QCD LCDA of Heavy Mesons from bHQET

魏焰冰

北京工业大学

兰州 · 兰州大学

2023 年 11 月 04 日

Beneke, Finauri, Vos and **YBW**: 2305.06401

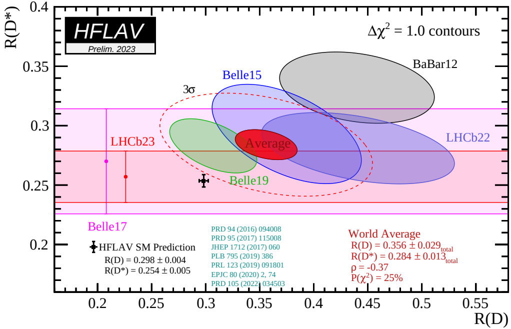
Outline

- ⁎ QCD LCDA of heavy mesons
- ⁎ Introduction to bHQET
- ⁎ Factorization of QCD LCDA
- ⁎ Numeric applications

New Physics

New physics beyond the SM

- Direct search: new particles
- Indirect search: $R(D^{(*)})$, $|V_{ub}|$, $|V_{cb}|$, $\cdots$



BaBar: [12']

Evidence for an excess of $\bar{B} \to D^{(*)}\tau^-\bar{\nu}_\tau$ decays

BaBar Collaboration • J.P. Lees (Annecy, LAPP) et al. (May, 2012)

Published in: *Phys.Rev.Lett.* 109 (2012) 101802 • e-Print: 1205.5442

pdf links DOI cite claim

reference search 1,133 citations

Belle-II, HL-LHC

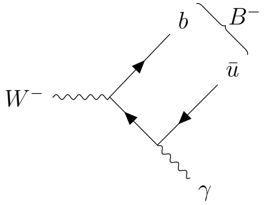
Non-perturbative LCDA of hadrons

Light-cone distribution amplitudes (LCDA): non-perturbative physics

- Calculate with LQCD [LPC collaboration, 22']
- Extract from the experiments: clean process

Rare decay $W^- \to B^- \gamma$ [Grossman, König and Neubert, 15']

$$A(W^- \to B^- \gamma) = \int_0^1 du\, T(u)\, \phi(u)$$



- Hard function T: m_W
- non-perturbative LCDA ϕ: below m_Q
- Power correction in m_Q/m_W: less than 1%

QCD LCDA of heavy meson ϕ

For fast moving meson, we need light-cone vectors n_- and n_+

$$n_-^2 = n_+^2 = 0, \qquad n_- \cdot n_+ = 2$$

The heavy meson QCD LCDA [Braun and Filyanov, 89']

$$\langle H(p_H)|\bar{Q}(0)\not{n}_+\gamma^5[0, tn_+]q(tn_+)|0\rangle = -if_H n_+ \cdot p_H \int_0^1 du\, e^{iutn_+\cdot p_H}\, \phi(u)$$

- Only large $n_+ \cdot p_H$ component appears in the hard function T
- ϕ: two scales m_Q and $\Lambda_{\rm QCD}$
- Large log resummation

Factorization of the QCD LCDA

- Momentum space matching of QCD to HQET @ NLO [Ishaq, Jia, Xiong and Yang, 19']

$$\phi(u) = \mathcal{J}(u,\omega) \otimes \varphi_+(\omega)\,, \qquad u \sim \Lambda_{\rm QCD}/m_Q$$

  Tree level

$$\mathcal{J}^{(0)}(u,\omega) = \delta\Big(u - \frac{\omega}{\omega + m_Q}\Big)$$

  ω and m_Q have different power counting

  NLO from [Bell and Feldmann, 08']

- Coordinate space matching @NLO [Zhao, 19']

Factorization of the QCD LCDA

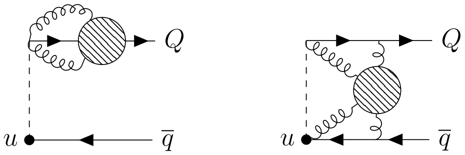
The leading-twist heavy-meson LCDA in (b)HQET [Grozin and Neubert, 96']

$$\langle H_v|\bar{h}_v(0)\not{n}_+\gamma^5[0,tn_+]q_s(tn_+)|0\rangle = -i\tilde{f}_H(\mu)\, n_+\cdot v\int_0^\infty d\omega\, e^{i\omega t n_+\cdot v}\varphi_+(\omega)$$

The factorization formula

$$\phi(u) = \begin{cases} \mathcal{J}_p(u,\omega)\otimes\varphi_+(\omega)\,, & u\sim\Lambda_{\rm QCD}/m_Q\,, \\ \mathcal{J}_{\rm tail}(u)\,, & u\sim 1\,, \end{cases}$$

- The jet function $\mathcal{J}$: m_Q, φ_+: $\Lambda_{\rm QCD}$



Momentum modes

A given momentum in the light-cone coordinate

$$l^\mu = (n_+ l,\ l_\perp,\ n_- l)$$

In the rest frame of the heavy meson

- heavy quark: $(1,1,1)m_Q$
- light degree of freedom: $(1,1,1)\Lambda_{\mathrm{QCD}}$

For a boosted heavy meson: $b \sim \frac{m_Q}{Q}$

- heavy quark (hard-collinear): $(\frac{1}{b}, 1, b)m_Q = (Q, m_Q, \frac{m_Q^2}{Q})$
- light degree of freedom (soft-collinear): $(\frac{Q}{m_Q}, 1, \frac{m_Q}{Q})\Lambda_{\mathrm{QCD}}$

Introduction to SCET

The collinear fields [Bauer, Fleming, Pirjol and Stewart, 01'], [Beneke, Chapovsky, Diehl and Feldmann, 02'], [Becher, Broggio and Ferroglia, 14']

$$q(x) = \frac{\not{n}_-\not{n}_+}{4} q(x) + \frac{\not{n}_+\not{n}_-}{4} q(x) = \xi(x) + \eta(x)$$

The power counting of the soft and collinear fields $\lambda = \sqrt{m_Q/Q}$

$$\xi \sim \lambda, \quad \eta \sim \lambda^2, \quad q_s \sim \lambda^3, \quad A_s \sim \lambda^2$$
$$n_+ A_c \sim 1, \quad A_{\perp c} \sim \lambda, \quad n_- A_c \sim \lambda^2,$$

The Lagrangian also has definite power counting

$$\mathcal{L}_{\rm SCET} = \mathcal{L}^{(0)} + \mathcal{L}_\xi^{(1)} + \mathcal{L}_\xi^{(2)} + \mathcal{L}_{\xi q}^{(1)} + \mathcal{L}_{\xi q}^{(2)} + \cdots$$

The QCD operator defining ϕ

$$\bar{Q}(0)\not{n}_+\gamma^5[0, tn_+]q(tn_+) = \bar{\xi}^{(Q)}(0)\,\not{n}_+\gamma^5\,[0, tn_+]\xi(tn_+)$$

Boosted HQET

The heavy quark field in bHQET [Fleming, Hoang, Mantry and Stewart, 07'], [Dai, Kim and Leibovich, 21']

$$h_n(x) \equiv \sqrt{\frac{2}{n_+ v}} e^{i m_Q v\cdot x} \xi^{(Q)}(x)$$

The relation between the HQET and bHQET heavy quark field

$$h_v(x) = \sqrt{\frac{n_+ v}{2}} \frac{1+\not{v}}{2} \left(1 - \frac{\not{n}_+}{2} \frac{i\not{D}_\perp + m_Q \not{v}_\perp - m_Q}{i n_+ D + m_Q n_+ v}\right) h_n(x)$$

The bHQET Lagrangian could be derived from the HQET one

$$\mathcal{L}_{\rm HQET} = \bar{h}_v(x) iv\cdot D h_v(x) = \underbrace{\bar{h}_n(x) iv\cdot D \frac{\not{n}_+}{2} h_n(x)}_{\mathcal{L}_{\rm bHQET}} + \mathcal{O}(\lambda)$$

Leading power bHQET operators

The operators must preserve the reparameterization invariance [Beneke, Chapovsky, Diehl and Feldmann, 02']

$$n_- \to \alpha\, n_- \,, \qquad n_+ \to \frac{1}{\alpha}\, n_+ \quad (\alpha \text{ real})$$

Then one find out the LP operators

$$\hat{\mathcal{O}}_k = \frac{1}{n_+ v}\sqrt{\frac{n_+ v}{2}}\bar{h}_n \not{n}_+ \left(n_+ v \frac{i\not{D}_\perp}{in_+ D}\right)^k \gamma^5 \xi_{sc}$$

From the EOM of the light quark

$$\not{n}_+ i\not{D}_\perp \frac{1}{in_+ D} i\not{D}_\perp \xi_{sc} = -\not{n}_+ in_- D \xi_{sc}$$

Only operators $\hat{\mathcal{O}}_0$ and $\hat{\mathcal{O}}_1$ will appear at LP

Boost invariance: decay constant

- From QCD to HQET

$$\underbrace{\bar{Q}\gamma^\alpha\gamma^5 q}_{f_H} = C_V(\mu)\,\underbrace{\bar{h}_v\gamma^\alpha\gamma^5 q_s}_{\tilde{f}_H} + C_S(\mu)\,v^\alpha\,\underbrace{\bar{h}_v\gamma^5 q_s}_{\tilde{f}_H}$$

Then one derive the relation

$$f_H = K(\mu)\,\tilde{f}_H(\mu)$$

- From SCET to bHQET

$$\underbrace{\bar{\xi}_C^{(Q)}\,\not{n}_+\gamma^5\xi_C}_{f_H} = C_+(\mu)\,\underbrace{\bar{h}_n\,\not{n}_+\gamma^5\xi_{sc}}_{\tilde{f}_H} + C_-(\mu)\,\underbrace{\bar{h}_n\,\not{n}_+\frac{i\not{D}_\perp}{in_+D}\gamma^5\xi_{sc}}_{\tilde{f}_H}$$

Boost invariance

$$f_H = K(\mu)\,\tilde{f}_H(\mu)$$

Inhomogeneous power counting of ϕ

Naive factorization formula $\delta \sim \Lambda_{\rm QCD}/m_Q$

$$\phi(u) = \mathcal{J}(u,\omega) \otimes \varphi_+(\omega)$$

- If u has definite power counting: $u \sim \mathcal{O}(1)$
- At the endpoint region we have $u \sim \mathcal{O}(\delta)$

We should distinguish the $u \sim \mathcal{O}(1)$ and $u \sim \mathcal{O}(\delta)$ cases

$$\phi(u) \sim \begin{cases} \delta^{-1}\,, & \text{for } u \sim \delta \quad (\text{“peak”}) \\ 1\,, & \text{for } u \sim 1 \quad (\text{“tail”}) \end{cases}$$

The light quark carries only a small fraction $\Lambda_{\rm QCD}/m_H$ of the total momentum

The peak region $u \sim \mathcal{O}(\delta)$

The operator level matching equation

$$\mathcal{O}_C(u) = \int_0^\infty d\omega\, \mathcal{J}_p(u,\omega)\, \mathcal{O}_h(\omega)$$

The SCET and bHQET non-local operators are

$$\mathcal{O}_C(u) = \int \frac{dt}{2\pi} e^{-iutn_+p} \bar{\xi}^{(Q)}(0)\, \not{n}_+ \gamma^5\, [0, tn_+] \xi(tn_+)$$

$$\mathcal{O}_h(\omega) = \frac{1}{m_H} \int \frac{dt}{2\pi} e^{-i\omega tn_+ v} \sqrt{\frac{n_+ v}{2}} \bar{h}_n(0)\, \not{n}_+ \gamma^5\, [0, tn_+] \xi_{sc}(tn_+) \sim \hat{\mathcal{O}}_0$$

$\mathcal{O}_C$ and $\mathcal{O}_h$ have the same IR but different UV behaviour

The hard-clooinear jet function up to NLO is

$$\mathcal{J}_p(u,\omega) = \theta(m_Q - \omega)\delta\left(u - \frac{\omega}{m_Q}\right)\left[1 + \frac{\alpha_s C_F}{4\pi}\left(\frac{L^2}{2} + \frac{L}{2} + \frac{\pi^2}{12} + 2\right) + \mathcal{O}(\alpha_s^2)\right]$$

with $L = \ln \frac{\mu^2}{m_Q^2}$

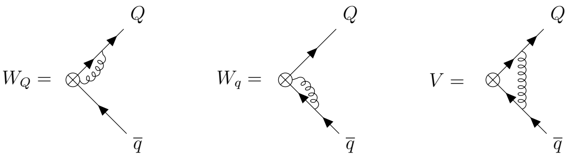
The peak region: matching procedure

On-shell partonic states

$$\langle Q(p_Q)\bar{q}(p_q)|\mathcal{O}_C(u)|0\rangle_{\rm SCET} = \int_0^\infty d\omega\, \mathcal{J}_p(u,\omega)\langle Q(p_Q)\bar{q}(p_q)|\mathcal{O}_h(\omega)|0\rangle_{\rm bHQET}$$

The SCET matrix element

$$\langle\mathcal{O}_C(u)\rangle_{\rm SCET} = \frac{1}{n_+ p_H}\bar{u}(p_Q)\not{n}_+\gamma^5 v(p_q)\left\{\delta(u-s) + \frac{\alpha_s C_F}{4\pi}M^{(1)}(u,s)\right\}$$



Perform the UV renormalization with the ERBL kernel, $M^{(1)}$ only has IR divergence [Lepage and Brodsky, 79'], [Efremov and Radyushkin, 80']

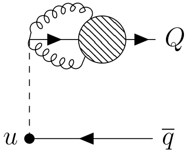
The peak region: matching procedure

The bHQET matrix element

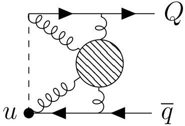
$$\langle \mathcal{O}_h(\omega)\rangle_{\rm bHQET} = \frac{1}{n_+ p_H}\bar{u}(p_Q)\not{n}_+\gamma^5 v(p_q)\left\{\delta\left(\frac{n_+ p_q}{n_+ v} - \omega\right) + \frac{\alpha_s C_F}{4\pi} N^{(1)}\left(\omega, \frac{n_+ p_q}{n_+ v}\right)\right\}$$

Perform the UV renormalization with the LN kernel, $N^{(1)}$ only has IR divergence [Lange and Neubert, 03']

- The UV renormalized $N^{(1)}$ cancels the IR contribution of $M^{(1)}$
- The jet function if free of both UV and IR divergences



peak region



tail region

The tail region: matching procedure

The matching coefficient will not dependent on ω, the SCET operator will match onto local operators

$$\mathcal{O}_C(u) = \mathcal{J}_+(u)\mathcal{O}_+ + \mathcal{J}_-(u)\mathcal{O}_-$$

The SCET matrix element starts at NLO

$$\langle \mathcal{O}_C(u) \rangle_{\rm SCET} = \frac{\alpha_s C_F}{4\pi} \sum_{\pm} M_{\pm}^{(1)}(u) \langle \mathcal{O}_\pm \rangle$$

The jet function at NLO

$$\mathcal{J}_{\rm tail}(u) = \mathcal{J}_+(u) + \mathcal{J}_-(u) = \frac{\alpha_s C_F}{4\pi} \frac{2\bar{u}}{u} \Big((1+u)[L - 2\ln u] - u + 1 \Big)$$

- $\mathcal{J}_{\rm tail}$ will depend on both u and m_Q

Large log resummation of QCD LCDA

- Evolve the HQET LCDA from 1 GeV to m_Q
- Evolve the QCD LCDA from m_Q to Q

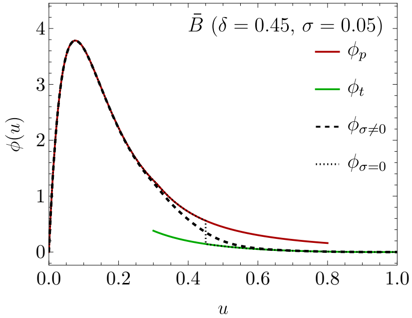
HQET LCDA with radiative tail [Lee and Neubert, 05']

$$\varphi_+(\omega;\mu_s) = \left(1 + \frac{\alpha_s(\mu_s) C_F}{4\pi}\left[\frac{1}{2} - \frac{\pi^2}{12}\right]\right)\varphi_+^{\text{mod}}(\omega;\mu_s) + \theta(\omega - \sqrt{e}\mu_s)\varphi_+^{\text{asy}}(\omega;\mu_s)$$

Two parameter models

$$\begin{aligned}
\varphi_+^{(\text{I})}(\omega;\mu_s) &= \left[1 - \beta + \frac{\beta}{2-\beta}\frac{\omega}{\omega_0}\right]\varphi_+^{\exp}\left(\omega,(1-\beta/2)\omega_0;\mu_s\right), \quad &&\text{for} \quad 0 \le \beta \le 1\,,\\
\varphi_+^{(\text{II})}(\omega;\mu_s) &= \frac{(1+\beta)^\beta}{\Gamma(2+\beta)}\left(\frac{\omega}{\omega_0}\right)^\beta \varphi_+^{\exp}\left(\omega,\frac{\omega_0}{1+\beta};\mu_s\right), &&\text{for} \quad -\frac{1}{2} < \beta < 1\,,\\
\varphi_+^{(\text{III})}(\omega;\mu_s) &= \frac{\sqrt{\pi}}{2\Gamma(3/2+\beta)} U\left(-\beta, \frac{3}{2}-\beta, (1+2\beta)\frac{\omega}{\omega_0}\right)\\
&\quad \times \varphi_+^{\exp}\left(\omega, \frac{\omega_0}{1+2\beta};\mu_s\right), &&\text{for} \quad 0 \le \beta < \frac{1}{2}
\end{aligned}$$

Initial condition of QCD LCDA at $\mu = m_Q$



Merging the tail and peak contributions

$$\phi(u) = \begin{cases} \phi_p(u) = \mathcal{J}_p(u,\omega) \otimes \varphi_+(\omega) \\ \phi_t(u) = \mathcal{J}_{\text{tail}}(u) \end{cases}$$

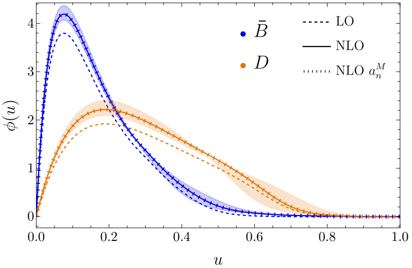
$$\phi_p(u)|_{u\sim1} = \phi_t(u)|_{u\ll1}$$

The Gegenbauer parameterization

$$\phi(u) = 6u\bar{u}[1 + \sum_{n=1}^{\infty} a_n(\mu) C_n^{(3/2)}(2u-1)]$$

with a_n from the merged LCDA

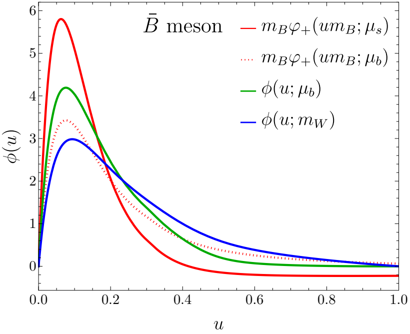
$$a_n^{\bar{B}}(\mu_b) = \{-1.08, 0.83, -0.51, 0.28, \dots\}$$



Evolution to the hard scale

Evolve of the Gegenbauer moments

$$\frac{a_n(\mu_h)}{a_n(\mu)} = \left(\frac{\alpha_s(\mu_h)}{\alpha_s(\mu)}\right)^{\frac{\gamma_n}{2\beta_0}}$$



Evolution steps: $\varphi_+(\mu_s) \to \varphi_+(m_Q) \to \phi(m_Q) \to \phi(Q)$

$$a_n^{\bar{B}}(\mu_b) = \{-1.082, 0.826, -0.513, 0.288, -0.157, 0.078, -0.030, \dots\}$$
$$a_n^{\bar{B}}(m_W) = \{-0.826, 0.542, -0.302, 0.156, -0.079, 0.037, -0.014, \dots\}$$

$W \to B\gamma$

The branch ratio

$$\mathrm{Br}(W \to B\gamma) = \frac{\Gamma(W \to B\gamma)}{\Gamma_W} = \frac{\alpha_{\rm em} m_W f_B^2}{48 v^2 \Gamma_W} |V_{ub}|^2 \left(|F_1^B|^2 + |F_2^B|^2 \right)$$

with

$$F_{1,2}^B \sim \int_0^1 du \, T_{1,2}(u) \, \phi(u)$$

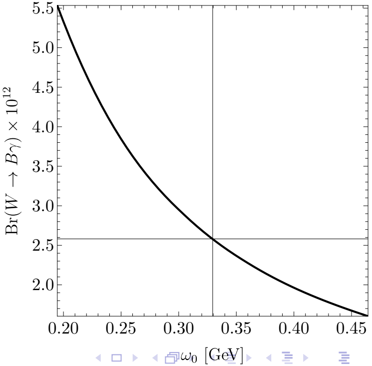
Without $\ln \Lambda_{\rm QCD}/m_Q$ Resummation

$$\mathrm{Br} = (1.99 \pm 0.17_{\rm in} \, {}^{+0.03}_{-0.06}\,{}_{\mu_h} \, {}^{+2.48}_{-0.80}\,{}_{\lambda_B}) \cdot 10^{-12}$$

Our result

$$\mathrm{Br} = (2.58 \pm 0.21_{\rm in} \, {}^{+0.05}_{-0.08}\,{}_{\mu_h} \, {}^{+2.95}_{-0.98}\,{}_{\lambda_B}) \cdot 10^{-12}$$

30% increase



Summary

※ Introduction to the SCET and bHQET

※ Match QCD LCDA to the HQET LCDA

- Peak region: $\mathcal{O}_C(u) = \mathcal{J}_p(u, \omega) \otimes \mathcal{O}_h(\omega)$
- Tail region: $\mathcal{O}_C(u) = \mathcal{J}_+(u)\mathcal{O}_+ + \mathcal{J}_-(u)\mathcal{O}_-$

※ $W^- \to B^-\gamma$ decay: 30% enhancement

Thank you!