$0\nu\beta\beta$ decay and nuclear many-body problems with operators from chiral effective field theory

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Outline

1. Lecture one: neutrinoless double-beta ($0\nu\beta\beta$) decay
   - Status of studies on $0\nu\beta\beta$ decay
   - Modeling the half-life of $0\nu\beta\beta$ decay phenomenologically
   - Uncertainties in the NMEs of $0\nu\beta\beta$ decay
2. Lecture two: $0\nu\beta\beta$ decay and nuclear structure within chiral EFT
   - Leading-order chiral EFT description of $nn \to ppe^-e^-$ transition
   - Preprocessing nuclear chiral forces
   - ab initio nuclear many-body methods
3. Lecture three: Recent studies with operators from chiral EFT
   - Advances in the ab initio studies of nuclear structure and decay
   - Uncertainty quantification of the NMEs of $0\nu\beta\beta$ decay
   - Correlation relations between $0\nu\beta\beta$ decay and DGT

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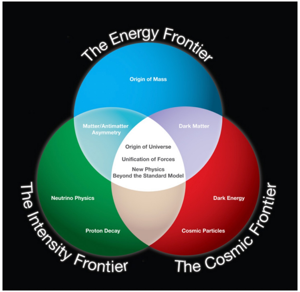
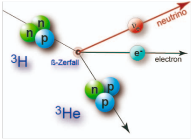
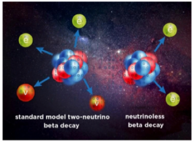
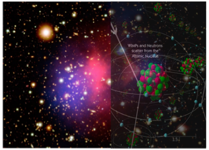




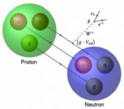
Single−beta decay



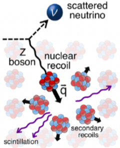
Neutrinoless double beta decay



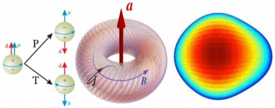
Dark matter direct detection



Superallowed Fermi transitions



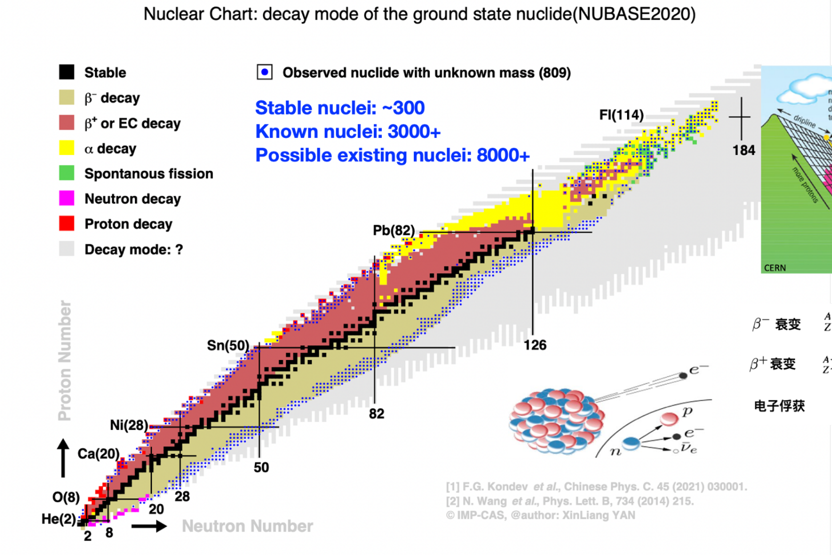
Neutrino scattering

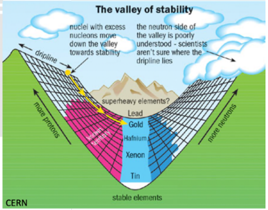


Symmetry-violating moments

- Three frontiers: for new physics
- Atomic nuclei: low-energy probes
- Fundamental interactions and symmetries.
- All about Nuclear Matrix Elements (NME)

Stability of atomic nuclei against single-β decay



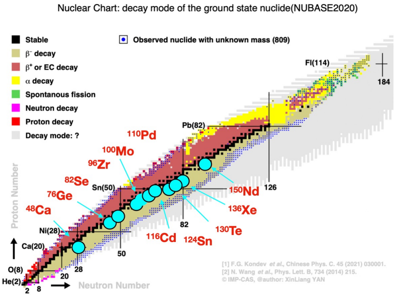


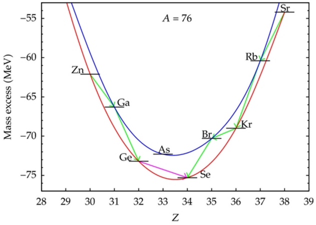
β^- 衰变 $\quad {}^A_Z X_N \to {}^A_{Z+1} Y_{N-1} + e^- + \bar{\nu}_e,$

β^+ 衰变 $\quad {}^A_Z X_N \to {}^A_{Z-1} Y_{N+1} + e^+ + \nu_e,$

电子俘获 $\quad {}^A_Z X_N + e^- \to {}^A_{Z-1} Y_{N+1} + \nu_e,$

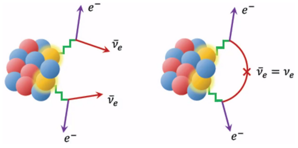
A special decay mode: $0\nu\beta\beta$ decay



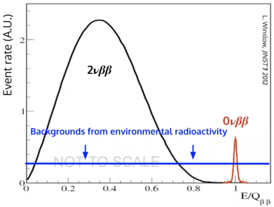


- The two modes of $\beta^-\beta^-$ decay:

$$(A, Z) \rightarrow (A, Z+2) + 2e^- + (2\bar{\nu}_e)$$



- Kinetic energy spectrum of electrons



Neutrino mass from oscillation experiments

$$\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} = \begin{pmatrix}1 & 0 & 0\\ 0 & c_{23} & s_{23}\\ 0 & -s_{23} & c_{23}\end{pmatrix}\begin{pmatrix}c_{13} & 0 & s_{13}e^{-i\delta_{\rm CP}}\\ 0 & 1 & 0\\ -s_{13}e^{i\delta_{\rm CP}} & 0 & c_{13}\end{pmatrix}\begin{pmatrix}c_{12} & s_{12} & 0\\ -s_{12} & c_{12} & 0\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix}$$

Global fit – Normal hierarchy

$\Delta m^2_{21} = 7.50^{+0.19}_{-0.17} \times 10^{-5}\mathrm{eV}^2$

$\Delta m^2_{31} = 2.457^{+0.047}_{-0.047} \times 10^{-3}\mathrm{eV}^2$

$\theta_{12} = 33.48^{+0.78}_{-0.75}(^\circ)$

$\theta_{23} = 42.3^{+3.0}_{-1.6}(^\circ)$

$\theta_{13} = 8.50^{+0.20}_{-0.21}(^\circ)$

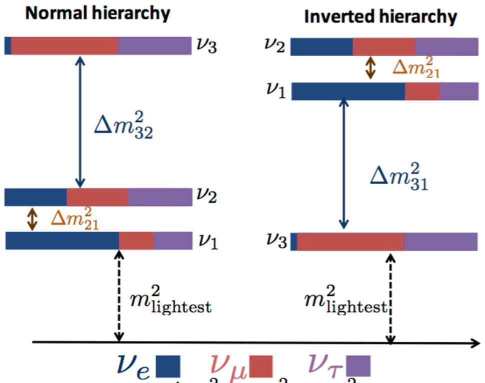
$\mathrm{sign}(\Delta m^2_{32}) = ?$

θ_{23} is maximal ?

$\delta_{\rm CP} = ?$

$m_{\rm lightest} = ?$

Gonzalez-Garcia *et al.*, arXiv:1512.06856



$\Delta m^2_{ij} = m^2_{\nu_i} - m^2_{\nu_j}$

Neutrino oscillations

- From mass to flavor states

$$|\nu_\alpha\rangle = \sum_{j=1}^{N=3} U^*_{\alpha j} |\nu_j\rangle \, .$$

- $\Delta m^2_{ij} (\neq 0)$, and $\theta_{ij} (\neq 0)$.

Open questions

- The nature of neutrinos.
- Neutrino mass m_j and its origin.

The observation of $0\nu\beta\beta$ decay would provide answers.

If $0\nu\beta\beta$ decay is driven by exchanging light massive Majorana neutrinos: (to be derived later)

$$\langle m_{\beta\beta}\rangle \equiv |\sum_{j=1}^{3} U^2_{ej} m_j| = \left[\frac{m_e^2}{g_A^4 G_{0\nu} T^{0\nu}_{1/2} |M^{0\nu}|^2}\right]^{1/2}$$

- U_{ej}: elements of the PMNS matrix
- $G_{0\nu}$: phase-space factor
- $M^{0\nu}$: the nuclear matrix element

$$M^{0\nu} = \langle \Psi_F | \hat{O}^{0\nu} | \Psi_I \rangle$$

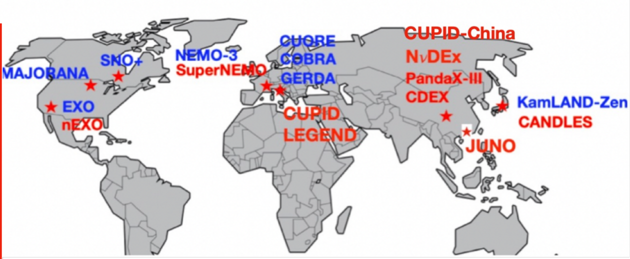
$$T^{0\nu}_{1/2} \simeq 10^{27-28} \left(\frac{0.01\mathrm{eV}}{\langle m_{\beta\beta}\rangle}\right)^2 \mathrm{yr}$$

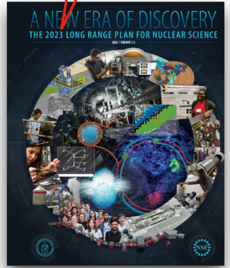
Current and next-generation of experiments

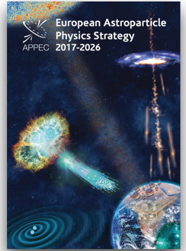
RECOMMENDATION 2

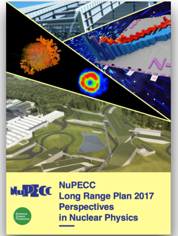
As the highest priority for new experiment construction, we recommend that the United States lead an international consortium that will undertake a neutrinoless double beta decay campaign, featuring the expeditious construction of ton-scale experiments, using different isotopes and complementary techniques.

One of the most compelling mysteries in all of science is how matter came to dominate over antimatter in the universe. Neutrinoless double beta decay, a process that spontaneously creates matter, may hold the key to solving this puzzle. Observation of this rare nuclear process would unambiguously demonstrate that neutrinos are their own antiparticles and would reveal the origin and scale of neutrino mass. The nucleus provides the only laboratory through which this fundamental physics can be addressed.



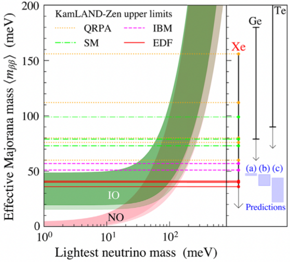






Constraints on neutrino mass from $0\nu\beta\beta$ decay

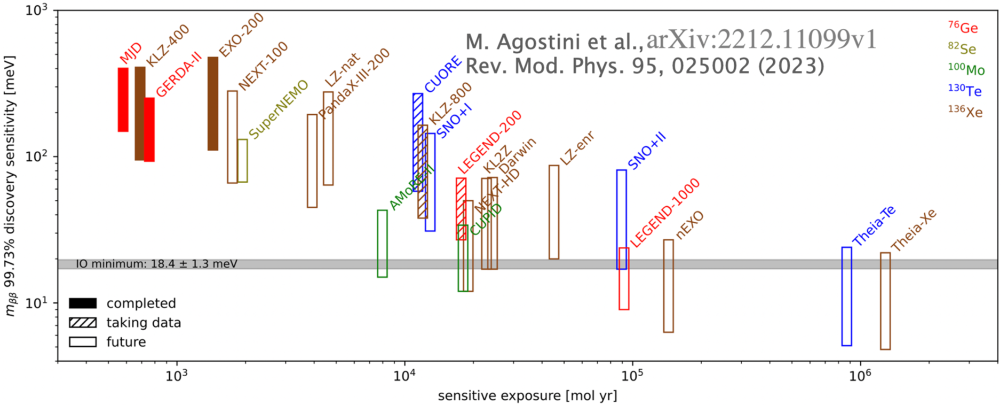
Isotope	$G_{0\nu}$ $[10^{-14}\ \mathrm{yr}^{-1}]$	$M^{0\nu}$ [min, max]	$T^{0\nu}_{1/2}$ [yr]	$\langle m_{\beta\beta}\rangle$ [meV]	Experiments References
^{48}Ca	2.48	[0.85, 2.94]	$> 5.8 \cdot 10^{22}$	[2841, 9828]	CANDLES: PRC78, 058501 (2008)
^{76}Ge	0.24	[2.38, 6.64]	$> 1.8 \cdot 10^{26}$	[73, 180]	GERDA: PRL125, 252502(2020)
^{82}Se	1.01	[2.72, 5.30]	$> 4.6 \cdot 10^{24}$	[277, 540]	CUPID-0: PRL129, 111801 (2023)
^{96}Zr	2.06	[2.86, 6.47]	$> 9.2 \cdot 10^{21}$	[3557, 8047]	NPA847, 168 (2010)
^{100}Mo	1.59	[3.84, 6.59]	$> 1.5 \cdot 10^{24}$	[310, 540]	CUPID-Mo: PRL126, 181802(2021)
^{116}Cd	0.48	[3.29, 5.52]	$> 2.2 \cdot 10^{23}$	[1766, 2963]	PRD 98, 092007 (2018)
^{130}Te	1.42	[1.37, 6.41]	$> 2.2 \cdot 10^{25}$	[90, 305]	CUORE: Nature 604, 53(2022)
^{136}Xe	1.46	[1.11, 4.77]	$> 2.3 \cdot 10^{26}$	[36, 156]	KamLAND-Zen: PRL130, 051801(2023)
^{150}Nd	6.30	[1.71, 5.60]	$> 2.0 \cdot 10^{22}$	[1593, 5219]	NEMO-3: PRD 94, 072003 (2016)



$$\langle m_{\beta\beta}\rangle = m_1 c_{12}^2 c_{13}^2 + m_2 c_{13}^2 s_{12}^2 e^{i\alpha_{21}} + m_3 s_{13}^2 e^{i(\alpha_{31}-2\delta)}$$

- The neutrino oscillation measurements: $\langle m_{\beta\beta}\rangle \in [20, 50]\ \mathrm{meV}$ for the IO case.
- An uncertainty of a factor of about 3 or even more (originated from the $M^{0\nu}$) in the $\langle m_{\beta\beta}\rangle$.

Next-generation of experiments

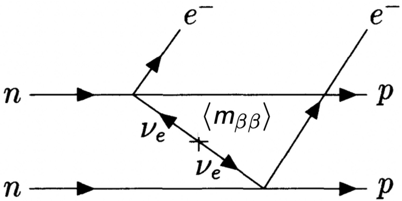


- Lifetime sensitivity of the ton-scale experiments: $T_{1/2}^{0\nu} > 10^{28}$yr.
- Covering the entire parameter space for the IO neutrino masses depending strongly on the employed NME.

JMY, J. Meng, Y.F. Niu, P. Ring, PPNP 126, 103965 (2022)

- The half-life of $0\nu\beta\beta$ decay:

$$[T^{0\nu}_{1/2}]^{-1} = \frac{1}{\ln 2}\Gamma^{0\nu} \qquad (1)$$



The decay width $\Gamma^{0\nu}$ is given by M. Fukugita, T. Yanagida, Physics of Neutrinos, and applications to astrophysics (2003)

$$\Gamma^{0\nu} = \frac{1}{2}\int \frac{d^3\boldsymbol{k}_1}{(2\pi)^3 2\epsilon_1}\int \frac{d^3\boldsymbol{k}_2}{(2\pi)^3 2\epsilon_2}(2\pi)\delta(E_I - E_F - \epsilon_1 - \epsilon_2)|\mathcal{M}_{fi}|^2. \qquad (2)$$

Here $\epsilon_{1,2} = k^0_{1,2}$ and $k_{1,2}$ are the energies and momenta of the two emitted electrons, respectively. The $E_{I,F}$ are the energies of initial and final nuclei, respectively.

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- The transition amplitude $\mathcal{M}_{fi}$ is determined by the S matrix

$$\langle f|S^{(2)}|i\rangle \quad = \quad i\mathcal{M}_{fi}(2\pi)\delta(E_I - E_F - \epsilon_1 - \epsilon_2), \tag{3}$$

where the S matrix is determined by the second-order effective weak interaction (energy scale $E \sim Q_{\beta\beta}$) S. M. Bilenky, S. T. Petcov, RMP59, 671 (1987)

$$\mathcal{H}_w(x) = \frac{G_\beta}{\sqrt{2}}\left[j_L^\mu(\mathcal{J}^\dagger_{L,\mu} + \kappa\mathcal{J}^\dagger_{R,\mu}) + j_R^\mu(\eta\mathcal{J}^\dagger_{L,\mu} + \lambda\mathcal{J}^\dagger_{R,\mu}\right] + \mathrm{h.c.} \tag{4}$$

The first term represents the *standard* V-A weak interaction, and others for non-standard interactions. The leptonic current

$$j_L^\mu(x) \equiv 2\bar{e}(x)\gamma^\mu\nu_{eL}(x) = \bar{e}\gamma^\mu(1-\gamma_5)\nu_e, \tag{5}$$

and quark (hadronic) current $\mathcal{J}^\dagger_{L,\mu}(x)$ (expression will be given later).

The S matrix (second-order) is given by

$$\langle f|S^{(2)}|i\rangle \equiv \frac{(-i)^2}{2!}\langle f|\int dt_1 \int dt_2\, T(H_w(x_1)H_w(x_2))|i\rangle \qquad (6)$$

where in the standard mechanism,

$$\begin{aligned} & H_w(x_1)H_w(x_2) \\ = & (\sqrt{2}G_\beta)^2[\bar{e}(x_1)\gamma^\mu \nu_{eL}(x_1)\mathcal{J}^\dagger_{L,\mu}(x_1)][\bar{e}(x_2)\gamma_\nu \nu_{eL}(x_2)\mathcal{J}^{\nu\dagger}_L(x_2)] \\ = & (\sqrt{2}G_\beta)^2\bar{e}(x_1)\gamma^\mu \nu_{eL}(x_1)\nu^T_{eL}(x_2)\gamma^T_\nu \bar{e}^T(x_2)\mathcal{J}^\dagger_{L,\mu}(x_1)\mathcal{J}^{\nu\dagger}_L(x_2). \end{aligned} \qquad (7)$$

- The S matrix (second-order) becomes

$$
\begin{aligned}
\langle f|S^{(2)}|i\rangle &= 4\frac{(-i)^2}{2!}\left(\frac{G_\beta}{\sqrt{2}}\right)^2 \int d^4x_1 \int d^4x_2 \\
&\times \bar{u}(k_1,s_1)e^{ik_1x_1}\gamma^\mu \langle 0|T\left(\nu_{eL}(x_1)\nu_{eL}^T(x_2)\right)|0\rangle \gamma^{\nu\,T}\bar{u}^T(k_2,s_2)e^{ik_2x_2} \\
&\times \langle \Psi_F|T\left(\mathcal{J}^\dagger_{L,\mu}(x_1)\mathcal{J}^\dagger_{L,\nu}(x_2)\right)|\Psi_I\rangle - (k_1 \leftrightarrow k_2). \qquad (8)
\end{aligned}
$$

Here $k_i x_i = \epsilon_i t_i - \boldsymbol{k}_i \cdot \boldsymbol{r}_i$ and $u(k,s)$ is a Dirac spinor for the electron. The $|\Psi_I\rangle$ and $|\Psi_F\rangle$ are wave functions of the initial and final nuclei. The second term $(k_1 \leftrightarrow k_2)$ arises due to the exchange between the two outgoing electrons, leading to the same contribution to the first term. S. M. Bilenky and S. T. Petcov, RMP 59, 671 (1987)

The hadronic current is defined in Heisenberg representation, i.e.,

$$\mathcal{J}^{\dagger}_{L,\mu}(x) = e^{iHt}\mathcal{J}^{\dagger}_{L,\mu}(\boldsymbol{r})e^{-iHt}. \tag{9}$$

By inserting the identity, $\sum_N |\Psi_N\rangle\langle\Psi_N| = 1$, one has

- case $t_1 > t_2$:

$$\begin{aligned} &\langle\Psi_F| T\left(\mathcal{J}^{\dagger}_{L,\mu}(x_1)\mathcal{J}^{\dagger}_{L,\nu}(x_2)\right)|\Psi_I\rangle \\ =& \sum_N e^{i(E_F-E_N)t_1} e^{i(E_N-E_I)t_2} \langle\Psi_F| \mathcal{J}^{\dagger}_{L,\mu}(\boldsymbol{r}_1)|\Psi_N\rangle\langle\Psi_N| \mathcal{J}^{\dagger}_{L,\nu}(\boldsymbol{r}_2)|\Psi_I\rangle. \end{aligned} \tag{10}$$

- case $t_1 < t_2$:

$$\begin{aligned} &\langle\Psi_F| T\left(\mathcal{J}^{\dagger}_{L,\mu}(x_1)\mathcal{J}^{\dagger}_{L,\nu}(x_2)\right)|\Psi_I\rangle \\ =& \sum_N e^{i(E_F-E_N)t_2} e^{i(E_N-E_I)t_1} \langle\Psi_F| J^{\dagger}_{L,\nu}(\boldsymbol{r}_2)|\Psi_N\rangle\langle\Psi_N| J^{\dagger}_{L,\mu}(\boldsymbol{r}_1)|\Psi_I\rangle. \end{aligned} \tag{11}$$

Neutrino mass and flavor eigenstates

The left-handed neutrino field is defined as (assuming only three light neutrinos)

$$\nu_{eL}(x) = \sum_{j=1}^{3} U_{ej}\nu_{jL}(x) = \sum_{j=1}^{3} U_{ej}P_L\nu_j(x), \tag{12}$$

where the projector $P_L = (1-\gamma_5)/2$. The PMNS matrix is defined as

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \mathcal{P}, \tag{13}$$

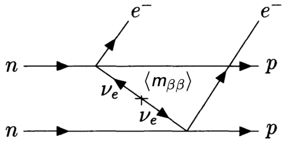
where $c_{ij} \equiv \cos\theta_{ij}$, and $s_{ij} \equiv \sin\theta_{ij}$ with three mixing angles $\theta_{12}, \theta_{23}, \theta_{13}$. The values of the diagonal matrix $\mathcal{P}$ depend on the nature of neutrinos:

$$\mathcal{P} = \begin{cases} \mathrm{diag}(1,1,1), & \text{Dirac}, \\ \mathrm{diag}(1, e^{i\alpha_{21}/2}, e^{i\alpha_{31}/2}), & \text{Majorana} \end{cases} \tag{14}$$

where α_{21}, α_{31} are the CP-violating Majorana phases affecting $0\nu\beta\beta$ decay.

The $\nu_j(x)$ is the field of a Majorana neutrino with mass mass m_j, satisfying the Majorana condition

$$\nu_j^c(x) = C\bar{\nu}_j^T = \nu_j(x), \qquad (15)$$



with C being the charge-conjugate operator.

The contraction (flavor eigenstate) and propagator (mass eigenstate) of neutrinos,

$$\begin{aligned}\langle 0| T\left(\nu_{eL}(x_1)\nu_{eL}^T(x_2)\right)|0\rangle &= -\sum_j U_{ej}^2 P_L \langle 0| T\left(\nu_j(x_1)\bar{\nu}_j(x_2)\right)|0\rangle P_L C \\ &= -\sum_j U_{ej}^2 m_j \int \frac{d^4q}{(2\pi)^4} e^{-iq(x_1-x_2)} \frac{i}{q^2-m_j^2} P_L C, \qquad (16)\end{aligned}$$

where $C^T = -C$, $C\gamma_5 C^{-1} = \gamma_5$ and $P_L\gamma^\mu q_\mu P_L = 0$ were used.

With the above considerations, the S matrix becomes

$$\begin{aligned}
\langle f|S^{(2)}|i\rangle &= 8\frac{(-i)^2}{2!}\left(\frac{G_\beta}{\sqrt{2}}\right)^2 \bar{u}(k_1,s_1)\gamma^\mu P_L C\gamma^{\nu T}\bar{u}^T(k_2,s_2)\sum_j U_{ej}^2 m_j \\
&\times \int_{-\infty}^{+\infty} dt_1 \left(\int_{-\infty}^{t_1} dt_2 + \int_{t_1}^{+\infty} dt_2\right) e^{ik_1^0 t_1} e^{ik_2^0 t_2} \frac{-i}{(2\pi)} \int \frac{e^{-iq^0(t_1-t_2)}}{q^2-m_j^2} dq^0 \\
&\times \int d^3x_1 d^3x_2 e^{-i\mathbf{k}_1\mathbf{x}_1} e^{-i\mathbf{k}_2\mathbf{x}_2} \frac{1}{(2\pi)^3} \int e^{i\mathbf{q}\cdot(\mathbf{x}_1-\mathbf{x}_2)} d^3q \\
&\times \langle \Psi_F| T\left(\mathcal{J}_{L,\mu}^\dagger(x_1)\mathcal{J}_{L,\nu}^\dagger(x_2)\right)|\Psi_I\rangle \\
&\equiv \langle f|S^{(2)}|i\rangle\Big|_{t_1>t_2} + \langle f|S^{(2)}|i\rangle\Big|_{t_1<t_2}. \qquad (17)
\end{aligned}$$

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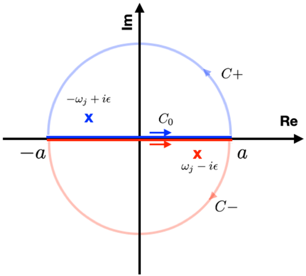
- In the first term

$$\int_{-\infty}^{+\infty} dt_1 \int_{-\infty}^{t_1} dx_2^0 e^{ik_1^0 t_1} e^{ik_2^0 t_2} \frac{-i}{(2\pi)} \int_{-\infty}^{+\infty} \frac{e^{-iq^0(t_1-t_2)}}{q^2 - m_j^2} dq^0 \langle \Psi_F | \mathcal{J}_{L,\mu}^\dagger(x_1) \mathcal{J}_{L,\nu}^\dagger(x_2) | \Psi_I \rangle$$
$$= \sum_N \int_{-\infty}^{+\infty} dt_1 e^{it_1(k_1^0 - q_j^0 + E_F - E_N)} \int_{-\infty}^{t_1} dt_2 e^{it_2(k_2^0 + q_j^0)} \frac{1}{2q_j^0} e^{i(E_N - E_I)t_2}$$
$$\times \langle \Psi_F | \mathcal{J}_{L,\mu}^\dagger(\boldsymbol{x}_1) | \Psi_N \rangle \langle \Psi_N | \mathcal{J}_{L,\nu}^\dagger(\boldsymbol{x}_2) | \Psi_I \rangle , \qquad (18)$$

where $q_j^0 \equiv (\boldsymbol{q}^2 + m_j^2)^{1/2}$, and the following relation (residue theorem) is used,

$$\int_{-\infty}^{+\infty} \frac{e^{-iq^0(t_1-t_2)}}{q^2 - m_j^2} dq^0 = \int_{-\infty}^{+\infty} \frac{e^{-iq^0(t_1-t_2)}}{q_0^2 - (\boldsymbol{q}^2 + m_j^2)} dq^0$$
$$= (-2\pi i) \frac{e^{-iq_j^0(t_1-t_2)}}{2q_j^0} \qquad (19)$$



Making use of the following relations,

$$\int_{-\infty}^{t_1} dt_2 e^{it_2(k_2^0+q_j^0)} e^{i(E_N-E_I)t_2} = \lim_{\epsilon\to 0} \int_{-\infty}^{t_1} dt_2 e^{it_2(k_2^0+q_j^0+E_N-E_I-i\epsilon)}$$
$$= \lim_{\epsilon\to 0} \frac{-ie^{it_1(k_2^0+q^0+E_N-E_I-i\epsilon)}}{k_2^0+q_j^0+E_N-E_I-i\epsilon} = \frac{-i}{k_2^0+q_j^0+E_N-E_I} e^{it_1(k_2^0+q_j^0+E_N-E_I)} \quad (20)$$

and $\int_{-\infty}^{+\infty} dt_1 e^{it_1 a} = 2\pi\delta(a)$, the first term of the S matrix is simplified further

$$\langle f|S^{(2)}|i\rangle\Big|_{t_1>t_2} = -i8\frac{(-i)^2}{2!}\left(\frac{G_\beta}{\sqrt{2}}\right)^2 \bar{u}(k_1,s_1)\gamma^\mu P_L C\gamma^{\nu T}\bar{u}^T(k_2,s_2)\sum_j U_{ej}^2 m_j \frac{1}{2q_j^0}$$
$$\times \int d^3x_1 d^3x_2 e^{-i\boldsymbol{k}_1\boldsymbol{x}_1} e^{-i\boldsymbol{k}_2\boldsymbol{x}_2} \frac{1}{(2\pi)^3}\int e^{i\boldsymbol{q}\cdot(\boldsymbol{x}_1-\boldsymbol{x}_2)} d^3q$$
$$\times \sum_N \frac{\langle\Psi_F|\mathcal{J}_{L,\mu}^\dagger(\boldsymbol{x}_1)|\Psi_N\rangle\langle\Psi_N|\mathcal{J}_{L,\nu}^\dagger(\boldsymbol{x}_2)|\Psi_I\rangle}{k_2^0+q_j^0+E_N-E_I}(2\pi)\delta(k_1^0+k_2^0+E_F-E_I).$$

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The second term in the S matrix can be carried out similarly. Combining the above two terms, one finally finds

$$\begin{aligned} &\langle f|S^{(2)}|i\rangle \\ = \;& i4\pi\delta(\epsilon_1+\epsilon_2+E_F-E_I)\left(\frac{G_\beta}{\sqrt{2}}\right)^2\int d^3r_1 d^3r_2 \bar{e}_{\boldsymbol{k}_1,s_1}(\boldsymbol{r}_1) P_R\gamma^\mu C\gamma^{\nu T} e^c_{\boldsymbol{k}_2,s_2}(\boldsymbol{r}_2) \\ & \times \sum_j U^2_{ej} m_j \frac{1}{q^0_j}\frac{1}{(2\pi)^3}\int d^3q e^{i\boldsymbol{q}\cdot\boldsymbol{r}_{12}} \\ & \times \sum_N \left[\frac{\langle\Psi_F|\mathcal{J}^\dagger_{L,\mu}(\boldsymbol{r}_1)|\Psi_N\rangle\langle\Psi_N|\mathcal{J}^\dagger_{L,\nu}(\boldsymbol{r}_2)|\Psi_I\rangle}{\epsilon_2+q^0_j+E_N-E_I} + \frac{\langle\Psi_F|\mathcal{J}^\dagger_{L,\nu}(\boldsymbol{r}_2)|\Psi_N\rangle\langle\Psi_N|\mathcal{J}^\dagger_{L,\mu}(\boldsymbol{r}_1)|\Psi_I\rangle}{\epsilon_1+q^0_j+E_N-E_I}\right] \end{aligned} \tag{21}$$

where k^0_i for the energy of electron is replaced with ϵ_i,

$$\gamma^\mu P_L = P_R\gamma^\mu. \tag{22}$$

With the approximations:

- The energy of neutrino is approximated as $q_j^0 = \sqrt{q^2 + m_j^2} \simeq q$
- $\epsilon_1 \simeq \epsilon_2 = (\epsilon_1 + \epsilon_2)/2 = (E_I - E_F)/2 \ll q$.
- For the s-wave electrons, only the $L = 0$ component is considered, $e^{-i\boldsymbol{k}\cdot\boldsymbol{r}} \to j_0(kr) \simeq 1$ as $kr \to 0$. Dirac spinor $e_{\boldsymbol{k},s} \equiv u(k,s)e^{-i\boldsymbol{k}\cdot\boldsymbol{r}} \simeq u(k,s)$.

The S matrix is simplified as

$$\langle f|S^{(2)}|i\rangle \simeq i(2\pi)\delta(\epsilon_1 + \epsilon_2 + E_F - E_I)\frac{g_A^2(0)G_\beta^2}{2\pi R_0}\int d^3r_1 d^3r_2 \bar{e}_{\boldsymbol{k}_1,s_1}(\boldsymbol{r}_1)P_R C\bar{e}^T_{\boldsymbol{k}_2,s_2}(\boldsymbol{r}_2)$$
$$\times \sum_j U_{ej}^2 m_j \frac{4\pi R_0}{g_A^2(0)}\int \frac{d^3q}{(2\pi)^3}e^{i\boldsymbol{q}\cdot\boldsymbol{r}_{12}}\sum_N \frac{\langle\Psi_F|\mathcal{J}^\dagger_{L,\mu}(\boldsymbol{r}_1)|\Psi_N\rangle\langle\Psi_N|\mathcal{J}^{\mu\dagger}_L(\boldsymbol{r}_2)|\Psi_I\rangle}{q\left[q + E_N - (E_I + E_F)/2\right]} \quad (23)$$

where the relations $C\gamma^{\nu T}C^{-1} = -\gamma^\nu$, $\gamma_5\gamma^\nu = -\gamma^\nu\gamma_5$, $\gamma^\mu\gamma^\nu = g^{\mu\nu} + \frac{1}{2}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)$ were used.

Thus, the transition amplitude $\mathcal{M}_{fi}$ becomes

$$\mathcal{M}_{fi} = \frac{g_A^2(0) G_\beta^2}{2\pi R_0} \langle m_{\beta\beta} \rangle \bar{u}(k_1, s_1) P_R C \bar{u}^T(k_2, s_2) M^{0\nu}, \tag{24}$$

where the effective electron neutrino mass $\langle m_{\beta\beta} \rangle$ is defined as

$$\langle m_{\beta\beta} \rangle \equiv \sum_j U_{ej}^2 m_j, \tag{25}$$

and the nuclear matrix element (NME) of $0\nu\beta\beta$ decay

$$\begin{aligned} M^{0\nu} &\equiv \frac{4\pi R_0}{g_A^2(0)} \int d^3 r_1 d^3 r_2 \int \frac{d^3 q}{(2\pi)^3} e^{i \boldsymbol{q}\cdot \boldsymbol{r}_{12}} \sum_N \frac{\langle \Psi_F | \mathcal{J}^\dagger_{L,\mu}(\boldsymbol{r}_1) | \Psi_N \rangle \langle \Psi_N | \mathcal{J}^{\mu\dagger}_L(\boldsymbol{r}_2) | \Psi_I \rangle}{q\left[q + E_N - (E_I + E_F)/2\right]} \\ &\equiv \langle \Psi_F | \hat{O}^{0\nu} | \Psi_I \rangle \end{aligned} \tag{26}$$

Here, $R_0 = 1.2 A^{1/3}$ is introduced to make the NME $M^{0\nu}$ dimensionless.

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One finally finds the expression for the $0\nu\beta\beta$-decay half-life

$$[T_{1/2}^{0\nu}]^{-1} = G_{0\nu} g_A^4(0) \left| \frac{\langle m_{\beta\beta} \rangle}{m_e} \right|^2 \left| M^{0\nu} \right|^2, \tag{27}$$

where the leptonic phase-space factor $G_{0\nu}$ is defined as

$$G_{0\nu} \equiv \frac{1}{(\ln 2)(2\pi)^5} \frac{G_\beta^4 m_e^2}{R_0^2} \frac{1}{4} \int \int \sum_{\text{spins}} \left| \bar{u}(k_1, s_1) P_R C \bar{u}^T(k_2, s_2) \right|^2 k_1 k_2 d\epsilon_1 d\cos\theta_{12} \tag{28}$$

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The effective one-body current operator takes the following form

$$\mathcal{J}_L^{\mu\dagger} = \bar{\psi}_N \gamma^\mu \left[g_V(\boldsymbol{q}^2) - g_A(\boldsymbol{q}^2)\gamma_5 + g_P(\boldsymbol{q}^2) q^\mu \gamma_5 - i g_W(\boldsymbol{q}^2)\sigma^{\mu\nu} q_\nu \right] \tau^+ \psi_N$$

The dipole form factors

$$\begin{aligned} g_V(\boldsymbol{q}^2) &= g_V(0)\left(1+\boldsymbol{q}^2/\Lambda_V^2\right)^{-2}, \\ g_A(\boldsymbol{q}^2) &= g_A(0)\left(1+\boldsymbol{q}^2/\Lambda_A^2\right)^{-2}, \\ g_P(\boldsymbol{q}^2) &= g_A(\boldsymbol{q}^2)\left(\frac{2m_p}{\boldsymbol{q}^2+m_\pi^2}\right), \\ g_W(\boldsymbol{q}^2) &= g_V(\boldsymbol{q}^2)\frac{\kappa_1}{2m_p}, \end{aligned}$$

where $g_V(0)=1$, $g_A(0)=1.27$, and the cutoff values are $\Lambda_V=0.85\,\text{GeV}$ and $\Lambda_A=1.09\,\text{GeV}$. According to the conserved vector current (CVC) hypothesis, $g_W(0)=\kappa_1/2m_p$ with κ_1 being the anomalous nucleon isovector magnetic moment $\kappa_1=\mu_n^{(a)}-\mu_p^{(a)}\simeq 3.7$.

The transition operators for the NME

- In the closure approximation $E_N \to \langle E_N \rangle$,

$$
\begin{aligned}
O^{0\nu} &= \frac{4\pi R_0}{g_A^2(0)} \int d^3 r_1 d^3 r_2 \int \frac{d^3 q}{(2\pi)^3} e^{i\boldsymbol{q}\cdot\boldsymbol{r}_{12}} \sum_N \frac{\mathcal{J}^\dagger_{L,\mu}(\boldsymbol{r}_1) \left|\Psi_N\right\rangle \left\langle \Psi_N\right| \mathcal{J}^{\mu\dagger}_L(\boldsymbol{r}_2)}{q\left[q + \langle E_N \rangle - (E_I + E_F)/2\right]} \\
&= \frac{4\pi R_0}{g_A^2(0)} \int d^3 r_1 d^3 r_2 \int \frac{d^3 q}{(2\pi)^3} e^{i\boldsymbol{q}\cdot\boldsymbol{r}_{12}} \frac{\mathcal{J}^\dagger_{L,\mu}(\boldsymbol{r}_1) \mathcal{J}^{\mu\dagger}_L(\boldsymbol{r}_2)}{q\left[q + \langle E_N \rangle - (E_I + E_F)/2\right]},
\end{aligned}
\tag{29}
$$

the transition operator becomes a product of two current operators composed of five terms (VV, AA, PP, AP and MM),

$$g_V^2(\boldsymbol{q}^2)(\bar\psi\gamma_\mu\tau^+\psi)^{(1)}(\bar\psi\gamma^\mu\tau^+\psi)^{(2)}, \qquad \mathrm{VV}$$

$$g_A^2(\boldsymbol{q}^2)(\bar\psi\gamma_\mu\gamma_5\tau^+\psi)^{(1)}(\bar\psi\gamma^\mu\gamma_5\tau^+\psi)^{(2)}, \qquad \mathrm{AA}$$

$$g_P^2(\boldsymbol{q}^2)(q_\mu\gamma_5\tau^+\psi)^{(1)}(\bar\psi q^\mu\gamma_5\tau^+\psi)^{(2)}, \qquad \mathrm{PP}$$

$$g_A(\boldsymbol{q}^2)g_P(\boldsymbol{q}^2)(\gamma\gamma_5\tau^+\psi)^{(1)}(\bar\psi\boldsymbol{q}\gamma_5\tau^+\psi)^{(2)}, \qquad \mathrm{AP}$$

The transition operators for the NME

In the impulse approximation (where only one nucleon in a nucleus is probed by an external source and the other nucleons act as spectators), the one-body hadronic current in the first quantization form

$$\mathcal{J}_L^{\mu\dagger}(\boldsymbol{r}) = \sum_{n=1}^{A} \left[\mathcal{J}_L^{\mu}(\boldsymbol{r}_n)\delta(\boldsymbol{r} - \boldsymbol{r}_n) \right]. \tag{30}$$

In the second quantization form,

$$\mathcal{J}_L^{\mu\dagger}(\boldsymbol{r}) = \sum_{p,p'} \langle N(p')| \mathcal{J}_L^{\mu\dagger}(\mathbf{r}) |N(p)\rangle c_{p'}^{\dagger} c_p, \tag{31}$$

where $\langle N(p')| \mathcal{J}_L^{\mu\dagger}(\mathbf{r}) |N(p)\rangle$ is the matrix element between the eigen states of momentum operator, The non-relativistic form of the current operator (truncation in terms of $1/m_p$, no nucleon recoil terms) reads Tomoda:1991

$$\mathcal{J}_L^{\mu\dagger}(\boldsymbol{r}) \equiv \sum_{n=1}^{A} \tau_n^+ \delta(\boldsymbol{r}-\boldsymbol{r}_n) \sum_{k=0,1,\cdots} \left[g_V V^{(k)\mu} - g_A A^{(k)\mu} - g_P P^{(k)\mu} + g_W W^{(k)\mu}\right]_n \tag{32}$$

Expansion in terms of $(1/m_p)^k$, and truncated up to $k=1$,

$$J_L^{0\dagger}(\boldsymbol{r}) = \sum_{n=1}^{A} \tau_n^+ \delta(\boldsymbol{r}-\boldsymbol{r}_n) \left(g_V - \frac{g_W}{2m_p}\boldsymbol{q}^2\right)_n, \tag{33}$$

$$\boldsymbol{J}_L^{\dagger}(\boldsymbol{r}) = -\sum_{n=1}^{A} \tau_n^+ \delta(\boldsymbol{r}-\boldsymbol{r}_n) \left(g_A\boldsymbol{\sigma} + ig_W\boldsymbol{\sigma}\times\boldsymbol{q} + ig_V\frac{(\boldsymbol{\sigma}\times\boldsymbol{q})}{2m_p} - g_P\frac{\boldsymbol{q}\boldsymbol{\sigma}\cdot\boldsymbol{q}}{2m_p}\right)_n \tag{34}$$

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Pions and Nuclei, Torleif Erik Oskar Ericson, W. Weise, 1988

(a) Dirac tensors

These are the bilinear combinations of Γ-matrices $\Gamma = \mathbb{1}, \gamma_5, \gamma^\mu, \gamma^\mu\gamma_5, \sigma^{\mu\nu}$ and Dirac fields

$$\begin{aligned} S &= \bar\psi\psi, \\ P &= \bar\psi\gamma_5\psi, \\ V^\mu &= \bar\psi\gamma^\mu\psi, \\ A^\mu &= \bar\psi\gamma^\mu\gamma_5\psi, \\ T^{\mu\nu} &= \bar\psi\sigma^{\mu\nu}\psi. \end{aligned} \tag{A6.1}$$

(b) Equivalent two-component spin representation

Using free positive energy Dirac spinors (eqn (A4.27)),

$$u(p,s) = \left(\frac{E_p + M}{2M}\right)^{\frac{1}{2}} \begin{pmatrix} \chi_s \\ \dfrac{\boldsymbol\sigma\cdot\mathbf{p}}{E_p + M}\chi_s \end{pmatrix} \tag{A6.2}$$

where χ_s are two-component Pauli spinors. The Dirac tensor matrix elements are given as

$$\bar u(p',s')\Gamma u(p,s) = \chi_{s'}^{+} M(p',p)\chi_s, \tag{A6.3}$$

where $M(p,p')$ have values according to Table A6.1.

(c) Non-relativistic expansion

The expressions in Table A.6.2 are obtained by expanding to order ω/M in the energy transfer $\omega = E' - E$, where $\bar E = \frac{1}{2}(E + E')$. They are particularly simple in the Breit frame defined by $\mathbf{p} = -\mathbf{p}' = \mathbf{q}/2$, $E' = E = (M^2 + \mathbf{q}^2/4)^{\frac{1}{2}}$. It follows that in the Breit frame $\mathbf{p} + \mathbf{p}' = 0$, $\omega = E' - E = 0$, and $\bar E = E$ in Table A6.2.

Table A6.1. Equivalent two-component free spinor matrix elements

Type	Γ	$M(p',p)$
S	$\mathbb{1}$	$N'N\left[1 - \dfrac{(\boldsymbol\sigma\cdot\mathbf{p}')(\boldsymbol\sigma\cdot\mathbf{p})}{(E'+M)(E+M)}\right]$
P	γ_5	$N'N\left[\dfrac{\boldsymbol\sigma\cdot\mathbf{p}}{E+M} - \dfrac{\boldsymbol\sigma\cdot\mathbf{p}'}{E'+M}\right]$
V^0	γ^0	$N'N\left[1 + \dfrac{(\boldsymbol\sigma\cdot\mathbf{p}')(\boldsymbol\sigma\cdot\mathbf{p})}{(E'+M)(E+M)}\right]$
$\mathbf{V}$	$\boldsymbol\gamma$	$N'N\left[\boldsymbol\sigma\dfrac{\boldsymbol\sigma\cdot\mathbf{p}}{E+M} + \dfrac{\boldsymbol\sigma\cdot\mathbf{p}'}{E'+M}\boldsymbol\sigma\right]$
A^0	$\gamma^0\gamma_5$	$N'N\left[\dfrac{\boldsymbol\sigma\cdot\mathbf{p}'}{E'+M} + \dfrac{\boldsymbol\sigma\cdot\mathbf{p}}{E+M}\right]$
$\mathbf{A}$	$\boldsymbol\gamma\gamma_5$	$N'N\left[\boldsymbol\sigma + \dfrac{\boldsymbol\sigma\cdot\mathbf{p}'}{E'+M}\boldsymbol\sigma\dfrac{\boldsymbol\sigma\cdot\mathbf{p}}{E+M}\right]$
T^{0j}	σ^{0j}	$N'Ni\left[\sigma^j\dfrac{\boldsymbol\sigma\cdot\mathbf{p}}{E+M} - \dfrac{\boldsymbol\sigma\cdot\mathbf{p}'}{E'+M}\sigma^j\right]$
T^{ij}	σ^{ij}	$N'N\left[\sigma^k - \dfrac{\boldsymbol\sigma\cdot\mathbf{p}'}{E'+M}\sigma^k\dfrac{\boldsymbol\sigma\cdot\mathbf{p}}{E+M}\right]\varepsilon_{ijk}$

Here $E' = E_{p'} = (\mathbf{p}'^2 + M^2)^{\frac{1}{2}}$, $E = E_p = (\mathbf{p}^2 + M^2)^{\frac{1}{2}}$, and the normalization constants are $N = \left(\dfrac{E+M}{2M}\right)^{\frac{1}{2}}$, $N' = \left(\dfrac{E'+M}{2M}\right)^{\frac{1}{2}}$.

The non-relativistic transition operators are usually adopted in most studies and they can also be derived by using a Foldy-Wouthuysen (FW) transformation.

L. L. Foldy and S. A. Wouthuysen, Phys. Rev. 78, 29 (1950).

Table: The non-relativistic reduction of the nuclear currents arranged in the order of $(1/m_p)^k$, where m_p is the nucleon mass, $q^\mu = p^\mu - p'^\mu$ and $\boldsymbol{Q} = \boldsymbol{p} + \boldsymbol{p}'$ with p^μ and $p^{\mu'}$ being the four-momenta of the initial and final nucleon states, respectively.

	$k=0$	$k=1$	$k=2$
$V^{(k)0}$	1	0	$-\frac{1}{8m_p^2}\boldsymbol{q}\cdot(\boldsymbol{q}-\mathrm{i}\boldsymbol{\sigma}\times\boldsymbol{Q})$
$W^{(k)0}$	0	$-\frac{1}{2m_p}\boldsymbol{q}\cdot(\boldsymbol{q}-\mathrm{i}\boldsymbol{\sigma}\times\boldsymbol{Q})$	0
$A^{(k)0}$	0	$\frac{1}{2m_p}\boldsymbol{\sigma}\cdot\boldsymbol{Q}$	0
$P^{(k)0}$	0	$-\frac{1}{2m_p}q^0\boldsymbol{\sigma}\cdot\boldsymbol{q}$	0
$\boldsymbol{V}^{(k)}$	0	$\frac{1}{2m_p}(\boldsymbol{Q}-\mathrm{i}\boldsymbol{\sigma}\times\boldsymbol{q})$	0
$\boldsymbol{W}^{(k)}$	$-\mathrm{i}\boldsymbol{\sigma}\times\boldsymbol{q}$	$-\frac{1}{2m_p}q^0(\boldsymbol{q}-\mathrm{i}\boldsymbol{\sigma}\times\boldsymbol{Q})$	$\frac{1}{8m_p^2}\left[\mathrm{i}\boldsymbol{q}^2(\boldsymbol{\sigma}\times\boldsymbol{q})+\mathrm{i}(\boldsymbol{\sigma}\cdot\boldsymbol{Q})(\boldsymbol{Q}\times\boldsymbol{q})-\boldsymbol{q}^2\boldsymbol{Q}+(\boldsymbol{q}\cdot\boldsymbol{Q})\boldsymbol{q}\right]$
$\boldsymbol{A}^{(k)}$	$\boldsymbol{\sigma}$	0	$-\frac{1}{8m_p^2}\left[\boldsymbol{\sigma}Q^2-\boldsymbol{Q}(\boldsymbol{\sigma}\cdot\boldsymbol{Q})+\boldsymbol{q}(\boldsymbol{\sigma}\cdot\boldsymbol{q})-\mathrm{i}\boldsymbol{q}\times\boldsymbol{Q}\right]$
$\boldsymbol{P}^{(k)}$	0	$-\frac{1}{2m_p}\boldsymbol{q}(\boldsymbol{\sigma}\cdot\boldsymbol{q})$	0

Thus, one finds the product of the currents,

$$\mathcal{J}^{\dagger}_{L,\mu}(\boldsymbol{r}_1)\mathcal{J}^{\mu\dagger}_{L}(\boldsymbol{r}_2) = -\sum_{m\neq n=1}^{A}\tau^{+}_{m}\tau^{+}_{n}\delta(\boldsymbol{r}_1-\boldsymbol{r}_m)\delta(\boldsymbol{r}_2-\boldsymbol{r}_n)\left(h_F(\boldsymbol{q})+h_{GT}\boldsymbol{\sigma}_m\cdot\boldsymbol{\sigma}_n-h_T S^{\boldsymbol{q}}_{mn}\right), \tag{35}$$

where the spin-tensor operator in momentum space is introduced

$$S^{\boldsymbol{q}}_{mn} = 3(\boldsymbol{\sigma}_m\cdot\boldsymbol{q})(\boldsymbol{\sigma}_n\cdot\boldsymbol{q})/\boldsymbol{q}^2-\boldsymbol{\sigma}_m\cdot\boldsymbol{\sigma}_n, \tag{36}$$

and

$$h_F(\boldsymbol{q}^2) = -g_V^2+g_Vg_W\frac{\boldsymbol{q}^2}{m_p}-g_W^2\frac{\boldsymbol{q}^4}{4m_p^2}, \tag{37}$$

$$h_{GT}(\boldsymbol{q}^2) = g_A^2-g_Ag_P\frac{\boldsymbol{q}^2}{3m_p}+g_P^2\frac{\boldsymbol{q}^4}{12m_p^2}+g_M^2\frac{\boldsymbol{q}^2}{6m_p^2}, \tag{38}$$

$$h_T(\boldsymbol{q}^2) = g_Ag_P\frac{\boldsymbol{q}^2}{3m_p}-g_P^2\frac{\boldsymbol{q}^4}{12m_p^2}+g_M^2\frac{\boldsymbol{q}^2}{12m_p^2}. \tag{39}$$

With the above expression, the transition operator $O^{0\nu}$ becomes

$$
\begin{aligned}
O^{0\nu} = & \sum_{m\neq n=1}^{A} \tau_m^+ \tau_n^+ \iint \mathrm{d}^3 r_1 \mathrm{d}^3 r_2 \delta(\boldsymbol{r}_1 - \boldsymbol{r}_{\boldsymbol{m}}) \delta(\boldsymbol{r}_2 - \boldsymbol{r}_{\boldsymbol{n}}) \\
& \times \frac{4\pi R_0}{g_A^2(0)} \int \frac{\mathrm{d}^3 q}{(2\pi)^3} \frac{e^{i\boldsymbol{q}\cdot\boldsymbol{r}_{12}}}{q(q+E_d)} \left[h_F(\boldsymbol{q}^2) + h_{GT}(\boldsymbol{q}^2) \boldsymbol{\sigma}_m \cdot \boldsymbol{\sigma}_n - h_T(\boldsymbol{q}^2) S^{\boldsymbol{q}}_{mn} \right] \\
\equiv & \sum_{m\neq n=1}^{A} \frac{4\pi R_0}{g_A^2(0)} \int \frac{\mathrm{d}^3 q}{(2\pi)^3} e^{i\boldsymbol{q}\cdot\boldsymbol{r}_{mn}} V^{0\nu}_{mn}(q, E_d),
\end{aligned}
\tag{40}
$$

where the neutrino potential $V^{0\nu}_{mn}(q, E_d)$ is defined as

$$
V^{0\nu}_{mn}(q, E_d) \equiv \frac{\tau_m^+ \tau_n^+}{q(q+E_d)} \left[h_F(\boldsymbol{q}^2) + h_{GT}(\boldsymbol{q}^2) \boldsymbol{\sigma}_m \cdot \boldsymbol{\sigma}_n - h_T(\boldsymbol{q}^2) S^{\boldsymbol{q}}_{mn} \right]
\tag{41}
$$

The transition operator can be written as a summation of Gamow-Teller (GT), Fermi, and tensor parts,

$$O^{0\nu} = \sum_{\alpha} O_{\alpha}^{0\nu} \tag{42}$$

where the GT, Fermi, and tensor parts are

$$O_{\rm F}^{0\nu} = \sum_{m\neq n=1}^{A} \frac{4\pi R_0}{g_A^2(0)} \int \frac{\mathrm{d}^3 q}{(2\pi)^3} e^{i\boldsymbol{q}\cdot\boldsymbol{r}_{mn}} \frac{\tau_m^+ \tau_n^+}{q(q+E_d)} h_F(\boldsymbol{q}^2), \tag{43a}$$

$$O_{\rm GT}^{0\nu} = \sum_{m\neq n=1}^{A} \frac{4\pi R_0}{g_A^2(0)} \int \frac{\mathrm{d}^3 q}{(2\pi)^3} e^{i\boldsymbol{q}\cdot\boldsymbol{r}_{mn}} \frac{\tau_m^+ \tau_n^+}{q(q+E_d)} h_{GT}(\boldsymbol{q}^2) \boldsymbol{\sigma}_m \cdot \boldsymbol{\sigma}_n, \tag{43b}$$

$$O_{\rm T}^{0\nu} = -\sum_{m\neq n=1}^{A} \frac{4\pi R_0}{g_A^2(0)} \int \frac{\mathrm{d}^3 q}{(2\pi)^3} e^{i\boldsymbol{q}\cdot\boldsymbol{r}_{mn}} \frac{\tau_m^+ \tau_n^+}{q(q+E_d)} h_T(\boldsymbol{q}^2)) S_{mn}^{\boldsymbol{q}}. \tag{43c}$$

With the plane wave expansion

$$e^{i\boldsymbol{q}\cdot\boldsymbol{r}_{mn}} = 4\pi \sum_{LM} i^L j_L(qr_{mn}) Y^*_{LM}(\hat{\boldsymbol{q}}) Y_{LM}(\hat{\boldsymbol{r}}_{mn}), \tag{44}$$

and the orthogonality relation

$$\int d\hat{\boldsymbol{q}} Y_{LM}(\hat{\boldsymbol{q}}) Y^*_{L'M'}(\hat{\boldsymbol{q}}) = \delta_{LL'}\delta_{MM'}, \quad Y_{00} = \frac{1}{\sqrt{4\pi}}, \tag{45}$$

one finds the expression for the Fermi part

$$\begin{aligned} O_F^{0\nu} &= \frac{4\pi R_0}{g_A^2(0)} \int \frac{dq}{(2\pi)^3} \frac{q^2}{q(q+E_d)} 4\pi \sum_{LM} i^L j_L(qr_{mn}) \sqrt{4\pi} \delta_{L0}\delta_{M0} Y_{LM}(\hat{\boldsymbol{r}}_{mn}) h_F(q^2) \tau_m^+ \tau_n^+ \\ &= \frac{2R_0}{\pi g_A^2(0)} \int dq q^2 \frac{h_F(q^2)}{q(q+E_d)} j_0(qr_{mn}) \tau_m^+ \tau_n^+ . \end{aligned} \tag{46}$$

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Similarly, one can derive an expression for the GT part

$$O_T^{0\nu} = -\frac{4\pi R_0}{g_A^2(0)} \int \frac{d^3q}{(2\pi)^3} \frac{h_T(\boldsymbol{q}^2)}{q(q+E_d)} 4\pi \sum_{LM} i^L j_L(q r_{mn}) Y^*_{LM}(\hat{\boldsymbol{q}}) Y_{LM}(\hat{\boldsymbol{r}}_{mn}) S^{\boldsymbol{q}}_{mn}. \tag{47}$$

By rewriting the spin-tensor operator in Eq.(36) into the following coupled form

Varshalovich:1988

$$S^{\boldsymbol{q}}_{mn} = 3\{\boldsymbol{\sigma}_1 \otimes \boldsymbol{\sigma}_2\}_2 \cdot \{\hat{\boldsymbol{q}} \otimes \hat{\boldsymbol{q}}\}_2, \tag{48}$$

and using the relation

$$\{\hat{\boldsymbol{q}} \otimes \hat{\boldsymbol{q}}\}_{2-\mu} = \sqrt{\frac{4\pi}{5}} \langle 1010|20\rangle Y_{2-\mu}(\hat{\boldsymbol{q}}), \quad \{\hat{\boldsymbol{r}} \otimes \hat{\boldsymbol{r}}\}_{2-\mu} = \sqrt{\frac{4\pi}{5}} \langle 1010|20\rangle Y_{2-\mu}(\hat{\boldsymbol{r}}). \tag{49}$$

One can simplify the tensor operator further

$$
\begin{aligned}
O_T^{0\nu} &= -3\frac{4\pi R_0}{g_A^2(0)}\int \frac{q^2 dq}{(2\pi)^3}\frac{h_T(q^2)}{q(q+E_d)}4\pi\sum_{LM}\sum_{\mu}(-1)^{\mu} i^L j_L(qr_{mn})Y_{LM}(\hat{\mathbf{r}}_{mn})\{\boldsymbol{\sigma}_m\otimes\boldsymbol{\sigma}_n\}_{2\mu} \\
&\quad \times \int d\hat{\mathbf{q}}\, Y^*_{LM}(\hat{\mathbf{q}})\{\hat{\mathbf{q}}\otimes\hat{\mathbf{q}}\}_{2-\mu} \\
&= -3\frac{4\pi R_0}{g_A^2(0)}\int \frac{q^2 dq}{(2\pi)^3}\frac{h_T(q^2)}{q(q+E_d)}4\pi\sum_{\mu}(-1)^{\mu} i^2 j_2(qr_{mn})\{\boldsymbol{\sigma}_m\otimes\boldsymbol{\sigma}_n\}_{2\mu}\{\hat{\mathbf{r}}_{mn}\otimes\hat{\mathbf{r}}_{mn}\}_{2-\mu} \\
&= \frac{2R_0}{\pi g_A^2(0)}\int q^2 dq\frac{h_T(q^2)}{q(q+E_d)} j_2(qr_{mn}) S^{\mathbf{r}}_{mn}, \qquad (50)
\end{aligned}
$$

where the minus sign is canceled by $i^2=-1$, and the spin-tensor operator in coordinate space

$$
S^{\mathbf{r}}_{mn} \equiv 3\{\boldsymbol{\sigma}_m\otimes\boldsymbol{\sigma}_n\}_2\cdot\{\hat{\mathbf{r}}_{mn}\otimes\hat{\mathbf{r}}_{mn}\}_2 = 3(\boldsymbol{\sigma}_m\cdot\hat{\mathbf{r}}_{mn})(\boldsymbol{\sigma}_n\cdot\hat{\mathbf{r}}_{mn}) - \boldsymbol{\sigma}_m\cdot\boldsymbol{\sigma}_n \qquad (51)
$$

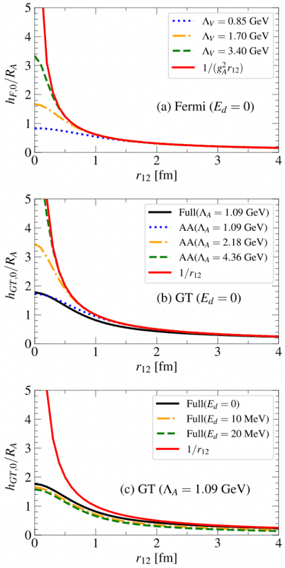
- Adding all the three terms together, one obtains the transition operator in coordinate space

$$O^{0\nu} = \sum_{m\neq n=1}^{A} \tau_m^+\tau_n^+ \Big[h^{0\nu}_{F,0}(r_{mn}, E_d) + h^{0\nu}_{GT,0}(r_{12}, E_d)\vec{\sigma}_m\cdot\vec{\sigma}_n + h^{0\nu}_{T,2}(r_{mn}, E_d)S^r_{mn} \Big],$$

where the neutrino potentials in coordinate space are defined as

$$h^{0\nu}_{\alpha,L}(r_{12}, E_d) = \frac{2R_0}{\pi g_A^2(0)}\int_0^\infty \mathrm{d}q\, q^2 \frac{h_\alpha(\boldsymbol{q}^2)}{q(q+E_d)} j_L(qr_{12})$$

where $E_d = \langle E_N\rangle - (E_I + E_F)/2 \simeq 1.12A^{1/2}$ MeV.



In the plane wave approximation for the electron wave functions

$$\sum_{s_1} u(k_1, s_1)\bar{u}(k_1, s_1) = \gamma_\mu k_1^\mu + m_e, \quad (52)$$

$$\sum_{s_2} u^c(k_2, s_2)\bar{u}^c(k_2, s_2) = \gamma_\mu k_2^\mu - m_e, \quad (53)$$

one finds the leptonic part

$$\begin{aligned} \sum_{s_1, s_2} \left|\bar{u}(k_1, s_1) P_R C \bar{u}^T(k_2, s_2)\right|^2 &= \frac{1}{4}\mathrm{Tr}\left[(1+\gamma_5)(\gamma_\mu k_2^\mu + m_e)(1-\gamma_5)(\gamma_\nu k_1^\nu - m_e)\right] \\ &= 2(\epsilon_1 \epsilon_2 - \boldsymbol{k}_1 \cdot \boldsymbol{k}_2). \quad (54) \end{aligned}$$

- The phase-space factor is simplified as below

$$G_{0\nu} = \frac{G_\beta^4 m_e^2}{2\ln(2)(2\pi)^5 R_0^2} \int\int (\epsilon_1\epsilon_2 - k_1 k_2 \cos\theta_{12}) k_1 k_2 d\epsilon_1 d\cos\theta_{12}$$
$$= \frac{G_\beta^4 m_e^2}{(2\pi)^5 R_0^2} \frac{1}{\ln(2)} \int_{m_e}^{m_e+Q_{\beta\beta}} k_1 k_2 \epsilon_1 \epsilon_2 d\epsilon_1 \quad (55)$$

where θ_{12} is the angle between two electron momenta $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$, and $\epsilon_2 = E_I - E_F - \epsilon_1$. In the above derivation, we have applied the relation $k_i^2 dk_i = k_i \epsilon_i d\epsilon_i$.

In the Fermi-Primakoff-Rosen (FPR) approximation, the Coulomb correction $F(Z_F, \epsilon)$ can be derived analytically under the approximation that the two electrons scatter off a point charge Z_F in non-relativistic kinematics

$$\begin{aligned} \mathcal{F}^{\rm NR}(Z_F, \epsilon) &= e^{\pi\eta} \mid \Gamma(1+i\eta) \mid^2 = \frac{2\pi\eta}{1-\exp(-2\pi\eta)} \\ &\approx \mathcal{F}^{\rm PR}(Z_F, \epsilon) = \left(\frac{\epsilon}{k}\right) F(Z_F), \end{aligned} \tag{56}$$

where the Sommerfeld parameter $\eta = \alpha Z_F \epsilon / k$ with $k = |\boldsymbol{k}|$ and ϵ being the momentum and energy of the electron, respectively, and

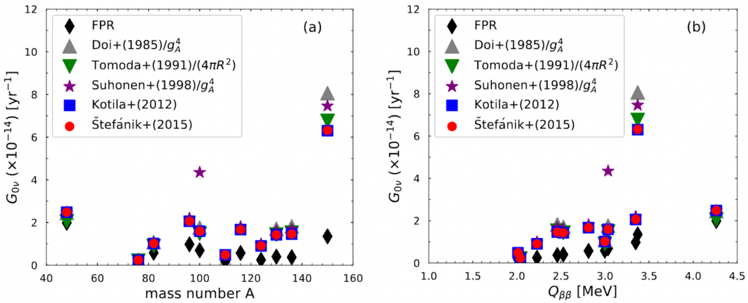
$$F(Z_F) \equiv \frac{2\pi\alpha Z_F}{1-\exp(-2\pi\alpha Z_F)} \tag{57}$$

for the energy-independent Fermi function.

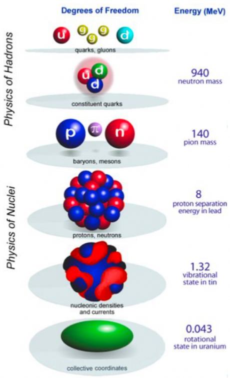
With the FPR approximation, the phase-space factor is simplified as

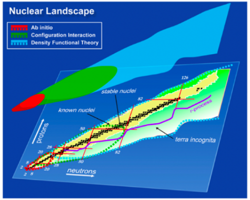
$$
\begin{aligned}
G_{0\nu}(\mathrm{FPR}) &= \frac{G_\beta^4 m_e^2}{(2\pi)^5 R_0^2}\frac{1}{\ln(2)}\int_{m_e}^{m_e+Q_{\beta\beta}} k_1 k_2 \epsilon_1 \epsilon_2 \mathcal{F}^{\mathrm{PR}}(Z_F,\epsilon_1)\mathcal{F}^{\mathrm{PR}}(Z_F,\epsilon_2) d\epsilon_1 \\
&\simeq \frac{F^2(Z_F)}{R_0^2}\frac{G_\beta^4 m_e^2}{(2\pi)^5}\frac{1}{\ln(2)}\int_0^{Q_{\beta\beta}} dT_1 \epsilon_1^2 \epsilon_2^2 \\
&= \frac{F^2(Z_F)}{R_0^2}\frac{G_\beta^4 m_e^2}{(2\pi)^5}\frac{1}{\ln(2)}\int_{m_e}^{m_e+Q_{\beta\beta}} (Q_{\beta\beta}+2m_e-\epsilon_1)^2 \epsilon_1^2 d\epsilon_1 \\
&= \frac{F^2(Z_F)}{R_0^2}\frac{G_\beta^4 m_e^7}{(2\pi)^5}\frac{1}{\ln(2)}\left(\frac{\tilde{T}_0^5}{30}+\frac{\tilde{T}_0^4}{3}+\frac{4\tilde{T}_0^3}{3}+2\tilde{T}_0^2+\tilde{T}_0\right)
\end{aligned}
\tag{58}
$$

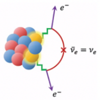
where $\tilde{T}_0 = Q_{\beta\beta}/m_e$.



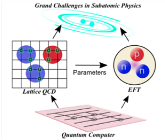
- The FPR approximation underestimates the $G_{0\nu}$, in particular for the heavier candidate nuclei.
- The $G_{0\nu}$ increases with the $Q^5_{\beta\beta}$ value except for ^{150}Nd (with a significant distortion Coulomb effect for which the FPR approximation is not valid).
- The use of a realistic proton density in isotopes brings changes less than 1%.



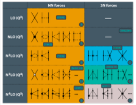




(Lattice)
QCD for NP

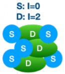


2010s — ab initio methods



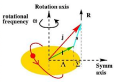
2000s — High-precision NN interactions — Entem, Machleidt (2003)

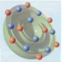
1990s — Chiral EFT for NN interaction — Weinberg (1991)



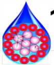
1970s — Self-consistent mean-field/EDF approaches; Interacting Boson Models — Arima, Iachello (1974) IBM; Hill, Griffin, Wheeler (1953,1957) GCM

1950s — Collective models; Shell models — Bohr and Mottelson (1953) PRM, BH; Bohm, Pines (1951-1953) RPA; Goeppert-Mayer (1949); Haxel, Jensen, and Suess (1949) ISM





1930s — Liquid drop model — von Weizsäcker (1935)

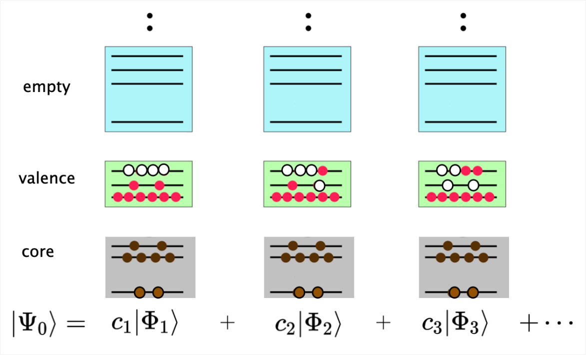


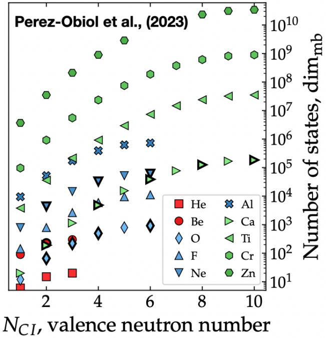
Modern studies: phenom. nuclear forces

- Interacting shell models (ISM) Vergados (1976), Haxton (1981), H.F.Wu (1985, 1993), Caurier (2008), Menéndez (2009), Horoi (2010), Coraggio (2020)
- Particle-number (and angular-momentum) projected BCS (HFB) with a schematic (PP+QQ) hamiltonian Grotz, Klapdor (1985), Chandra (2008), Rath (2010), Hinohara (2014)
- Quasi-particle random-phase approx. (QRPA) with a G-matrix residual interaction Vogel-2ν (1986), Engel (1988), Rodin (2003), Faessler (1998), Simkovic (1999), Fang (2010) or EDF Mustonen (2013), Terasaki (2015), Lv(2023), Bai (2023?)
- Interacting Boson Models (IBM) Barea (2009, 2012)
- GCM+EDFs Rodríguez (2010), Song (2014), Yao (2015)
- Angular momentum projected interacting shell model based on an effective interaction Iwata, Shimizu (2016) or EDF Wang (2021, 2023)
- Others: Generalized-seniority scheme Engel, Vogel, Ji, Pittel (1989)

Recent ab initio studies: chiral nuclear forces

- Quantum MC/NCSM for light nuclei: Pastore(2018); Yao(2021)
- Basis-expansion methods for candidates: Yao(2020), Belley(2021), Navorio(2021)





- Dimension of the model space: $d \sim C_M^{Z_v} C_M^{N_v}$, where M is the number of s.p. states in the valence space for neutrons and protons.
- Include all correlations, but in a limited model space determined by the combination of valence nucleons and valence orbits.

The basic building blocks of IBM are s and d bosons

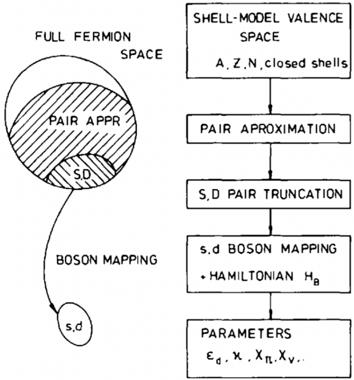
J. Barea and F. Iachello, PRC79, 044301 (2009)

$$|\Psi_0(N;\alpha_\mu)\rangle \propto \left(s^\dagger + \sum_\mu \alpha_\mu d^\dagger_\mu\right)^N |vac\rangle, \quad N = n_s + n_d$$

$$s^\dagger = \sum_j \alpha_j \sqrt{\frac{\Omega_j}{2}} \left(c^\dagger_j \times c^\dagger_j\right)^{(0)}$$

$$d^\dagger_\mu = \sum_{j\leqslant j'} \beta_{jj'} \frac{1}{\sqrt{1+\delta_{jj'}}} \left(c^\dagger_j \times c^\dagger_{j'}\right)^{(2)}_\mu$$

where s, d are Cooper pairs formed by two nucleons in the valence shell coupled to angular momenta $J = 0$ and $J = 2$, respectively. The structure coefficients α, β are to be determined.



The pnQRPA in a nutshell

The proton-neutron quasiparticle random-phase approximation (pn-QRPA) as a method for small-amplitude (charge-changing) excitations.

- From g.s. to excited state

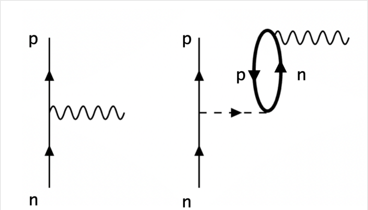
$$|\Psi_\nu(N_{I/F})\rangle = Q_\nu^\dagger|\mathrm{QRPA}\rangle, \quad Q_v|\mathrm{QRPA}\rangle = 0$$

where (quasi-boson approx.)

$$Q_\nu^\dagger = \sum_{pn} X_{pn}^\nu \beta_p^\dagger \beta_n^\dagger - Y_{pn}^\nu \beta_n \beta_p, \quad [Q_\nu, Q_{\nu'}^\dagger] \simeq \delta_{\nu\nu'}$$

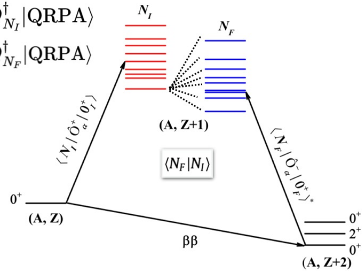
- (Q)RPA equation P. Ring, P. Schuck, The nuclear many-body problem, 1980

$$\begin{pmatrix} A & B \\ B^* & A^* \end{pmatrix} \begin{pmatrix} X_{pn}^\nu \\ Y_{pn}^\nu \end{pmatrix} = \hbar\Omega_\nu \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} X_{pn}^\nu \\ Y_{pn}^\nu \end{pmatrix}$$



$$|N_I\rangle = Q_{N_I}^\dagger|\mathrm{QRPA}\rangle$$

$$|N_F\rangle = Q_{N_F}^\dagger|\mathrm{QRPA}\rangle$$



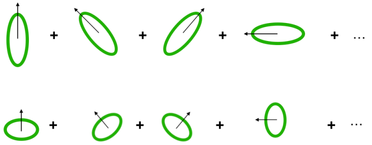
The GCM in a nutshell

The wave function in generator coordinate method (GCM) is constructed as a superposition of mean-field configurations with different intrinsically deformed shapes

$$|\Psi_{JM\pi}\rangle = \sum_{q_i,K} f_K^J(q_i) \hat{P}^J_{MK} \hat{P}^{N,Z,\pi}|\Phi(q_i)\rangle,$$

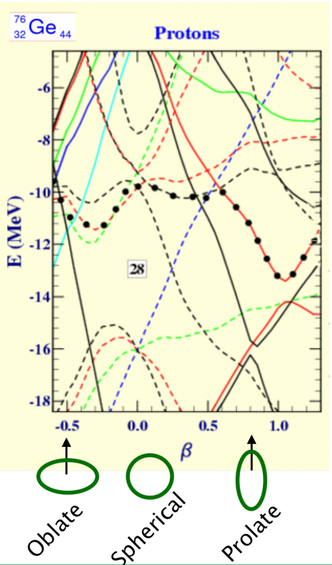
where the angular momentum projection operator

$$\hat{P}^J_{MK} = \frac{2J+1}{8\pi^2}\int d\Omega D^{J*}_{MK}(\Omega)\hat{R}(\Omega).$$



$+ \cdots = \hat{R}(\Omega)|\Phi(q_i)\rangle$

$+ \cdots = \hat{R}(\Omega)|\Phi(q_j)\rangle$



NME of $0\nu\beta\beta$ decay

中山大學 SUN YAT-SEN UNIVERSITY

- Nuclear models produce many-body wave functions $\left|\Psi_{I/F}\right\rangle$
- By writing the transition operator in second quantization form, the NME becomes

$$M^{0\nu} \equiv \langle\Psi_F|\,\hat{O}^{0\nu}\,|\Psi_I\rangle = \sum_{pp'nn'} \langle pp'|O^{0\nu}|nn'\rangle \langle\Psi_F|c_p^\dagger c_{p'}^\dagger c_{n'} c_n|\Psi_I\rangle$$

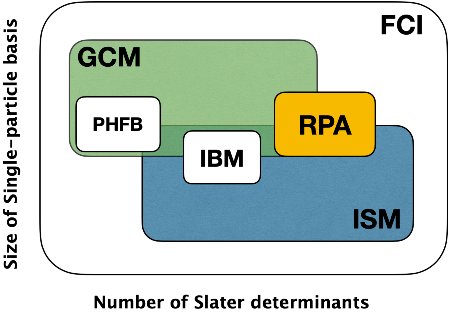
- The two-body matrix element $\langle pp'|O^{0\nu}|nn'\rangle$ depends on transition operators.
- The two-body transition density $\langle\Psi_F|c_p^\dagger c_{p'}^\dagger c_{n'} c_n|\Psi_I\rangle$ depends on nuclear models.

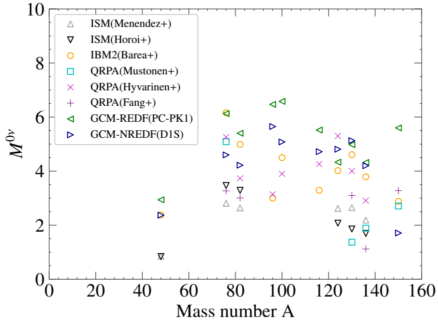
Extended Wick Theorem: R. Balian, E. Brezin, Nuovo Cim.B 64, 37 (1969)

For two arbitrary non-orthogonal quasiparticle vacua, $|\Phi_I\rangle, |\Phi_F\rangle$, one has

$$\frac{\langle\Phi_F|c_p^\dagger c_{p'}^\dagger c_{n'} c_n|\Phi_I\rangle}{\langle\Phi_F|\Phi_I\rangle} = \langle\Phi_F|c_p^\dagger c_{p'}^\dagger|\Phi_I\rangle\langle\Phi_F|c_{n'} c_n|\Phi_I\rangle + \langle\Phi_F|c_p^\dagger c_n|\Phi_I\rangle\langle\Phi_F|c_p^\dagger c_{n'}|\Phi_I\rangle - \langle\Phi_F|c_p^\dagger c_{n'}|\Phi_I\rangle\langle\Phi_F|c_p^\dagger c_{n'}|\Phi_I\rangle. \quad (59)$$

Comparison of nuclear models

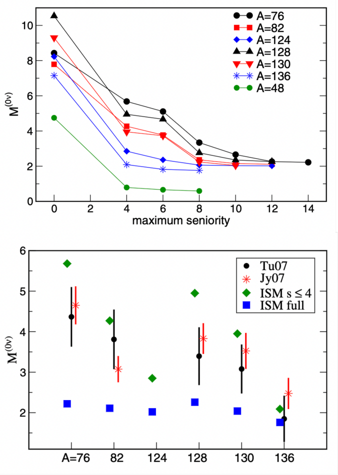




- A general argument: nuclear-structure properties reasonably reproduced (to be checked quantitatively).
- These nuclear models are not equivalent! Different schemes (model spaces and interactions): apples v.s. oranges
- ISM predicts small NMEs, while IBM and EDF predict large NMEs. Efforts in resolving the discrepancy: very difficult or even impossible?

Comparison of ISM with QRPA

- Seniority number: number of particles that are not in pairs.
- Enlarging the model space of ISM (with the nonzero seniority numbers) reduces significantly the NMEs.
- The NME by the spherical QRPA is comparable to that of ISM with $s \leq 4$.
  Caveat: different interactions are employed!

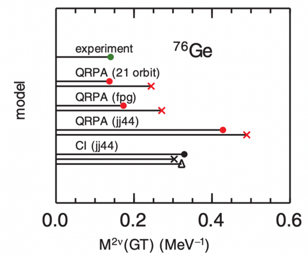


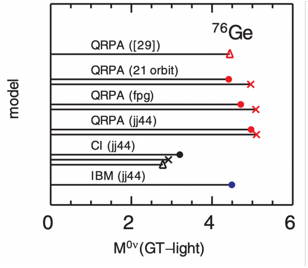
E. Caurier et al., PRL 100, 052503 (2008)

To understand the discrepancy between ISM (IBM) and QRPA, the same operators should be employed.

- With the same interaction *jj*44 (pf5g9), the QRPA and IBM produce systematically larger $M^{0\nu}$ values for the NMEs than the ISM (CI).

Brown, Fang, Horoi, PRC 92, 041301(R) (2015)





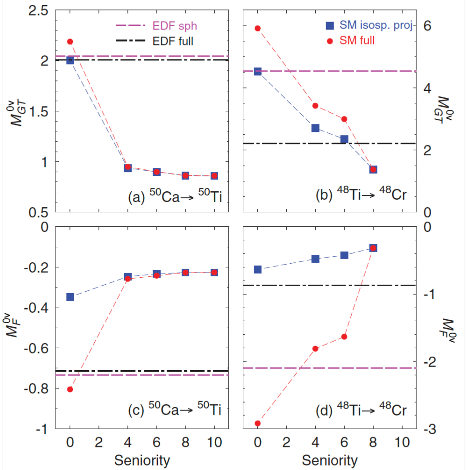
Conclusion

One of the main sources of discrepancy between ISM and QRPA is the different numbers of pairing-broken configurations which decreases the NME significantly.

- The NMEs by the spherical EDF calculation approximately equal to the ISM with the seniority number $s = 0$.
- The full EDF (with deformation and other effects) produces a much smaller NME, much still larger than that by the ISM.
- Caveat: different interactions (Gogny D1S in EDF and KB3G in ISM) are used.

Conclusion

In the EDF studies, it is necessary to include pairing-broken configurations with higher-seniority numbers, which together with deformation effects quench the NME.



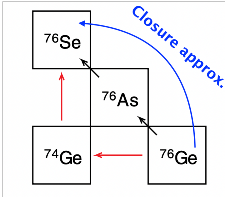
J. Menéndez et al., PRC90, 024311 (2014)

- Re-writing the transition operator by inserting intermediate states $|N\rangle$

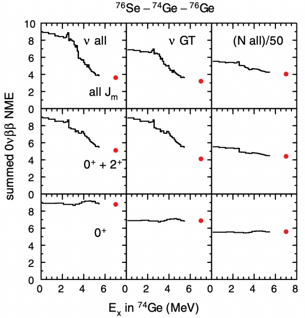
$$M^{0\nu} = \sum_{pp'nn'} \langle pp'|O^{0\nu}|nn'\rangle \sum_N \langle \Psi_F|c_p^\dagger c_{p'}^\dagger|\Psi_N\rangle \langle \Psi_N|c_{n'} c_n|\Psi_I\rangle$$

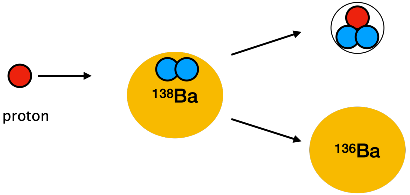
where the intermediate state is chosen differently.

- The NME is decomposed into sums of products over the intermediate nucleus with two less nucleons.
- Pairing interaction enhances the two-nucleon transfer cross sections and thus the NME. G. Potel et al., PRC87, 054321 (2013)
- The QRPA and IBM treat the pairing correlation differently.

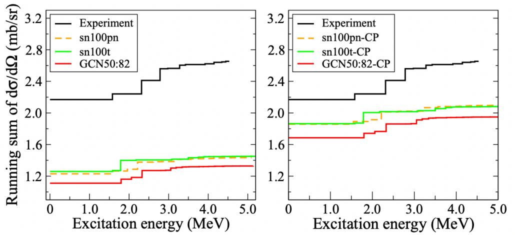


Two-nucleon transfer

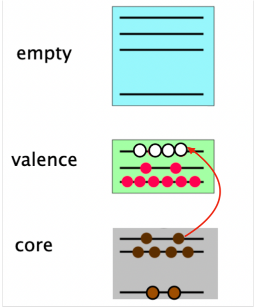




ISM (w/o core polarization) ISM (w/ core polarization)

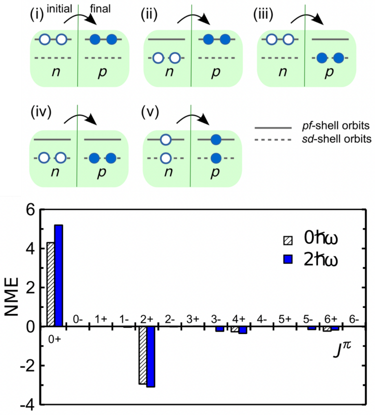


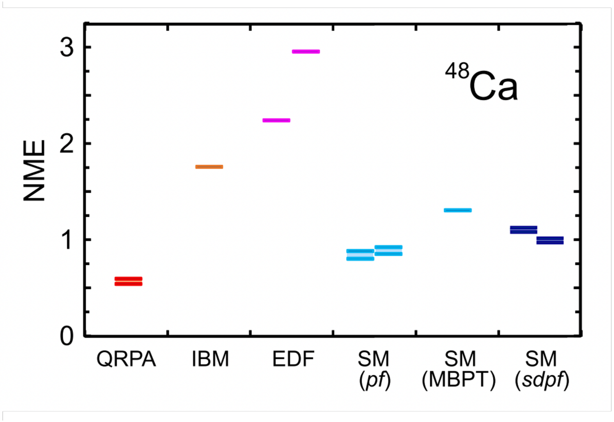
B.M. Rebeiro et al., Phys.Lett.B 809 (2020) 135702



Core polarization

- Core polarization effect is necessary for the ISM to reproduce the $^{138}\mathrm{Ba}(p,t)^{136}\mathrm{Ba}$ reaction cross section.
- With this effect, the $J=0$ component of $M^{0\nu}_{\mathrm{GT}}$ increases from 5.67 to 8.96.





- The NME increases by about 30% ($M^{0\nu} = 1.1$), which is due to cross-shell $sd - pf$ pairing correlations. Y. Iwata et al., Phys Rev Lett 116, 112502 (2016)

Conclusion: enlarge the model space of ISM generally increases the NME.

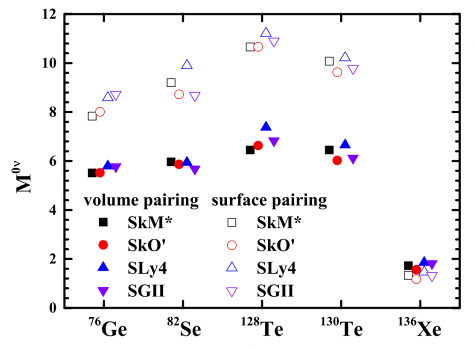
The isovector pairing interaction,

$$V^{pp}(\boldsymbol{r}_1, \boldsymbol{r}_2) = \left[t_0' + \frac{t_3'}{6}\rho\left(\frac{\boldsymbol{r}_1 + \boldsymbol{r}_2}{2}\right)\right]\delta(\boldsymbol{r}_1 - \boldsymbol{r}_2)$$

Parameters t_i' are fitted to pairing gaps determined from odd-even mass difference and

- Volume type: $t_3' = 0$
- Surface type: $t_3' \neq 0$, reducing pairing in nuclear interior.

Two different choices of isovector pairing interactions leads to quite different NMEs.



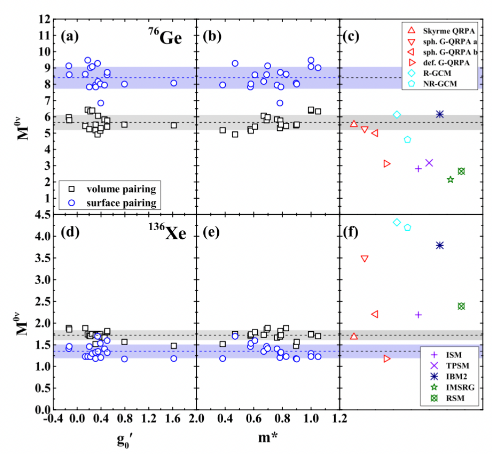
W. L. Lv, Y.-F. Niu, D.-L. Fang, JMY, C.-L. Bai, J. Meng, PRC108, L051304 (2023)

Uncertainties from different treatments of pairing correlations

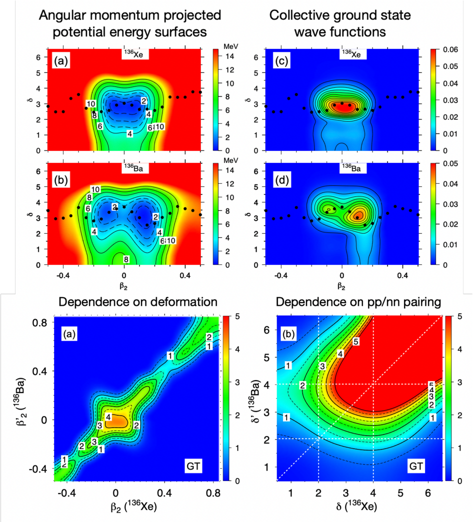
18 Skyrme EDFs characterized with

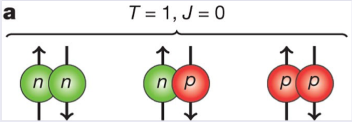
- different nucleon effective mass m^* (single-particle properties),
- different Landau parameter g_0' (spin-isospin excitation properties)

and two types of isovector pairing interactions are employed in the spherical QRPA calculation.



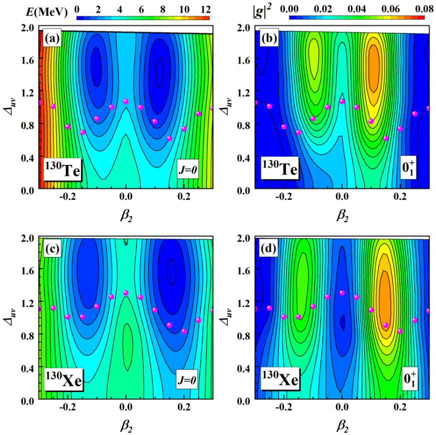
W. L. Lv, Y.-F. Niu, D.-L. Fang, JMY, C.-L. Bai, J. Meng, PRC108, L051304 (2023)

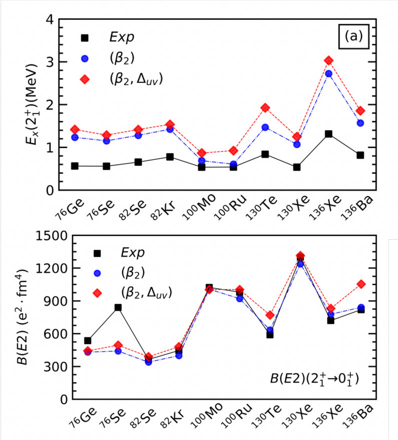




- The inclusion of isovector pairing fluctuation increases the NMEs of candidate nuclei by 10%–40%

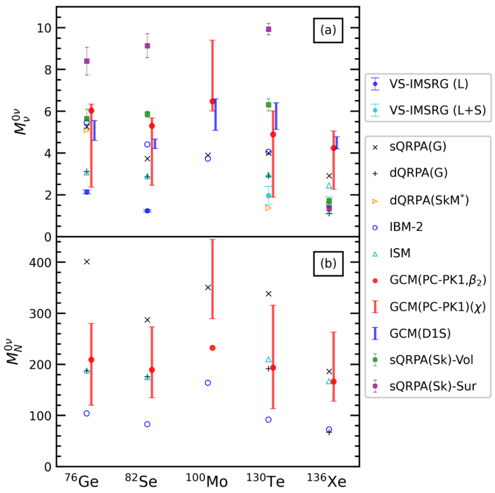
N. López-Vaquero et al., PRL111, 142501 (2013)





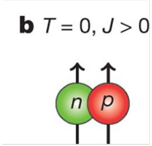
- The inclusion of isovector pairing fluctuation enhances the pairing correlation, increasing the excitation energies.

C.R. Ding, X. Zhang, JMY, P. Ring, J. Meng, PRC108, 054304 (2023)



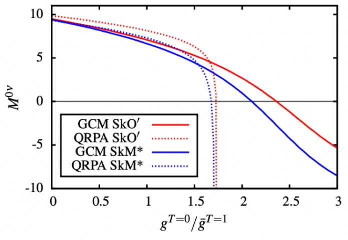
C.R. Ding, X. Zhang, JMY, P. Ring, J. Meng, PRC108, 054304 (2023)

- The predicted NMEs are reduced by about 12% – 62% with pairing strengths determined by excitation energies.
- The NMEs increase by about 56% – 218% with isovector pairing fluctuation (an uncertainty of a factor up to three), comparable to the observed discrepancy among various nuclear models.
- Including isoscalar pairing and cranked states to reduce the uncertainty.

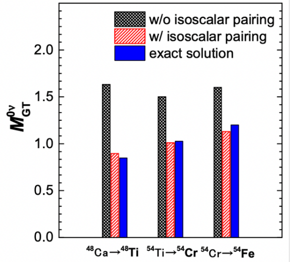


Isoscalar pairing

N. Hinohara & J. Engel, PRC90, 031301(R) (2014)



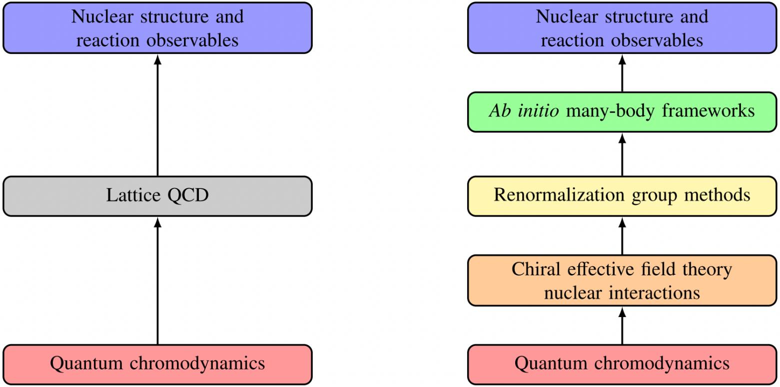
C. F. Jiao, J. Engel, and J. D. Holt, PRC96, 054310 (2017)



- The NME decreases with the increase of the strength of isoscalar pairing.
- The isoscalar pairing fluctuation decreases the NMEs of candidate nuclei.

Conclusion: The strength parameters of pairing correlations between nucleons in QRPA and MR-EDF need to be further constrained by other data than odd-even mass difference.

Contents



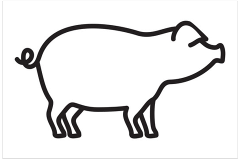
K. Hebeler, Phys. Rep. 890, 1 (2021)

Basic ideas of constructing an EFT

S. Weinberg, Physica 96A, 327 (1979); S. Weinberg, Phys. Lett. B 251, 288 (1990); Nucl. Phys. B363, 3 (1991)

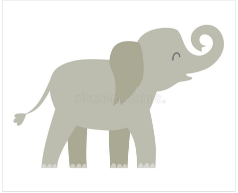
- Symmetry consideration (chiral symmetry of QCD)
- Identification of important energy scales (active and break down), and the effective degrees of freedom (pions and nucleons)
- Writing down a most general Lagrangian (order by order convergence)

EFT for an elephant:



LO

NLO



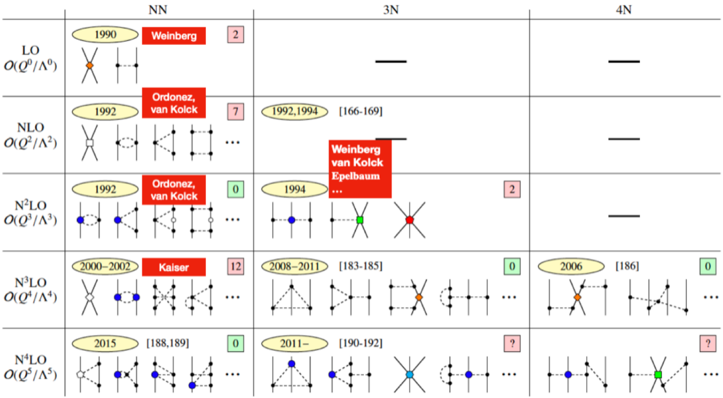
N2LO

Strategy for developing ab initio methods for $0\nu\beta\beta$ decay

- Operator forms: (Chiral) effective field theory (EFT) to specify the forms of nuclear forces and transition operators at different orders.
- Parametrization: Scattering data or Lattice QCD calculations to determine the low-energy constants (LECs) of the operators.
- Many-body solvers: A systematically improvable nuclear-structure theory to solve the nuclear many-body problem and compute observables.

Nuclear forces from chiral EFT

- Non-relativistic chiral 2N+3N interactions (Weinberg power counting and others)



K. Hebeler, Phys. Rep. 890, 1 (2020)

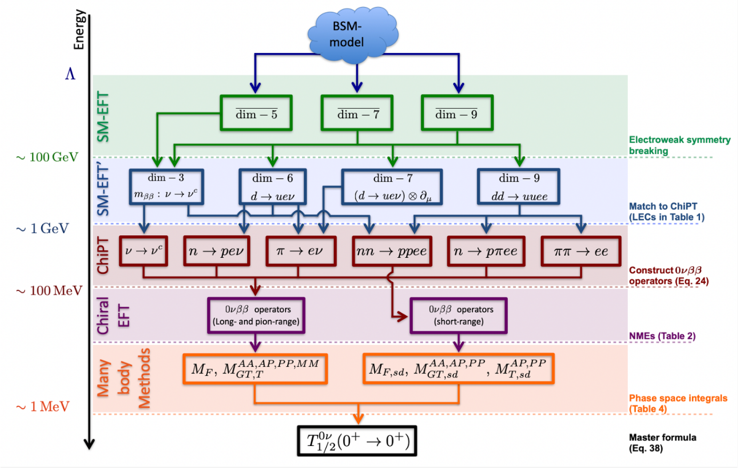
- Relativistic chiral 2N interaction (up to N^{2}LO)

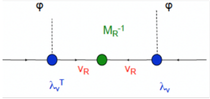
J.-X. Lu, C.-X. Wang, Y. Xiao, L.-S. Geng, J. Meng, P. Ring, PRL128, 142002 (2022)

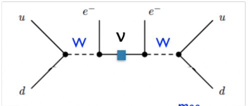
EFT for the $0\nu\beta\beta$ transition operators

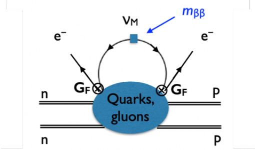
Model-independent analysis of operators at different energy scales

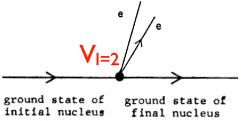
Cirigliano (2018)











- In the SM-EFT, the weak scale effective Lagrangian with the Weinberg dim-5 ($\Delta L = 2$) operator

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm SM} + \frac{u_{\alpha\beta}}{\Lambda_{\rm LNV}} \epsilon_{ij}\epsilon_{mn} L_i^{T\alpha} C L_m^{\beta} H_j H_n + H.c. \tag{60}$$

where $L^T = (\nu_L, e_L)$, $\alpha, \beta = e, \mu, \tau$, and i, j, m, n are SU(2) indices.

- The quark-level Lagrangian for the $0\nu\beta\beta$ decay induced by the $m_{\beta\beta}$

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm QCD} - \frac{4G_F}{\sqrt{2}} V_{ud} (\bar{u}_L \gamma^\mu d_L)(\bar{e}_L \gamma_\mu \nu_{eL}) - \frac{m_{\beta\beta}}{2} \nu_{eL}^T C \nu_{eL} + H.c. \tag{61}$$

where G_F for Fermi constant, and V_{ud} for the CKM matrix.

Chiral EFT for the $0\nu\beta\beta$ transition operators

- For low-energy hadronic and nuclear processes ($E \sim$100 MeV), the above $\mathcal{L}_{\rm eff}$ needs to be mapped onto a theory of hadrons, which can be organized using chiral EFT according to the scaling of operators in powers of Q/Λ_χ,

$$Q \sim m_\pi \sim 0.14\text{GeV}, \quad \Lambda_\chi \simeq 4\pi F_\pi \simeq 1.1\text{GeV}. \tag{62}$$

  Operators in the chiral EFT are expressed in terms of the following degrees of freedom: N, π, e^-, ν_e.
- Naive dimension analysis (NDA): Feynman rule for the propagators

$$N(Q^{-1}), \quad \pi(Q^{-2}), \quad \partial(Q^1), \quad \int d^4p(Q^4). \tag{63}$$

Chiral EFT for the $0\nu\beta\beta$ transition operators

- The power of a connected irreducible diagram involving A nucleons S. Weinberg, Phys. Lett. B 251 (1990) 288; 295 (1992) 114;Nuclear Phys. B363 (1991) 3

$$\nu = 2A - 2C + 2L - 2 + \sum_i \Delta_i \tag{64}$$

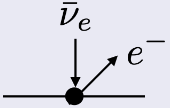
  where A, C, L are for the numbers of nucleons, separately connected pieces, and loops in the diagram, Δ_i for the interaction vertex.

- For the NN system, $A = 2, C = 1, L = 0$, the power is simply determined by (V. Cirigliano+, PRC2018)

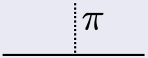
$$\nu = \sum_i \Delta_i = \sum_i (\frac{n_f}{2} + d - 2 + n_e), \tag{65}$$

  where n_f, d count the numbers of nucleon fields and derivatives, n_e for numbers of charged leptons in the vertex.

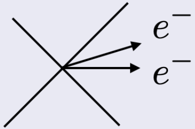
Counting the power of vertices



$$\Delta(n_f = 2, d = 0, n_e = 1) = \frac{n_f}{2} + d - 2 + n_e = 0$$



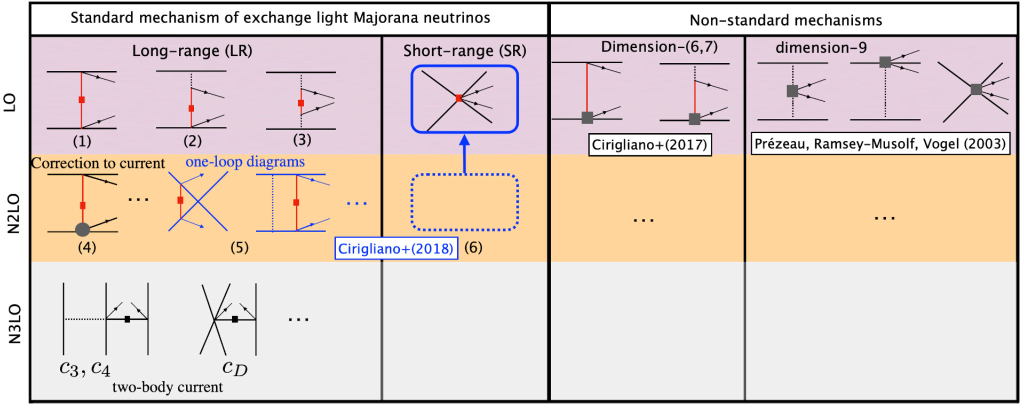
$$\Delta(n_f = 2, d = 1, n_e = 0) = \frac{n_f}{2} + d - 2 + n_e = 0$$



$$\Delta(n_f = 4, d = 0, n_e = 2) = \frac{n_f}{2} + d - 2 + n_e = 2$$

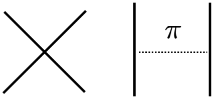
For the 3N system, $A = 3$, $C = 1$, $L = 0$, the power ν becomes

$$\nu = 2 + \sum_i \Delta_i = 2 + \sum_i \left(\frac{n_f}{2} + d - 2 + n_e\right). \quad (66)$$



LO description of $nn \to ppe^- e^-$ process

- Diagrams for nuclear force at the LO:



- Nuclear potential

$$\begin{aligned} V_{NN}^{(\mathrm{LO})}(\mathbf{q}) &= C_S + C_T \boldsymbol{\sigma}^{(1)} \cdot \boldsymbol{\sigma}^{(2)} - \frac{g_A^2}{4F_\pi^2} \frac{(\boldsymbol{\sigma}^{(1)} \cdot \mathbf{q})(\boldsymbol{\sigma}^{(2)} \cdot \mathbf{q})}{\mathbf{q}^2 + m_\pi^2} \boldsymbol{\tau}^{(1)} \cdot \boldsymbol{\tau}^{(2)} \\ &= C_S + C_T \boldsymbol{\sigma}^{(1)} \cdot \boldsymbol{\sigma}^{(2)} - \frac{g_A^2}{12F_\pi^2} \left[\left(1 - \frac{\boldsymbol{q}^2}{\mathbf{q}^2 + m_\pi^2}\right) \boldsymbol{\sigma}^{(1)} \cdot \boldsymbol{\sigma}^{(2)} - \frac{\boldsymbol{q}^2}{\mathbf{q}^2 + m_\pi^2} S_{\boldsymbol{q}}^{(12)} \right] \end{aligned}$$

where $\boldsymbol{q} = \boldsymbol{p} - \boldsymbol{p}'$ for transferred momentum and $C_{S,T}$ for the LECs of the contact interaction and

$$S_{\boldsymbol{q}}^{(12)} = 3 \frac{(\boldsymbol{\sigma}^{(1)} \cdot \mathbf{q})(\boldsymbol{\sigma}^{(2)} \cdot \mathbf{q})}{\mathbf{q}^2} - \boldsymbol{\sigma}^{(1)} \cdot \boldsymbol{\sigma}^{(2)}. \tag{67}$$

- For the nn/pp and $^1S_0$ channel,

$$\boldsymbol{\sigma}^{(1)} \cdot \boldsymbol{\sigma}^{(2)} = -3, \quad S_{\boldsymbol{q}}^{(12)} = 0, \tag{68}$$

the LO nuclear potential is simplified as

$$\begin{aligned} V_{NN}^{1S0}(p, p') &= C_S - 3C_T - \frac{g_A^2}{4F_\pi^2}\left[\left(1 - \frac{\boldsymbol{q}^2}{\mathbf{q}^2 + m_\pi^2}\right)\right] \\ &\equiv \frac{g_A^2}{4F_\pi^2}\left(C - \frac{m_\pi^2}{\mathbf{q}^2 + m_\pi^2}\right). \end{aligned} \tag{69}$$

- The high-momentum part of the nuclear potential is regulated

$$V_{NN}^{1S0}(p, p') \to f_\Lambda^{n_{\rm exp}}(p) V_{NN}^{1S0}(p, p') f_\Lambda^{n_{\rm exp}}(p') \tag{70}$$

where the regulator

$$f_\Lambda^{n_{\rm exp}} = \exp\left[- \left(\frac{p^2}{\Lambda^2} \right)^{n_{\rm exp}} \right]. \tag{71}$$

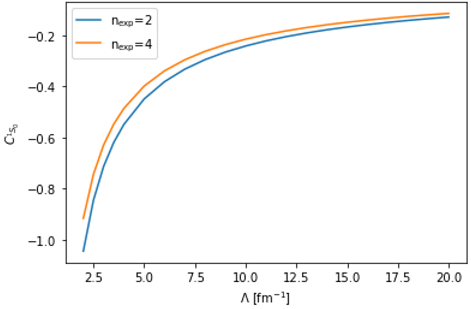
The NN potential of the partial wave ${}^1S_0$ in coordinate-space is given by

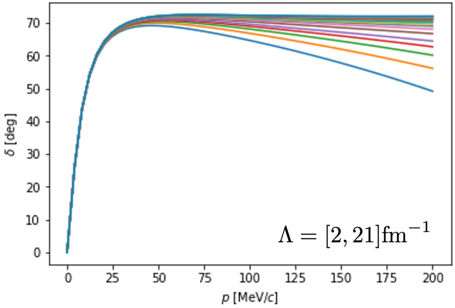
$$V_{NN}^{1S0}(r, r') = \frac{2}{\pi} \int dp p^2 \int dp' (p')^2 j_0(pr) j_0(p'r') V_{NN}^{1S0}(p, p'). \tag{72}$$

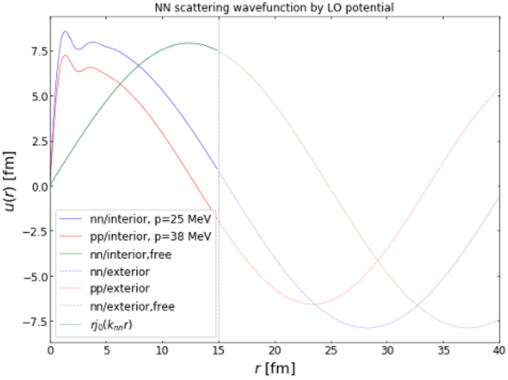
- The LEC C is fitted to NN scattering length for a given regulator $(\Lambda, n_{\rm exp})$.
- At low energies NN scattering, the s-wave phase shift δ can be written as

$$p \cot\delta = -\frac{1}{a} + \frac{1}{2} r_0 p^2$$

where r_0 is the effective range, and a the scattering length.







- The external wave function of two-nucleon

$$u_\ell^{ext}(r) = \frac{e^{-i\delta_\ell}}{2ik}\left[I_\ell(\eta, kr) - e^{2i\delta_\ell} O_\ell(\eta, kr)\right] \quad (73)$$

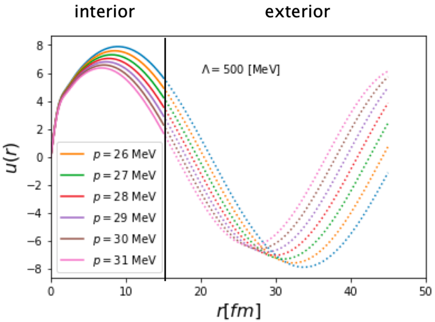
where the incoming and outgoing wave functions

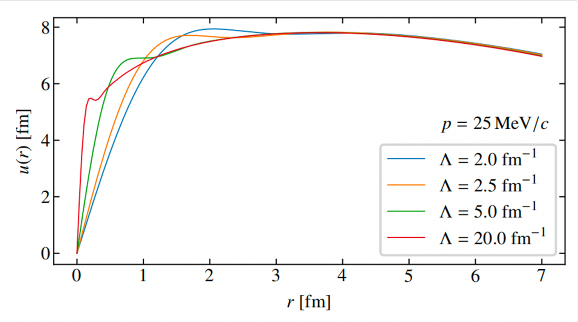
$$I_\ell(\eta, x = kr) \underset{x\to\infty}{\longrightarrow} \mathrm{e}^{-\mathrm{i}\left(x - \frac{1}{2}\ell\pi - \eta \ln 2x + \sigma_\ell\right)}$$
$$O_\ell(\eta, x = kr) \underset{x\to\infty}{\longrightarrow} \mathrm{e}^{\mathrm{i}\left(x - \frac{1}{2}\ell\pi - \eta \ln 2x + \sigma_\ell\right)} \quad (74)$$

We neglect the Coulomb force, i.e., $\eta = 0$.

- The internal wave function is obtained using the R matrix. P. Descouvemont and D. Baye 2010 Rep. Prog. Phys. 73, 036301

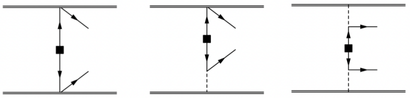
The impact of momentum p and cutoff Λ on the wave function of two nucleons.





- At the LO, the hadronic current (V+A+P)

$$\mathcal{J}_L^{\mu\dagger} = \bar{\psi}_N \left[g_V\gamma^\mu - g_A\gamma^\mu\gamma_5 + g_A \frac{2m_p}{(\boldsymbol{q}^2 + m_\pi^2)} q^\mu \gamma_5 \right] \tau^+ \psi_N \quad (75)$$



Again, the transition operator of $0\nu\beta\beta$ decay is decomposed into three terms

$$O^{0\nu} = \sum_{m\neq n=1}^{A} \tau_m^+\tau_n^+ \left[h_{F,0}^{0\nu}(r_{mn},0) + h_{GT,0}^{0\nu}(r_{12},0)\boldsymbol{\sigma}_m\cdot\boldsymbol{\sigma}_n + h_{T,2}^{0\nu}(r_{mn},0)S_{mn}^{\boldsymbol{r}} \right],$$

$$h_{\alpha,L}^{0\nu}(r_{12},0) = \frac{2R_0}{\pi g_A^2(0)} \int_0^\infty \mathrm{d}q\, q^2 \frac{h_\alpha(\boldsymbol{q}^2)}{\boldsymbol{q}^2} j_L(qr_{12})$$

But the neutrino potential at the LO is simplified as,

$$
\begin{aligned}
h_F(\boldsymbol{q}^2) &= -g_V^2, \qquad (76)\\
h_{GT}(\boldsymbol{q}^2) &= g_A^2 - g_A^2\left(\frac{2m_p}{\boldsymbol{q}^2+m_\pi^2}\right)\frac{\boldsymbol{q}^2}{3m_p} + g_A^2\left(\frac{2m_p}{\boldsymbol{q}^2+m_\pi^2}\right)^2\frac{\boldsymbol{q}^4}{12m_p^2}\\
&= g_A^2\left(1-\frac{2}{3}\frac{\boldsymbol{q}^2}{\boldsymbol{q}^2+m_\pi^2}+\frac{1}{3}\frac{\boldsymbol{q}^4}{(\boldsymbol{q}^2+m_\pi^2)^2}\right) \qquad (77)\\
h_T(\boldsymbol{q}^2) &= g_A^2\left(\frac{2m_p}{\boldsymbol{q}^2+m_\pi^2}\right)\frac{\boldsymbol{q}^2}{3m_p} - g_A^2\left(\frac{2m_p}{\boldsymbol{q}^2+m_\pi^2}\right)^2\frac{\boldsymbol{q}^4}{12m_p^2}\\
&= g_A^2\left(\frac{2}{3}\frac{\boldsymbol{q}^2}{\boldsymbol{q}^2+m_\pi^2}-\frac{1}{3}\frac{\boldsymbol{q}^4}{(\boldsymbol{q}^2+m_\pi^2)^2}\right). \qquad (78)
\end{aligned}
$$

We only consider the transition $nn(^1S_0) \to pp(^1S_0)$, for which $\boldsymbol{\sigma}_m \cdot \boldsymbol{\sigma}_n = -3$, $S^r_{mn} = 0$,

$$\begin{aligned} O^{0\nu} &= \sum_{m\neq n=1}^{A} \tau_m^+ \tau_n^+ \left[h^{0\nu}_{F,0}(r_{mn}, 0) - 3h^{0\nu}_{GT,0}(r_{12}, 0) \right] \\ &= -\sum_{m\neq n=1}^{A} \tau_m^+ \tau_n^+ \frac{2R_0}{\pi g_A^2(0)} \int_0^\infty \mathrm{d}q\, q^2 V^{1S0}_{0\nu}(\boldsymbol{q}^2) j_0(qr_{12}), \end{aligned} \tag{79}$$

where

$$\begin{aligned} V^{1S0}_{0\nu}(\boldsymbol{q}^2) &= \frac{1}{\boldsymbol{q}^2} \left[g_V^2 + 3g_A^2 \left(1 - \frac{2}{3} \frac{\boldsymbol{q}^2}{\boldsymbol{q}^2 + m_\pi^2} + \frac{1}{3} \frac{\boldsymbol{q}^4}{(\boldsymbol{q}^2 + m_\pi^2)^2} \right) \right] \\ &= \frac{1}{\boldsymbol{q}^2} \left[g_V^2 + g_A^2 \left(2 + \frac{m_\pi^4)}{(\boldsymbol{q}^2 + m_\pi^2)^2} \right) \right] \end{aligned} \tag{80}$$

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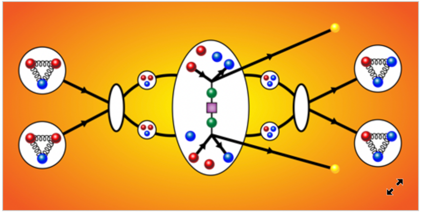
New Leading Contribution to Neutrinoless Double-β Decay

Vincenzo Cirigliano, Wouter Dekens, Jordy de Vries, Michael L. Graesser, Emanuele Mereghetti, Saori Pastore, and Ubirajara van Kolck

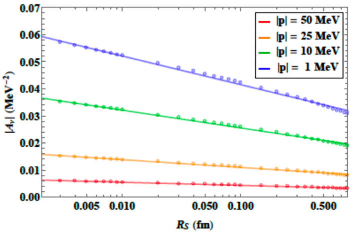
Phys. Rev. Lett. **120**, 202001 – Published 16 May 2018

Physics See Synopsis: A Missing Piece in the Neutrinoless Beta-Decay Puzzle

The inclusion of short-range interactions in models of neutrinoless double-beta decay could impact the interpretation of experimental searches for the elusive decay.



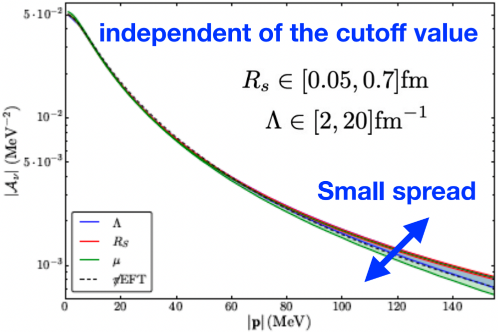
J. de Vries/Nikhef; adapted by APS/Alan Stonebraker

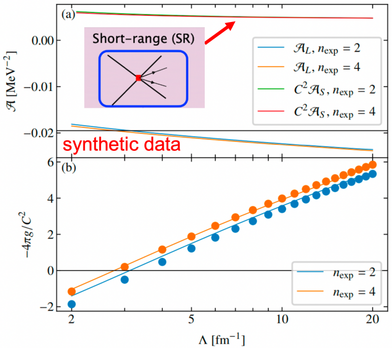


Lines fitted to $A_\nu = a + b \ln R_S$

logarithmic dependence on R_S

LO description of $nn \to ppe^-e^-$ process

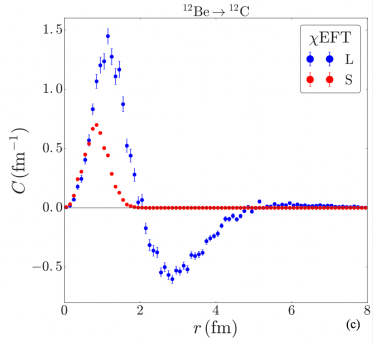




Toward Complete Leading-Order Predictions for Neutrinoless Double β Decay

Vincenzo Cirigliano, Wouter Dekens, Jordy de Vries, Martin Hoferichter, and Emanuele Mereghetti
Phys. Rev. Lett. **126**, 172002 – Published 30 April 2021

Synthetic data $\mathcal{A}_\nu(|\mathbf{p}|,|\mathbf{p}'|)e^{-i(\delta_{{}^1S_0}(|\mathbf{p}|)+\delta_{{}^1S_0}(|\mathbf{p}'|))} = -0.0195(5)\,\text{MeV}^{-2}$



A	Model	M_L	M_S	Uncertainty
6	AV18	7.45	0.48	
	χEFT	7.82	1.15	+/-16%
12	AV18	0.653	0.518	
	χEFT	0.725	0.533	+/-73%

V. CIRIGLIANO *et al*. PRC **100**, 055504 (2019)

According to the VMC calculation, the contribution of the contact transition operator

$$V_{\nu,S} = -2g_\nu^{NN}\tau^{(1)+}\tau^{(2)+}$$

to the NME of $0\nu\beta\beta$ decay of

- ^{6}He could be up to $\sim \pm 16\%$
- ^{12}Be could be up to $\sim \pm 73\%$

The actual contribution depends on the value of the LEC g_ν^{NN}, which should be determined by the data of the process or the calculation of a more fundamental theory for the process.

A relativistic description of the $0\nu\beta\beta$ decay at LO

A recent study in the relativistic chiral EFT shows that

- the $nn \to ppe^-e^-$ transition amplitude $\mathcal{A}_\nu$ is regulator-independent, thus no need to introduce the contact transition operator.
- The predicted $\mathcal{A}_\nu = 0.02085\mathrm{MeV}^{-2}$, about 10% larger than the value by Cirigliano (2021).
- The discrepancy could be attributed to the different power counting: the LO of relativistic chiral EFT contains partial N2LO contribution of non-relativstic EFT.

Y.L. Yang and P. W. Zhao, arXiv:2308.03356v1 (2023)

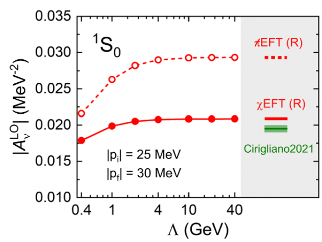
Bethe-Salpeter equation

$$T(\vec{p}\,',\vec{p}) = V(\vec{p}\,',\vec{p}) + \int \frac{d^3p''}{(2\pi)^3} V(\vec{p}\,',\vec{p}\,'')\frac{M_N^2}{E_{p''}}\frac{1}{p^2 - p''^2 + i\epsilon} T(\vec{p}\,'',\vec{p})$$

$$E_{p''} \equiv \sqrt{M_N^2 + p''^2}.$$

Lippmann-Schwinger equation

$$\widehat{T}(\vec{p}\,',\vec{p}) = \widehat{V}(\vec{p}\,',\vec{p}) + \int d^3p'' \widehat{V}(\vec{p}\,',\vec{p}\,'')\frac{M_N}{p^2 - p''^2 + i\epsilon}\widehat{T}(\vec{p}\,'',\vec{p})$$



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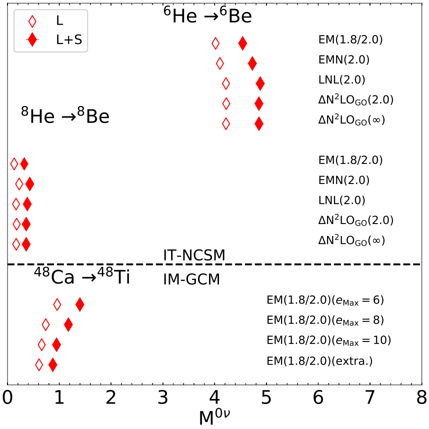
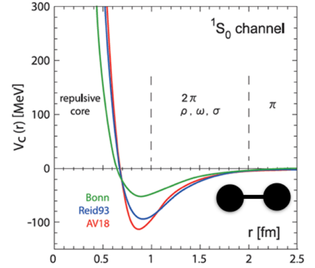
Challenge to extend from two-nucleon system to finite (candidate) nuclei

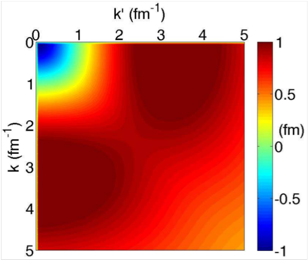
Two main difficulties: repulsive hard core and quantum many-body problem.

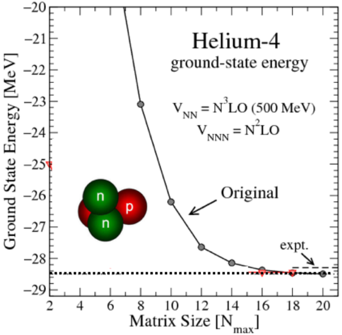
- The LEC g_ν^{NN} consistent with the employed chiral interaction (EM1.8/2.0) is determined based on the synthetic data.
- The contact term turns out to enhance (instead of qunech) the NME for ^{48}Ca by 43(7)%, thus the half-life $T_{1/2}^{0\nu\beta\beta}$ is only half of the previously expected value.
- The uncertainty (7%) is due to the synthetic data which can be reduced by using an accurate value of the LEC (g_ν^{NN}).



R. Wirth, JMY, H. Hergert, PRL127, 242502 (2021)

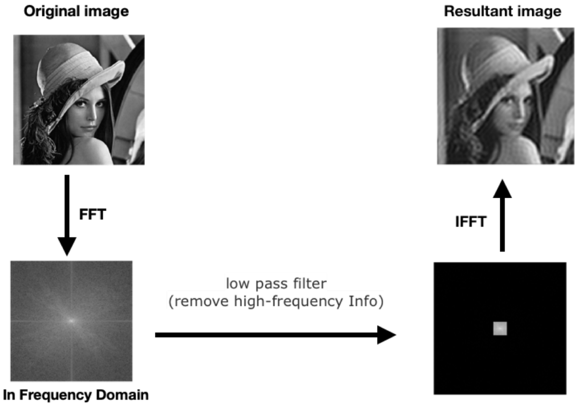




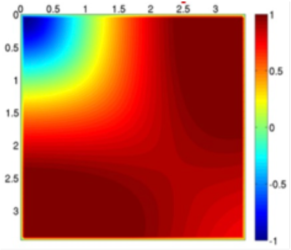


- Repulsive core & strong tensor force: low and high k modes strongly coupled.
- non-perturbative, poorly convergence in basis expansion methods.

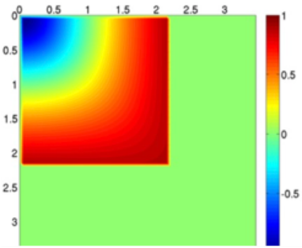
Low-pass filter

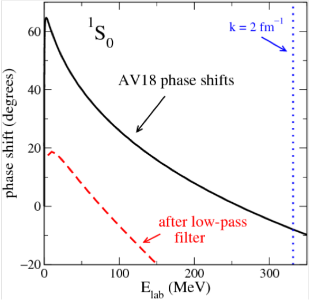


- Low-pass filter: passes signals with a frequency lower than a selected cutoff frequency and attenuates signals with frequencies higher than the cutoff frequency.
- Long-wavelength (low-E) information is preserved.



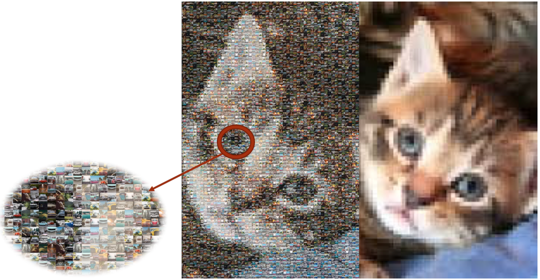
low pass filter
(remove high-frequency Info)





- Cut at $\Lambda = 2.2$ fm^{-1}
- Fails to reproduce the phase shift
- because low and high k are coupled

R. J. Furnstahl, K. Hebeler, RPP(2013)



- Renormalization group (RG) is an iterative coarse-graining procedure designed to tackle difficult physics problems involving many length scales.
- To extract relevant features of a physical system for describing phenomena at large length scales by integrating out short distance degrees of freedom.
- The effects of high-E physics can be absorbed into LECs with the RG.

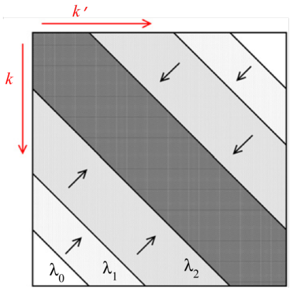
K. G. Wilson (1983); S. D. Glazek & K. G. Wilson (1993); F. Wegner (1994)

- Apply unitary transformations to Hamiltonian

$$H_s = U_s H U_s^\dagger \equiv T_{\rm rel} + V_s$$

from which one finds the flow equation

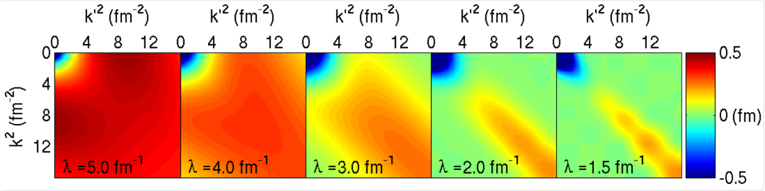
$$\frac{dH_s}{ds} = [\eta_s, H_s], \quad \eta_s = [T_{\rm rel}, H_s]$$

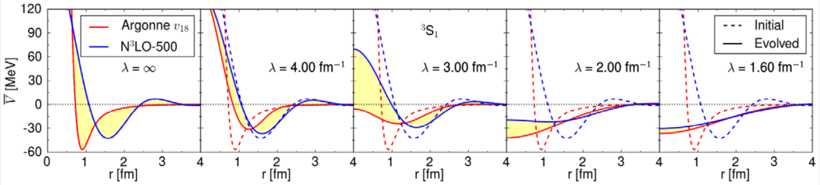


The flow parameter s is usually replaced with $\lambda = s^{-1/4}$ in units of fm^{-1} (a measure of the spread of off-diagonal strength).

Evolution of the potential

$$\frac{dV_s(k,k')}{ds} = -(k^2 - k'^2)V_s(k,k') + \frac{2}{\pi}\int_0^\infty q^2 dq (k^2 + k'^2 - 2q^2) V_s(k,q) V_s(q,k')$$





Local projection of AV18 and N^3LO(500 MeV) potentials $V(r)$.

- The hard core "disappears" in the softened interactions

S. K. Bogner et al. (2010); Wendt et al. (2012)

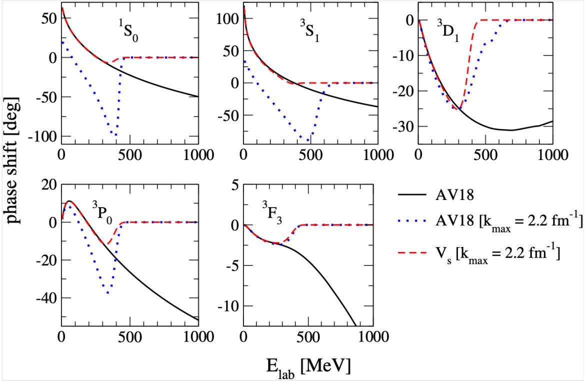
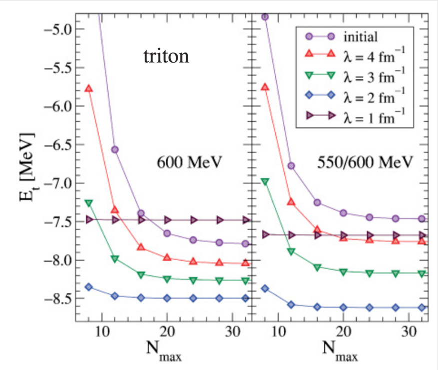
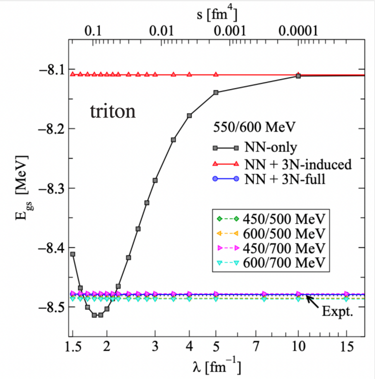


Figure: Blue dotted (red dashed) lines for the low-pass filter (SRG-softened) of AV18; black solid for the full AV18, on top of that for the SRG-softened AV18.

The *NN* phase shifts are preserved in the SRG. S. K. Bogner et al. (2007); D. Jurgenson et al. (2008)

SRG evolution of nucleon-nucleon interaction





- Convergence becomes faster as the decreases of the λ.
- importance of (induced) three-body forces, NO2B approximation

Bogner et al. PRC75, 061001(R) (2007); Furnstahl et al. (2013)

A toolbox for nuclear ab initio methods

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- Quantum Monte-Carlo (QMC) methods
- Lattice effective field theory (LEFT)

Basis expansion methods

- Full configuration interaction (FCI) or no-core shell model (NCSM)
- Coupled-cluster (CC) theory
- In-medium similarity renormalization group (IMSRG)
- Self-consistent Green's function (SCGF) theory
- Many-body perturbation theory (MBPT)
- (Relativistic) Brueckner-Hartree-Fock (BHF) theory
- ...

- All A nucleons are considered active.
- The nuclear wave function in the FCI is expanded in a set of Slater determinant basis functions,

$$|\Psi^{(\mathrm{FCI})}\rangle = \sum_k C_k |\Phi_k\rangle$$

where the many-body basis $|\Phi_k\rangle$ consists of all Slater determinants constructed from the single-particle basis set

$$\left\{ |\Phi_k\rangle = \hat{\mathcal{A}}\left(\phi_{k_1} \dots \phi_{k_A}\right) \right\}.$$

- The expansion coefficients are obtained from a large-scale Hamiltonian matrix diagonalization.

$$\sum_l H_{kl} C_l = E C_k, \quad H_{kl} = \langle \Phi_k | H | \Phi_l \rangle$$

The Full Configuration Interaction (FCI) Method

- Starting from a reference state

$$|\Phi_0\rangle = \hat{\mathcal{A}}\left(\phi_{i_1}\dots\phi_{i_A}\right)$$

which is a single Slater determinant build from the set of single-particle orbitals that minimize the energy functional $E_{\rm ref}[\phi_{i_1},\dots,\phi_{i_A}]$, such as a HF state.

- The FCI wave function can be parametrized by the linear ansatz

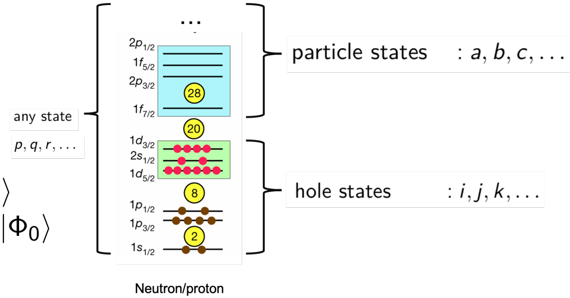
$$|\Psi^{\rm (FCI)}\rangle = \left(1+\hat{C}^{\rm (FCI)}\right)|\Phi_0\rangle, \quad \hat{C}^{\rm (FCI)} = \sum_{n=1}^{A}\hat{C}_n^{\rm (FCI)}$$

where the $np-nh$ excitation operator generating all possible $np-nh$ excitations reads

$$\hat{C}_n = \frac{1}{(n!)^2}\sum_{\substack{i_1,i_n\\ a_1,\dots,a_n}} c_{i_1,\dots i_n}^{a_1,a_n}\hat{a}_{a_1}^\dagger\dots\hat{a}_{a_n}^\dagger\hat{a}_{i_n}\dots\hat{a}_{i_1}.$$

The orbitals occupied by the reference state (referred to as hole states) and the unoccupied (particle) states

hole states	: $i, j, k, \ldots \in$	occupied in $|\Phi_0\rangle$
particle states	: $a, b, c, \ldots \in$	unoccupied in $|\Phi_0\rangle$
any state	: $p, q, r, \ldots$	



Dimension of model space

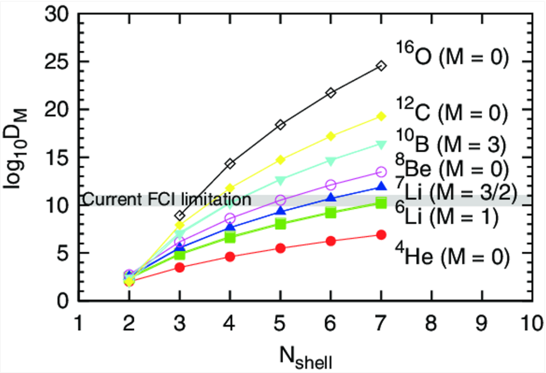
Considering n neutrons distributed among N single-particle states

$$\begin{pmatrix} N \\ n \end{pmatrix} = \frac{N!}{(N-n)!n!}.$$

^{16}O: 4 major HO shells only ($0s, 0p, 1s, 0d$ and $1p, 0f$ shells), total 40 single particle states for neutrons and protons.

$$\begin{pmatrix} 40 \\ 8 \end{pmatrix}^2 = (\frac{40!}{(32)!8!})^2 \sim 1 \times 10^{15},$$

possible Slater determinants.



In practical calculation, the wave function is truncated up to the $Mp - Mh$ and the FCI in this case is called CIM

$$\hat{C}^{(\mathrm{FCI})} \simeq \sum_{n=1}^{M} \hat{C}_n^{(\mathrm{CIM})}$$

and

$$\hat{H}\left(1+\sum_{n=1}^{M} \hat{C}_n^{(\mathrm{CIM})}\right)|\Phi_0\rangle = E^{(\mathrm{CIM})}\left(1+\sum_{n=1}^{M} \hat{C}_n^{(\mathrm{CIM})}\right)|\Phi_0\rangle$$

Notice: the wave functions of configurations are orthogonormal, and thus,

$$\begin{aligned}
\langle\Phi_0|\,\hat{H}\left(1+\sum_{n=1}^{M} \hat{C}_n^{(\mathrm{CIM})}\right)|\Phi_0\rangle &= E^{(\mathrm{CIM})}\langle\Phi_0|\left(1+\sum_{n=1}^{M} \hat{C}_n^{(\mathrm{CIM})}\right)|\Phi_0\rangle = E^{(\mathrm{CIM})}\langle\Phi_0|\,\Phi_0\rangle \\
&= E^{(\mathrm{CIM})}. \qquad (81)
\end{aligned}$$

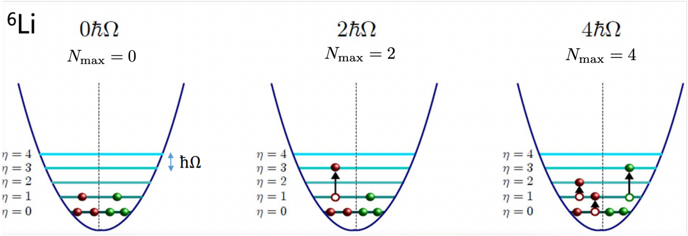
A set of coupled equations are obtained for the unknown amplitudes $c_{i_1,\ldots 1_k}^{a_1\ldots a_k}$ by left-projecting the CIM Schrödinger equation onto the reference $|\Phi\rangle$ and excited determinants $\left|\Phi_{i_1,\ldots i_k}^{a_1\ldots a_k}\right\rangle$,

$$\left\langle \Phi_{i_1}^{a_1} \left| \hat{H}\left(1+\sum_{n=1}^{M} \hat{C}_n^{(\mathrm{CIM})}\right) \right| \Phi_0 \right\rangle = E^{(\mathrm{CIM})} c_{i_1}^{a_1} \quad , \forall a_1, i_1$$

$$\left\langle \Phi_{i_1\ldots i_M}^{a_1\ldots a_M} \left| \hat{H}\left(1+\sum_{n=1}^{M} \hat{C}_n^{(\mathrm{CIM})}\right) \right| \Phi_0 \right\rangle = E^{(\mathrm{CIM})} c_{i_1\ldots i_M}^{a_1\ldots a_M} \quad , \forall a_1, \ldots, i_M$$

where $np - nh$ excitation $\left|\Phi_{i_1,\ldots i_n}^{a_1\ldots a_n}\right\rangle$ of the reference determinant is defined as the Slater determinant in which, relative to the reference state $|\Phi\rangle$, n hole states have been replaced by n particle states, i.e.,

$$\begin{aligned}\left|\Phi_{i_1,\ldots i_n}^{a_1, a_n}\right\rangle &= \left(\hat{a}_{a_1}^{\dagger}\hat{a}_{i_1}\right)\left(\hat{a}_{a_2}^{\dagger}\hat{a}_{i_2}\right)\ldots\left(\hat{a}_{a_n}^{\dagger}\hat{a}_{i_n}\right)|\Phi_0\rangle \\ &= \hat{a}_{a_1}^{\dagger}\ldots\hat{a}_{a_n}^{+}\hat{a}_{i_n}\ldots\hat{a}_{i_1}|\Phi_0\rangle\end{aligned}$$



- Wave function in the NCSM

$$\left|\Psi^{(\mathrm{NCSM})}\right\rangle = \left(1 + \sum_{n=1}^{A} \hat{C}_n^{(\mathrm{NCSM})}\right) |\Phi\rangle,$$

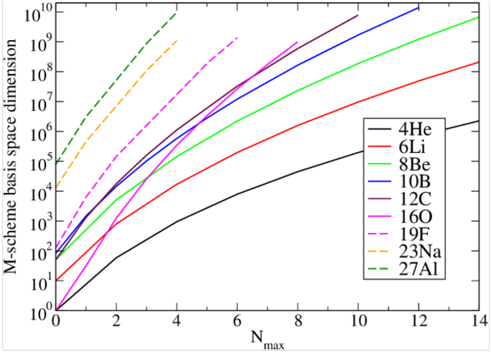
with excitation operators

$$\hat{C}_n^{(\mathrm{NCSM})} = \frac{1}{(n!)^2} \sum_{\substack{i_1,\dots,i_n \\ a_1,\dots,a_n}}^{\prime} c_{i_1,\dots i_n}^{a_1\dots a_n} \hat{a}_{a_1}^{\dagger} \dots \hat{a}_{a_n}^{\dagger} \hat{a}_{i_n} \dots \hat{a}_{i_1}$$

- Truncation on excitation energy of a Slater determinant relative to the unperturbed reference state defined by

$$e^{a_1\dots a_n}_{i_1\dots i_n} \equiv \sum_{k=1}^{n} (e_{a_k} - e_{i_k}) \leq N_{\rm max},$$

- Dimension of the configurations still increases exponentially with $N_{\rm Max}$.



The Exponential Ansatz

- The wave function is constructed as

$$|\Psi\rangle = e^{\hat{T}}|\Phi\rangle, \quad \hat{T} = \sum_{n=1}^{A} \hat{T}_n$$

where the cluster operator $\hat{T}$ is defined in close analogy to the CI case.

$\hat{T}_1 =$  $= \frac{1}{(1!)^2} \sum_{ai} t_i^a \{\hat{a}_a^\dagger \hat{a}_i\}$

$\hat{T}_2 =$  $= \frac{1}{(2!)^2} \sum_{abij} t_{ij}^{ab} \{\hat{a}_a^\dagger \hat{a}_b^\dagger \hat{a}_j \hat{a}_i\}$

$\vdots$

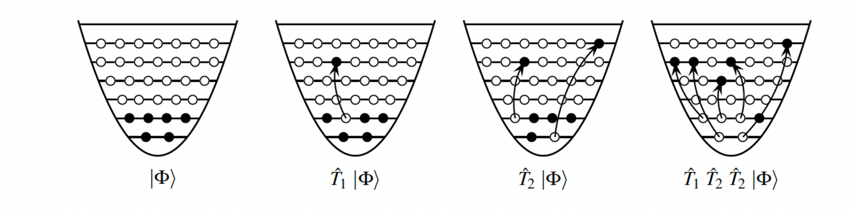
$\hat{T}_n =$  $= \frac{1}{(n!)^2} \sum_{\substack{a_1 \dots a_n \\ i_1 \dots i_n}} t_{i_1 \dots i_n}^{a_1 \dots a_n} \{\hat{a}_{a_1}^\dagger \dots \hat{a}_{a_n}^\dagger \hat{a}_{i_n} \dots \hat{a}_{i_1}\}\,.$

Coupled-Cluster method (CCM)

In this method, the cluster operator is truncated to some excitation rank M ,

$$\hat{T}^{(M)} = \sum_{n=1}^{M} \hat{T}_n.$$

For $M = 2$, it is called *CCSD*, and so on. Due to its nonlinear nature, the Coupled-Cluster Ansatz allows to generate higher-order excitations from products of lower-order excitation operators.



For a truncated CCM with the cluster operator

$$\hat{T} \approx \hat{T}^{(\mathrm{M})} = \hat{T}_1 + \hat{T}_2 + \ldots + \hat{T}_M$$

the expression for the correlation energy $\Delta E^{(\mathrm{M})} = \Delta E\left(\boldsymbol{t}^{(\mathrm{M})}\right)$ as function of the cluster amplitudes

$$\boldsymbol{t}^{(M)} \equiv \left\{\{t_i^a\}, \left\{t_{ij}^{ab}\right\}, \ldots, \left\{t_{i_1,\ldots,i_M}^{a_1,a_M}\right\}\right\},$$

can be derived by left-projecting the similarity-transformed Schrödinger equation

$$\hat{\mathcal{H}}^{(\mathrm{M})}|\Phi\rangle = \Delta E^{(\mathrm{M})}|\Phi\rangle$$

with

$$\hat{\mathcal{H}}^{(\mathrm{M})} \equiv e^{-\hat{T}^{(\mathrm{M})}} \hat{H}_N e^{\hat{T}^{(\mathrm{M})}}$$

onto the reference state.

A coupled set of algebraic equations for the determination of the amplitudes $t^{(\mathrm{M})}$ is obtained by left-projecting the similarity-transformed Schrödinger equation onto the excited determinants $\left|\Phi_{i_1,\dots i_n}^{a_1\dots a_n}\right\rangle$ with $n \leq M$, i.e.,

$$\left\langle \Phi \left| \hat{\mathcal{H}}^{(\mathrm{M})} \right| \Phi \right\rangle = \Delta E^{(\mathrm{M})} \tag{82}$$

$$\left\langle \Phi_i^a \left| \hat{\mathcal{H}}^{(\mathrm{M})} \right| \Phi \right\rangle = 0, \quad \forall a, i \tag{83}$$

$$\left\langle \Phi_{ij}^{ab} \left| \hat{\mathcal{H}}^{(\mathrm{M})} \right| \Phi \right\rangle = 0, \quad \forall a, b, i, j \tag{84}$$

$$\vdots \tag{85}$$

$$\left\langle \Phi_{i_1,\dots M}^{a_1,\dots M} \left| \hat{\mathcal{H}}^{(\mathrm{M})} \right| \Phi \right\rangle = 0, \quad \forall a_1, \dots, a_M, i_1, \dots, i_M. \tag{86}$$

In the case of CCSD, for example, the $\hat{T}_1$ and $\hat{T}_2$ amplitudes can be determined by solving the system of the first three equations.

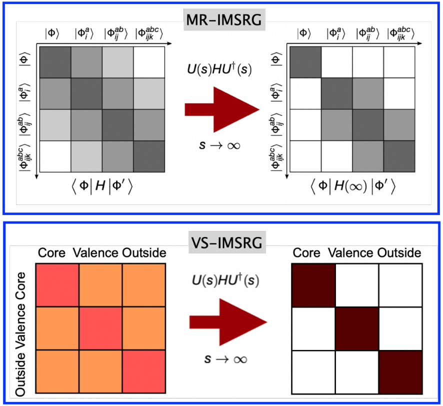
- Unitary transformations

$$\hat{H}(s) = \hat{U}(s)\hat{H}_0\hat{U}^\dagger(s)$$

Flow equation

$$\frac{d\hat{H}(s)}{ds} = [\hat{\eta}(s), \hat{H}(s)]$$

- Generator $\eta(s)$: chosen either to decouple a given reference state from its excitations or to decouple the valence space from the excluded spaces.
- Not necessary to construct the whole H matrix, computation complexity scales polynomially with nuclear size.



H. Hergert et al., Phys. Rep. 621, 165 (2016); S. R. Stroberg et al., Annu. Rev. Nucl. Part. Sci. 69, 307 (2019)

- Taking the derivative $\hat{H}(s)$ with respect to the flow parameter s, one finds the SRG flow equation for the Hamiltonian

$$\frac{d\hat{H}(s)}{ds} = \frac{d\hat{U}(s)}{ds}\hat{H}(0)\hat{U}^\dagger(s) + \hat{U}(s)\hat{H}(0)\frac{d\hat{U}^\dagger(s)}{ds}$$
$$= \frac{d\hat{U}(s)}{ds}\hat{U}^\dagger(s)\hat{H}(s) + \hat{H}(s)\hat{U}(s)\frac{d\hat{U}^\dagger(s)}{ds} = [\hat{\eta}(s), \hat{H}(s)]$$

where one defines the anti-Hermitian operator

$$\hat{\eta}(s) \equiv \frac{d\hat{U}(s)}{ds}\hat{U}^\dagger(s) = -\hat{\eta}^\dagger(s)$$

The solution to the SRG flow equation is

$$\frac{dU(s)}{ds} = \eta(s)U(s) \Rightarrow U(s) = \mathcal{S}\exp\left[\int_0^s ds'\eta(s')\right]. \tag{87}$$

K. Tsukiyama, S. Bogner and A. Schwenk, PRL 106 (2011) 222502

This expression is defined equivalently either as a product of infinitesimal unitary transformations,

$$U(s) = \lim_{N\to\infty} \prod_{i=0}^{N} e^{\eta(s_i)\delta s_i}, \quad s_{i+1} = s_i + \delta s_i, \quad \sum_i \delta s_i = s$$

or through a series of expansions:

$$U(s) = \sum_n \frac{1}{n!} \int_0^s ds_1 \int_0^s ds_2 \ldots \int_0^s ds_n \mathcal{S}\left\{\eta\left(s_1\right) \ldots \eta\left(s_n\right)\right\}.$$

Here, the S-ordering operator $\mathcal{S}$ ensures that the flow parameters appearing in the integrands are always in descending order, $s_1 > s_2 > \ldots$.

- Wegner proposed the generator

$$\hat{\eta}(s) = [\hat{H}_d(s), \hat{H}_{od}(s)]$$

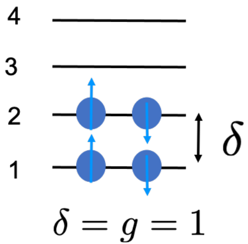
which will be able to drive the $\hat{H}_{od}(s) \to 0$ as the flow parameter $s \to \infty$.

Application of IMSRG to the pairing model

- The pairing Hamiltonian:

$$\hat{H} = \delta \sum_{p\sigma} (p-1) a^{\dagger}_{p\sigma} a_{p\sigma} - \frac{1}{2} g \sum_{pq} a^{\dagger}_{p+} a^{\dagger}_{p-} a_{q-} a_{q+},$$

where δ controls the spacing of single-particle levels that are indexed by a principal quantum number $p = 1, \ldots, 4$ and their spin projection σ, and g the strength of the pairing interaction.



state	p	$2s_z$	ε
0	1	1	0
1	1	−1	0
2	2	1	δ
3	2	−1	δ
4	3	1	2δ
5	3	−1	2δ
6	4	1	3δ
7	4	−1	3δ

- Let's only consider the $S_z = 0$ sub block with two particle pairs. In this block, the Hamiltonian is represented by the six-dimensional ($C_4^2 = 6$) matrix

$$H = \begin{pmatrix} 2\delta - g & -g/2 & -g/2 & -g/2 & -g/2 & 0 \\ -g/2 & 4\delta - g & -g/2 & -g/2 & -0 & -g/2 \\ -g/2 & -g/2 & 6\delta - g & 0 & -g/2 & -g/2 \\ -g/2 & -g/2 & 0 & 6\delta - g & -g/2 & -g/2 \\ -g/2 & 0 & -g/2 & -g/2 & 8\delta - g & -g/2 \\ 0 & -g/2 & -g/2 & -g/2 & -g/2 & 10\delta - g \end{pmatrix}.$$

We split the Hamiltonian matrix into diagonal and off-diagonal parts:

$$H_d(s) = \text{diag}\left(E_0(s), \ldots, E_5(s)\right), \quad H_{od}(s) = H(s) - H_d(s)$$

The flow equation in the configuration basis in which one has $\hat{H}_d|i\rangle = E_i|i\rangle$, and

$$\frac{d}{ds}\langle i|\hat{H}|j\rangle = \sum_k (\langle i|\hat{\eta}|k\rangle\langle k|\hat{H}|j\rangle - \langle i|\hat{H}|k\rangle\langle k|\hat{\eta}|j\rangle) \tag{88}$$

$$= -\left(E_i - E_j\right)\langle i|\hat{\eta}|j\rangle + \sum_k \left(\langle i|\hat{\eta}|k\rangle \left\langle k\left|\hat{H}_{od}\right|j\right\rangle - \left\langle i\left|\hat{H}_{od}\right|k\right\rangle \langle k|\hat{\eta}|j\rangle\right) \tag{89}$$

where $\left\langle i\left|\hat{H}_{od}\right|i\right\rangle = 0$ and block indices as well as the s-dependence have been suppressed for brevity.

The Wegner generator is given by

$$\langle i|\hat{\eta}|j\rangle = \left\langle i\left|\left[\hat{H}_d, \hat{H}_{od}\right]\right|j\right\rangle = (E_i - E_j)\left\langle i\left|\hat{H}_{od}\right|j\right\rangle,$$

and inserting this into the flow equation, we obtain

$$\begin{aligned}\frac{d}{ds}\langle i|\hat{H}|j\rangle &= -(E_i - E_j)^2\left\langle i\left|\hat{H}_{od}\right|j\right\rangle \\ &\quad + \sum_k (E_i + E_j - 2E_k)\left\langle i\left|\hat{H}_{od}\right|k\right\rangle\left\langle k\left|\hat{H}_{od}\right|j\right\rangle \end{aligned} \tag{90}$$

If $\left\|\hat{H}_{od}(s_0)\right\| \ll 1$ in some suitable norm, the second term in the flow equation can be neglected compared to the first one. For the diagonal and off-diagonal matrix elements, this implies

$$\frac{dE_i}{ds} = \frac{d}{ds}\left\langle i\left|\hat{H}_d\right|i\right\rangle = 2\sum_k (E_i - E_k)\left\langle i\left|\hat{H}_{od}\right|k\right\rangle\left\langle k\left|\hat{H}_{od}\right|i\right\rangle \approx 0$$

and

$$\frac{d}{ds}\langle i|\hat{H}|j\rangle \approx -(E_i - E_j)^2\left\langle i\left|\hat{H}_{od}\right|j\right\rangle$$

respectively. Thus, the diagonal matrix elements will be (approximately) constant in the asymptotic region,

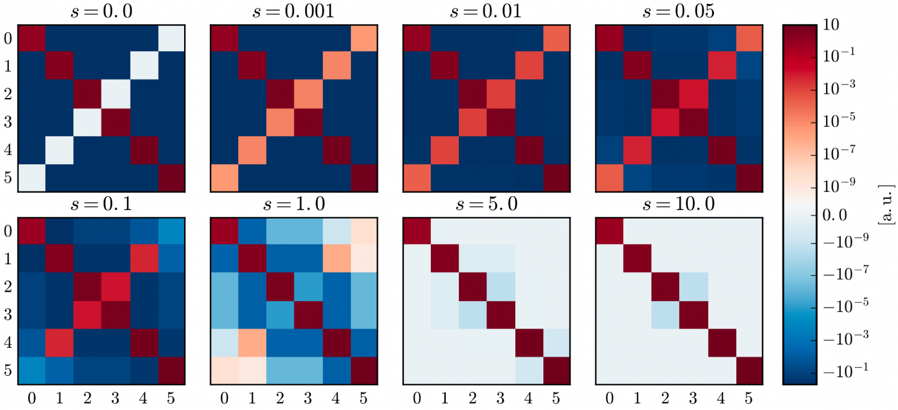
$$E_i(s) \approx E_i(s_0), \quad s > s_0,$$

which in turn allows us to integrate the flow equation for the off-diagonal matrix elements.

We obtain

$$\left\langle i \left| \hat{H}_{od}(s) \right| j \right\rangle \approx \left\langle i \left| \hat{H}_{od}(s_0) \right| j \right\rangle e^{-(E_i - E_j)^2 (s - s_0)}, \quad s > s_0$$

i.e., the off-diagonal matrix elements are suppressed exponentially.



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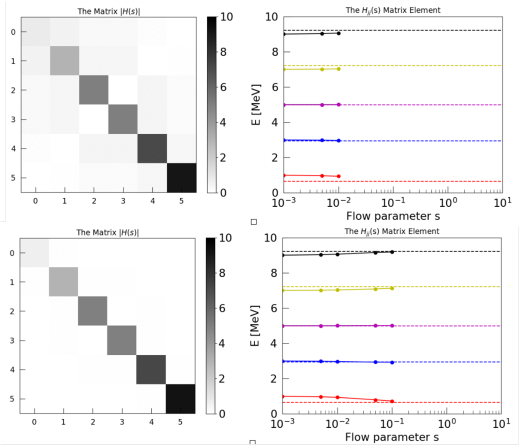
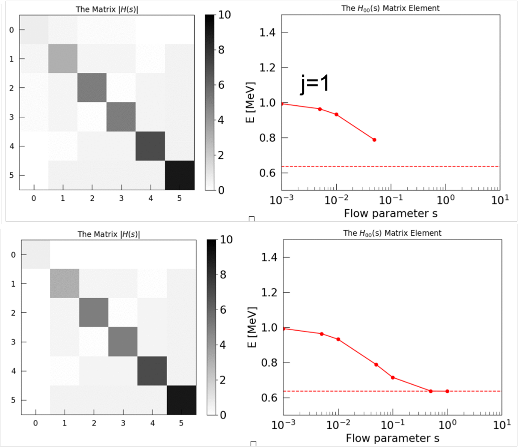
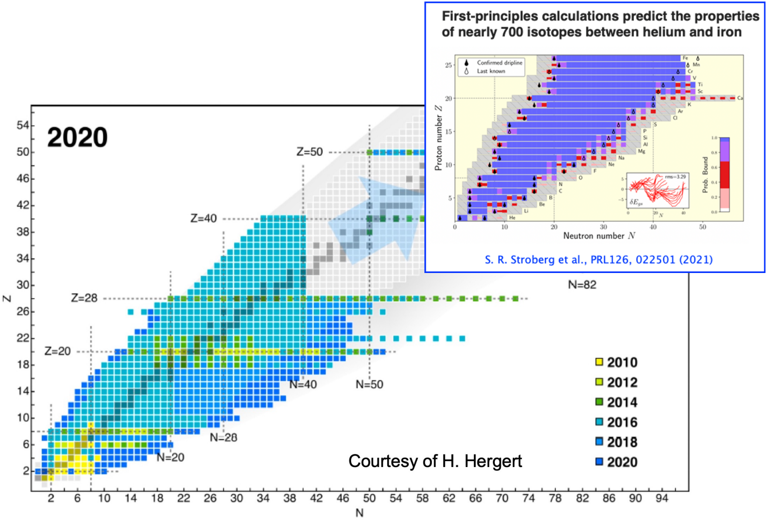
$$H_{od}(s) = H(s) - H_d(s)$$



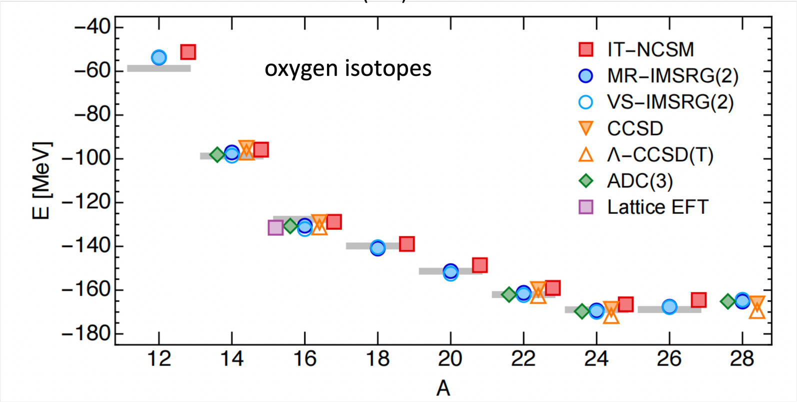
$$H_{od}(s) = H_{0j}(s) + H_{j0}(s)$$





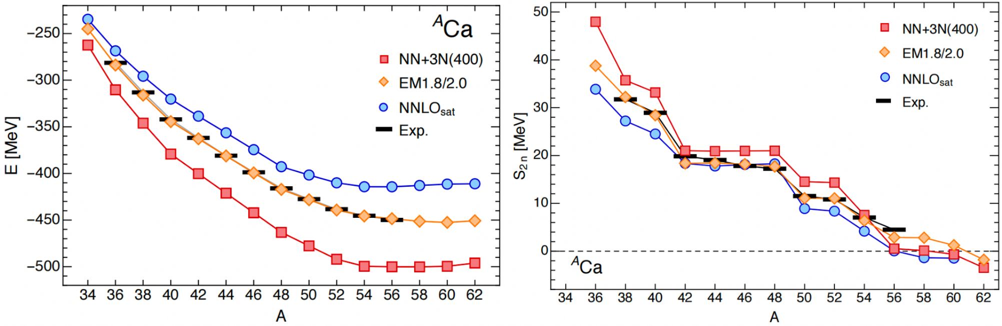
2020
Z=50
Z=40
Z=28
Z=20
N=20
N=28
N=40
N=50
N=82
Z
N
2010
2012
2014
2016
2018
2020
Courtesy of H. Hergert
First-principles calculations predict the properties of nearly 700 isotopes between helium and iron
Confirmed dripline
Last known
Proton number Z
Neutron number N
Prob. Bound
S. R. Stroberg et al., PRL126, 022501 (2021)

chiral NN+3N(400) interaction at $\lambda = 1.88$ fm^{-1}



H. Hergert, Font. Phys. 8, 379 (2020)

Benchmark calculations for Ca isotopes



H. Hergert, Font. Phys. 8, 379 (2020)

Beta decay and axial-vector coupling strength g_A

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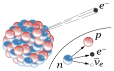
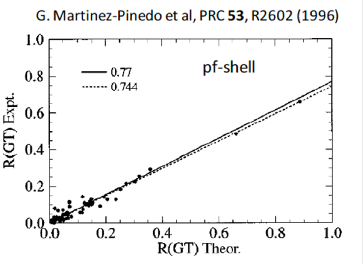
- The half-life of single-beta decay

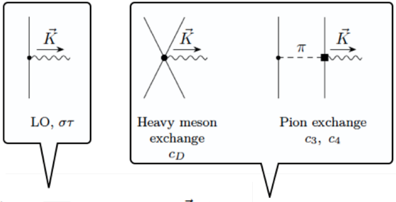


$$t_{1/2} = \frac{\kappa}{f_0(B_F + B_{\rm GT})},$$

$$B_F = \frac{g_V^2}{2J_i+1}|M_F|^2, \quad B_{\rm GT} = \frac{g_A^2}{2J_i+1}|M_{\rm GT}|^2$$



- The charge-changing axial-vector current



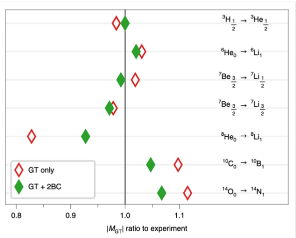
$$\vec{J}^A(\vec{K}) = \sum_j \imath g_A \boldsymbol{\sigma}_j \boldsymbol{\tau}_j^{\pm} e^{\imath \vec{K}\cdot\vec{r}_j}.$$

2B currents

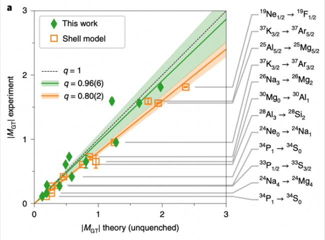
Park, T.-S. et al. PRC67, 055206 (2003);
M. Hoferichter et al., PRD102, 074018 (2020)

$$g_A^{\rm eff}(q,0,\rho) \equiv g_A \left\{1 - \frac{\rho}{f_\pi^2}\left[-\frac{c_D}{4}\frac{1}{g_A \Lambda_\chi} + \frac{2c_3}{3}\frac{q^2}{4m_\pi^2+q^2} + \frac{I(\rho,0)}{3}\left(2c_4 - c_3 + \frac{1}{2m_p}\right)\right]\right\}$$

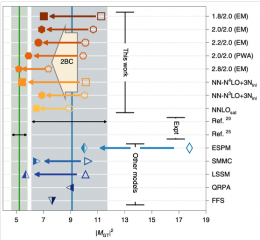
Ab initio methods for nuclear single-β decay



NCSM



VS–IMSRG



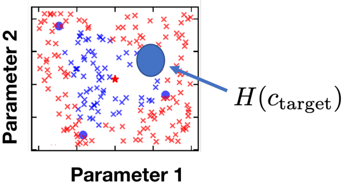
CC

- **VS-IMSRG**: a unitary transformation is constructed to decouple a valence-space Hamiltonian H_{vs}. The eigenstates are obtained by a subsequent diagonalization of the H_{vs}.
- A proper treatment of strong nuclear correlations and the consistency between two-body currents (2BCs) and three-nucleon forces explain the g_A-quenching puzzle in conventional valence-space shell-model calculations.

P. Gysbers et al., Nature Physics 15, 428 (2019)

The Hamiltonian with a set of LEC parameters (c_i)

$$H(c_i)|\psi(c_i)\rangle = E(c_i)|\psi(c_i)\rangle$$

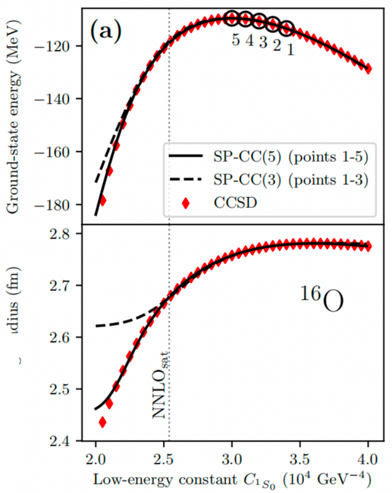


Basic idea: $|\psi(c_{\rm target})\rangle = \sum_{i=1}^{N_{\rm EC}} f_i|\psi(c_i)\rangle$

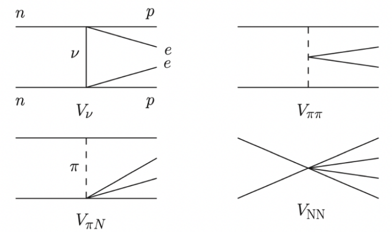
Procedure

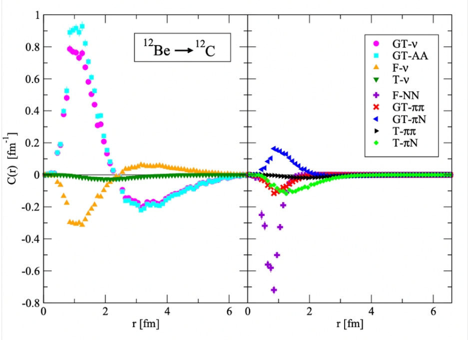
- calculate $|\psi(c_i)\rangle$, $i = 1, \ldots N_{\rm EC}$ in "easy" regime
- solve generalized eigenvalue problem $H|\psi\rangle = \lambda N|\psi\rangle$ with
  - $H_{ij} = \langle\psi_i|H(c_{\rm target})|\psi_j\rangle$
  - $N_{ij} = \langle\psi_i|\psi_j\rangle$

Frame et al., PRL 121 032501 (2018)



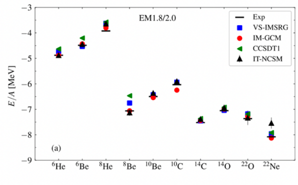
A. Ekstrom, G. Hagen, PRL123, 252501(2019)

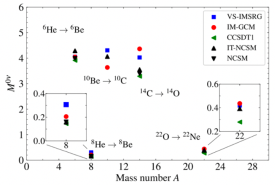




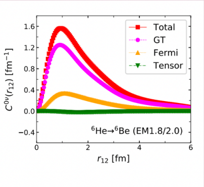
- The variational Monte Carlo (VMC) with the NN(AV18) + 3N(Illinois-7).
- Light Majorana neutrino exchange + multi-TeV (dim9) mechanisms of LNV.
- The N^{2}LO effects captured by nucleon form factors impact the matrix elements at 10% level.
- The non-factorizable terms at N2LO may lead to O(10%) corrections, indicating that the NME converges with the chiral expansion order for the transition operators.
- Difficult to extend to the candidate nuclei of $0\nu\beta\beta$ decay.

S. Pastore et al., PRC97, 014606 (2018)

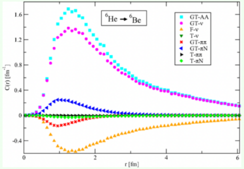




IM-GCM (EM1.8/2.0)



S. Pastore et al (2018)



Note: A factor of $-g_A^2$ has been multiplied into the Fermi part.

- IT-NCSM and NCSM are quasi-exact methods, but limited to light nuclei.
- VS-IMSRG, IM-GCM, and CCSDT1 with some kinds of truncations can be applied to heavier candidate nuclei.
- Using different ab initio methods but the same input to estimate the truncation errors of the many-body methods.

Ab initio methods for the lightest candidate ^{48}Ca

- Multi-reference in-medium generator coordinate method (IM-GCM)

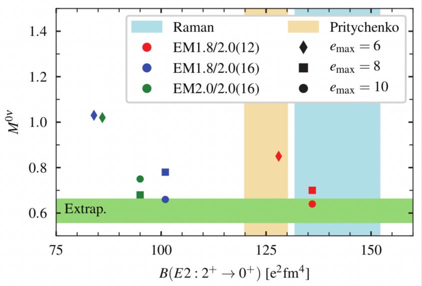
  JMY et al., PRL124, 232501 (2020)

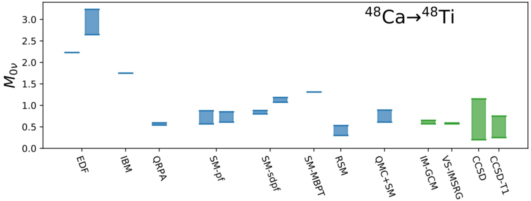
- IMSRG+ISM (VS-IMSRG)

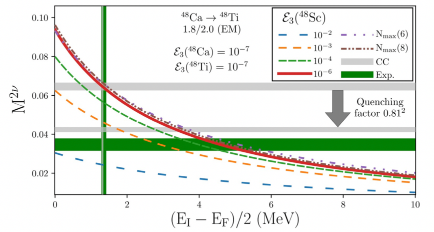
  A. Belley et al., PRL126, 042502 (2021)

- Coupled-cluster with singlets, doublets, and partial triplets (CCSDT1) .

  S. Novario et al., PRL126, 182502 (2021)







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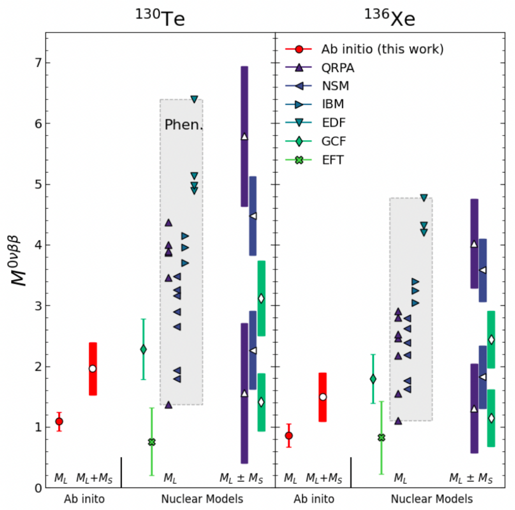
With both the long- and short-range transition operators, the VS-IMSRG method is applied to study the NMEs of heavier candidates:

- For ^{130}Te, $M^{0\nu}_{L+S} \in [1.52, 2.40]$
- For ^{136}Xe, $M^{0\nu}_{L+S} \in [1.08, 1.90]$

The uncertainty is composed of different sources: nuclear interaction, reference-state, basis extrapolation, closure approximation, and the LEC for the short-range transition operators.

The values are generally smaller than those from phenomenological nuclear models.

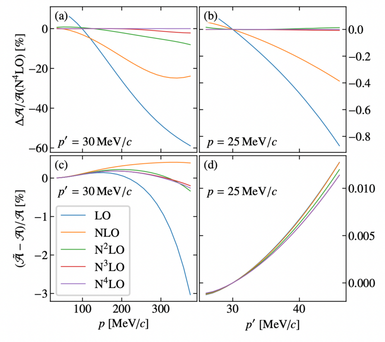
A. Belley et al, arXiv:2307.15156 (2023)



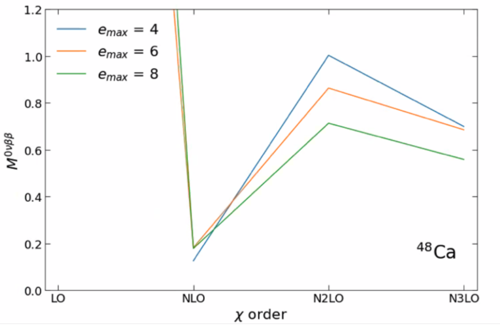
A more comprehensive quantification analysis

different nuclear many-body solvers, convergence of NMEs with chiral expansion orders, etc.

$2n \to 2p + 2e^-$

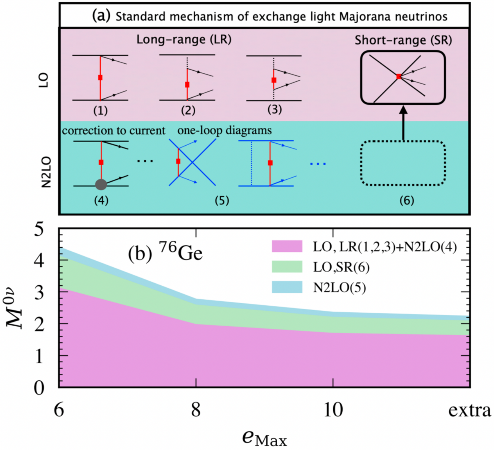


$^{48}\mathrm{Ca} \to ^{48}\mathrm{Ti} + 2e^-$



- The $\mathcal{A}_\nu(2n \to 2p + 2e^-)$ converges quickly w.r.t. the chiral expansion order of nuclear interactions.
- Convergence is slightly slower in candidate nucleus ^{48}Ca.

Convergence of NMEs w.r.t. chiral expansion: ^{76}Ge



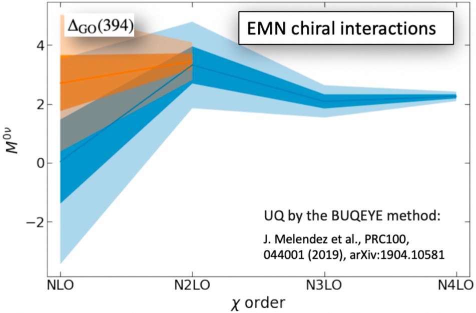
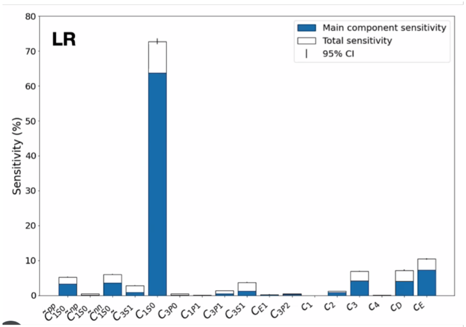
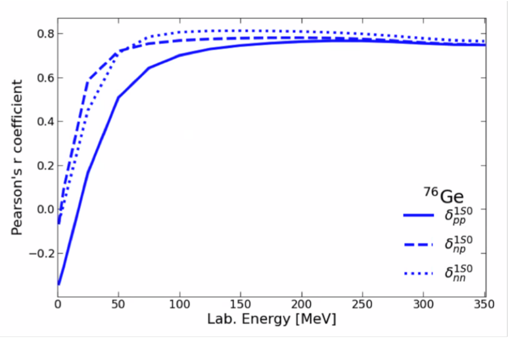


TABLE I. **The recommended value for the total NME of $0\nu\beta\beta$ decay in ^{76}Ge, together with the uncertainties from different sources.**

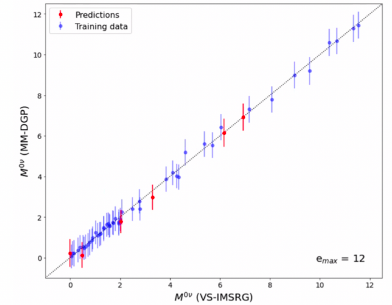
$M^{0\nu}$	$\epsilon_{\rm LEC}$	$\epsilon_{\chi\rm EFT}$	$\epsilon_{\rm MBT}$	$\epsilon_{\rm OP}$	$\epsilon_{\rm EM}$
$2.60^{+1.28}_{-1.36}$	0.75	0.3	0.88	0.55	<0.06

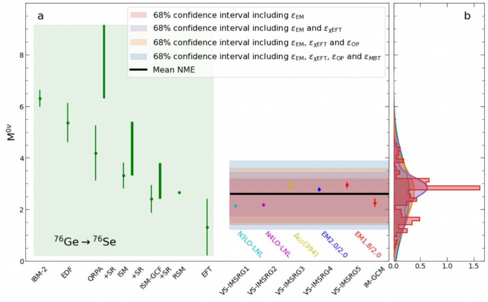
Uncertainty quantification of NME for ^{76}Ge





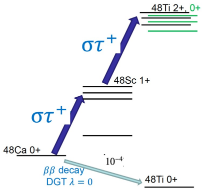
- The long-range part of the NME is sensitive to the LEC $C_{^1S_0}$.
- The phase shift of the $^1S_0$ channel is linearly correlated to the NME.
- The neutron-proton phase-shift δ_{np}^{1S0} at 50 MeV is used to weight the samples.



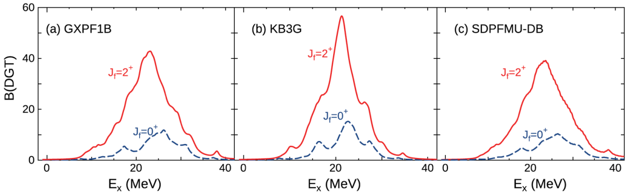


- Emulator, 8188 samples of chiral interactions, phase shift, $M^{0\nu} = 3.44^{+1.33}_{-1.56}$.
- Including the g.s. energies of $A = 2, 3, 4, 16$ and phase shift: $M^{0\nu} = 2.60^{+1.28}_{-1.36}$, which gives the effective neutrino mass $\langle m_{\beta\beta}\rangle = 187^{+205}_{-62}$ meV.
- The next-generation ton-scale Germanium experiment ($\sim 1.3\times 10^{28}$ yr): $\langle m_{\beta\beta}\rangle = 22^{+24}_{-7}$ meV, covering almost the entire range of IO hierarchy.

A. Belley, JMY et al, arXiv:2308.15634 (2023)



N. Shimizu, J. Menéndez and K. Yako, PRL(2018)



- Double Gamow-Teller (DGT) transition is allowed in the SM of particle physics and this transition is to be measured in Osaka Univ.
- The NME for the ground-state to ground-state DGT transition is defined as

$$M^{\rm DGT} = \left\langle 0_f^+ \left| \sum_{1,2} [\sigma_1 \otimes \sigma_2]^0 \tau_1^+ \tau_2^+ \right| 0_i^+ \right\rangle$$

Compared with $0\nu\beta\beta$ decay: No neutrino potential.

In conventional nuclear models, the neutrino potential is evaluated in a harmonic oscillator basis,

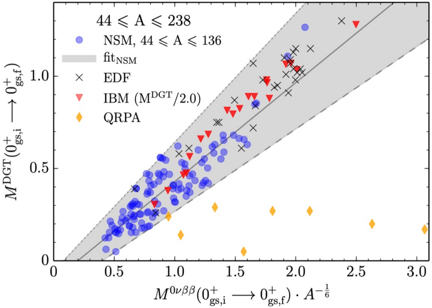
oscillator length $b = \sqrt{\hbar/(M_N\omega)}$ the frequency ω is scaled as $A^{-1/3}$

$$R \sim A^{1/3}$$

$$M^{0\nu\beta\beta} \propto R/b \sim A^{1/6}$$

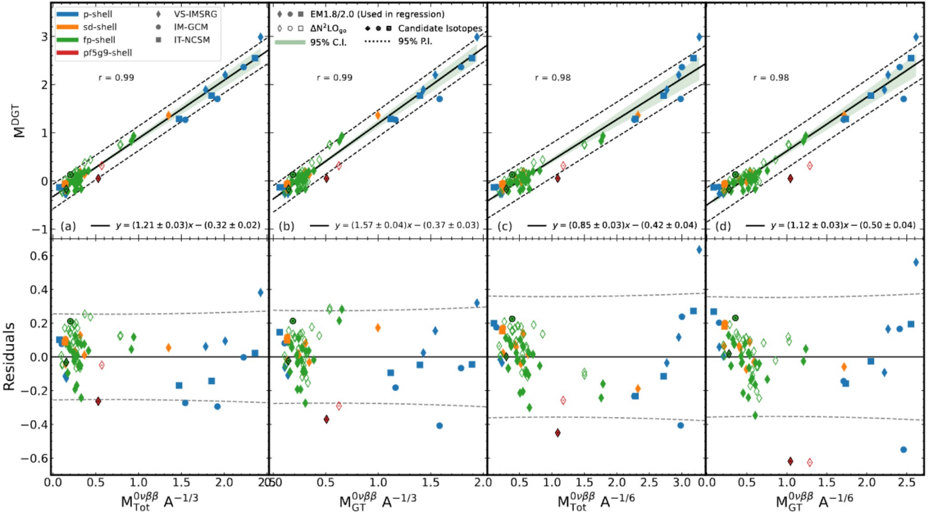
$$M^{0\nu\beta\beta} = A^{1/6}\,\frac{M^{\rm DGT}/q^2 - n}{m},$$

n = 0.180; m = 0.447,
n = 0.106; m = 0.536,
n = 0.056; m = 0.699

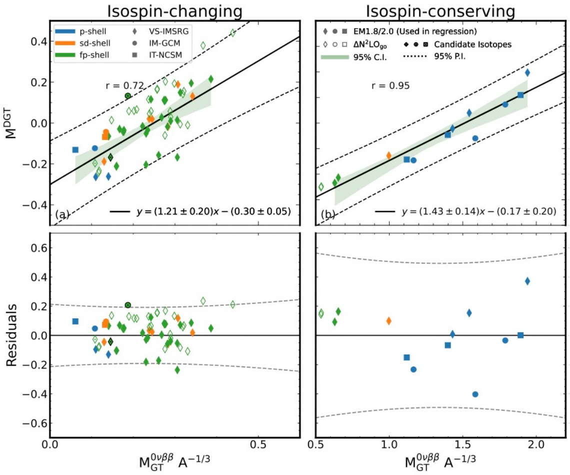


C. Brase, J. Menéndez, E. A. Coello Pérez, A. Schwenk, 2108.11805 [nucl-th] (2021)

Correlation relations between DGT transition and $0\nu\beta\beta$ decay



- The isospin of the nuclear ground state is $T = |N - Z|/2$.
- The $0\nu\beta\beta$ decays are isospin-changing transitions (weak correlation, relatively light nuclei, more careful studies for heavy nuclei).



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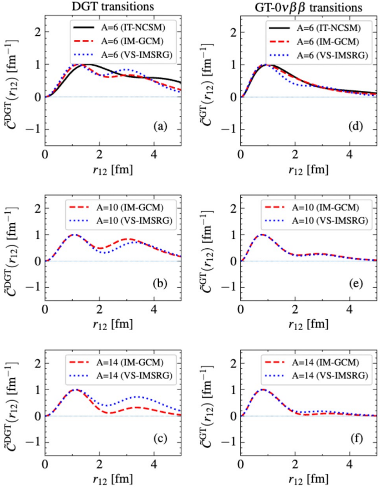
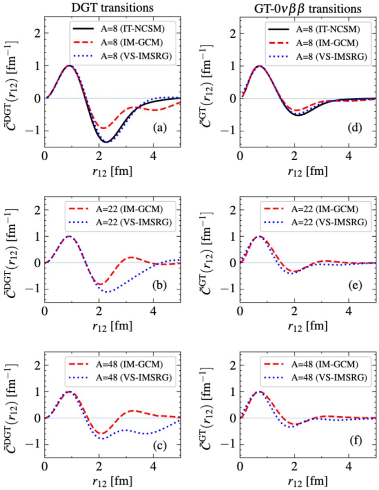
Isospin-conserving transitions:



Isospin-changing transitions:



Thank you for your attention