Baryons in the light-front approach: The three-quark picture

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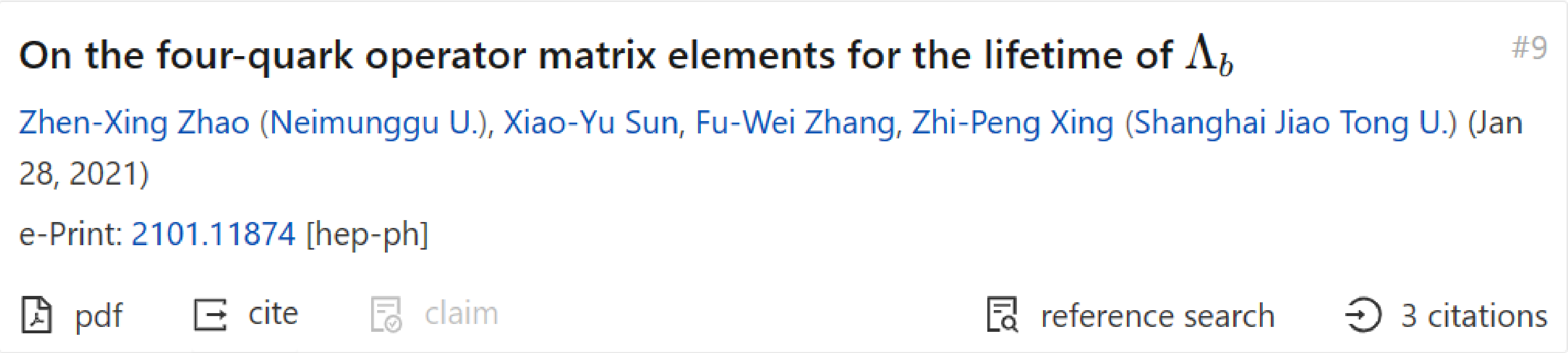
Some Progress in QCDSR and HQE

Submission rejected

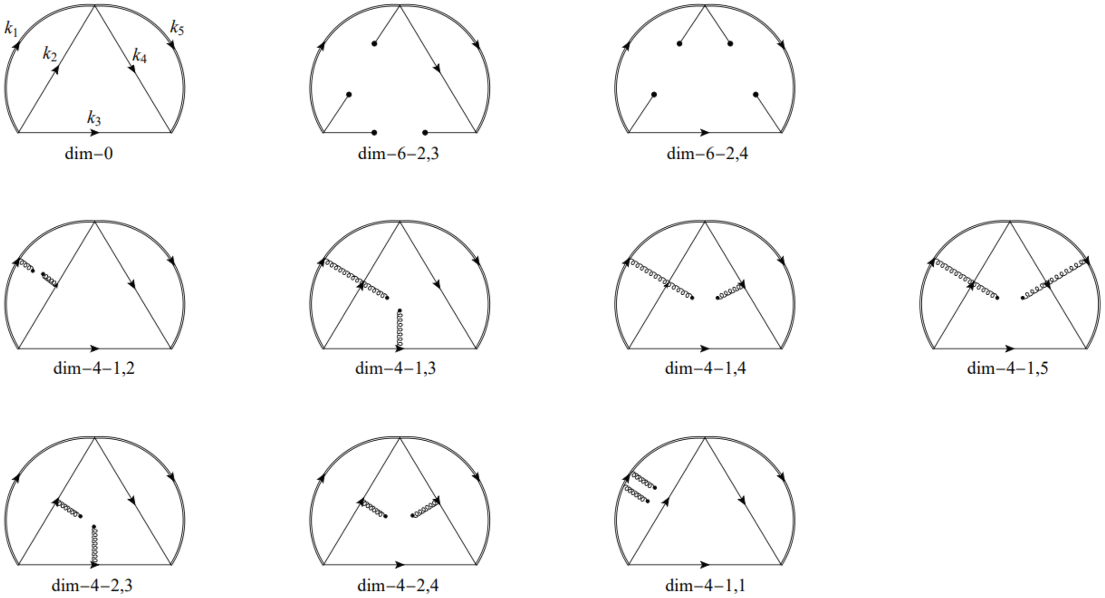


On the four-quark operator matrix elements for the lifetime of Λ_b

#9

Zhen-Xing Zhao (Neimunggu U.), Xiao-Yu Sun, Fu-Wei Zhang, Zhi-Peng Xing (Shanghai Jiao Tong U.) (Jan 28, 2021)

e-Print: 2101.11874 [hep-ph]



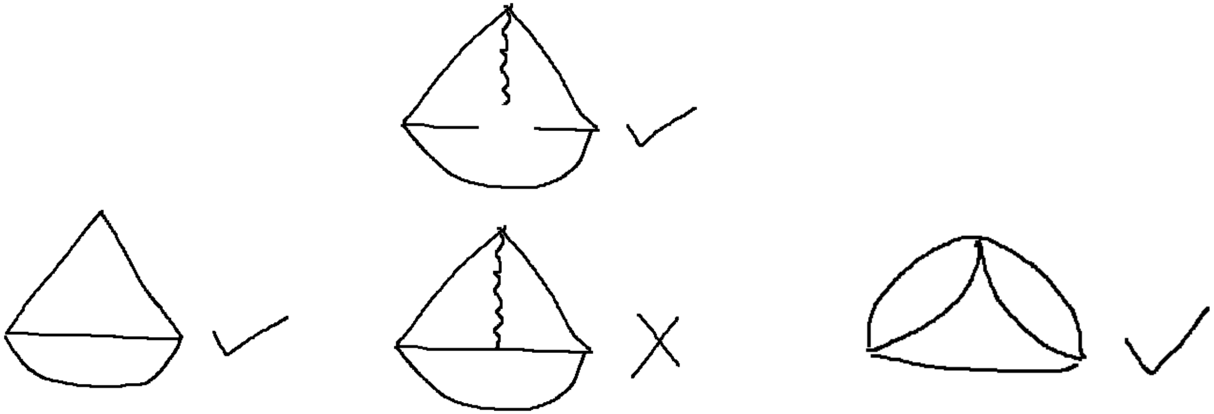
√ HQET matrix element

× Full QCD matrix element

may not necessarily be a bad thing

Way 1: Full QCD is OK

Lenz: 1405.3601



$$\boxed{\Gamma = \frac{G_F^2 m_b^5}{192\pi^3}|V_{cb}|^2 \left[c_{3,b}\frac{\langle B|\bar{b}b|B\rangle}{2M_B} + \frac{c_{5,b}}{m_b^2}\frac{\langle B|\bar{b}g_s\sigma_{\mu\nu}G^{\mu\nu}b|B\rangle}{2M_B} + \frac{c_{6,b}}{m_b^3}\frac{\langle B|(\bar{b}q)_\Gamma(\bar{q}b)_\Gamma|B\rangle}{M_B} + \ldots \right]} . \tag{2.52}$$

Full QCD!

⇓

$$\boxed{\Gamma = \frac{G_F^2 m_b^5}{192\pi^3} V_{cb}^2 \left[c_{3,b} - c_{3,b}\frac{\mu_\pi^2}{2m_b^2} + c_{G,b}\frac{\mu_G^2}{2m_b^2} + \frac{c_{6,b}}{m_b^3}\frac{\langle B|(\bar{b}q)_\Gamma(\bar{q}b)_\Gamma|B\rangle}{M_B} + \ldots \right]} . \tag{2.65}$$

HQET

Way 2: Full QCD→HQET

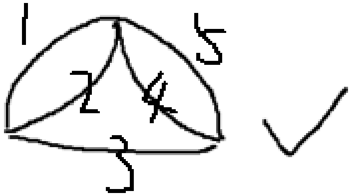
$$\rho^{1,2,3,4} \sim \Big\{ \frac{\pi^3 \mathrm{m1}^3 \left(\mathrm{m1}^2-\mathrm{m12s}\right)^2 \left(\mathrm{m1}^2-\mathrm{m45s}\right)^2 \left(2\mathrm{m12s}^2\mathrm{s2}+\mathrm{m12s}(\mathrm{s2}(\mathrm{s2}-\mathrm{s1})-2\mathrm{m45s}(\mathrm{s1}+\mathrm{s2}))+\mathrm{m45ss1}(2\mathrm{m45s}+\mathrm{s1}-\mathrm{s2})\right)}{8\mathrm{m12s}^2\mathrm{m45s}^2\left((\mathrm{s1}-\mathrm{s2})^2\right)^{3/2}},$$

$$-\frac{\pi^3\mathrm{m1}\left(\mathrm{m1}^2-\mathrm{m12s}\right)^2\left(\mathrm{m1}^2-\mathrm{m45s}\right)^2\left(-\mathrm{m12ss1}\left(2\mathrm{m1}^2\mathrm{s2}+2\mathrm{m45s}^2+\mathrm{m45s}(\mathrm{s1}-\mathrm{s2})\right)+\mathrm{m1}^2\mathrm{s1}(\mathrm{m45s}(\mathrm{s1}+\mathrm{s2})+\mathrm{s2}(\mathrm{s1}-\mathrm{s2}))+\mathrm{m12s}^2\mathrm{m45s}(\mathrm{s1}+\mathrm{s2})\right)}{8\mathrm{m12s}^2\mathrm{m45s}^2\left((\mathrm{s1}-\mathrm{s2})^2\right)^{3/2}},$$

$$-\frac{\pi^3\mathrm{m1}\left(\mathrm{m1}^2-\mathrm{m12s}\right)^2\left(\mathrm{m1}^2-\mathrm{m45s}\right)^2\left(\mathrm{m12s}\left(\mathrm{m1}^2\mathrm{s2}(\mathrm{s1}+\mathrm{s2})+\mathrm{m45s}^2(\mathrm{s1}+\mathrm{s2})+\mathrm{m45ss2}(\mathrm{s1}-\mathrm{s2})\right)+\mathrm{m1}^2\mathrm{s1s2}(-2\mathrm{m45s}-\mathrm{s1}+\mathrm{s2})-2\mathrm{m12s}^2\mathrm{m45ss2}\right)}{8\mathrm{m12s}^2\mathrm{m45s}^2\left((\mathrm{s1}-\mathrm{s2})^2\right)^{3/2}},$$

$$-\frac{\pi^3\mathrm{m1}\left(\mathrm{m1}^2-\mathrm{m12s}\right)^2\left(\mathrm{m1}^2-\mathrm{m45s}\right)^2\left(\mathrm{m45ss1s2}(-4\mathrm{m12s}+\mathrm{s1}-\mathrm{s2})+\mathrm{m12ss2}(\mathrm{m12s}(\mathrm{s1}+\mathrm{s2})+\mathrm{s1}(\mathrm{s2}-\mathrm{s1}))+\mathrm{m45s}^2\mathrm{s1}(\mathrm{s1}+\mathrm{s2})\right)}{8\mathrm{m12s}^2\mathrm{m45s}^2\left((\mathrm{s1}-\mathrm{s2})^2\right)^{3/2}}\Big\}$$

$$\times\,\theta(\mathrm{m12s}^2(-\mathrm{s2})+\mathrm{m12s}(\mathrm{m45s}(\mathrm{s1}+\mathrm{s2})+\mathrm{s2}(\mathrm{s1}-\mathrm{s2}))-\mathrm{m45ss1}(\mathrm{m45s}+\mathrm{s1}-\mathrm{s2}))$$



$s_1 < s_2,$

$m_Q \to \infty$

⇩

Integrate out

m_{12}, m_{45}

Different

Dirac structures:

be of equal

importance

$$\Big\{\frac{\pi^3\sigma^5\left(2\sigma^2-7\sigma\sigma\mathrm{p}+7\sigma\mathrm{p}^2\right)}{105\mathrm{m1}^2},\frac{\pi^3\sigma^5\left(2\sigma^2-7\sigma\sigma\mathrm{p}+7\sigma\mathrm{p}^2\right)}{105\mathrm{m1}^2},$$

$$\frac{\pi^3\sigma^5\left(2\sigma^2-7\sigma\sigma\mathrm{p}+7\sigma\mathrm{p}^2\right)}{105\mathrm{m1}^2},\frac{\pi^3\sigma^5\left(2\sigma^2-7\sigma\sigma\mathrm{p}+7\sigma\mathrm{p}^2\right)}{105\mathrm{m1}^2}\Big\}$$

$$\rho_{\Pi}^{pert}(\sigma,\sigma') = \frac{3}{32\pi^6}(1+b^2)\{\boxed{\theta(\sigma-\sigma')}\sigma'^5\,(\frac{\sigma'^2}{105}-\frac{\sigma\sigma'}{30}+\frac{\sigma^2}{30})+\boxed{(\sigma\leftrightarrow\sigma')}\}\ . \qquad (32)$$

HQET sum rules

Colangelo and De Fazio: 9604425

Experiences and lessons

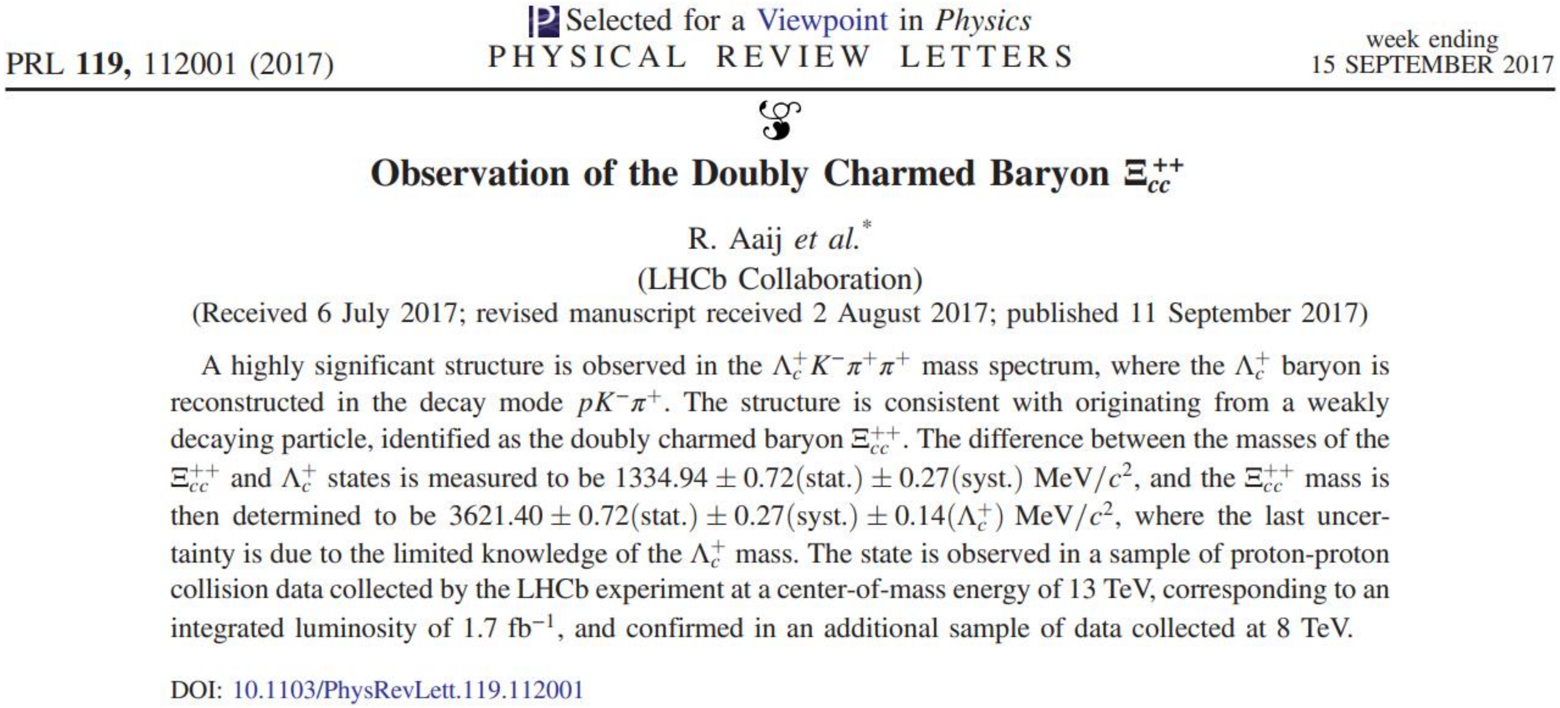
- Full theory contains more information
- EFT is usually simple

Outline

- Introduction
- Framework and some applications
- Numerical results
- Summary and outlook

Introduction

Observation of Ξ_{cc}^{++}



Selected for a Viewpoint in *Physics*
PRL **119,** 112001 (2017) PHYSICAL REVIEW LETTERS week ending 15 SEPTEMBER 2017

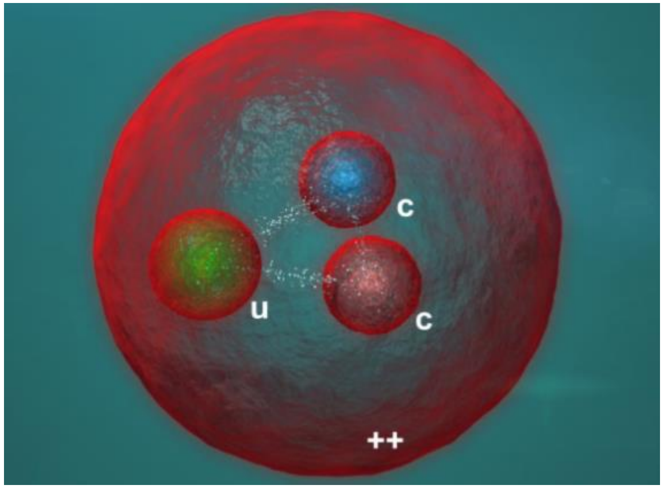
Observation of the Doubly Charmed Baryon Ξ_{cc}^{++}

R. Aaij *et al.*[*]
(LHCb Collaboration)

(Received 6 July 2017; revised manuscript received 2 August 2017; published 11 September 2017)

A highly significant structure is observed in the $\Lambda_c^+ K^- \pi^+ \pi^+$ mass spectrum, where the Λ_c^+ baryon is reconstructed in the decay mode $pK^-\pi^+$. The structure is consistent with originating from a weakly decaying particle, identified as the doubly charmed baryon Ξ_{cc}^{++}. The difference between the masses of the Ξ_{cc}^{++} and Λ_c^+ states is measured to be $1334.94 \pm 0.72(\text{stat.}) \pm 0.27(\text{syst.})\ \text{MeV}/c^2$, and the Ξ_{cc}^{++} mass is then determined to be $3621.40 \pm 0.72(\text{stat.}) \pm 0.27(\text{syst.}) \pm 0.14(\Lambda_c^+)\ \text{MeV}/c^2$, where the last uncertainty is due to the limited knowledge of the Λ_c^+ mass. The state is observed in a sample of proton-proton collision data collected by the LHCb experiment at a center-of-mass energy of 13 TeV, corresponding to an integrated luminosity of $1.7\ \text{fb}^{-1}$, and confirmed in an additional sample of data collected at 8 TeV.

DOI: 10.1103/PhysRevLett.119.112001



$$\Xi_{cc}^{++} \rightarrow \Lambda_c^+ K^- \pi^+ \pi^+$$

Observation of the doubly charmed baryon Ξ_{cc}^{++} #1

LHCb Collaboration • Roel Aaij (CERN) et al. (Jul 5, 2017)

Published in: *Phys.Rev.Lett.* 119 (2017) 11, 112001 • e-Print: 1707.01621 [hep-ex]

Observation of Ξ_{cc}^{++}

#21

Weak decays of doubly heavy baryons: the $1/2 \to 1/2$ case

Wei Wang (Shanghai Jiao Tong U. and Shanghai Jiaotong U.), Fu-Sheng Yu (Lanzhou U.), Zhen-Xing Zhao (Shanghai Jiaotong U. and Shanghai Jiao Tong U.) (Jul 10, 2017)

Published in: *Eur.Phys.J.C* 77 (2017) 11, 781 • e-Print: 1707.02834 [hep-ph]

#22

Discovery Potentials of Doubly Charmed Baryons

Fu-Sheng Yu (Lanzhou U. and Lanzhou, Inst. Modern Phys.), Hua-Yu Jiang (Lanzhou U.), Run-Hui Li (Neimunggu U.), Cai-Dian Lü (Beijing, Inst. High Energy Phys. and Beijing, GUCAS), Wei Wang (Shanghai Jiao Tong U. and Shanghai Jiaotong U.) et al. (Mar 27, 2017)

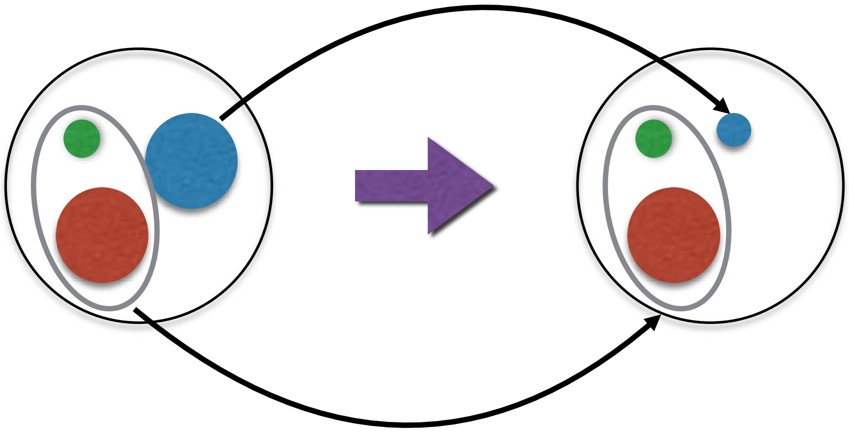
Published in: *Chin.Phys.C* 42 (2018) 5, 051001 • e-Print: 1703.09086 [hep-ph]

-- Test the standard model

-- Search for the origin of CP violation and new physics

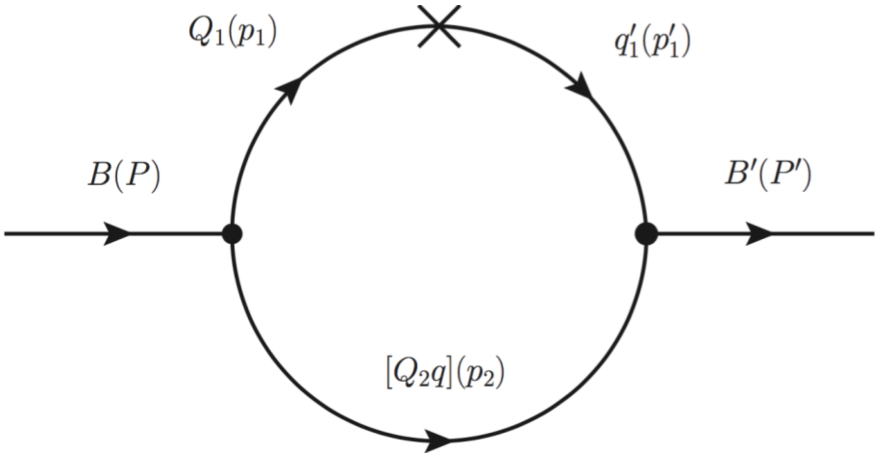
-- Understand the strong interactions

Light-front quark model—the diquark picture



diquark

0^+ or 1^+



H. W. Ke, X. Q. Li and Z. T. Wei, Phys. Rev. D 77, 014020 (2008)
H. W. Ke, X. H. Yuan, X. Q. Li, Z. T. Wei and Y. X. Zhang, Phys. Rev. D 86, 114005 (2012)

Some defects:

-- $\Xi_{bc}(bcq)$,

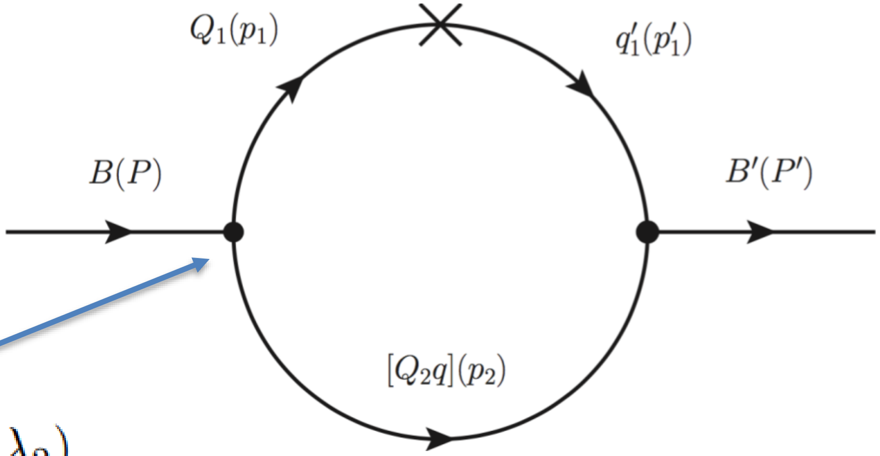
for c decay, bq -- diquark

for b decay, cq -- diquark

-- more parameters, such as m_{di}-- $m_{[ud]}$ and $m_{\{ud\}}$

Light-front quark model—the diquark picture

A Lorentz boost between p_2 and $\bar{P}$



$$\Gamma = -\frac{1}{\sqrt{3}}\gamma_5 \not{\epsilon}^*(p_2,\lambda_2),$$

H. W. Ke, X. H. Yuan, X. Q. Li, Z. T. Wei and Y. X. Zhang, Phys. Rev. D 86, 114005 (2012)

$$\begin{aligned}\Gamma &= \frac{1}{\sqrt{3}}\gamma_5 \not{\epsilon}^*(\bar{P},\lambda_2)\\ &= \frac{1}{\sqrt{3}}\gamma_5\left(\not{\epsilon}^*(p_2,\lambda_2) - \frac{M_0+m_1+m_2}{p_2\cdot\bar{P}+m_2M_0}\epsilon^*(p_2,\lambda_2)\cdot\bar{P}\right),\end{aligned}$$

Chun-Khiang Chua, Phys. Rev. D 99, 014023 (2019)

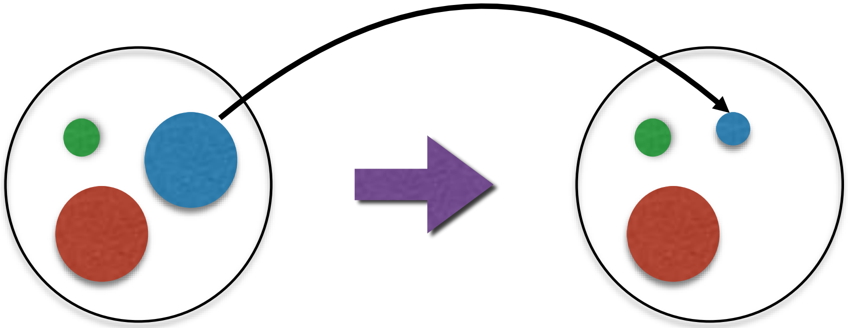
Light-front quark model – the three-quark picture

S. Tawfiq, P. J. O'Donnell, and J. G. Körner, Phys. Rev. D 58, 054010 (1998)

H.-W. Ke, N. Hao, and X.-Q. Li, Eur. Phys. J. C 79, 540 (2019), ...

C.-Q. Geng, C.-W. Liu, and T.-H. Tsai, Phys.Lett.B 815, 136125 (2021), …

Y.-S. Li, and X. Liu, Phys.Rev.D 107, 033005 (2023), …



-- lack a proof of spin wavefunctions

-- shape parameters cannot be well determined

-- relation between the diquark picture and the three-quark picture

Framework and some applications

The baryon state

$$
\begin{aligned}
|\mathcal{B}(P,S,S_z)\rangle &= \int \{d^3\tilde{p}_1\}\{d^3\tilde{p}_2\}\{d^3\tilde{p}_3\}2(2\pi)^3\delta^3(\tilde{P}-\tilde{p}_1-\tilde{p}_2-\tilde{p}_3)\frac{1}{\sqrt{P^+}} \\
&\times \sum_{\lambda_1,\lambda_2,\lambda_3} \Psi^{SS_z}(\tilde{p}_1,\tilde{p}_2,\tilde{p}_3,\lambda_1,\lambda_2,\lambda_3)C^{ijk}|q_1^i(p_1,\lambda_1)q_2^j(p_2,\lambda_2)q_3^k(p_3,\lambda_3)\rangle,
\end{aligned}
$$

spin and momentum　　color　　flavor

Λ_Q $$A_0\bar{u}(p_3,\lambda_3)(\bar{\not{P}}+M_0)(-\gamma_5)C\bar{u}^T(p_2,\lambda_2)\bar{u}(p_1,\lambda_1)u(\bar{P},S_z)\Phi(x_i,k_{i\perp}),$$

Σ_Q $$A_1\bar{u}(p_3,\lambda_3)(\bar{\not{P}}+M_0)(\gamma^\mu-v^\mu)C\bar{u}^T(p_2,\lambda_2)\bar{u}(p_1,\lambda_1)(\frac{1}{\sqrt{3}}\gamma_\mu\gamma_5)u(\bar{P},S_z)\Phi(x_i,k_{i\perp}),$$

Σ_Q^* $$A_1'\bar{u}(p_3,\lambda_3)(\bar{\not{P}}+M_0)(\gamma^\mu-v^\mu)C\bar{u}^T(p_2,\lambda_2)\bar{u}(p_1,\lambda_1)u_\mu(\bar{P},S_z)\Phi(x_i,k_{i\perp}),$$

(udQ)

Three different flavors $$\left(\frac{1}{2}\otimes\frac{1}{2}\right)\otimes\frac{1}{2}=(0\oplus 1)\otimes\frac{1}{2}=\frac{1}{2}\oplus\frac{1}{2}\oplus\frac{3}{2}.$$

Spin wavefunction

Take $\Sigma_Q(udQ)$ as an example

where ud are considered as an axial-vector "diquark"

Step 1: In the rest frame of quark 2 and 3 – "diquark"

$$I^\mu \equiv \bar{u}(p_3, s_3)\frac{(\bar{\not{P}} + M_0)}{2M_0}\gamma_\perp^\mu(p_{23})(-C)\bar{u}^T(p_2, s_2)$$

$$\gamma_\perp^\mu(p_{23}) = \gamma_\perp^\mu(\bar{P}) - \frac{M_0 p_{23}^\mu + m_{23}\bar{P}^\mu}{m_{23}M_0}\frac{\gamma_\perp(\bar{P})\cdot p_{23}}{e_{23}+m_{23}},$$

$$p_{23} = p_2 + p_3, \qquad m_{23}^2 = p_{23}^2,$$

$$\gamma_\perp^\mu(\bar{P}) = \gamma^\mu - \not{v}v^\mu, \qquad v^\mu = \bar{P}^\mu/M_0.$$

$$I^\mu \sim \langle\frac{1}{2}\frac{1}{2}; s_3 s_2|\frac{1}{2}\frac{1}{2}; 1, s_{23}\rangle\epsilon^{*\mu}(p_{23}, s_{23}).$$

Spin wavefunction

Step 2: Couple the “diquark” to quark 1

$$T \equiv I^\mu \cdot \bar{u}(p_1, s_1)\Gamma_{1,23\mu} u(\bar{P}, S_z)$$

$$\Gamma_{1,23\mu} = \frac{\gamma_5}{\sqrt{3}}(\gamma_\mu - \frac{M_0 + m_1 + m_{23}}{M_0(e_{23} + m_{23})}\bar{P}_\mu).$$

$$T \sim \langle\frac{1}{2}\frac{1}{2}; s_3 s_2|\frac{1}{2}\frac{1}{2}; 1 s_{23}\rangle\langle\frac{1}{2}1; s_1 s_{23}|\frac{1}{2}1; \frac{1}{2}S_z\rangle.$$

Step 3: Tensor simplification

$$\begin{aligned}
& \bar{u}(p_3, \lambda_3)(\bar{\not{P}} + M_0)\underline{\gamma^\mu_\perp(p_{23})}(-C)\bar{u}^T(p_2, \lambda_2)\bar{u}(p_1, \lambda_1)\underline{\Gamma_{1,23\mu}}u(\bar{P}, S_z) \\
= \quad & \ldots \\
= \quad & \bar{u}(p_3, \lambda_3)(\bar{\not{P}} + M_0)(\gamma^\mu - v^\mu)C\bar{u}^T(p_2, \lambda_2)\bar{u}(p_1, \lambda_1)(\frac{1}{\sqrt{3}}\gamma_\mu\gamma_5)u(\bar{P}, S_z)
\end{aligned}$$

- Lorentz boost plays a crucial role
- Same method can be applied to multi-quark states!

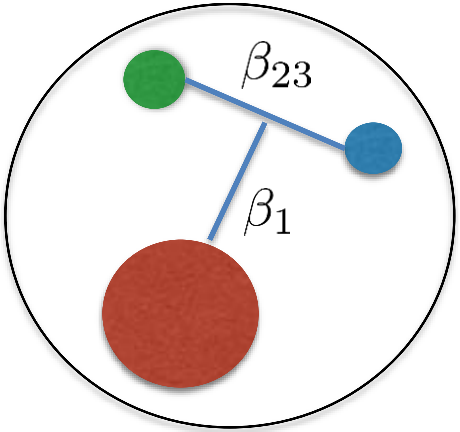
Momentum wavefunction

$$\Phi(x_1, x_2, x_3, k_{1\perp}, k_{2\perp}, k_{3\perp}) = \sqrt{\frac{e_1 e_2 e_3}{x_1 x_2 x_3 M_0}}\varphi(\vec{k}_1, \beta_1)\varphi(\frac{\vec{k}_2 - \vec{k}_3}{2}, \beta_{23})$$

$$\varphi(\vec{k}, \beta) \equiv 4\left(\frac{\pi}{\beta^2}\right)^{3/4} \exp\left(\frac{-k_\perp^2 - k_z^2}{2\beta^2}\right)$$

shape parameters

$$\int (\prod_{i=1}^{3} \frac{dx_i d^2 k_{i\perp}}{2(2\pi)^3}) 2(2\pi)^3 \delta(1 - \sum x_i)\delta^2(\sum k_{i\perp})|\Phi(x_i, k_{i\perp})|^2 = 1$$



To determine the shape parameters

Take Λ_Q as an example

$$\langle 0|J_{\Lambda_Q}|\Lambda_Q(P,S_z)\rangle$$

Step 1: Calculate it in LFQM

Step 2: Use the definition of

$$\langle 0|J_{\Lambda_Q}|\Lambda_Q(P,S_z)\rangle = \lambda_{\Lambda_Q} u(P,S_z).$$

Step 3: Extract the pole residue

$$\lambda_{\Lambda_Q} = \int \frac{dx_2 d^2k_{2\perp}}{2(2\pi)^3}\frac{dx_3 d^2k_{3\perp}}{2(2\pi)^3}\frac{1}{\sqrt{x_1x_2x_3}}\Phi(x_i,k_{i\perp})\sqrt{6}A_0 \times \frac{\mathrm{Tr}[...]\mathrm{Tr}[\gamma^+(\not p_1+m_1)(\bar{\not P}+M_0)]}{\mathrm{Tr}[\gamma^+(\not P+M)]},$$

$$\lambda_{\Lambda_Q} = \text{the above equation with } \gamma^+ \to \gamma^+\gamma^-$$

M can be extracted!

Here contains β_1 and β_{23}

$$\mathrm{Tr}[...] = \mathrm{Tr}[C\gamma_5(\not p_3+m_3)(\bar{\not P}+M_0)(-\gamma_5)C(\not p_2+m_2)^T]$$

Form factors

$\Lambda_b \to \Lambda_c$ Form factors

Step 1: Calculate the matrix elements in LFQM

Step 2: Write the matrix elements in terms of form factors

$$\langle \Lambda_c(P', S_z') | \bar{c}\gamma^\mu b | \Lambda_b(P, S_z) \rangle = \bar{u}(P', S_z') \left[\gamma^\mu f_1(q^2) + i\sigma^{\mu\nu} \frac{q_\nu}{M} f_2(q^2) + \frac{q^\mu}{M} f_3(q^2) \right] u(P, S_z),$$

$$\langle \Lambda_c(P', S_z') | \bar{c}\gamma^\mu \gamma_5 b | \Lambda_b(P, S_z) \rangle = \bar{u}(P', S_z') \left[\gamma^\mu g_1(q^2) + i\sigma^{\mu\nu} \frac{q_\nu}{M} g_2(q^2) + \frac{q^\mu}{M} g_3(q^2) \right] \gamma_5 u(P, S_z),$$

Step 3: Extract the form factors

$$f_1 = \frac{1}{8P^+P'^+} \int \{d^3\tilde{p}_2\}\{d^3\tilde{p}_3\} \frac{\Phi'^*\Phi}{\sqrt{P^+P'^+p_1^+p_1'^+}} A_0' A_0 \mathrm{Tr}[...]$$

$$\times \mathrm{Tr}[(\bar{\not{P}} + M_0)\gamma^+(\bar{\not{P}}' + M_0')(\not{p}_1' + m_1')\gamma^+(\not{p}_1 + m_1)],$$

$$\mathrm{Tr}[...] = \mathrm{Tr}[(\bar{\not{P}} + M_0)(-\gamma_5)C(\not{p}_2 + m_2)^T C\gamma_5(\bar{\not{P}}' + M_0')(\not{p}_3 + m_3)]$$

f2, g1, g2 can also be obtained in a similar way

$\Sigma_b \to \Sigma_c$, $\Xi_{cc} \to \Lambda_c$ Form factors

The relation between the two pictures

Λ_Q $\quad \psi_0(321) \equiv \bar{u}(\underline{p_3, \lambda_3})(\bar{\not{P}} + M_0)(-\gamma_5)C\bar{u}^T(\underline{p_2, \lambda_2})\bar{u}(\underline{p_1, \lambda_1})u(\bar{P}, S_z),$

Σ_Q $\quad \psi_1(321) \equiv \bar{u}(\underline{p_3, \lambda_3})(\bar{\not{P}} + M_0)(\gamma^\mu - v^\mu)C\bar{u}^T(\underline{p_2, \lambda_2})\bar{u}(\underline{p_1, \lambda_1})(\frac{1}{\sqrt{3}}\gamma_\mu\gamma_5)u(\bar{P}, S_z)$

Quark 3 and 2 form a diquark

$$\begin{cases} \psi_0(321) = -\psi_0(231), \\ \psi_1(321) = \psi_1(231) \end{cases}$$

A diquark bases

normalization factor $\quad \dfrac{1}{4\sqrt{M_0^3(e_1+m_1)(e_2+m_2)(e_3+m_3)}}$

orthogonal $\quad \sum_{\lambda_1\lambda_2\lambda_3} \psi_0^\dagger(32\underline{1'})\psi_1(321) = 0$

T matirx

$$\begin{pmatrix} \psi_0(312) \\ \psi_1(312) \end{pmatrix} = T \begin{pmatrix} \psi_0(321) \\ \psi_1(321) \end{pmatrix} \qquad T = \begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}$$

$$T^{-1} = T$$

The relation between the two pictures

1. Calculate the overlap factors of $\Xi_{bc}^{+}(cbu) \to \Lambda_b(dbu)$

$$\psi_1(bcu) = \underline{-\frac{\sqrt{3}}{2}}\psi_0(buc) \underline{-\frac{1}{2}}\psi_1(buc),$$

$$\begin{aligned}\psi_0(udb) &= \frac{1}{2}\psi_0(ubd) - \frac{\sqrt{3}}{2}\psi_1(ubd) \\ &= \underline{-\frac{1}{2}}\psi_0(bud) \underline{-\frac{\sqrt{3}}{2}}\psi_1(bud)\end{aligned}$$

$$\langle\psi_0(udb)|\psi_1(bcu)\rangle = \underline{\frac{\sqrt{3}}{4}}\langle\psi_0(bud)|\psi_0(buc)\rangle + \underline{\frac{\sqrt{3}}{4}}\langle\psi_1(bud)|\psi_1(buc)\rangle,$$

2. Calculate the overlap factors of $\Xi_{cc}^{++}(ccu) \to \Lambda_c(dcu)$

$$\boxed{\frac{2}{\sqrt{2}}}\{\frac{\sqrt{3}}{4}, \frac{\sqrt{3}}{4}\} = \{\frac{\sqrt{6}}{4}, \frac{\sqrt{6}}{4}\}$$

W. Wang, F.-S. Yu, and Z.-X. Zhao, Eur. Phys. J. C 77, 781 (2017)

Possibly better definitions of interpolating currents

Hermite conjugate $\psi_{0,1}$

$$J^{\text{new}}_{\Lambda_Q} = \epsilon_{abc}[u_a^T C\gamma_5(1+\not{v})d_b]Q_c,$$
$$J^{\text{new}}_{\Sigma_Q} = \epsilon_{abc}[u_a^T C(\gamma^\mu - v^\mu)(1+\not{v})d_b]\frac{1}{\sqrt{3}}\gamma_\mu\gamma_5 Q_c$$

$$\boxed{v^\mu \equiv p^\mu/\sqrt{p^2}}$$

Traditional definitions

$$J_{\Lambda_Q} = \epsilon_{abc}[u_a^T C\gamma_5 d_b]Q_c,$$
$$J_{\Sigma_Q} = \epsilon_{abc}[u_a^T C\gamma^\mu d_b]\gamma_\mu\gamma_5 Q_c$$

Comments:

1. The factor $1/\sqrt{3}$

$$\lambda_{\Sigma_Q} \approx \lambda_{\Lambda_Q}$$

2. Let $v \to 0$ in $J^{\text{new}} \to J$ -- in fact, we cannot do that.

Numerical results

Inputs and shape parameters

Inputs $m_u = m_d = 0.25 \text{ GeV}, \quad m_c = 1.4 \text{ GeV}, \quad m_b = 4.8 \text{ GeV}.$

$$\lambda_{\Lambda_b} = 0.030 \pm 0.009, \quad \lambda_{\Lambda_c} = 0.022 \pm 0.008,$$
$$\lambda_{\Sigma_b} = 0.062 \pm 0.018, \quad \lambda_{\Sigma_c} = 0.045 \pm 0.015,$$
$$\lambda_{\Xi_{cc}} = 0.115 \pm 0.027.$$

Z.-G. Wang, Eur. Phys. J. C 68, 479 (2010)
Z.-G. Wang, Phys. Lett. B 685, 59 (2010)
Z.-G. Wang, Eur. Phys. J. A 45, 267 (2010)

$\Lambda_b \to \Lambda_c$ $\quad \beta_{b,[ud]} = 0.63 \pm 0.05 \text{ GeV}, \quad \beta_{[ud]} = 0.27 \pm 0.03 \text{ GeV},$
$\quad \beta_{c,[ud]} = 0.45 \pm 0.05 \text{ GeV};$

$\Sigma_b \to \Sigma_c$ $\quad \beta_{b,\{ud\}} = 0.66 \pm 0.04 \text{ GeV}, \quad \beta_{\{ud\}} = 0.28 \pm 0.03 \text{ GeV},$
$\quad \beta_{c,\{ud\}} = 0.49 \pm 0.04 \text{ GeV};$

$\Xi_{cc} \to \Lambda_c$ $\quad \beta_{u,\{cc\}} = 0.490 \pm 0.040 \text{ GeV}, \quad \beta_{\{cc\}} = 0.400 \pm 0.025 \text{ GeV}.$

$$\boxed{\lambda_1 \approx \lambda_2 \approx \lambda_{\text{QCDSR}}}$$

Form factors and comparison

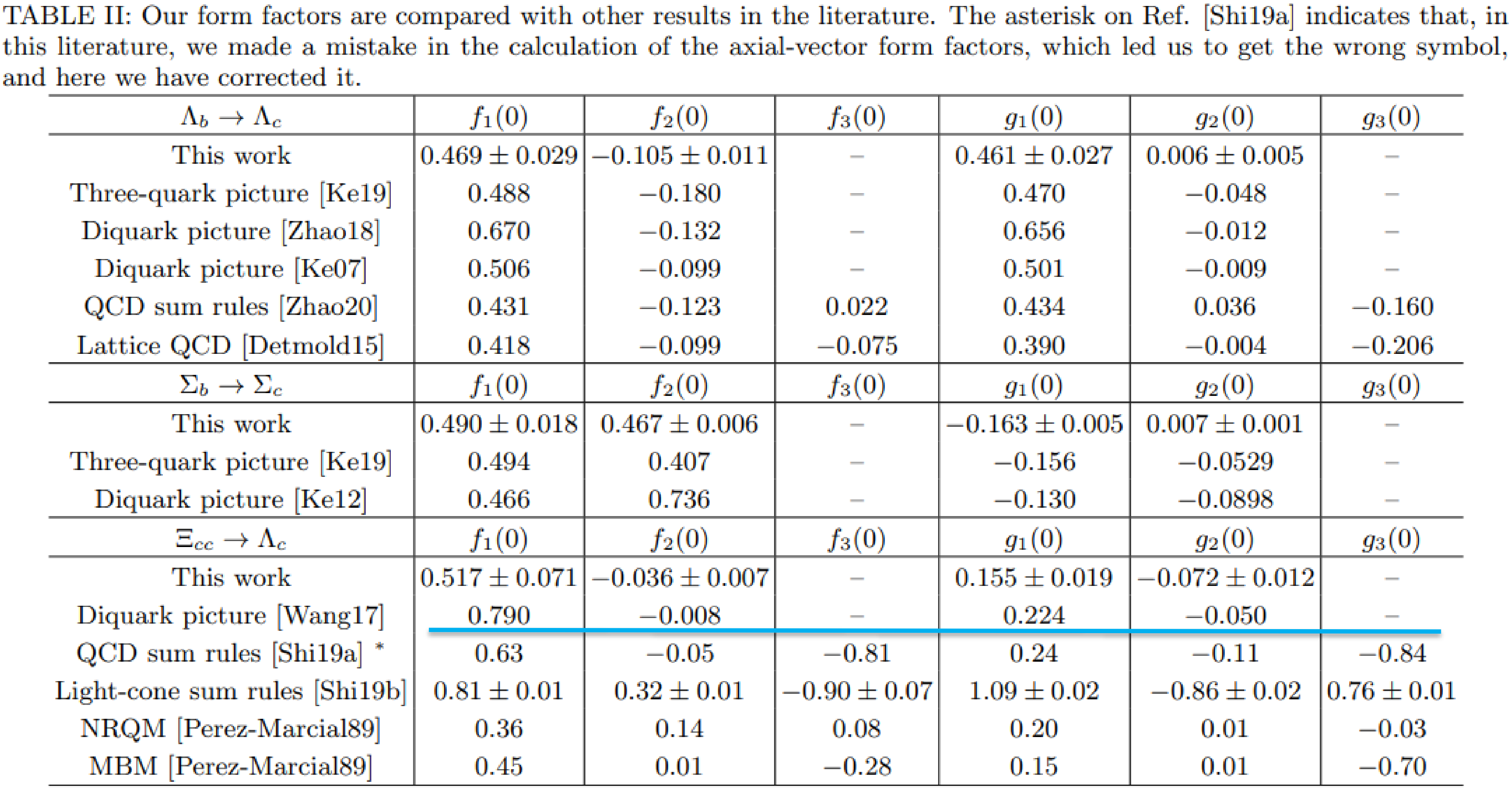
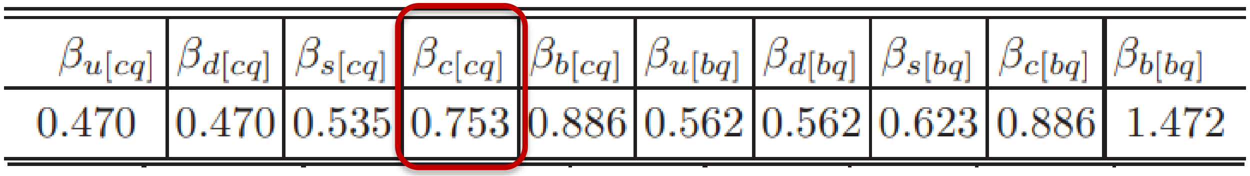


TABLE II: Our form factors are compared with other results in the literature. The asterisk on Ref. [Shi19a] indicates that, in this literature, we made a mistake in the calculation of the axial-vector form factors, which led us to get the wrong symbol, and here we have corrected it.

$\Lambda_b \to \Lambda_c$	$f_1(0)$	$f_2(0)$	$f_3(0)$	$g_1(0)$	$g_2(0)$	$g_3(0)$
This work	0.469 ± 0.029	-0.105 ± 0.011	–	0.461 ± 0.027	0.006 ± 0.005	–
Three-quark picture [Ke19]	0.488	-0.180	–	0.470	-0.048	–
Diquark picture [Zhao18]	0.670	-0.132	–	0.656	-0.012	–
Diquark picture [Ke07]	0.506	-0.099	–	0.501	-0.009	–
QCD sum rules [Zhao20]	0.431	-0.123	0.022	0.434	0.036	-0.160
Lattice QCD [Detmold15]	0.418	-0.099	-0.075	0.390	-0.004	-0.206
$\Sigma_b \to \Sigma_c$	$f_1(0)$	$f_2(0)$	$f_3(0)$	$g_1(0)$	$g_2(0)$	$g_3(0)$
This work	0.490 ± 0.018	0.467 ± 0.006	–	-0.163 ± 0.005	0.007 ± 0.001	–
Three-quark picture [Ke19]	0.494	0.407	–	-0.156	-0.0529	–
Diquark picture [Ke12]	0.466	0.736	–	-0.130	-0.0898	–
$\Xi_{cc} \to \Lambda_c$	$f_1(0)$	$f_2(0)$	$f_3(0)$	$g_1(0)$	$g_2(0)$	$g_3(0)$
This work	0.517 ± 0.071	-0.036 ± 0.007	–	0.155 ± 0.019	-0.072 ± 0.012	–
Diquark picture [Wang17]	0.790	-0.008	–	0.224	-0.050	–
QCD sum rules [Shi19a] *	0.63	-0.05	-0.81	0.24	-0.11	-0.84
Light-cone sum rules [Shi19b]	0.81 ± 0.01	0.32 ± 0.01	-0.90 ± 0.07	1.09 ± 0.02	-0.86 ± 0.02	0.76 ± 0.01
NRQM [Perez-Marcial89]	0.36	0.14	0.08	0.20	0.01	-0.03
MBM [Perez-Marcial89]	0.45	0.01	-0.28	0.15	0.01	-0.70

Diquark picture



$\beta_{u[cq]}$	$\beta_{d[cq]}$	$\beta_{s[cq]}$	$\beta_{c[cq]}$	$\beta_{b[cq]}$	$\beta_{u[bq]}$	$\beta_{d[bq]}$	$\beta_{s[bq]}$	$\beta_{c[bq]}$	$\beta_{b[bq]}$
0.470	0.470	0.535	0.753	0.886	0.562	0.562	0.623	0.886	1.472

Decay widths and comparison

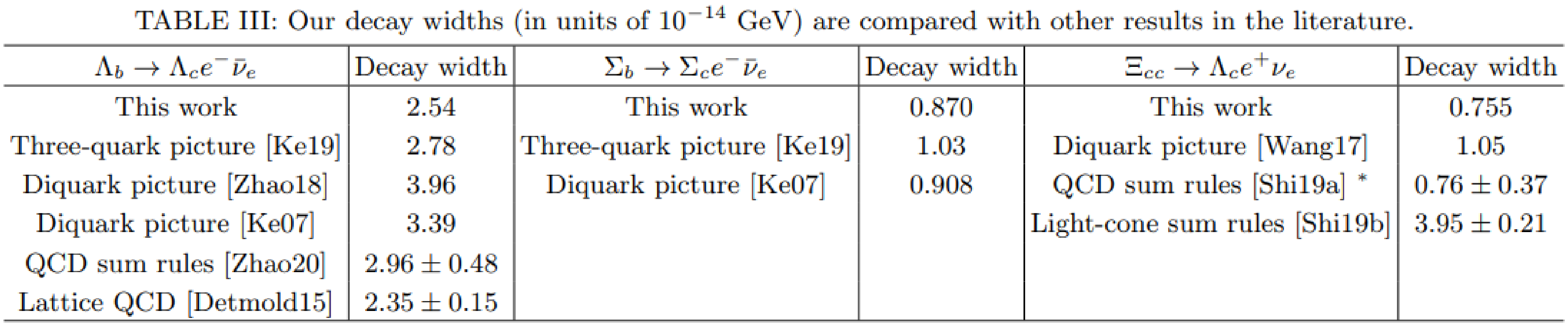


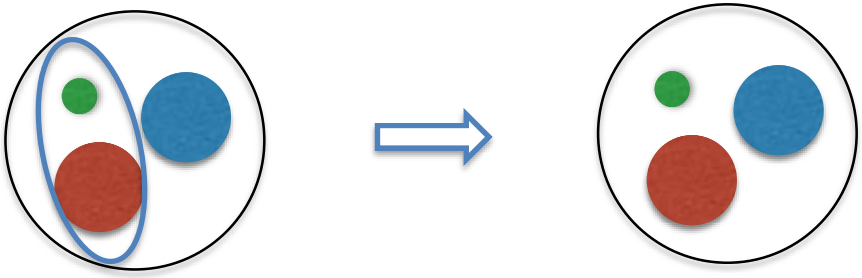
TABLE III: Our decay widths (in units of 10^{-14} GeV) are compared with other results in the literature.

$\Lambda_b \to \Lambda_c e^- \bar{\nu}_e$	Decay width	$\Sigma_b \to \Sigma_c e^- \bar{\nu}_e$	Decay width	$\Xi_{cc} \to \Lambda_c e^+ \nu_e$	Decay width
This work	2.54	This work	0.870	This work	0.755
Three-quark picture [Ke19]	2.78	Three-quark picture [Ke19]	1.03	Diquark picture [Wang17]	1.05
Diquark picture [Zhao18]	3.96	Diquark picture [Ke07]	0.908	QCD sum rules [Shi19a] *	0.76 ± 0.37
Diquark picture [Ke07]	3.39			Light-cone sum rules [Shi19b]	3.95 ± 0.21
QCD sum rules [Zhao20]	2.96 ± 0.48				
Lattice QCD [Detmold15]	2.35 ± 0.15				

Summary and outlook

Summary

- A three-quark picture in LFQM is built up using the quark spinors and Dirac matrices
- The shape parameters are determined with the help of pole residue
- The relation between the diquark picture and the three-quark picture is clarified



A small flaw?

$$\boxed{\lambda_1 \approx \lambda_2 \approx \lambda_{\rm QCDSR}}$$

Outlook

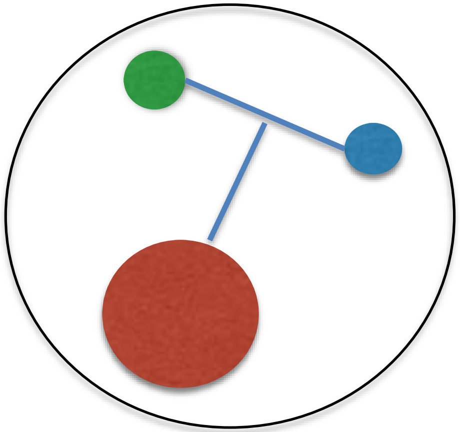
- Various applications
- Lorentz boost plays a crucial role when building the model-- Multiquark states? – the most famous X(3872)
- Interpolating currents of baryons

To make a tetraquark state

Spin wf: S-wave

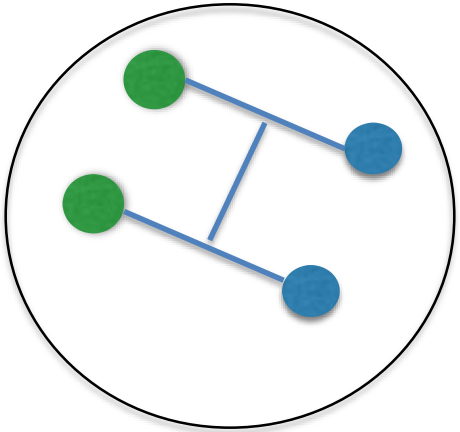
Building blocks:

- Quark
- Diquark



baryon

- RF of 23
- RF of 123



tetraquark state

- RF of 12
- RF of 34
- RF of 1234

- 0^+ Diquark $$I \equiv \bar{u}(p_3, s_3)\frac{(\bar{\not{P}} + M_0)}{2M_0}\gamma_5(-C)\bar{u}^T(p_2, s_2)$$
- 1^+ Diquark $$I^\mu \equiv \bar{u}(p_3, s_3)\frac{(\bar{\not{P}} + M_0)}{2M_0}\gamma_\perp^\mu(p_{23})(-C)\bar{u}^T(p_2, s_2)$$

To make a tetraquark state

- Momentum wf: $\beta_{12}, \beta_{34}, \beta_{12,34}$
- Color wf: 3*3*3*3
- Flavor wf: SU(3)

Thank you for your attention!