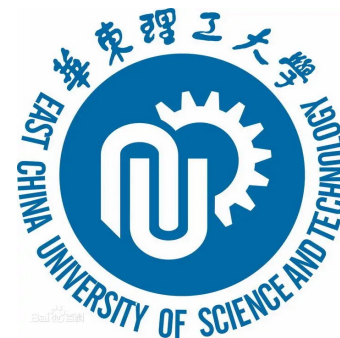




QED Contributions to $\Xi_c - \Xi_c'$ mixing

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2023年11月4日

Cooperated with Zhi-Fu Deng, Jun Zeng, and Wei Wang

arXiv:2309.16386

Outline

- Ξ_c semi-leptonic decays from experiments and theory
- Interpretation by SU(3) breaking effect
- $\Xi_c - \Xi_c'$ mixing from SU(3) breaking
- $\Xi_c - \Xi_c'$ mixing angle in LFQM
- Conclusion

Ξ_c semi-leptonic decays

Deviation between exp & th

Ξ_c semi-leptonic decays

Belle: *Phys.Rev.Lett.* 127 (2021) 12, 121803

$$\mathcal{B}(\Xi_c^0 \rightarrow \Xi^- e^+ \nu_e) = 1.31 \pm 0.04 \pm 0.07 \pm 0.38\%,$$

$$\mathcal{B}(\Xi_c^0 \rightarrow \Xi^- \mu^+ \nu_\mu) = 1.27 \pm 0.06 \pm 0.1 \pm 0.37\%,$$

$$\mathcal{B}(\Xi_c^0 \rightarrow \Xi^- e^+ \nu_e) / \mathcal{B}(\Xi_c^0 \rightarrow \Xi^- \mu^+ \nu_\mu) = 1.03 \pm 0.05 \pm 0.01.$$

Alice: *Phys.Rev.Lett.* 127 (2021) 27, 272001

$$\mathcal{B}(\Xi_c^0 \rightarrow \Xi^- e^+ \nu_e) = 2.5 \pm 0.8\%$$

$$\text{from } \begin{cases} \mathcal{B}(\Xi_c^0 \rightarrow \Xi^- e^+ \nu_e) / \mathcal{B}(\Xi_c^0 \rightarrow \Xi^- \pi^+) = 1.38 \pm 0.14 \pm 0.22 \\ \mathcal{B}(\Xi_c^0 \rightarrow \Xi^- \pi^+) = 1.8 \pm 0.7\% \end{cases}$$

Lattice:

$$\mathcal{B}(\Xi_c^0 \rightarrow \Xi^- e^+ \nu_e) = 2.38(0.30)_{\text{stat.}}(0.32)_{\text{syst.}}\%$$

$$\mathcal{B}(\Xi_c^0 \rightarrow \Xi^- \mu^+ \nu_\mu) = 2.29(0.29)_{\text{stat.}}(0.31)_{\text{syst.}}\%$$

$$\mathcal{B}(\Xi_c^+ \rightarrow \Xi^0 e^+ \nu_e) = 7.18(0.90)_{\text{stat.}}(0.98)_{\text{syst.}}\%,$$

$$\mathcal{B}(\Xi_c^+ \rightarrow \Xi^0 \mu^+ \nu_\mu) = 6.91(0.87)_{\text{stat.}}(0.93)_{\text{syst.}}\%$$

Chinese Physics C Vol. 46, No. 1 (2022) 011002

Editors' Suggestion

**First lattice QCD calculation of semileptonic decays
of charmed-strange baryons Ξ_c^***

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Ξ_c semi-leptonic decays

LCSR: *Phys.Rev.D* 104 (2021) 5, 054030

Semileptonic Ξ_c baryon decays in the light cone QCD sum rules

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Eur. Phys. J. A (2012) 48: 2

Light cone QCD sum rules study of the semileptonic heavy Ξ_Q and Ξ'_Q transitions to Ξ and Σ baryons

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QCDSR:

arXiv: [2103.09436](https://arxiv.org/abs/2103.09436) $\mathcal{B}(\Xi_c^0 \rightarrow \Xi^- e^+ \nu_e) = 3.4 \pm 1.7\%$. by Zhen-Xing Zhao

Decay Channel	Present Work
$\Xi_c^0 \rightarrow \Xi^- e^+ \nu_e$	1.85 ± 0.56
$\Xi_c^0 \rightarrow \Xi^- \mu^+ \nu_\mu$	1.79 ± 0.54
$\Xi_c^+ \rightarrow \Xi^0 e^+ \nu_e$	5.51 ± 1.65
$\Xi_c^+ \rightarrow \Xi^0 \mu^+ \nu_\mu$	5.34 ± 1.61
$\Xi_c^+ \rightarrow \Lambda^0 e^+ \nu_e$	0.092 ± 0.028
$\Xi_c^+ \rightarrow \Lambda^0 \mu^+ \nu_\mu$	0.089 ± 0.027

Ξ_c semi-leptonic decays

SU(3) analysis:

Branching ratio	$SU(3)_f$ (a)	$SU(3)_f$ (b)	$SU(3)_f$ (c)
$10^2 \mathcal{B}(\Lambda_c^+ \rightarrow \Lambda e^+ \nu_e)$	3.6 ± 0.4	3.6 ± 0.4	3.2 ± 0.3
$10^2 \mathcal{B}(\Lambda_c^+ \rightarrow \Lambda \mu^+ \nu_\mu)$	3.5 ± 0.5	3.6 ± 0.4	3.2 ± 0.3
$10^2 \mathcal{B}(\Xi_c^+ \rightarrow \Xi^0 e^+ \nu_e)$	11.9 ± 1.3	9.8 ± 1.1	10.7 ± 0.9
$10^2 \mathcal{B}(\Xi_c^+ \rightarrow \Xi^0 \mu^+ \nu_\mu)$	11.6 ± 1.7	9.8 ± 1.1	10.8 ± 0.9
$10^2 \mathcal{B}(\Xi_c^0 \rightarrow \Xi^- e^+ \nu_e)$	3.0 ± 0.3	2.4 ± 0.3	2.7 ± 0.2
$10^2 \mathcal{B}(\Xi_c^0 \rightarrow \Xi^- \mu^+ \nu_\mu)$	2.9 ± 0.4	2.4 ± 0.3	2.7 ± 0.2

(a). Equal masses of initial and final baryons;

(b). Constant helicity amplitude

$$H_{\lambda_2 \lambda_W}^{V(A)} = \underline{a_{\lambda_2 \lambda_W}^{V(A)}(q^2)} (\mathbf{B}_n)_j^i T^j(\bar{\mathbf{3}}) (\mathbf{B}_c)_i$$

(c). transition form factors in the heavy quark limit

Chao-Qiang Geng, Chia-Wei Liu, Tien-Hsueh Tsai, Shu-Wei Yeh
Phys.Lett.B 792 (2019) 214-218

Ξ_c semi-leptonic decays

SU(3) analysis with SU(3) breaking:

Xiao-Gang He, Fei Huang, Wei Wang, Zhi-Peng Xing

Phys.Lett.B 823 (2021) 136765

Mass matrix:
$$M = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix} \sim m_s \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} = m_s \times \omega.$$

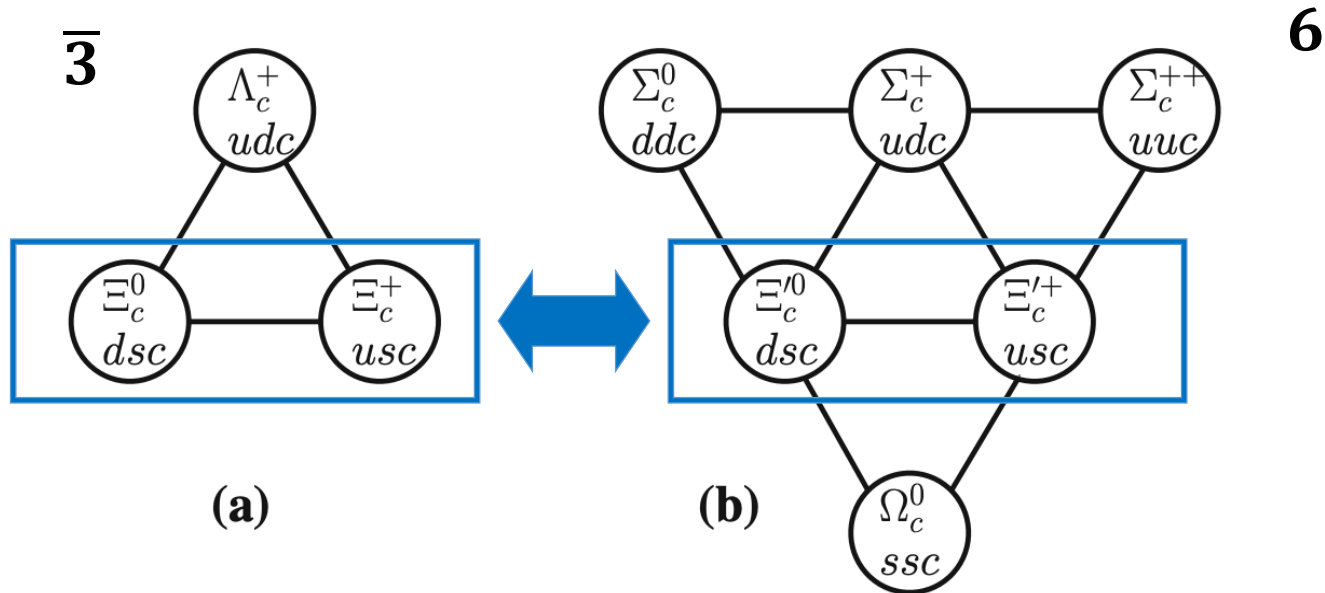
channel	branching ratio(%)	
	experimental data	SU(3) symmetry
$\Lambda_c^+ \rightarrow \Lambda^0 e^+ \nu_e$	3.6 ± 0.4 [33]	3.6 ± 0.4 (input)
$\Lambda_c^+ \rightarrow \Lambda^0 \mu^+ \nu_\mu$	3.5 ± 0.5 [33]	3.5 ± 0.5 (input)
$\Xi_c^+ \rightarrow \Xi^0 e^+ \nu_e$	2.3 ± 1.5 [33]	12.17 ± 1.35
$\Xi_c^0 \rightarrow \Xi^- e^+ \nu_e$	1.54 ± 0.35 [4, 5]	4.10 ± 0.46
$\Xi_c^0 \rightarrow \Xi^- \mu^+ \nu_\mu$	1.27 ± 0.44 [4]	3.98 ± 0.57

No SU(3) breaking

channel	branching ratio(%)		
	experimental data	fit data(pole model)	fit data(constant).
$\Lambda_c^+ \rightarrow \Lambda^0 e^+ \nu_e$	3.6 ± 0.4	3.61 ± 0.32	3.62 ± 0.32
$\Lambda_c^+ \rightarrow \Lambda^0 \mu^+ \nu_\mu$	3.5 ± 0.5	3.48 ± 0.30	3.45 ± 0.30
$\Xi_c^+ \rightarrow \Xi^0 e^+ \nu_e$	2.3 ± 1.5	3.89 ± 0.73	3.92 ± 0.73
$\Xi_c^0 \rightarrow \Xi^- e^+ \nu_e$	1.54 ± 0.35	1.29 ± 0.24	1.31 ± 0.24
$\Xi_c^0 \rightarrow \Xi^- \mu^+ \nu_\mu$	1.27 ± 0.44	1.24 ± 0.23	1.24 ± 0.23
fit parameter (pole model)	$f_1 = 1.01 \pm 0.87, \delta f_1 = -0.51 \pm 0.92$ $f'_1 = 0.60 \pm 0.49, \delta f'_1 = -0.23 \pm 0.41$		$\chi^2/d.o.f = 1.6$
fit parameter (constant)	$f_1 = 0.86 \pm 0.92, \delta f_1 = -0.25 \pm 0.88$ $f'_1 = 0.85 \pm 0.36, \delta f'_1 = -0.43 \pm 0.50$		$\chi^2/d.o.f = 1.9$

SU(3) breaking

SU(3) breaking $\rightarrow \Xi_c - \Xi'_c$ mixing



$$\begin{pmatrix} |\Xi_Q\rangle \\ |\Xi'_Q\rangle \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} |\Xi_Q^{\bar{3}}\rangle \\ |\Xi_Q^6\rangle \end{pmatrix}$$

Mixing angle

$\Xi_c - \Xi_c'$ mixing to match experiments

From semi-leptonic decays

$$|\theta_c| = 0.137(5)\pi (\approx 24^\circ) \quad \text{from } \Xi_c^0 \rightarrow \Xi^- e^+ \nu_e \quad (\text{LFQM})$$

Chao-Qiang Geng, Xiang-Nan Jin, Chia-Wei Liu
Phys.Lett.B 838 (2023) 137736

From non-leptonic decays

$$\text{LHCb: } \frac{\mathcal{B}(\Xi_{cc}^{++} \rightarrow \Xi_c^{+'} \pi^+)}{\mathcal{B}(\Xi_{cc}^{++} \rightarrow \Xi_c^+ \pi^+)} \equiv \frac{\mathcal{B}'}{\mathcal{B}} = 1.41 \pm 0.17 \pm 0.1$$

Phys. Rev. Lett. 121, 162002 (2018) JHEP 05 (2022) 038

To fit this ratio:

$$\theta = 16.27^\circ \pm 2.30^\circ \text{ or } 85.54^\circ \pm 2.30^\circ \quad (\text{LFQM})$$

Hong-Wei Ke, Xue-Qian Li, *Phys.Rev.D* 105 (2022) 9, 096011

$\Xi_c - \Xi_c'$ mixing in QCDSR

$$\Pi^{(\prime)}(p) = i \int d^4x e^{ip \cdot x} \langle 0 | T \{ J^{(\prime)}(x) \bar{J}^{(\prime)}(0) \} | 0 \rangle,$$

$$\begin{pmatrix} J \\ J' \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} J_0 \\ J_1 \end{pmatrix} \quad \begin{aligned} i \int d^4x e^{ip \cdot x} \langle 0 | T \{ J(x) \bar{J}'(0) \} | 0 \rangle &= 0, \\ i \int d^4x e^{ip \cdot x} \langle 0 | T \{ J'(x) \bar{J}(0) \} | 0 \rangle &= 0. \end{aligned}$$

$$\left\{ \begin{array}{l} \theta_c = (1.2 \sim 2.8)^\circ \text{ for the } Q = c \\ \theta_b = (0.28 \sim 0.34)^\circ \text{ for the } Q = b \end{array} \right. \quad \text{Heavy quark limit: } \theta \rightarrow 0$$

Eur.Phys.J.C 83 (2023) 10, 961

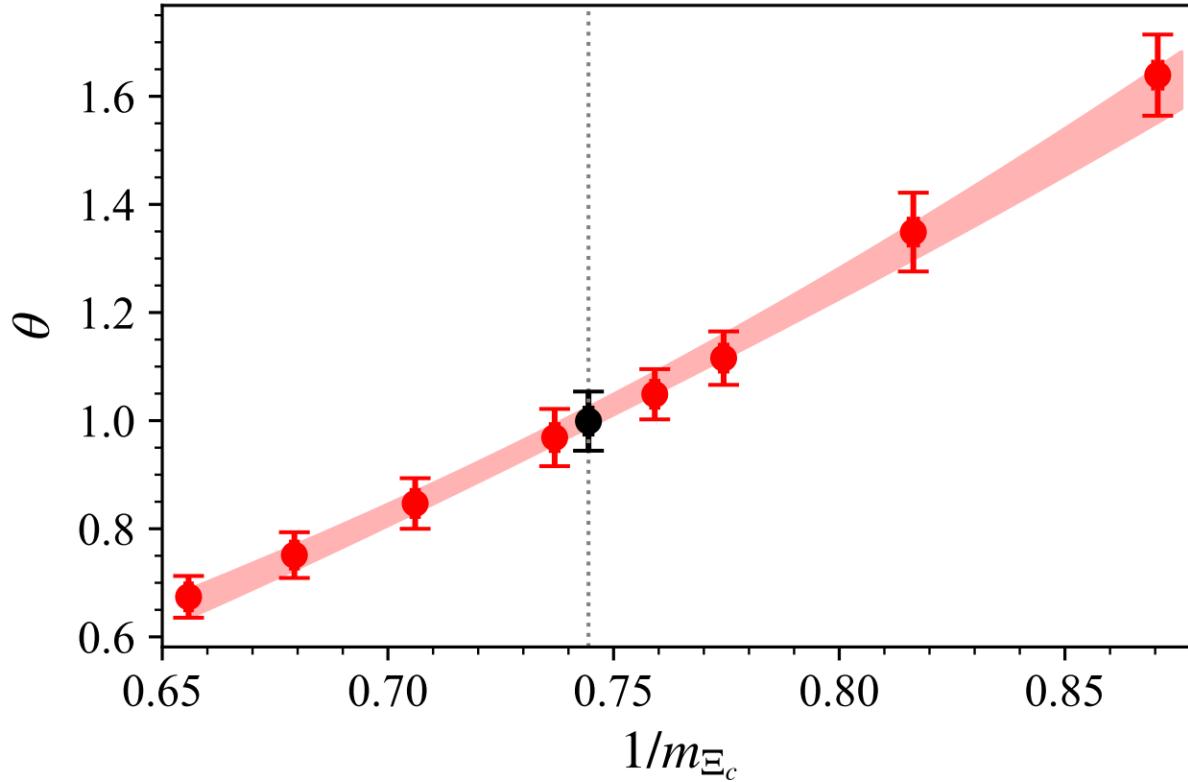
Revisiting $\Xi_Q - \Xi_Q'$ mixing in QCD sum rules

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² School of Physics, East China University of Science and Technology, Shanghai 200237, China

$\Xi_c - \Xi_c'$ mixing in LQCD



$$\theta = (1.22 \pm 0.13 \pm 0.01)^\circ$$

$\Xi_c - \Xi_c'$ mixing From Lattice QCD

Phys.Lett.B 841 (2023) 137941

Hang Liu,¹ Liuming Liu,^{2,3} Peng Sun,^{2,3} Wei Sun,⁴ Jin-Xin Tan,¹ Wei Wang,^{1,5,*} Yi-Bo Yang,^{6,7,8,3} and Qi-An Zhang^{9,†}

$\Xi_c - \Xi_c'$ mixing from SU(3) breaking

Extracting $\Xi_c - \Xi_c'$ mixing angle

The Lagrangian can be decomposed into SU(3) conserving and breaking terms:

$$\mathcal{L}_{\text{QCD+QED}} = \mathcal{L}_0 + \Delta\mathcal{L}$$

$$\begin{aligned} \mathcal{L}_0 = & \sum_q \bar{\psi}_q (i\not{D} - m_u) \psi_q + e \sum_q e_s \bar{\psi}_q \not{A} \psi_q \\ & + e e_c \bar{\psi}_c \not{A} \psi_c, \end{aligned}$$


The same mass and charge for u, d, s,
SU(3) conserving

$$\Delta\mathcal{L} = \bar{\psi}_s \underline{(m_u - m_s)} \psi_s + e \underline{(e_u - e_s)} \bar{\psi}_u \not{A} \psi_u$$

The mass and charge differences are attributed to the
SU(3) breaking terms

Extracting $\Xi_c - \Xi_c'$ mixing angle

Hamiltonian: $H = H_0 + \Delta H$

The SU(3) conserving Hamiltonian is diagonalized by $\Xi_c^{\bar{3}}$ and Ξ_c^6 :

$$H_0|\Xi_c^{\bar{3}}\rangle = m_{\Xi_c^{\bar{3}}}|\Xi_c^{\bar{3}}\rangle, \quad H_0|\Xi_c^6\rangle = m_{\Xi_c^6}|\Xi_c^6\rangle$$

The full Hamiltonian is diagonalized by Ξ_c and Ξ_c' :

$$H|\Xi_c\rangle = m_{\Xi_c}|\Xi_c\rangle, \quad H|\Xi_c'\rangle = m_{\Xi_c'}|\Xi_c'\rangle$$

Transformation between doublets: $|S\rangle = (|\Xi_c^{\bar{3}}\rangle, |\Xi_c^6\rangle)^T$, $|P\rangle = (|\Xi_c\rangle, |\Xi_c'\rangle)^T$

$$|P\rangle = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} |S\rangle = U|S\rangle.$$

Extracting $\Xi_c - \Xi_c'$ mixing angle

In the basis of $|S\rangle = (|\Xi_c^{\bar{3}}\rangle, |\Xi_c^6\rangle)^T$:

$$\mathbf{H} = \begin{pmatrix} \langle \Xi_c^{\bar{3}}(S'_z) | H | \Xi_c^{\bar{3}}(S_z) \rangle & \langle \Xi_c^6(S'_z) | H | \Xi_c^{\bar{3}}(S_z) \rangle \\ \langle \Xi_c^{\bar{3}}(S'_z) | H | \Xi_c^6(S_z) \rangle & \langle \Xi_c^6(S'_z) | H | \Xi_c^6(S_z) \rangle \end{pmatrix}$$



$$|S\rangle = U^T |P\rangle$$



$$= 2(2\pi)^3 \delta^{(3)}(\vec{0}) \delta_{S_z S'_z}$$

$$\times \begin{pmatrix} m_{\Xi_c}^2 \cos^2 \theta + m_{\Xi_c'}^2 \sin^2 \theta & (m_{\Xi_c}^2 - m_{\Xi_c'}^2) \cos \theta \sin \theta \\ (m_{\Xi_c}^2 - m_{\Xi_c'}^2) \cos \theta \sin \theta & m_{\Xi_c}^2 \sin^2 \theta + m_{\Xi_c'}^2 \cos^2 \theta \end{pmatrix}$$



$$\begin{aligned} & \langle \Xi_c^6(S'_z) | H | \Xi_c^{\bar{3}}(S_z) \rangle \\ = & (2\pi)^3 \delta^{(3)}(\vec{0}) \delta_{S_z S'_z} (m_{\Xi_c}^2 - m_{\Xi_c'}^2) \sin 2\theta. \end{aligned}$$

Extracting $\Xi_c - \Xi_c'$ mixing angle

On the another hand: $H = H_0 + \Delta H$



$$\langle \Xi_c^6 | H_0 | \Xi_c^{\bar{3}} \rangle = 0$$

H_0 conserves SU(3) symmetry

$$\begin{aligned} & \langle \Xi_c^6(S'_z) | H | \Xi_c^{\bar{3}}(S_z) \rangle \\ = & (2\pi)^3 \delta^{(3)}(\vec{0}) \langle \Xi_c^6(S'_z) | \Delta \mathcal{H}(0) | \Xi_c^{\bar{3}}(S_z) \rangle \end{aligned}$$

An unknown
matrix element



$$\begin{aligned} & \langle \Xi_c^6(S'_z) | H | \Xi_c^{\bar{3}}(S_z) \rangle \\ = & (2\pi)^3 \delta^{(3)}(\vec{0}) \delta_{S_z S'_z} (m_{\Xi_c}^2 - m_{\Xi_c'}^2) \sin 2\theta. \end{aligned}$$

Mixing angle

Extracting θ

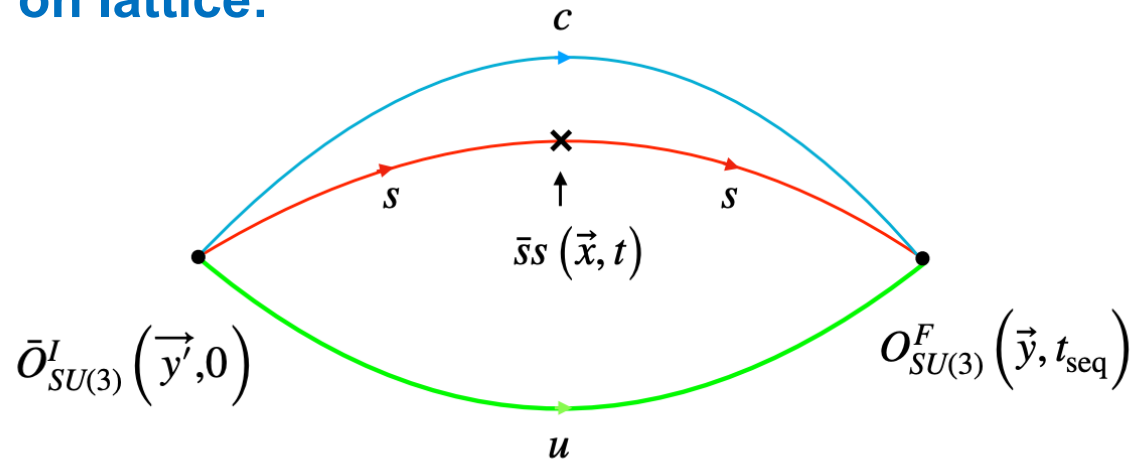


$$\sin 2\theta = \frac{1}{2} \sum_{S_z} \frac{\langle \Xi_c^6(S_z) | \Delta \mathcal{H}(0) | \Xi_c^{\bar{3}}(S_z) \rangle}{m_{\Xi_c}^2 - m_{\Xi_c'}^2}$$

QCD contribution to $\Xi_c - \Xi_c'$

$$\Delta\mathcal{H}_m = (m_s - m_u)\bar{s}s$$

Calculate a three-point correlation function on lattice:

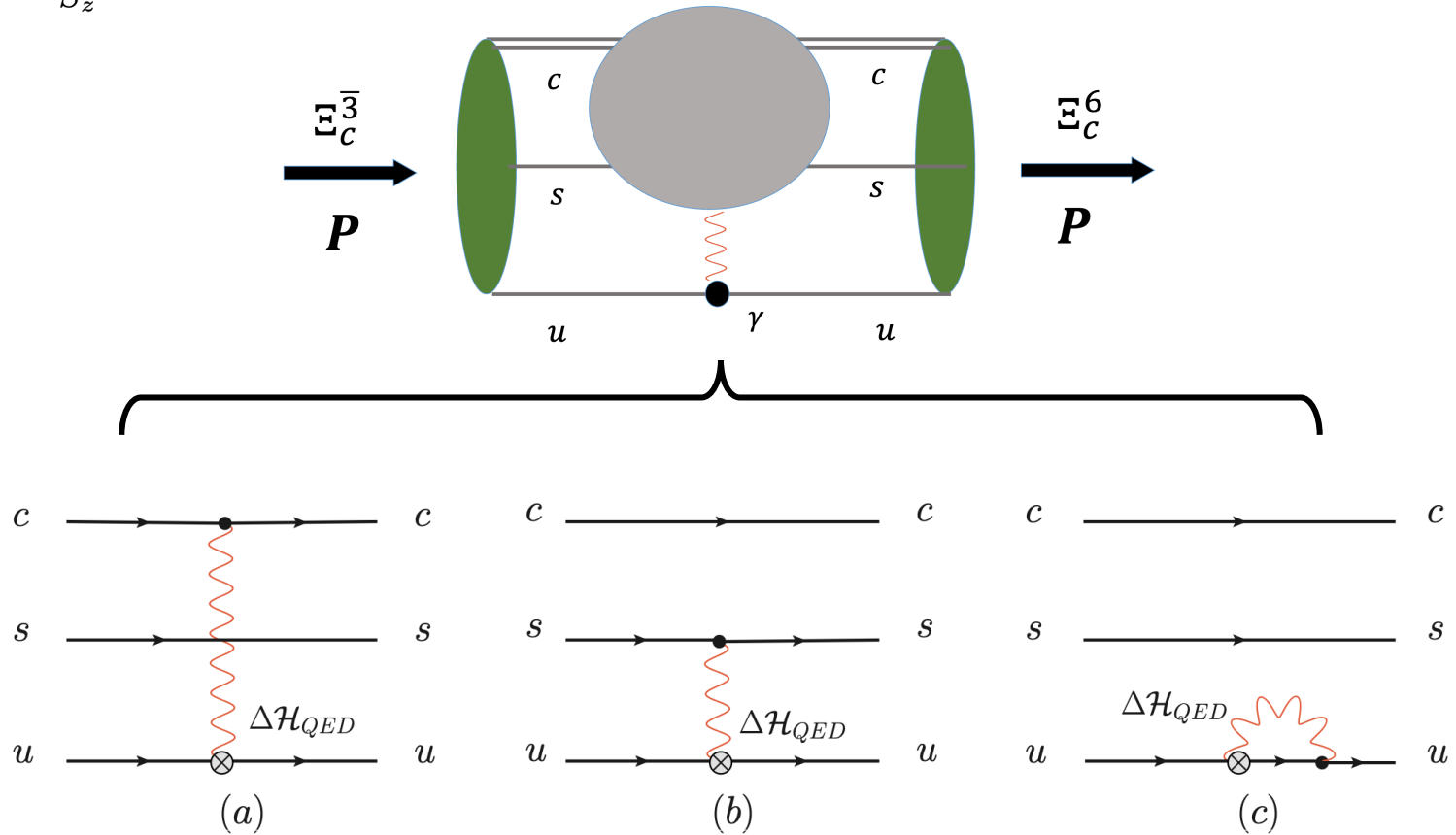


	$ M_{\bar{s}s}^{6-\bar{3}} $	$ \theta $
C11P29S	0.155(14)GeV	$(0.97 \pm 0.08 \pm 0.25)^\circ$

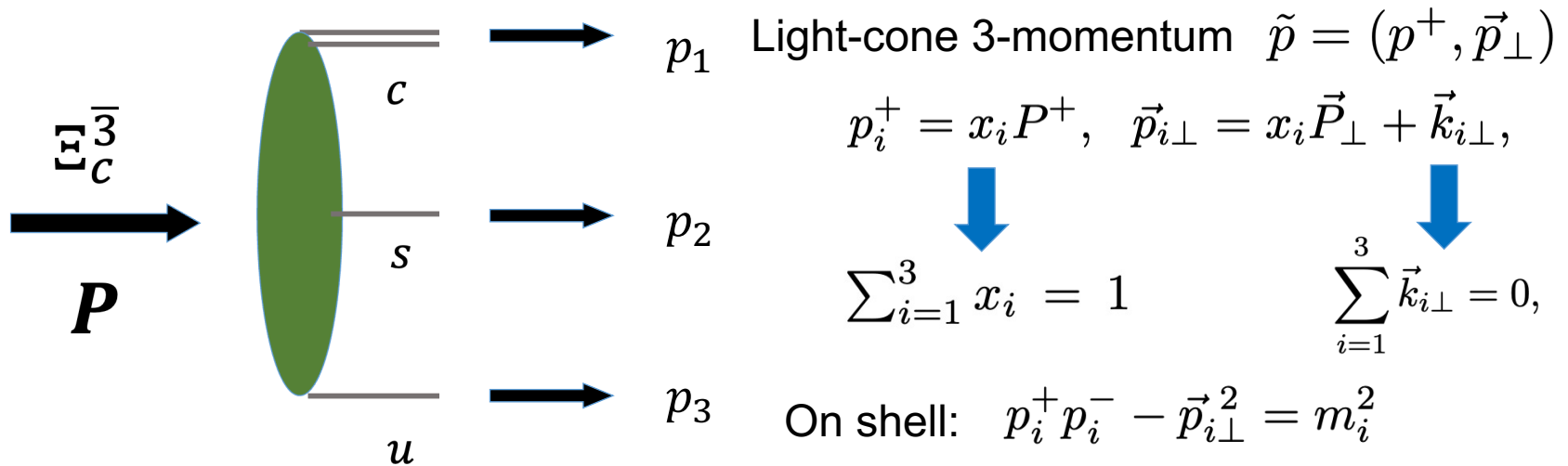
Hang Liu, Wei Wang, Qi-An Zhang, arXiv:[2309.05432](https://arxiv.org/abs/2309.05432)

QCD contribution to $\Xi_c - \Xi_c'$

$$\frac{1}{2} \sum_{S_z} \langle \Xi_c^6(P, S_z) | \Delta \mathcal{H}_{\text{QED}}(0) | \Xi_c^{\bar{3}}(P, S_z) \rangle \quad \Delta \mathcal{H}_{\text{QED}} = e(e_u - e_s) \bar{\psi}_u A \psi_u$$



The three-quark picture of LFQM



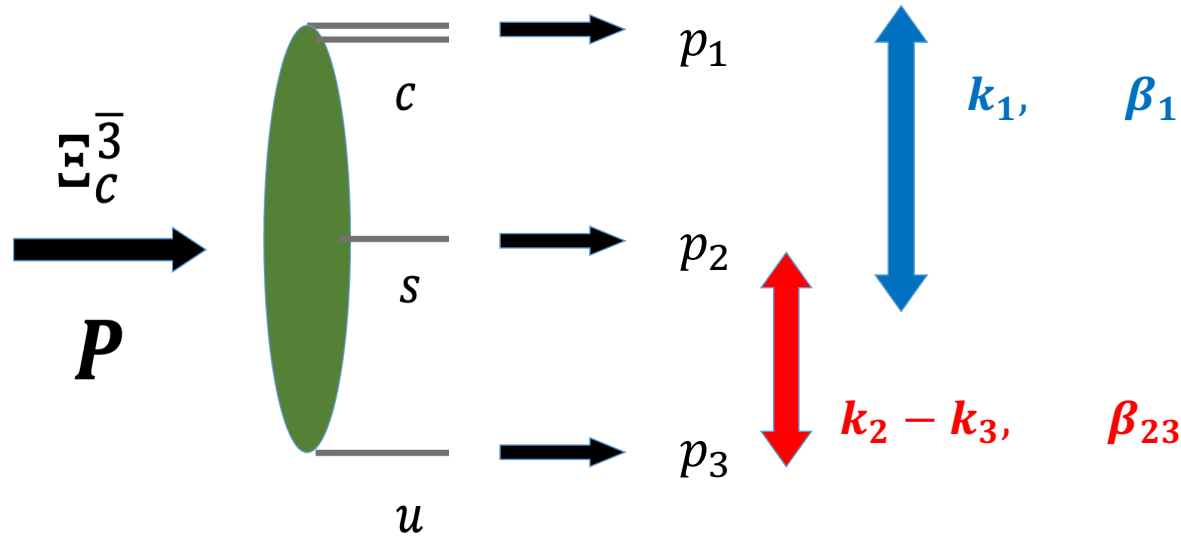
Light-cone 3-momentum conservation

$$|\Xi_c^{\bar{3}/6}\rangle = \int \left(\prod_{i=1}^3 \{d^3 \tilde{p}_i\} \right) 2(2\pi)^3 \frac{\delta^{(3)}(\tilde{P} - \tilde{p}_1 - \tilde{p}_2 - \tilde{p}_3)}{\sqrt{P^+}} \sum_{\lambda_i} \Psi_{\bar{3}/6}^{S, S_z} \frac{\epsilon^{ijk}}{\sqrt{6}} |c^i(p_1, \lambda_1) s^j(p_2, \lambda_2) u^k(p_3, \lambda_3)\rangle$$

$$\{d^3 \tilde{p}\} \equiv \frac{dp^+ d^2 p_\perp}{2(2\pi)^3 \sqrt{p^+}}$$

Wave function

Three-quark picture in LFQM



Spin: $\Psi_{\bar{3}} = A \bar{u}_{\lambda_3}(p_3) (\vec{P} + M_0) (-\gamma_5) C \bar{u}_{\lambda_2}^T(p_2) \bar{u}_{\lambda_1}(p_1) u(\bar{P}) \Phi(x_i, k_{i\perp})$

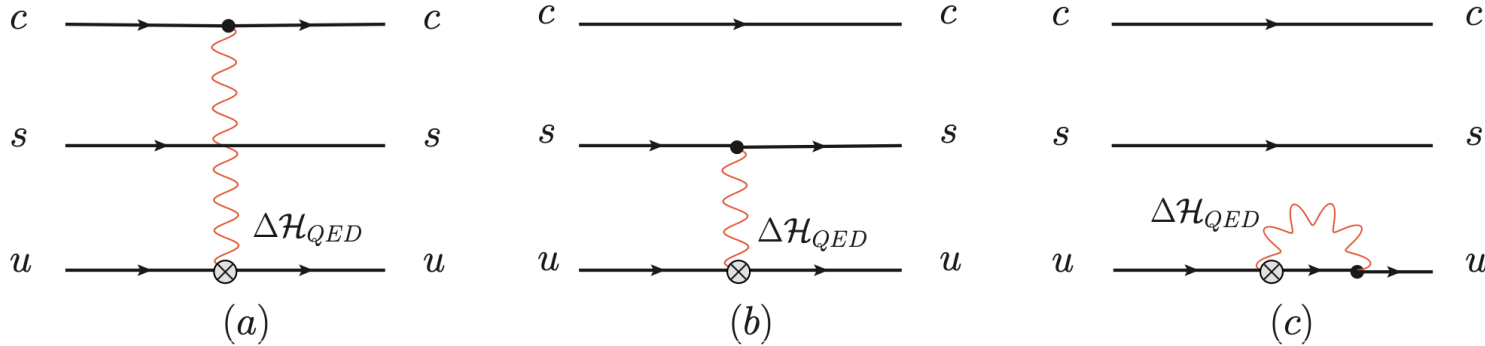
$$\Phi(x_i, k_{i\perp}) = \sqrt{\frac{e_1 e_2 e_3}{x_1 x_2 x_3 M_0}} \phi(\vec{k}_1, \beta_1) \phi\left(\frac{\vec{k}_2 - \vec{k}_3}{2}, \beta_{23}\right),$$

Momentum:

$$\phi(\vec{k}, \beta) = 4 \left(\frac{\pi}{\beta^2} \right)^{\frac{3}{4}} e^{\frac{-k_{\perp}^2 - k_z^2}{2\beta^2}},$$

Zhen-Xing Zhao, Fu-Wei Zhang, Xiao-Hui Hu, Yu-Ji Shi
Phys.Rev.D 107 (2023) 11, 116025

LFQM Calculation



Contributes

Vanishes

Vanishes



The charm quark spin is changed by photon, a $1/m_c$ effect.

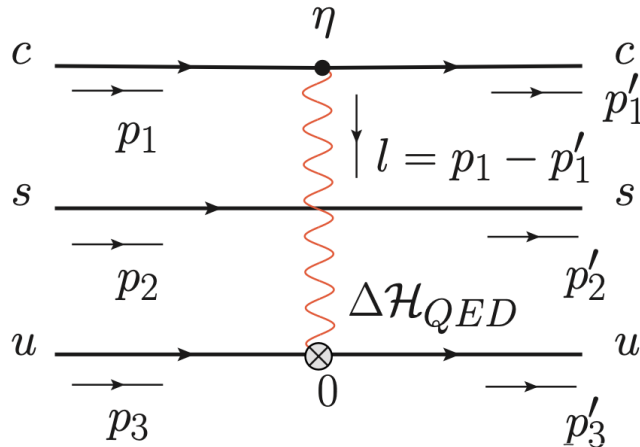
$$\bar{h}_v i(v \cdot D) h_v - \frac{1}{2m_Q} \bar{h}_v \not{D}^\perp \sum_{n=0}^N \left(\frac{-i(v \cdot D)}{2m_Q} \right)^n \not{D}^\perp h_v$$

Spin conserved

Spin changed

$$m_Q \uparrow \longrightarrow \theta_{QED} \downarrow$$

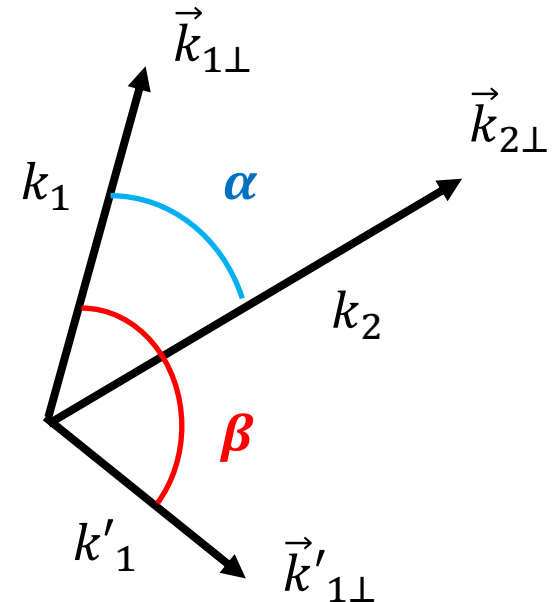
LFQM Calculation



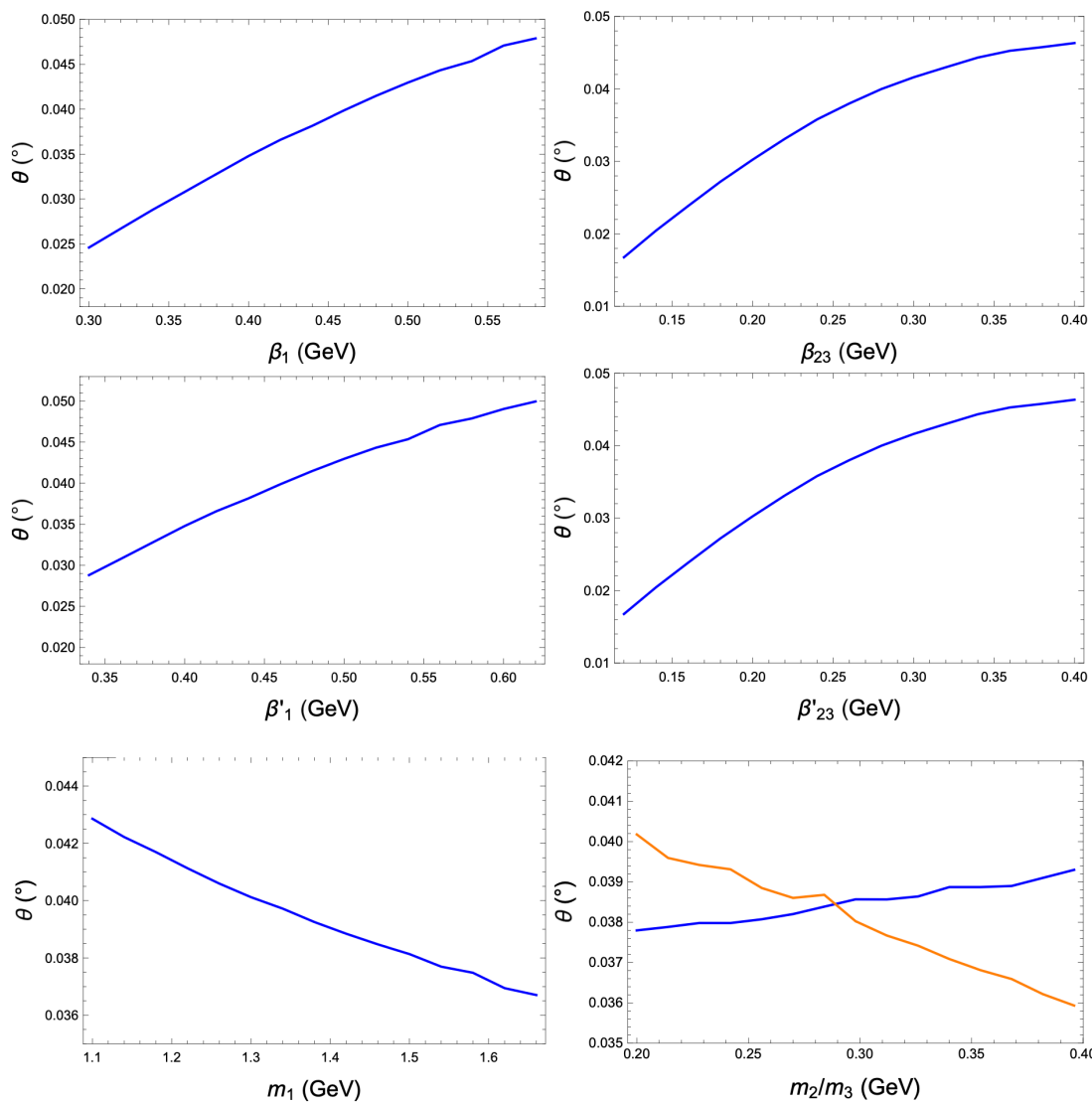
The only possible IR region is $l^2 = 0$

$$\propto \frac{l^\mu l^\nu}{l^2} \quad \text{No IR divergence}$$

$$\begin{aligned} & e(e_u - e_s) \langle \Xi_c(1) | \bar{\psi}_u(0) A(0) \psi_u(0) | \Xi_c(0) \rangle \\ &= -\frac{2e^2}{3} \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \int_0^{1-x_2} dx'_1 \int_0^\infty \frac{dk_1 dk_2 dk'_1}{[2(2\pi)^3]^3} \\ & \times 2\pi \int_0^{2\pi} d\alpha d\beta \frac{k_1 k_2 k'_1 A^2}{\sqrt{x_1 x'_1 (1-x_1-x_2)(1-x'_1-x_2)}} \\ & \times \Phi(x_i, k_{i\perp}) \Phi(x'_i, k'_{i\perp}) \frac{\text{Tr}_A \text{Tr}_B}{l^2 + i\epsilon} \end{aligned}$$



Mixing Angle



$$\theta_{\text{QED}} \approx 0.04^\circ$$

Very small !

Conclusion

- To match the measured semi-leptonic decay BF, the required $\Xi_c - \Xi_c'$ mixing angle is around $10^\circ - 30^\circ$
- QCD contributes only $\sim 1^\circ$
- QED contribution is much smaller: 0.04°
- It seems that the $\Xi_c - \Xi_c'$ mixing effect is too small to affect the Ξ_c decays

Any other mechanisms to explain the gap between exp. & th. ?

$K_1(1270) - K_1(1400)$ mixing

$$\begin{pmatrix} |K_1(1270)\rangle \\ |K_1(1400)\rangle \end{pmatrix} = \begin{pmatrix} \cos \theta_{K_1} & \sin \theta_{K_1} \\ -\sin \theta_{K_1} & \cos \theta_{K_1} \end{pmatrix} \begin{pmatrix} |K_{1B}\rangle \\ |K_{1A}\rangle \end{pmatrix}$$

$$\theta_{K_1} = 22^\circ \pm 7^\circ \text{ or } \theta_{K_1} = 68^\circ \pm 7^\circ$$

Yu-Ji Shi, Jun Zeng, Zhi-Fu Deng, arXiv:[2310.20429](https://arxiv.org/abs/2310.20429)

Thank you for your attention !