PQCD of Λ_b decays : progress

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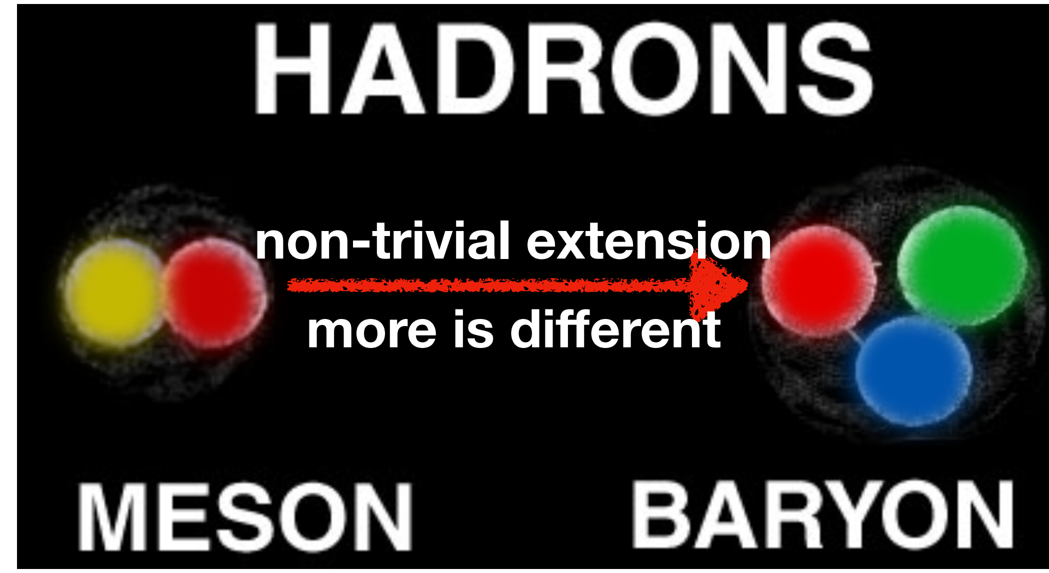
Zhen-Jun Xiao and Fu-Sheng Yu

Outline

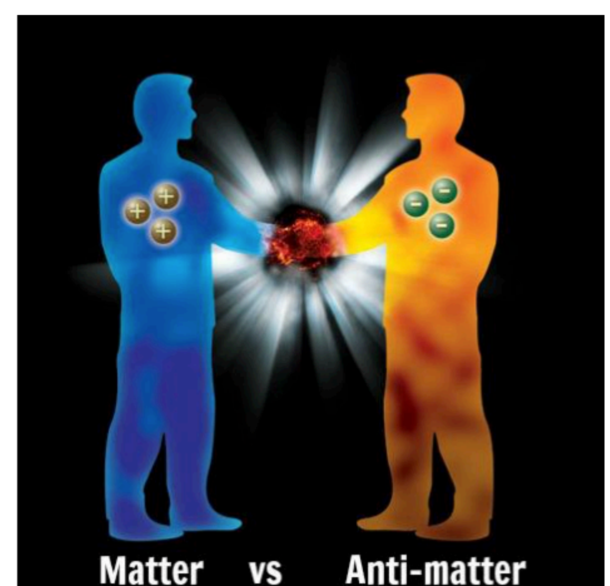
- Why baryon physics?
- PQCD framework in baryon
- PQCD of non-leptonic Λ_b decays
- PQCD of two-body Λ_b decays
- Summary

Heavy flavor physics and CPV

- Heavy flavor physics has achieved great progress in meson systems,
- KM mechanism for the CPV has established in B meson decays,
- But the studies on heavy flavor baryons are still limited.



- SM and cosmology require CPV,
- Which is well established in K, B and D mesons, but never established in any baryon.
- Comparison between predictions and measurements is helpful to test SM and search NP.



Opportunities and Challenges

- LHCb is a baryon factory, has large Λ_b production: $\dfrac{N_{\Lambda_b}}{N_{B^{0,-}}} \sim 0.5$
- Baryon CPV measurements in LHCb have reached to order of 1 % *[LHCb,2018]*

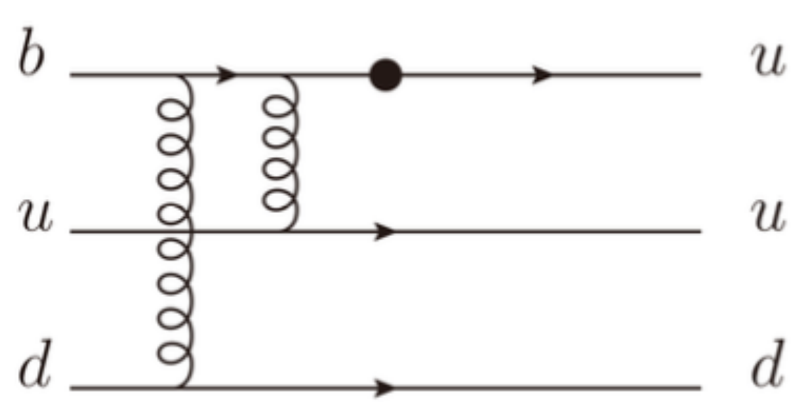
$$A_{CP}(\Lambda_b \to p\pi^-) = (-3.5 \pm 1.7 \pm 2.0)\,\%\,,\quad A_{CP}(\Lambda_b \to pK^-) = (-2.0 \pm 1.3 \pm 1.0)\,\%$$

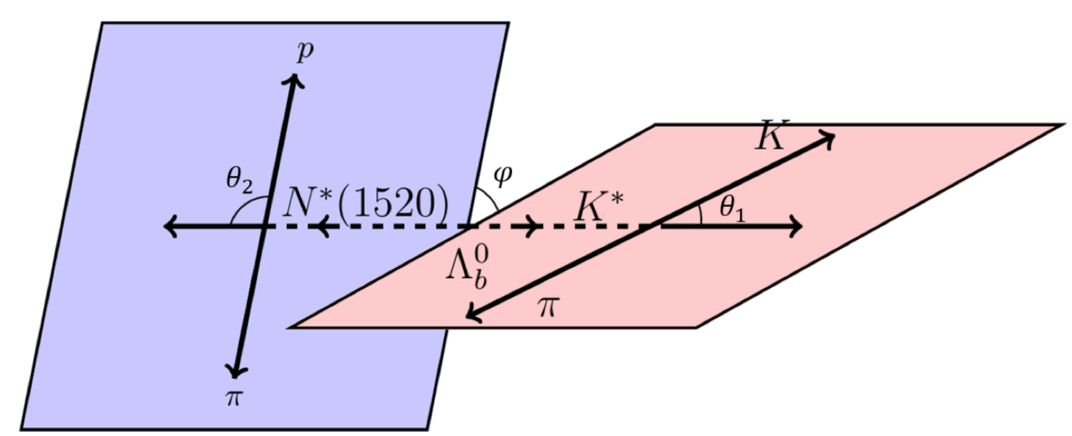
- CPV in some B meson decays are as large as 10 % *[PDG,2022]* :

$$A_{CP}(B^0 \to K^+\pi^-) = (-8.34 \pm 0.32)\,\%\,,\quad A_{CP}(B^0 \to K^{*0}\eta) = (19 \pm 5)\,\%\,,\quad A_{CP}(B_s \to K^-\pi^+) = (22.4 \pm 1.2)\,\%$$

- The CPV in b-baryon can be observed soon.

- QCD dynamics for baryon decays:
  - one more hard gluon; power counting rule; Why CPV of $\Lambda_b \to p\pi, pK$ are so small
- Non-perturbative inputs
  - Theoretical uncertainties are dominated by non-perturbative inputs, such as LCDAs
- Observables
  - T-odd triple products $(\overrightarrow{p_1} \times \overrightarrow{p_2}) \cdot \overrightarrow{p_3}$, defined by kinematics, but unclear related to decay amplitudes.
- ……





Theoretical progresses

- QCD studies on baryons are limited

✓ Generalized factorization *[Hsiao,Geng,2015; Liu,Geng,2021]*;

lost of non-factorizable contributions, such as types of W-exchange diagrams

✓ QCDF *[Zhu,Ke,Wei,2016;2018]*

based on diquark picture, no W-exchange diagrams

✓ PQCD *[Lü,Wang,Zou,Ali,Kramer,2009]*

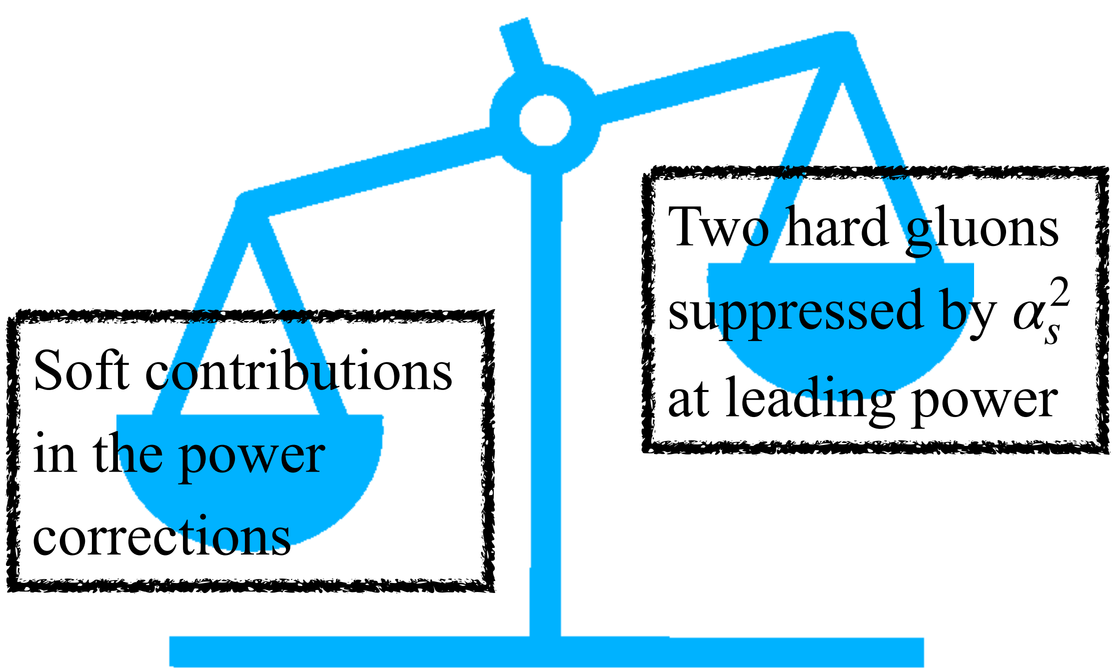
only considering the leading twist of LCDAs

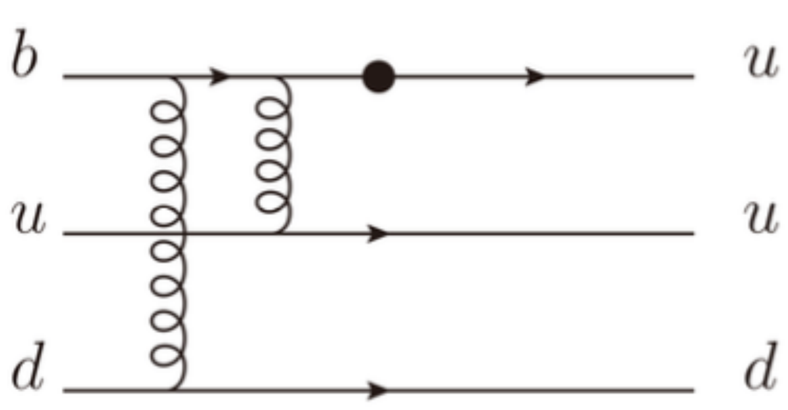
	measurement	Generalized factorization	QCDF	PQCD
$Br(\Lambda_b \to p\pi^-) \times 10^{-6}$	4.5 ± 0.8	4.2 ± 0.7	$4.66^{+2.22}_{-1.81}$	$4.11 \sim 4.57$
$Br(\Lambda_b \to pK^-) \times 10^{-6}$	5.4 ± 1.0	4.8 ± 0.7	$1.82^{+0.97}_{-1.07}$	$1.70 \sim 3.15$
$A_{CP}(\Lambda_b \to p\pi^-)\,\%$	-2.5 ± 2.9	-3.9 ± 0.2	-32^{+49}_{-1}	$-3.74 \sim -3.08$
$A_{CP}(\Lambda_b \to pK^-)\,\%$	-2.5 ± 2.2	5.8 ± 0.2	-3^{+25}_{-4}	$8.1 \sim 11.4$

- More is different, baryons are very different from mesons!
- Factorization: heavy-to-light form factor is factorizable at leading power in SCET and no end-point singularity appears! *[Wei Wang,1112.0237]*

$$\xi_{\Lambda_b\to\Lambda} = f_{\Lambda_b}\Phi_{\Lambda_b}(x_i)\otimes J(x_i,y_i)\otimes f_{\Lambda}\Phi_{\Lambda}(y_i)$$

- However, the leading-power result is one order smaller than the total one
  - Leading-power: $\xi_{\Lambda_b\to\Lambda}(0) = -0.012$ *[W.Wang,2011]*
  - Total form factor: $\xi_{\Lambda_b\to\Lambda}(0) = 0.18$ *[Y.L.Shen,Y.M.Wang,2016]*





PQCD approach

- PQCD has successfully predicted CPV in B meson decays

$$A_{CP}(B \to \pi^+\pi^-) = (30 \pm 20)\,\% \,, \quad A_{CP}(B \to K^+\pi^-) = (-17 \pm 5)\,\%$$

[Keum,H-n.Li,Sanda,2000; C.D.Lü,Ukai,M.Z.Yang,2000]

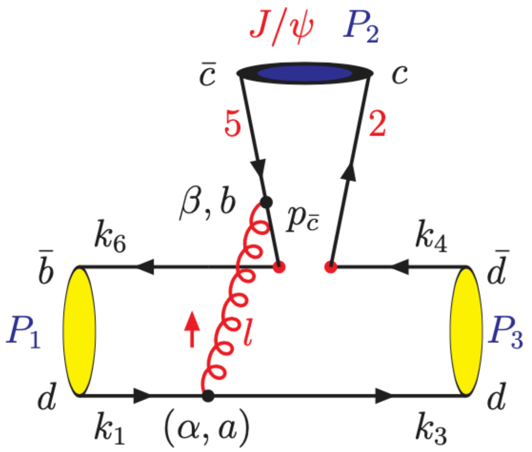
$$A_{CP}(B \to \pi^+\pi^-) = (32 \pm 4)\,\% \,, \quad A_{CP}(B \to K^+\pi^-) = (-8.3 \pm 0.4)\,\%$$

[PDG,2022; first measurements were made in 2001]

Factorization hypothesis:

$$\mathscr{A} = \langle M_2 M_3 \;\; \mathscr{H} \;\; B \rangle$$

$$\sim \int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\frac{d^4k_3}{(2\pi)^4}\Psi_B(k_1,\mu)\Psi_2(k_2,\mu)\Psi_3(k_3,\mu)\cdot H(k_1,k_2,k_3,\mu)C_i(\mu)$$



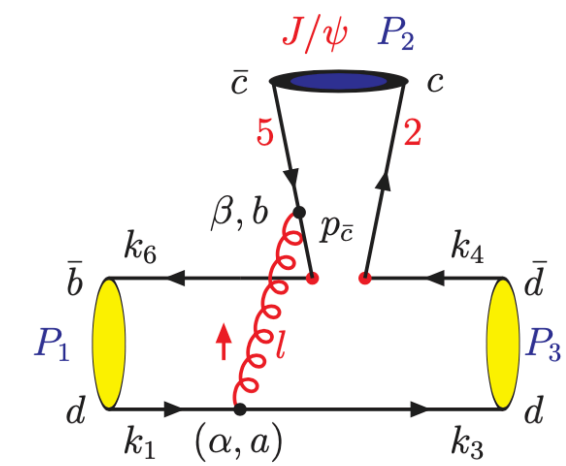
- Under collinear factorization:

endpoint singularity: propagator $\sim \dfrac{1}{x_1 x_2 Q^2} \to \infty$ when $x_{1,2} \to 0{,}1$

$$\mathscr{A} \sim \int_0^1 dx_1 dx_2 dx_3 \phi_B(x_1,\mu) * H\left(x_1, x_2, x_3, \mu, \alpha_s(x_i,\mu)\right) * \phi_\eta(x_2,\mu)\phi_{J/\psi}(x_3,\mu)$$

- PQCD approach (based on k_T factorization): retain transverse momentum of parton k_T

  ◉propagator $\sim \dfrac{1}{x_1x_2Q^2 + \;\; k_{iT}\;^2}$



$$\mathscr{A} = \langle M_2M_3 \;\; \mathscr{H} \;\; B\rangle$$

$$\sim \int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\frac{d^4k_3}{(2\pi)^4}\Psi_B(k_1,\mu)\Psi_2(k_2,\mu)\Psi_3(k_3,\mu)\cdot H(k_1,k_2,k_3,\mu)C_i(\mu)$$

$$\sim \int_0^1 dx_2dx_2dx_3 \int \frac{d^2k_{1T}}{(2\pi)^2}\frac{d^2k_{2T}}{(2\pi)^2}\frac{d^2k_{3T}}{(2\pi)^2}\phi_B(x_1,k_{1T},\mu)\phi_2(x_2,k_{2T},\mu)\phi_3(x_3,k_{3T},\mu)\cdot H(x_1,x_2,x_3,k_{1T},k_{2T},k_{3T},\mu)C_i(\mu)$$

$$H(x_i,k_T,\mu) \sim \frac{N_1(x_1,x_2,x_3)N_2(x_1,x_2,x_3)}{l^2 \;\; p_c^2} = \frac{N_1(x_1,x_2,x_3)}{x_1x_3M_B^2 - \;\; k_{1T}-k_{3T} \;\;^2}\frac{N_2(x_1,x_2,x_3)}{M_B^2(1-x_3) - \;\; k_{3T} \;\;^2}$$

- Resum double-log radiative correction, obtain k_T Sudakov factor $S(x_i, b_i)$ and threshold Sudakov factor $S_t(x_i)$.

[NPB (Collins, 1981)
NPB (Botts, Sterman, 1989)
PRD (Hsiang-nan Li, 1995)
PRL (Hsiang-nan Li, 1995)
PRD (Hsiang-nan Li, 1996)
PRD (Hsiang-nan Li, 1998)
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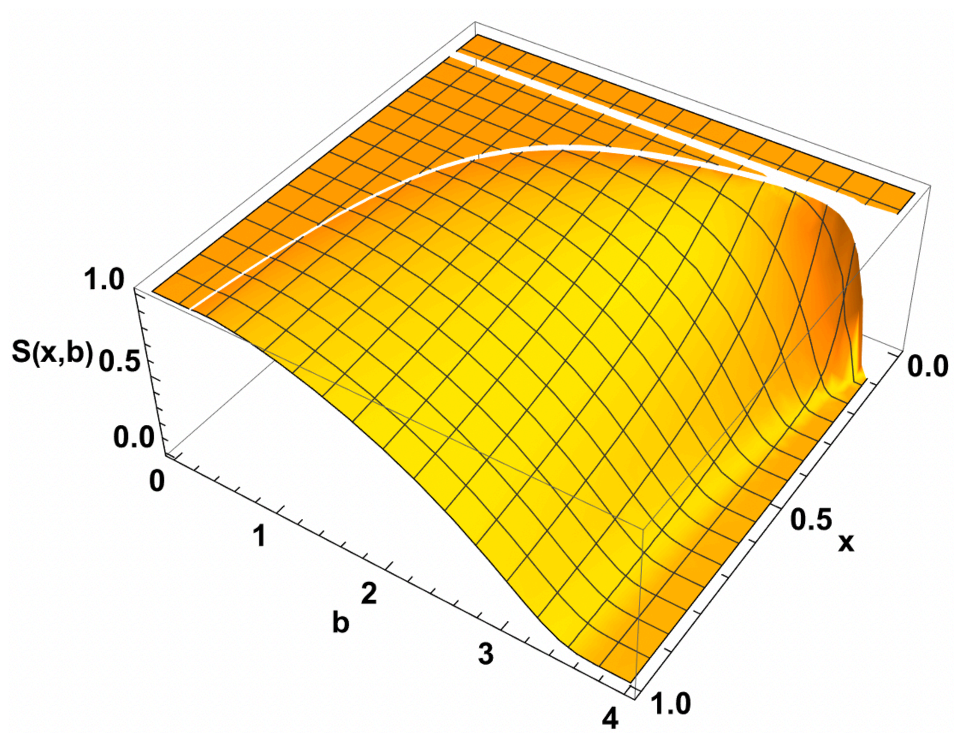
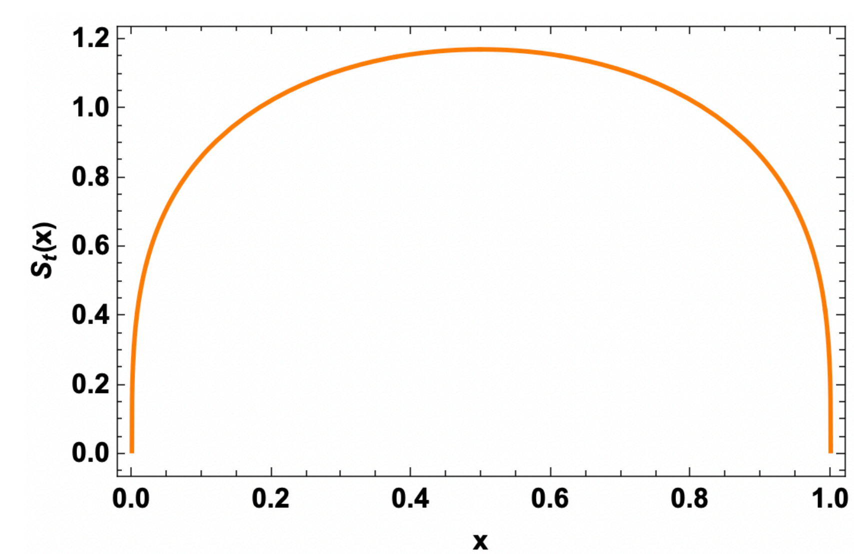
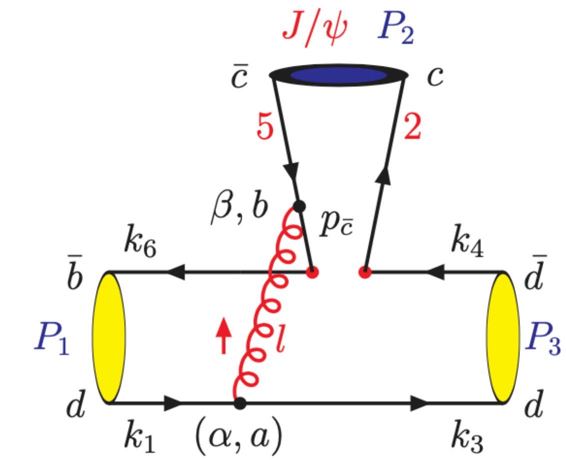




- PQCD approach (based on k_T factorization): retain transverse momentum of parton k_T

  ◉ propagator $\sim \dfrac{1}{x_1 x_2 Q^2 + \;\; k_{iT} \;\;^2}$



$$\mathscr{A} = \langle M_2 M_3 \;\; \mathscr{H} \;\; B \rangle$$

$$\sim \int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\frac{d^4k_3}{(2\pi)^4}\Psi_B(k_1,\mu)\Psi_2(k_2,\mu)\Psi_3(k_3,\mu)\cdot H(k_1,k_2,k_3,\mu)C_i(\mu)$$

$$\sim \int_0^1 dx_2 dx_2 dx_3 \int \frac{d^2k_{1T}}{(2\pi)^2}\frac{d^2k_{2T}}{(2\pi)^2}\frac{d^2k_{3T}}{(2\pi)^2}\phi_B(x_1,k_{1T},\mu)\phi_2(x_2,k_{2T},\mu)\phi_3(x_3,k_{3T},\mu)\cdot H(x_1,x_2,x_3,k_{1T},k_{2T},k_{3T},\mu)C_i(\mu)$$

$$H(x_i,b_i) \sim \int \frac{d^2k_{1T}}{(2\pi)^2}\frac{d^2k_{3T}}{(2\pi)^2}e^{ib\cdot k_{1T}}e^{ib\cdot k_{3T}}\frac{N_1(x_1,x_2,x_3)}{x_1x_3M_B^2 - \;\; k_{1T}-k_{3T} \;\;^2}\frac{N_2(x_1,x_2,x_3)}{M_B^2(1-x_3) - \;\; k_{3T} \;\;^2}$$

$$\sim N_1(x_1,x_2,x_3)N_2(x_1,x_2,x_3)\cdot\left[K_0(\sqrt{x_1x_3M_B^2}b_1)I_0(\sqrt{(1-x_3)M_B^2}b_3)K_0(\sqrt{(1-x_3)M_B^2}b_1)\Theta(b_3-b_1) + K_0(\sqrt{x_1x_3M_B^2}b_1)I_0(\sqrt{(1-x_3)M_B^2}b_1)K_0(\sqrt{(1-x_3)M_B^2}b_3)\Theta(b_1-b_3)\right]$$

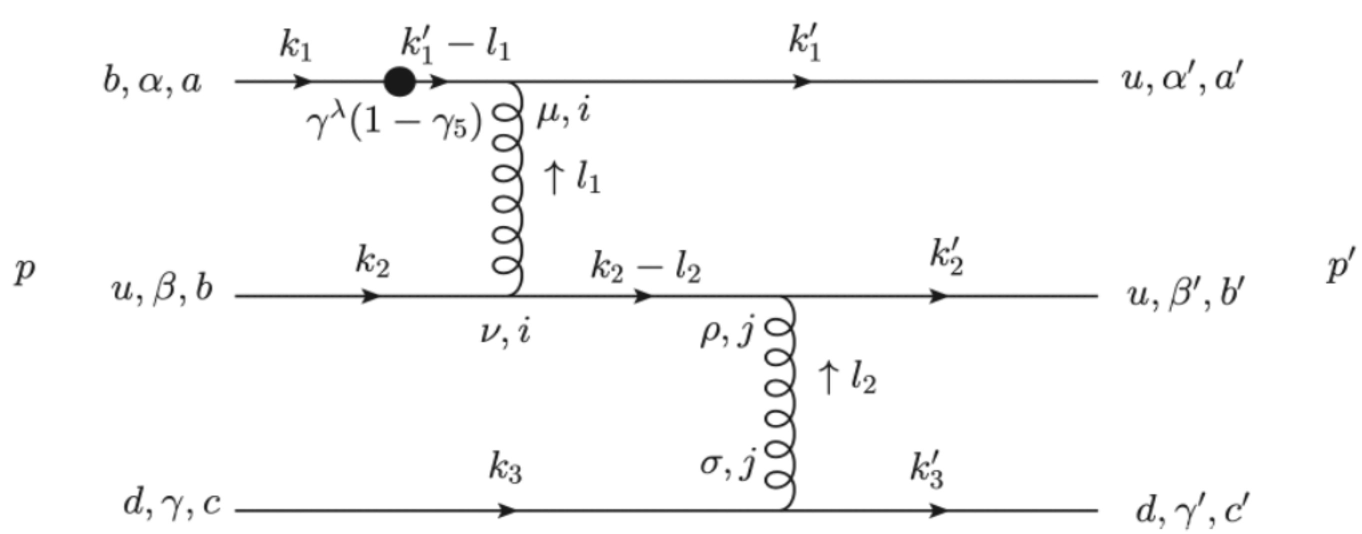
after Fourier tramsform

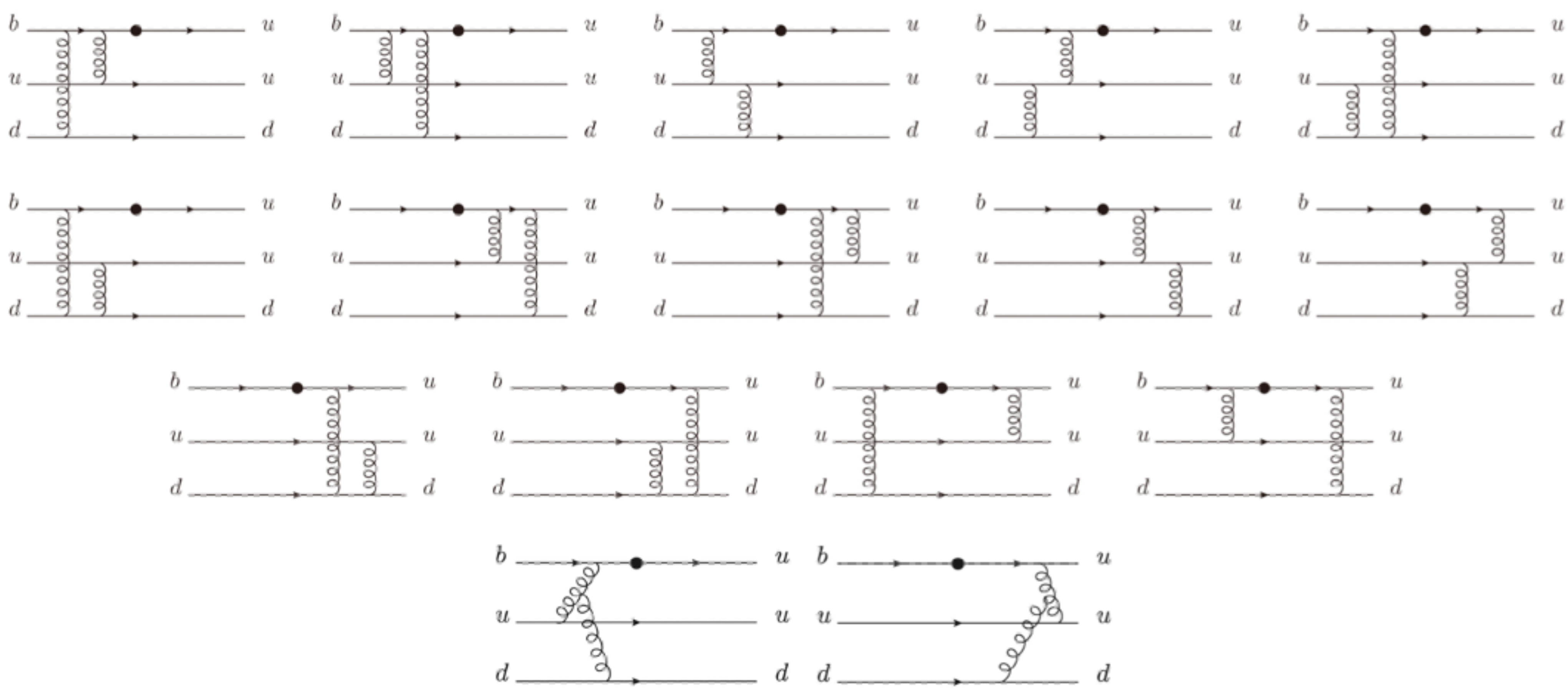
$$\sim \int_0^1 dx_1dx_2dx_3\int d^2b_1d^2b_2d^2b_3\phi_B(x_1,b_1,\mu)\phi_2(x_2,b_2,\mu)\phi_3(x_3,b_3,\mu)\cdot H(x_1,x_2,x_3,b_1,b_2,b_3,\mu)C_i(\mu)\times\Pi_i S(x_i,b_i)\times S_t(x_i)$$

$\Lambda_b \to p$ form factors in PQCD

$$F_i(q^2) \sim \int_0^1 d[x]d[x'] \int d^2[b]d^2[b']\phi_{\Lambda_b}([x],[b],\mu) \cdot H([x],[x'],[b],[b'],\mu)C_i(\mu) \cdot \phi_p([x'],[b'],\mu) \cdot \Pi_i S(x_i,b_i)S_t(x_i)$$

$$\langle p \; \bar{u}\gamma_\mu(1-\gamma_5)b \; \Lambda_b\rangle = \bar{p}(f_1\gamma_\mu - if_2\sigma_{\mu\nu}q^\nu + f_3 q_\mu)\Lambda_b - \bar{p}(g_1\gamma_\mu - ig_2\sigma_{\mu\nu}q^\nu + g_3 q_\mu)\gamma_5\Lambda_b$$

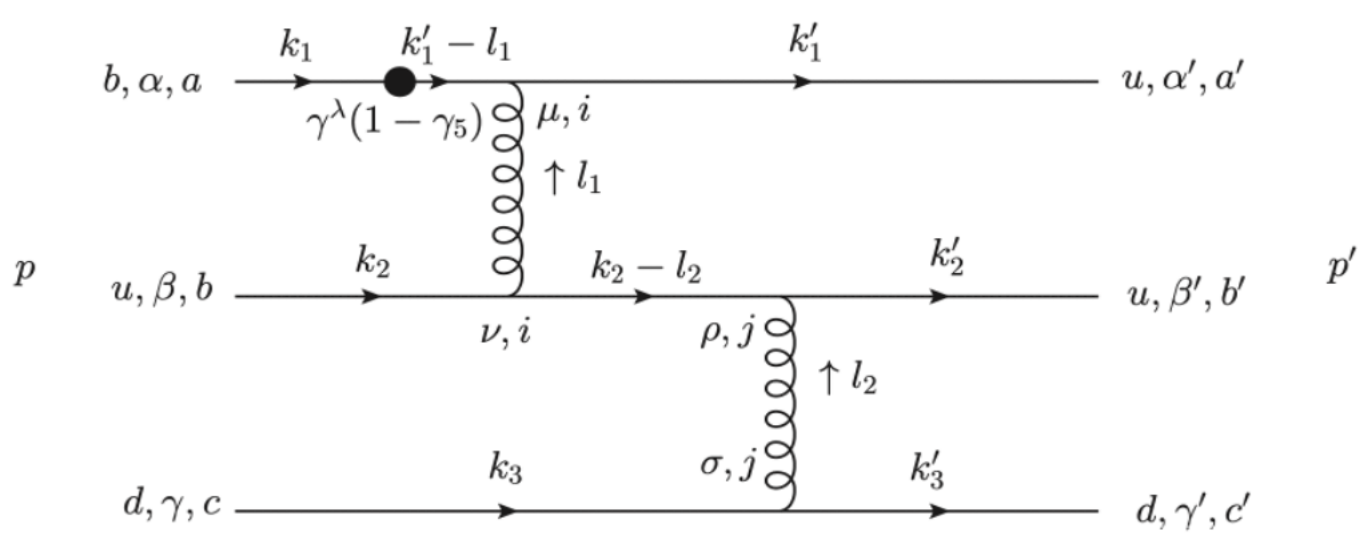


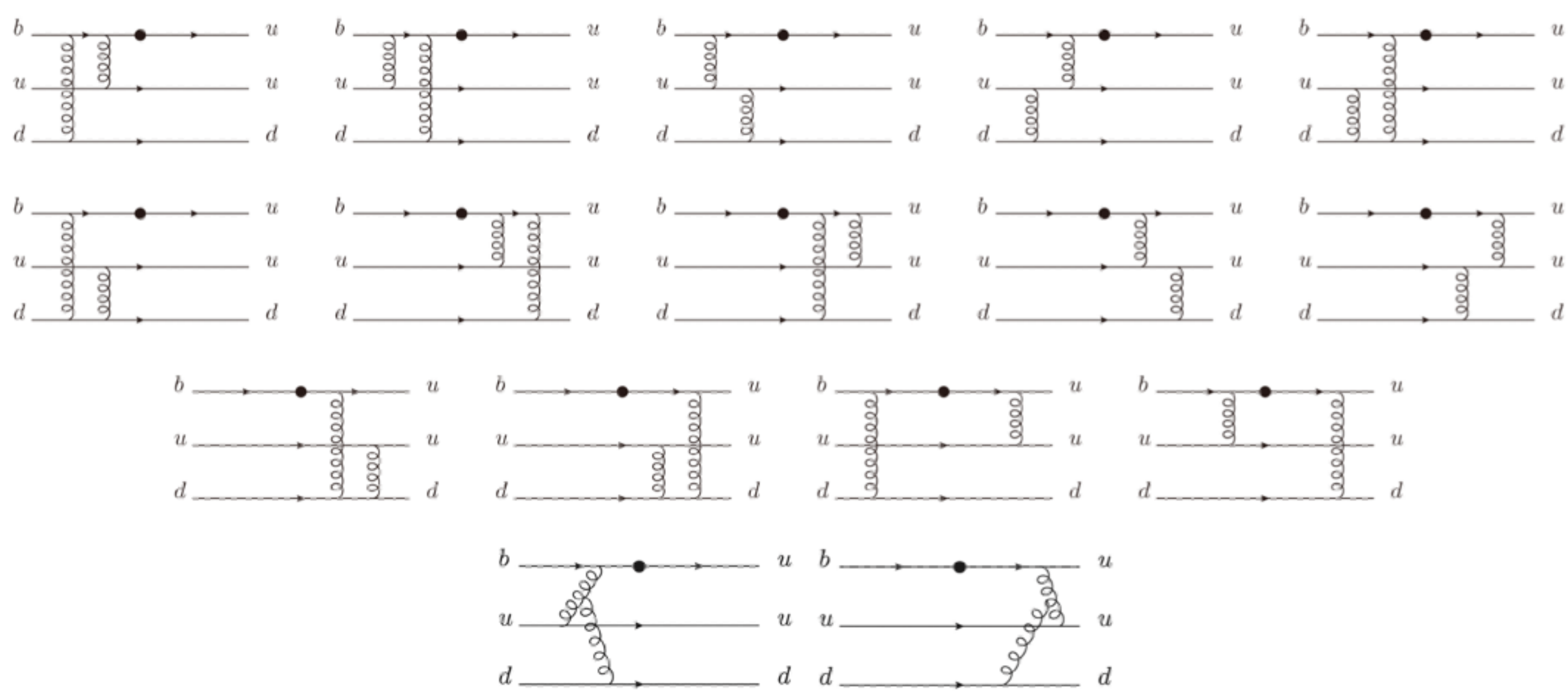


$\Lambda_b \to p$ form factors in PQCD

$$F_i(q^2) \sim \int_0^1 d[x]d[x'] \int d^2[b]d^2[b']\phi_{\Lambda_b}([x],[b],\mu) \cdot H([x],[x'],[b],[b'],\mu)C_i(\mu) \cdot \phi_p([x'],[b'],\mu) \cdot \Pi_i S(x_i,b_i)S_t(x_i)$$

$$\langle p \;\; \bar{u}\gamma_\mu(1-\gamma_5)b \;\; \Lambda_b\rangle = \bar{p}(f_1\gamma_\mu - if_2\sigma_{\mu\nu}q^\nu + f_3 q_\mu)\Lambda_b - \bar{p}(g_1\gamma_\mu - ig_2\sigma_{\mu\nu}q^\nu + g_3 q_\mu)\gamma_5\Lambda_b$$





$\Lambda_b \to p$ form factors in PQCD

- LCDAs for Λ_b

$$\Phi_{\Lambda_b}^{\alpha\beta\delta}(t_1,t_2) \equiv \epsilon_{ijk}\langle 0|[u_i^T(t_1\bar{n})]_\alpha[0,t_1\bar{n}][d_j(t_2\bar{n})]_\beta[0,t_2\bar{n}][b_k(0)]_\delta|\Lambda_b(v)\rangle$$
$$= \frac{1}{4}\Big\{f_{\Lambda_b}^{(1)}(\mu)[\tilde{M}_1(v,t_1,t_2)\gamma_5 C^T]_{\beta\alpha} + f_{\Lambda_b}^{(2)}(\mu)[\tilde{M}_2(v,t_1,t_2)\gamma_5 C^T]_{\beta\alpha}\Big\}[\Lambda_b(v)]_\delta \quad (23)$$

$$M_2(\omega_1,\omega_2) = \frac{\not{n}}{\sqrt{2}}\psi_2(\omega_1,\omega_2) + \frac{\not{\bar{n}}}{\sqrt{2}}\psi_4(\omega_1,\omega_2)$$
$$M_1(\omega_1,\omega_2) = \frac{\not{\bar{n}}\not{n}}{4}\psi_3^{+-}(\omega_1,\omega_2) + \frac{\not{n}\not{\bar{n}}}{4}\psi_3^{-+}(\omega_1,\omega_2)$$

$$\psi_2(x_2,x_3) = \frac{x_2 x_3}{\omega_0^4} m_{\Lambda_b}^4 e^{-(x_2+x_3)m_{\Lambda_b}/\omega_0},$$
$$\psi_3^{+-}(x_2,x_3) = \frac{2x_2}{\omega_0^3} m_{\Lambda_b}^3 e^{-(x_2+x_3)m_{\Lambda_b}/\omega_0},$$
$$\psi_3^{-+}(x_2,x_3) = \frac{2x_3}{\omega_0^3} m_{\Lambda_b}^3 e^{-(x_2+x_3)m_{\Lambda_b}/\omega_0},$$
$$\psi_4(x_2,x_3) = \frac{1}{\omega_0^2} m_{\Lambda_b}^2 e^{-(x_2+x_3)m_{\Lambda_b}/\omega_0},$$

- ***P.Ball, V.M.Braun, E.Gardi (2008)***
- ***G.Bell, T.Feldmann, Yu-Ming Wang, M.W.Y.Yip (2013)***
- ***Yu-Ming Wang, Yue-Long Shen (2016)***

$\Lambda_b \to p$ form factors in PQCD

- LCDAs for proton

➤ ***V.M.Braun, R.J.Fries, N.Mahnke, E.Stein (2001)***

$$
\begin{aligned}
\bar{\Phi}^{\alpha\beta\gamma}_{proton} \equiv& \langle \mathcal{P}(p')|\bar{u}^i_\alpha(0)\bar{u}^j_\beta(z_1)\bar{d}^k_\gamma(z_2)|0\rangle \\
=&\frac{1}{4}\{S_1 m_p C_{\beta\alpha}(\bar{N}^+\gamma_5)_\gamma + S_2 m_p C_{\beta\alpha}(\bar{N}^-\gamma_5)_\gamma + P_1 m_p (C\gamma_5)_{\beta\alpha}\bar{N}^+_\gamma + P_2 m_p (C\gamma_5)_{\beta\alpha}\bar{N}^-_\gamma + V_1(C\not{P})_{\beta\alpha}(\bar{N}^+\gamma_5)_\gamma \\
&+ V_2(C\not{P})_{\beta\alpha}(\bar{N}^-\gamma_5)_\gamma + V_3\frac{m_p}{2}(C\gamma_\perp)_{\beta\alpha}(\bar{N}^+\gamma_5\gamma^\perp)_\gamma + V_4\frac{m_p}{2}(C\gamma_\perp)_{\beta\alpha}(\bar{N}^-\gamma_5\gamma^\perp)_\gamma + V_5\frac{m_p^2}{2Pz}(C\not{z})_{\beta\alpha}(\bar{N}^+\gamma_5)_\gamma \\
&+ V_6\frac{m_p^2}{2Pz}(C\not{z})_{\beta\alpha}(\bar{N}^-\gamma_5)_\gamma + A_1(C\gamma_5\not{P})_{\beta\alpha}(\bar{N}^+)_\gamma + A_2(C\gamma_5\not{P})_{\beta\alpha}(\bar{N}^-)_\gamma + A_3\frac{m_p}{2}(C\gamma_5\gamma_\perp)_{\beta\alpha}(\bar{N}^+\gamma^\perp)_\gamma \\
&+ A_4\frac{m_p}{2}(C\gamma_5\gamma_\perp)_{\beta\alpha}(\bar{N}^-\gamma^\perp)_\gamma + A_5\frac{m_p^2}{2Pz}(C\gamma_5\not{z})_{\beta\alpha}(\bar{N}^+)_\gamma + A_6\frac{m_p^2}{2Pz}(C\gamma_5\not{z})_{\beta\alpha}(\bar{N}^-)_\gamma - T_1(iC\sigma_{\perp P})_{\beta\alpha}(\bar{N}^+\gamma_5\gamma^\perp)_\gamma \\
&- T_2(iC\sigma_{\perp P})_{\beta\alpha}(\bar{N}^-\gamma_5\gamma^\perp)_\gamma - T_3\frac{m_p}{Pz}(iC\sigma_{Pz})_{\beta\alpha}(\bar{N}^+\gamma_5)_\gamma - T_4\frac{m_p}{Pz}(iC\sigma_{zP})_{\beta\alpha}(\bar{N}^-\gamma_5)_\gamma - T_5\frac{m_p^2}{2Pz}(iC\sigma_{\perp z})_{\beta\alpha}(\bar{N}^+\gamma_5\gamma^\perp)_\gamma \\
&- T_6\frac{m_p^2}{2Pz}(iC\sigma_{\perp z})_{\beta\alpha}(\bar{N}^-\gamma_5\gamma^\perp)_\gamma + T_7\frac{m_p}{2}(C\sigma_{\perp\perp'})_{\beta\alpha}(\bar{N}^+\gamma_5\sigma^{\perp\perp'})_\gamma + T_8\frac{m_p}{2}(C\sigma_{\perp\perp'})_{\beta\alpha}(\bar{N}^-\gamma_5\sigma^{\perp\perp'})_\gamma\}
\end{aligned}
$$

TABLE I: Twist classification of proton distribution amplitudes.

	twist-3	twist-4	twist-5	twist-6
Vector	V_1	V_2, V_3	V_4, V_5	V_6
Pseudo-Vector	A_1	A_2, A_3	A_4, A_5	A_6
Tensor	T_1	T_2, T_3, T_7	T_4, T_5, T_8	T_6
Scalar		S_1	S_2	
Pesudo-Scalar		P_1	P_2	

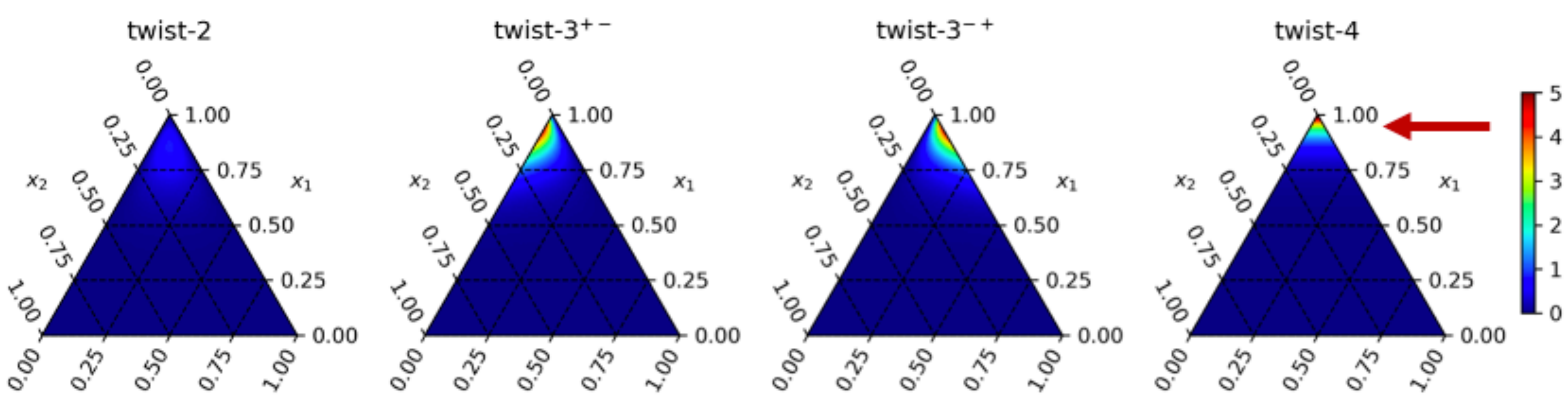
$\Lambda_b \to p$ form factors in PQCD

- Result of form factor f_1

	proton contribution				
	twist-3	twist-4	twist-5	twist-6	total
Λ_b exponential					
twist-2	0.0007	-0.00007	-0.0005	-0.000003	0.0001
twist-3^{+-}	-0.0001	0.002	0.0004	-0.000004	0.002
twist-3^{-+}	-0.0002	0.0060	0.000004	0.00007	0.006
twist-4	0.01	0.00009	0.25	0.0000007	0.26
total	0.01	0.008	0.25	0.00007	$0.27 \pm 0.09 \pm 0.07$

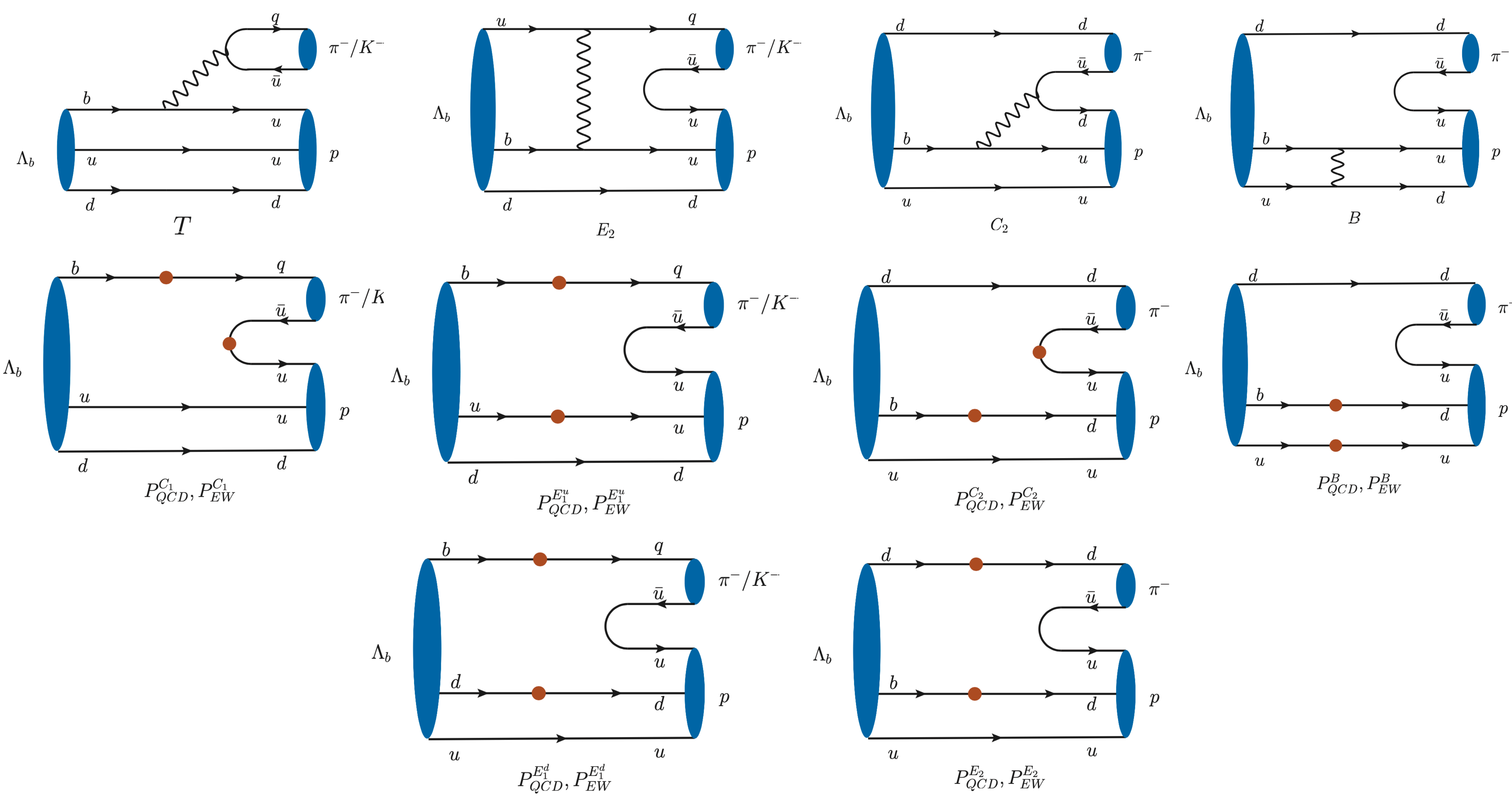
$r = \frac{m_p}{M_{\Lambda_b}}$

D_7	twist-3	twist-4	twist-5	twist-6
twist-2	~ 0	$r \cdot 2\sqrt{2}(1-x_1)x_3$	$r^2 \cdot 2\sqrt{2}x_3$	$r^3 \cdot 4\sqrt{2}(1-x_1)(1-x_2')$
twist-3^{+-}	$x_3(1-x_1)$	$r \cdot x_3$	$r^2 \cdot (1-x_1)(1-x_2')$	~ 0
twist-3^{-+}	~ 0	$r \cdot x_3$	$r^2 \cdot (1-x_1)(1-x_2')$	$r^3 \cdot (1-x_2')$
twist-4	$4\sqrt{2}x_3$	$r \cdot 2\sqrt{2}(1-x_1)(1-x_2')$	$r^2 \cdot 2\sqrt{2}(1-x_2')$	~ 0



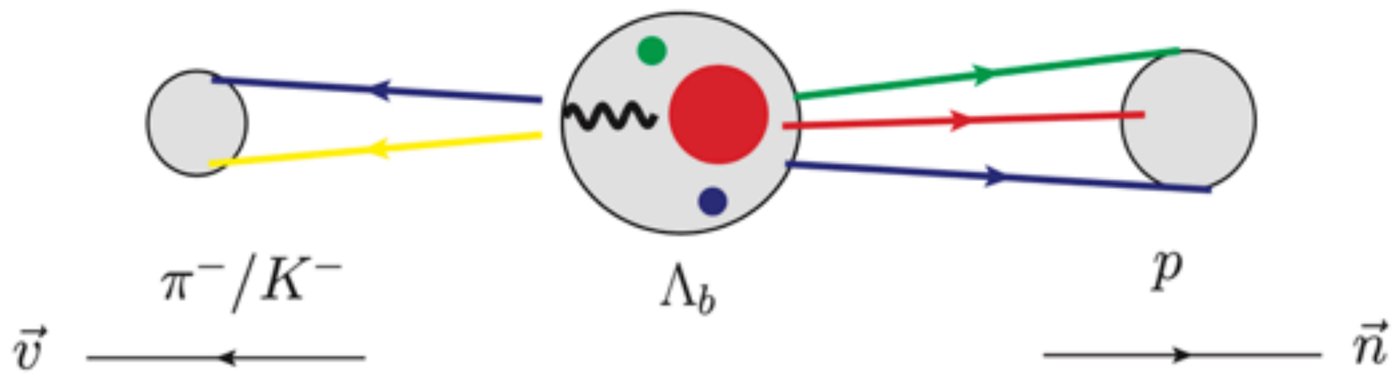
Two-body non-leptonic Λ_b decays

- $\Lambda_b \to p\pi^-,\ \ p\rho^-,\ \ pa_1(1260),\ \ pK^-,\ \ pK^{*-},\ \ pK_1(1270),\ \ pK_1(1400)$



Two-body non-leptonic Λ_b decays

- Kinematics



$$p = (\frac{M_{\Lambda_b}}{\sqrt{2}}, \frac{M_{\Lambda_b}}{\sqrt{2}}, 0), \qquad p' = (\frac{M_{\Lambda_b}}{\sqrt{2}}\eta_1, \frac{M_{\Lambda_b}}{\sqrt{2}}\eta_2, 0), \qquad p = (\frac{M_{\Lambda_b}}{\sqrt{2}}(1-\eta_1), \frac{M_{\Lambda_b}}{\sqrt{2}}(1-\eta_2), 0)$$

$$\begin{aligned} k_1 &= (p^+, x_1 p^-, \mathbf{k}_{1T}), & k_1' &= (x_1' p'^+, 0, \mathbf{k}_{1T}), & q_1 &= (0, yq^-, \mathbf{q}_T), \\ k_2 &= (0, x_2 p^-, \mathbf{k}_{2T}), & k_2' &= (x_2' p'^+, 0, \mathbf{k}_{2T}), & q_1 &= (0, (1-y)q^-, \mathbf{q}_T), \\ k_1 &= (0, x_3 p^-, \mathbf{k}_{3T}), & k_1' &= (x_3' p'^+, 0, \mathbf{k}_{3T}). \end{aligned}$$

$$\eta_1 = \left(M_{\Lambda_b}^2 - M_{\mathcal{M}}^2 + M_p^2 + \sqrt{(-M_{\Lambda_b}^2 + M_{\mathcal{M}}^2 - M_p^2)^2 - 4M_{\Lambda_b}^2 M_p^2}\right) / (2M_{\Lambda_b}^2)$$

$$\eta_2 = \left(M_{\Lambda_b}^2 - M_{\mathcal{M}}^2 + M_p^2 - \sqrt{(-M_{\Lambda_b}^2 + M_{\mathcal{M}}^2 - M_p^2)^2 - 4M_{\Lambda_b}^2 M_p^2}\right) / (2M_{\Lambda_b}^2)$$

- PQCD formula for two-body decays

$$F_i(q^2) \sim \int_0^1 d[x]d[x']dy \int d^2[b]d^2[b']db_y \phi_{\Lambda_b}([x],[b],\mu) \cdot H([x],[x'],y,[b],[b'],b_y,\mu)C_i(\mu) \cdot \phi_p([x'],[b'],\mu)\phi_M(y,b_y,\mu) \cdot \Pi_i S(x_i,b_i)S_t(x_i)$$

Two-body non-leptonic Λ_b decays

- LCDAs for meson

$$\Phi_\pi(p,x,\zeta) \equiv \frac{i}{\sqrt{2N_C}}\gamma_5\left[\not p \phi_\pi^A(x) + m_0^\pi \phi_\pi^P(x) + \zeta m_0^\pi(\not p \not n - 1)\phi_\pi^T(x)\right].$$

[P.Ball, 2005,2006]

$$\begin{aligned}
\phi^A_{\pi(K)}(x) &= \frac{f_{\pi(K)}}{2\sqrt{2N_c}} 6x(1-x)\left[1 + a_1^{\pi(K)} C_1^{3/2}(2x-1) + a_2^{\pi(K)} C_2^{3/2}(2x-1)\right. \\
&\quad \left. + a_4^{\pi(K)} C_4^{3/2}(2x-1)\right],
\end{aligned}$$

$$\begin{aligned}
\phi^P_{\pi(K)}(x) &= \frac{f_{\pi(K)}}{2\sqrt{2N_c}}\left[1 + \left(30\eta_3 - \frac{5}{2}\rho^2_{\pi(K)}\right) C_2^{1/2}(2x-1)\right. \\
&\quad \left. - 3\left\{\eta_3\omega_3 + \frac{9}{20}\rho^2_{\pi(K)}(1 + 6a_2^{\pi(K)})\right\} C_4^{1/2}(2x-1)\right], \\
\phi^T_{\pi(K)}(x) &= \frac{f_{\pi(K)}}{2\sqrt{2N_c}}(1-2x)\left[1\right. \\
&\quad \left. + 6\left(5\eta_3 - \frac{1}{2}\eta_3\omega_3 - \frac{7}{20}\rho^2_{\pi(K)} - \frac{3}{5}\rho^2_{\pi(K)} a_2^{\pi(K)}\right)(1 - 10x + 10x^2)\right]
\end{aligned}$$

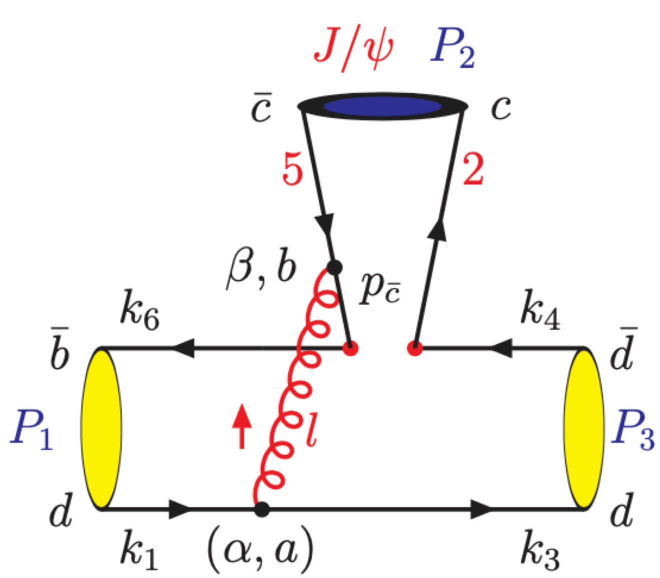
$$\begin{aligned}
C_1^{3/2}(t) &= 3\,t\,, \\
C_2^{1/2}(t) &= \frac{1}{2}(3\,t^2 - 1)\,, \quad C_2^{3/2}(t) = \frac{3}{2}(5\,t^2 - 1)\,, \\
C_4^{1/2}(t) &= \frac{1}{8}(3 - 30\,t^2 + 35\,t^4)\,, \quad C_4^{3/2}(t) = \frac{15}{8}(1 - 14\,t^2 + 21\,t^4)\,.
\end{aligned}$$

$$\begin{aligned}
a_1^\pi &= 0, \quad a_2^{\pi,K} = 0.25 \pm 0.15, \quad a_4^\pi = -0.015, \quad a_1^K = 0.06, \\
\rho_\pi &= m_\pi/m_0^\pi, \quad \rho_K = m_K/m_0^K, \quad \eta_3^{\pi,K,\eta} = 0.015, \quad \omega_3^{\pi,K,\eta} = -3,
\end{aligned}$$

$$m_0^\pi = 1.4 \pm 0.1 \text{ GeV}\,, \quad m_0^K = 1.6 \pm 0.1 \text{ GeV} \qquad \rho_{\pi(K)} = m_{\pi(K)}/m_0^{\pi(K)}$$

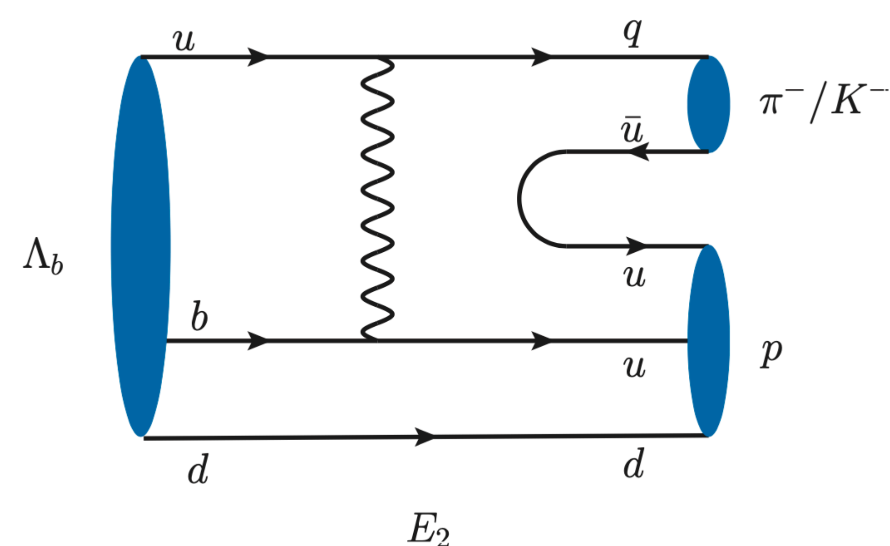
- The 18th W-exchange diagram, contribution from leading-twist LCDAs for S-wave

$$
\begin{aligned}
f_1^{E_{18}} &= G_F \frac{\pi^2}{54\sqrt{3}} f_{\Lambda_b} f_p \int [dx] \int [dx'] \int dy\, [\alpha_s(t^{E_{18}})]^2 \psi_{\Lambda_b}(x) \\
&\times \Big\{ \Big[16 m_0 M_{\Lambda_b}^4 [(C_1 - C_2) V_{ub} V_{ud}^* + ((C_3 + C_9) - (C_4 + C_{10})) V_{tb} V_{td}^*](y-1)(\phi_M^P(y) + \phi_M^T(y)) \\
&\quad + 16 m_0 M_{\Lambda_b}^4 ((C_5 + C_7) - (C_6 + C_8)) V_{tb} V_{td}^* (y-1)(\phi_M^P(y) + \phi_M^T(y)) \Big] \psi_p^V(x') \\
&\quad + \Big[16 m_0 M_{\Lambda_b}^4 [(C_1 - C_2) V_{ub} V_{ud}^* + ((C_3 + C_9) - (C_4 + C_{10})) V_{tb} V_{td}^*](y-1)(\phi_M^P(y) + \phi_M^T(y)) \\
&\quad - 16 m_0 M_{\Lambda_b}^4 ((C_5 + C_7) - (C_6 + C_8)) V_{tb} V_{td}^* (y-1)(\phi_M^P(y) + \phi_M^T(y)) \Big] \psi_p^A(x') \Big\} \\
&\times \frac{1}{16\pi^2} \int b_2 db_2 \int b_3 db_3 \int b_q db_q \int d\theta_1 \int d\theta_2 \exp[-S^{E_{18}}(x, x', y, b, b', b_q)] \\
&\quad \{K_0(\sqrt{C^{E_{18}}}|b_2'|)\theta(C^{E_{18}}) + \frac{\pi i}{2}[J_0(\sqrt{|C^{E_{18}}|}|b_2'|) + i N_0(\sqrt{|C^{E_{18}}|}|b_2'|)]\theta(-C^{E_{18}})\} \int_0^1 \frac{dz_1 dz_2}{z_1(1-z_1)} \\
&\quad \sqrt{\frac{X_2^{E_{18}}}{|Z_2^{E_{18}}|}} \Big\{ K_1(\sqrt{X_2^{E_{18}} Z_2^{E_{18}}})\Theta(Z_2^{E_{18}}) + \frac{\pi}{2}[J_1(\sqrt{X_2^{E_{18}}|Z_2^{E_{18}}|}) + i N_1(\sqrt{X_2^{E_{18}}|Z_2^{E_{18}}|})]\Theta(-Z_2^{E_{18}}) \Big\},
\end{aligned}
$$



6-dimensional integration

sample= $\gtrapprox 10^5$



12-dimensional integration

sample $\gtrapprox 10^{10}$

- Numerical integration, VEGAS, Parallel computing, GPU-accelerated

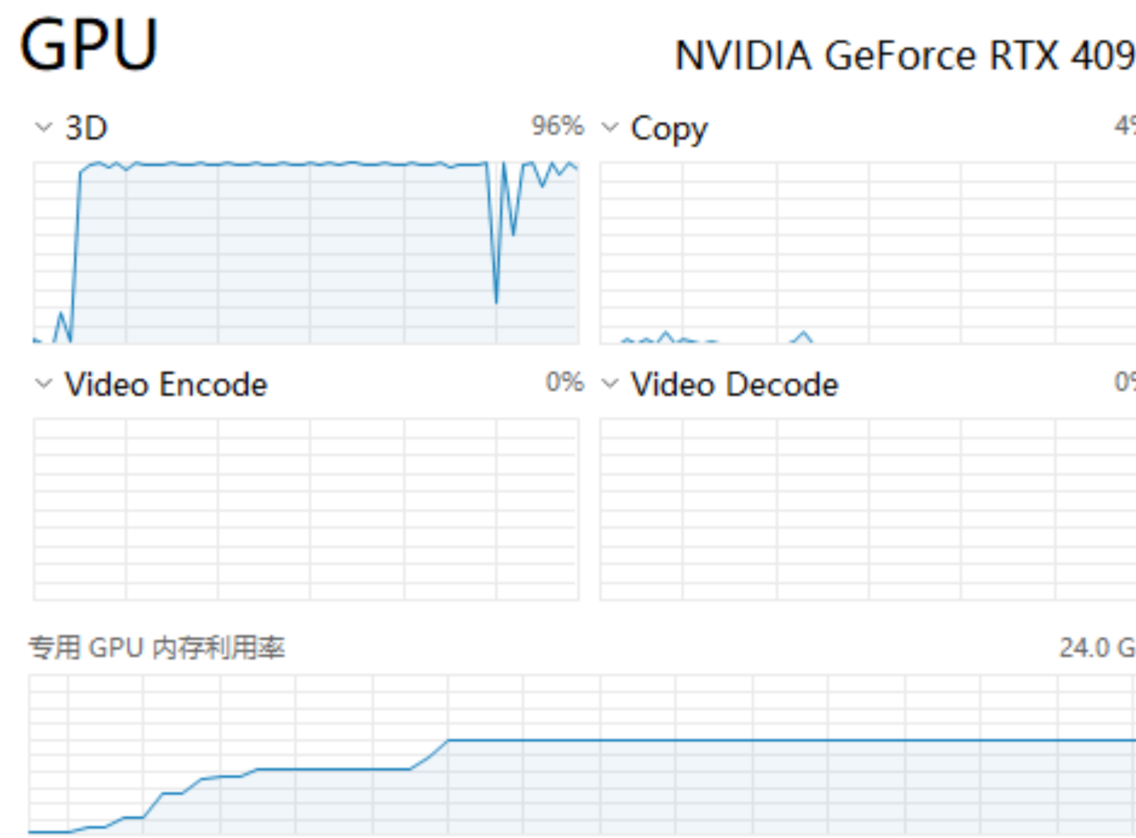
PyTorch torchquad

```
(base) PS C:\Users\ZaynH\Desktop\Lb2pK> conda activate mytorch
(mytorch) PS C:\Users\ZaynH\Desktop\Lb2pK> python .\test.py
resultEII7SwaveReal tensor(0.9127)
resultEII7SwaveImag tensor(0.5339)
resultEII7PwaveReal tensor(-0.2512)
resultEII7PwaveImag tensor(-0.7203)
(mytorch) PS C:\Users\ZaynH\Desktop\Lb2pK> python .\test.py
resultEII7SwaveReal tensor(-2.7334)
resultEII7SwaveImag tensor(1.6898)
resultEII7PwaveReal tensor(-0.8296)
resultEII7PwaveImag tensor(1.4193)
```

sample=3千万

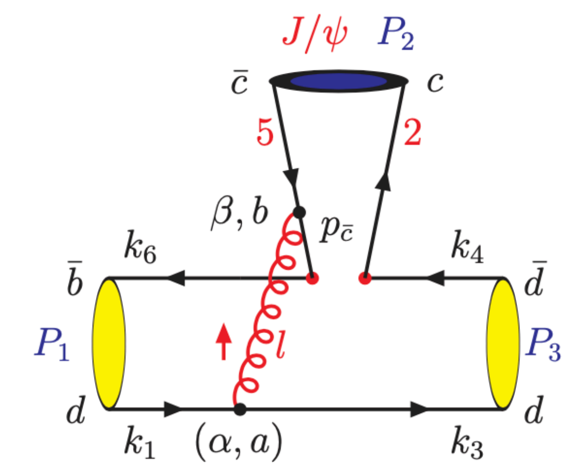
```
(mytorch) PS C:\Users\ZaynH\Desktop\Lb2pK> python .\test.py
resultEII7SwaveReal tensor(0.2935)
resultEII7SwaveImag tensor(1.9887)
resultEII7PwaveReal tensor(-1.0786)
resultEII7PwaveImag tensor(2.7952)
(mytorch) PS C:\Users\ZaynH\Desktop\Lb2pK> python .\test.py
resultEII7SwaveReal tensor(-1.7840)
resultEII7SwaveImag tensor(2.0037)
resultEII7PwaveReal tensor(0.7348)
resultEII7PwaveImag tensor(-1.3421)
(mytorch) PS C:\Users\ZaynH\Desktop\Lb2pK>
```

sample=3亿



- PQCD approach (based on k_T factorization): retain transverse momentum of parton k_T

  propagator $\sim \dfrac{1}{x_1 x_2 Q^2 + \; k_{iT} \; ^2}$



$$\mathscr{A} = \langle M_2 M_3 \;\; \mathscr{H} \;\; B \rangle$$

$$\sim \int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\frac{d^4k_3}{(2\pi)^4}\Psi_B(k_1,\mu)\Psi_2(k_2,\mu)\Psi_3(k_3,\mu)\cdot H(k_1,k_2,k_3,\mu)C_i(\mu)$$

$$\sim \int_0^1 dx_2 dx_2 dx_3 \int \frac{d^2k_{1T}}{(2\pi)^2}\frac{d^2k_{2T}}{(2\pi)^2}\frac{d^2k_{3T}}{(2\pi)^2}\phi_B(x_1,k_{1T},\mu)\phi_2(x_2,k_{2T},\mu)\phi_3(x_3,k_{3T},\mu)\cdot H(x_1,x_2,x_3,k_{1T},k_{2T},k_{3T},\mu)C_i(\mu)$$

$$H(x_i,b_i) \sim \int \frac{d^2k_{1T}}{(2\pi)^2}\frac{d^2k_{3T}}{(2\pi)^2}e^{ib\cdot k_{1T}}e^{ib\cdot k_{3T}}\frac{N_1(x_1,x_2,x_3)}{x_1x_3M_B^2 - \;\; k_{1T}-k_{3T} \;\; ^2}\frac{N_2(x_1,x_2,x_3)}{M_B^2(1-x_3) - \;\; k_{3T} \;\; ^2}$$

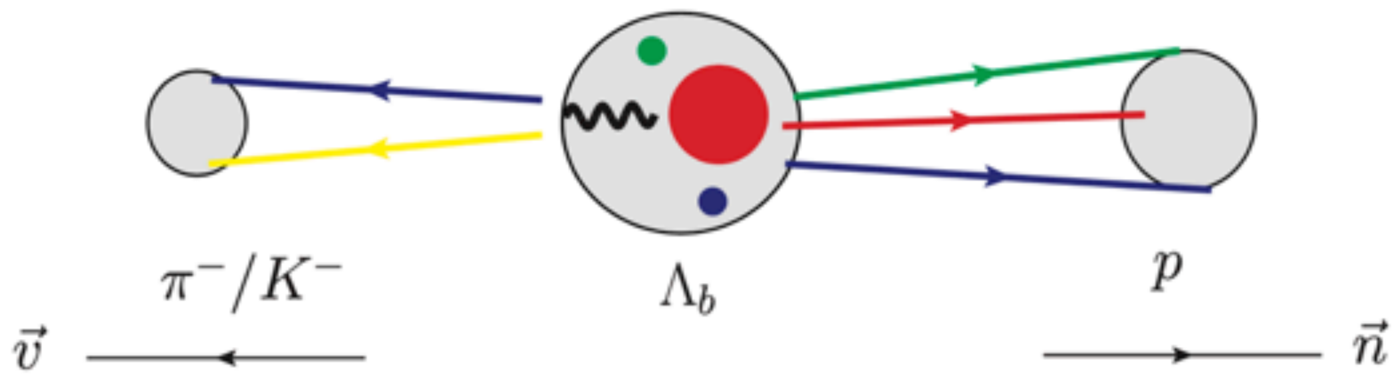
$$\sim N_1(x_1,x_2,x_3)N_2(x_1,x_2,x_3)\cdot\left[K_0(\sqrt{x_1x_3M_B^2}b_1)I_0(\sqrt{(1-x_3)M_B^2}b_3)K_0(\sqrt{(1-x_3)M_B^2}b_1)\Theta(b_3-b_1)+K_0(\sqrt{x_1x_3M_B^2}b_1)I_0(\sqrt{(1-x_3)M_B^2}b_1)K_0(\sqrt{(1-x_3)M_B^2}b_3)\Theta(b_1-b_3)\right]$$

after Fourier tramsform

$$\sim \int_0^1 dx_1 dx_2 dx_3 \int d^2b_1 d^2b_2 d^2b_3 \phi_B(x_1,b_1,\mu)\phi_2(x_2,b_2,\mu)\phi_3(x_3,b_3,\mu)\cdot H(x_1,x_2,x_3,b_1,b_2,b_3,\mu)C_i(\mu)\times\Pi_i S(x_i,b_i)\times S_t(x_i)$$

Two-body non-leptonic Λ_b decays

- Kinematics



$$p = (\frac{M_{\Lambda_b}}{\sqrt{2}}, \frac{M_{\Lambda_b}}{\sqrt{2}}, 0), \quad p' = (\frac{M_{\Lambda_b}}{\sqrt{2}}\eta_1, \frac{M_{\Lambda_b}}{\sqrt{2}}\eta_2, 0), \quad p = (\frac{M_{\Lambda_b}}{\sqrt{2}}(1-\eta_1), \frac{M_{\Lambda_b}}{\sqrt{2}}(1-\eta_2), 0)$$

$$k_1 = ((1-x_3)p^+, (1-x_2)p^-, k_{1T})$$
$$k_2 = (0, x_2 p^-, k_{2T})$$
$$k_3 = (x_3 p^+, 0, k_{3T})$$

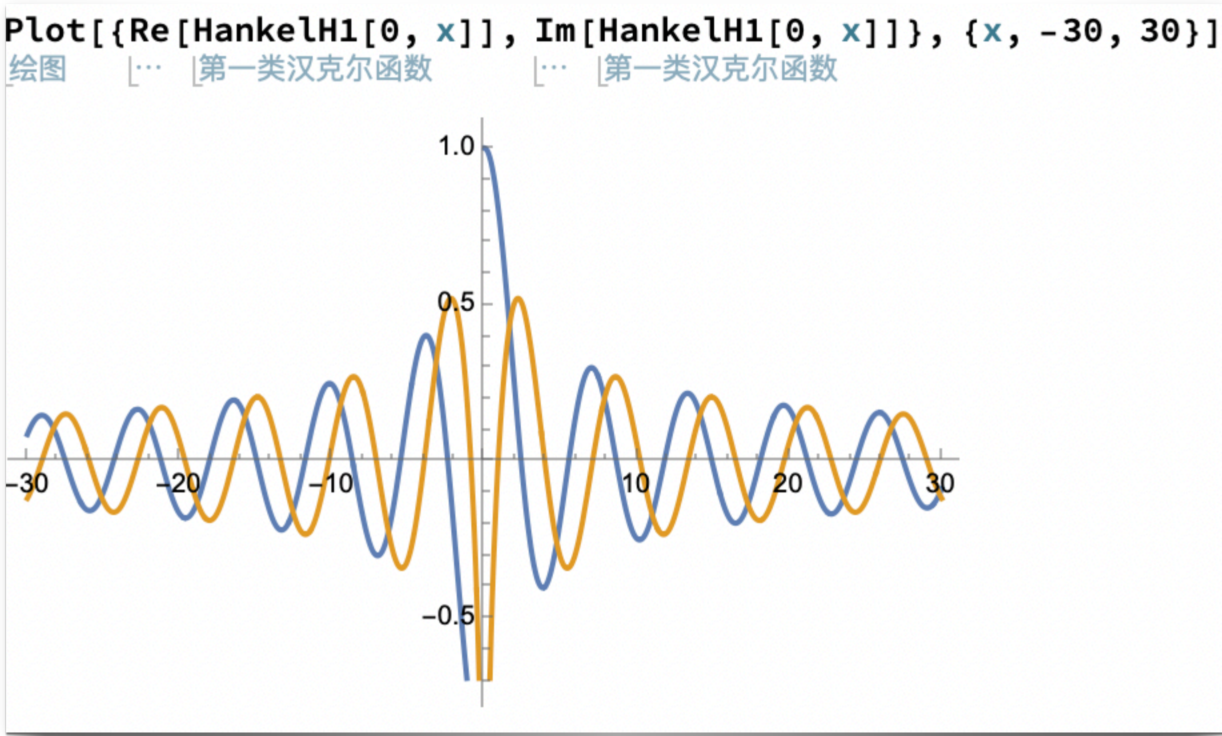
$$k_1 = (p^+, x_1 p^-, \mathbf{k}_{1T}), \quad k_1' = (x_1' p'^+, 0, \mathbf{k}_{1T}), \quad q_1 = (0, y q^-, \mathbf{q}_T),$$
$$k_2 = (0, x_2 p^-, \mathbf{k}_{2T}), \quad k_2' = (x_2' p'^+, 0, \mathbf{k}_{2T}), \quad q_1 = (0, (1-y) q^-, \mathbf{q}_T),$$
$$k_1 = (0, x_3 p^-, \mathbf{k}_{3T}), \quad k_1' = (x_3' p'^+, 0, \mathbf{k}_{3T}).$$

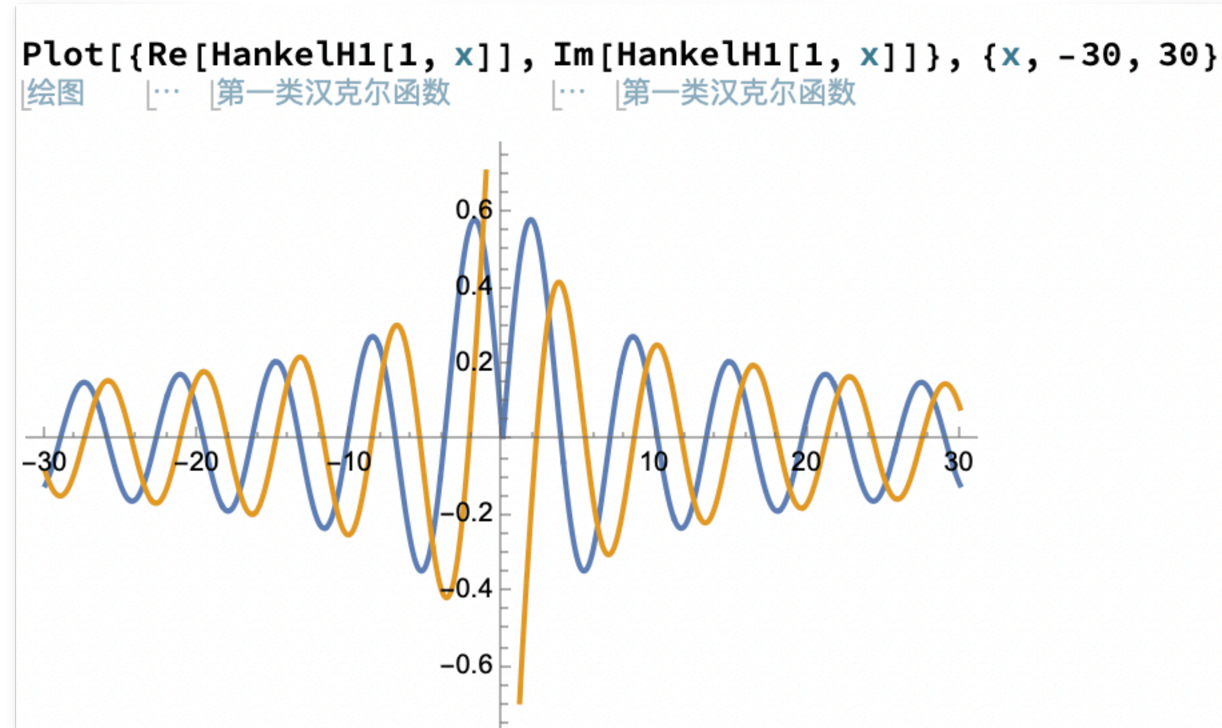
$$\eta_1 = \left(M_{\Lambda_b}^2 - M_{\mathcal{M}}^2 + M_p^2 + \sqrt{(-M_{\Lambda_b}^2 + M_{\mathcal{M}}^2 - M_p^2)^2 - 4M_{\Lambda_b}^2 M_p^2}\right)/(2M_{\Lambda_b}^2)$$
$$\eta_2 = \left(M_{\Lambda_b}^2 - M_{\mathcal{M}}^2 + M_p^2 - \sqrt{(-M_{\Lambda_b}^2 + M_{\mathcal{M}}^2 - M_p^2)^2 - 4M_{\Lambda_b}^2 M_p^2}\right)/(2M_{\Lambda_b}^2)$$

- PQCD formula for two-body decays

$$F_i(q^2) \sim \int_0^1 d[x]d[x']dy \int d^2[b]d^2[b']db_y \phi_{\Lambda_b}([x],[b],\mu) \cdot H([x],[x'],y,[b],[b'],b_y,\mu)C_i(\mu) \cdot \phi_p([x'],[b'],\mu)\phi_M(y,b_y,\mu) \cdot \Pi_i S(x_i,b_i)S_t(x_i)$$

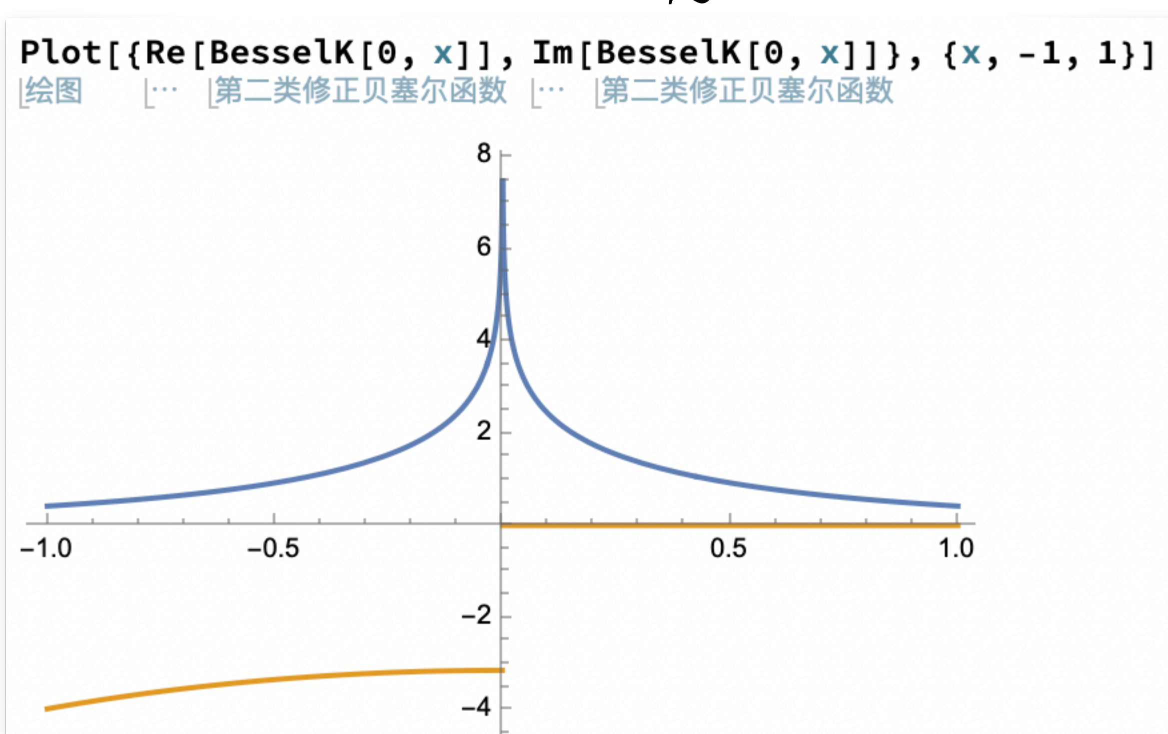
- Bessel functions

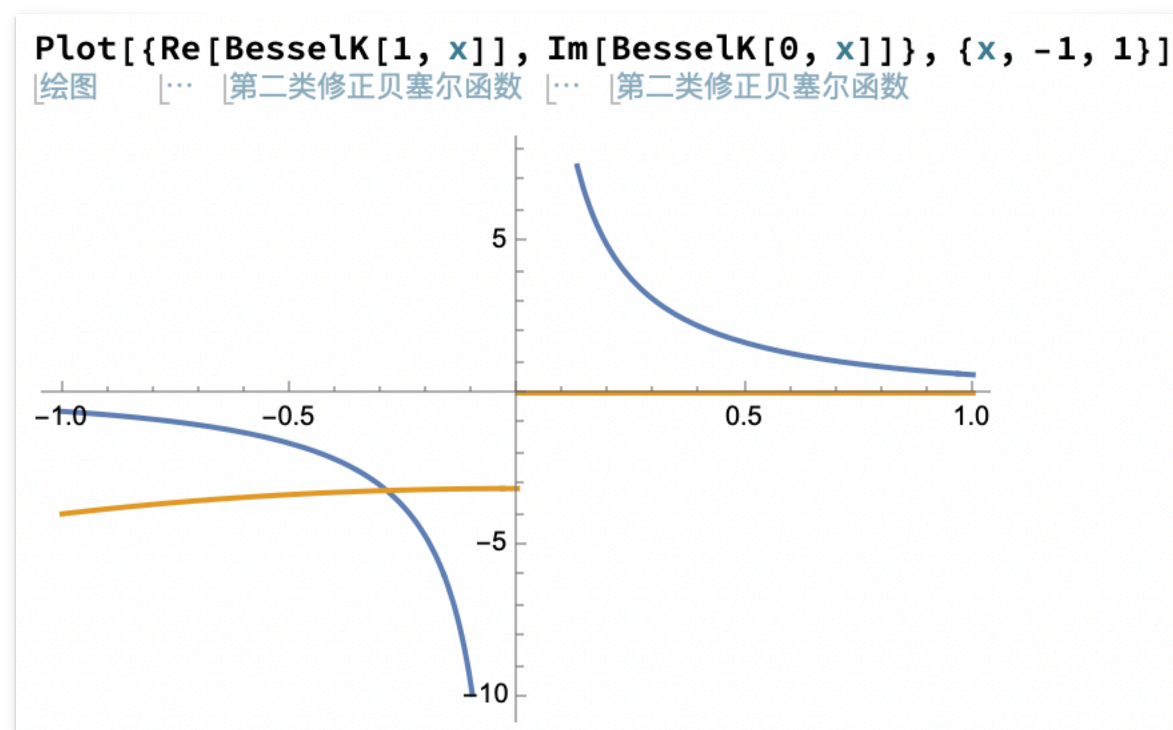




12-dimensional integration
sample $\gtrsim 10^{10}$

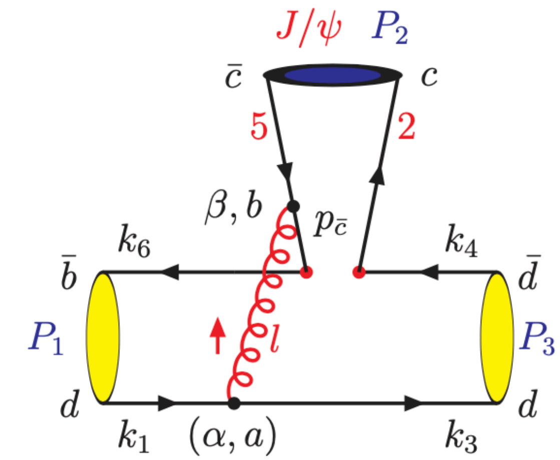
1-dimensional integration
sample5 $\sim$ 6





- PQCD approach (based on k_T factorization): retain transverse momentum of parton k_T

  - propagator $\sim \dfrac{1}{x_1 x_2 Q^2 + \ \ k_{iT}\ \ ^2}$



$$\mathscr{A} = \langle M_2 M_3 \ \ \mathscr{H} \ \ B \rangle$$

$$\sim \int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\frac{d^4k_3}{(2\pi)^4}\Psi_B(k_1,\mu)\Psi_2(k_2,\mu)\Psi_3(k_3,\mu)\cdot H(k_1,k_2,k_3,\mu)C_i(\mu)$$

$$\sim \int_0^1 dx_2 dx_2 dx_3 \int \frac{d^2k_{1T}}{(2\pi)^2}\frac{d^2k_{2T}}{(2\pi)^2}\frac{d^2k_{3T}}{(2\pi)^2}\phi_B(x_1,k_{1T},\mu)\phi_2(x_2,k_{2T},\mu)\phi_3(x_3,k_{3T},\mu)\cdot H(x_1,x_2,x_3,k_{1T},k_{2T},k_{3T},\mu)C_i(\mu)$$

$$H(x_i,b_i) \sim \int \frac{d^2k_{1T}}{(2\pi)^2}\frac{d^2k_{3T}}{(2\pi)^2}e^{ib\cdot k_{1T}}e^{ib\cdot k_{3T}}\frac{N_1(x_1,x_2,x_3)}{x_1x_3M_B^2 - \ \ k_{1T}-k_{3T}\ \ ^2}\frac{N_2(x_1,x_2,x_3)}{M_B^2(1-x_3) - \ \ k_{3T}\ \ ^2}$$

$$\sim N_1(x_1,x_2,x_3)N_2(x_1,x_2,x_3)\cdot\left[K_0(\sqrt{x_1x_3M_B^2}b_1)I_0(\sqrt{(1-x_3)M_B^2}b_3)K_0(\sqrt{(1-x_3)M_B^2}b_1)\Theta(b_3-b_1)+K_0(\sqrt{x_1x_3M_B^2}b_1)I_0(\sqrt{(1-x_3)M_B^2}b_1)K_0(\sqrt{(1-x_3)M_B^2}b_3)\Theta(b_1-b_3)\right]$$

after Fourier tramsform

$$\sim \int_0^1 dx_1dx_2dx_3\int d^2b_1d^2b_2d^2b_3\phi_B(x_1,b_1,\mu)\phi_2(x_2,b_2,\mu)\phi_3(x_3,b_3,\mu)\cdot H(x_1,x_2,x_3,b_1,b_2,b_3,\mu)C_i(\mu)\times\Pi_i S(x_i,b_i)\times S_t(x_i)$$

after resum double-log term

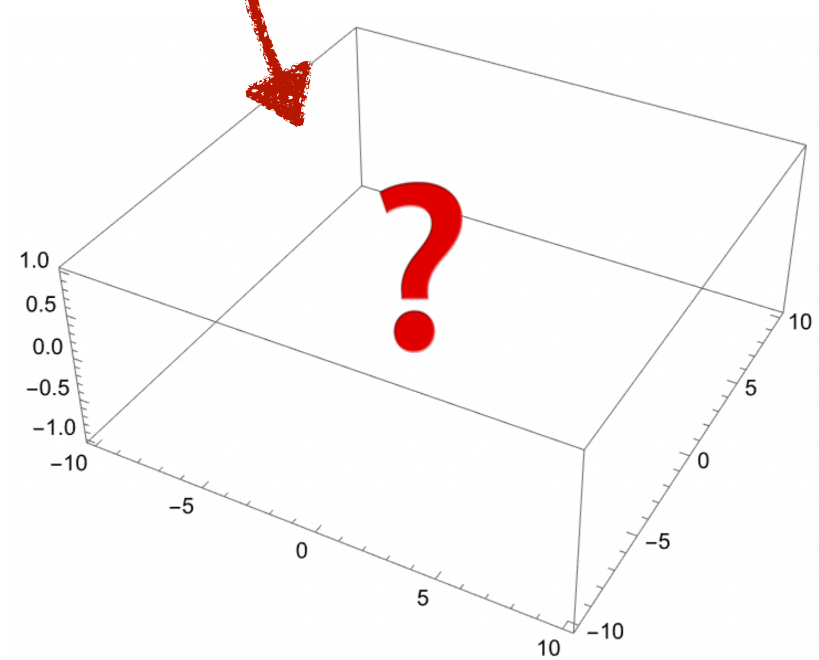
$$\sim \int_0^1 dx_2 dx_2 dx_3 \int \frac{d^2k_{1T}}{(2\pi)^2}\frac{d^2k_{2T}}{(2\pi)^2}\frac{d^2k_{3T}}{(2\pi)^2}\phi_B(x_1,k_{1T},\mu)\phi_2(x_2,k_{2T},\mu)\phi_3(x_3,k_{3T},\mu)\cdot H(x_1,x_2,x_3,k_{1T},k_{2T},k_{3T},\mu)C_i(\mu)\ \times\Pi_i S(x_i,k_{iT})$$

Determine $S(Q, k_{iT})$ from $S(Q, b_i)$

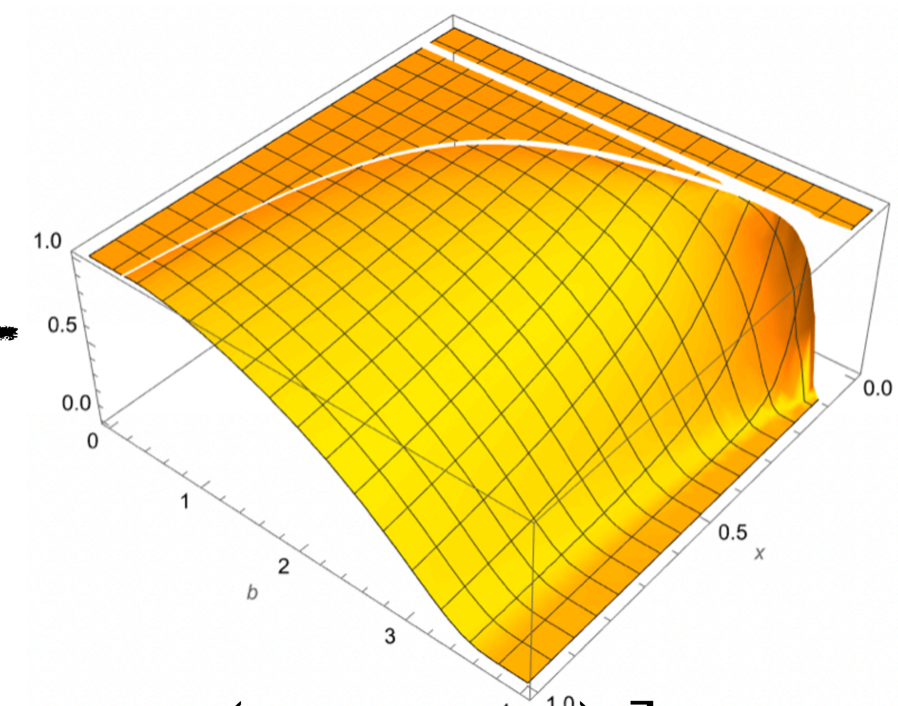
$$S(Q, b_i, \alpha, \beta, \ldots) = \int \frac{d^2k_T}{(2\pi)^2} e^{ib\cdot k_T} S(Q, k_T, \alpha, \beta, \ldots)$$

$$= \int_0^\infty k_T dk_T \int_0^{2\pi} d\theta e^{ibk_T cos\theta} S(Q, k_T, \alpha, \beta, \ldots)$$

$$= \int_0^\infty dk_T 2\pi k_T J(0, bk_T) S(Q, kT, \alpha, \beta, \ldots)$$



fit $\alpha, \beta, \gamma \ldots$



$$S(Q, k_T) = \gamma(Q) \cdot Exp\left[-\alpha(Q) ln^2\left(\frac{ln(Q/\Lambda_{QCD}}{ln(k_T/\Lambda_{QCD}}\right) + \beta(Q) ln\left(\frac{ln(Q/\Lambda_{QCD}}{ln(k_T/\Lambda_{QCD}}\right)\right]$$

$$S(Q, k_T) \rightarrow 0 \qquad when\ k_T \rightarrow 0\ or\ \infty$$

Summary

- Heavy baryon physics plays an important role
- PQCD approach is powerful to explain and predict measurements
- But we still have a long way to realize
  ◉Sudakov factor in k_T space; ◉Threshold Sudakov factor; ◉factorization; ◉……