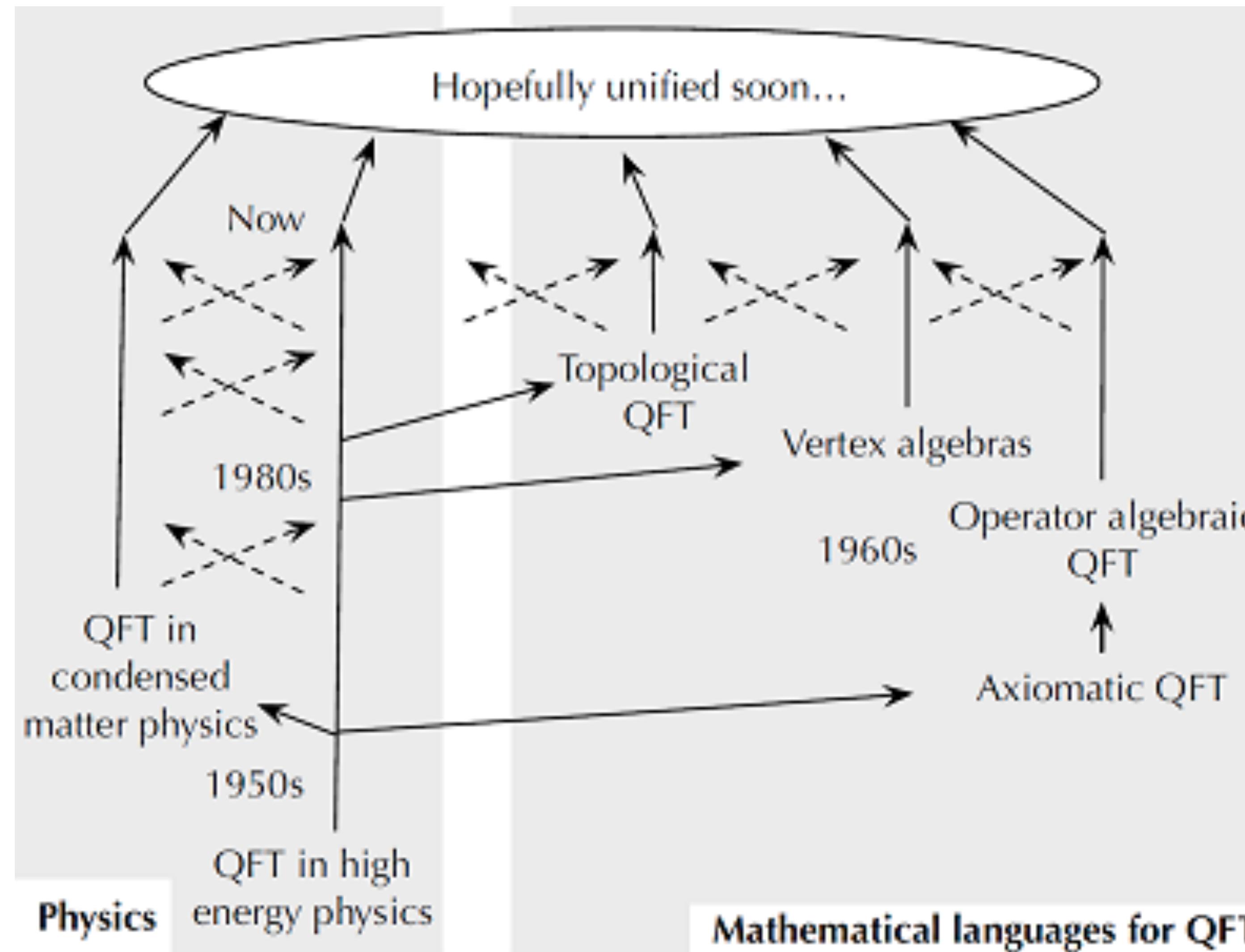


From Conformal Collider to the LHC and beyond

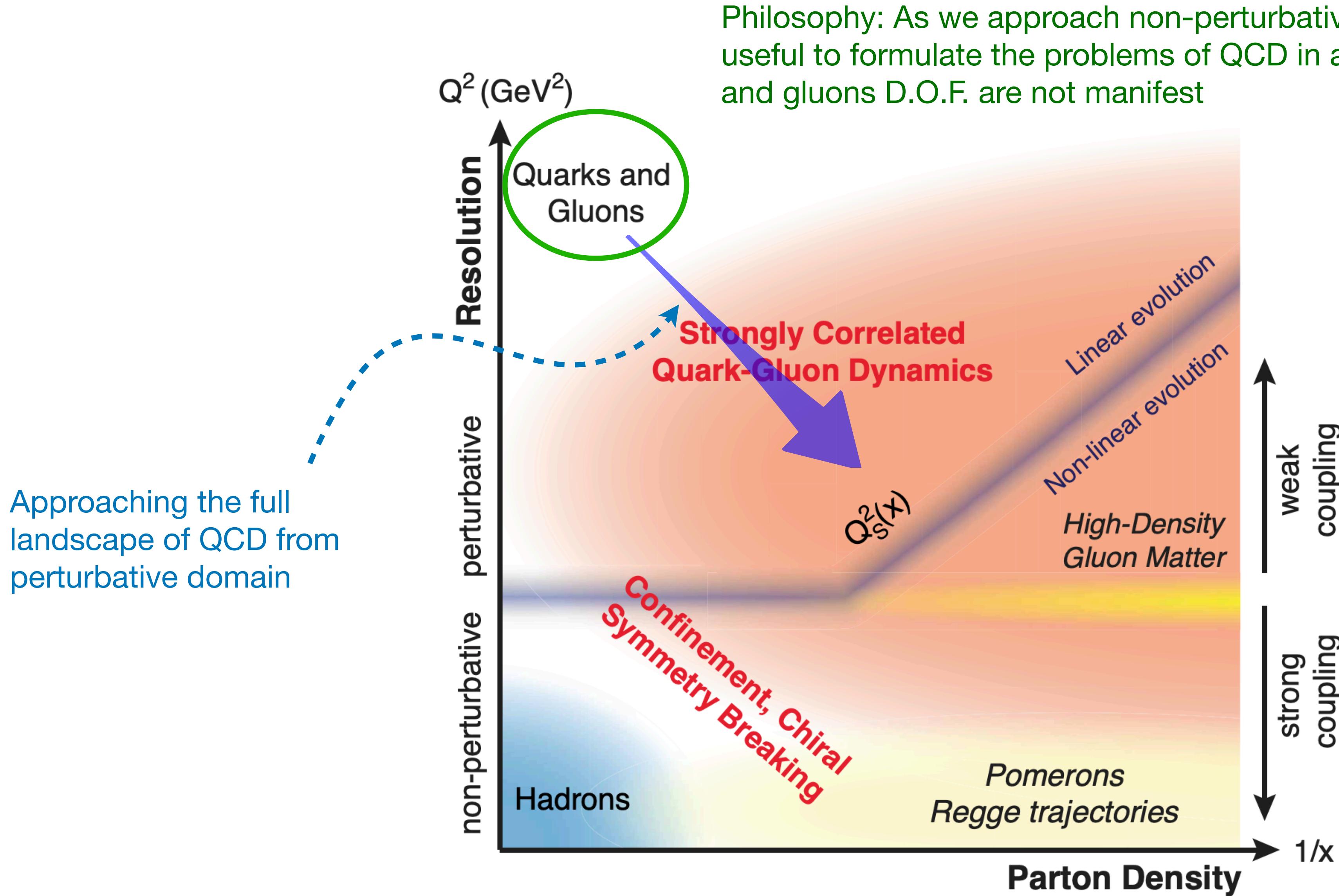
Hua Xing Zhu
Peking University

高能理论论坛第51期
中科院高能所
2023年9月4日

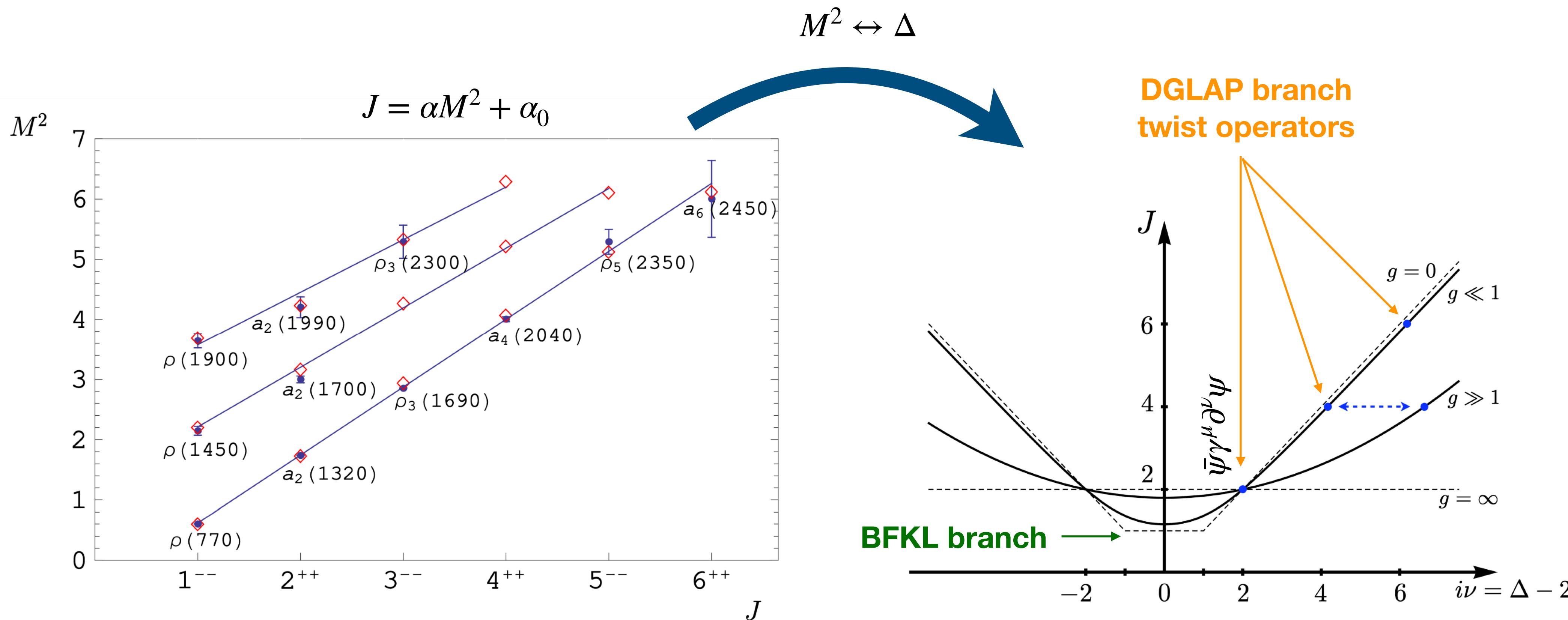
The landscape of Quantum Field Theory



The landscape of QCD



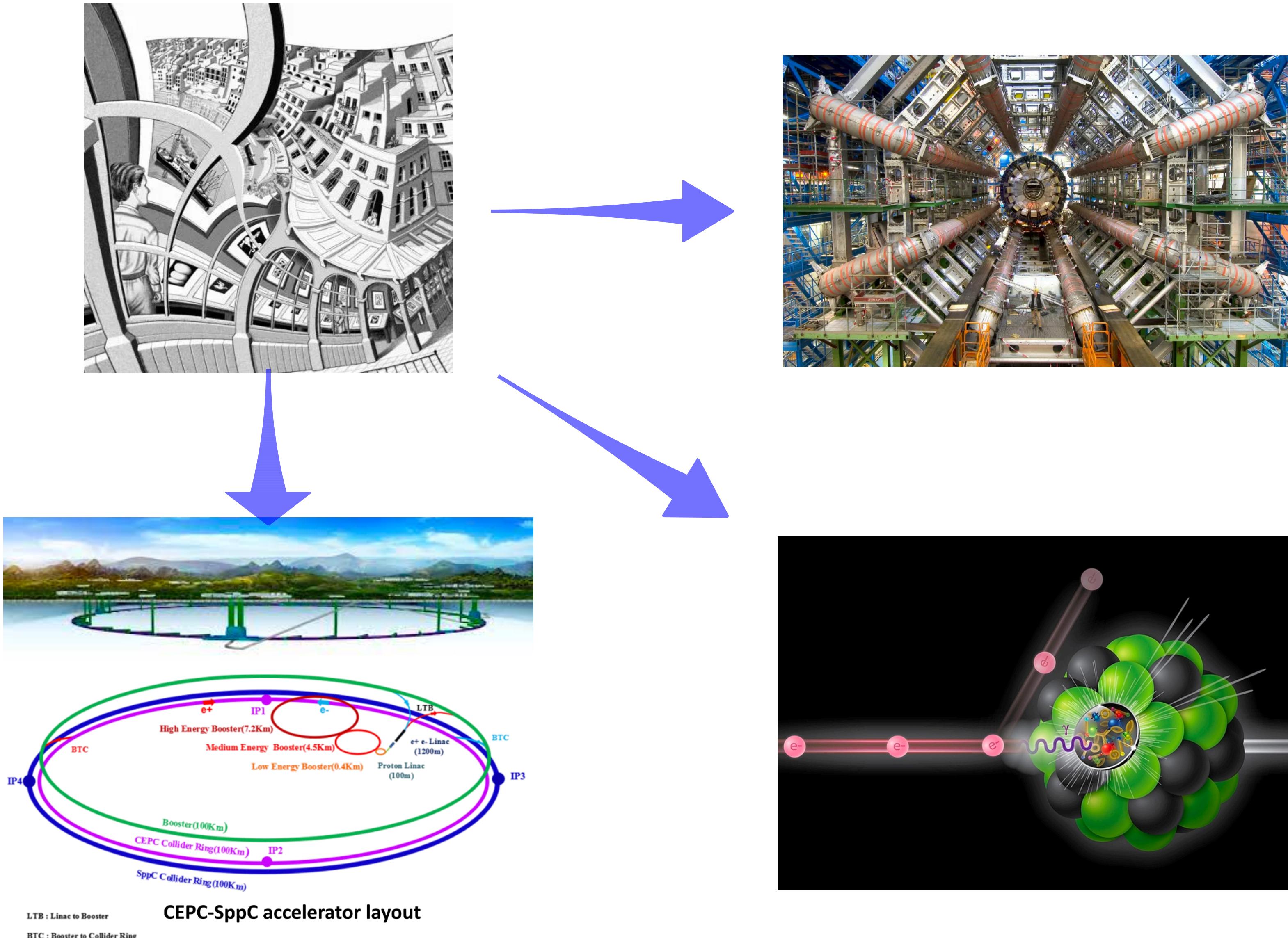
Regge trajectory: hadrons and partons



The dimension of DGLAP operators govern evolution PDFs and FFs.

Understanding the both the DGLAP and BFKL branch is a central task of perturbative QCD, and might shed light on non-perturbative dynamics of QCD.

Reformulating QCD measurements using energy correlators



Energy Correlations in Electron-Positron Annihilation: Testing Quantum Chromodynamics

C. Louis Basham, Lowell S. Brown, Stephen D. Ellis, and Sherwin T. Love

Department of Physics, University of Washington, Seattle, Washington 98195

(Received 21 August 1978)

An experimental measure is presented for a precise test of quantum chromodynamics. This measure involves the asymmetry in the energy-weighted opening angles of the jets of hadrons produced in the process $e^+e^- \rightarrow$ hadrons at energy W . It is special for several reasons: It is reliably calculable in asymptotically free perturbation theory; it has rapidly vanishing (order $1/W^2$) corrections due to nonperturbative confinement effects; and it is straightforward to determine experimentally.

L. Dixon, M.X. Luo, V. Shtabovenko, T.Z.
Yang, HXZ, 1801.03219

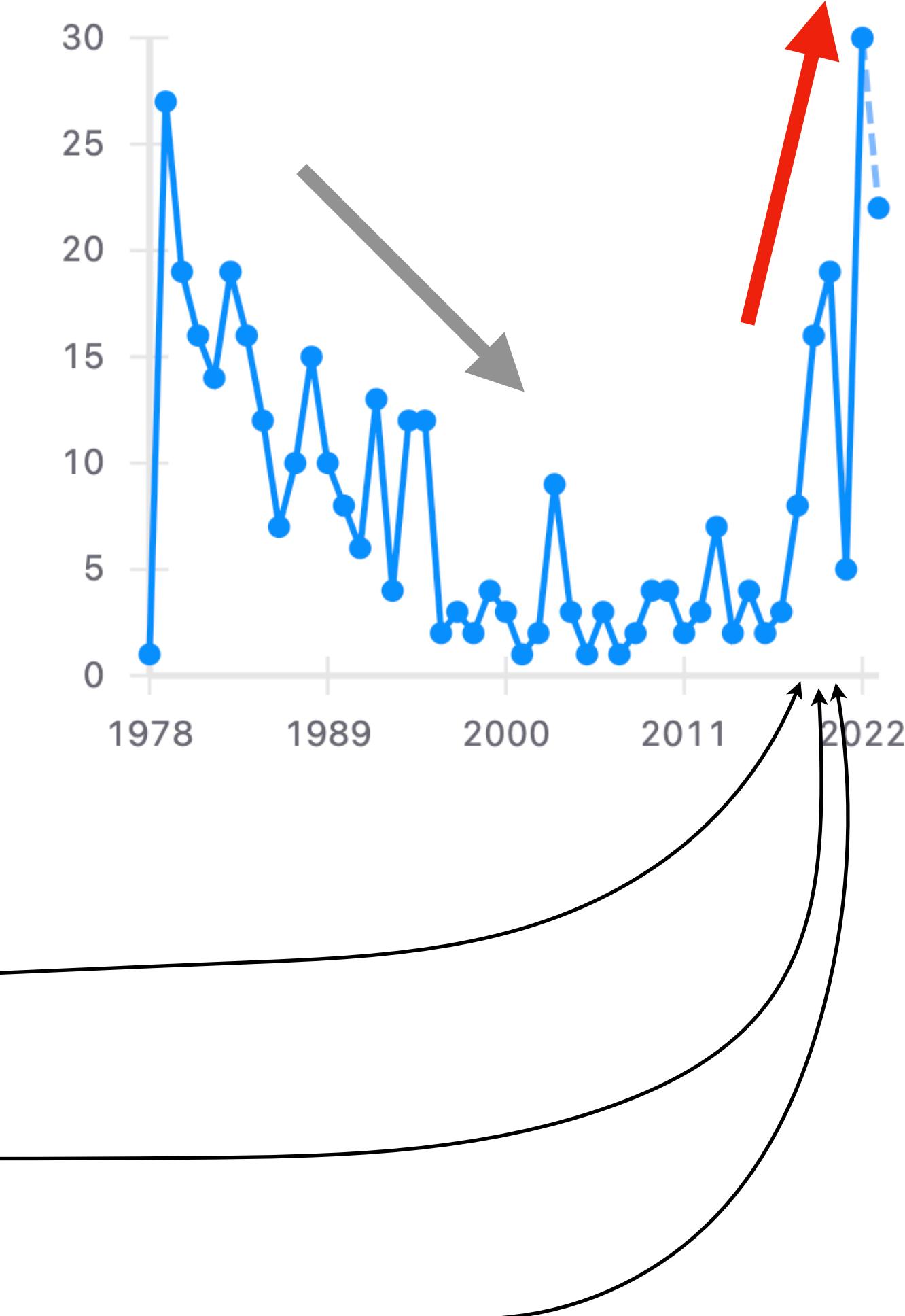
I. Moult, HXZ, 1801.02627

H. Chen, M.X. Luo, I. Moult, T.Z. Yang, X.Y.
Zhang, HXZ, 1912.11050

L. Dixon, I. Moult, HXZ, 1905.01310

M.X. Luo, V. Shtabovenko, T.Z. Yang, HXZ,
1903.07277

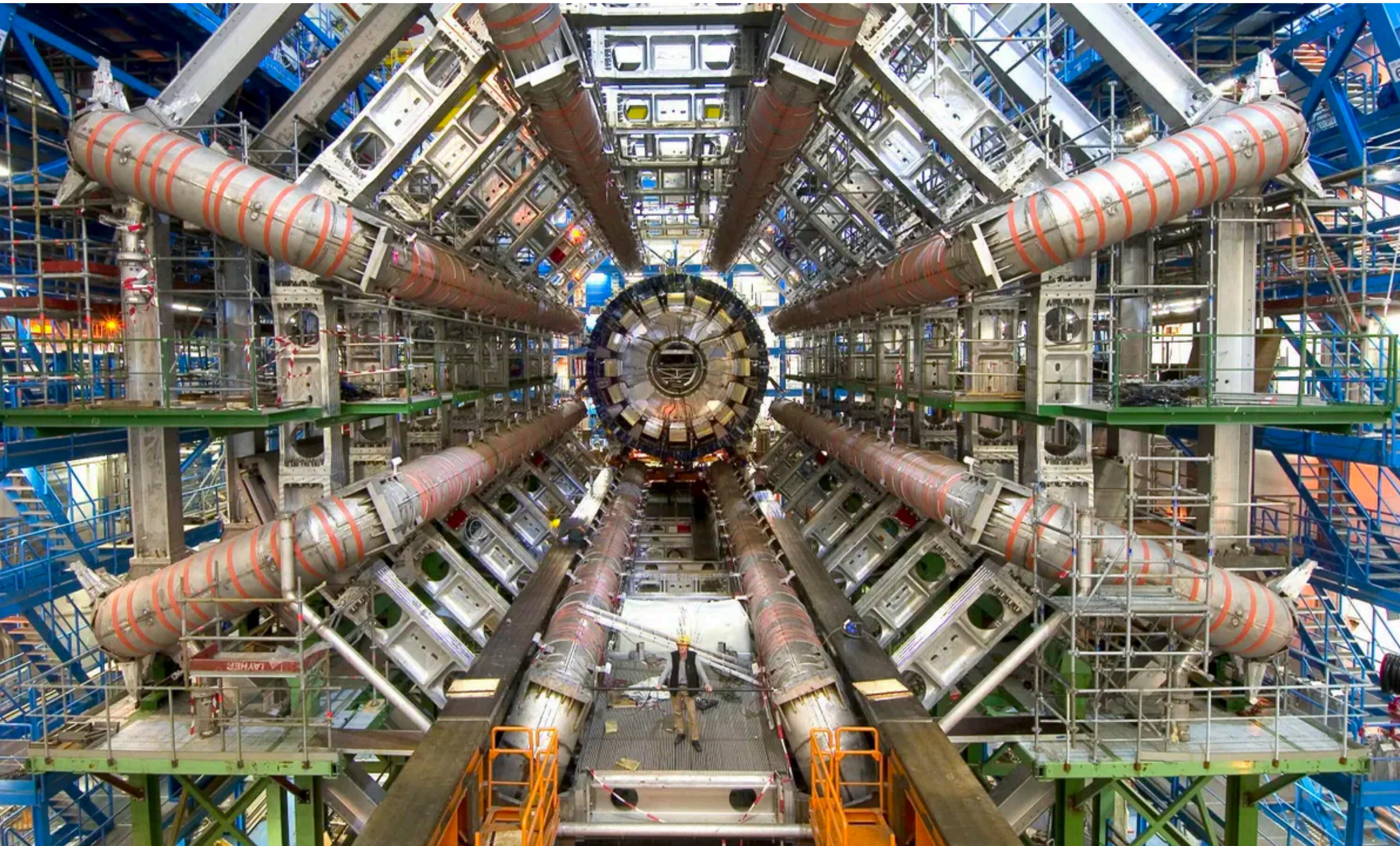
A.J. Gao, H.T. Li, I. Moult, HXZ, 1901.04497

Citations per year

H. Chen, I. Moult, X.Y. Zhang, HXZ,
2011.02492

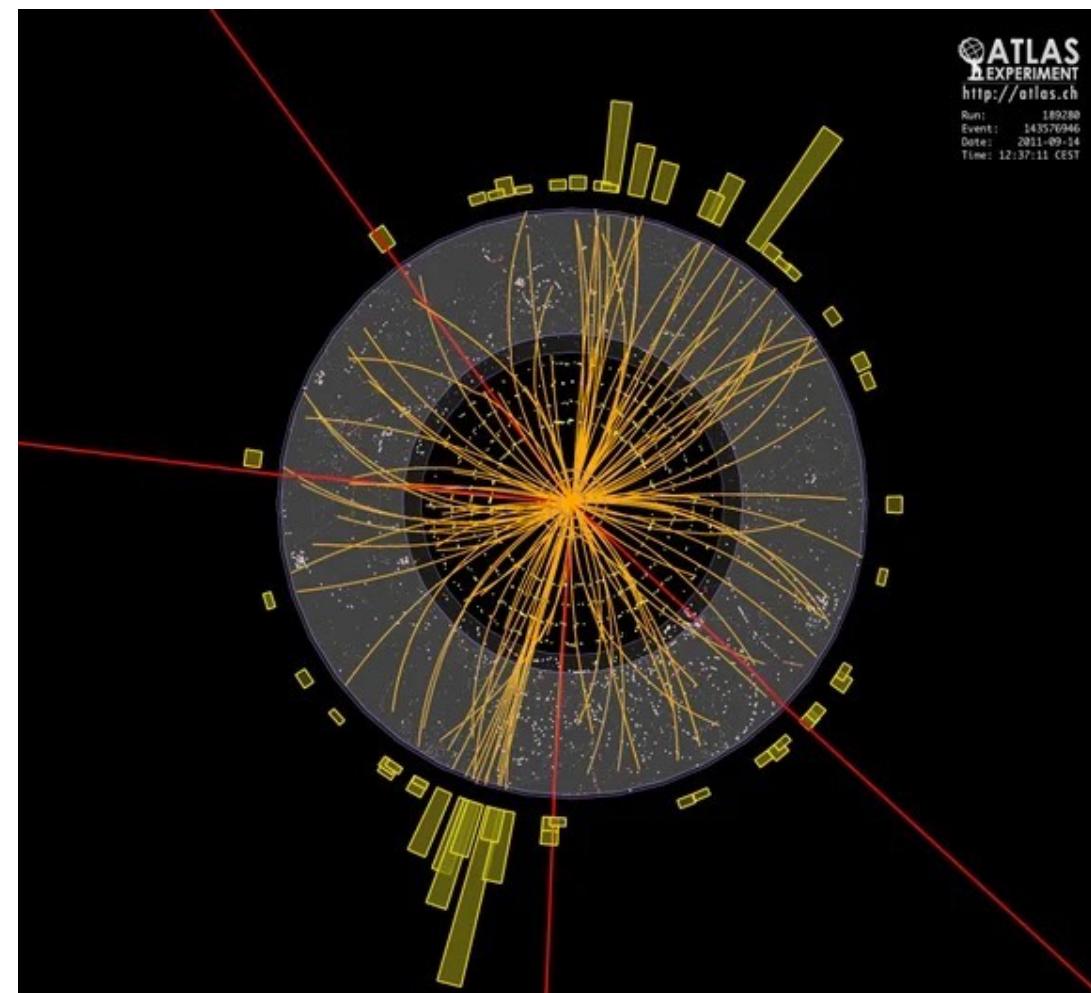
H. Chen, I. Moult, X.Y. Zhang, HXZ,
2004.11381

Energy correlators for jets at the LHC



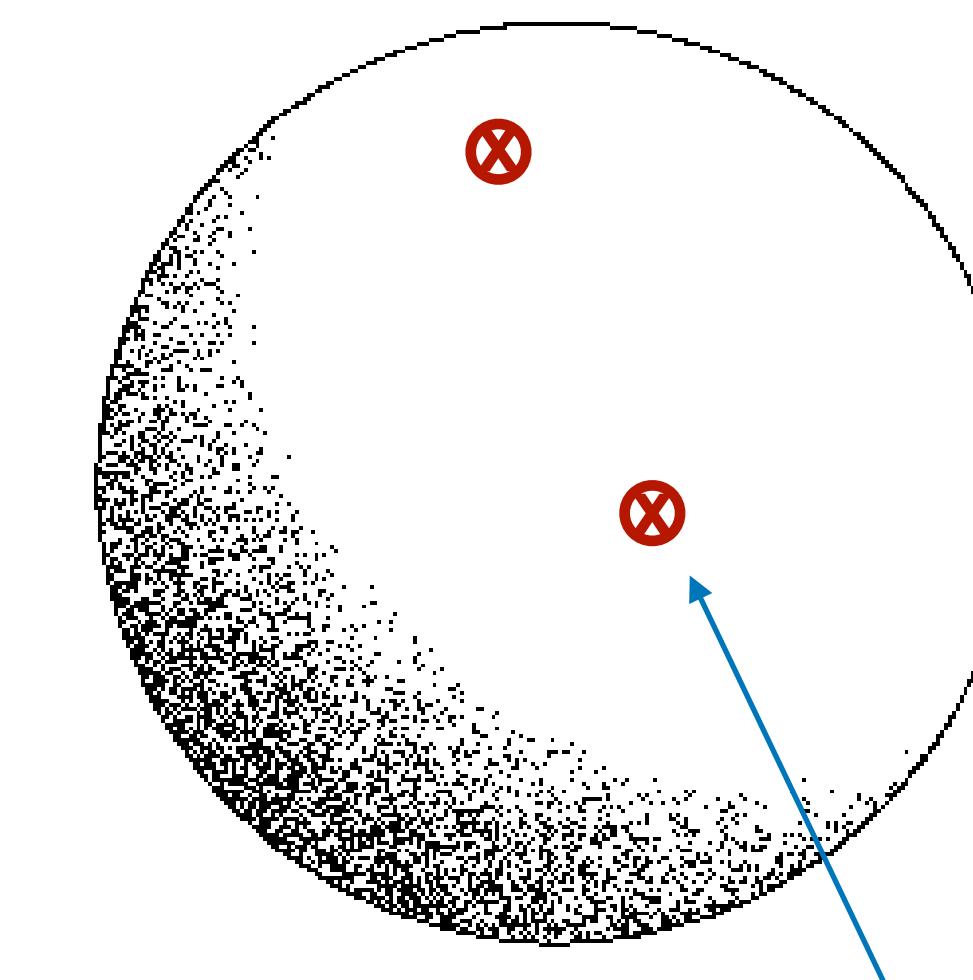
Energy correlators as N-point correlation function on celestial sphere

Scattering in momentum eigenstate

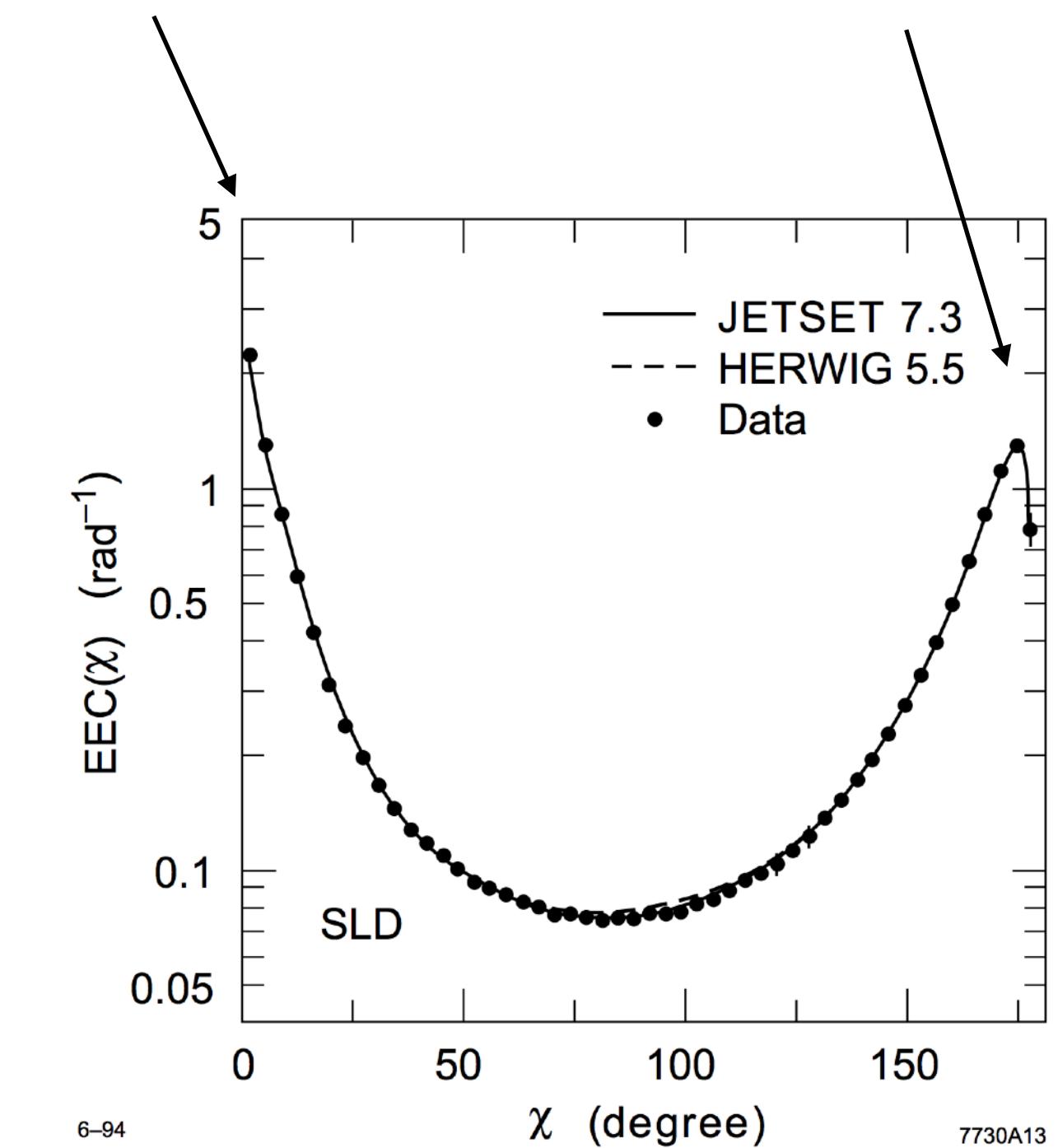


Measurement in boost eigenstate

$$\langle \mathcal{E}(\vec{n}_1) \mathcal{E}(\vec{n}_2) \rangle$$



DGLAP singularity

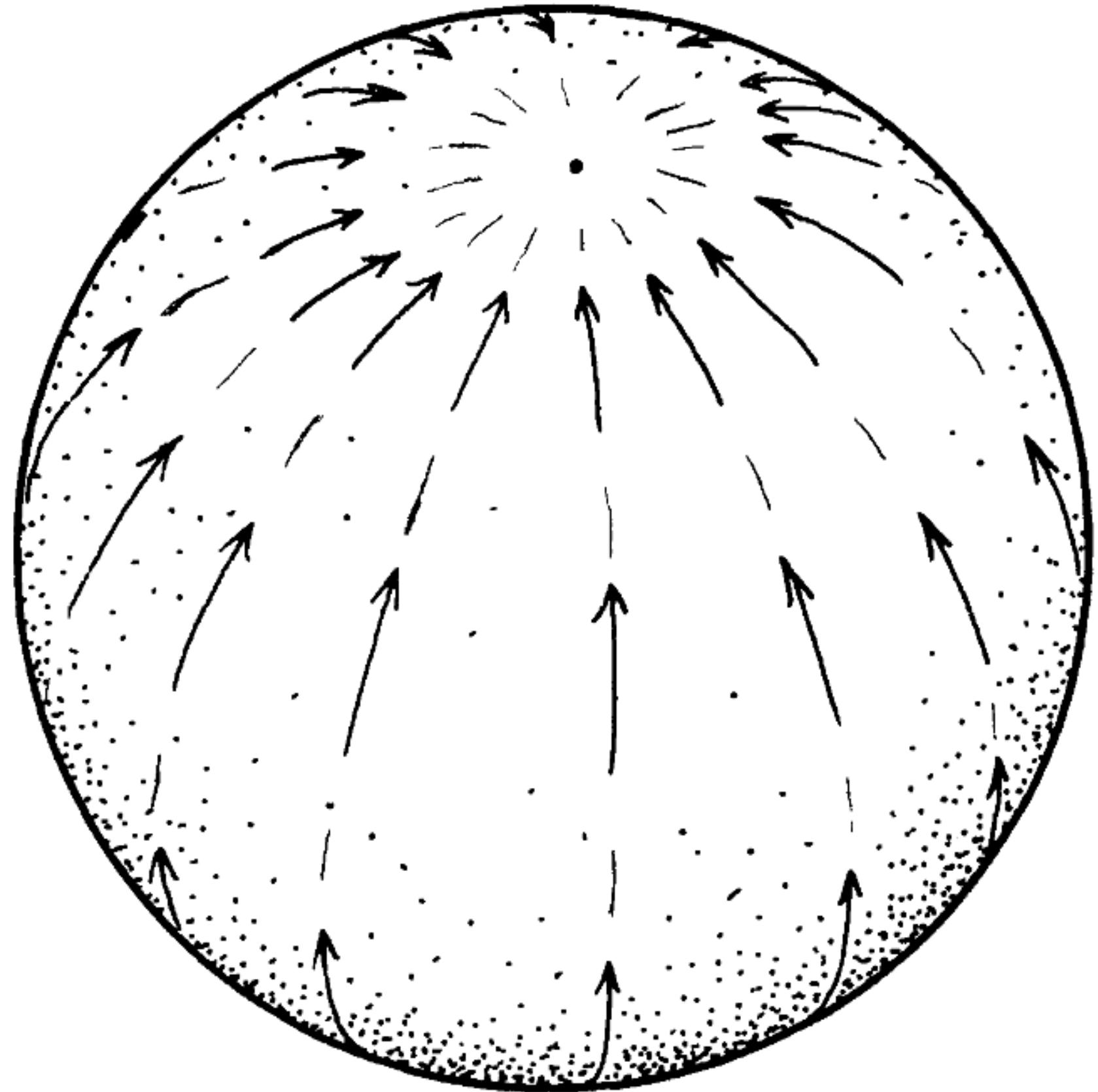


Why conformal symmetry?

1. Lorentz symmetry as a conformal symmetry on the celestial sphere.
2. Classical (massless) QCD lagrangian is conformal invariant!

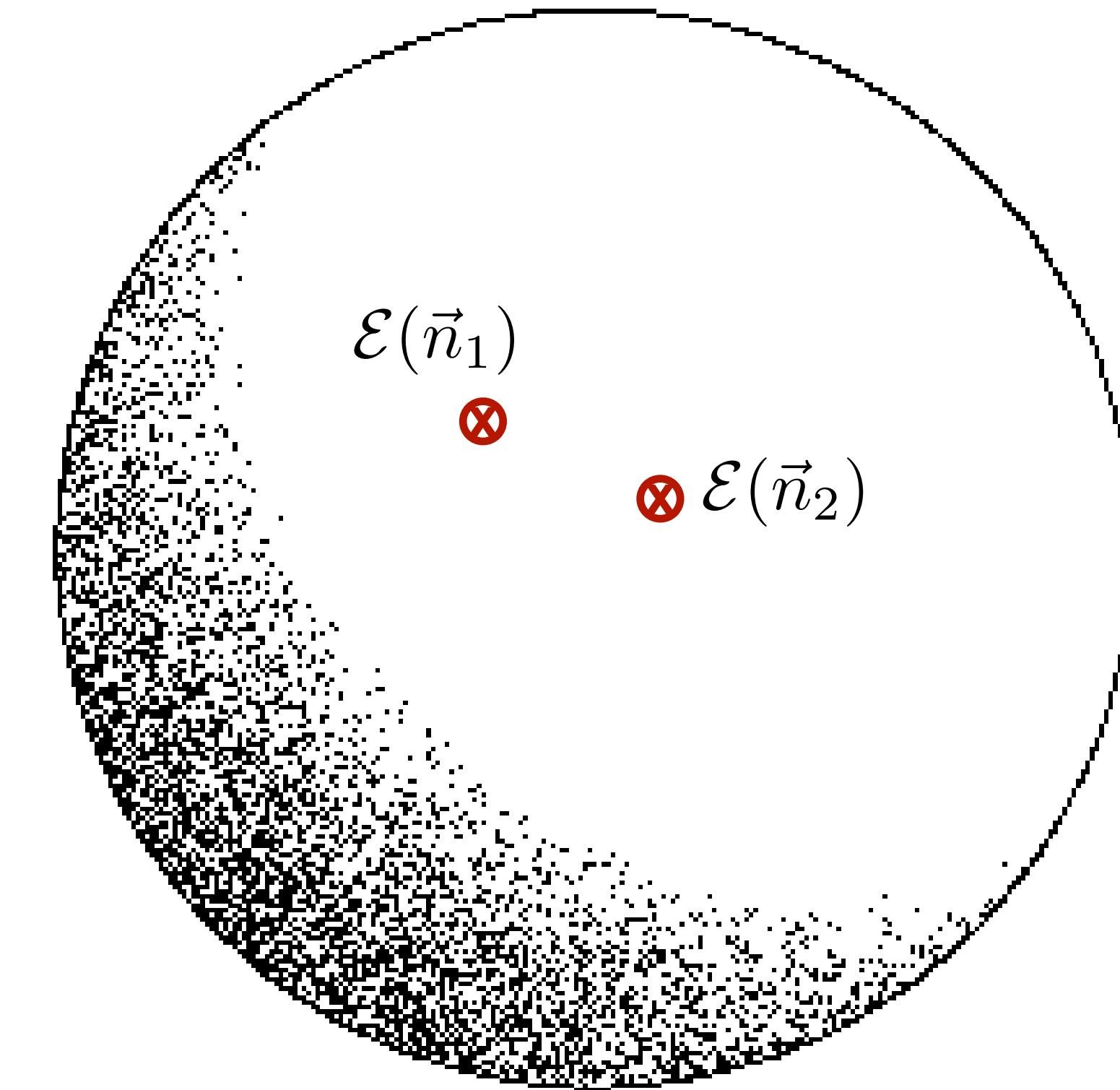
Lorentz transformation and conformal transformation on celestial sphere

Lorentz group = $SL(2, \mathbb{C})/Z_2$



Boost in the z direction becomes dilation around the north pole

$$\lim_{\hat{n}_2 \rightarrow \hat{n}_1}$$



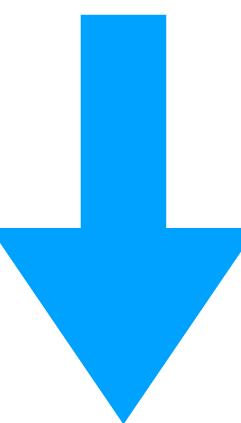
Scaling behavior of small angle correlation determined by conformal symmetry on the celestial sphere

Energy flow operator

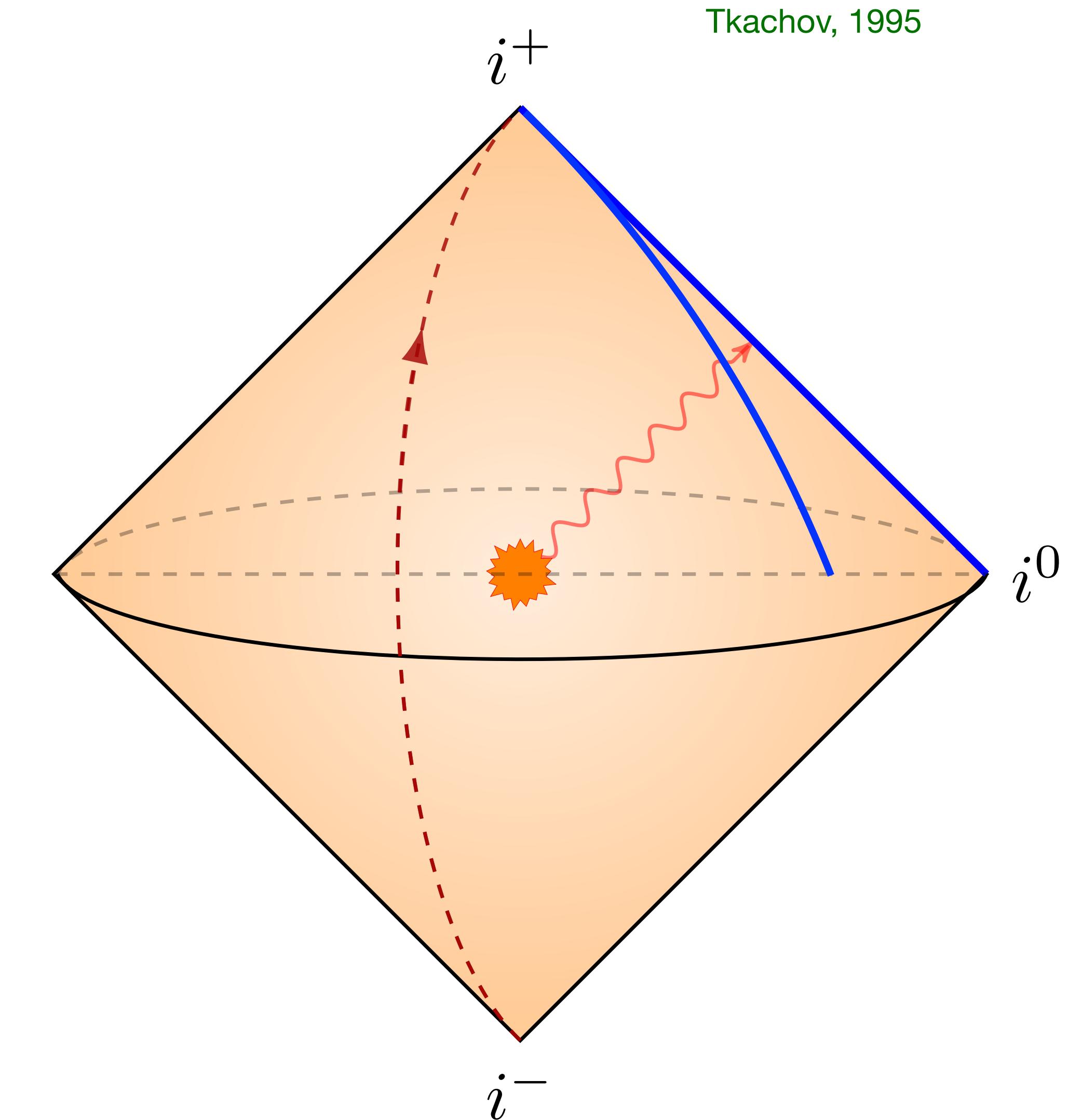
The energy flow operator measures the energy deposition on a detector at direction \vec{n}

$$\mathcal{E}(\vec{n}) = \lim_{r \rightarrow \infty} r^2 \int_0^\infty dt \vec{n}_i T^{0i}(t, r\vec{n})$$

$$\mathcal{E}(\vec{n})|p\rangle = p^0 \delta^{(2)}(\hat{p} - \hat{n})|p\rangle$$



$$\mathcal{E}(\vec{n}) = \int_{-\infty}^\infty d(n \cdot x) \lim_{\bar{n} \cdot x \rightarrow \infty} (\bar{n} \cdot x)^2 \bar{n}^\mu \bar{n}^\nu T_{\mu\nu}(x)$$



The energy flow operator (ANEC) also found important application in black hole physics and quantum information!

General lightray operator

Hofman, Maldacena 08; Kologlu, Kravchuk, Simmons-Duffin, Zhiboedov, 19

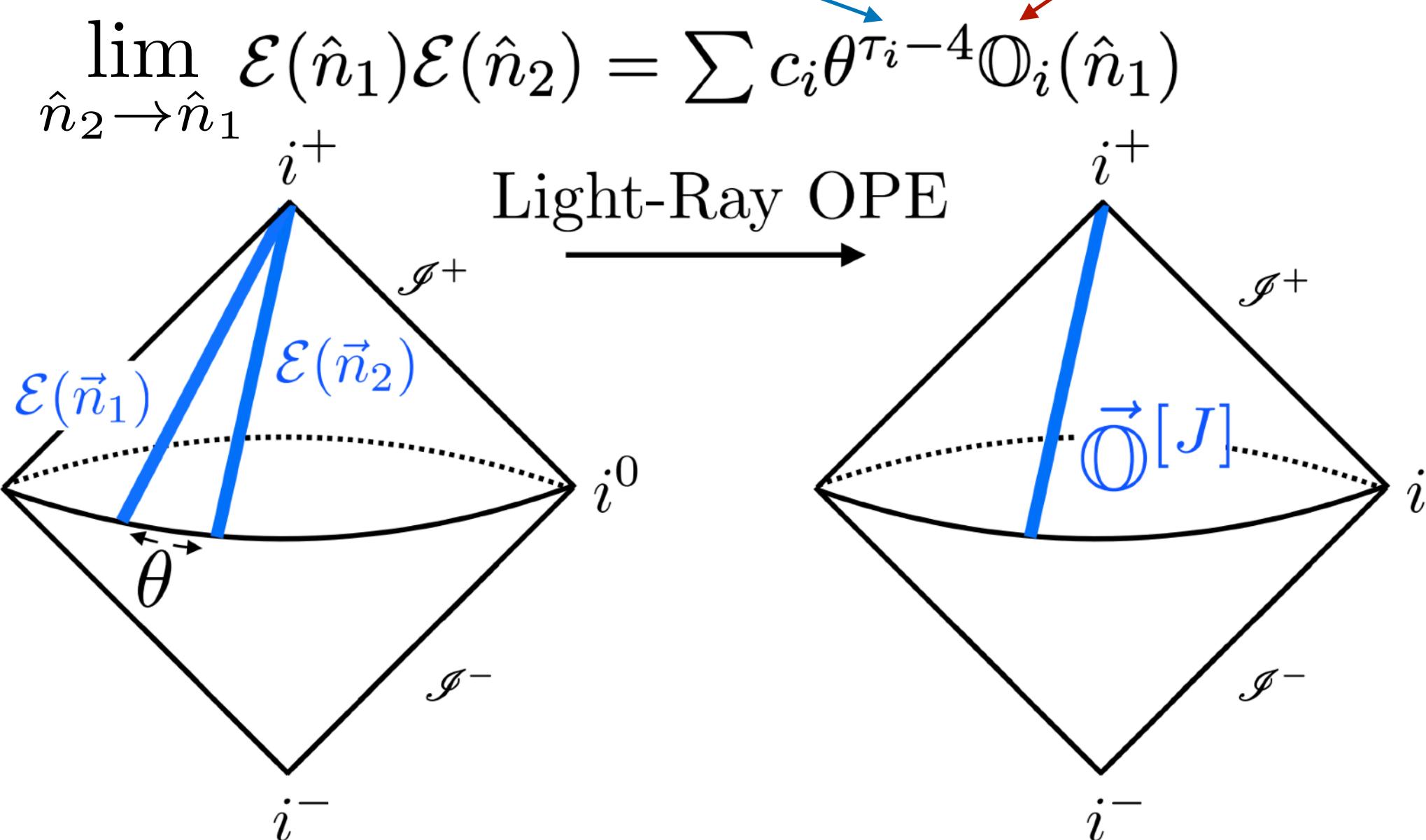
$$\mathbb{O}_{J-1, \Delta-1}(\vec{n}) = \int_{-\infty}^{\infty} d(n \cdot x) \lim_{\bar{n} \cdot x \rightarrow \infty} (\bar{n} \cdot x)^{\Delta-J} \bar{n}_{\mu_1} \cdots \bar{n}_{\mu_J} O_{\Delta, J}^{\mu_1 \cdots \mu_J}(x)$$

↑
lightray operator living on
the celestial sphere

↑
local twist operator of
dimension Δ and spin J

**determined by celestial
dimension**

**determined by bulk
dimension**



At twist 2 the relevant unpolarized operators are

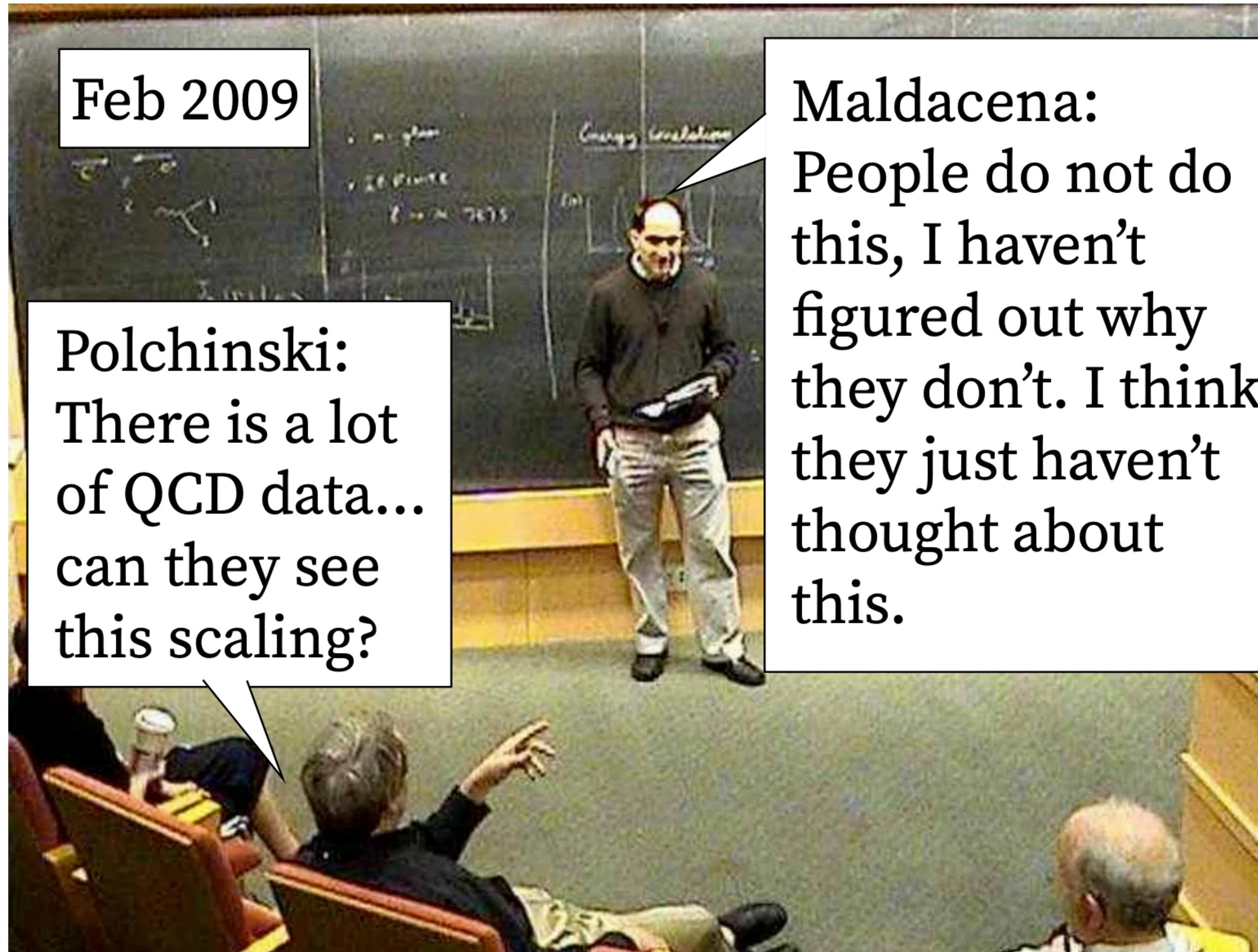
$$\mathcal{O}_q^{[J]} = \frac{1}{2^J} \bar{\psi} \gamma^+ (iD^+)^{J-1} \psi$$

$$\mathcal{O}_g^{[J]} = -\frac{1}{2^J} F_a^{\mu+} (iD^+)^{J-2} F_a^{\mu+}$$

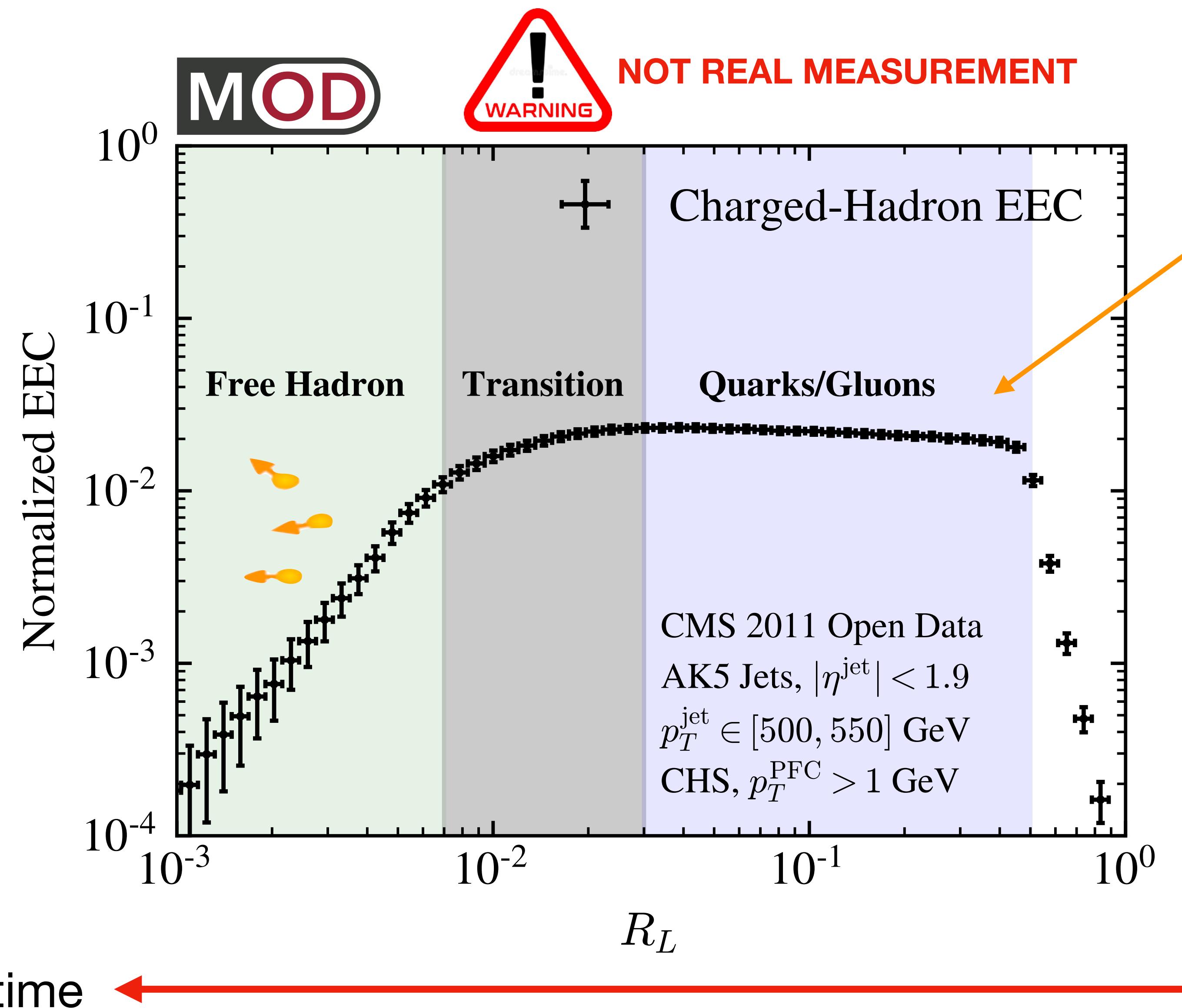
CFT picture receives controllable logarithmic corrections in QCD

Dixon, Moult, HXZ, 2019

Scaling is one of the most profound phenomena in physics

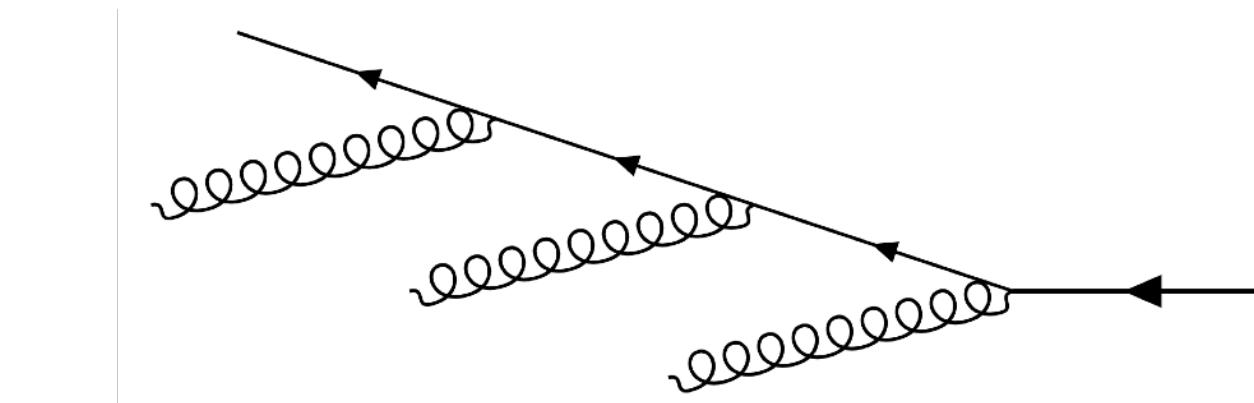


EEC in CMS opendata



Scaling behavior determines by lightray OPE (modulo logarithmic running effect)

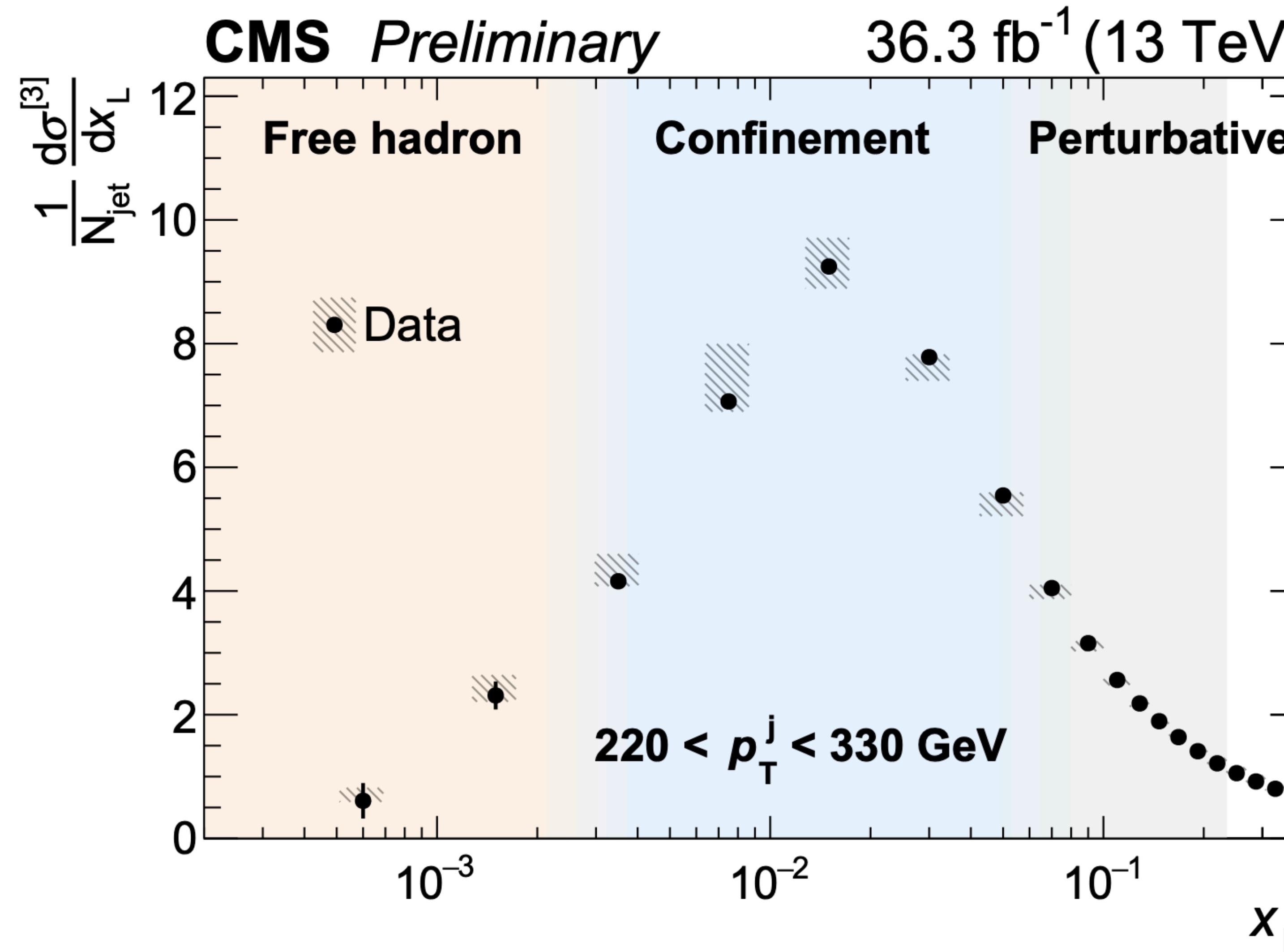
Perturbative quark/gluon splitting



QCD cascade: large angle correlation resulted from early time interaction

Real measurement from CMS

<https://cds.cern.ch/record/2866560>, 2023

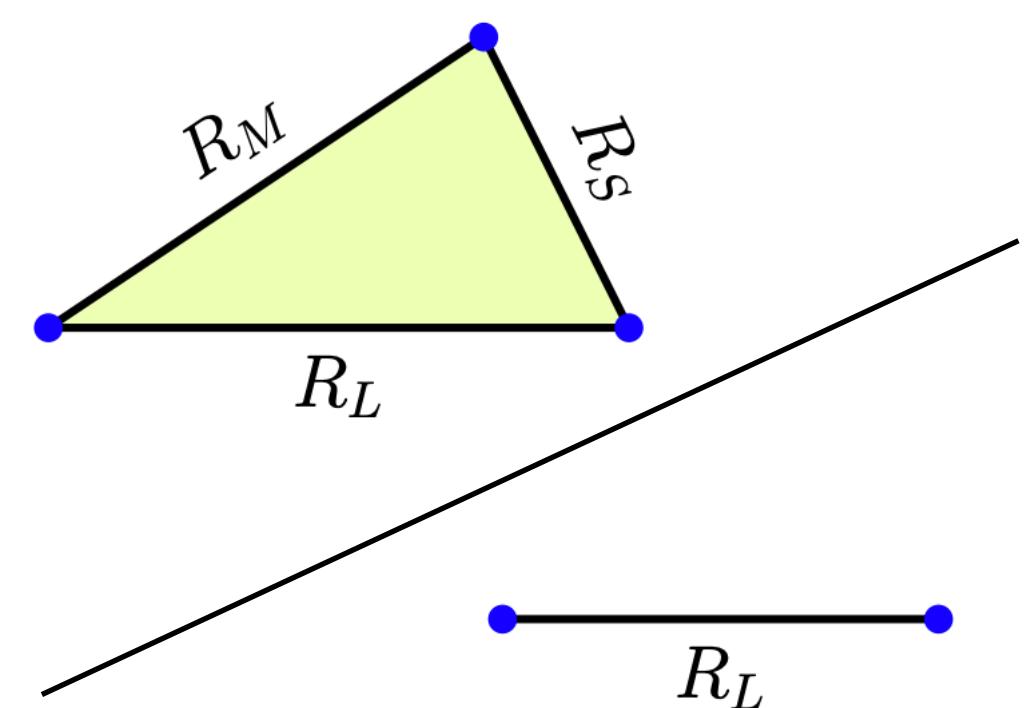


First ever measurement of the time evolution of quark/gluon to hadrons fragmentation!

Projected N-point energy correlator

H. Chen, Moult, X.Y. Zhang, HXZ, 2020
W. Chen, J. Gao, Y. Li, Z. Xu, X.Y. Zhang, HXZ, 2023

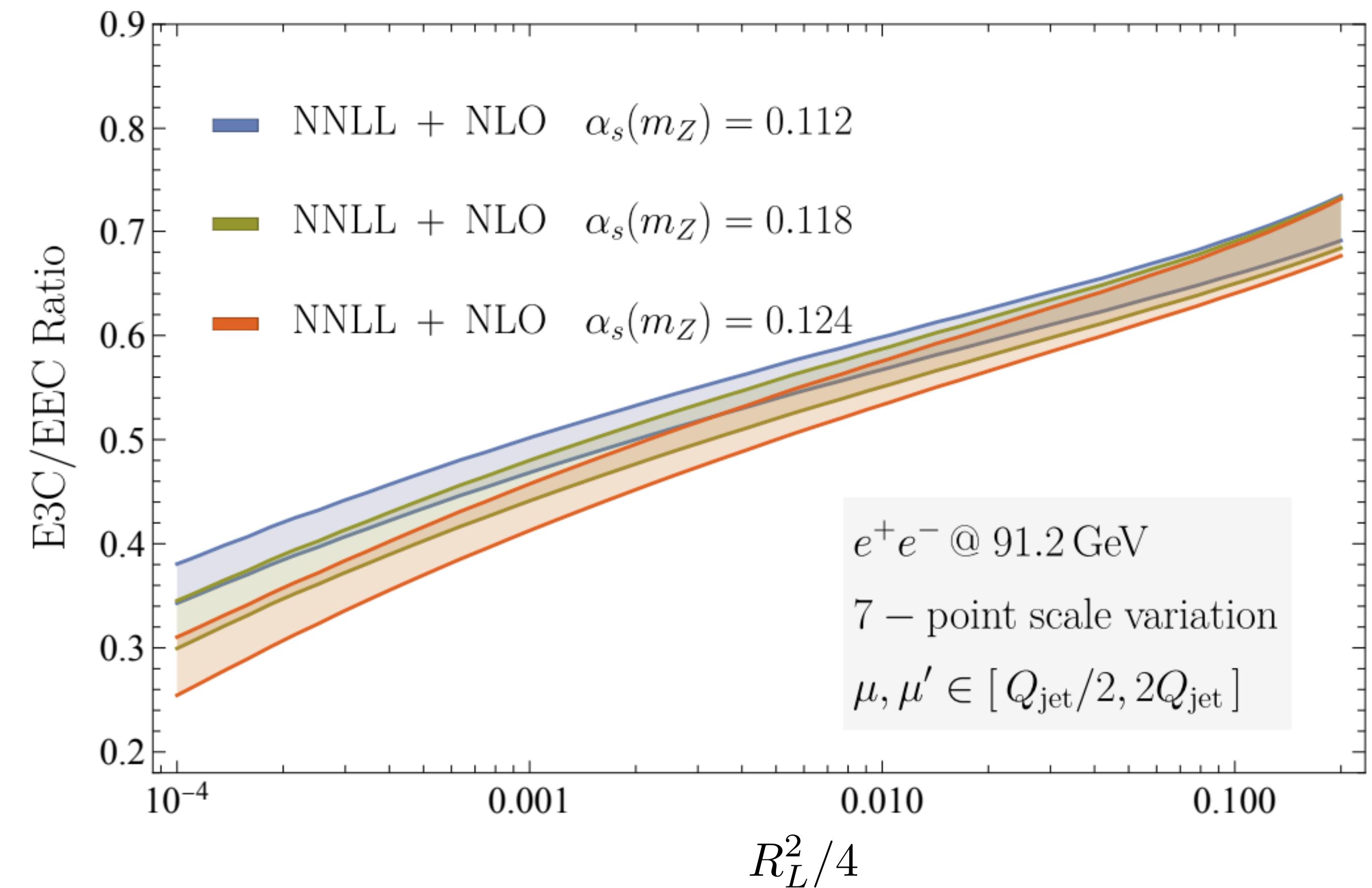
Ratio of projected three-point energy correlator over EEC



$$\sim \frac{(R_L)^{\Delta_{J=4}(\alpha)}}{(R_L)^{\Delta_{J=3}(\alpha)}}$$

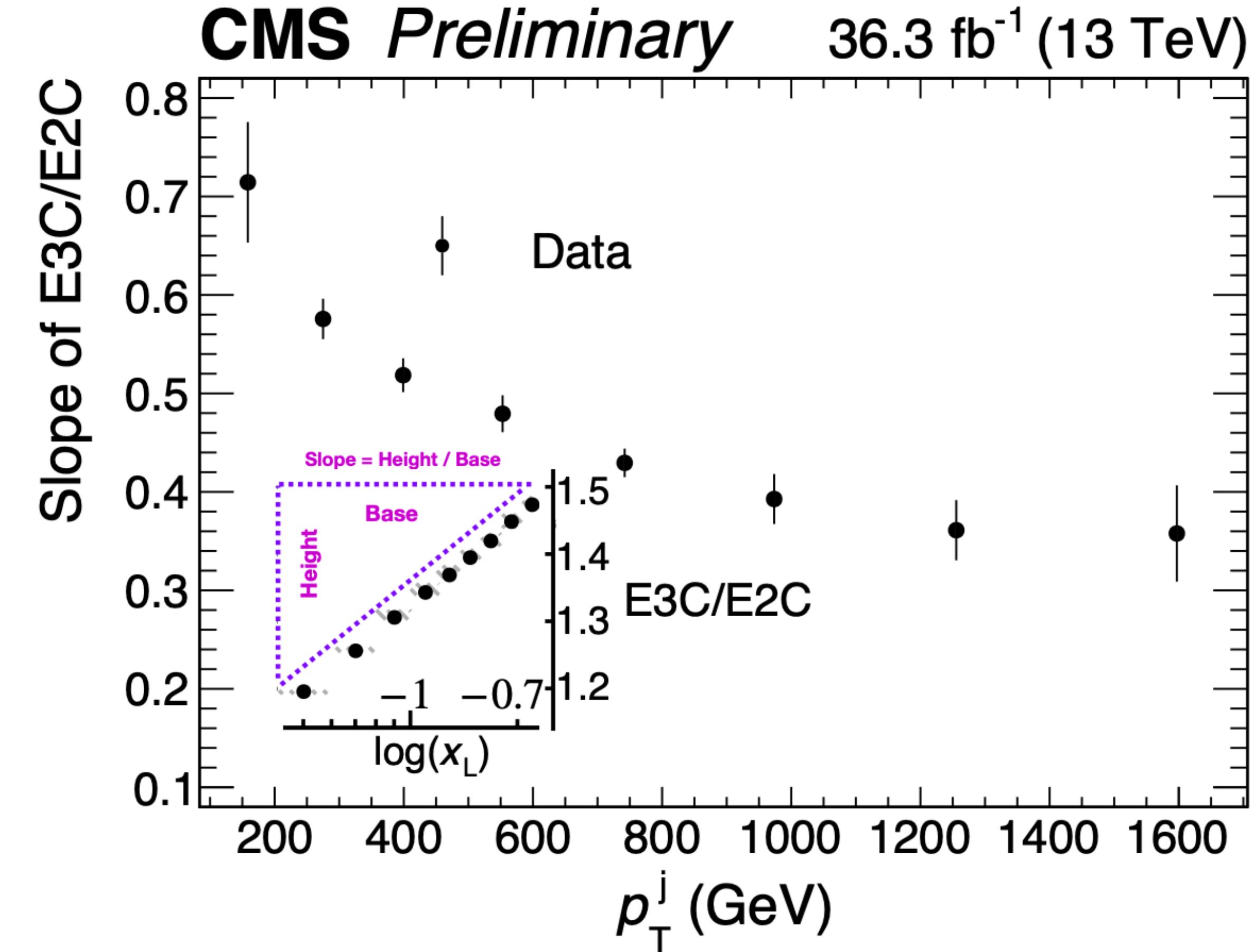
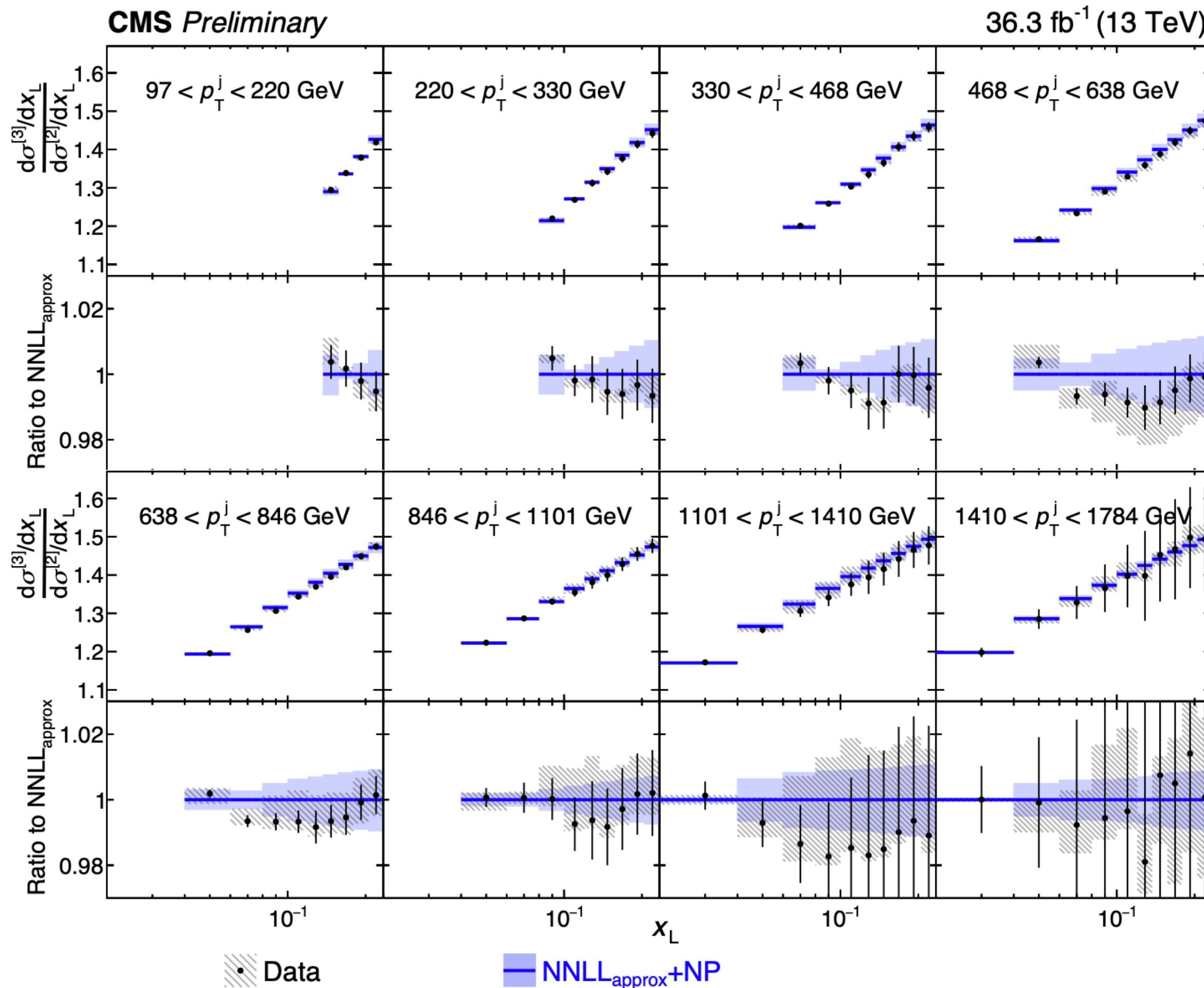
Lots of uncertainties cancelled after the ratio

Sensitive to α_s !



Precision α_s measurement from CMS

<https://cds.cern.ch/record/2866560>, 2023



$$\alpha_s(m_Z) = 0.1229^{+0.0014}_{-0.0012} (\text{stat.})^{+0.0030}_{-0.0033} (\text{theo.})^{+0.0023}_{-0.0036} (\text{exp.})$$

Energy correlator and gluon transverse spin

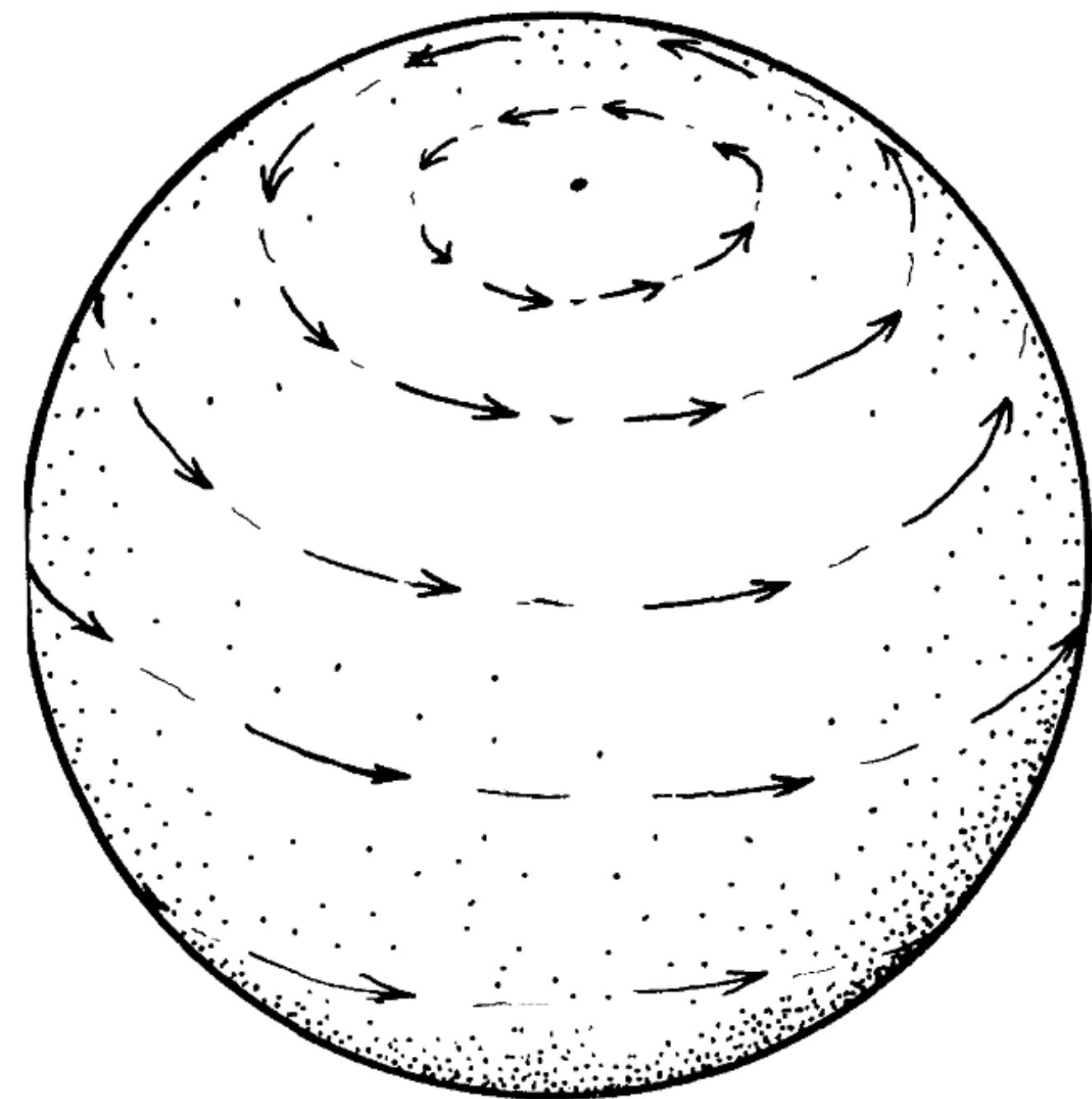


Fig. 1-6. The effect of a rotation on S^+ .

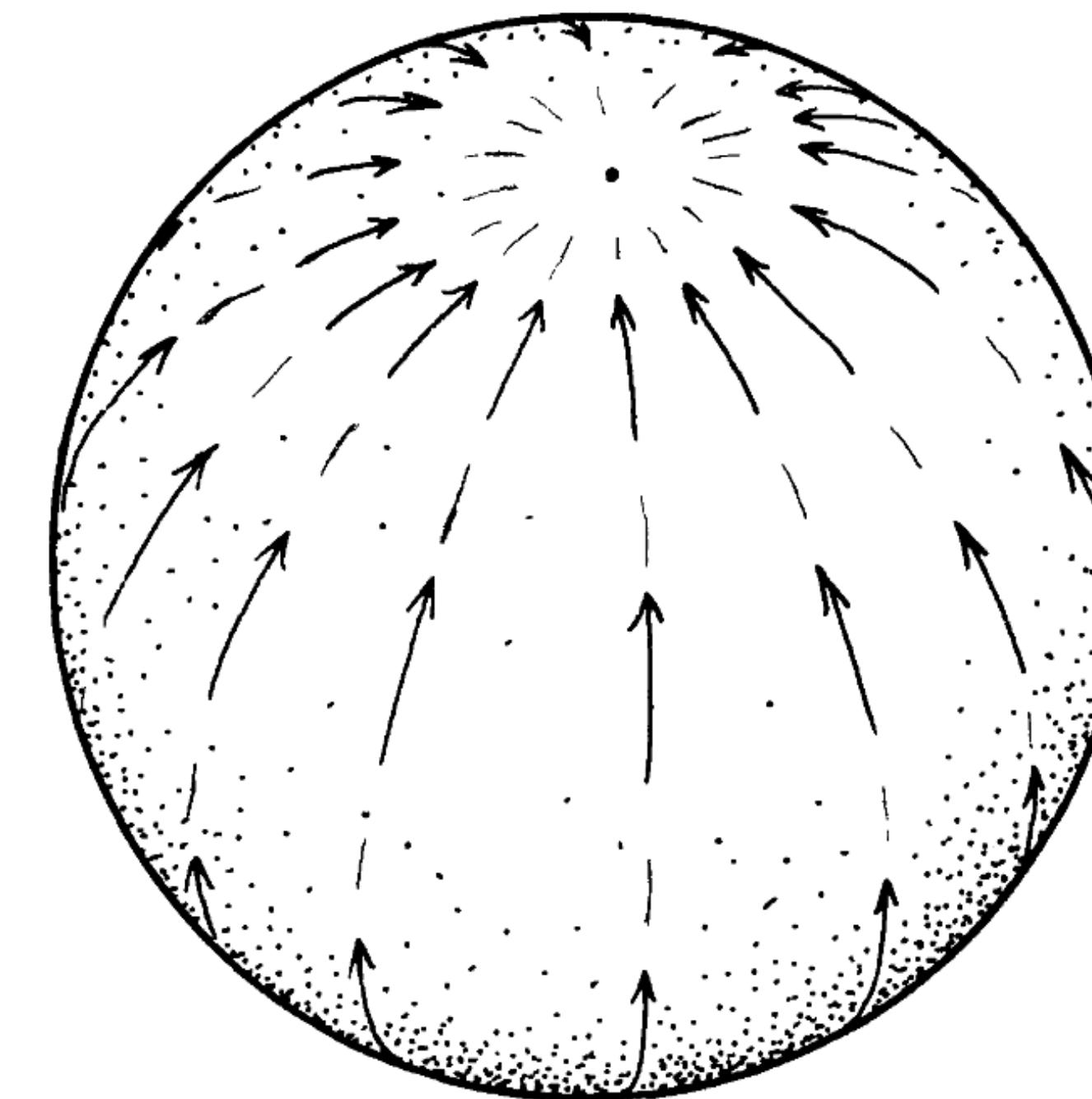
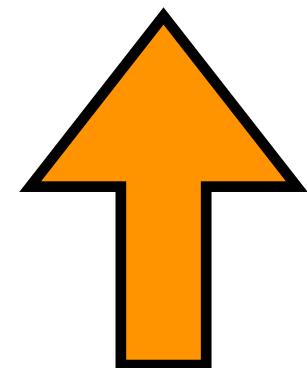
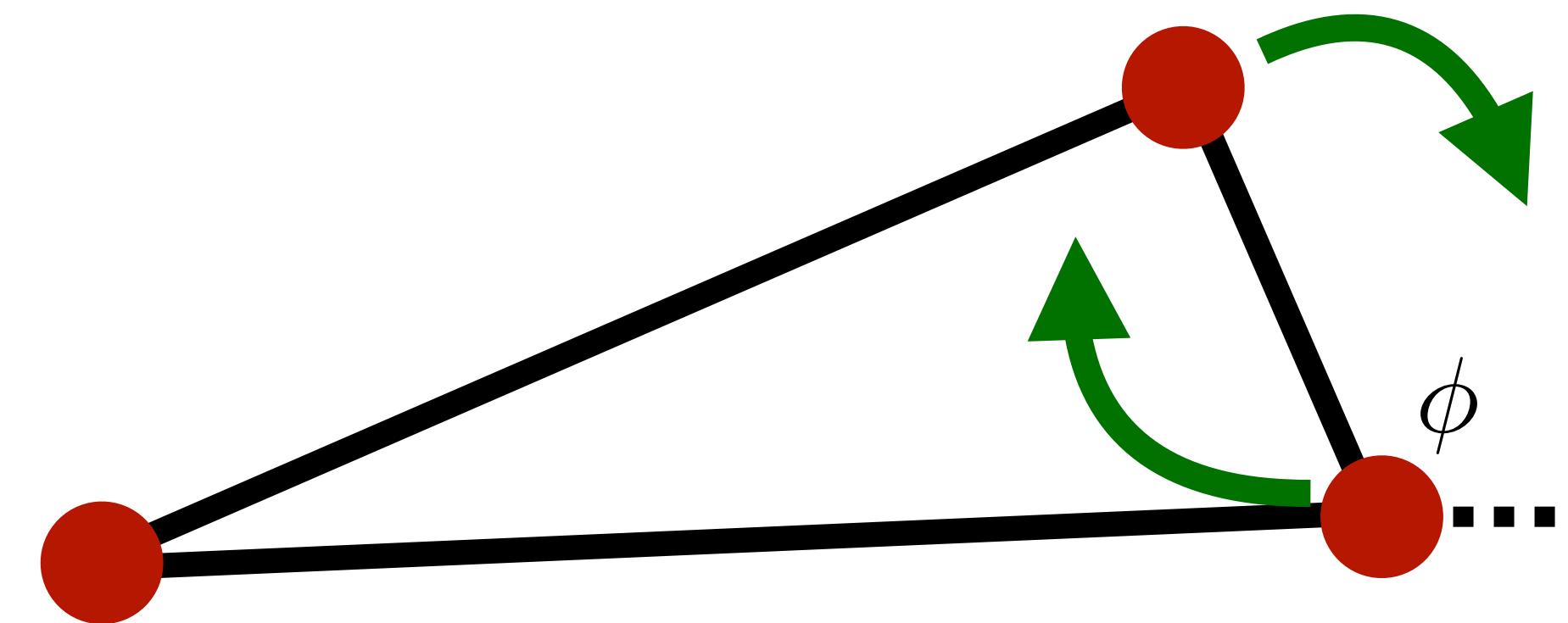


Fig. 1-7. The effect of a boost on S^+ .

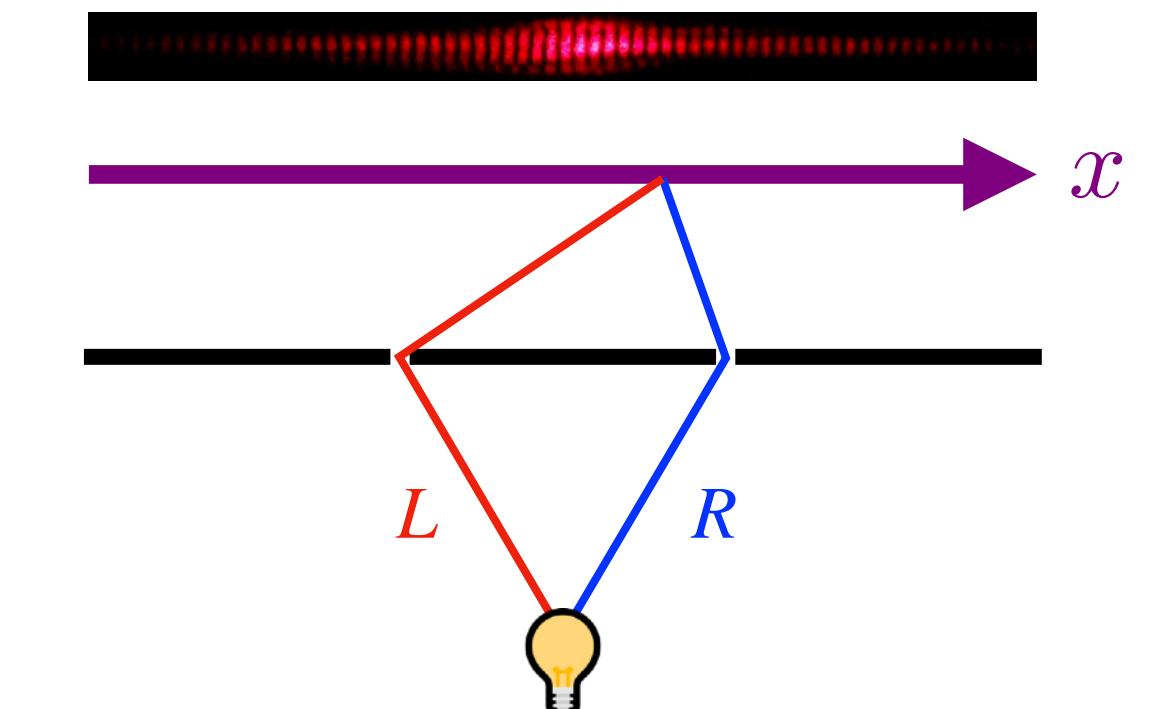


How light-ray operator
insertion rotate on the
celestial sphere

Spin interference from energy correlators



Interpretation in terms of gluon spin interference

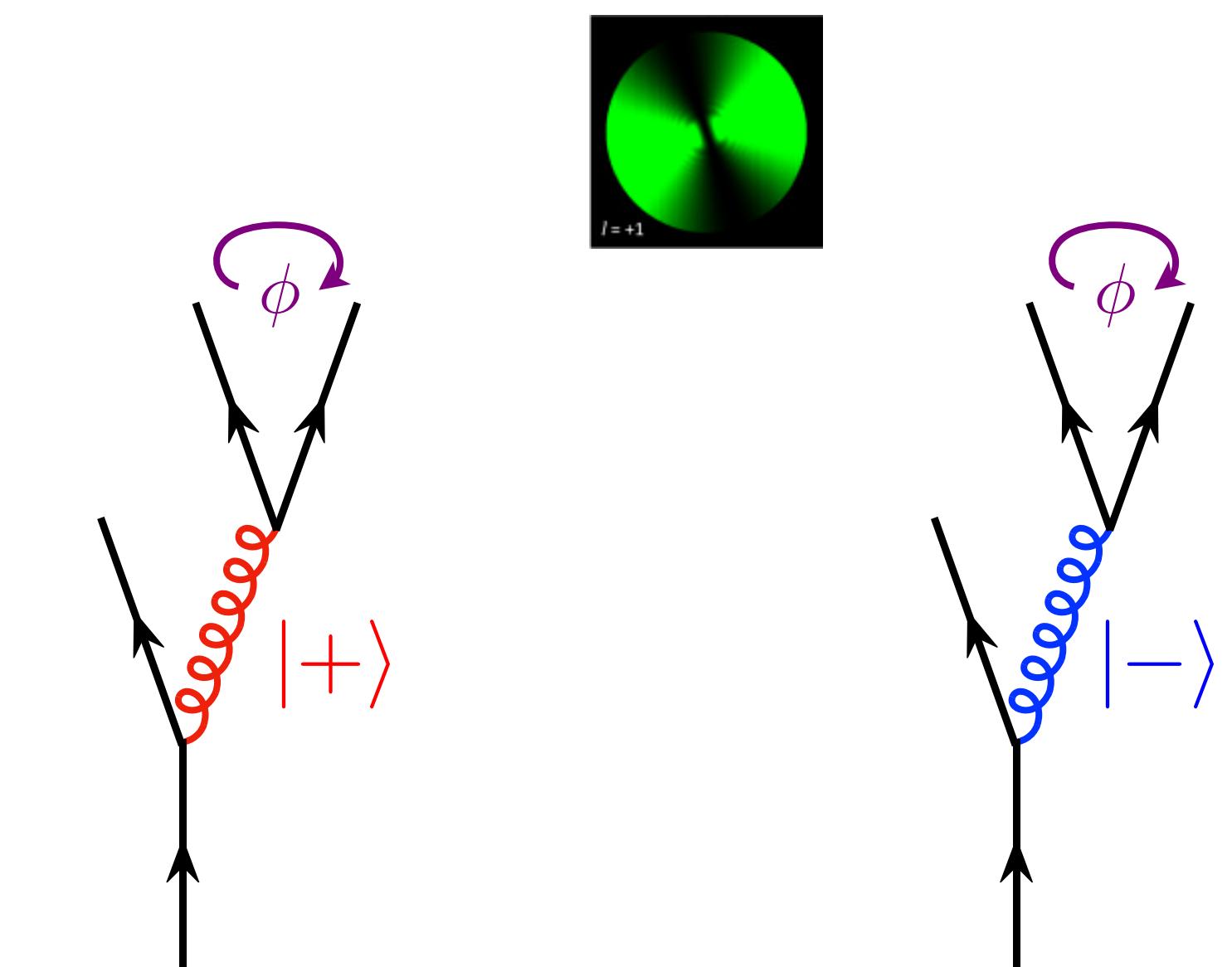


One-loop perturbation theory calculation:

$$S q_q^{(0)}(\phi) = C_F n_f T_F \left(\frac{39 - 20 \cos(2\phi)}{225} \right) + C_F C_A \left(\frac{273 + 10 \cos(2\phi)}{225} \right) + C_F^2 \frac{16}{5}$$

$$S q_g^{(0)}(\phi) = C_A n_f T_F \left(\frac{126 - 20 \cos(2\phi)}{225} \right) + C_A^2 \left(\frac{882 + 10 \cos(2\phi)}{225} \right) + C_F n_f T_F \frac{3}{5}$$

H. Chen, M.X. Luo, Moult, T.Z. Yang, X.Y. Zhang, HXZ, 2019



RG evolution for spin interference

$$\mathcal{E}(\vec{n}_1) \cdot \mathcal{E}(\vec{n}_2) \sim \sum_i c_i \theta^{\tau_i - 4} \mathbb{O}_i(\vec{n}_2)$$

$$\mathcal{O}_q^{[J]} = \frac{1}{2^J} \bar{\psi} \gamma^+ (iD^+)^{J-1} \psi$$

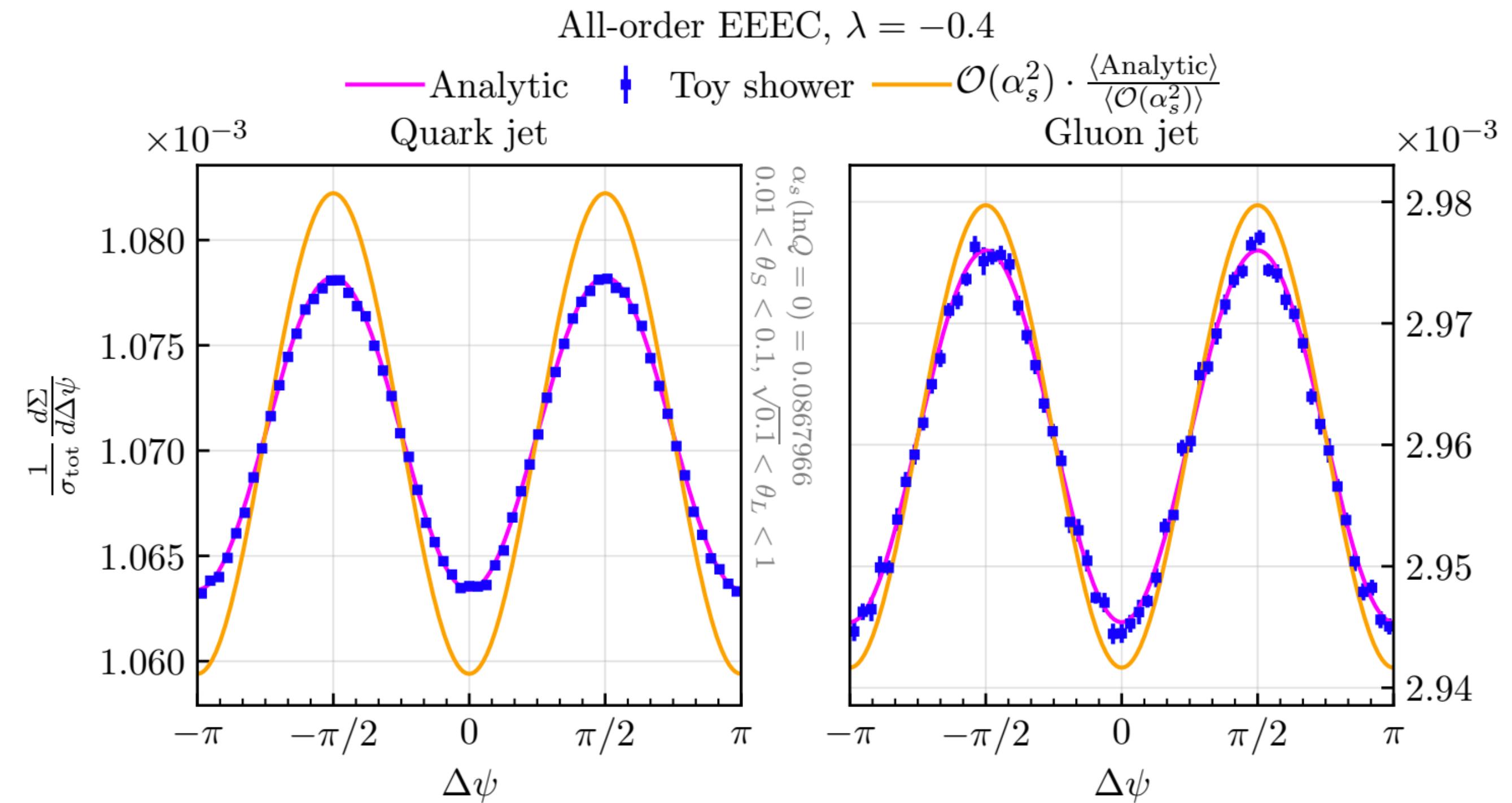
$$\mathcal{O}_g^{[J]} = -\frac{1}{2^J} F_a^{\mu+} (iD^+)^{J-2} F_a^{\mu+}$$

$$\mathcal{O}_{\tilde{g}, \lambda}^{[J]} = -\frac{1}{2^J} F_a^{\mu+} (iD^+)^{J-2} F_a^{\nu+} \epsilon_{\lambda, \mu} \epsilon_{\lambda, \nu}$$

helicity \pm

$$\frac{d}{d \ln \mu^2} \vec{\mathcal{O}}^{[J]} = -\hat{\gamma}(J) \cdot \vec{\mathcal{O}}^{[J]}$$

First all-order analytic results with spin correlation in the scaling limit



Explicitly verified by a Monte Carlo simulation incorporating spin interference
 Open the door for spin probes of new physics at the LHC

But Lorentz symmetry has a more profound implication than JUST rotation and scaling

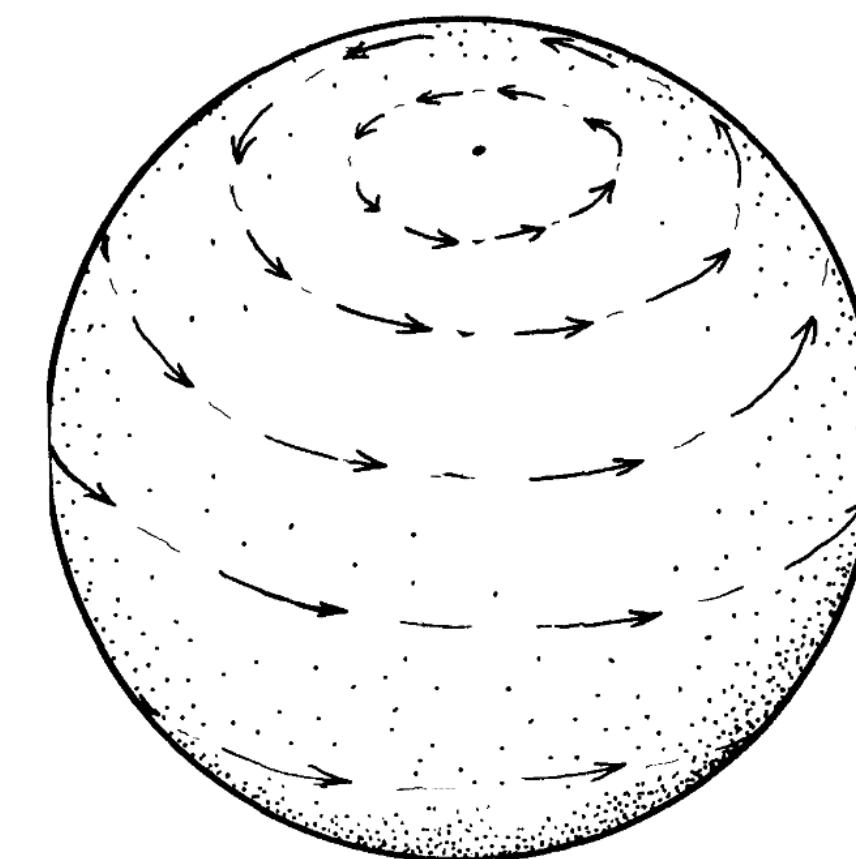


Fig. 1-6. The effect of a rotation on S^+ .

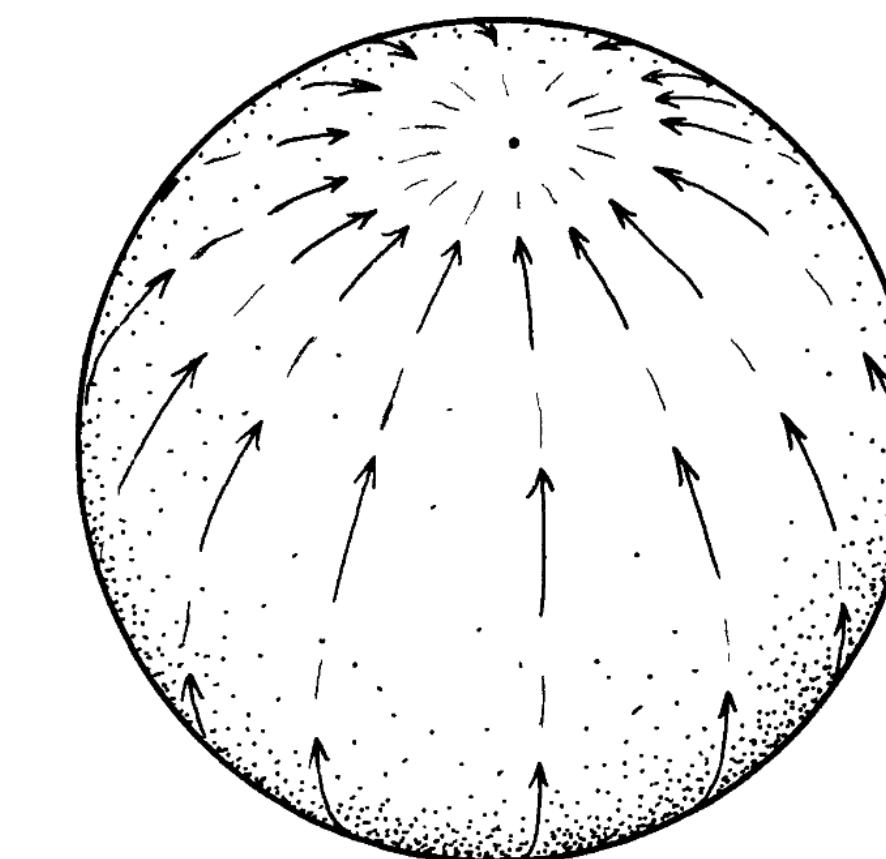


Fig. 1-7. The effect of a boost on S^+ .

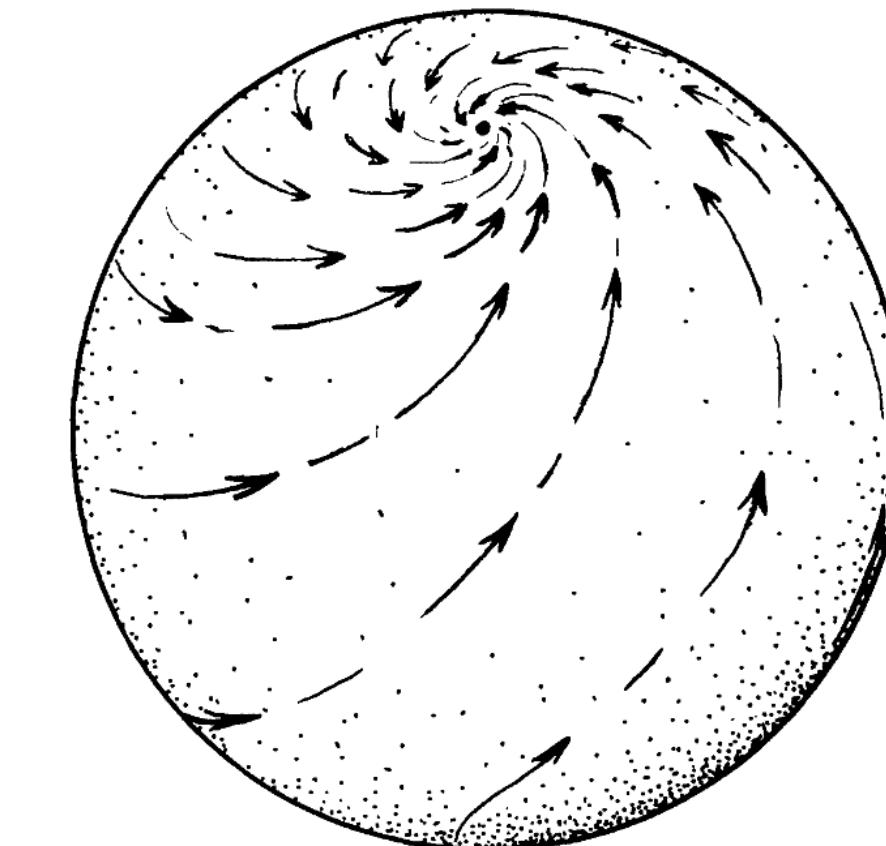
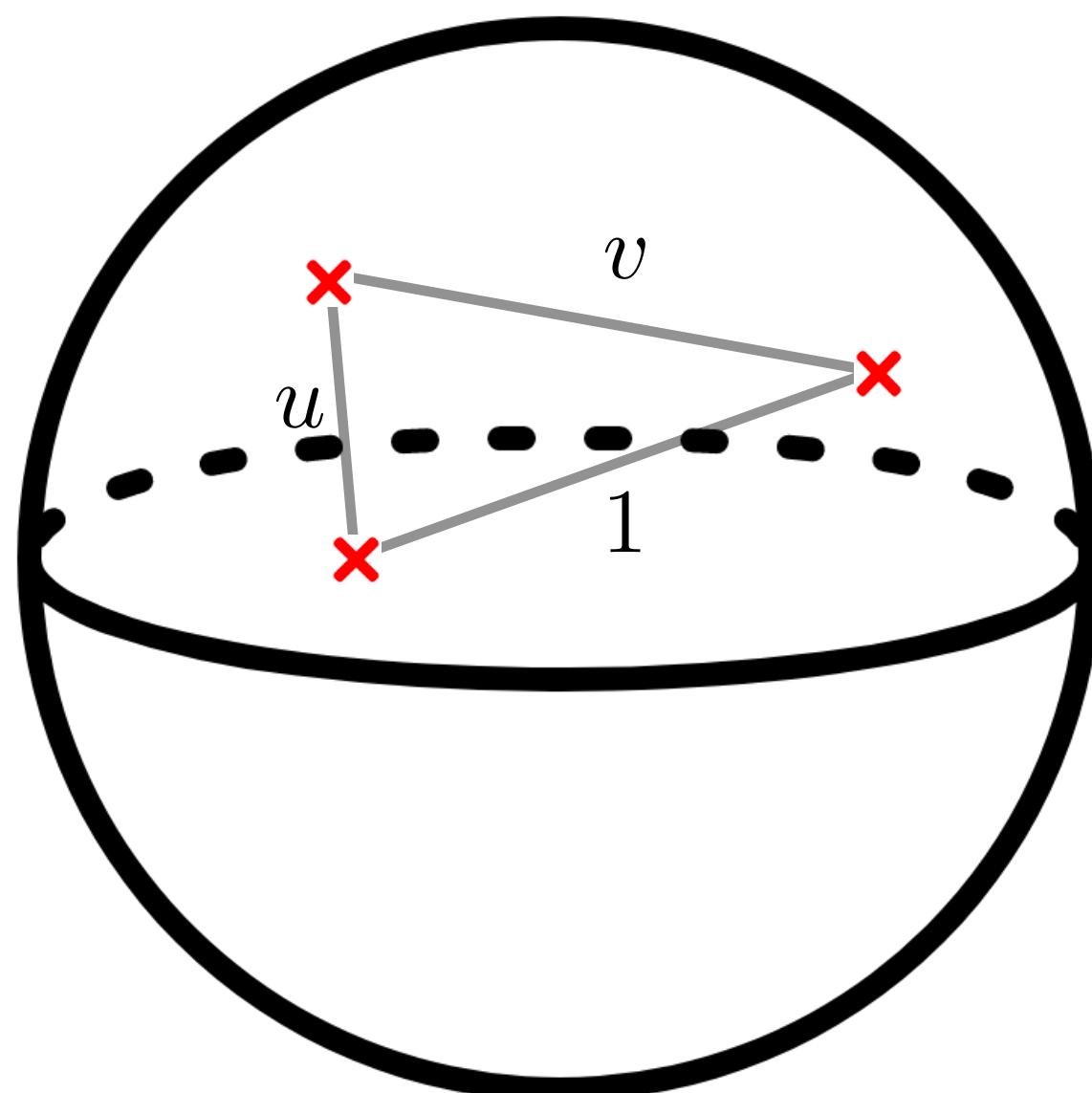


Fig. 1-8. The effect of a four-screw on S^+ .

Three-point energy correlator in the weak coupling limit



Weak coupling limit



$$u \rightarrow 0$$

Scaling limit:

$$z \rightarrow 0$$

H. Chen, M.X. Luo, Moult, T.Z. Yang, X.Y. Zhang, HXZ, 2019

$$\begin{aligned} G_{\mathcal{N}=4}(z) = & \frac{1+u+v}{2uv}(1+\zeta_2) - \frac{1+v}{2uv}\log(u) - \frac{1+u}{2uv}\log(v) \\ & - (1+u+v)(\partial_u + \partial_v)\Phi(z) + \frac{(1+u^2+v^2)}{2uv}\Phi(z) + \frac{(z-\bar{z})^2(u+v+u^2+v^2+u^2v+uv^2)}{4u^2v^2}\Phi(z) \\ & + \frac{(u-1)(u+1)}{2uv^2}D_2^+(z) + \frac{(v-1)(v+1)}{2u^2v}D_2^+(1-z) + \frac{(u-v)(u+v)}{2uv}D_2^+\left(\frac{z}{z-1}\right), \end{aligned}$$

$$u = z\bar{z}$$

$$v = (1-z)(1-\bar{z})$$

$$\Phi(z) = \frac{2}{z-\bar{z}} \left(\text{Li}_2(z) - \text{Li}_2(\bar{z}) + \frac{1}{2} (\log(1-z) - \log(1-\bar{z})) \log(z\bar{z}) \right)$$

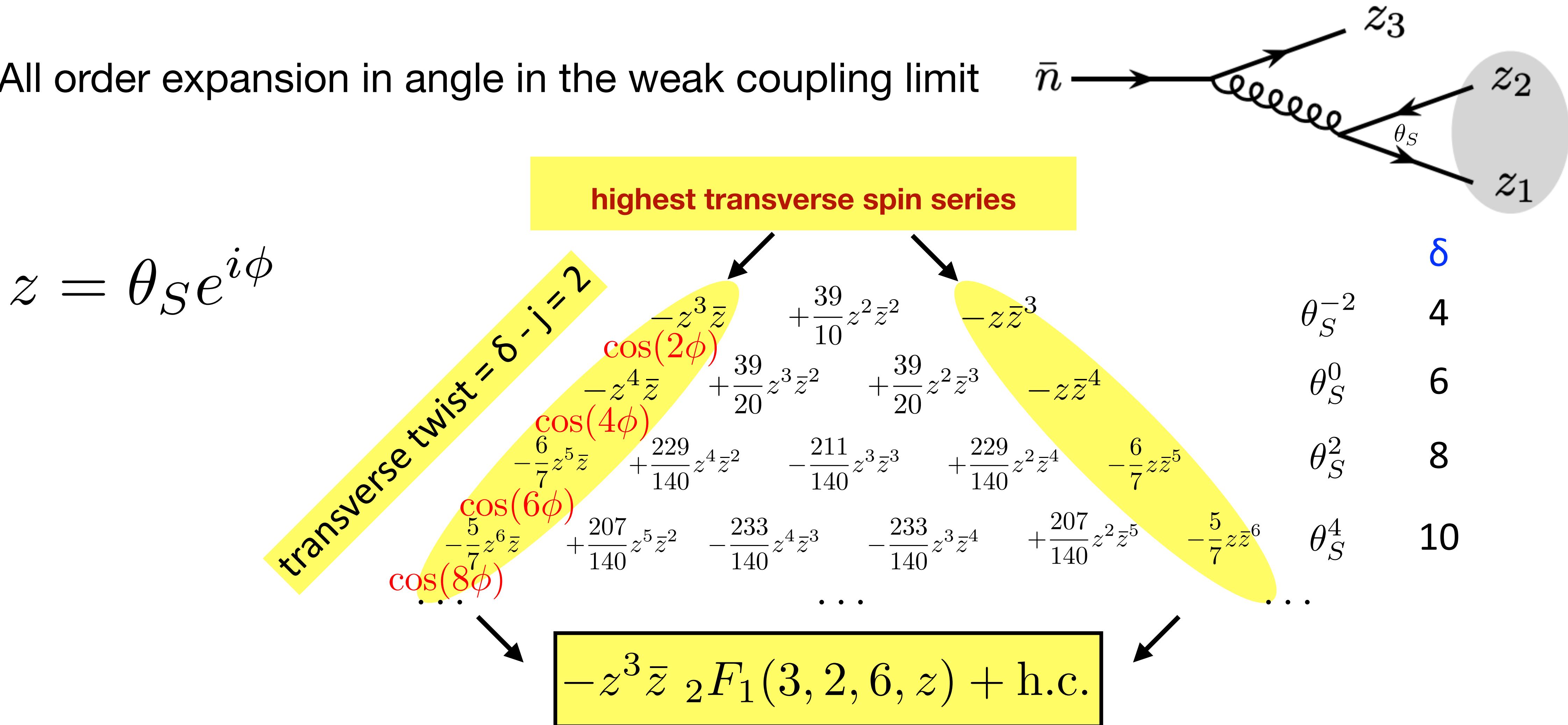
$$D_2^+(z) = \text{Li}_2(1-|z|^2) + \frac{1}{2} \log(|1-z|^2) \log(|z|^2)$$

QCD is more complicated and structurally the same

Resemble a conformal 4-pt correlation function

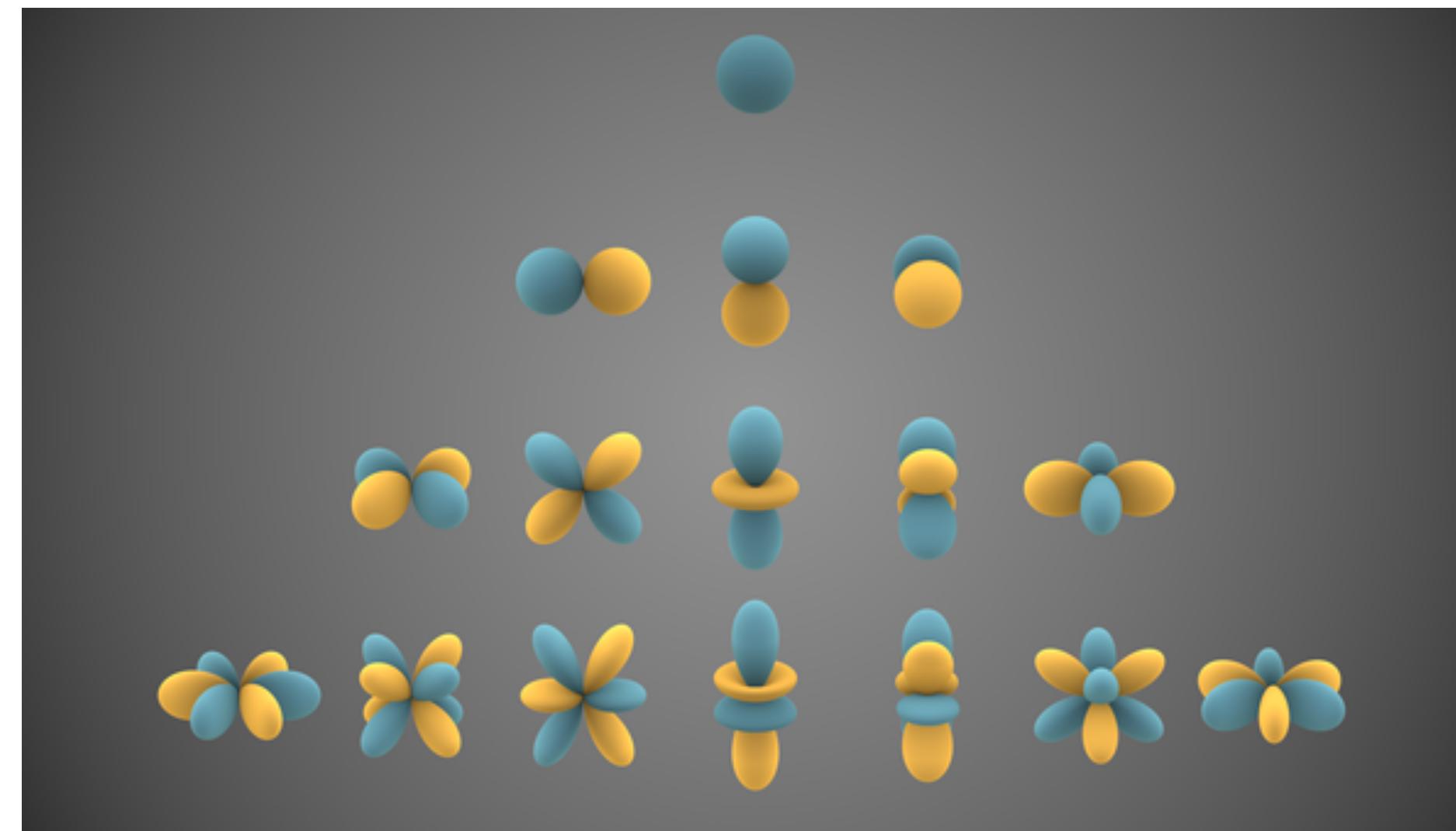
Power expansion in the scaling limit

All order expansion in angle in the weak coupling limit



Remarkably simple structure that calls for a better organization of the expansion

Two-dimensional conformal block expansion



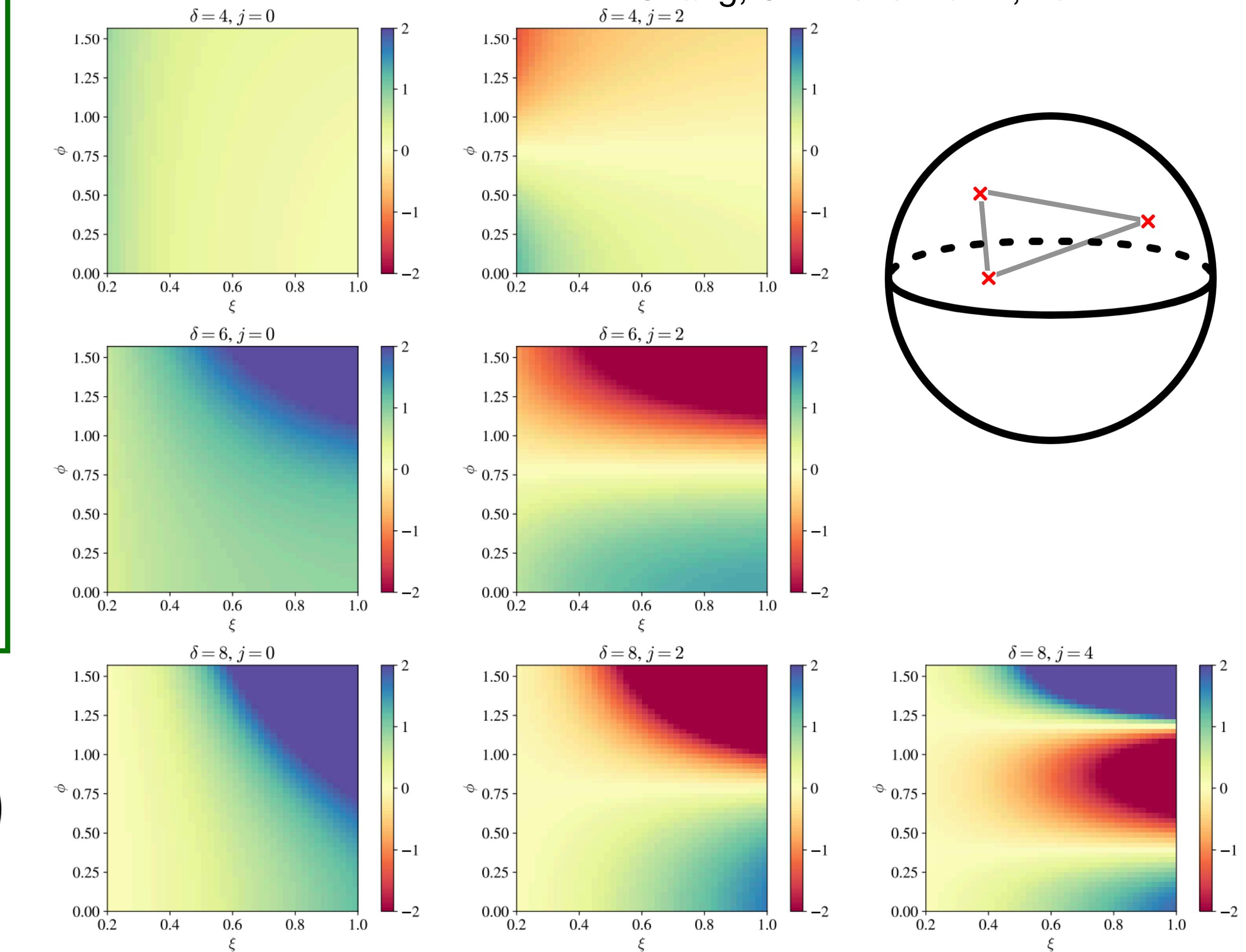
non-relativistic spherical harmonic expansion

$$G_{\delta,j}(u,v) \equiv G_{\delta,j}(z,\bar{z}) = \frac{1}{1 + \delta_{j,0}} \left(k_{\frac{\delta-j}{2}}(z) k_{\frac{\delta+j}{2}}(\bar{z}) + k_{\frac{\delta+j}{2}}(z) k_{\frac{\delta-j}{2}}(\bar{z}) \right)$$

$$k_h(x) \equiv x^h {}_2F_1(h+a, h+b, 2h, x)$$

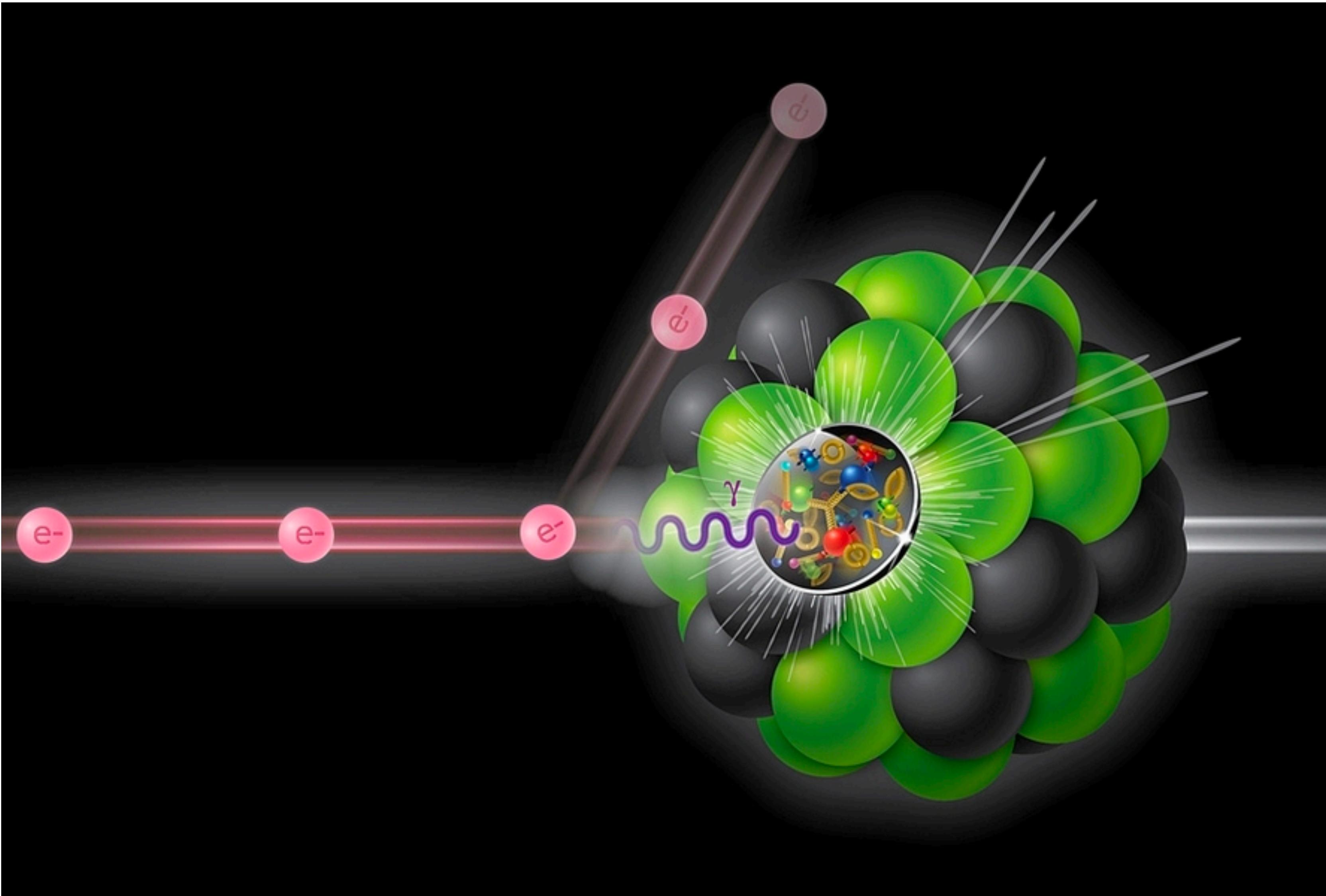
Dolan, Osborn

H. Chen, Moult, Sandor, HXZ, 2022
Chang, Simmons-Duffin, 2022

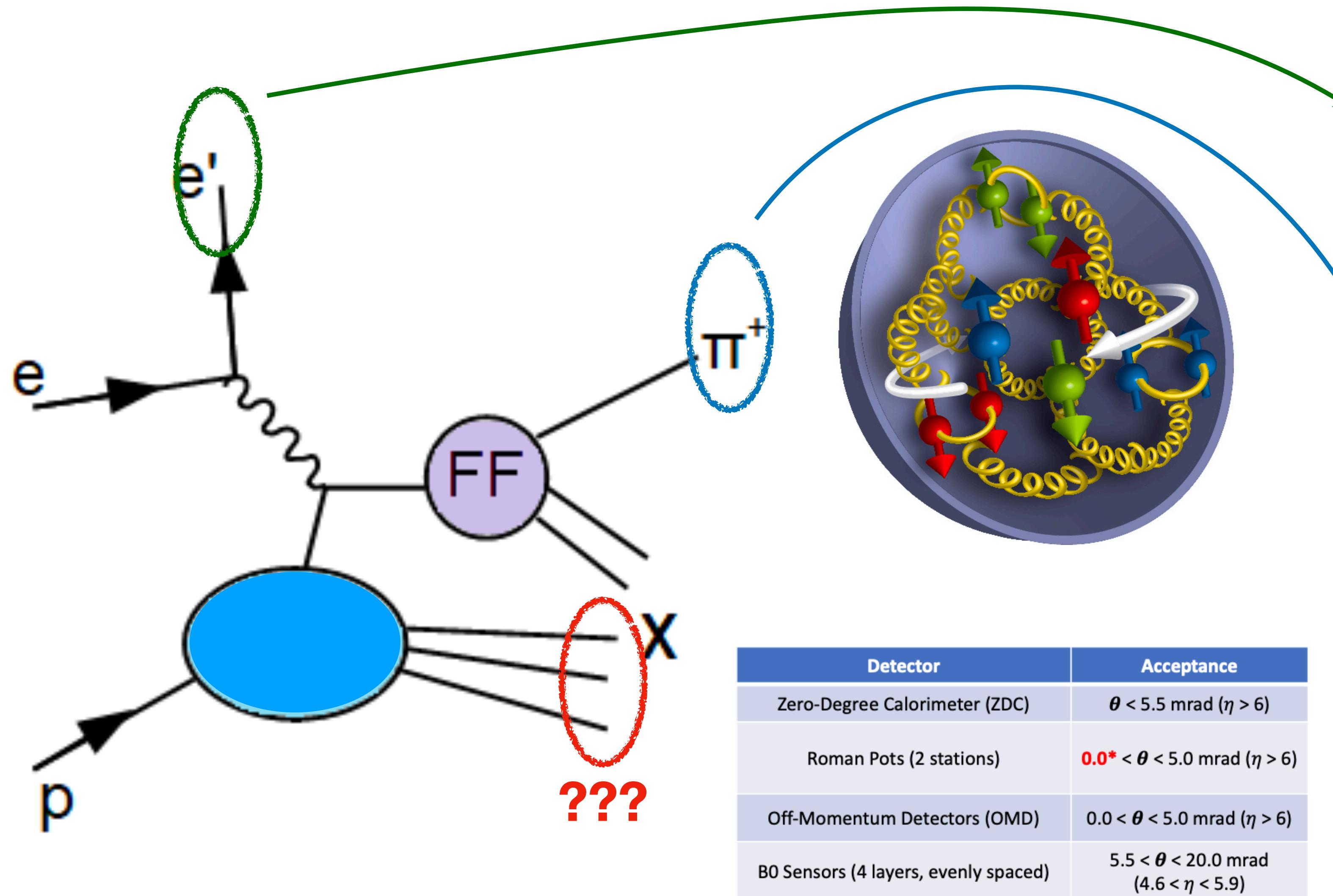


Celestial expansion using conformal block

Energy correlators for jets at the EIC/EICC



ep and nucleon structure

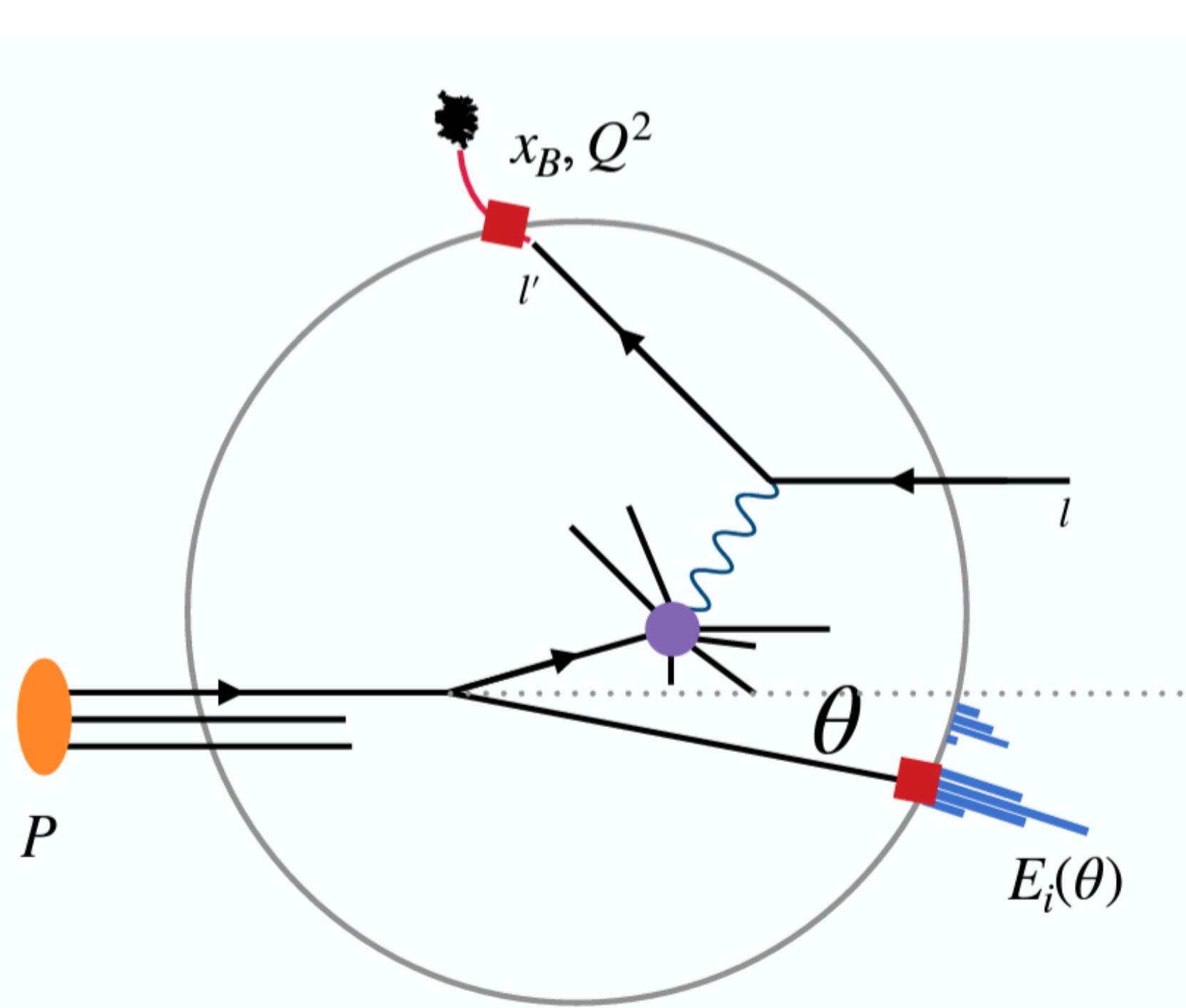


- Structure function measurement: PDFs(x)
- SIDIS:
 - TMD
 - spin
- How can we utilize the forward information and what does it probes?

The nucleon EEC

X.H. Liu, HXZ, 2022

Energy weighted correlation of forward hadron with beam



$$f_{\text{EEC}}(x, \theta) = \int_{-\infty}^{\infty} \frac{dy^-}{2\pi} e^{ixp^+y^-} \frac{\gamma^+}{2} \langle P | \bar{\psi}(0) \mathcal{E}(\theta) \psi(y^-) | P \rangle$$

Insertion of energy flow operator between lightcone separated field
Naturally generalize to N energy flow operator insertion

Compare with

collinear PDF: $f_q(x) = \int_{-\infty}^{\infty} \frac{dy^-}{2\pi} e^{ixp^+y^-} \frac{\gamma^+}{2} \langle P | \bar{\psi}(0) \psi(y^-) | P \rangle$

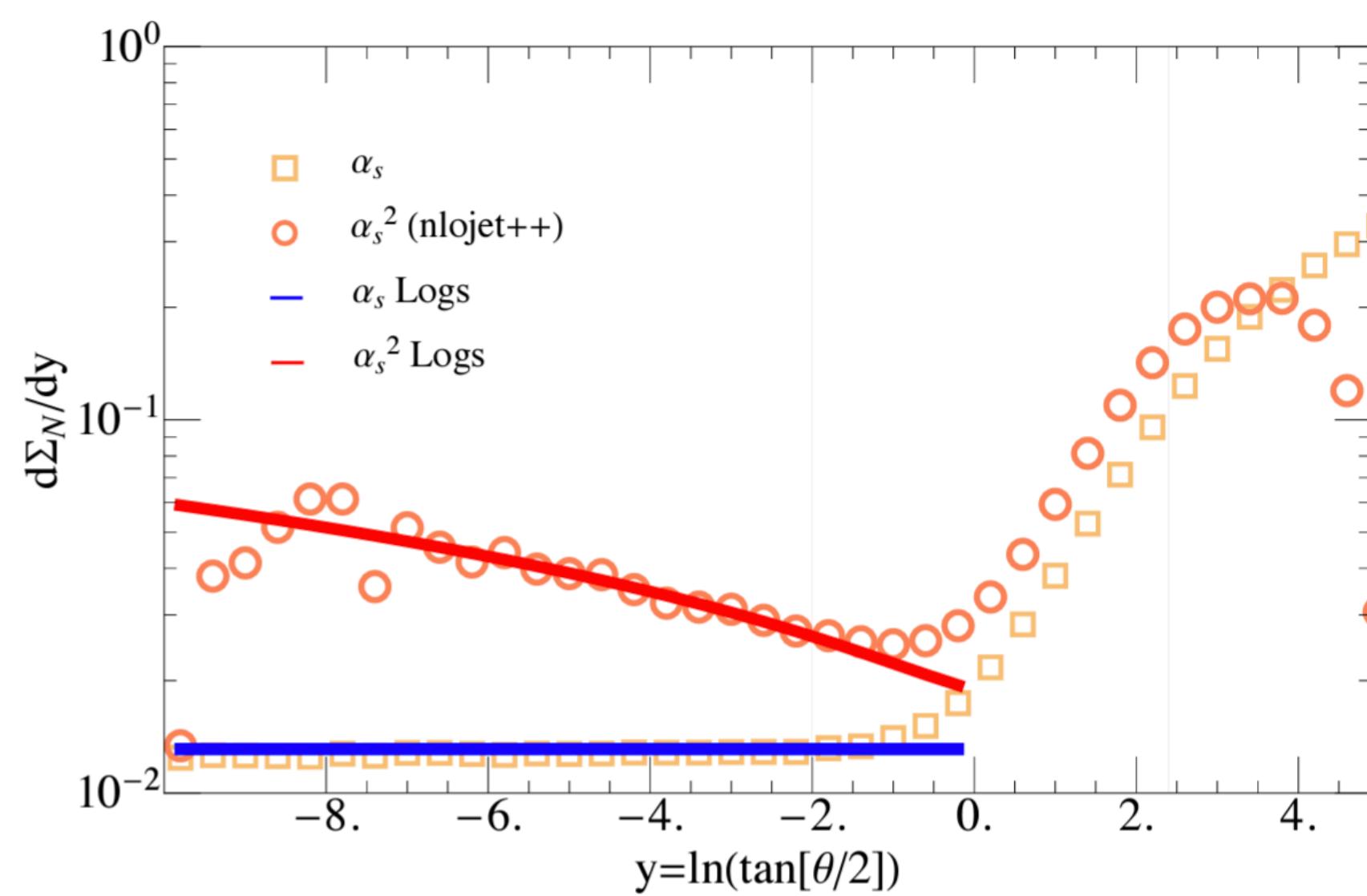
TMD PDF: $f_q(x) = \int_{-\infty}^{\infty} \frac{dy^-}{2\pi} e^{ixp^+y^-} \frac{\gamma^+}{2} \langle P | \bar{\psi}(0) \psi(y_\perp, y^-) | P \rangle$

What can we learn from the nucleon EEC?

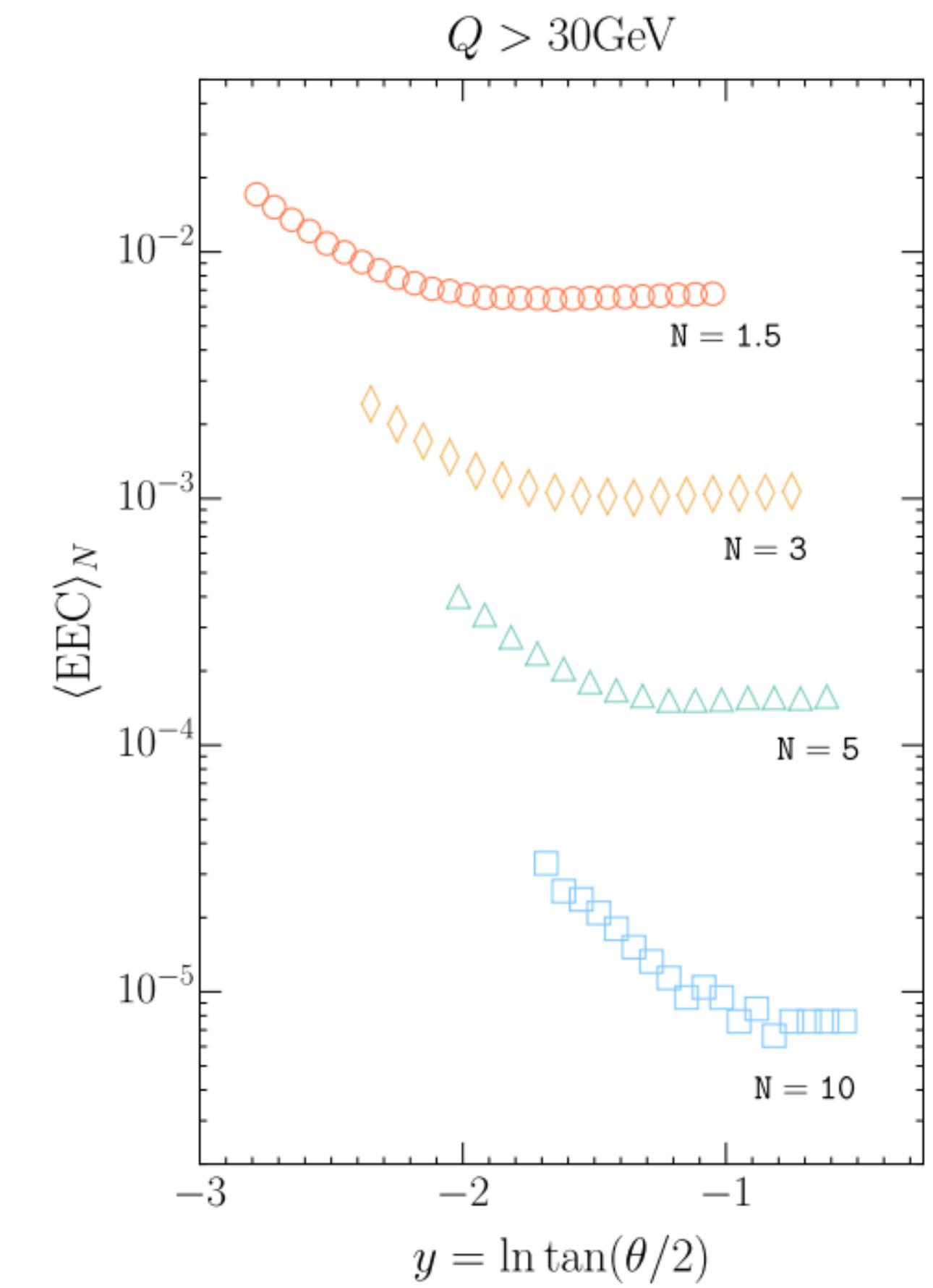
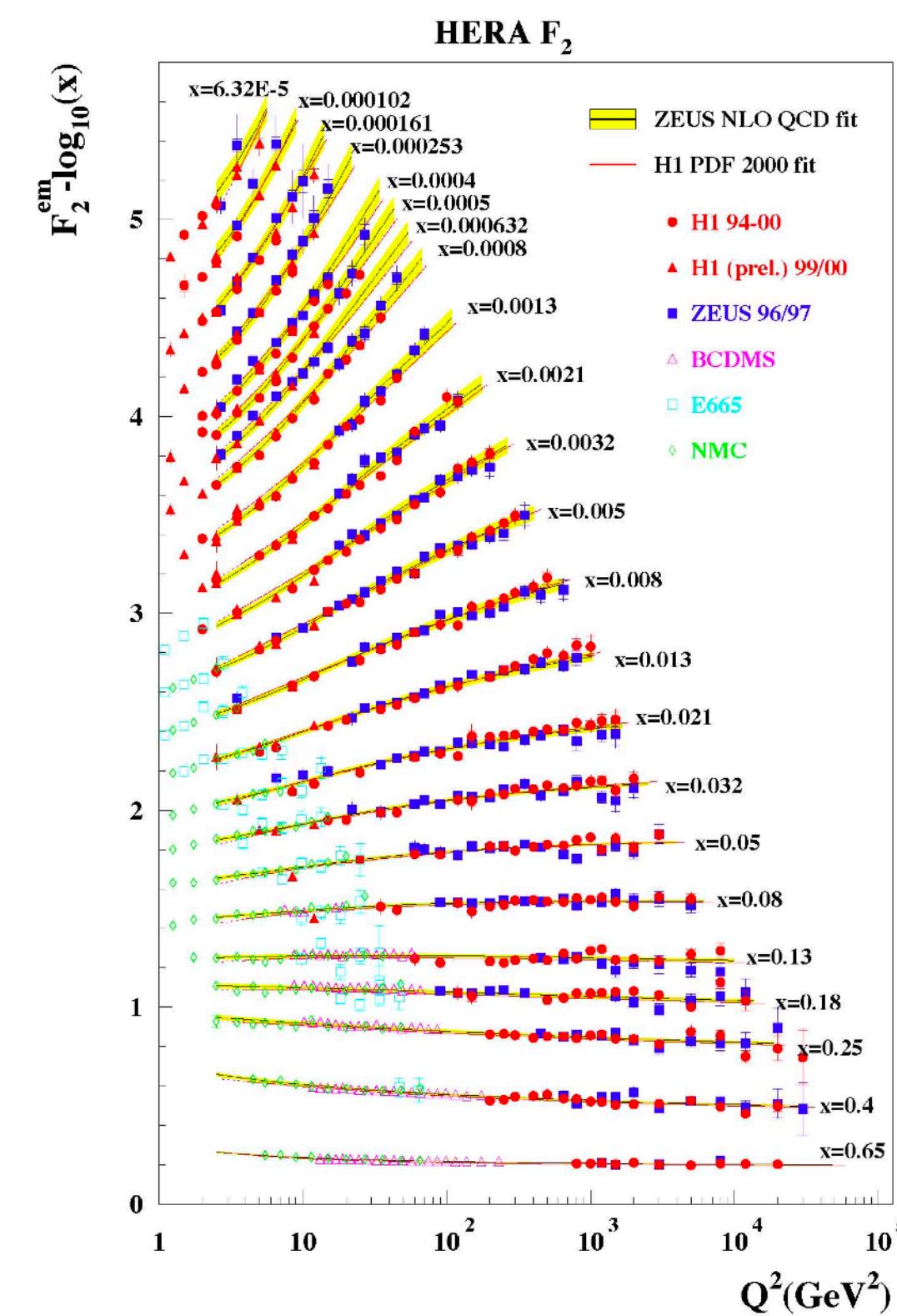
Modified DGLAP evolution

X.H. Liu, HXZ, 2022
 H.T. Cao, Liu, HXZ, 2023

$$\frac{d}{d \ln \mu^2} f_{i,\text{EEC}}(N, \ln \frac{Q\theta}{u\mu}) = \sum_j \int d\xi \xi^{N-1} P_{ij}(\xi) f_{j,\text{EEC}}(N, \ln \frac{Q\theta}{\xi u\mu})$$

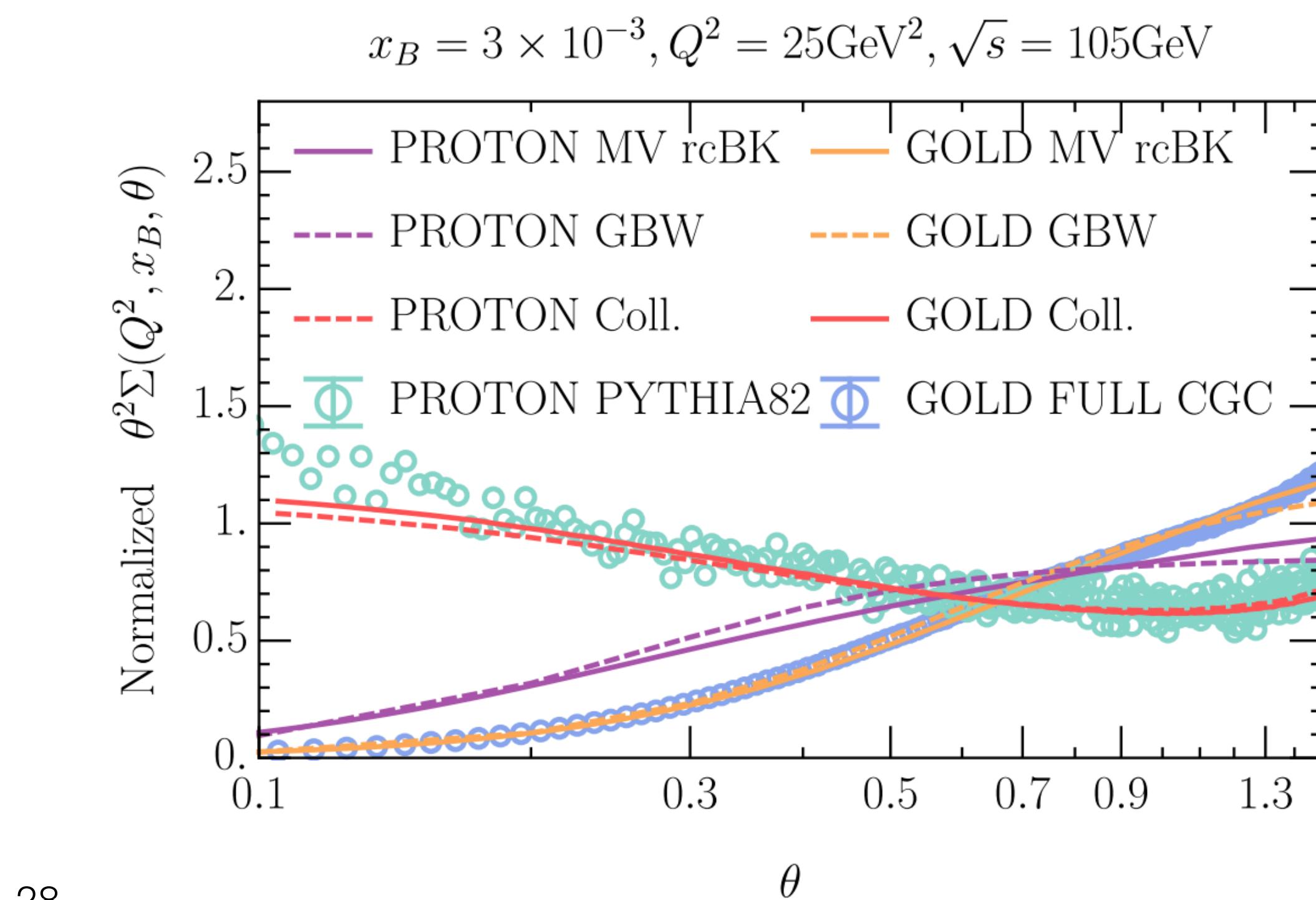
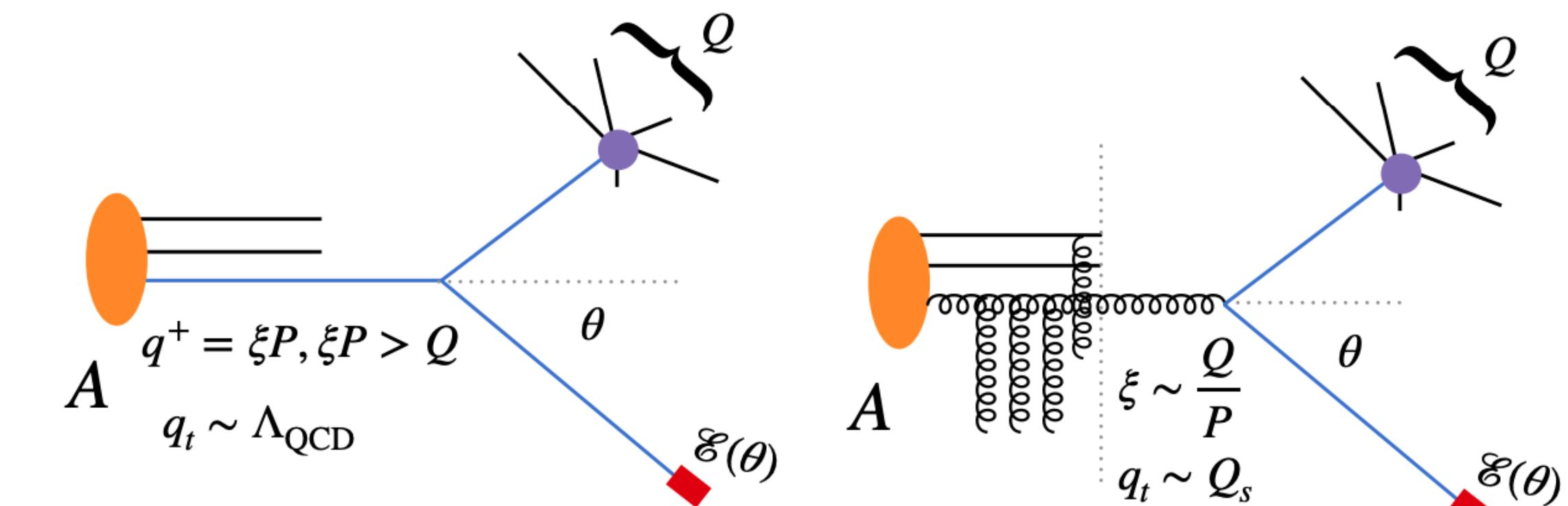
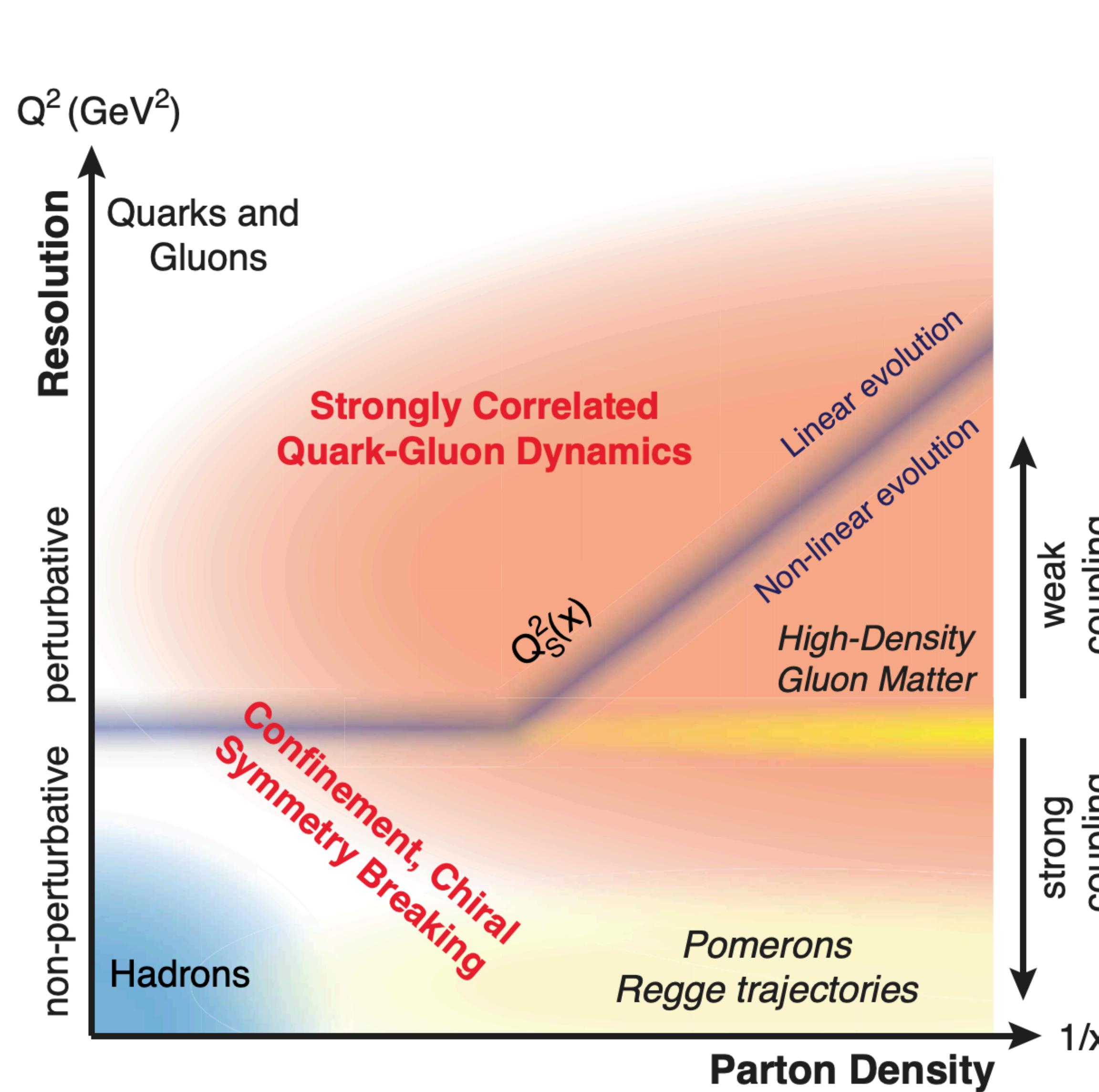


Bjorken scaling and scaling violation, not in Q evolution but in angle!

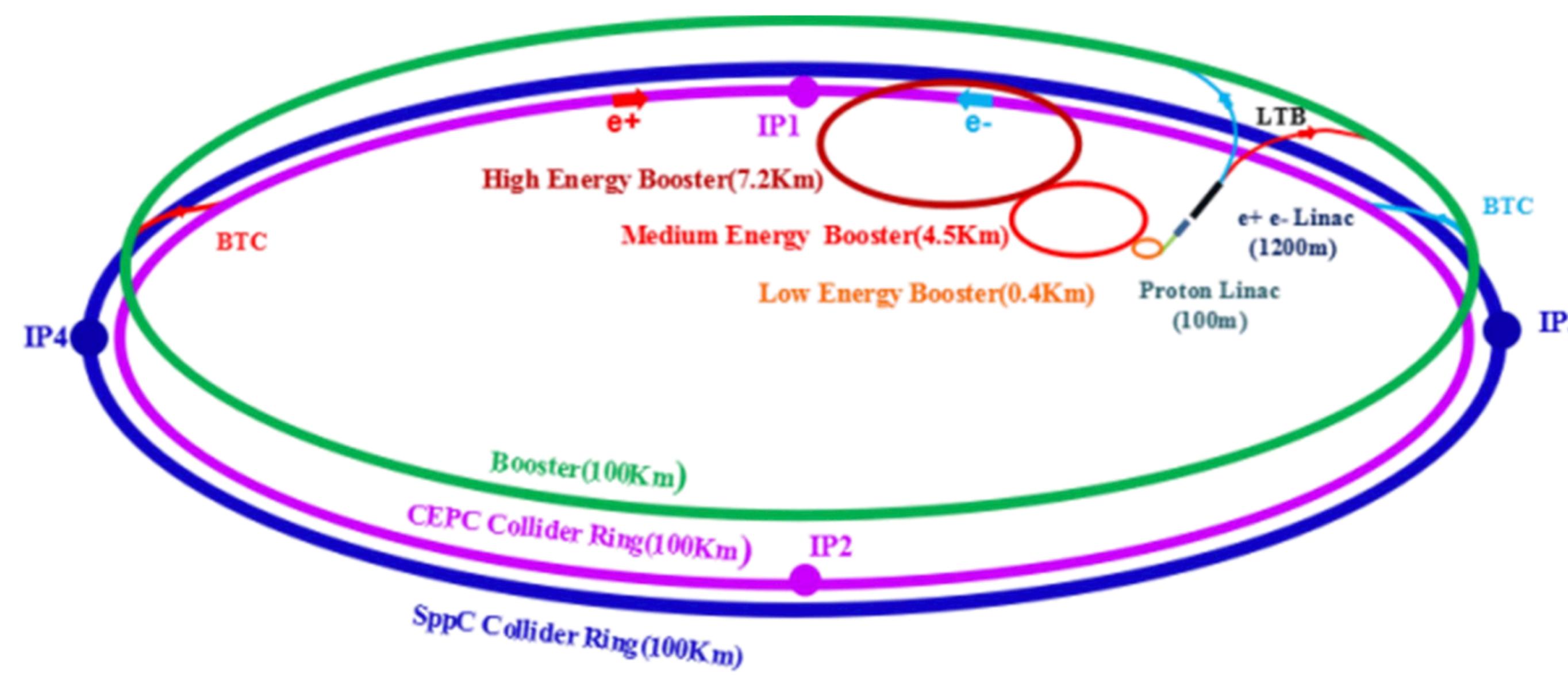


Probing gluon saturation

H.Y. Liu, X.H. Liu, J.C. Pan, F. Yuan, HXZ, 2023



Energy correlators for jets at the CEPC/FCC-ee

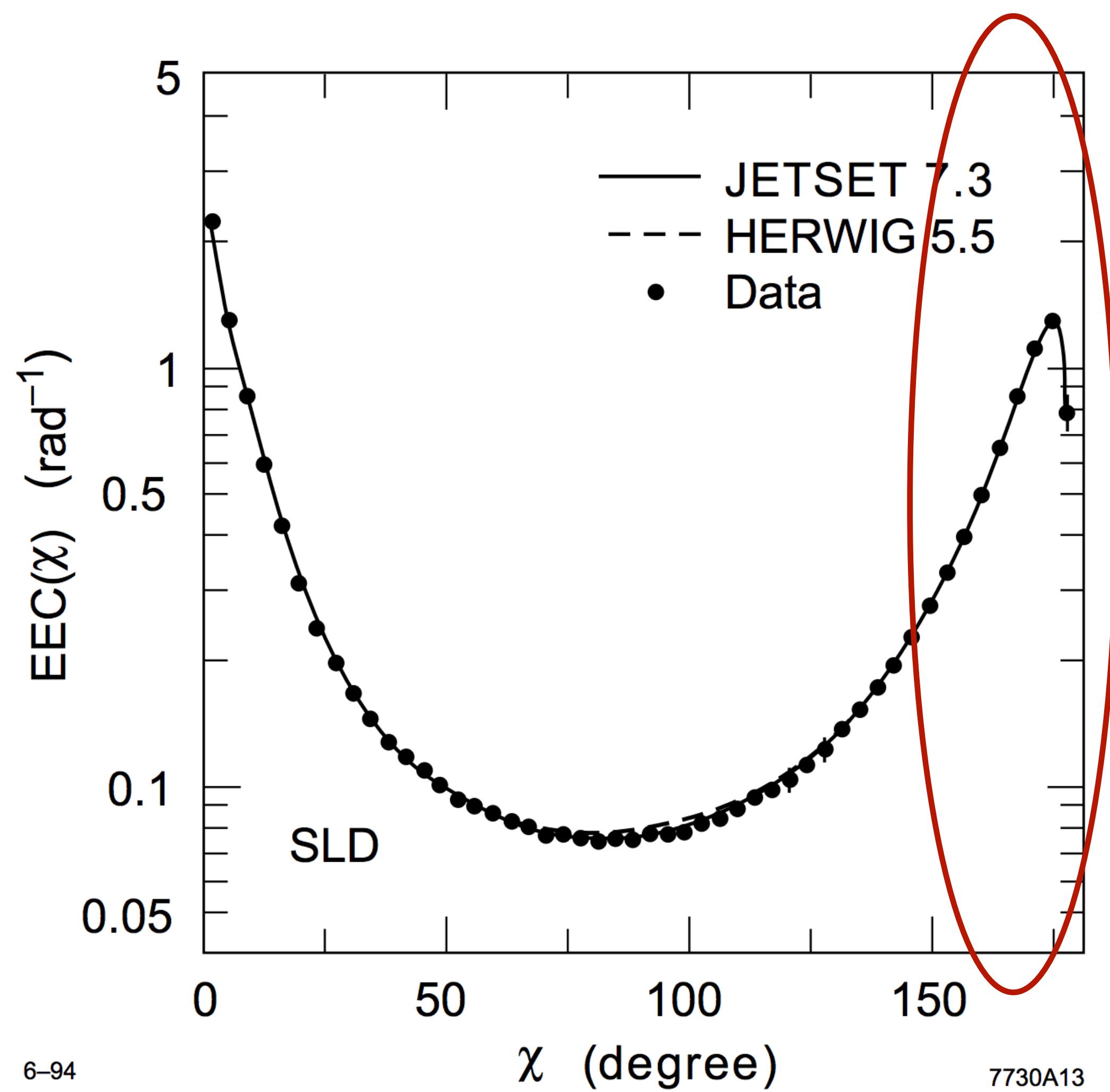


CEPC-SppC accelerator layout

LTB : Linac to Booster

BTC : Booster to Collider Ring

Sudakov logarithms in EEC



Double logarithmic series in perturbation theory

$$\text{EEC}(\chi) = \alpha_s L^2 + \alpha_s^2 L^4 + \alpha_s^3 L^6 + \dots$$

$$L = \ln \left(\frac{1 + \cos \chi}{2} \right)$$

Emergent Sudakov peak structure

Similar structure appears in many different contexts: Sudakov form factor, threshold resummation, TMD resummation, ...

Resummation in SCET

$$\text{EEC}(\chi) = H J^2 S$$

moment of TMD fragmentation function

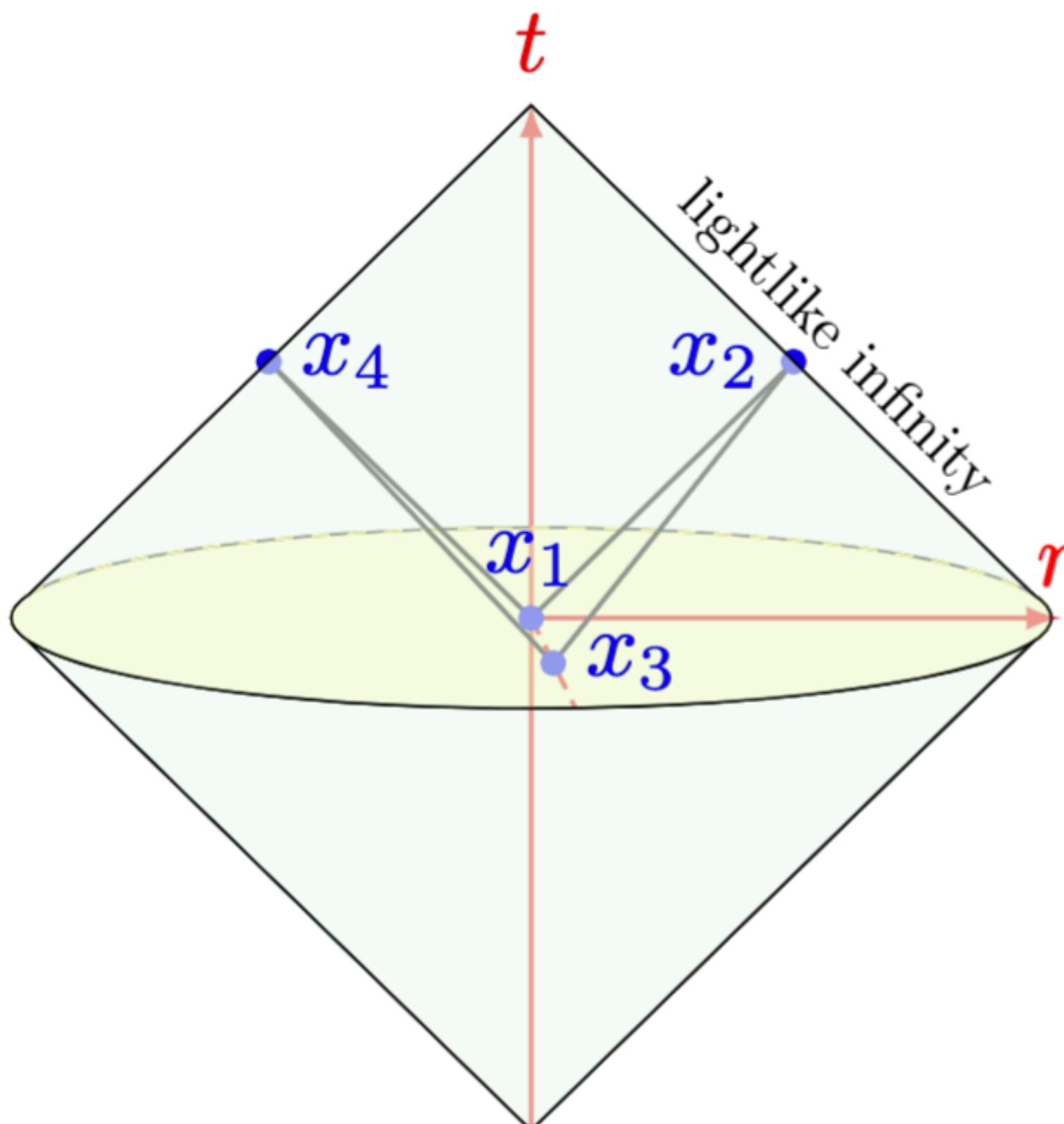
TMD soft function

Novel method for Sudakov resummation in EEC

H. Chen, X.N. Zhou, HXZ, 2023

EEC can be written as a four point Wightman correlation function

$$\text{EEC}(y) = \frac{8\pi^2}{q^2\sigma_0} \int d^4x e^{iq \cdot x_{13}} \langle J^\mu(x_1) \mathcal{E}(n_2) \mathcal{E}(n_4) J_\mu^\dagger(x_3) \rangle$$



$$u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2} = z\bar{z}, \quad v = \frac{x_{23}^2 x_{14}^2}{x_{13}^2 x_{24}^2} = (1-z)(1-\bar{z})$$

Double lightcone limit: $u \rightarrow 0, v \rightarrow 0$ ($z \rightarrow 0, \bar{z} \rightarrow 1$)

$$\text{one loop} = \left[-\frac{1}{4} \log u \log v + 0 \cdot \log(uv) + \dots \right] - \left[\frac{1}{4}(u+v) \log u \log v + \frac{1}{2}(u \log u + v \log v) + \dots \right] + \dots,$$

$$\begin{aligned} \text{two loop} = & \left[\frac{1}{16} \log^2 u \log^2 v + 0 \cdot \log u \log v \log(uv) + \dots \right] + \left[\frac{1}{8}(u+v) \log^2 u \log^2 v \right. \\ & \left. + \frac{3}{16} \log u \log v (u \log u + v \log v) + \frac{1}{8} \log u \log v (v \log u + u \log v) + \dots \right] + \dots, \end{aligned}$$

$$\text{three loop} = \left[-\frac{1}{96} \log^3 u \log^3 v + 0 \cdot \log^2 u \log^2 v \log(uv) + \dots \right] - \left[\frac{1}{48}(u+v) \log^3 u \log^3 v \right]$$

Origin of Sudakov double logarithms

H. Chen, X.N. Zhou, HXZ, 2023

$$\langle J_\mu(x_1) T^{\rho\sigma}(x_2) T^{\lambda\kappa}(x_4) J^\mu(x_3) \rangle$$

S-channel OPE

$$\sum_{\tau, l} \text{truncated in } \tau, \text{ but infinite in } l = \sum_{\tau, l} a_{\tau, l} G_{\Delta, l}(x_1, x_2, x_3, x_4)$$

partial wave decomposition of 4-pt function

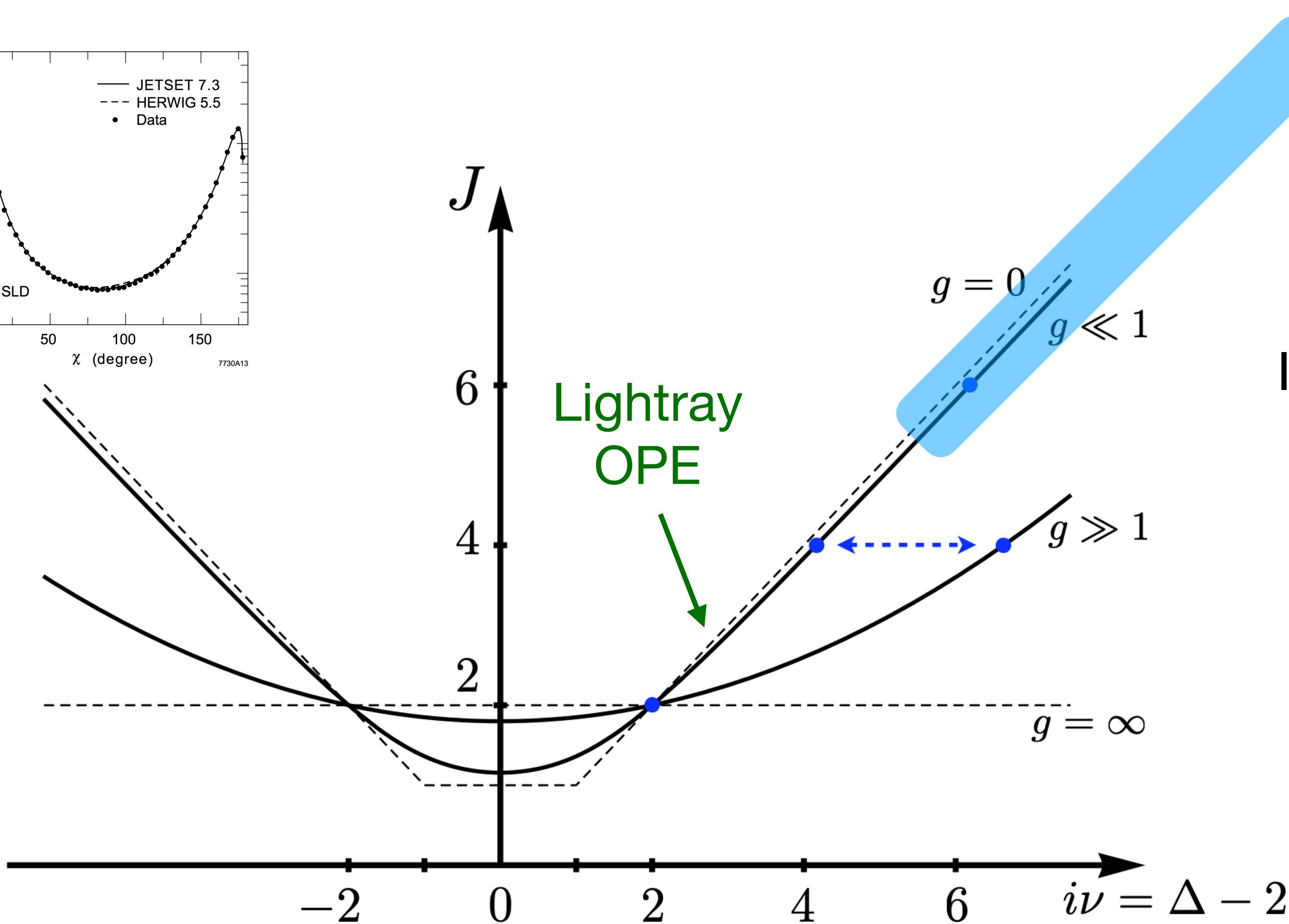
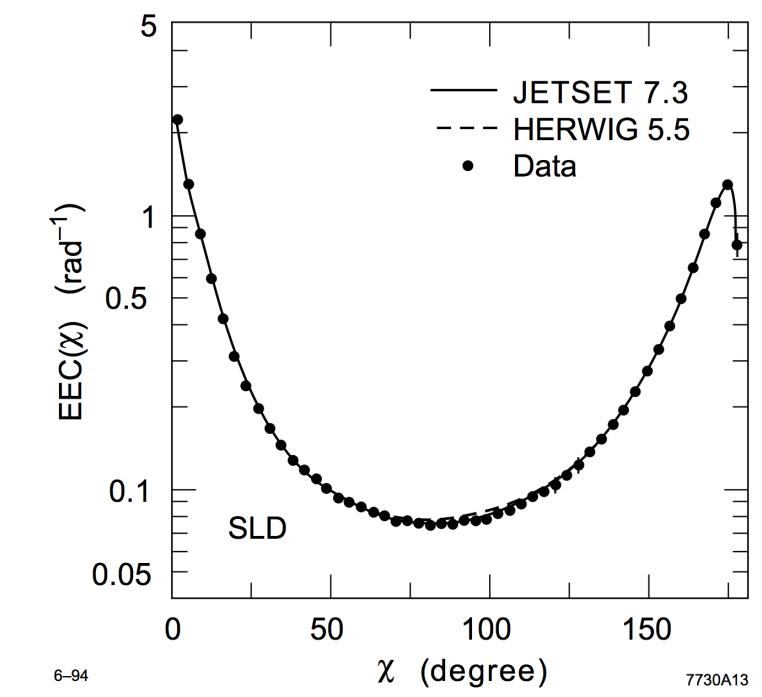
4-pt conformal block

$$G_{\Delta, \ell}(u, v) = \frac{z\bar{z}}{\bar{z} - z} [k_{\Delta-\ell-2}(z)k_{\Delta+\ell}(\bar{z}) - (z \leftrightarrow \bar{z})]$$

$$k_\beta(x) = x^{\beta/2} {}_2F_1(\beta/2, \beta/2; \beta; x)$$

- Logarithmic divergence in $u > 0$: anomalous dimension of $\mathcal{O}_{\Delta, l}$ \Leftrightarrow collinear divergence
- Logarithmic divergence in $v > 0$: sum over infinite number of $\mathcal{O}_{\Delta, l}$ \Leftrightarrow soft divergence

role of analyticity in spin



Back-to-back EEC
Bulk OPE

Infinite spin summation: Casimir equation

$$\mathcal{C}_\tau G_{\Delta,l}(z, \bar{z}) = J_{\tau,l}^2 G_{\Delta,l}(z, \bar{z})$$

$$J_{\tau,\ell}^2 = (\ell + \frac{\tau}{2})(\ell + \frac{\tau}{2} - 1)$$

Analyticity in spin plays crucial role in understanding small angle and large angle expansion!

Resummation of EEC in N=4 SYM

H. Chen, X.N. Zhou, HXZ, 2023

$$\text{EEC}(y) \sim \sum_{n=1}^{\infty} \sum_{m=0}^{2n-1} \alpha_s^n \left(c_{n,m} \frac{\log^m y}{y} + d_{n,m} \log^m y \right)$$

“NLL”: $m \geq 2n - 2$

	Power Corrections		Perturbative Corrections	
	twist	large spin	LL	“NLL”
LP	2	$\mathcal{O}(\ell^0)$	$a_{2,\ell}^{(0)}, \gamma_{2,\ell}^{(1)}$	$a_{2,\ell}^{(1)}, \gamma_{2,\ell}^{(2)}$
NLP	2	$\mathcal{O}(\ell^{-2})$	$a_{2,\ell}^{(0)}, \gamma_{2,\ell}^{(1)}$	$a_{2,\ell}^{(1)}, \gamma_{2,\ell}^{(2)}$
	4	$\mathcal{O}(\ell^0)$	$a_{4,\ell}^{(0)}, \gamma_{4,\ell}^{(1)}$	$a_{4,\ell}^{(1)}, \gamma_{4,\ell}^{(2)}$

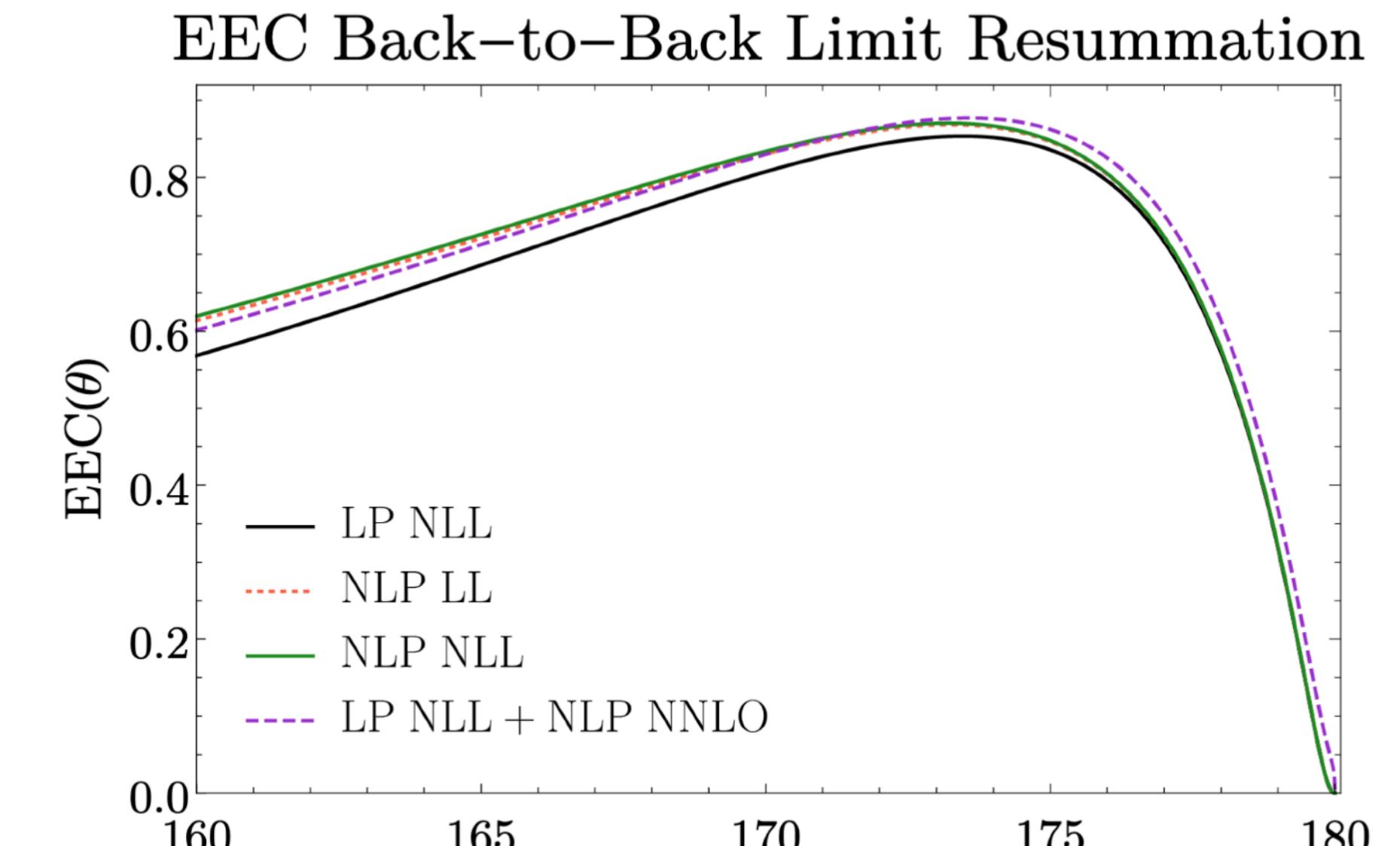
$$\text{EEC}(y) = -\frac{aL_y e^{-\frac{aL_y^2}{2}}}{4y} - \frac{1}{4} \left[\sqrt{\frac{\pi}{2}} \sqrt{a} \operatorname{erf} \left(\sqrt{\frac{a}{2}} L_y \right) + aL_y e^{-\frac{aL_y^2}{2}} \right] + \frac{a}{48} (7aL_y^2 - 4)e^{-\frac{aL_y^2}{2}} + \frac{a}{12} + \dots$$

$$y = \frac{1 + \cos \chi}{2}$$

$$\operatorname{erf}(z) = \frac{2}{\sqrt{\pi}} \int_0^z e^{-t^2} dt$$

Results disagree with a previous calculation based on factorization ansatz in SCET !

I. Moult, G. Vita, K. Yan, 2020



Summary

- Resurgence of interests in EEC and its generalization inspired by conformal collider physics
 - Remarkably rich QCD dynamics can be probed by EECs
 - Scaling behavior in jet evolution and real time hadronization
 - Spinning gluon effects in jet substructure
 - Celestial block expansion of multiple-point EEC
 - Scaling in DIS through angular correlation
 - Probing Gluon saturation
 - Theory: back-to-back expansion in EEC at e+e- through local OPE
-
- The diagram consists of two green curly braces. The top brace groups the first four items in the list: 'Scaling behavior in jet evolution and real time hadronization', 'Spinning gluon effects in jet substructure', 'Celestial block expansion of multiple-point EEC', and 'Scaling in DIS through angular correlation'. To the right of this group is the text 'LHC/e+e-'. The bottom brace groups the last three items: 'Probing Gluon saturation', 'Theory: back-to-back expansion in EEC at e+e- through local OPE', and 'LHC/e+e-'. To the right of this group is the text 'EIC/EICC'.

Energy Correlators at the Collider Frontier

Jul 8–19, 2024

MITP - Mainz Institute for Theoretical Physics, Johannes Gutenberg University Mainz
Europe/Berlin timezone

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Some keys areas of focus include:

- Identifying new/unique phenomenological applications
- Developing techniques for computations of energy correlators in QCD
- Extending the links between EECs in conformal theories and EECs in QCD.
- Finding synergy between jet physics and heavy-ion physics within the EEC framework.
- Identifying how recent EEC developments can feedback into broader collider phenomenology and Monte Carlo generators.

<https://indico.mitp.uni-mainz.de/event/358/>

Thank you very much for your attention!