SMEFT phenomenology Homework

Consider the Higgs sector in the SM, which we write as

$$\mathcal{L}_{\text{Higgs}} = (D_\mu H)^\dagger (D^\mu H) - V(H)\,, \qquad V(H) = -\mu^2 H^\dagger H + \lambda (H^\dagger H)^2\,. \tag{1}$$

With $\mu^2 > 0$ and $\lambda > 0$, the Higgs field H develops a vacuum expectation value (vev). Without loss of generality, and in the unitary gauge, we can write $H = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h \end{pmatrix}$, where v is the vev and h is the "Higgs particle". We can also write V in terms of h in the following form:

$$V(h) = \frac{1}{2} m_h^2\, h^2 + \lambda_3\, h^3 + \lambda_4\, h^4 + \text{constant}\,. \tag{2}$$

You may find the following expressions useful:

$$(1+x)^4 = 1 + 4\,x + 6\,x^2 + \ldots\,, \qquad (1+x)^6 = 1 + 6\,x + 15\,x^2 + 20\,x^3 + \ldots\,. \tag{3}$$

(a) Find the vev v by writing $\phi = v + h$ and solving $\frac{dV}{d\phi}|_{\phi=v} = 0$, and show that V is at its minimum at the vacuum. Express v and m_h in terms of μ^2 and λ. From measurements we know that $v = 246\,\text{GeV}$ and $m_h = 125\,\text{GeV}$. For now, we can ignore their measurement uncertainties. Work out the values of μ^2 and λ. Then express λ_3 and λ_4 in terms of v and m_h and work out their numerical values.

(b) Let's now add a dimension-6 operator $\mathcal{O}_6 = (H^\dagger H)^3$ to the Lagrangian. We can write the potential as

$$V(H) = -\mu^2 H^\dagger H + \lambda (H^\dagger H)^2 + \frac{c_6}{\Lambda^2}(H^\dagger H)^3\,, \tag{4}$$

where c_6 is a dimensionless parameter and Λ is the scale of some new physics. Let's define $\frac{\bar{c}_6}{v^2} \equiv \frac{c_6}{\Lambda^2}$. Working with $\bar{c}_6$ will turn out to be much simpler. If we require the potential to have a minimum, what does it imply on the sign of $\bar{c}_6$? Assuming we still have $\mu^2 > 0$ and $\lambda > 0$, find out the new vev v and write it in terms of μ^2, λ and $\bar{c}_6$.

(c) Express μ^2 and λ in terms of v, m_h and $\bar{c}_6$. For λ, you may find it helpful to first express m_h^2 in terms of v, λ and $\bar{c}_6$. Note that $v = 246\,\text{GeV}$ and $m_h = 125\,\text{GeV}$ are physical parameters which are fixed by experiments. However, the presence of $\bar{c}_6$ changes the relation between μ^2, λ and v, m_h, so μ^2 and λ will take different values from the SM ones if $\bar{c}_6$ is non-zero.

(d) With 3 parameters we need at least 3 measurements to determine their values. In addition to v and m_h, we can measure the so-called "triple Higgs coupling", which is λ_3 in Eq. (2). It is a very difficult measurement. To see how the triple Higgs coupling is related to c_6, it is most convenient to define a quantity $\kappa_3 \equiv \frac{\lambda_3}{\lambda_{3,\text{SM}}}$, where $\lambda_{3,\text{SM}}$ is the value of λ_3 in the SM without $\mathcal{O}_6$, which you have worked out in part a). Write κ_3 in terms of v, m_h and $\bar{c}_6$. You don't need to plugging in the numerical values of v and m_h.

(e) The quartic Higgs coupling, λ_4, is even more difficult to measure. However, with just the $\mathcal{O}_6$ operator, λ_4 is not an independent parameter. Work out the relation between $\kappa_4 \equiv \frac{\lambda_4}{\lambda_{4,\text{SM}}}$ and κ_3. Does this relation still hold if we add an additional dimension-8 operator $\mathcal{O}_8 = (H^\dagger H)^4$? (For the last question, you don't need to work out the details, just state your reasons.)