Form factors of light pseudoscalar mesons

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2023 年 11 月 5 日

1. Measurement of timelike form factor in the resonant regions, which makes up the piece lacking solid **QCD-based calculation**.

2. With the perturbative QCD calculation up to next-to-leading-order of strong coupling and twist four level of meson distribution amplitudes, compare with the experimental results and extract relevant information about the **meson structure**.

3. The results of the transition form factors of η and η' are experimentally given. Based on this, we give the **mixing angle** related to the flavor eigenstate.

outline

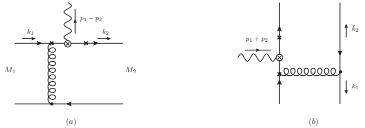


Fig 1: Feynman diagrams of $\pi\gamma^\star \to \pi$ (left) and $\gamma^\star \to \pi\pi$ (right) form factors at leading-order. which the EM vertexes are denoted by $\otimes$ and the possible attachments of internal hard gluons are denoted by $\times$.

- $p_1 = (Q/\sqrt{2}, 0, \mathbf{0})$, $p_2 = (0, Q/\sqrt{2}, \mathbf{0})$; $k_1 = (xQ/\sqrt{2}, 0, \mathbf{K_{1T}})$, $k_2 = (0, xQ/\sqrt{2}, \mathbf{K_{2T}})$
  The momentum transfer squared is $q^2 = (p_1 - p_2)^2$ in the spacelike form factor, whereas it reads as $q^2 = (p_1 + p_2)^2$ in the timelike.
- The invariant amplitudes of transitions with EM curents can be defined by the nonlocal matrix element:

$$\langle \pi^-(p_2)|j^{\text{em}}_{\mu,q}|\pi^-(p_1)\rangle = \langle \pi^-(p_2)|\left(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d\right)|\pi^-(p_1)\rangle \equiv e_q(p_1+p_2)\mathcal{F}_\pi(Q^2)\,,$$

- separating the contributions from short and longdistance interactions, the matrix element is written in the factorizable formulism as:
  $\langle \pi^-(p_2)|J^{\text{e.m.}}_\mu|\pi^-(p_1)\rangle =$

$$\oint dz_1\, dz_2\, H^{ijkl}_{\gamma\beta\alpha\delta}(z_2, z_1)\langle \pi^-(p_2)\Big|\left\{\overline{d}_\gamma(z_2)\exp\left(ig_s\int_0^{z_2} d\sigma_{\nu'}\, A_{\nu'}(\sigma)\right) u_\beta(0)\right\}_{kj}\Big|0\rangle_{\mu_t}$$

$$\langle 0\Big|\left\{\overline{u}_\alpha(0)\exp\left(ig_s\int_{z_1}^{0} d\sigma_\nu\, A_\nu(\sigma)\right) d_\delta(z_1)\right\}_{il}\Big|\pi^-(p_1)\rangle_{\mu_t}$$

$$\langle \pi^-(p_2)| J_\mu^{\text{e.m.}} |\pi^-(p_1)\rangle =$$

$$\oint dz_1\, dz_2\, H^{ijkl}_{\gamma\beta\alpha\delta}(z_2, z_1) \langle \pi^-(p_2) \Big| \left\{ \overline{d}_\gamma(z_2) \exp\left(i g_s \int_0^{z_2} d\sigma_{\nu'} A_{\nu'}(\sigma) \right) u_\beta(0) \right\}_{kj} \Big| 0 \rangle_{\mu_t}$$
$$\langle 0 \Big| \left\{ \overline{u}_\alpha(0) \exp\left(i g_s \int_{z_1}^{0} d\sigma_\nu A_\nu(\sigma) \right) d_\delta(z_1) \right\}_{il} \Big| \pi^-(p_1) \rangle_{\mu_t}$$

- $H^{ijkl}_{\gamma\beta\alpha\delta}(z_2, z_1)$ is the perturbative hard kernel

$$H^{ijkl}_{\gamma\beta\alpha\delta}(z_1, z_2) = (-1) \left[i g_s \gamma_m \right]_{\alpha\beta} T^{ij} \left[(i e_q \gamma_\mu) S_0(0 - z_1)(i g_s \gamma_n) \right]_{\gamma\delta} T^{kl} \left[-i D^0_{mn}(z_1 - z_2) \right],$$

- $\langle 0 \Big| \left\{ \overline{u}_\alpha(0) \exp\left(i g_s \int_{z_1}^{0} d\sigma_\nu A_\nu(\sigma) \right) d_\delta(z_1) \right\}_{il} \Big| \pi^-(p_1) \rangle_{\mu_t}$ is the LCDAs

$$\langle 0 \Big| \left\{ \overline{u}_\alpha(0) \exp\left(i g_s \int_{z_1}^{0} d\sigma_\nu A_\nu(\sigma) \right) d_\delta(z_1) \right\}_{il} \Big| \pi^-(p_1) \rangle_{\mu_t}$$
$$= \frac{\delta_{il}}{3} \left\{ \frac{1}{4} (\gamma_5 \gamma^\rho)_{\delta\alpha} \langle 0 | \overline{u}(0) \exp\left(i g_s \int_{z_1}^{0} d\sigma_\nu A_\nu(\sigma) \right) (\gamma_\rho \gamma_5)\, d(z_1) | \pi^-(p_1) \rangle_{\mu_t} + \cdots \right\}.$$

Then, we can get space-like $\mathcal{F}_\pi(q^2) = \mathcal{F}_\pi^{2\text{p}}(q^2) + \mathcal{F}_\pi^{3\text{p}}(q^2)$.
q^2 is converted to $-q^2$, we can get time-like form factors $\mathcal{F}_\pi(s)$.

The standard dispersion relation is

$$\mathcal{F}_P(q^2) = \frac{1}{\pi}\int_{s_0}^{\infty} ds \frac{\mathrm{Im}\mathcal{F}_P(s)}{s - q^2 - i\epsilon}, \qquad q^2 > s_0$$

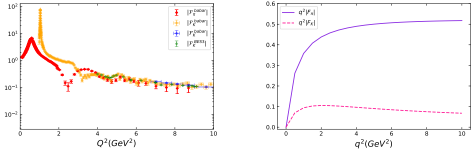


Fig 2: data(left) Space-like form factor(right) derived from standard dispersion relation.

The measurement of timelike form factor is carried out for the modulus square $|F_\pi(s)|^2$, and the information of imaginary part is usually obtained by a parameterization of $F_\pi(s)$, which brings an inevitable model dependence which results in an additional uncertainty to the spacelike form factor.

The modulus squared dispersion integral is

[S. Cheng, A. Khodjamirian and A. V. Rusov, PRD 102 (2020) 074022
J. Chai, S. Cheng and J. Hua, EPJC 83 (2023) no.7, 556.]

$$\mathcal{F}_\pi^{\mathrm{DR}}(\mathbf{q^2}) = \exp\left[\frac{\mathbf{q^2}\sqrt{\mathbf{s_0} - \mathbf{q^2}}}{\mathbf{2\pi}} \int\limits_{\mathbf{4m_\pi^2}}^{\infty} \mathbf{ds} \frac{\ln|\mathcal{F}_\pi(\mathbf{s})|^2}{\mathbf{s}\sqrt{\mathbf{s} - \mathbf{s_0}}\,(\mathbf{s} - \mathbf{q^2})}\right].$$

Space-like form factor derived from modulus squared dispersion relation

$$\mathcal{F}_\pi^{\mathrm{DR}}(\mathbf{q^2}) = \exp\left[\frac{\mathbf{q^2}\sqrt{\mathbf{s_0}-\mathbf{q^2}}}{\mathbf{2\pi}}\int_{\mathbf{4m_\pi^2}}^{\infty}\mathbf{ds}\,\frac{\ln|\mathcal{F}_\pi(\mathbf{s})|^2}{\mathbf{s}\sqrt{\mathbf{s}-\mathbf{s_0}}\,(\mathbf{s}-\mathbf{q^2})}\right].$$

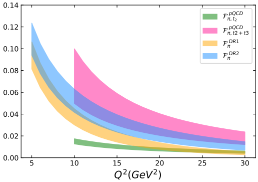


Fig 3: Space-like form factor.

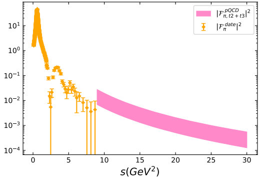


Fig 4: Time-like form factor.

$$|\mathcal{F}_\pi(s)|^2 = \Theta(s_{\max}-s)|\mathcal{F}_\pi^{\mathrm{data}}(s)|^2 + \Theta(s-s_{\max})|\mathcal{F}_\pi^{\mathrm{tail}}(s)|^2$$

$$\Downarrow$$

$$|\mathcal{F}_\pi(s)|^2 = \Theta(s_{\max}-s)\,|\mathcal{F}_{\pi,\mathrm{Inter.}}^{\mathrm{data}}(s)|^2 + \Theta(s-s_{\max})\,|\mathcal{F}_\pi^{\mathrm{pQCD}}(s)|^2$$

Space-like form factor derived from pQCD

$$\mathcal{F}_\pi^{\rm pQCD}(\mathbf{Q^2}) = (\mathbf{m_0^\pi})^2\mathbf{F_1}(\mathbf{Q^2}) + \mathbf{m_0^\pi}\mathbf{F_2}(\mathbf{Q^2}) + \mathbf{F_3}(\mathbf{Q^2}) + \mathbf{m_0^\pi}\mathbf{a_2^\pi}\mathbf{F_4}(\mathbf{Q^2}) + \mathbf{a_2^\pi}\mathbf{F_5}(\mathbf{Q^2}) + (\mathbf{a_2^\pi})^2\mathbf{F_6}(\mathbf{Q^2})\,.$$

The form factor with the contributions from 2p-to-2p and 3p-to-3p scattering:

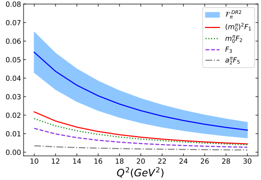


Fig 5: The spacelike form factor obtained from the modified dispersion relation and the pQCD calculation.

$$\mathcal{F}_\pi^{\rm pQCD}(Q^2) = \mathcal{F}_{\pi,\rm t2}^{\rm pQCD}(Q^2) + \mathcal{F}_{\pi,\rm t3}^{\rm pQCD}(Q^2) + \mathcal{F}_{\pi,\rm t2\otimes t4}^{\rm pQCD}(Q^2) + \mathcal{F}_{\pi,\rm 3p}^{\rm pQCD}(Q^2)\,.$$

$$\begin{aligned}
\mathcal{F}_{\pi,\rm t2}^{\rm pQCD}(Q^2) &= \mathcal{F}_1^{\rm t2}(Q^2) + a_2^\pi\,\mathcal{F}_2^{\rm t2}(Q^2) + (a_2^\pi)^2\,\mathcal{F}_3^{\rm t2}(Q^2)\,,\\
\mathcal{F}_{\pi,\rm t3}^{\rm pQCD}(Q^2) &= (m_0^\pi)^2\mathcal{F}_1^{\rm 2p,t3}(Q^2) + m_0^\pi\mathcal{F}_2^{\rm 2p,t3}(Q^2)\\
&\quad + (a_2^\pi)^2\mathcal{F}_3^{\rm 2p,t3}(Q^2) + a_2^\pi\mathcal{F}_4^{\rm 2p,t3}(Q^2)\\
&\quad + m_0^\pi\,a_2^\pi\,\mathcal{F}_5^{\rm 2p,t3}(Q^2) + \mathcal{F}_6^{\rm 2p,t3}(Q^2)\,,\\
\mathcal{F}_{\pi,\rm t2\otimes t4}^{\rm pQCD}(Q^2) &= \mathcal{F}_1^{\rm 2p,t2\otimes t4}(Q^2) + a_2^\pi\,\mathcal{F}_2^{\rm 2p,t2\otimes t4}(Q^2)\,.
\end{aligned}$$

Result and discussion

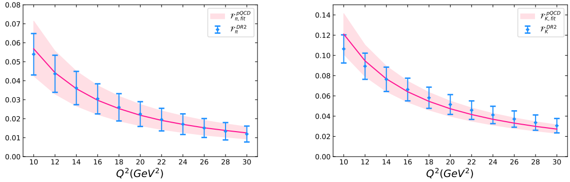


Fig 6: The spacelike form factor obtained from the modified dispersion relation and the pQCD calculation with the new obtained parameters in scenario II.

Tab 1: Fitting results of m_0^π and a_2^π at the default scale 1 GeV.

Scenario	I	II	IIA	IIB
m_0^π (GeV)	$1.37^{+0.29}_{-0.32}$	$1.31^{+0.27}_{-0.30}$	$0.93^{+0.24}_{-0.27}$	$1.59^{+0.30}_{-0.34}$
a_2^π	0.25 ± 0.25	0.23 ± 0.25	0.25 ± 0.25	0.26 ± 0.25

Tab 2: Results of m_0^K by the fit, the $a_1^K = -0.108$ and $a_2^K = 0.170$ form lattice.

Scenario	I	II	IIA	IIB
m_0^π (GeV)	$1.62^{+0.18}_{-0.20}$	$1.72^{+0.23}_{-0.25}$	$1.43^{+0.20}_{-0.22}$	$1.97^{+0.25}_{-0.18}$

Result and discussion

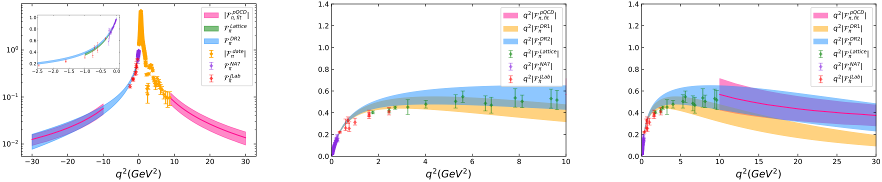


Fig 7: The magnitude of π electromagnetic form factor

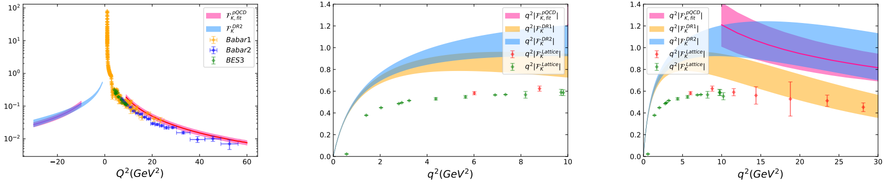


Fig 8: The magnitude of K electromagnetic form factor

outline

Transition form factors of π^0, η and η'

Fig 9: Feynman diagrams of Pseudoscalar meson transition form factors at leading-order. Which '•' represent virtual photon vertices.

$$\mathcal{F}_{\pi\gamma}(Q^2) = \frac{\sqrt{2}f_\pi}{3}\int_0^1 dx \int_0^\infty bdb\phi_\pi(x)\exp[-S(x,b,Q,\mu)]S_t(x,Q)K_0(\sqrt{x}Qb)$$
$$\times\left[1-\frac{\alpha_s(\mu)}{4\pi}C_F\left(ln\frac{\mu^2 b}{2\sqrt{x}Q}+\gamma_E+2lnx+3-\frac{\pi^2}{3}\right)\right]$$

[H. n. Li and S. Mishima, PRD 80 (2009), 074024]

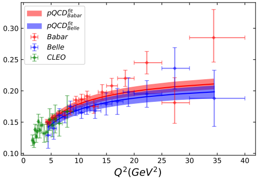


Fig 10: T f-f, the vertical axis $Q^2F(Q^2)$.

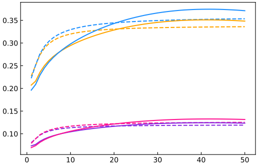
Tab 3: Fit a_2^π and a_2^π, a_4^π form data.

G-m	Belle	Babar
a_2	$0.156^{+0.098}_{-0.099}$	$0.287^{+0.084}_{-0.083}$
a_2	0.170 ± 0.080	0.280 ± 0.070
a_4	$-0.03^{+0.008}_{-0.003}$	$0.009^{-0.006}_{-0.019}$

Transition form factors of π^0, η and η'

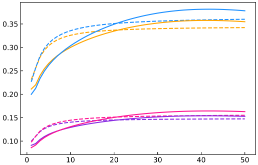
Considering the difference in charge factors between π(In the transition form factor, π refers to π^0), η_q and η_s, $C_{\pi^0} = \frac{e_u^2 - e_d^2}{\sqrt{2}} = \frac{3}{9\sqrt{2}}$, $C_{\eta_q} = \frac{e_u^2 + e_d^2}{\sqrt{2}} = \frac{5}{9\sqrt{2}}$ and $C_{\eta_s} = e_s^2 = \frac{1}{9}$, so we have

$$\mathcal{F}_{\eta_q\gamma}(Q^2) = \frac{5}{3}\mathcal{F}_{\pi\gamma}(Q^2), \qquad \mathcal{F}_{\eta_s\gamma}(Q^2) = \frac{\sqrt{2}}{3}\mathcal{F}_{\pi\gamma}(Q^2).$$



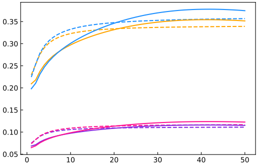
$$f_{\eta_q} = (1.07 \pm 0.02) f_\pi ,$$
$$f_{\eta_s} = (1.34 \pm 0.06) f_\pi ,$$
$$\phi = 39.3^\circ \pm 1.0^\circ$$

[PRD 58 (1998), 114006]



$$f_{\eta_q} = (1.09 \pm 0.03) f_\pi ,$$
$$f_{\eta_s} = (1.66 \pm 0.06) f_\pi ,$$
$$\phi = 40.7^\circ \pm 1.4^\circ$$

[JHEP 06 (2005), 029]



$$f_{\eta_q} = (1.08 \pm 0.04) f_\pi ,$$
$$f_{\eta_s} = (1.25 \pm 0.08) f_\pi ,$$
$$\phi = 37.7^\circ \pm 0.7^\circ$$

[PRD 85 (2012), 057501]

Transition form factors of π^0, η and η'

quark flavor basis $\eta_q = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$, $\eta_s = s\bar{s}$

singlet-octet basis $\eta_8 = \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s})$, $\eta_1 = \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s})$

$$\begin{pmatrix} \eta_8 \\ \eta_1 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{3}} & -\frac{\sqrt{2}}{\sqrt{3}} \\ \frac{\sqrt{2}}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix} \begin{pmatrix} \eta_q \\ \eta_s \end{pmatrix} = \begin{pmatrix} \cos 54.7^\circ & -\sin 54.7^\circ \\ \sin 54.7^\circ & \cos 54.7^\circ \end{pmatrix} \begin{pmatrix} \eta_q \\ \eta_s \end{pmatrix}$$

Physical state η, η' is mixed with η_8, η_1 or η_q, η_s

$$\begin{aligned} \begin{pmatrix} \eta \\ \eta' \end{pmatrix} &= \begin{pmatrix} \cos\theta_8 & -\sin\theta_1 \\ \sin\theta_8 & \cos\theta_1 \end{pmatrix} \begin{pmatrix} \eta_8 \\ \eta_1 \end{pmatrix} \\ &= \begin{pmatrix} \cos\theta_8 & -\sin\theta_1 \\ \sin\theta_8 & \cos\theta_1 \end{pmatrix} \begin{pmatrix} \cos 54.7^\circ & -\sin 54.7^\circ \\ \sin 54.7^\circ & \cos 54.7^\circ \end{pmatrix} \begin{pmatrix} \eta_q \\ \eta_s \end{pmatrix} \\ &= \begin{pmatrix} \cos\theta_8 \cos 54.7^\circ - \sin\theta_1 \sin 54.7^\circ & -\cos\theta_8 \sin 54.7^\circ - \sin\theta_1 \cos 54.7^\circ \\ \sin\theta_8 \cos 54.7^\circ + \cos\theta_1 \sin 54.7^\circ & -\sin\theta_8 \sin 54.7^\circ + \cos\theta_1 \cos 54.7^\circ \end{pmatrix} \begin{pmatrix} \eta_q \\ \eta_s \end{pmatrix} \end{aligned}$$

$$\mathcal{F}_{\eta\gamma}(Q^2) = (\cos\theta_8 \cos 54.7^\circ - \sin\theta_1 \sin 54.7^\circ)\mathcal{F}_{\eta_q\gamma}(Q^2) - (\cos\theta_8 \sin 54.7^\circ + \sin\theta_1 \cos 54.7^\circ)\mathcal{F}_{\eta_s\gamma}(Q^2),$$

$$\mathcal{F}_{\eta'\gamma}(Q^2) = (\sin\theta_8 \cos 54.7^\circ + \cos\theta_1 \sin 54.7^\circ)\mathcal{F}_{\eta_q\gamma}(Q^2) + (-\sin\theta_8 \sin 54.7^\circ + \cos\theta_1 \cos 54.7^\circ)\mathcal{F}_{\eta_s\gamma}(Q^2).$$

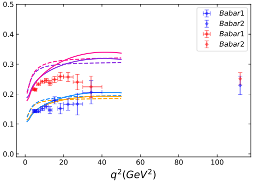
Transition form factors of π^0, η and η'

If $\theta_8 = \theta_1 = \theta$, then

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \eta_8 \\ \eta_1 \end{pmatrix} = \begin{pmatrix} \cos(\theta+54.7^\circ) & -\sin(\theta+54.7^\circ) \\ \sin(\theta+54.7^\circ) & \cos(\theta+54.7^\circ) \end{pmatrix} \begin{pmatrix} \eta_q \\ \eta_s \end{pmatrix}$$
$$= \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} \eta_q \\ \eta_s \end{pmatrix}$$

Therefore, we can get the well-known relationship $\phi = \theta + 54.7^\circ$.

$$\mathcal{F}_{\eta\gamma}(Q^2) = \cos\phi \mathcal{F}_{\eta_q\gamma}(Q^2) - \sin\phi \mathcal{F}_{\eta_s\gamma}(Q^2),$$
$$\mathcal{F}_{\eta'\gamma}(Q^2) = \sin\phi \mathcal{F}_{\eta_q\gamma}(Q^2) + \cos\phi \mathcal{F}_{\eta_s\gamma}(Q^2).$$



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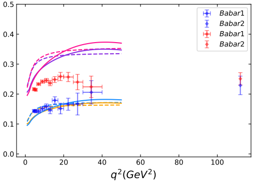
[PRD 58 (1998), 114006]



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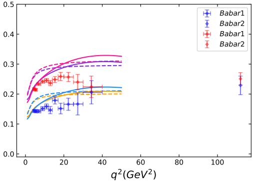

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[PRD 85 (2012), 057501]

Transition form factors of π^0, η and η'

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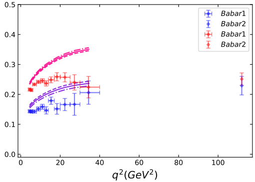

Fig 11: Fit with an angle ϕ

$$\phi = 40.6^{\circ}{}^{+0.9^{\circ}}_{-0.8^{\circ}}, \qquad \phi = 39.1^{\circ}{}^{+1.4^{\circ}}_{-1.1^{\circ}}$$

$$\begin{aligned}
&\mathcal{F}_{\eta\gamma}(Q^2)\\
&= (\cos\theta_8 \cos 54.7^{\circ} - \sin\theta_1 \sin 54.7^{\circ})\mathcal{F}_{\eta_q\gamma}(Q^2)\\
&- (\cos\theta_8 \sin 54.7^{\circ} + \sin\theta_1 \cos 54.7^{\circ})\mathcal{F}_{\eta_s\gamma}(Q^2),\\
&\mathcal{F}_{\eta'\gamma}(Q^2)\\
&= (\sin\theta_8 \cos 54.7^{\circ} + \cos\theta_1 \sin 54.7^{\circ})\mathcal{F}_{\eta_q\gamma}(Q^2)\\
&+ (-\sin\theta_8 \sin 54.7^{\circ} + \cos\theta_1 \cos 54.7^{\circ})\mathcal{F}_{\eta_s\gamma}(Q^2).
\end{aligned}$$

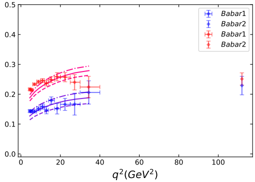

Fig 12: Fit from two angles θ_8 and θ_1

$$\theta_8 = -45.9^{\circ}{}^{+6.4^{\circ}}_{-6.3^{\circ}}, \ \theta_1 = -13.7^{\circ}{}^{+0.0^{\circ}}_{+0.5^{\circ}}$$
$$\theta_8 = -53.3^{\circ}{}^{+10.3^{\circ}}_{-12.1^{\circ}}, \ \theta_1 = -15.5^{\circ}{}^{-1.4^{\circ}}_{-1.1^{\circ}}$$

outline

Summary

1. we obtain the chiral mass of light pseudoscalar mesons $m_0^\pi = 1.31^{+0.27}_{-0.30}$ GeV,
$m_0^K = 1.72^{+0.23}_{-0.25}$ GeV.

2. Due to the chiral enhancement contribution from 3-twist distribution amplitudes in the pQCD, we can not extract the lowest Gegenbauer moments of mesons from the electromagnetic form factor.
   This problem could be settled down with the foresee Belle-II and BESIII measurement of the transition form factor especially in the large momentum transfers regions.

3. The mixing angles in η and η' should be two different mixing angles to better describe the experimental data. $\theta_8 = -53.3^{\circ}{}^{+10.3^\circ}_{-12.1^\circ}$, $\theta_1 = -15.5^{\circ}{}^{-1.4^\circ}_{-1.1^\circ}$

Thanks for your patience...