\Lambda^4}$ strictly speaking, need to calculate $d8$

~~what if Λ is not that large, shall we keep $d6^2$?~~

Typically:

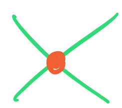
If measurement is very precise $\Rightarrow$ can constrain $\Lambda \gg E$ ✓

$\Rightarrow$ ok to just keep $\frac{1}{\Lambda^2}$ (ideal case!)

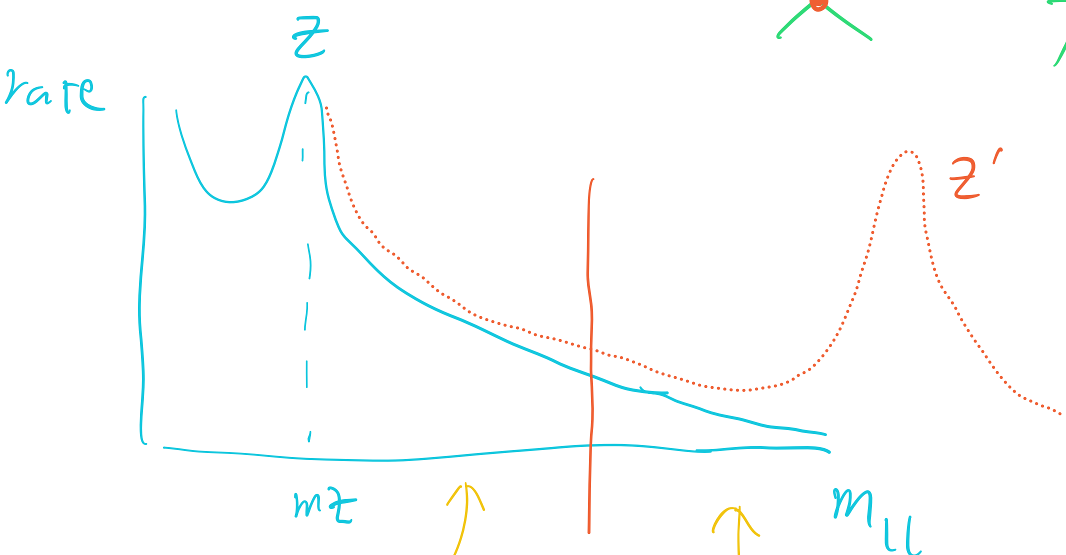
What if it's not the case?

typical LHC measurement

Drell-Yan $pp \to l^+ l^-$



$\sim \frac{E^2}{\Lambda^2}$



small E, large statistics

cut here and throw away $\Rightarrow$? EFT valid?

large E, small statistics

(or do something else)

easier to constrain

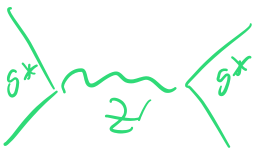
$\Lambda \gg E$

ok to truncate at $\frac{1}{\Lambda^2}$

(Lepton colliders usually don't have this problem.)

When / where is EFT invalid?

This depends on the coupling strength!



$$\sim \frac{(g^*)^2}{p^2 - m_{Z'}^2} \sim \frac{(g^*)^2}{m_{Z'}^2} + \frac{(g^*)^2 p^2}{m_{Z'}^4} + \cdots$$

d6: we only measure the combination

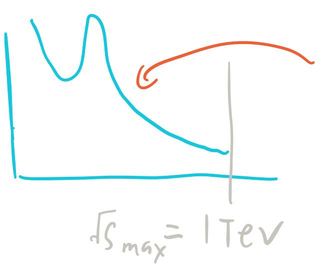
$$\frac{(g^*)^2}{m_{Z'}^2} \sim \frac{c}{\Lambda^2}$$

$m_{Z'}$ is the scale near which EFT breaks down!

g^* is small / large $\lesssim 4\pi$ $\Rightarrow$ $m_{Z'}$ is small / large

$d6^2$: $\frac{(g^*)^4}{m_{Z'}^4}$ Sm.d8 : $\frac{g^2 g^{*2}}{m_{Z'}^4}$ $d6^2 \gg d8$ if g_* is large!

e.g.



from the measurement I have

$$\left|\frac{c}{\Lambda^2}\right| \lesssim 1\ \mathrm{TeV}^{-2}$$

is the EFT valid?

$c = 1$: $\frac{\Lambda}{\sqrt{s}_{max}} \simeq 1$ maybe not?

$c = 9$: $\frac{\Lambda}{\sqrt{s}_{max}} \simeq 3$ maybe yes.

Important exceptions of the $\frac{1}{\Lambda^2}$ power counting

- SM contribution is absent or suppressed

  leading order: $d6^2 \sim \frac{1}{\Lambda^4}$

  SM·d6 is very small, SM·d8 << $d6^2$ ....

- rare process

  flavor violation ...

  proton decay $\quad T_p \sim \frac{m_p^5}{\Lambda^4} \qquad \Lambda \gtrsim 10^{15}$ GeV

- Fermi's theory: Weak interaction

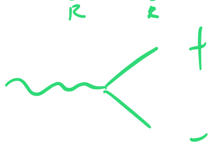
$$T_\mu \sim \frac{m_\mu^5}{\Lambda_{EW}^4} \;!$$

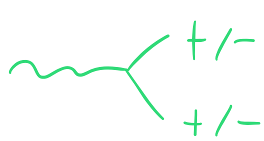
no interference with QED!

- The interference term with SM is suppressed.

  Different helicity amplitudes ....

  e.g. $Z_\mu \bar{f}_{L/R} \gamma^\mu f_{L/R}$ $\qquad Z_{\mu\nu} \bar{f}_L \sigma^{\mu\nu} f_R$





no interference in the $m_f \to 0$ limit!

SM input parameters, and how SMEFT modifies them.

The SM has a set of free parameters to be fixed by experiments.

$$g \quad g' \quad v \quad \lambda \quad y_t \quad \cdots\cdots$$

which are related to

$$m_Z, \quad m_W, \quad G_F, \quad \alpha, \quad m_h, \quad m_t, \quad \cdots\cdots$$

muon decay (for G_F)

electron magnetic moment (for α)

These relations can be modified by higher dimensional operators!

e.g.

3rd generation

$$\mathcal{L} \supset y_t \bar{Q}_L^{(3)} t_R \tilde{H} + \frac{C_t}{\Lambda^2} |H|^2 \bar{Q}_L^{(3)} t_R \tilde{H} + h.c.$$

$$= \frac{y_t}{\sqrt{2}} (v+h) \bar{t}_L t_R + \frac{C_t}{\Lambda^2} \frac{(v+h)^3}{2\sqrt{2}} \bar{t}_L t_R + h.c. + \cdots$$

SM $C_t = 0$:

$$\mathcal{L} = \underbrace{\frac{y_t v}{\sqrt{2}}}_{m_t} \bar{t}_L t_R + \underbrace{\frac{y_t}{\sqrt{2}}}_{g_{htt}} h \bar{t}_L t_R + h.c. + \cdots$$

with c_t : $\mathcal{L} = \underbrace{\left(\frac{y_t v}{\sqrt{2}} + \frac{c_t v^3}{2\sqrt{2}\Lambda^2}\right)}_{m_t} \bar{t}_L t_R + \underbrace{\left(\frac{y_t}{\sqrt{2}} + \frac{c_t 3v^2}{2\sqrt{2}\Lambda^2}\right)}_{g_{htt}} h \bar{t}_L t_R + h.c. + \dots$

Question: Does the measurement of m_t gives us a constraint on c_t ?

No! Because a nonzero c_t only changes the "inferred value" of y_t.

We need 2 measurements to fix 2 parameters.

In other words, c_t changes the relation between m_t & g_{htt}

$$m_t = \frac{y_t v}{\sqrt{2}} + \frac{c_t v^3}{2\sqrt{2}\Lambda^2} \qquad \frac{y_t}{\sqrt{2}} = \frac{m_t}{v} - \frac{c_t v^2}{2\sqrt{2}\Lambda^2}$$

$$\underline{\underline{g_{htt} = \frac{m_t}{v} + \frac{2 c_t v^2}{2\sqrt{2}\Lambda^2}}}$$

More generally, any operator of the form $|H|^2 \mathcal{O}_{SM}$ can only be probed with the "Higgs particle"!

$g_{SM}\mathcal{O}_{SM}$ vs. $g_{SM}\mathcal{O}_{SM} + \frac{c}{\Lambda^2}|H|^2\mathcal{O}_{SM}$

$$= g_{SM}\mathcal{O}_{SM} + \frac{c}{\Lambda^2}\frac{v^2}{2}\mathcal{O}_{SM} + \text{terms with } h$$

$$= \left(g_{SM} + \frac{cv^2}{2\Lambda^2}\right)\mathcal{O}_{SM} + \text{terms with } h$$

redefine $\bar{g} = g_{SM} + \frac{cv^2}{2\Lambda^2}$

$$= \bar{g}\,\mathcal{O}_{SM} + \text{terms with } h$$

(can also be h in the loop)

Similarly, $\mathcal{O}_6 \equiv (H^\dagger H)^3$ can only be probed by measuring the Higgs self coupling!

HW

SM EW sector W γ Z light fermions (ignore Higgs)

tree level 3 parameters

g g' v

$SU(2)$ $U(1)_Y$

usually fixed by 3 very precise measurements

so precise that we simply ignore the error!

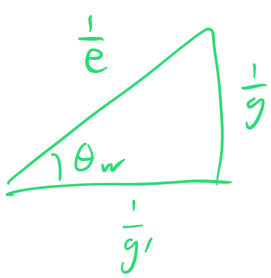
(input scheme. (m_Z, G_F, m_W) is also a common choice)

$$m_Z = 91.1876 \pm 0.0021 \text{ GeV}$$

$$G_F = 1.166378\underset{\pm 6}{7} \times 10^{-5} \text{ GeV}^{-2}$$ muon decay

α $\alpha^{-1}(m_Z) = 127.952 \pm 0.009$ e magnetic moment

mainly from had contribution



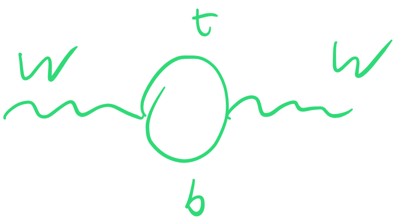
$$G_F = \frac{1}{\sqrt{2} v^2} \qquad \alpha = \frac{e^2}{4\pi} \qquad \tan\theta_W = \frac{g'}{g}$$

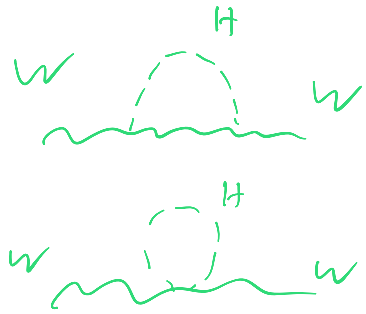
$$m_Z = \frac{1}{2} \frac{g v}{c_W} \qquad m_W = \frac{1}{2} g v$$

SM tree level: m_W is totally fixed! $\frac{m_W}{m_Z} = \cos\theta_W$

precision >> tree

loop:





m_t , m_H uncertainties $\overset{\text{1-loop}}{\Longrightarrow}$ m_W

"SM prediction" of m_W (with input measurements m_Z G_F α m_t m_H)

$$M_{W(\text{theory})} = 80357 \pm \overbrace{4_{\text{input}} \pm 4_{\text{theory}}}^{\sim 6\ \text{MeV}}\ \text{MeV}$$

m_t

missing higher order corrections

(CDF M_W measurement is significantly different from it....)

Add to $\mathcal{L}_{SM}$ the following dimension 6 operators

$$\mathcal{L} = \frac{\hat{S}}{m_W^2} \frac{1}{4} g g' H^\dagger \sigma^a H W^a_{\mu\nu} B^{\mu\nu} + \frac{\hat{T}}{v^2} \frac{1}{2} \left(H^\dagger \overleftrightarrow{D_\mu} H \right)^2$$

$$+ \frac{C_{ll}^{1221}}{v^2} (\bar{l}_1 \gamma^\mu l_2)(\bar{l}_2 \gamma_\mu l_1)$$

They can modify m_W ! (by changing the inferred values of SM parameters)

$$O_T : \frac{1}{2} \overbrace{\left(\frac{g v}{2 c_W} \right)^2}^{m_Z^{0\,2}} Z_\mu Z^\mu \Rightarrow \frac{1}{2} \left(\frac{g v}{2 c_W} \right)^2 (1 - \hat{T}) Z_\mu Z^\mu$$

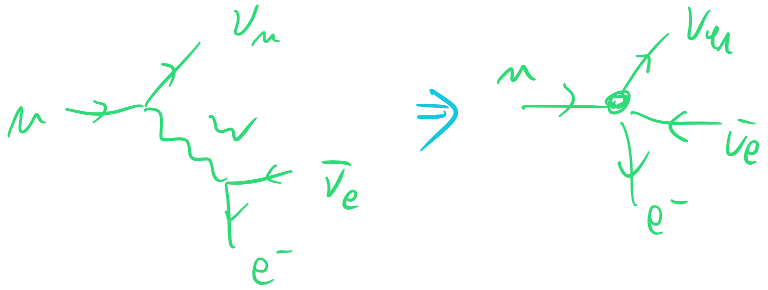
$$m_Z^2 = m_Z^{0\,2} (1 + \delta m_Z^2) \qquad \delta m_Z^2 = -\hat{T}$$

$\frac{g v}{2 c_W}$ is changed

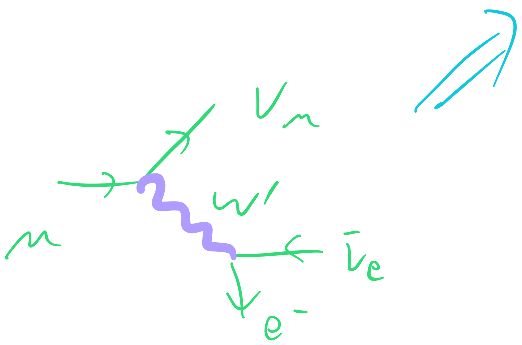
91.1876 ± 0.0021 GeV

$O_T : \frac{m_W}{m_Z} \neq \cos\theta_W$!

muon decay



$$G_F \sim \sqrt{\Gamma} \sim \frac{g^2}{m_W^2} \propto \frac{1}{v^2}$$



$$G_F \sim \frac{g^2}{m_W^2} + \frac{g_{W'}^2}{m_{W'}^2}$$

$$G_F = G_F^{SM}(1 + \delta G_F)$$

$$C_{ll}^{1221} = -2\,\delta G_F$$

measured value

$1.1663787_{\pm 6} \times 10^{-5}\ \mathrm{GeV}^{-2}$

inferred SM value

$\delta G_F \neq 0$

$\Downarrow$

$v \neq 246\ \mathrm{GeV}$

O_{WB} : kinetic mixing $\Rightarrow$ redefine fields $\Rightarrow \delta m_Z^2 \Rightarrow$ modify SM parameters

$$\delta m_W = \frac{1}{2C_{2w}}\left[C_w^2\,\hat{T} - S_w^2\left(\delta G_F + 2\hat{S}\right)\right]$$

These operators also modify Z pole observables....

Global fit (simplest case)

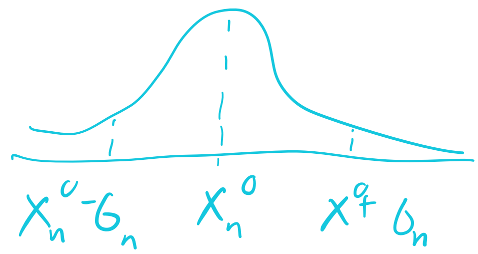
(we'll now use σ to denote the standard deviation!)

suppose we measure a set of discrete observables X_n s,
X can be rate measurements (cross sections, decay rates, # of events....)
or inferred measurements (mass ....)
each has a Gaussian error (Possion large N limit)

when we write

$$X_n = X_n^0 \pm \sigma_n$$

we mean



σ_n 1 standard deviation
1 sigma

Gaussian error $\Longrightarrow$ $-\log$ likelihood $\propto \chi^2$

theory prediction

$$\chi^2 = \sum_n \frac{(X_n - X_n^0)^2}{\sigma_n^2}$$

$$C_i^{(6)} \equiv \frac{c_i^{(6)}}{\Lambda^2}$$

truncate at $\frac{1}{\Lambda^2}$ $\Rightarrow$ $\underbrace{X_n = X_n^{SM} + a_{ni}}_{\text{calculated by hand or MC simulation}} \cancel{\frac{c_i^{(6)}}{\Lambda^2}}\ C_i^{(6)}$

minimize $\chi^2 \Leftrightarrow$ maximize liklihood $\rightarrow$ gives best-fit values for $C_i^{(6)}$

$$\Rightarrow \quad \chi^2 = \sum_{ij} (c_i - c_i^0)\,[\sigma^{-2}]_{ij}\,(c_j - c_j^0) + \chi^2_{min}$$

$$\sigma^{-2} \equiv (\delta\vec{c}\,\rho\,\delta\vec{c})^{-1}$$ inverse covariance matrix

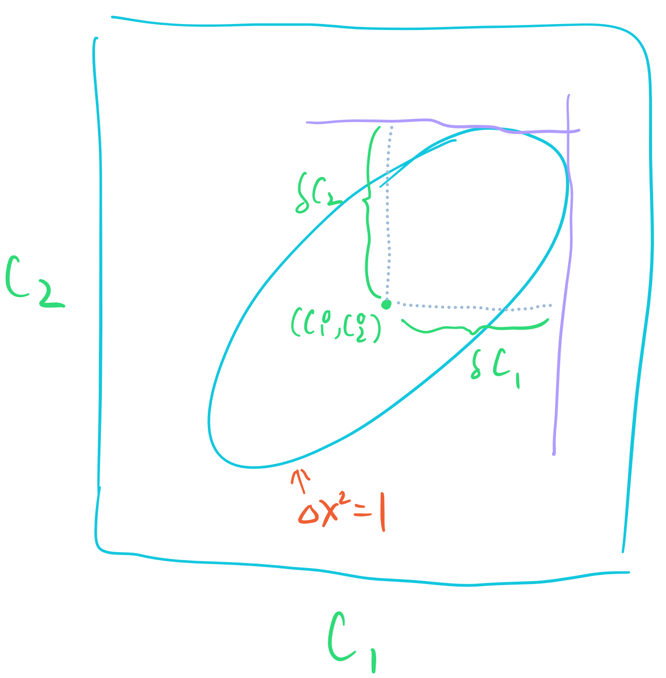
c_i^0 : best-fit values

δc_i : one-sigma precion, ρ_{ij} : correlation matrix

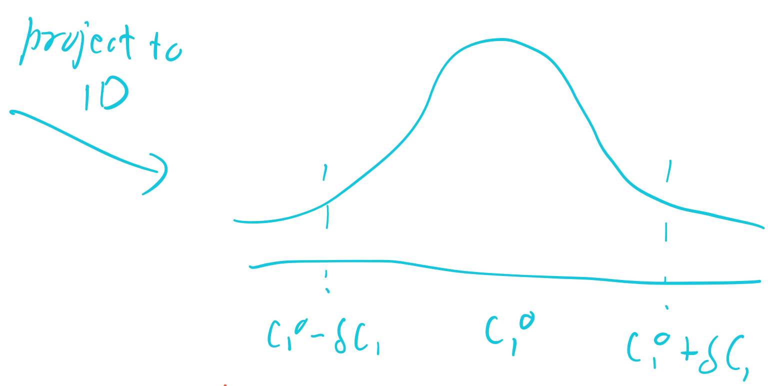
results of the Global fit

$\Delta\chi^2$ gives a measure of the "goodness of fit".

fixed $\Delta\chi^2$ $\Leftrightarrow$ qudratic equations of c_i $\Leftrightarrow$ ellipses!



Imagine this in n-dimensional parameter space . . . .



Beyond $\frac{1}{\Lambda^2}$ $\Rightarrow$ not Gaussian anymore!