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ZHENGZHOU UNIVERSITY

The subleading-twist LCDA of B-meson

ShuMan Hu (胡书曼)

LanZhou University

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In collaboration with Wei Wang, Ji Xu, Shuai Zhao

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arXiv:2308.13977

OUTLINE

1. Introduction to LaMET
2. Our work
3. Summary
4. Outlook

1 Introduction to LaMET

- Hard scattering cross sections of hadrons can be calculated as the convolution of the basic parton scattering cross sections and parton distribution function.

➢ scattering cross sections can be calculated in perturbation theory.

➢ parton distribution function cannot be evaluated in perturbation theory.

In 2013, a new approach was proposed which is LaMET.

light-cone correlation function ⟶ static correlation function in large-momentum hadron

Lorentz boost

✓ [X.Ji, Sci.China Phys.Mech.Astron.57 (2014)]
✓ [Phys.Rev.Lett.110.262002]

- The parton physics can be evaluated in lattice QCD by LaMET.
- The heavy flavor physics in LaMET frame, e.g., B-meson DAs.
- Quasi-DAs are defined with matrix elements of equal-time nonlocal operators.

"The quasi-DA and its light-cone counterpart are related by a matching relation and the matching coefficient can be calculated with perturbative QCD."

quasi DA ⟶ LCDA

- LCDA : light-cone distribution amplitude

✓ the momentum distribution of quarks and anti-quarks inside a meson

✓ cannot be evaluated in perturbation theory

✓ cannot be simulated directly on the lattice

02 Our work

[Phys. Rev. D 102, 011502(R)]

- In this paper, a new method for the model-independent determination of the light-cone distribution amplitude of the B-meson in heavy quark effective theory (HQET) is proposed, and the perturbative matching coefficient is derived.

The LCDAs of B-meson: inherent parts of factorization theorems

↓

the leading-twist LCDA : dominant contribution in the heavy-quark expansion

- Considering only the leading power contribution is not accurate.

- ✓ The higher-twist distribution amplitudes give rise to power-suppressed contributions to B decays. [Phys. Rev. D 89, 094004 (2014)] , [Adv.High Energy Phys. 2022 (2022) 2755821]
- ✓ The utility of QCD factorization theorem depends on the possibility to estimate the power corrections involving higher-twist DAs. [Phys. Rev. D 92, 074044 (2015)]
- ✓ The subleading-twist LCDA governs the leading-power contribution to $B \to D$ form factors. [JEHP05, 024 (2022)]
- ✓ The structure of subleading-twist DA is simpler than assumed. [JHEP0804:061 (2008)]

→ the subleading-twist LCDA and quasi-DA of B-meson

[arXiv:2308.13977]

➢ The B-meson LCDA in coordinate space

[Grozin, Neubert, 1997; Beneke, Feldmann, 2000]

$$\langle 0\,|\bar{q}_\beta(\eta n_+)W(\eta n_+,0)h_{v\alpha}(0)|\,\bar{B}(v)\rangle = -\frac{i\tilde{f}_B(\mu)M}{4}\left[\frac{1+\not{v}}{2}\left\{2\tilde{\phi}_B^+(\eta,\mu)+\left(\tilde{\phi}_B^-(\eta,\mu)-\tilde{\phi}_B^+(\eta,\mu)\right)\frac{\not{n}_+}{n_+\cdot v}\right\}\gamma_5\right]_{\alpha\beta}$$

the leading-twist LCDA

the subleading-twist LCDA

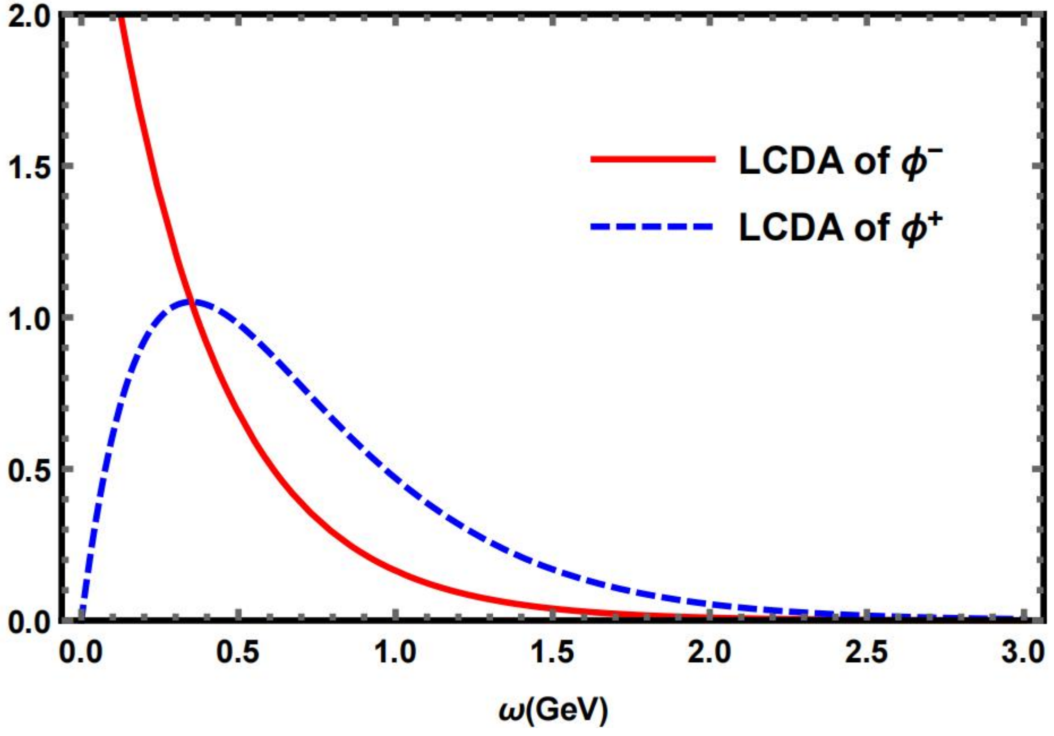
the subleading-twist LCDA: $$\tilde{\phi}_B^-(\eta,\mu) = \frac{1}{i\tilde{f}_B M v^+}\langle 0\,|\bar{q}(\eta n_+)\not{n}_-\gamma_5 W(\eta n_+,0)h_v(0)|\,\bar{B}(v)\rangle$$

Fourier transformation

$$\phi_B^-(\omega,\mu) = \frac{v^+}{2\pi}\int_{-\infty}^{+\infty} d\eta\, e^{i\omega v^+\eta}\,\tilde{\phi}_B^-(\eta,\mu)$$ in momentum space

➢ The exponential models of B-meson LCDA in momentum space

$$\phi_B^+(\omega) = \frac{\omega}{\omega_0^2} e^{-\frac{\omega}{\omega_0}} \qquad \phi_B^-(\omega) = \frac{1}{\omega_0} e^{-\frac{\omega}{\omega_0}}$$



[Phys.Rev. D55 (1997) 272-290]
[Phys. Rev. D 69, 034014(2004)]
[J.Phys.Conf.Ser.1690 (2020)1, 012081]

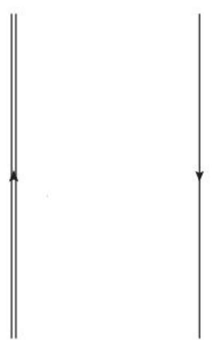
➢ the subleading-twist LCDA of B-meson

$$\phi_B^-(\omega,\mu) = v^+ \int_{-\infty}^{+\infty} \frac{d\eta}{2\pi}\, e^{i\omega v^+ \eta} \frac{\langle 0\,|\bar{q}(\eta n_+)\not{n}_- \gamma_5 W(\eta n_+,0) h_v(0)|\,\bar{B}(v)\rangle}{\langle 0\,|\bar{q}(0)\not{n}_- \gamma_5 h_v(0)|\,\bar{B}(v)\rangle}$$

➢ the suleading-twist quasi-DA of B-meson

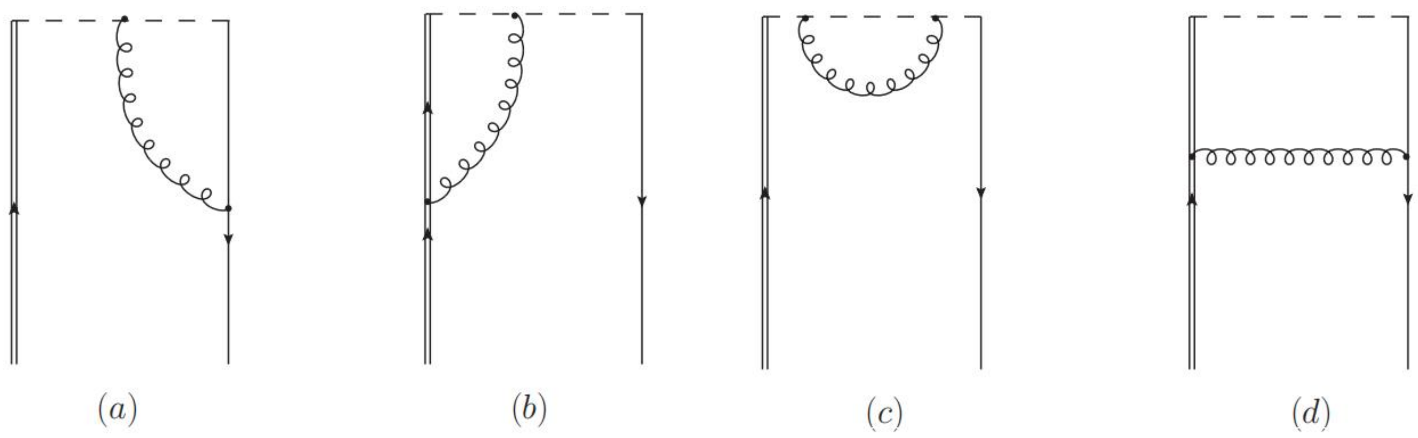
$$\varphi_B^-(\xi,\mu) = v^z \int_{-\infty}^{+\infty} \frac{d\tau}{2\pi}\, e^{i\xi v^z \tau} \frac{\langle 0\,|\bar{q}(\tau n_z)(\gamma^t-\gamma^z)\gamma_5 W(\tau n_z,0) h_v(0)|\,\bar{B}(v)\rangle}{\langle 0\,|\bar{q}(0)(\gamma^t-\gamma^z)\gamma_5 h_v(0)|\,\bar{B}(v)\rangle}$$

- tree-level



- In calculation process, we replace B-meson state with a heavy b quark plus a off-shell light quark.

- one-loop



- To determine the perturbative matching coefficient entering the hard-collinear factorization formula for quasidistribution amplitude.

➢ the hard-collinear factorization formula

$$\varphi_B^-(\xi,\mu) = \int_0^\infty d\omega H(\xi,\omega,v^z,\mu)\,\phi_B^-(\omega,\mu) + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}}{v^z\xi}\right)$$

➢ the matching coefficient

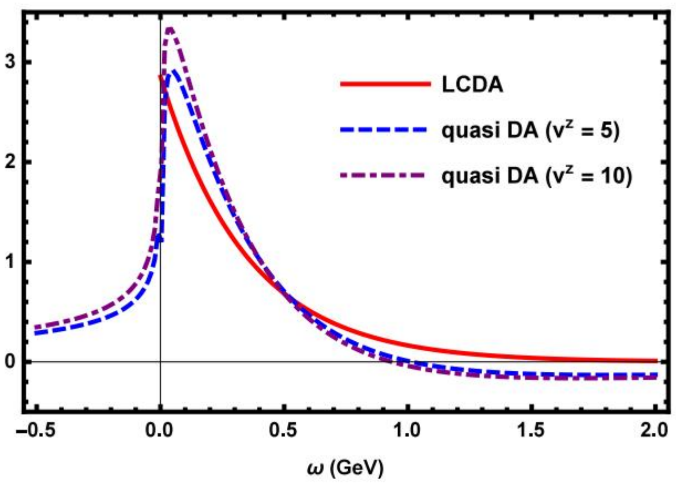
$$\begin{aligned}
H(\xi,\omega,v^z,\mu) \;=\; & \delta(\xi-\omega) + \frac{\alpha_s C_F}{4\pi}\Bigg\{\left[\frac{1}{\omega-\xi}\left(3-\ln\frac{\mu^2}{4v^{z2}(\xi-\omega)^2}-\ln\frac{\xi^2}{(\xi-\omega)^2}\right)\right]\theta(-\xi)\theta(\omega) \\
& +\left[\frac{1}{\omega(\omega-\xi)}\left(3\omega-2\xi-3\omega\ln\frac{\mu^2}{4v^{z2}(\omega-\xi)^2}+\omega\ln\frac{\xi^2}{(\omega-\xi)^2}\right)\right]_{\oplus}\theta(\xi)\theta(\omega-\xi) \\
& +\left[\frac{1}{\xi-\omega}\left(-3+\ln\frac{\mu^2}{4v^{z2}(\xi-\omega)^2}+\ln\frac{\xi^2}{(\xi-\omega)^2}\right)\right]_{\oplus}\theta(\omega)\theta(\xi-\omega) \\
& +\left[\frac{1}{2}\ln^2\frac{\mu^2}{4v^{z2}\xi^2}-\ln\frac{\mu^2}{4v^{z2}\xi^2}-2-6\ln 3+5\ln 4+\frac{7\pi^2}{12}\right]\delta(\xi-\omega)\Bigg\}.
\end{aligned}$$

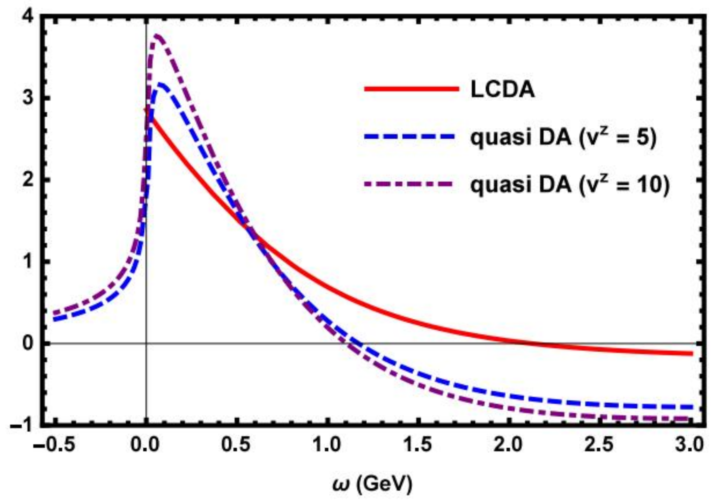
➢ Perspectives for lattice calculations

- two phenomenological models of $\phi_B^-(\xi,\mu)$

$$\phi^-_{B,\mathrm{I}}(\omega) = \frac{1}{\lambda_B} e^{-\omega/\lambda_B}$$

$$\phi^-_{B,\mathrm{II}}(\omega,\mu) = -\frac{2}{\pi\lambda_B}\left(\frac{\omega\mu}{\omega^2+\mu^2} + \arctan\frac{\omega}{\mu} - \frac{\pi}{2} + \frac{4(\sigma_B-1)}{\pi^2}\left\{\mathrm{Im}\left[\mathrm{Li}_2\left(\frac{i\omega}{\mu}\right)\right] - \arctan\frac{\omega}{\mu}\ln\frac{\omega}{\mu}\right\}\right)$$





03 Summary

- We proposed a hard-collinear factorization formula to extract the subleading-twist B-meson LCDA from the quasi-DA calculated on the lattice.

- We derived the perturbative matching coefficient.

- We compared the LCDA and quasi-DA of B-meson.

04 Outlook

- ✓ Studying the simulation of quasidistribution amplitude on the lattice according to these results.

- ✓ Repeating inverse moment of quasi-DA in *[Phys. Rev. D 106, L011503]*, and calculate quasilogarithmic moment, providing the complete mixing matrix.

- ✓ Expanding this approach to other physical quantities, such as the LCDA of heavy baryons and doubly heavy baryons.

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Thank you

Back up

Page 3:

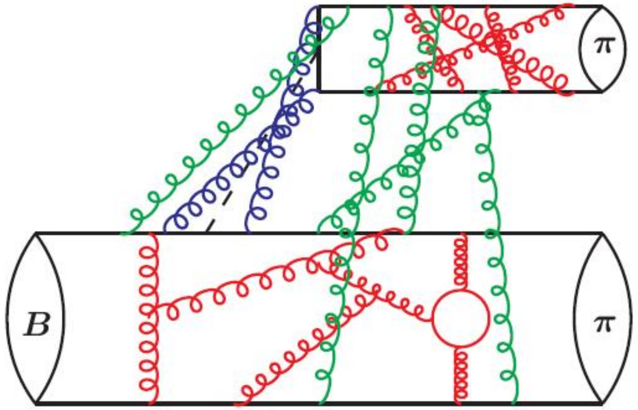
(1)

$$\langle \pi(p')\,\pi(q)\,|Q_i|\,\bar{B}(p)\rangle = f^{B\to\pi}(q^2)\int_0^1 dx T_i^{\mathrm{I}}(x)\phi_\pi(x) + \int_0^1 d\xi dx dy T_i^{\mathrm{II}}(\xi,x,y)\phi_B(\xi)\phi_\pi(x)\phi_\pi(y)$$

$B \to \pi$ form factor

Hard kernel

B-meson LCDA



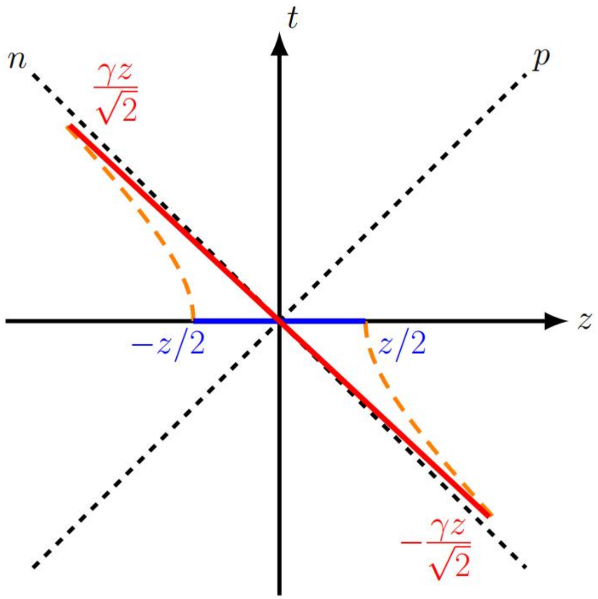
Page 3:

(2)

$$\tilde{q}\left(x, \mu^2, P^z\right) = \int \frac{dz}{4\pi} e^{izk^z} \langle P|\bar{\psi}(z)\gamma^z \times \exp\left(-ig\int_0^z dz' A^z\left(z'\right)\right)\psi(0)|P\rangle$$

$$q\left(x, \mu^2\right) = \int \frac{d\xi^-}{4\pi} e^{-ix\xi^- P^+} \langle P|\bar{\psi}\left(\xi^-\right)\gamma^+ \times \exp\left(-ig\int_0^{\xi^-} d\eta^- A^+\left(\eta^-\right)\right)\psi(0)|P\rangle$$

The momentum distribution defined above has been called *quasi-PDF*, but in reality it is a physical momentum distribution in a proton of momentum P .



The PDFs describe the probability distributions of quarks and gluon inside nucleon.

Page 6:

The distribution amplitude in (3.37) obeys the evolution equation

$$\frac{d}{d\ln\mu}\,\phi_B^-(\omega;\mu) = -\,\frac{\alpha_s C_F}{4\pi}\int_0^\infty d\omega'\,\gamma_-^{(1)}(\omega,\omega';\mu)\,\phi_B^-(\omega';\mu) - \frac{\alpha_s C_F}{4\pi}\int_0^\infty d\omega'\,\gamma_{-+}^{(1)}(\omega,\omega';\mu)\,\phi_B^+(\omega';\mu) + \mathcal{O}(\alpha_s^2)\,, \tag{3.38}$$

where the anomalous dimension kernels $\gamma_-^{(1)}(\omega,\omega';\mu)$ and $\gamma_{-+}^{(1)}(\omega,\omega';\mu)$ can be read off the UV-divergent terms in (3.36) (see appendix B for details)

$$\gamma_-^{(1)}(\omega,\omega';\mu) = \gamma_+^{(1)}(\omega,\omega';\mu) - \Gamma_{\rm cusp}^{(1)}\,\frac{\theta(\omega'-\omega)}{\omega'}\,, \tag{3.39}$$

$$\gamma_{-+}^{(1)}(\omega,\omega';\mu) = -\Gamma_{\rm cusp}^{(1)}\left[\frac{m\,\theta(\omega'-\omega)}{\omega'^2}\right]_+\,. \tag{3.40}$$

Among others, the knowledge of γ_- is essential to check the factorization of certain correlation functions appearing in sum-rule calculations for $B\to\pi$ form factors within SCET [40].

Page 8: