

Hybrid Renormalization for Quasi Distribution Amplitudes of A Light Baryon

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Contents

1 Introduction to LCDA\Quasi-DA

2 Introduction to LaMET

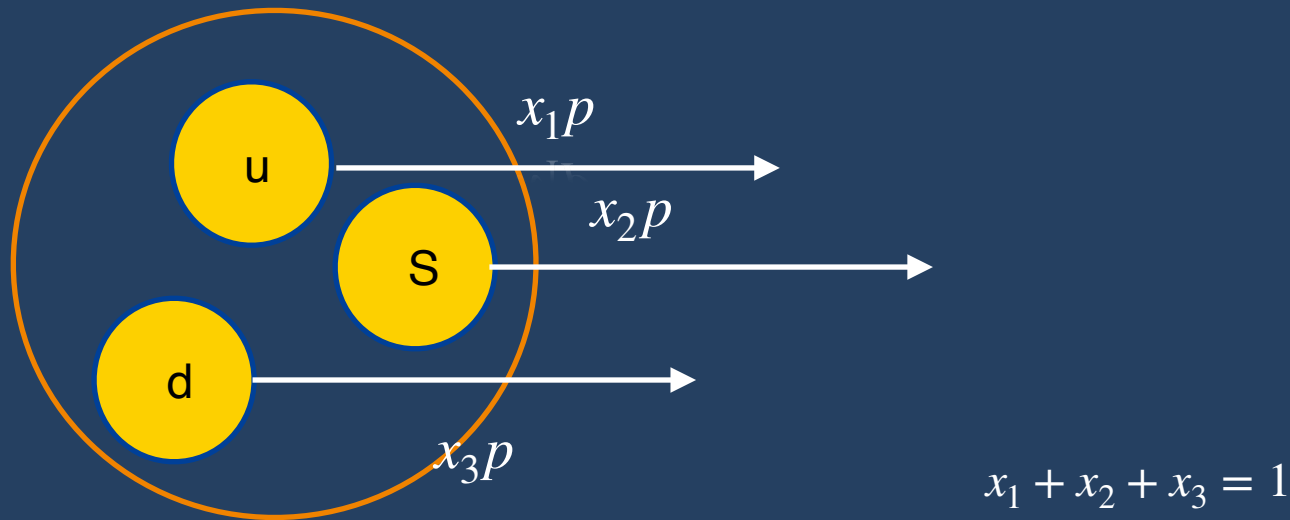
3 Hybrid Renormalization

4 Result and Outlook

Introduction to LCDA\Quasi-DA

Physical Meaning of LCDA:

It describes the momentum fraction of each parton in the baryon.



The partonic interpretation has not been proven beyond tree level.

Introduction to LCDA\Quasi-DA

Operator definition of baryon LCDA in coordinate space:

$$M(z_1, z_2, z_3, P, \mu) = \left\langle 0 \left| \varepsilon^{ijk} u_{\alpha}^{i'}(\vec{z}_1) U_{i'i}(\vec{z}_1, \vec{z}_0) d_{\beta}^{j'}(\vec{z}_2) U_{j'j}(\vec{z}_2, \vec{z}_0) s_{\gamma}^{k'}(\vec{z}_3) U_{k'k}(\vec{z}_3, \vec{z}_0) \right| P(P, \lambda) \right\rangle$$

$$[a_1 n, a_0 n] = P \exp \left[i g \int_0^1 dt (x - y)_{\mu} A^{\mu}(tx + (1 - x)y) \right]$$

f, g, h : quark flavour

$|P(P, \lambda)\rangle$: Baryon state

$$\vec{z}_1 = \frac{1}{\sqrt{2}}(z_1, 0, 0, z_1)$$

Introduction to LCDA\Quasi-DA

$$\left\langle 0 \left| u_{\alpha}(z_1) d_{\beta}(z_2) s_{\gamma}(z_3) \right| P(P, \lambda) \right\rangle$$

leading twist

$$= f_N \{ (\not{P} C)_{\alpha\beta} (\gamma_5 u_{\Lambda})_{\gamma} V(z_i P \cdot n) + (\not{P} \gamma_5 C)_{\alpha\beta} (u_{\Lambda})_{\gamma} \underline{A(z_i P \cdot n)} + (i \sigma_{\mu\nu} P^{\nu} C)_{\alpha\beta} (\gamma_{\mu} \gamma_5 u_{\Lambda})_{\gamma} T(z_i P \cdot n) \}$$

$$M(z_1, z_2, z_3, P^z, \mu) = \left\langle 0 \left| u^T(z_1) \Gamma d(z_2) s(z_3) \right| P(P, \lambda) \right\rangle_R \quad \Gamma = C \gamma_5 \hbar$$

Difficulty of LCDA: cannot be calculated perturbatively

Introduction to LCDA\Quasi-DA

A good way to calculate non-perturbative physics:

Lattice QCD

Flaw of Lattice QCD: only cannot solve light-like correlator

Matrix element which is calculable on Lattice: **Quasi-DA**

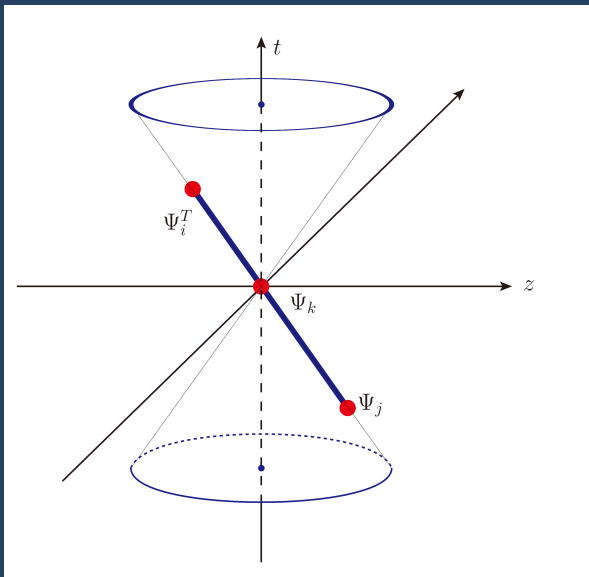
$$\tilde{M}(z_1, z_2, z_3, P^z, \mu) = \left\langle 0 \left| u^T(z_1) \tilde{\Gamma} d(z_2) s(z_3) \right| P(P, \lambda) \right\rangle_R \quad \tilde{\Gamma} = C\gamma_5 \not{z}$$

$$\vec{z} = (0, 0, 0, z)$$

Large Momentum Effective theory

10.1103/PhysRevLett.110.262002

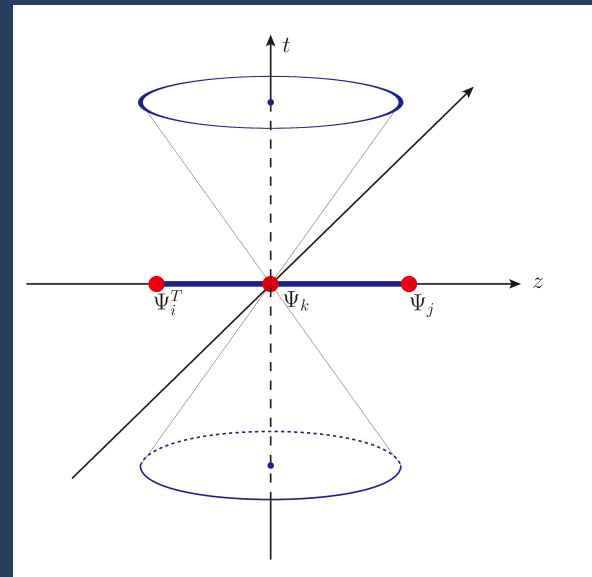
LCDA
Light-cone operator



relation ?



Quasi-DA
Equal-time operator



Large Momentum Effective theory

Baryon-DA

$$\tilde{\Phi}(x_1, x_2, P^z, \mu) = \int dy_1 dy_2 \mathcal{C}(x_1, x_2, y_1, y_2, P^z, \mu) \Phi(y_1, y_2, \mu) + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}}{x_1 P^z}, \frac{\Lambda_{\text{QCD}}}{x_2 P^z}, \frac{\Lambda_{\text{QCD}}}{(1-x_1-x_2)P^z}\right)$$

The matching kernel $\mathcal{C}$ is perturbative calculable.

The matching kernel is valid for the same renormalization.

Large Momentum Effective theory

Procedure to calculate LCDA:

1. Using pQCD to calculate the matching kernel.
2. Calculate the quasi-DA using Lattice QCD.
3. Do the same renormalization on Quasi-DA
4. Convert the Quasi-DA calculated on lattice to LCDA.

Hybrid Renormalization

arXiv:2008.03886v2

Difficulty1: Regularization

Perturbative QCD.	v.s.	Lattice QCD
DR		Hard-cutoff

Difficulty2: Cannot introduce infrared effect

We need to extract perturbative information on Lattice.

Difficulty3: Perturbative calculation of Quasi-DA is divergent

even after $\overline{MS}$ subtraction.

Hybrid Renormalization

arXiv:2008.03886v2

The previous method: RI/MOM

$$M^{RI/MOM}(x_1, x_2, \mu, P^2 = 0) = \frac{M^{\bar{MS}}(x_1, x_2, \mu, P^2 = 0)}{M^{\bar{MS}}(x_1, x_2, \mu, P^2 = -\rho)}$$

Linear divergence is not eliminated throughly.

It introduces extra infrared structure.

Hybrid Renormalization

1. Do 'Self-renormalization' to Lattice element

$$\tilde{M}^{\bar{M}S}(z_1, z_2, P^z) = \frac{\tilde{M}(z_1, z_2, a, P^z)}{Z_R(z_1, z_2, a)}$$

$$Z_R(z_1, z_2, a, \mu) = \exp\left[\left(\frac{k}{a \ln[a \Lambda_{QCD}]} - m_0\right)\tilde{z} + \frac{\gamma_0}{b_0} \ln\left[\frac{\ln[1/(a \Lambda_{QCD})]}{\ln[\mu/\Lambda_{\bar{M}S}]}\right] + \ln\left[1 + \frac{d}{\ln(a \Lambda_{QCD})}\right] + f(z_1, z_2)a\right]$$

2. Do 'Ratio-renormalization' to both lattice matrix element and perturbative calculation.

$$\tilde{M}^{hybrid}(z_1, z_2, P^z) = \frac{\tilde{M}^{\bar{M}S}(z_1, z_2, P)}{\tilde{M}^{\bar{M}S}(z'_1(z_1, z_2), z'_2(z_1, z_2), P = 0)}$$

Hybrid Renormalization

arXiv:2008.03886v2

A scheme which deal with short and long length scale differently.

On lattice QCD side: Self-renormalization

A scheme convert factor should be included on Lattice QCD to convert to $\bar{M}S$ scheme

$$Z_R(z_1, z_2, a, \mu) = \exp \left[\underbrace{\left(\frac{k}{a \ln[a\Lambda_{\text{QCD}}]} - m_0 \right) \tilde{z}}_{\text{Linear divergence}} + \underbrace{\frac{\gamma_0}{b_0} \ln \left[\frac{\ln[1/(a\Lambda_{\text{QCD}})]}{\ln[\mu/\Lambda_{\bar{M}S}]} \right]}_{\text{Log divergence}} + \underbrace{\ln \left[1 + \frac{d}{\ln(a\Lambda_{\text{QCD}})} \right] + f(z_1, z_2)a}_{\text{Discrete effect}} \right]$$

Linear divergence

Log divergence

Discrete effect

Hybrid Renormalization

arXiv:2008.03886v2

A scheme which deal with short and long length scale differently.

On lattice QCD side: **Self-renormalization**

A scheme convert factor should be included on Lattice QCD to convert to $\bar{M}S$ scheme

$$Z_R(z_1, z_2, a, \mu) = \exp \left[\underbrace{\left(\frac{k}{a \ln[a\Lambda_{\text{QCD}}]} - m_0 \right) \tilde{z}}_{\text{Linear divergence}} + \underbrace{\frac{\gamma_0}{b_0} \ln \left[\frac{\ln[1/(a\Lambda_{\text{QCD}})]}{\ln[\mu/\Lambda_{\bar{M}S}]} \right]}_{\text{Log divergence}} + \underbrace{\ln \left[1 + \frac{d}{\ln(a\Lambda_{\text{QCD}})} \right] + f(z_1, z_2)a}_{\text{Discrete effect}} \right]$$

Linear divergence

Log divergence

Discrete effect

Claim:
$$\lim_{a \rightarrow 0} \frac{\tilde{M}(z_1, z_2, a, P^z)}{Z_R(z_1, z_2, a)} = \lim_{\epsilon \rightarrow 0} \frac{\tilde{M}(z_1, z_2, \epsilon, P^z)}{Z_{\bar{M}S}(z_1, z_2, \epsilon)} = \tilde{M}^{\bar{M}S}(z_1, z_2, P^z)$$

Hybrid Renormalization

On pQCD side:

Quasi-DA is a plus function which contains divergence even after $\bar{MS}$

$$M(x)_{\oplus}^{x_0} = M(x) - \delta(x - x_0) \int M(y) dy$$

$$\lim_{x_{1/2} \rightarrow \infty} \tilde{M}(x_1, x_2, P, \epsilon) \simeq \frac{C}{x_{1/2}} \rightarrow \int dx_{1/2} M(x_{1/2}) \text{ is divergent at large } x$$

We need to eliminate the $\frac{1}{x}$ behavior at large momentum.

Hybrid Renormalization

Where does $\frac{1}{x}$ come from ?

$$\tilde{M}(z_1, z_2, P^z, \epsilon) \supset \log z_1^2, \log z_2^2, \log(z_1 - z_2)^2$$

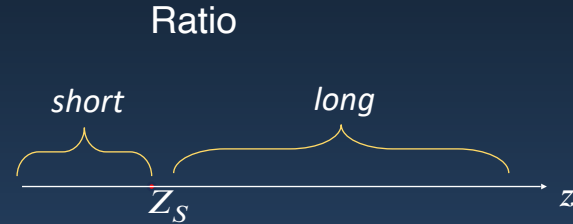
$$\log z_1^2 \rightarrow \frac{1}{|x_1|}, \log z_2^2 \rightarrow \frac{1}{|x_2|}, \log(z_1 - z_2)^2 \rightarrow \frac{1}{|x_1 - x_2|}$$

A simple solution: Ratio with 0 momentum matrix element when z is small

$$\tilde{M}^{ratio}(z_1, z_2, P, \epsilon) = \frac{\tilde{M}(z_1, z_2, P, \epsilon)}{\tilde{M}(z_1, z_2, 0, \epsilon)}$$

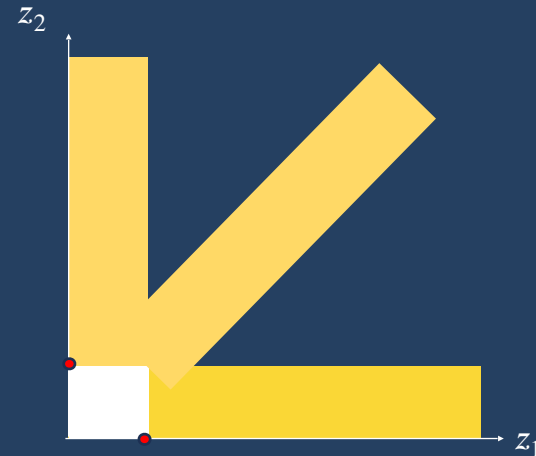
Hybrid Renormalization

In PDF case: only one length scale $\bar{\psi}(z)\Gamma\psi(0)$



In baryon-LCDA case: three length scale $\psi^T(z_1)\Gamma\psi(z_2)\psi(0) \rightarrow z_1 \ z_2 \ (z_1 - z_2)$

Short and long distances are mixed in yellow area.
Matrix element should be renormalized separately.

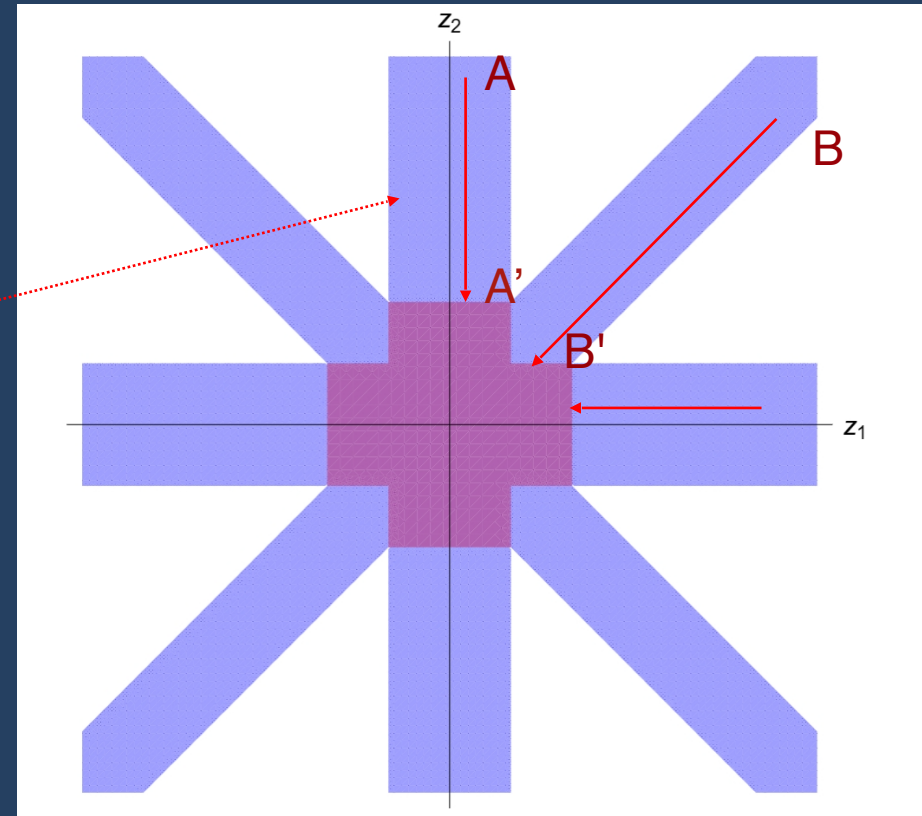


Hybrid Renormalization

Up to one loop, the scale is separated: $A \log z_1^2 + B \log z_2^2 + C \log(z_1 - z_2)^2$

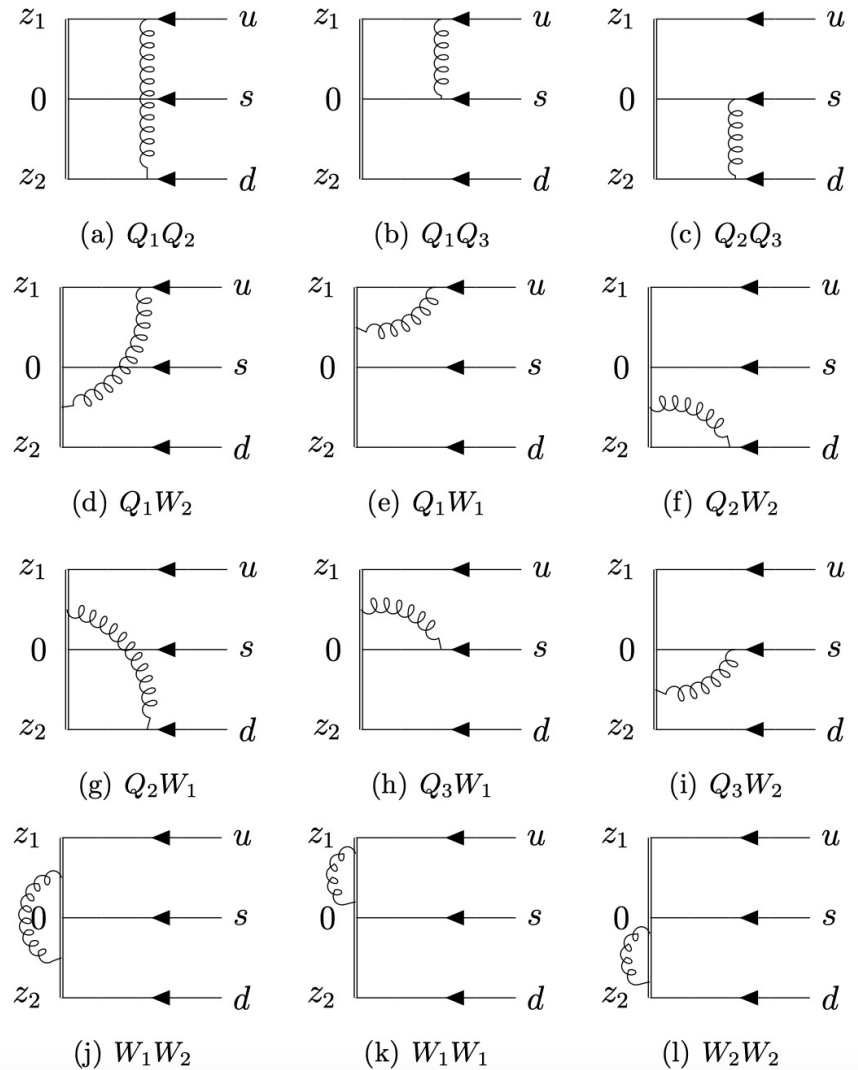
$$\tilde{q}^{hybrid}(z_1, z_2, P) = \frac{\tilde{q}^{\bar{M}S}(z_1, z_2, P)}{\tilde{q}^{\bar{M}S}(z'_1(z_1, z_2), z'_2(z_1, z_2), P = 0)}$$

$$\frac{\hat{M}_{\overline{\text{MS}}}(z_1, z_2, 0, P^z, \mu)}{\hat{M}_{\overline{\text{MS}}}(z_1, \text{sign}(z_2)2z_s, 0, 0, \mu)} \theta(z_s - |z_1|) \theta(|z_2| - 2z_s)$$



Result and Outlook

One-loop calculation:



Result and Outlook

One-loop calculation:

$$L_1 = \ln \frac{z_1^2 \mu^2 e^{2\gamma_E}}{4} \ \& \ L_2 = \ln \frac{z_2^2 \mu^2 e^{2\gamma_E}}{4} \ \& \ .L_{12} = \ln \frac{(z_1 - z_2)^2 \mu^2 e^{2\gamma_E}}{4}$$

$$\begin{aligned}
 M_p(z_1, z_2, 0, P^z, \mu) = & \left\{ 1 + \frac{\alpha_s C_F}{\pi} \left(\frac{1}{2} L_1^{\text{UV}} + \frac{1}{2} L_2^{\text{UV}} + \frac{1}{2} L_{12}^{\text{UV}} + \frac{3}{2} \right) \right\} M_0(z_1, z_2, 0, P^z, \mu) \\
 & - \frac{\alpha_s C_F}{8\pi} \int_0^1 d\eta_1 \int_0^{1-\eta_1} d\eta_2 \\
 & \times \left\{ \left(L_1^{\text{IR}} - 1 + \frac{1}{\epsilon_{\text{IR}}} \right) M_0((1-\eta_1)z_1, z_2, \eta_2 z_1, P^z, \mu) + \left(L_2^{\text{IR}} - 1 + \frac{1}{\epsilon_{\text{IR}}} \right) M_0(z_1, (1-\eta_1)z_2, \eta_2 z_2, P^z, \mu) \right. \\
 & \left. + 2 \left(L_{12}^{\text{IR}} - 3 + \frac{1}{\epsilon_{\text{IR}}} \right) M_0((1-\eta_1)z_1 + \eta_1 z_2, (1-\eta_2)z_2 + \eta_2 z_1, 0, P^z, \mu) \right\} \\
 & - \frac{\alpha_s C_F}{4\pi} \int_0^1 d\eta \times \left\{ M_0((1-\eta)z_1 + \eta z_2, z_2, 0, P^z, \mu) \left\{ \left(L_{12}^{\text{IR}} + 1 + \frac{1}{\epsilon_{\text{IR}}} \right) \left(\frac{1-\eta}{\eta} \right)_+ + 2 \left(\frac{\ln \eta}{\eta} \right)_+ \right\} \right. \\
 & + M_0(z_1, (1-\eta)z_2 + \eta z_1, 0, P^z, \mu) \left\{ \left(L_{12}^{\text{IR}} + 1 + \frac{1}{\epsilon_{\text{IR}}} \right) \left(\frac{1-\eta}{\eta} \right)_+ + 2 \left(\frac{\ln \eta}{\eta} \right)_+ \right\} \\
 & + M_0((1-\eta)z_1, z_2, 0, P^z, \mu) \left\{ \left(L_1^{\text{IR}} + 1 + \frac{1}{\epsilon_{\text{IR}}} \right) \left(\frac{1-\eta}{\eta} \right)_+ + 2 \left(\frac{\ln \eta}{\eta} \right)_+ \right\} \\
 & + M_0(z_1, (1-\eta)z_2, 0, P^z, \mu) \left\{ \left(L_2^{\text{IR}} + 1 + \frac{1}{\epsilon_{\text{IR}}} \right) \left(\frac{1-\eta}{\eta} \right)_+ + 2 \left(\frac{\ln \eta}{\eta} \right)_+ \right\} \\
 & - M_0(z_1, z_2, \eta z_1, P^z, \mu) \left\{ \left(L_1^{\text{IR}} + 1 + \frac{1}{\epsilon_{\text{IR}}} \right) \left(\frac{1-\eta}{\eta} \right)_+ + 2 \left(\frac{\ln \eta}{\eta} \right)_+ \right\} \\
 & \left. - M_0(z_1, z_2, \eta z_2, P^z, \mu) \left\{ \left(L_2^{\text{IR}} + 1 + \frac{1}{\epsilon_{\text{IR}}} \right) \left(\frac{1-\eta}{\eta} \right)_+ + 2 \left(\frac{\ln \eta}{\eta} \right)_+ \right\} \right\}, \tag{2.10}
 \end{aligned}$$

Result and Outlook

One-loop calculation of ratio element:

$$\begin{aligned}\delta\hat{M}_H^{(1)}(z_1, z_2, \mu) = & \hat{M}_p^{(1)}(z_1, z_2, 0, 0, \mu) (\theta(2z_s - |z_1|)\theta(z_s - |z_2|) + \theta(z_s - |z_1|)\theta(|z_2| - z_s)\theta(2z_s - |z_2|)) \\ & + \hat{M}_p^{(1)}(z_1, \text{sign}(z_2)2z_s, 0, 0, \mu) \theta(z_s - |z_1|)\theta(|z_2| - 2z_s) + \hat{M}_p^{(1)}(\text{sign}(z_1)2z_s, z_2, 0, 0, \mu) \theta(|z_1| - 2z_s)\theta(z_s - |z_2|) \\ & + \hat{M}_p^{(1)}(z_s + (z_1 - z_2)\theta(z_1 - z_2), z_s + (z_2 - z_1)\theta(z_2 - z_1), 0, 0, \mu) \theta(|z_1| - z_s)\theta(|z_2| - z_s)\theta(z_s - |z_1 - z_2|) \\ & + \hat{M}_p^{(1)}(z_s + (z_1 + z_2)\theta(z_1 + z_2), -z_s + (z_2 + z_1)\theta(-z_2 - z_1), 0, 0, \mu) \theta(|z_1| - z_s)\theta(|z_2| - z_s)\theta(z_s - |z_1 + z_2|) \\ & + \hat{M}_p^{(1)}(\text{sign}(z_1)z_s, \text{sign}(z_2)2z_s, 0, 0, \mu) \theta(|z_1| - z_s)\theta(|z_2| - z_s)\theta(|z_1 - z_2| - z_s)\theta(|z_1 + z_2| - z_s).\end{aligned}$$

Result and Outlook

$$C^{hybrid}(x_1, x_2, y_1, y_2, \mu) = C^{\bar{M}S}(x_1, x_2, y_1, y_2, \mu) - \delta C(x_1, x_2, y_1, y_2, \mu)$$

$$\begin{aligned} \delta C_H^{(1)}(x_1, x_2, y_1, y_2, P^z, \mu) = (P^z)^2 \frac{\alpha_s C_F}{2\pi} & \left[I_H[(x_1 - y_1)P^z, (x_2 - y_2)P^z] + I_{HSI}[(x_1 - y_1)P^z, (x_2 - y_2)P^z] \right. \\ & + I_{HSII}[(x_1 - y_1)P^z, (x_2 - y_2)P^z] + I_{HSIII}[(x_1 - y_1)P^z, (x_2 - y_2)P^z] + I_{HSIV}[(x_1 - y_1)P^z, (x_2 - y_2)P^z] \\ & \left. + I_S[(x_1 - y_1)P^z, (x_2 - y_2)P^z] + \delta[(x_1 - y_1)P^z] \delta[(x_2 - y_2)P^z] \left(\frac{5}{2} \ln \left(\frac{\mu^2 e^{2\gamma_E}}{4} \right) + 4 \right) \right], \end{aligned} \quad (3.22)$$

$$\begin{aligned} I_H[p_1, p_2] & \equiv \int \frac{dz_1}{2\pi} \frac{dz_2}{2\pi} e^{ip_1 z_1 + ip_2 z_2} \left[\frac{7}{8} \ln(z_1^2) + \frac{7}{8} \ln(z_2^2) + \frac{3}{4} \ln((z_1 - z_2)^2) \right] \quad (B.7) \\ & \times (\theta(2z_s - |z_1|) \theta(z_s - |z_2|) + \theta(z_s - |z_1|) \theta(|z_2| - z_s) \theta(2z_s - |z_2|)) \\ & = \frac{7}{8} [I_0(\{2z_s, -2z_s\}, p_2) I_1(\{z_s, -z_s\}, p_1) + (I_0(\{2z_s, -2z_s\}, p_1) - I_0(\{z_s, -z_s\}, p_1)) I_1(\{z_s, -z_s\}, p_2) \\ & + I_0(\{z_s, -z_s\}, p_2) (I_1(\{2z_s, -2z_s\}, p_1) - I_1(\{z_s, -z_s\}, p_1)) + I_0(\{z_s, -z_s\}, p_1) I_1(\{2z_s, -2z_s\}, p_2)] \\ & + \frac{3}{8\pi(p_1 + p_2)} [\sin((p_1 + p_2)z_s) [I_1(\{z_s, -z_s\}, p_1) + I_1(\{z_s, -z_s\}, -p_2) \\ & - I_1(\{2z_s, -2z_s\}, p_1) - I_1(\{2z_s, -2z_s\}, -p_2) + I_1(\{3z_s, -3z_s\}, p_1) + I_1(\{3z_s, -3z_s\}, -p_2)] \\ & + \sin(2(p_1 + p_2)z_s) [-I_1(\{z_s, -z_s\}, p_1) - I_1(\{z_s, -z_s\}, -p_2) \\ & + I_1(\{3z_s, -3z_s\}, p_1) + I_1(\{3z_s, -3z_s\}, -p_2)] \\ & + \cos(2(p_1 + p_2)z_s) [-I_1 t(\{z_s, -z_s\}, p_1) + I_1 t(\{z_s, -z_s\}, -p_2) \\ & + I_1 t(\{3z_s, -3z_s\}, p_1) - I_1 t(\{3z_s, -3z_s\}, -p_2)] \\ & + \cos((p_1 + p_2)z_s) [-I_1 t(\{z_s, -z_s\}, p_1) + I_1 t(\{z_s, -z_s\}, -p_2) - I_1 t(\{2z_s, -2z_s\}, p_1) \\ & + I_1 t(\{2z_s, -2z_s\}, -p_2) + I_1 t(\{3z_s, -3z_s\}, p_1) - I_1 t(\{3z_s, -3z_s\}, -p_2)]], \end{aligned}$$

Result and Outlook

Apply hybrid scheme to all leading twist LCDA:

$$f_N\{(PC)_{\alpha\beta}(\gamma_5 u_\Lambda)_\gamma V(z_i P \cdot n) + (P\gamma_5 C)_{\alpha\beta}(u_\Lambda)_\gamma A(z_i P \cdot n) + (i\sigma_{\mu\nu}P^\nu C)_{\alpha\beta}(\gamma_\mu\gamma_5 u_\Lambda)_\gamma T(z_i P \cdot n)$$

Does scales separate to all order? —A prerequisite to hybrid scheme

$$\hat{M}_p(z_1, z_2, 0, 0, \mu) = \left(1 + \sum_{n=1} \sum_{m=0}^n \alpha_s^n a_{n,m} L_1^m\right) \left(1 + \sum_{n=1} \sum_{m=0}^n \alpha_s^n b_{n,m} L_2^m\right) \left(1 + \sum_{n=1} \sum_{m=0}^n \alpha_s^n c_{n,m} L_{12}^m\right)$$

$$L_1 = \ln \frac{z_1^2 \mu^2 e^{2\gamma_E}}{4} \text{ \& } L_2 = \ln \frac{z_2^2 \mu^2 e^{2\gamma_E}}{4} \text{ \& } L_{12} = \ln \frac{(z_1 - z_2)^2 \mu^2 e^{2\gamma_E}}{4}$$

Operator mixing problem.

谢谢！

